Maths Primer
Supplementary questions

CMEE MSc/MRes 2022-23

12th December 2022

1. Solve the following system of simultaneous equations. (This means that both equations are true – using rearranging and substitution you should be able to find out the values of x and y.

$$2x + 3y = 15$$
$$x - y = 5$$

2. Suppose that

$$A = \begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix} \text{ and } \boldsymbol{z} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Find the vector $\boldsymbol{b} = A\boldsymbol{z}$ (write it in terms of x and y which are unknown).

3. Show that the system of simultaneous equations and the matrix equation written here are equivalent. (Show that one is the same as the other).

Simultaneous equation:

$$5x + 2y = 3$$
$$5x - 4y - 9 = 0$$

Matrix equation:

$$A\boldsymbol{z} = \boldsymbol{b}$$

where $A = \begin{pmatrix} 5 & 2 \\ 5 & -4 \end{pmatrix}$, $\boldsymbol{z} = \begin{pmatrix} x \\ y \end{pmatrix}$, and $\boldsymbol{b} = \begin{pmatrix} 3 \\ 9 \end{pmatrix}$.

4. a) Write the following system of simultaneous equations as a matrix equation in the same form as the one in Question 3; $A\boldsymbol{z} = \boldsymbol{b}$, where $\boldsymbol{z}$ is the vector $\begin{pmatrix} x \\ y \end{pmatrix}$ and the matrix A and vector $\boldsymbol{b}$ are to be found.

$$3x - 7y = 4$$
$$x + 7y = 10$$

b) Find the inverse A^{-1} of the matrix A.

c) Show that if you multiply both sides of the equation $A\boldsymbol{z} = \boldsymbol{b}$ by A^{-1}, you get an equation with only the vector $\boldsymbol{z}$ on the left hand side. (Hint: Multiplying both sides by the inverse of A will give: $A^{-1}A\boldsymbol{z} = A^{-1}\boldsymbol{b}$.

d) Hence, find the value of x and y.

5. a) Write down the following system of simultaneous equations as a matrix equation. This time the matrix will be 3x3 and the vector of variables will be (x, y, z) in a column vector.

$$x + 5z = 3$$
$$2x + y + 6z = 4$$
$$3x + 4y = 6$$

b) Either by hand or using functions in R or Python, compute the inverse of the matrix.

c) Hence, find the values of x, y, and z.