
Submission instructions as in previous homeworks.

Any dynamic programming solution should be done using an iterative algorithm.

25 (100 PTS.) Rainbow walk

We are given a directed graph with n vertices and m edges ($m \geq n$), where each edge e has a color $c(e)$ from $\{1, \dots, k\}$.

- 25.A.** (20 PTS.) Describe an algorithm, as fast as possible, to decide whether there exists a closed walk that uses all k colors. (In a walk, vertices and edges may be repeated. In a closed walk, we start and end at the same vertex.)
- 25.B.** (80 PTS.) Now, assume that there are only 3 colors, i.e., $k = 3$. Describe an algorithm, as fast as possible, to decide whether there exists a walk that uses all 3 colors. (The start and end vertex may be different.)

26 (100 PTS.) Stay safe

We are given an *undirected* graph with n vertices and m edges ($m \geq n$), where each edge e has a positive real weight $w(e)$, and each vertex is marked as either “safe” or “dangerous”.

- 26.A.** (35 PTS.) Given safe vertices s and t , describe an $O(m)$ -time algorithm to find a path from s to t that passes through the smallest number of dangerous vertices.
- 26.B.** (65 PTS.) Given safe vertices s and t and a value W , describe an algorithm, as fast as possible, to find a path from s to t that passes through the smallest number of dangerous vertices, subject to the constraint that the total weight of the path is at most W .

27 (100 PTS.) Stay stable

We are given a directed graph with n vertices and m edges ($m \geq n$), where each edge e has a weight $w(e)$ (you can assume that no two edges have the same weight). For a cycle C with edge sequence $e_1 e_2 \dots e_\ell e_1$, define the *fluctuation* of C to be

$$f(C) = |w(e_1) - w(e_2)| + |w(e_2) - w(e_3)| + \dots + |w(e_\ell) - w(e_1)|.$$

- 27.A.** (10 PTS.) Show that the cycle with the minimum fluctuation cannot have repeated vertices or edges, i.e., it must be a simple cycle.
- 27.B.** (90 PTS.) Describe a polynomial-time algorithm, as fast as possible, to find the cycle with the minimum fluctuation.