Maths Primer

This list of exercises has the objective of reviewing some of the preliminary requirements for the topics that will be covered during the *Maths for Biologists* module. Textbook content for these topics can be found on the module's main reference:

- [Calculus for biology and medicine](#)
  Neuhauser, Claudia

Copies of this book are available at the Silwood Library and also as ebooks through the Imperial College Library.

Functions

1. Suppose that a fungal disease originates in the middle of an orchard, initially affecting only one tree. The disease spreads out radially at a constant speed of 3 meters per day. What area will be affected after 2 days, 4 days, and 8 days? Write an equation that expresses the affected area as a function of time, measured in days, and show that this function is a polynomial of degree 2. If time is measured in weeks, how will your equation for the affected area be written?

> **Answer**
>
> With a velocity of radial spread equal to $v_R = 3\ m/day$, after t days the radius of the infected orchard area as a function of t, $R(t)$ will be equal to:
>
> $R(t) = 3t$
>
> Assuming a circular spread, the infected area after t days, $A(t)$, will be equal to the area of a circle with radius $R(t)$. Thus:
>
> $A(t) = \pi[R(t)]^2 = \pi(3t)^2 = 9\pi t^2\ m^2$,
> which is a polynomial of degree 2 in t.
>
> After 2 days, 4 days, and 8 days the infected area will be given by:
>
> | t | $A(t)$ |
> | --- | --- |
> | 2 | $36\pi\ m^2\ (\approx 113.1\ m^2)$ |
> | 4 | $144\pi\ m^2\ (\approx 452.4\ m^2)$ |
> | 8 | $576\pi\ m^2\ (\approx 1809.6\ m^2)$ |
>
> If t is measured in weeks, then $R(t) = (3 \times 7)t = 21t\ m/week$, and the affected area shall be given by $A(t) = \pi[R(t)]^2 = \pi(21t)^2 = 441\pi t^2$.

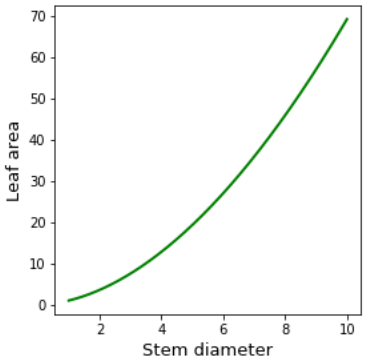
2. In the problems below, use *Python* or *R* to sketch each scaling relation (K. J. Niklas (1994). Plant Allometry: The Scaling of Form and Process. University of Chicago Press).

   (a) In a sample based on 46 species, leaf area was found to be proportional to $(stem\ diameter)^{1.84}$. On the basis of your graph, as stem diameter increases, does leaf area increase or decrease?

   (b) In a sample based on 28 species, the volume fraction of spongy mesophyll was found to be proportional to $(leaf\ thickness)^{-0.49}$ (the spongy mesophyll is part of the internal tissue of a leaf blade). On the basis of your graph, as leaf thickness increases, does the volume fraction of spongy mesophyll increase or decrease?

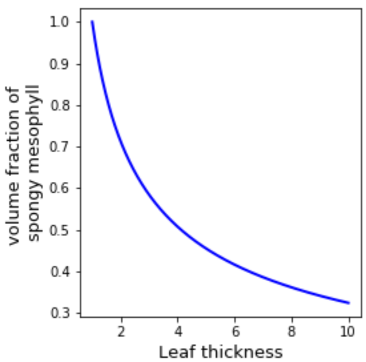
Answer

(a) Leaf area



As stem diameter increases, leaf area increases.

(b) Volume fraction of spongy mesophyll



As leaf thickness increases, the volume fraction of spongy mesophyll decreases.

3. I. Simplify the following expressions:
(a) $3^{4\log_3 x}$
(b) $4^{-\log_{1/2} x}$
(c) $\log_{1/2} 4^x$
(d) $\log_3 9^{-x}$

II. Show that the function $y = (1/2)^x$ can be written in the form $y = \mathrm{e}^{-\mu x}$, where μ is a positive constant. Determine μ.

Tip

Remember the following identities:

$$\log_a x^r = r\log_a x$$
$$\log_a a^x = x$$
$$a^{\log_a x} = x$$
$$a^{-r} = \frac{1}{a^r}$$
$$(a^r)^s = a^{rs},$$

assuming a positive and different from 1)

Answer

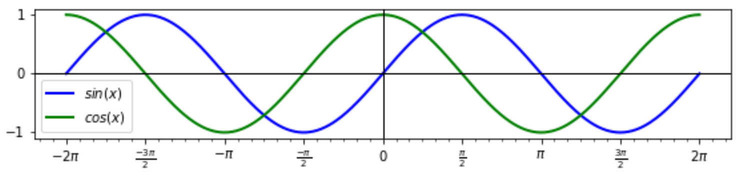
I.
(a) x^4
(b) x^2
(c) $-2x$
(d) $-2x$

II.
$y = (1/2)^x = e^{\ln(1/2)^x} = e^{x\ln(1/2)} = e^{-\mu x}$,
where $\mu = -\ln(1/2) > 0$

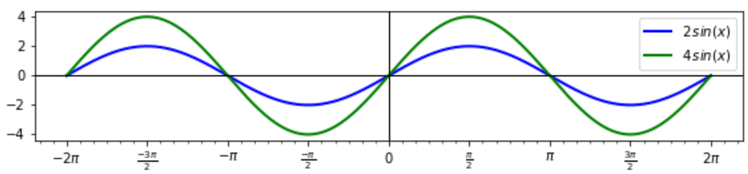
4. Using *Python* or *R*, plot the following graphs:
   (a) $y = sin(x)$ and $y = cos(x)$, with $-2\pi \leq x \leq 2\pi$.
   (b) $y = 2sin(x)$ and $y = 4sin(x)$, with $-2\pi \leq x \leq 2\pi$. What changes in comparison to $y = sin(x)$?
   (c) $y = cos(2x)$ and $y = cos(4x)$, with $-2\pi \leq x \leq 2\pi$. What changes in comparison to $y = cos(x)$?
   (d) $y = cos(x + \frac{\pi}{4})$ and $y = cos(x + \frac{\pi}{2})$, with $-2\pi \leq x \leq 2\pi$. What changes in comparison to $y = cos(x)$?

Answer

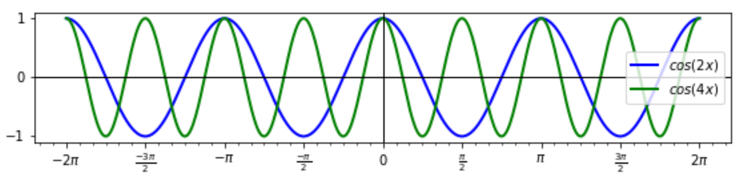
(a)



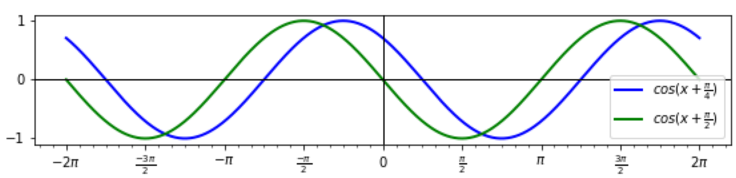
(b)



The amplitudes of the functions (maximum and minimum values of the oscillation) are multiplied by 2 and 4.
(c)



The frequencies of the functions (number of complete cycles per interval) are multiplied by 2 and 4.
(d)



The function $y = cos(x)$ is dislocated to the left by $\pi/4$ and $\pi/2$.

Matrix algebra

1. Let

$$A = \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 2 & 1 & 3 & 0 \end{pmatrix}$$

   (a) If possible, compute AB
   (b) If possible, compute BA

Answer

(a) $AB = \begin{pmatrix} 7 & 5 & 9 & -1 \\ -4 & -2 & -6 & 0 \end{pmatrix}$

(b) It is not possible to calculate BA because the number of columns of B is defferent from the number of lines of A.

2. Let

$$A = \begin{pmatrix} 1 & 4 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

(a) Compute AB
(b) Compute BA

Answer

(a) $AB = \begin{pmatrix} 1 \end{pmatrix}$

(b) $BA = \begin{pmatrix} -1 & -4 & 2 \\ 2 & 8 & -4 \\ 3 & 12 & -6 \end{pmatrix}$

3. Let

$$A = \begin{pmatrix} 2 & 1 \\ -1 & -3 \end{pmatrix}$$

Find A^2, A^3, and A^4

Answer

(a) $A^2 = \begin{pmatrix} 3 & -1 \\ 1 & 8 \end{pmatrix}$

(b) $A^3 = \begin{pmatrix} 7 & 6 \\ -6 & -23 \end{pmatrix}$

(c) $A^4 = \begin{pmatrix} 8 & -11 \\ 11 & 63 \end{pmatrix}$

4. Suppose that

$$A = \begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

(a) Compute $\det(A)$. Is A invertible? If yes, find the inverse of A.
(b) Compute $\det(B)$. Is B invertible? If yes, find the inverse of B.

Answer

(a) $\det(A) = 0$. A is not invertible.

(b) $\det(B) = -3$.

$B^{-1} = \begin{pmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{pmatrix}$