CS/ECE 374 FALL 2018
Homework 3 Problem 3

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Given languages L_1 and L_2 we define *insert*(L_1, L_2) to be the language $\{uvw \mid v \in L_1, uw \in L_2\}$ to be the set of strings obtained by "inserting" a string of L_1 into a string of L_2. For example if $L_1 = \{isfun\}$ and $L_2 = \{0, CS\}$ then

$$insert(L_1, L_2) = \{isfun0, 0isfun, isfunCS, CisfunS, CSisfun\}$$

- The goal is to show that if L_1 and L_2 are regular languages then *insert*(L_1, L_2) is also regular. In particular you should describe how to construct an NFA $N = (Q, \Sigma, \delta, s, A)$ from two DFAs $M_1 = (Q_1, \Sigma, \delta_1, s_1, A_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, s_2, A_2)$ such that $L(N) = insert(L(M_1), L(M_2))$. You do not need to prove the correctness of your construction but you should explain the ideas behind the construction (see lab 3 solutions).

Solution:

1. Let $M_1 = (Q_1, \Sigma, \delta_1, s_1, A_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, s_2, A_2)$ such that $L(N) = insert(L(M_1), L(M_2))$, We construct an NFA $N = (Q, \Sigma, \delta, s, A)$ that accepts $insert(L(M_1), L(M_2))$ as follows:
$Q := Q_1 * Q_2$
$\Sigma := \Sigma$
$\delta((q_2, before), a) := \begin{cases} \{\delta((q_2, a), before), (q_2, after)\} \text{ a} \in \text{L}(\text{M}_1) \\ \{\delta((q_2, a), before)\} otherwise \end{cases}$
$\delta((q_2, after), a) := \{\delta((q_2, a), after)\}$ a $\in L(M_2)$
$s := \{s_1, s_2\}$
$A := A_1 \cap A_2$
For the states in the NFA, Q is going to be $Q_1 * Q_2$ without any doubts. Since we can start at both old state s_1 and s_2, the start state for NFA is just the set contain both s_1 and s_2. Same reason apply for accepting state, we can end at both old accepting states, the accepting states for NFA is the union of A_1 and A_2. As for the language it accepts, doesn't matter if the input language that start with $L(M_1)$ or $L(M_2)$, once it receive the language in $L(M_1)$, because $L(N) = insert(L(M_1), L(M_2))$, only breaks the language that is in $L(M_2)$ in half, that means once we pass or enter the start state in $L(M_1)$, there is only one way to the accepting state (only one accepting state as there is only one route).

   - The state (q_2, before) means (the simulation of) M_2 is in state q_2 and N has not yet enter the start state for M_1.
   - The state (q_2, after) means (the simulation of) M_2 is in state q_2 and N has passed the accepting state for M_1.

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