HW 11

Due on **Wednesday, April 24, 2019 at 10am**

CS/ECE 374: Algorithms & Models of Computation, Spring 2019 Version: **1.1**

Submission instructions as in previous homeworks.

31 (100 PTS.) Walk with me

31.A. (50 PTS.) We are given a *directed* graph G with n vertices and m edges ($m \geq n$), where each vertex v has a height $h(v)$. The *cost* of traversing an edge (u, v) is $c(u, v) = |h(v) - h(u)|$. The cost of a walk in G is the sum of the costs of edges in the walk. Prove that finding the minimum cost walk that visits all the vertices om G is NP-HARD. (In a walk, vertices and edges may be repeated, and the start and end vertices may be different.)

31.B. (50 PTS.) We are given a directed graph G with n vertices and m edges ($m \geq n$), where each edge e has a set of colors $C(e) \subseteq \{1, ..., k\}$. Prove that deciding whether there exists a walk that uses all k colors (i.e. the union of the sets of colors of the edges of walk covers all colors.) is NP-HARD. (Hint: Reduce from Set Cover.)

32 (100 PTS.) Things are hard.

32.A. (20 PTS.) Suppose we have n prisoners $P_1, \ldots, P_n$ that we want to place in some disconnected blocks of a prison. Each prisoner is assigned to one block, and he/she will not be able to access other blocks. However, some prisoners are bitter enemies (going all the way back to kindergarten) and cannot be placed in the same block. Given integers n and k and a list of enemies for each of the n prisoners, we want to determine whether k blocks are sufficient to house all the prisoners? Prove that this problem is NP-HARD. You can safely assume that every block has unbounded capacity.

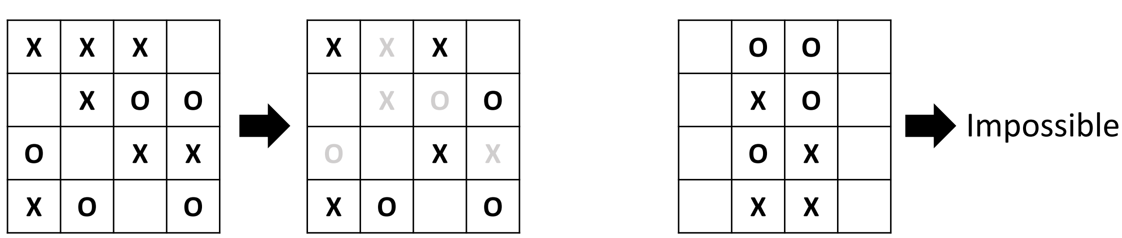
32.B. (40 PTS.) Let G be an arbitrary directed weighted graph with n vertices and m edges such that no edge weight is zero (weights can be positive or negative). Prove that finding a *zero-length* (i.e., zero weight) Hamiltonian cycle in G is NP-COMPLETE.

32.C. (40 PTS.) Consider the following **XO** puzzle. You are given an $n \times m$ grid of squares where each square has an **X**, an **O** or is empty. Your goal is to erase some of the **X**s and **O**s so that

(i) every row contains at least one **X** or one **O**, and

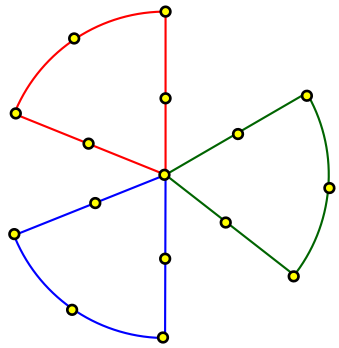
(ii) no column contains both **X** and **O**.

For some input grids, it is impossible to solve the puzzle. The figure below shows two examples: a grid that is solvable and a grid that is impossible to solve. Prove that, given a grid, it is NP-HARD to determine whether the puzzle is solvable. (Hint: Reduce from 3SAT.)



33 (100 PTS.) Fan, fan, fan.

An undirected graph a ***3-blade-fan*** if it consists of three cycles C_1, C_2, and C_3 of k nodes each and they all share exactly one node. Hence, the graph has $3k - 2$ nodes. The figure below shows a ***3-blade-fan*** of 16 nodes.



Given an undirected graph G with n vertices and m edges and an integer k, the FAN problem asks whether or not there exists a subgraph of G which is a ***3-blade-fan***. Prove that FAN is NP-COMPLETE.