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Homework 0 Problem 1

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1. Suppose S is a set of 103 integers. Prove that there is a subset $S' \subseteq S$ of at least 15 numbers such that the difference of any two numbers in S' is a multiple of 7.

Solution:

1. Since S is a set, by definition of set, all elements of S are different. We can order these integer elements by ascendance and name them as $x_1, x_2, x_3, ..., x_{103}$. For any of these integers, x_i, x_i mod 7 range in [0, 6]. Let $r_0, r_1, ..., r_6$ be sets of integers and r_j contains x_is where x_i mod 7 = j. Then by the pigeon hole principle, since 103/7 is greater than 14, there must be one set of r_is to have at least $14 + 1 = 15$ elements. Also, elements of each r_i increase by 7. Therefore, there is a subset $S' \subseteq S$ of at least 15 numbers such that the difference of any two numbers in S' is a multiple of 7.

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