Maths Primer

This list of exercises has the objective of reviewing some of the preliminary requirements for the topics that will be covered during the *Maths for Biologists* module. Textbook content for these topics can be found on the module's main reference:

- Calculus for biology and medicine
  Neuhauser, Claudia

Copies of this book are available at the Silwood Library and also as ebooks through the Imperial College Library.

Functions

1. Suppose that a fungal disease originates in the middle of an orchard, initially affecting only one tree. The disease spreads out radially at a constant speed of 3 meters per day. What area will be affected after 2 days, 4 days, and 8 days? Write an equation that expresses the affected area as a function of time, measured in days, and show that this function is a polynomial of degree 2. If time is measured in weeks, how will your equation for the affected area be written?

2. In the problems below, use *Python* or *R* to sketch each scaling relation (K. J. Niklas (1994). Plant Allometry: The Scaling of Form and Process. University of Chicago Press).

   (a) In a sample based on 46 species, leaf area was found to be proportional to $(stem\ diameter)^{1.84}$. On the basis of your graph, as stem diameter increases, does leaf area increase or decrease?

   (b) In a sample based on 28 species, the volume fraction of spongy mesophyll was found to be proportional to $(leaf\ thickness)^{-0.49}$ (the spongy mesophyll is part of the internal tissue of a leaf blade). On the basis of your graph, as leaf thickness increases, does the volume fraction of spongy mesophyll increase or decrease?

3. I. Simplify the following expressions:
   (a) $3^{4\log_3 x}$
   (b) $4^{-\log_{1/2} x}$
   (c) $\log_{1/2} 4^x$
   (d) $\log_3 9^{-x}$

   II. Show that the function $y = (1/2)^x$ can be written in the form $y = \mathrm{e}^{-\mu x}$, where μ is a positive constant. Determine μ.

> **Tip**
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> Remember the following identities:
> $$\log_a x^r = r\log_a x$$
> $$\log_a a^x = x$$
> $$a^{\log_a x} = x$$
> $$a^{-r} = \frac{1}{a^r}$$
> $$(a^r)^s = a^{rs},$$
> assuming a positive and different from 1)

4. Using *Python* or *R*, plot the following graphs:
   (a) $y = sin(x)$ and $y = cos(x)$, with $-2\pi \leq x \leq 2\pi$.
   (b) $y = 2sin(x)$ and $y = 4sin(x)$, with $-2\pi \leq x \leq 2\pi$. What changes in comparison to $y = sin(x)$?
   (c) $y = cos(2x)$ and $y = cos(4x)$, with $-2\pi \leq x \leq 2\pi$. What changes in comparison to $y = cos(x)$?
   (d) $y = cos(x + \frac{\pi}{4})$ and $y = cos(x + \frac{\pi}{2})$, with $-2\pi \leq x \leq 2\pi$. What changes in comparison to $y = cos(x)$?

> **Review**
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> On the reference textbook, a short review on *trigonometry* is presented on section $1.1.4$. *Exponentials and logarithms* are found on section $1.1.5$. Section $1.3.4$ on *verbal description of graphs* is also worth reviewing.

Matrix algebra

1. Let

$$A = \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 2 & 1 & 3 & 0 \end{pmatrix}$$

(a) If possible, compute AB

(b) If possible, compute BA

2. Let

$$A = \begin{pmatrix} 1 & 4 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

(a) Compute AB

(b) Compute BA

3. Let

$$A = \begin{pmatrix} 2 & 1 \\ -1 & -3 \end{pmatrix}$$

Find A^2, A^3, and A^4

4. Suppose that

$$A = \begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

(a) Compute $\det(A)$. Is A invertible? If yes, find the inverse of A.

(b) Compute $\det(B)$. Is B invertible? If yes, find the inverse of B.

Review

Be sure to review matrix operations. Sections $9.2.1 - 9.2.3$ cover the basics.
