4.1.3: Recognizing well-defined algebraic functions and their ranges.

Which of the following are functions from R to R? If f is a function, give its range.

(b) not a function from R to R.

X can’t be 2 or -2

(c)a function from R to R. The range of f(x) is f(x)>=0

4.1.5: Range of a function.

b. f(x) = {4,9,16,25}

d. {0,1}⁵ = (00000),(00001),(00010),(00011),(00100),(00101),(00110), (00111),

(01000),(01001), (01010), (01011),(01100),(01101),(01110), (01111),

(10000),(10001),(10010),(10011),(10100),(10101),(10110), (10111),

(11000),(11001), (11010), (11011),(11100),(11101),(11110), (11111)

f(x) = {0,1,2}

maximum value of f(x) is 2.

h. AxA={11,12,13,21,22,23,31,32,33}

f(x,y) = {11,21,31,12,22,32,13,23,33}

i. AxA={11,12,13,21,22,23,31,32,33}

f(x,y) = {12,13,14,22,23,24,32,33,34}

l. P(A) = { ∅,{1},{2},{3},{1,2},{1,3},{2,3},{1,2,3}}

f(X) = { ∅,{2},{3},{2,3}}

4.2.2: Properties of algebraic functions.

For each of the functions below, indicate whether the function is onto, one-to-one, neither or both. If the function is not onto or not one-to-one, give an example showing why.

(c)　h: Z → Z. h(x) = x3

　　one-to-one and not onto.

If a != b, then a3 != b3, so h is one-to-one.

When h(x) = -3, there is not an x in Z can map it, so h is not onto.

(g)　f: Z × Z → Z × Z, f(x, y) = (x+1, 2y)

one-to-one and not onto.

If x1 != x2, y1 != y2, then x1+1 != x2+1, 2y1 ­!= 2y2, so f is one-to-one.

Not all the y in Z can be expressed as 2y, for example, when 2y = 1, there is not a y in Z can map it.

(k)　f: Z+ × Z+ → Z+, f(x, y) = 2x + y.

Both one-to-one and onto.

If x1 != x2, then 2x1 != 2x2, so 2x1 + y1 != 2x2 + y2, so f is one-to-one.

For every 2x+y in Z, there are integer x and y to map it, so f is onto.

4.2.4: Properties of functions on strings and power sets.

(b)f: {0, 1}3→{0, 1}3. The output of f is obtained by taking the input string and replacing the first bit by 1, regardless of whether the first bit is a 0 or 1. For example, f(001) = 101 and f(110) = 110.

Onto not one-to-one. Neither

The value of f can be 100,101,110,111, every element in the range of f can be mapped in the domain of f, so f is onto.

f(001)=f(101)=101, so f is not one-to-one.

001

(c)　f: {0, 1}3→{0, 1}3. The output of f is obtained by taking the input string and reversing the bits. For example f(011) = 110.

Both onto and one-to-one.

All the elements in the range of f can be mapped in the domain of f, so f is onto.

If x1 != x2 in {0, 1}3, then f(x1) != f(x2), so f is one-to-one.

(d)　f: {0, 1}3→{0, 1}4. The output of f is obtained by taking the input string and adding an extra copy of the first bit to the end of the string. For example, f(100) = 1001.

One-to-one and not onto.

When f(x)=0101, there is not an element in {0, 1}3 can map it, so f is not onto.

If x1 != x2 in {0, 1}3, then f(x1) != f(x2), so f is one-to-one.

(g)　Let A be defined to be the set {1, 2, 3, 4, 5, 6, 7, 8} and let B = {1}. f: P(A) → P(A). For X ⊆ A, f(X) = X - B. Recall that for a finite set A, P(A) denotes the power set of A which is the set of all subsets of A.

Neither one-to-one or onto.

f(∅)=f({1})=∅, so f is not one-to-one.

When f = {1, 2, 3, 4, 5, 6, 7, 8}, there is not a P(A) can map it, so f is not onto.

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1. f : Z→Z+, f(x)=2x.
2. f: Z→Z+, f(x)=x^2.
3. f: Z→Z+, f(x)=x.

f(x) = -2x -1 if x<0

f(x) = 2(x+1) if x>= 0

1. f: Z→Z+, f(x)=x^2+5

4.3.2: Finding inverses of functions.

c. f(x) is a bijection, it has a well-defined inverse.

f-1(x)=(x-3)/2

d.f({1})=f({2})=1, f is not one-to-one, so f has no well-defined inverse.

g. f(x) is a bijection, it has a well-defined inverse.

The output of f-1 is obtained by taking the input string and reversing the bits.

f-1(X) = {0,1}^3

i.f(x) is a bijection, it has a well-defined inverse.

f(x,y)=(x+5,y-2)

f-1(x,y) =(x-5,y+2)

4.4.8: Explicit formulas for compositions of functions.

c. f o h = f(h(x)) = f(x^2+1) = 2\*x^2+5

d. h o f = h(f(x)) = h(2x+3) = (2x+3)^2+1=4\*x^2+12x+10

4.4.2: Composition of functions on integers.

b. (f ο h)(52) = f(h(52))=f(「52/5」)＝f(11)=121

c. (g ο h ο f)(x) = g(h(f(x)))

f(4)=16

h(f(4))=h(16)=4

g(h(f(4)))=g(4)=16

d.(h ο f )(x)= h(f(x))= 「x^2/5」

4.4.6: Composition of functions on sets of strings.

c. (h ο f)(010)=h(f(010))=h(110)=111

d. the range of f: {100,101,110,111}

the range of h o f: {101,111}

1. the range of f: {100,101,110,111}

the range of g o f: {001,101,011,111}

4.4.4: Composition of onto and one-to-one functions.

c. yes.

f: R→R g:R→R

f(x)=x^2

g(x)=x^3/2

g o f = x^6

(x^2)^3-2

d. yes

f: R→R g:R→R

f(x)=x^3/2

g(x)=x^2