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Homework 0 Problem 1

(a) Suppose S is a set of 103 integers. Prove that there is a subset $S' \subseteq S$ of at least 15 numbers such that the difference of any two numbers in S' is a multiple of 7.

Solution: (a) Since S is a set, by definition of set, there is no duplicate in set S. S contains 103 integers, then we can have x1, x2, x3 ... x103 from smallest to largest in S. For any integer k, k mod 7 will result in [0,6]. By pigeon hole principle, let b0, b1, b2, b3, b4, b5, b6 where bi = xi mod 7. Now 103 / 7 > 14, means that there is at least one b containing 15 x. Since integers in one hole will have same result mod 7, then the difference between them will be a multiple of 7. Hence, there is always a subset S' of S containing 15 integers such that the difference between any two numbers in the set S' will be a multiple of 7.

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