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1. Suppose $M_1 = (Q_1, \Sigma, \delta_1, s_1, A_1)$ is a DFA and $N_2 = (Q_2, \Sigma, \delta_2, s_2, A_2)$ is an NFA. Formally describe a DFA that accepts the language $L(M_1) \setminus L(N_2)$. To be more specific, letting $M = (Q, \Sigma, \delta, s, A)$ be the DFA, describe the components Q, δ, s, A in terms of the components of M_1 and N_2. This combines subset construction and product construction to give you practice with formalism. Be aware of the distinction between the transition function of a DFA and that of a NFA. You can use δ_1^* and δ_2^* in your construction. You do not need to prove the correctness of your construction.
2. For a language L let SUFFIX(L) $= \{y \mid \exists x \in \Sigma^*, xy \in L\}$ be the set of suffixes of strings in L. Let PSUFFIX(L) $= \{y \mid \exists x \in \Sigma^*, |x| \geq 1, xy \in L\}$ be the set of proper suffixes of strings in L. Prove that if L is regular then PSUFFIX(L) is regular via the following technique. Let $M = (Q, \Sigma, \delta, s, A)$ be a DFA accepting L. Describe a NFA N in terms of M that accepts PSUFFIX(L). Explain the construction of your NFA.

Solution: 1.Based on the Theorem, For every NFA N, there is a DFA $det(N)$ such that $L(N) = L(det(N))$. We define the DFA $det(M_2) = (Q_2', \Sigma, \delta_2', s_2', A_2')$ as follows:

$Q_2' = P(Q_2)$

$s_2' = \delta_{N_2}^*(s_2, \epsilon)$

$\delta_2'(X, a) = \bigcup\limits_{q \in X} \delta_{N_2}^*(q, a)$ for $X \subseteq Q_2$, $a \in \Sigma$.

$A_2' = \{X \subseteq Q_2 | X \cap A_2 \neq \emptyset\}$

Then we define a DFA $M = L(M_1) \setminus L(N_2) = (Q, \Sigma, \delta, s, A)$ as follows:

$Q = Q_1 \times P(Q_2)$

$s = (s_1, \delta_{N_2}^*(s_2, \epsilon))$

$A = \{(p, X) | p \in A_1 butX \notin Q_2 | X \cap A_2 \neq \emptyset\}$

$\delta((r, X), a) = (\delta_1(r, a), \bigcup\limits_{q \in X} \delta_{N_2}^*(q, a))$ for $r \in Q_1$, $X \subseteq Q_2$, $a \in \Sigma$

2.Since the definition of PSUFFIX(L) is $\exists x \in \Sigma^*, |x| \geq 1, xy \in L$, the length of the string x is at least 1. There must be an intermediate state $q \in Q$ such that $\delta^*(s, x) = q$ and $\delta^*(q, y) \in A$.

Let $M = (Q, \Sigma, \delta, s, A)$ be a DFA accepting L. We define a new NFA $N' = (Q', \Sigma', \delta', s', A')$ with ϵ-transitions that accepts PSUFFIX(L).

$Q' = Q \cup \{s'\}$

s' is an explicit state in Q'

$\Sigma' = \Sigma$

$A' = A$

$\delta'(q, a) = \delta(q, a)$ for $q \in Q$

$\delta'(s', a) = q \in Q \setminus \{s\}$

Let y be an arbitrary string that accepted by the NFA N'. Since we make the transitions from the new starting state to all the states in Q except for the old start state, there is no path between the new starting state and the old starting state. Besides, there is no ϵ-transition in the DFA.

Therefore we can make sure that $x \in \Sigma^*$, $|x|$ is at least great than or equal to 1 and $xy \in L$. Hence $y \in \text{PSUFFIX}(L)$. Our NFA N' correctly recognizes the language PSUFFIX(L).

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