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Sensitivity of Subtraction

10 points

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x, y) = x - y.$$

Measure the size of the input (x, y) by $|x| + |y|$ (i.e. use the 1-norm to measure distances on the input side of the function), and assume that $|x| + |y| \approx 1$ and $x - y \approx \epsilon$.

- Show that $\kappa(f)$ has an approximate upper bound of $1/\epsilon$.
- What can you conclude about the sensitivity of subtraction?

Hint 1: "$x - y \approx \epsilon$" means that the difference between x and y is small, where ϵ is that small difference. To compute the condition number, you will be examining a perturbation $(x + \Delta x, y + \Delta y)$ of the input (x, y). The size of the perturbation is independent of ϵ, but also assumed small, so that you can assume that $(x + \Delta x) - (y + \Delta y)$ is also small, but not necessarily equal to ϵ.

Hint 2: To simplify the analysis, the problem lets you also assume that $|x| + |y| \approx 1$. Since we assume $|\Delta x| + |\Delta y|$ is also assumed small, you may also assume $|x + \Delta x| + |y + \Delta y| \approx 1$ if needed.

Please submit your response to this written problem as a PDF file below. You may do either of the following:

- write your response out by hand, scan it, and upload it as a PDF.

  We will not accept unprocessed pictures taken with your phone.

  If you decide to use your phone for scanning, make sure to use an app such as CamScanner (https://www.camscanner.com/) to get a readable PDF. Alternatively, there's a fast and convenient scanner in the Engineering IT office in 2302 Siebel that can just email you a PDF. (It's the Fax-machine-looking thing--not the scanner that's attached to one of the computers.)

- create the PDF using software.

  If you're looking for an easy-ish way to type math, check out TeXmacs (http://texmacs.org/) or LyX (http://www.lyx.org/). Both are installed in the virtual machine. (Under "Applications / Accessories / GNU TeXmacs editor" and "Applications / Office / LyX document processor" respectively.)

Submit your response to each problems in this homework as a separate PDF. If you have multiple PDFs that you need to merge into one, try PDF Split and Merge (http://www.pdfsam.org/download/).

NOTE: Please make sure your solutions are legible and easy to follow. If they are not, we may deduct up to five points *per problem*.

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Your answer is correct.

The following feedback was provided:

Good job!

$$f(x,y) = x - y$$

$$\begin{aligned}\text{Relative change in } f &= \frac{|f(x+\Delta x, y+\Delta y) - f(x,y)|}{|f(x,y)|} \\ &= \frac{|((x+\Delta x)-(y+\Delta y))-(x-y)|}{|x-y|} \\ &= \frac{|\Delta x - \Delta y|}{|x-y|}.\end{aligned}$$

$$\begin{aligned}\text{Relative change in input} &= \frac{\Delta \boldsymbol{x}}{\boldsymbol{x}} \\ &= \frac{|\Delta x| + |\Delta y|}{|x| + |y|}.\end{aligned}$$

Given that $|x| + |y| \approx 1$ and $x - y \approx \epsilon$:

$$\begin{aligned}\text{cond}(f) &= \frac{\text{Relative change in } f}{\text{Relative change in input}} \\ &= \frac{\dfrac{|\Delta x - \Delta y|}{|x-y|}}{\dfrac{|\Delta x| + |\Delta y|}{|x| + |y|}} \\ &\approx \frac{\dfrac{|\Delta x - \Delta y|}{\epsilon}}{\dfrac{|\Delta x| + |\Delta y|}{1}} \\ &= \frac{1}{\epsilon}\,\frac{|\Delta x - \Delta y|}{|\Delta x| + |\Delta y|} \leq \frac{1}{\epsilon}.\end{aligned}$$

If $x - y \approx \epsilon$, the condition number will be large. Thus, subtraction is sensitive if the two numbers are of nearly the same magnitude.