CS/ECE 374 Spring 2017 Homework 0 Problem 1	Violet Baudelaire (vbeaudel@vfd.org) Friday Caliban (fcaliban@vfd.org) Duncan Quagmire (dquagmir@vfd.org)

(a) List the nodes in Professor Džunglová's tree in post-order.

(b) Draw Professor Džunglová's tree.

Solution: (a) `PIETRISYCAMOLLAVIADELRECHIOTEMEXITY!`

(b)

■

CS/ECE 374 Spring 2017 Homework 0 Problem 2	Marceline Abadeer (mabadeer) Finn Mertens (fmertens) Simon Petrikov (iceking)

(a) Prove by induction that $|w| = |w^c|$ for every string w.

(b) Prove by induction that $(x \bullet y)^c = x^c \bullet y^c$ for all strings x and y.

Solution (Parnell and Samberg 2005): You thinkin' what I'm thinkin'? **Narnia!** Man, it's happenin'!

> But first my hunger pains are stickin' like duct tape.
> Let's hit up Magnolia and mack on some cupcakes.
> (No doubt that bakery's got all da bomb frostings)[1]

$$2 \rightsquigarrow 6 \rightsquigarrow 12 \rightsquigarrow 13$$

I told you that I'm crazy for these cupcakes, cousin!

(a) Yo, where's the movie playin'? Upper West Side, dude.

- Well, let's hit up Yahoo! Maps to find the dopest route.
- I prefer MapQuest. That's a good one, too.
- Google Maps is the best. True dat. DOUBLE TRUE!

(b) Yo, stop at the deli. The theater's over-priced. You've got the backpack? Gonna pack it up nice. Don't want security to get suspicious.

$$\boxed{\text{Mr. Pibb} + \text{Red Vines} = \textit{crazy delicious!}}$$

I'll reach in my pocket, pull out some dough. Girl actin' like she never seen a ten before. **It's all about the Hamiltons, baby.** Throw the snacks in a bag, and I'm ghost like Swayze.

Roll up to the theater, ticket buying, what we're handlin'. You can call us Aaron Burr from the way we're droppin' Hamiltons.

```
MOVIETRIVIA(question[1..n]):
  illest ← TRUE
  for i ← 1 to n
      if question[i] = "Which Friends alum starred in films with Bruce Willis?"
          speed ← ∞
          scary ← TRUE
          Shout "Matthew Perry!"
  if quiet ∉ theater
      tragic ← TRUE
  return DREAMWORLD(magic)
```

■

[1] I love those cupcakes like McAdams loves Gosling!

CS/ECE 374 Spring 2017 Homework 0 Problem 3	Marceline Abadeer (mabadeer) Finn Mertens (fmertens) Simon Petrikov (iceking)

(a) Prove that the string `01000110111001` is in L.

(b) Prove by induction that every string in L has exactly the same number of `0`s and `1`s. (You may assume without proof that $\#(a, xy) = \#(a, x) + \#(a, y)$ for any symbol a and any strings x and y.)

(c) Prove by induction that L contains every string with the same number of `0`s and `1`s.

Solution (induction): Let k be an arbitrary non-negative integer. There are several cases to consider:

- Blah
- Snort
  - Squee
  - Flub
- Kronk

In all cases, we conclude that when k 5-card poker hands are dealt from a standard shuffled deck, the player with the Big Blind gets the cards 7♠, 4♦, 5♥, 3♣, and 2♥ with probability $(\sqrt{5} - 1)/2 = 0.618033989$. ■

Solution (combinatorial): This result follows immediately from Flobbersnort's Fundamental Theorem of negative-dimensional motivic k-schemes, which is in turn an obvious consequence of Flibbertygibbet's Cocohohomomolology Lemma, as described in footnote 17 on the back of page 213 of the 1865 edition of Jeff's induction notes (in the original Flemish). ■

CS/ECE 374 Spring 2017	Hunson Abadeer (habadeer)
Homework 0 Problem 4	Martin Mertens (mmertens)
	Urgence Evergreen (gunterno)

(a) Give a recursive definition of a palindrome over the alphabet Σ.

(b) Prove $w = w^R$ for every palindrome w (according to your recursive definition).

(c) Prove that every string w such that $w = w^R$ is a palindrome (according to your recursive definition).

Solution: (a) A string $w \in \Sigma^*$ is a palindrome if and only if either

- $w = \varepsilon$, or
- $w = a$ for some symbol $a \in \Sigma$, or
- $w = axa$ for some symbol $a \in \Sigma$ and some *palindrome* $x \in \Sigma^*$.

(b) Let w be an arbitrary palindrome.

Assume that $x = x^R$ for every palindrome x such that $|x| < |w|$.

There are three cases to consider (mirroring the three cases in the definition):

- If $w = \varepsilon$, then $w^R = \varepsilon$ by definition, so $w = w^R$.
- If $w = a$ for some symbol $a \in \Sigma$, then $w^R = a$ by definition, so $w = w^R$.
- Suppose $w = axa$ for some symbol $a \in \Sigma$ and some palindrome $x \in P$. Then

$$
\begin{aligned}
w^R &= (a \cdot x \bullet a)^R \\
&= (x \bullet a)^R \bullet a && \text{by definition of reversal} \\
&= a^R \bullet x^R \bullet a && \text{You said we could assume this.} \\
&= a \bullet x^R \bullet a && \text{by definition of reversal} \\
&= a \bullet x \bullet a && \text{by the inductive hypothesis} \\
&= w && \text{by assumption}
\end{aligned}
$$

In all three cases, we conclude that $w = w^R$.

(c) Let w be an arbitrary string such that $w = w^R$.

Assume that every string x such that $|x| < |w|$ and $x = x^R$ is a palindrome.

There are three cases to consider (mirroring the definition of "palindrome"):

- If $w = \varepsilon$, then w is a palindrome by definition.
- If $w = a$ for some symbol $a \in \Sigma$, then w is a palindrome by definition.

- Otherwise, we have $w = ax$ for some symbol a and some *non-empty* string x.
  The definition of reversal implies that $w^R = (ax)^R = x^R a$.
  Because x is non-empty, its reversal x^R is also non-empty.
  Thus, $x^R = by$ for some symbol b and some string y.
  It follows that $w^R = bya$, and therefore $w = (w^R)^R = (bya)^R = ay^R b$.

  [At this point, we need to prove that $a = b$ and that y is a palindrome.]

  Our assumption that $w = w^R$ implies that $bya = ay^R b$.
  The recursive definition of string equality immediately implies $a = b$.

  Because $a = b$, we have $w = ay^R a$ and $w^R = aya$.
  The recursive definition of string equality implies $y^R a = ya$.
  It immediately follows that $(y^R a)^R = (ya)^R$.
  Known properties of reversal imply $(y^R a)^R = a(y^R)^R = ay$ and $(ya)^R = ay^R$.
  It follows that $ay^R = ay$, and therefore $y = y^R$.
  The inductive hypothesis now implies that y is a palindrome.

  We conclude that w is a palindrome by definition.

In all three cases, we conclude that w is a palindrome.

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