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```

1 Binary Search and Ternary Search

1.1 LowerBound

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace gnu pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 1000001
#define mod 1000000007
// first element >= x
vector<int> k (MAXN);
int lower(int 1, int r, int x) // first element >= x
  while (1 < r)
    int mid = (1 + r) >> 1;
    (x \le k[mid]) ? r = mid : 1 = mid + 1;
  return k[1];
```

1.2 Aplications

```
#include <bits/stdc++.h>
using namespace std;
#define lli long long int
```

```
#define pb push_back
#define in insert
#define pi pair<int, int>
#define pii pair<int, pi>
#define mp make pair
#define fir first
#define sec second
#define MAXN 1001
// 1 - ts para double
long double ts()
  long double 1 = 0, r = DBL MAX;
  for (int i = 0; i < 2000; i++)
    long double 11 = (1 * 2 + r) / 3.0;
    long double 12 = (1 + 2 * r) / 3.0;
    if (possible(l1))
      r = 12;
    else
      1 = 11:
  return 1;
// 2- bb para double
long double bb()
  long double i = 0, f = DBL MAX, m;
  while (f - i > 0.000000001)
    m = (i + f) / 2.0;
    if (possible(m))
      f = m;
    else
      i = m;
  return i;
// 3 - bb pra int
lli bb()
  lli i = 0, f = INT_MAX, m;
  while (i < f)
    \mathbf{m} = (\mathbf{i} + \mathbf{f}) / 2;
    if (possible(m))
      f = m:
    else
      i = m + 1;
```

```
return i;
// 4 - ts pra int (valor minimo da funcao f(x)), sendo x
    um inteiro
int l = 1, r = INT MAX;
while (r - 1 > 15)
  int 11 = (1 * 2 + r) / 3;
  int 12 = (1 + 2 * r) / 3;
  (calc(11) < calc(12)) ? r = 12 : 1 = 11;
for (int i = 1; i \le r; i++)
// vejo qual a melhor opcao de l ate r em o(n)
// busca ternaria para int, usando busca binaria:
int 1 = 0, r = 1e9;
while (1 < r)
  int mid = (1 + r) >> 1;
  (calc(mid) < calc(mid + 1))? r = mid : l = mid + 1;
return calc(1);
```

1.3 TS

```
// busca ternaria
// divide em 3 partes, 2 mids
// mid1 = 1 + (r-1)/3
// mid2 = r - (r-1)/3
#include <bits/stdc++.h>
using namespace std;
#define lli long long int
#define pb push_back
#define in insert
#define pi pair<int, int>
#define pii pair<int, pi>
#define mp make pair
#define fir first
#define sec second
#define MAXL 100001
int n, key;
vector<int> ar;
int ts()
  int 1 = 0, r = n - 1;
  while (r >= 1)
```

```
int mid1 = 1 + (r - 1) / 3;
    int mid2 = r - (r - 1) / 3;
    if (ar[mid1] == key)
      return mid1;
    if (ar[mid2] == key)
      return mid2;
    if (key < ar[mid1])</pre>
      r = mid1 - 1;
    else if (key > ar[mid2])
     1 = mid2 + 1;
    else
      1 = mid1 + 1;
      r = mid2 - 1;
  return -1; // nao encontrado
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  cin >> n:
  ar.resize(n);
  for (int i = 0; i < n; i++)
    cin >> ar[i];
  sort(ar.begin(), ar.end());
  cin >> key;
  cout << ts() << endl;</pre>
  return 0:
```

1.4 UpperBound

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;

#define int long long int
#define pb push_back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
```

```
#define sec second
#define DEBUG 0
#define MAXN 1000001
#define mod 1000000007
// last element <= x
vector<int> k (MAXN);
int upper(int 1, int r, int x)
{
   while (1 < r)
   {
      int mid = (1 + r + 1) >> 1;
      (k[mid] <= x) ? 1 = mid : r = mid - 1;
   }
   return k[1];
}</pre>
```

1.5 STL

```
// lower - primeiro maior ou igual a x
// upper - ultimo menor ou iqual a x
#include <bits/stdc++.h>
using namespace std;
#define lli long long int
#define pb push_back
vector <int> v ;
int main()
    int n , aux ;
    cin >> n;
    for (int i = 0; i < n; i++)
       cin >> aux ;
       v.pb(aux);
    }
    sort(v.begin() , v.end());
    int q;
    cin >> q;
    while (q--)
        cin >> aux ;
        vector <int> :: iterator low = lower_bound (v.
           begin() , v.end() , aux) ;
```

1.6 BS

```
#include <bits/stdc++.h>
using namespace std;
#define lli long long int
#define pb push_back
vector <int> v;
int binarysearch (int n , int x)
    int i = 0;
    int f = n - 1;
    int m ;
    while(i <= f)</pre>
        \mathbf{m} = (\mathbf{i} + \mathbf{f}) / 2 ;
        if(v[m] == x) return m + 1;
        if(v[m] < x) i = m + 1;
        if (v[m] > x)   f = m - 1;
    return 0 ;
int main ()
    int n , aux , m ;
    cin >> n ;
    for (int i = 0; i < n; i++)
        cin >> aux ;
        v.pb(aux);
   sort(v.begin() , v.end());
```

```
cin >> m;
cout << binarysearch(n , m) << endl;
return 0;
}</pre>
```

2 Miscellaneous

2.1 meetinthemiddle

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace gnu pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 1000001
int n, t;
vector<int> v;
vector<int> a;
vector<int> b;
void solve2(int i, int j, int k)
  if (i == i)
   b.pb(k);
   return;
  solve2(i + 1, j, k);
  solve2(i + 1, j, k + v[i]);
void solve(int i, int j, int k)
  if (i == j)
    a.pb(k);
```

```
return;
  solve(i + 1, j, k);
  solve(i + 1, j, k + v[i]);
int upper(int 1, int r, int x)
 while (1 < r)
    int mid = (1 + r + 1) >> 1;
    (b[mid] \le x) ? 1 = mid : r = mid - 1;
  return b[1];
int meetinthemiddle()
  solve(0, (n >> 1) + 1, 0);
  solve2((n >> 1) + 1, n, 0);
  sort(b.begin(), b.end());
  int ans = 0;
  for (auto const &i : a)
    if (i > t)
      continue;
    ans = max(ans, i);
    int kappa = i + upper(0, b.size() - 1, t - i);
    if (kappa <= t)</pre>
      ans = max(ans, kappa);
  return ans;
signed main()
 ios_base::sync_with_stdio(false);
 cin.tie(NULL);
 cin >> n >> t;
 v.resize(n);
 for (int i = 0; i < n; i++)
    cin >> v[i];
  cout << meetinthemiddle() << endl;</pre>
  return 0;
```

2.2 bitmasks

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
```

```
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 200005
#define mod 998244353
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int n, mask;
  vector<int> masks;
  // quantidade de bits setados na mask
  cout << builtin popcount(mask) << endl;</pre>
  // para printar o valor do bit i
  for (int i = 0; i < n; i++)
    cout << ((mask >> i) & 1) << " ";
  cout << endl:
  // quando eh necessario percorrer todas as submasks
     ate (1 << n)
  // e fazer algo com todas as submasks dessa mask
  // util em problemas de dp com mask por exemplo
  for (int i = 0; i < n; i++)
    for (int j = 0; j < (1 << n); j++)
      if ((\dot{j} >> \dot{i} \& 1) == 0)
        //alguma coisa aqui sabendo que a mask(j) eh uma
            submask de(i ^1 << i)
  // para percorrer por todas as submasks de uma mask
  for (int s = mask; s; s = (s - 1) \& mask)
    // alguma coisa agui sabendo que s eh uma submask de
        mask
```

```
// quando eh necessario percorrer todas as submasks
   ate (1 \ll n)
// e fazer algo com todas as submasks dessa mask O(3^n
// util em problemas de dp com mask por exemplo
for (int m = 0; m < (1 << n); m++)
  for (int s = m; s; s = (s - 1) \& m)
    // alguma coisa aqui sabendo que mask s eh uma
        submask de m
// comprimindo as masks de um vector baseada em uma
   mask qualquer
for (int i = 0; i < masks.size(); i++)</pre>
  int compressed = 0, curr_bit = 0;
  for (int j = 0; j < n; j++)
    if (!(mask & (1LL << j)))</pre>
      continue;
    if (masks[i] & (1LL << j))</pre>
      compressed |= (1LL << curr_bit);</pre>
    curr bit++;
  // alguma coisa sabendo que a mask compressed eh a
     mask comprimida da mask atual
return 0;
```

2.3 coordinate compression

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;

#define int long long int
#define pb push_back
#define pi pair<int, int>
```

```
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 500005
#define mod 1000000007

void compress(vector<int> &v)

{
  vector<int> val;
  for (auto const &i : v)
    val.pb(i);
  sort(val.begin(), val.end());
  val.erase(unique(val.begin(), val.end()), val.end());
  for (auto &i : v)
    i = lower_bound(val.begin(), val.end(), i) - val.
        begin();
}
```

2.4 two pointers

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace __qnu_pbds;
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb tree tag, tree order statistics node update>;
#define PI acos(-1)
#define pb push_back
#define int long long int
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 100001
#define mod 1000000007
signed main()
 ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int n, m;
  cin >> n >> m;
  vector<int> v(n);
  vector<int> vv(m);
  for (int i = 0; i < n; i++)
    cin >> v[i];
```

```
for (int i = 0; i < m; i++)
    cin >> vv[i];
  int ans = 0, prev = LLONG MAX, curr = 0;
  for (int 1 = 0, r = 0; 1 < m; 1++)
    if (vv[1] != prev)
      curr = 0;
    while (r < n \&\& v[r] <= vv[1])
     if (v[r] == vv[l])
        curr++;
      r++;
    ans += curr;
    prev = vv[1];
  cout << ans << endl;
//You are given two arrays a and b, sorted in non-
   decreasing order. Find the number of pairs (i, j) for
    which ai=bi.
```

2.5 inversion count

```
// seja S = a1, a2 , ... , an
// uma inversao S e um par (i,j) com i < j e ai > aj
// Solucao O(n2) nao ideal:
//for(int i=0;i<n;i++)
//
      for(int j=i+1; j<n; j++)
               if(v[i]>v[i]) ans++;
// Em vez de trabalharmos com o vetor inteiro(n2), vamos
    dividir o vetor ao meio e trabalhar com suas
   metades.
// que chamaremos de u1 e u2.
// Queremos saber o valor de inv, o numero de inversoes
   em v. Ha tres tipos de inversoes (i, j) (i, j) em v:
// aquelas em que i e j estao ambos em u1, aquelas em
   que i e j estao ambos em u2 e aquelas
// em que i esta em u1 e j esta em u2.
#include <bits/stdc++.h>
using namespace std;
#define lli long long int
#define pb push back
#define in insert
#define pi pair<int, int>
#define pii pair<int, pi>
```

```
#define mp make_pair
#define fir first
#define sec second
#define MAXN 100001
#define INF 1000000000
int merge_sort(vector<int> &v)
  int ans = 0;
  if (v.size() == 1)
    return 0;
 vector<int> u1, u2;
  for (int i = 0; i < v.size() / 2; i++)
    u1.pb(v[i]);
  for (int i = v.size() / 2; i < v.size(); i++)</pre>
    u2.pb(v[i]);
  ans += merge_sort(u1);
  ans += merge_sort(u2);
  u1.pb(INF);
  u2.pb(INF);
  int ini1 = 0, ini2 = 0;
  for (int i = 0; i < v.size(); i++)</pre>
    if (u1[ini1] <= u2[ini2])
     v[i] = u1[ini1];
     ini1++;
    else
     v[i] = u2[ini2];
     ini2++;
      ans += u1.size() - ini1 - 1;
  return ans;
```

```
}
signed main()
{
   int n;
   cin >> n;
   vector<int> v(n);
   for (int i = 0; i < n; i++)
      cin >> v[i];
   cout << merge_sort(v) << endl;
   return 0;
}</pre>
```

2.6 sprague grundy

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __qnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define endl '\n'
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 500009
#define mod 1000000001
vector<int> v = \{2, 3, 4, 5, 6\};
unordered map<int, bool> vis;
unordered map<int, int> dp;
int q(int x) // achar o grundy number na marra
    if (x == 0)
        return 0;
    vector<br/>bool> ok(4, 0);
    int mex = 0;
    for (auto const &i : v)
        int curr = q(x / i);
        if (curr < 4)
            ok[curr] = 1;
        while (ok[mex])
```

```
mex++;
    vis[x] = 1;
    return dp[x] = mex;
int solve(int x) // padraozin
    vector<int> ini = {0, 1, 2, 2, 3, 3, 0, 0, 0, 0, 0,
       0 } ;
    while (x >= 12)
       x /= 12;
    return ini[x];
signed main()
    ios_base::sync_with_stdio(false);
    cin.tie(NULL);
    int q;
    cin >> q;
    while (q--)
        int n;
        cin >> n;
        int x = 0;
        for (int i = 0; i < n; i++)
            int k;
           cin >> k;
           x = solve(k);
        (x > 0) ? cout << "Henry\n" : cout << "Derek\n";
    return 0;
}
/*
game theory (um exemplo simples de problema pra ficar no
    repo)
- pro nim classico
- existem n pilhas cada uma possui x[i] blocos
- em uma play posso escolher uma pilha e tirar uma
   quantidade qualquer de blocos dela
- quem ganha?
- o jogador que comeca ganha se o xor dos tamanhos das
   pilhas for != 0
- teorema sprague-grundy (transformar um jogo qualquer
   em nim)
- seja v um estado que eu tou do jogo, podemos calcular
   o grundy number desse estado
```

```
- seja o conjuntos de estados adjacentes a v {u1, u2,
   ..., un}
- g(v) = mex(g(u1), g(u2), ..., g(un))
- se v nao tem nenhum extado adjacente, entao g(v) = 0
-g(v) -> grundy number do estado v
- com isso se tivemos varios estados iniciais (varias
   pilhas)
- podemos simplesmente achar o grundy number de cada um
   deles e depois saber quem ganha
- pelo valor do xor dos grundy numbers
- exemplo: floor division game
- existem n numeros e em uma play posso escolher um
   deles e dividir por 2, 3, 4, 5 ou 6
- quem ganha?
- achar o grundy number de cada um dos n numeros
- se o xor for != 0, ganha quem comeca jogando
- caso contrario, o outro jogador ganha
- as vzs e util tbm ver se existe um padrao (em caso de
   altas constantes)
- notando o padrao, da pra achar o grundy number de
   forma mais eficiente e resolver o problema
```

2.7 stack trick

```
#include <bits/stdc++.h>
using namespace std;
#define PI acos(-1)
#define int long long int
#define pb push_back
#define pi pair<int, int>
#define fir first
#define sec second
#define MAXN 300001
#define mod 1000000007
int n;
vector<int> v;
vector<int> ans;
void solve()
  stack<pi> s;
  for (int i = n - 1; i >= 0; i--)
    while (!s.empty() && s.top().fir <= v[i])</pre>
      s.pop();
```

```
(!s.empty()) ? ans[i] = s.top().sec : ans[i] = -1;
    s.push(\{v[i], i\});
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  cin >> n;
 v.resize(n);
  ans.resize(n);
  for (int i = 0; i < n; i++)
   cin >> v[i];
  solve();
  for (auto const &i : ans)
    cout << i << " ";
  cout << endl;</pre>
// WITHOUT SEGMENT TREE
// for each index (0 <= i < n), find another index (0 <=
// which v[j] > v[i] and j > i and j is as close as
   possible to i.
// if this index does not exist, print -1
/*
5
1 3 3 4 5
*/
/*
1 \ 3 \ 3 \ 4 \ -1
*/
```

3 STL

3.1 ordered set

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

#define int long long int
#define pb push_back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec_second
```

```
#define DEBUG false
#define MAXN 200002
template <class T> // template do ordered set
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  ordered_set<int> s; // ordered_set
  s.insert(1):
  s.insert(1);
  s.insert(2);
  s.insert(4);
  s.insert(3);
  for (auto const &i : s) // nao adiciona elementos
     repetidos, que nem o set normal
    cout << i << " ";
  cout << endl;</pre>
  cout << *(s.find_by_order(0)) << endl; // iterator do</pre>
     elemento 0
  cout << *(s.find by order(1)) << endl; // iterator do</pre>
     elemento 1
  cout << s.order_of_key(4) << endl;</pre>
                                          // quantidade
     de elementos que sao menores do que 4
  cout << s.order_of_key(6) << endl;</pre>
                                       // quantidade
     de elementos que sao menores do que 4
// find by order : O(log n), retorna (um iterator) qual
   o k-esimo elemento do set
// order_of_key: O(log n), retorna qual a quantidade de
   elementos menores do que x no set
```

3.2 STL

```
1) Vector
vector <int> v; //Criacao o vector
v.push_back(10); //Adiciono o elemento 10 no final do
    vector v
v.size() // retorna o tamanho do vector
v.resize(10); //Muda o tamanho do vector v para 10.
v.pop_back(); //Apaga o ultimo elemento do vector V.
v.clear(); // apaga todos os elementos do vector v .
sort(v.begin(), v.end()); //Ordena todo o vector v
```

```
2)Pair
pair <string, int> p; // criando a pair relacionando um
   first com um second
p.first = "Joao"; // adicionando elementos
p.second = 8 ; //adicionando elementos
// utilidade: vector de pair
vector< pair <int, string> > v; // criando o vector v de
v.push_back(make_pair(a,b)); // dando push back em uma
   pair no vector usando make pair
sort(v.begin(), v.end()); // tambem e possivel ordenar o
    vector de pair
3) Oueue / FIla
queue <int> f; // criando a queque
f.push(10); // adiciona alguem na fila
f.pop(); // remove o elemento que esta na frente da fila
f.front(); // olha qual o elemento esta na frete da fila
f.emptv() // retorna true se a fila estiver vazia e
   false se nao estiver vazia
4) Stack / Pilha
stack <int> p ; // criando a stack
pilha.push(x); //Adiciona o elemento x no topo da pilha
pilha.pop(); //Remove elemento do topo da pilha
pilha.top(); // retorna o elemento do topo da pilha
pilha.empty(); // verifica se a pilha esta vazia ou nao
5) Set
set <int> s ; // criando a set
// obs: a set nao adiciona elementos repetidos
s.insert(10); //Adiciona o elemento 10 no set
s.find(10) // Para realizar uma busca no set utilizamos
   o comando find.
o find retorna um ponteiro que aponta para o elemento
   procurado caso o elemento esteja no set ou para o
   final do set, caso o elemento procurado nao esteja
   no set , em complexidade O(log n)
```

if(s.find(10) != s.end()) // procurando pelo 10, se ele

```
s.erase(10); //Apaga o elemento 10 do set em O(log n)
s.clear(); // Apaga todos os elementos
s.size(); // Retorna a quantidade de elementos
s.begin(); // Retorna um ponteiro para o inicio do set
s.end(); // Retorna um ponteiro para o final do set
6) Map
map <string, int> m; //Cria uma variavel do tipo map que
    mapeia strings em int
// Em um map cada elemento esta diretamente ligado a um
   valor, ou seja, cada elemento armazenado no map
   possui um valor correspondente
// Se tivermos um map de strings em inteiros e inserimos
    os pair ("Joao", 1), ("Alana", 10), ("Rodrigo", 9)
// Caso facamos uma busca pela chave "Alana" receberemos
    o numero 10 como retorno.
m.insert(make_pair("Alana", 10)); //Inserimos uma
   variavel do tipo pair diretamente no map, O(log n)
M["Alana"] = 10; //Relacionando o valor 10 a chave "
   Alana"
if(m.find("Alana") != m.end()){ //Se a chave "Alana" foi
    inserida no map
cout << m["Alana"] << endl; //Imprime o valor</pre>
   correspondente a chave "Alana", no caso, o valor 10.
m.erase("Alana"); //Apaga o elemento que possui a chave
   "Alana" do map
m.clear(); // Apaga todos os elementos
m.size(); // Retorna a quantidade de elementos
m.begin(); // Retorna um ponteiro para o inicio do map
m.end(); // Retorna um ponteiro para o final do map
7) Priority Queue
priority_queue <int> q; // declarando a priority queue
// Para utilizar a priority_queue do C++ e importante
   apenas saber que o maior elemento sempre estara na
   primeiro posicao.
// Com execao disso, todos os outros metodos sao
   semelhantes ao uso de uma queue comum, porem para
   manter a estrutura organizada, a complexidade da
   operacao de insercao e O(logn).
p.push(i) // adiciono o elemento i na priority_queue
p.pop(); // apago o primeiro da fila
p.top(); // vejo quem esta no topo
```

estiver no set

4 Utils

4.1 runner2

```
# This script does the following:
# 1 - Run a code with all inputs files from a folder
# 2 - Compare the output for each test case with the
   answer
import os
code = "a.cpp" # Path to your code
input_folder = "input" # Path to folder which the input
   files are
output_folder = "output" # Path to folder which the
   output files are
input_prefix = "L_" # prefix of all input files names
output_prefix = "L_" # prefix of all input files names
tests = 56 # Number of test cases
def compile code():
    os.system('q++' + code + ' -o code -02')
def get_ans(output):
    out = open(output, "r")
    ret = out.read()
    out.close()
    return ret
def get code output(input):
    output = os.popen('./code <' + input).read()</pre>
    return output
def main():
    compile code()
    # tests indexed from 1
    for i in range (1, tests + 1):
        ans = get ans(output folder + '/' +
           output_prefix + str(i))
        code_output = get_code_output(input_folder + '/'
            + input_prefix + str(i))
        print('Case' + str(i) + ': ')
        if ans == code_output:
            print('ACCEPTED')
        else :
            print('FAILED\n')
            print('ANSWER:')
            print(ans)
            print('\nCODE OUTPUT:')
```

```
print(code_output)
print()

if __name__ == '__main__':
    main()
```

$4.2 \quad \text{int} 128$

```
// https://codeforces.com/blog/entry/75044
// functions to print and read a __int128 in c++
__int128 read()
  _{\text{int}128} x = 0, f = 1;
  char ch = getchar();
  while (ch < '0' || ch > '9')
    if (ch == '-')
     f = -1;
    ch = getchar();
  while (ch >= '0' && ch <= '9')
    x = x * 10 + ch - '0';
    ch = getchar();
  return x * f;
void print( int128 x)
  if (x < 0)
    cout << "-";
    x = -x;
  if (x > 9)
    print(x / 10);
  char at = (x % 10) + '0';
  cout << at;
```

4.3 execution time

```
// https://codeforces.com/blog/entry/57647
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
```

```
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 300005
#define mod 1000000007
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
 // just call clock in the beginning
  clock_t time = clock();
  // ...
  // ...
  // some code here ...
  // ...
  // ...
  // execution time:
  cout << setprecision(3) << fixed << (double)(clock() -</pre>
      time) / CLOCKS_PER_SEC << endl;</pre>
  return 0:
```

4.4 runner

```
# This script does the following:
# 1 - Generate a random testcase
# 2 - Run some "naive" code with this input
# 3 - Run your code with this input
# 4 - Compare the outputs
import os

naive = "brute.cpp" # path to naive code
code = "d.cpp" # path to your code
generator = "g.cpp" # path to test generator

def compile_codes():
    os.system('g++' + generator + ' -o generator -O2')
    os.system('g++' + naive + ' -o naive -O2')
    os.system('g++' + code + ' -o code -O2')
```

```
def generate_case():
    os.system('./generator > in');
def get naive output():
    output = os.popen('./naive <in').read()</pre>
    return output
def get_code_output():
    output = os.popen('./code <in').read()</pre>
    return output
def main():
    compile_codes()
    while True:
        generate case()
        naive output = get naive output()
        code_output = get_code_output()
        if naive_output == code_output:
            print('ACCEPTED')
        else :
            print('FAILED\n')
            print('ANSWER:')
            print(naive output)
            print('\nCODE OUTPUT:')
            print (code_output)
            break
if name == ' main ':
    main()
```

5 Math

5.1 totient

```
#define MAXN 100000
int phi[MAXN];

void calc()
{
  for (int i = 0; i < MAXN; i++)
    phi[i] = i;
  for (int i = 2; i < MAXN; i++)
  {
    if (phi[i] == i)
    }
}</pre>
```

5.2 iterative fft

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace gnu pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define PI acos (-1)
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first.
#define sec second
#define MAXN 125001
#define mod 1000000007
#define cd complex<double>
namespace fft
  int n;
  void fft(vector<cd> &a, bool invert)
```

```
int n = a.size();
  for (int i = 1, j = 0; i < n; i++)
    int bit = n >> 1;
    for (; j & bit; bit >>= 1)
     i ^= bit;
    j ^= bit;
    if (i < j)
      swap(a[i], a[i]);
  for (int len = 2; len <= n; len <<= 1)</pre>
    double ang = 2 * PI / len * (invert ? -1 : 1);
    cd wlen(cos(ang), sin(ang));
    for (int i = 0; i < n; i += len)
      cd w(1);
      for (int j = 0; j < len / 2; j++)
        cd u = a[i + j], v = a[i + j + len / 2] * w;
        a[i + j] = u + v;
        a[i + j + len / 2] = u - v;
        w \star = wlen;
 if (invert)
    for (cd &x : a)
      x /= n;
vector<double> mul(vector<double> a, vector<double> b)
 vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.
     end());
 n = 1:
  while (n < a.size() + b.size())
    n <<= 1;
  fa.resize(n);
  fb.resize(n);
  fft(fa, false);
 fft(fb, false);
 for (int i = 0; i < n; i++)
   fa[i] \star = fb[i];
  fft(fa, true);
 vector<double> ans(n);
  for (int i = 0; i < n; i++)
    ans[i] = fa[i].real();
 return ans;
```

```
} // namespace fft
signed main()
{
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
}
```

5.3 fraction

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace ___gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define endl '\n'
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 200006
#define mod 1000000007
struct fraction
  int x, y; // x / y
  fraction() {}
  fraction(int x, int y) : x(x), y(y) {}
  bool operator==(fraction o) { return (x * o.y == o.x *
  bool operator!=(fraction o) { return (x * o.y != o.x *
      v); }
  bool operator>(fraction o) { return (x * o.y > o.x * y
  bool operator>=(fraction o) { return (x * o.y >= o.x *
      y); }
  bool operator<(fraction o) { return (x * o.y < o.x * y</pre>
  bool operator<=(fraction o) { return (x * o.y <= o.x *</pre>
  fraction operator+(fraction o)
    fraction ans:
```

```
ans.y = (y == o.y) ? y : y * o.y;
    ans.x = (x) * (ans.y / y) + (o.x) * (ans.y / o.y);
    // ans.simplify();
   return ans;
  fraction operator*(fraction o)
   fraction ans;
   ans.x = x * o.x;
   ans.y = y * o.y;
   // ans.simplify();
   return ans;
  fraction inv()
    fraction ans = fraction(x, y);
    swap(ans.x, ans.y);
    return ans;
  fraction neg()
    fraction ans = fraction(x, y);
    ans.x *=-1:
   return ans;
 void simplify()
   if (abs(x) > 1e9 \mid | abs(y) > 1e9) // slow
       simplification
      int g = \underline{gcd}(y, x);
     x /= q;
     y /= q;
  // substraction and division can be easily done
};
signed main()
 ios_base::sync_with_stdio(false);
 cin.tie(NULL);
 return 0;
```

5.4 gaussian elimination2

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
```

```
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define endl '\n'
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 230
#define mod 100000001
#define EPS 1e-9
bitset<MAXN> ans;
int gauss(vector<bitset<MAXN>> &a)
  ans.reset();
  int n = a.size(), m = a[0].size() - 1, ret = 1;
  vector<int> where (m, -1);
  for (int col = 0, row = 0; col < m && row < n; col++)</pre>
    for (int i = row; i < n; i++)</pre>
      if (a[i][col])
        swap(a[i], a[row]);
        break;
    if (!a[row][col])
      continue;
    where [col] = row;
    for (int i = 0; i < n; i++)
      if (i != row && a[i][col])
        a[i] ^= a[row];
    ++row;
  for (int i = 0; i < m; i++)
    if (where[i] != -1)
      ans[i] = (a[where[i]][m] / a[where[i]][i]);
    else
      ret = 2;
  for (int i = 0; i < n; i++)
```

```
{
    double sum = 0;
    for (int j = 0; j < m; j++)
        sum += (ans[j] * a[i][j]);
    if (abs(sum - a[i][m]) > EPS)
        ret = 0;
    }
    return ret;
}
signed main()
{
}
```

5.5 modular arithmetic

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 500001
#define mod 1000000007
struct modint
  int val;
 modint(int v = 0) { val = v % mod; }
  int pow(int y)
    modint x = val;
    modint z = 1;
    while (y)
      if (y & 1)
       z \star = x;
      x \star = x;
      y >>= 1;
    return z.val;
```

```
int inv() { return pow(mod - 2); }
  void operator=(int o) { val = o % mod; }
  void operator=(modint o) { val = o.val % mod; }
  void operator+=(modint o) { *this = *this + o; }
  void operator-=(modint o) { *this = *this - o; }
  void operator*=(modint o) { *this = *this * o; }
  void operator/=(modint o) { *this = *this / o; }
  bool operator==(modint o) { return val == o.val; }
  bool operator!=(modint o) { return val != o.val; }
  int operator*(modint o) { return ((val * o.val) % mod)
  int operator/(modint o) { return (val * o.inv()) % mod
  int operator+(modint o) { return (val + o.val) % mod;
  int operator-(modint o) { return (val - o.val + mod) %
      mod; }
};
modint f[MAXN];
void fat()
  f[0] = 1;
  for (int i = 1; i < MAXN; i++)</pre>
    f[i] = f[i - 1] * i;
modint ncr(int n, int k)
  modint d = f[k] * f[n - k];
  modint ans = f[n] / d;
  return ans;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  return 0;
```

5.6 crivo

```
#include <bits/stdc++.h>
using namespace std;

#define lli long long int
#define pb push_back
#define in insert
#define pi pair<int, int>
```

```
#define pd pair<double, int>
#define pii pair<int, pi>
#define mp make pair
#define fir first
#define sec second
#define MAXN 100001
#define mod 1000000007
bitset <MAXN> prime;
void crivo ()
  prime.set();
  prime[0] = false;
  prime[1] = false;
  for (int i = 2; i < MAXN; i++)
    if(prime[i])
      for (int j = 2; j * i < MAXN; j++)
        prime[j * i] = false;
signed main()
  crivo();
  int q;
  cin >> q;
  while (q--)
    int n;
    cin >> n;
    (prime[n]) ? cout << "YES\n" : cout << "NO\n" ;</pre>
  return 0;
```

5.7 divisors

```
#include <bits/stdc++.h>
using namespace std;

#define PI acos(-1)
#define int long long int
#define pb push_back
#define pi pair<int, int>
#define fir first
#define sec second
#define MAXN 5001
#define mod 1000000007
signed main()
```

```
ios_base::sync_with_stdio(false);
cin.tie(NULL);
int n;
cin >> n;
int ans = 0;
for (int i = 1; i <= sqrt(n); i++)
{
   if (!(n % i))
   {
      ans++;
      if (n / i != i)
           ans++;
   }
}
cout << ans << endl;</pre>
```

5.8 gaussian elimination

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define double long double
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first.
#define sec second
#define DEBUG 1
#define MAXN 2001
#define mod 1000000007
#define EPS 1e-9
vector<double> ans;
int gauss(vector<vector<double>> a)
  int n = a.size(), m = a[0].size() - 1, ret = 1;
  ans.assign(m, 0);
  vector<int> where(m, -1);
  for (int col = 0, row = 0; col < m && row < n; col++,</pre>
     row++)
```

```
int sel = row;
    for (int i = row; i < n; i++)</pre>
      if (abs(a[i][col]) > abs(a[sel][col]))
        sel = i:
    if (abs(a[sel][col]) < EPS)</pre>
      continue;
    for (int i = col; i <= m; i++)</pre>
      swap(a[sel][i], a[row][i]);
    where [col] = row;
    for (int i = 0; i < n; i++)
      if (i != row)
        double c = a[i][col] / a[row][col];
        for (int j = col; j <= m; j++)</pre>
          a[i][j] = a[row][j] * c;
  for (int i = 0; i < m; i++)
    if (where[i] != -1)
      ans[i] = (a[where[i]][m] / a[where[i]][i]);
    else
      ret = 2;
  for (int i = 0; i < n; i++)
    double sum = 0;
    for (int j = 0; j < m; j++)
      sum += (ans[j] * a[i][j]);
    if (abs(sum - a[i][m]) > EPS)
      ret = 0;
  return ret; // 0 = nao existe solucao, 1 = existe uma
     solucao, 2 = existem multiplas solucoes
signed main()
 ios base::sync with stdio(false);
  cin.tie(NULL);
 vector < vector < double >> a = \{\{1.0, 1.0, 20.0\}, // 1x + \}
      1y = 20
                                \{3.0, 4.0, 72.0\}\}; // 3x +
                                     4v = 72
  cout << gauss(a) << endl;</pre>
  for (auto const &i : ans) // x = 8 e y = 12
    cout << i << " ";
  cout << endl;</pre>
```

```
// eliminacao gaussiana
// para resolver sistemas com n equacoes e m incognitas
// para isso iremos utilizar uma representacao usando
// matrizes, no qual uma coluna extra e adicionada,
// representando os resultados de cada equacao.
// algoritimo:
// ideia: qualquer equacao pode ser reescrita como uma
   combinacao linear dela mesma
// 1- dividir a primeira linha(primeira equacao) por a
   [0][0]
// 2- adicionar a primeira linha as linhas restantes, de
    modo que, os
// coeficientes da primeira coluna se tornem todos
   zeros, para que
// isso aconteca, na i-esima linha devemos adicionar
   a primeira linha
// multiplicada por (a[i][0] * -1)
// 3- com isso, o elemento a[0][0] = 1 e os demais
   elementos da primeira coluna
// serao iguais a zero
// 4- continuamos o algoritimo a partir da etapa 1
   novamente, dessa vez
// com a segunda coluna e a segunda linha, dividindo
   a linha por a[1][1]
// e assim sucessivamente
// 5- ao final, teremos a resposta
// complexidade O(min(n, m) * n * m);
// se n == m, logo a complexidade sera O(n^3)
```

5.9 primefactors2

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;

#define PI acos(-1)
#define pb push_back
#define int long long int
#define pi pair<int, int>
#define pii pair<pi, int>
```

```
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 1000001
#define mod 1000000007
namespace primefactors
 bitset<MAXN> prime;
  vector<int> nxt(MAXN);
  vector<int> factors;
  void crivo()
    prime.set();
    prime[0] = false, prime[1] = false;
    for (int i = 2; i < MAXN; i++)</pre>
      if (prime[i])
        nxt[i] = i;
        for (int j = 2; j * i < MAXN; j++)
          prime[j * i] = false;
          nxt[j * i] = i;
  void fact(int n)
    factors.clear();
    while (n > 1)
      factors.pb(nxt[n]);
      n = n / nxt[n];
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  return 0;
```

5.10 matrix exponentiation2

```
// count some \{f(0) + f(1) + \dots + f(n)\} with just one
   matrix exponentiation
// creates an extra dimension in the matrix and
   initializes that column with 1s
#include <bits/stdc++.h>
using namespace std;
#define PI acos(-1)
#define pb push back
#define mp make pair
#define int long long int
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define MAXN 100001
#define MAXL 20
#define INF 200001
#define mod 1000000007
const int n = 11;
vector<vector<int>> ans(n, vector<int>(n));
vector<vector<int>> multiply(vector<vector<int>> a,
   vector<vector<int>> b)
  vector<vector<int>> res(n, vector<int>(n));
  for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
     res[i][i] = 0;
      for (int k = 0; k < n; k++)
        res[i][j] = (res[i][j] + (((a[i][k] % mod) * (b[
           k][j] % mod)) % mod)) % mod;
  return res;
vector<vector<int>> expo(vector<vector<int>> mat, int m)
  for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
      ans[i][j] = (i == j);
  while (m > 0)
    if (m & 1)
     ans = multiply(ans, mat);
    m = m / 2;
```

```
mat = multiply(mat, mat);
  return ans;
bool is prime(int n)
  for (int i = 2; i < n; i++)
    if (!(n % i))
      return false;
  return true;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int q;
  cin >> q;
  while (q--)
    int k;
    cin >> k;
    int resp = 0;
    vector<vector<int>> mat(n, vector<int>(n, 0));
    for (int i = 1; i \le 9; i++)
      for (int j = 1; j \le 9; j++)
        if (is prime(i + j))
          mat[i][j] = 1;
    for (int i = 0; i \le 10; i++)
      mat[i][10] = 1;
    vector<vector<int>> ans = expo(mat, k - 1);
    for (int i = 0; i < n; i++)
      for (int j = 0; j < n; j++)
        resp = (resp + ans[i][i]) % mod;
    cout << resp - 7 << endl;
  return 0;
```

5.11 segmentedsieve

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;
```

```
#define PI acos(-1)
#define pb push_back
#define int long long int
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 1000003
#define mod 1000000007
vector<int> prime;
void segmentedsieve(int 1, int r)
  int lim = sqrt(r);
  vector<bool> mark(lim + 1, false);
  vector<int> primes;
  for (int i = 2; i <= lim; ++i)</pre>
    if (!mark[i])
      primes.pb(i);
      for (int j = i * i; j <= lim; j += i)
        mark[i] = true;
    }
  vector<bool> isprime(r - 1 + 1, true);
  for (int i : primes)
    for (int j = \max(i * i, (l + i - 1) / i * i); j <= r
       ; i += i)
      isprime[j - 1] = false;
  if (1 == 1)
    isprime[0] = false;
  for (int i = 0; i < isprime.size(); i++)</pre>
    if (isprime[i])
      prime.pb(i + 1);
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int 1, r;
  cin >> 1 >> r;
  segmentedsieve(l, r);
  for (auto const &i : prime)
    cout << i << " ";
  return 0;
```

5.12 pollard rho

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int int128
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 1000001
#define mod 998244353
int read() // int128 functions
  int x = 0, f = 1;
  char ch = getchar();
  while (ch < '0' || ch > '9')
    if (ch == '-')
     f = -1;
    ch = getchar();
  while (ch >= '0' && ch <= '9')
   x = x * 10 + ch - '0';
    ch = getchar();
  return x * f;
void print(__int128 x) // __int128 functions
  if (x < 0)
    cout << "-";
   x = -x;
  stack<char> s;
  while (x)
    s.push((x % 10) + '0');
```

```
x = x / 10;
  while (!s.empty())
   cout << s.top();</pre>
    s.pop();
namespace pollard_rho
  int multiplicate(int x, int y, int m)
   return (x * y) % m;
  int modpow(int x, int y, int m)
    int z = 1;
    while (y)
      if (y & 1)
      z = (z * x) % m;
     x = (x * x) % m;
      y >>= 1;
    return z;
  bool is_composite(int n, int a, int d, int s)
    int x = modpow(a, d, n);
    if (x == 1 \text{ or } x == n - 1)
     return false;
    for (int r = 1; r < s; r++)
     x = multiplicate(x, x, n);
      if (x == n - 1LL)
        return false;
    return true;
  };
  int miller_rabin(int n)
    if (n < 2)
      return false;
    int r = 0, d = n - 1LL;
    while ((d \& 1LL) == 0)
     d >>= 1;
     r++;
    for (int a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29,
```

```
31, 37})
      if (n == a)
        return true;
      if (is_composite(n, a, d, r))
        return false;
    return true;
  int f (int x, int m)
    return multiplicate(x, x, m) + 1;
  int rho(int n)
    int x0 = 1, t = 0, prd = 2;
    int x = 0, y = 0, q;
    while (t % 40 || __gcd(prd, n) == 1)
      if (x == y)
       x^{0++};
       x = x0;
        y = f(x, n);
      q = multiplicate(prd, max(x, y) - min(x, y), n);
      if (q != 0)
       prd = q;
     x = f(x, n);
      y = f(y, n);
      y = f(y, n);
      t++;
    return __gcd(prd, n);
  vector<int> fact(int n)
    if (n == 1)
      return {};
    if (miller_rabin(n))
      return {n};
    int x = rho(n);
    auto l = fact(x), r = fact(n / x);
    l.insert(l.end(), r.begin(), r.end());
    return 1;
 }
signed main()
 //ios_base::sync_with_stdio(false);
```

```
#include <bits/stdc++.h>
  //cin.tie(NULL);
                                                              #include <ext/pb_ds/assoc_container.hpp>
  while (1)
                                                              #include <ext/pb ds/tree policy.hpp>
    int n = read();
                                                              using namespace std;
                                                              using namespace __gnu_pbds;
    if (n == 0)
     break;
                                                              template <class T>
    vector<int> factors = pollard_rho::fact(n);
    sort(factors.begin(), factors.end());
                                                              using ordered set = tree<T, null type, less<T>,
    int prev = -1, cnt = 0;
                                                                 rb_tree_tag, tree_order_statistics_node_update>;
    for (auto const &i : factors)
                                                              #define int long long int
      if (prev != i)
                                                              #define endl '\n'
                                                              #define pb push_back
                                                              #define pi pair<int, int>
        if (prev != -1)
                                                              #define pii pair<int, pi>
          print (prev);
                                                              #define fir first
          printf("^");
                                                              #define sec second
          print(cnt);
                                                              #define MAXN 500001
          printf(" ");
                                                              #define mod 1000000007
        prev = i;
                                                              struct node
        cnt = 0;
                                                                int me, cnt, id;
                                                                int down[2];
      cnt++;
                                                                node(int c = 0) : me(c)
    if (prev !=-1)
                                                                  cnt = 0;
     print (prev);
                                                                  id = -1;
     printf("^");
                                                                  fill(begin(down), end(down), -1);
     print(cnt);
                                                                }
     printf(" ");
                                                              };
                                                              struct trie xor
    printf("\n");
                                                                vector<node> t;
  return 0;
                                                                trie_xor()
// sources:
// https://github.com/PauloMiranda98/Competitive-
                                                                  t.resize(1);
   Programming-Notebook/blob/master/code/math/prime.h
// https://github.com/brunomaletta/Biblioteca/blob/
                                                                void add(int n, int id)
   master/Codigo/Matematica/pollardrho.cpp
// fast integer factorization with pollard-rho
                                                                  int v = 0;
// https://www.spoj.com/problems/FACT0/ - ok
                                                                  for (int i = 30; i >= 0; i--)
// https://www.spoj.com/problems/FACT1/ - ok
// https://www.spoj.com/problems/FACT2/ - sigkill
                                                                    int bit = (n \& (1 << i)) ? 1 : 0;
// since the limit is at most 29 digits(in FACT2), we
                                                                    if (t[v].down[bit] == -1)
   need to use int128
                                                                      t[v].down[bit] = t.size();
                                                                      t.emplace back(bit);
```

```
v = t[v].down[bit];
      t[v].cnt++;
    t[v].id = id;
  void rem(int n, int id)
    int v = 0;
    for (int i = 30; i >= 0; i--)
      int bit = (n \& (1 << i)) ? 1 : 0;
      v = t[v].down[bit];
      t[v].cnt--;
   }
  int qry(int n) // maximum xor with n
    int v = 0;
    for (int i = 30; i >= 0; i--)
      int bit = (n \& (1 << i)) ? 0 : 1;
      int nxt = t[v].down[bit];
      if (nxt != -1 && t[nxt].cnt > 0)
       v = nxt;
      else
       v = t[v].down[bit ^ 1];
    return t[v].id;
};
signed main()
// alguns problemas:
// https://codeforces.com/problemset/problem/706/D
// https://codeforces.com/contest/1625/problem/D
// https://codeforces.com/contest/888/problem/G
```

5.14 mobius

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;
```

```
#define int long long int
#define endl '\n'
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 5000005
#define mod 100000001
int lpf[MAXN];
int mobius[MAXN];
int g[MAXN];
void calc lpf()
  for (int i = 2; i < MAXN; i++)
    if (!lpf[i])
      for (int j = i; j < MAXN; j += i)</pre>
        if (!lpf[j])
          lpf[j] = i;
void calc_mobius()
  calc_lpf();
  mobius[1] = 1;
  for (int i = 2; i < MAXN; i++)</pre>
    if (lpf[i / lpf[i]] == lpf[i])
      mobius[i] = 0;
    else
      mobius[i] = -1 * mobius[i / lpf[i]];
int count pairs(int n)
  // f(n) \rightarrow contar pares (i, j) com __gcd(i, j) == 1 e
     1 <= i, j <= n
  int ans = 0;
  for (int d = 1; d <= n; d++)</pre>
    // quadrado pq sao pares (2 caras)
    // mas se fossem x caras seria (n / d) x
    int sq = (n / d) * (n / d);
```

```
int x = mobius[d] * sq;
    ans += x;
  return ans;
int gcd_sum(int n)
 // soma de todos os gcd(i, j) com 1 <= i, j <= n
  int ans = 0;
  for (int k = 1; k \le n; k++) // fixa o valor do gcd(i),
      j) e conta quantos pares com gcd(i, j) == k
    int \lim = n / k;
    int curr = k * count_pairs(lim);
    ans += curr;
  return ans;
int lcm_sum(int n)
 // soma de todos os lcm(i, j) com 1 <= i, j <= n
  for (int i = 1; i <= n; i++)
    g[i] = 0;
  for (int i = 1; i \le n; i++)
    for (int j = i; j <= n; j += i)</pre>
      q[j] += (mobius[i] * j * i);
  int ans = 0;
  for (int 1 = 1; 1 <= n; 1++)
    int cima = (1 + n / 1) * (n / 1);
    int f = (cima / 2) * (cima / 2);
   f *= q[1];
    ans += f;
  return ans;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int q;
  cin >> q;
  calc_mobius();
  for (int i = 1; i \le q; i++)
    int n;
    cin >> n;
    int ans = lcm_sum(n);
```

```
for (int i = 1; i <= n; i++)
      ans -= i;
    ans /= 2;
    cout << "Case " << i << ": " << ans << endl;</pre>
  return 0;
// https://codeforces.com/blog/entry/53925
// mobius inversion
// sejam f(x) e g(x) funcoes
// e q(x) e definida da seguinte maneira
// q(x) = soma \ dos \ f(d), no qual d eh um divisor de x
// temos que:
// f(n) = soma dos (g(d) * u(n / d)), no qual d eh um
   divisor de x
// u(x) \rightarrow mobius function
// propiedade legal:
// seja l(x) \rightarrow soma de u(d), para cada divisor d de x
// 1(1) = 1
// 1(x) = 0, x > 1
// problemas iniciais:
// https://vjudge.net/problem/AtCoder-abc162_e
// https://vjudge.net/problem/CodeChef-SMPLSUM
```

5.15 fft

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define PI acos(-1)
#define int long long int
#define pb push_back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 100001
#define mod 1000000007
#define cd complex<double> // numeros complexos na STL
```

```
void dft(vector<cd> &a)
  int n = a.size();
  if (n == 1)
    return;
  vector<cd> a0(n / 2), a1(n / 2);
  for (int i = 0; 2 * i < n; i++)
    a0[i] = a[2 * i];
    a1[i] = a[2 * i + 1];
  dft(a0);
  dft(a1);
  double ang = 2 * PI / n;
  cd w(1), wn(cos(ang), sin(ang));
  for (int i = 0; 2 * i < n; i++)
    a[i] = a0[i] + w * a1[i];
    a[i + n / 2] = a0[i] - w * a1[i];
    w \star = wn;
void inverse dft(vector<cd> &a)
  int n = a.size();
  if (n == 1)
    return;
  vector\langle cd \rangle a0 (n / 2), a1 (n / 2);
  for (int i = 0; 2 * i < n; i++)
    a0[i] = a[2 * i];
    a1[i] = a[2 * i + 1];
  inverse_dft(a0);
  inverse dft(a1);
  double ang = 2 * PI / n * -1;
  cd w(1), wn(cos(ang), sin(ang));
  for (int i = 0; 2 * i < n; i++)
    a[i] = a0[i] + w * a1[i];
    a[i + n / 2] = a0[i] - w * a1[i];
    a[i] /= 2;
    a[i + n / 2] /= 2;
    w \star = wn;
  }
vector<int> fft(vector<int> a, vector<int> b)
  vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end
```

```
());
  int n = 1;
  while (n < a.size() + b.size())
    n <<= 1;
  fa.resize(n);
  fb.resize(n);
  dft(fa);
                               // DFT(A)
                               // DFT(B)
  dft(fb);
  for (int i = 0; i < n; i++) // DFT(A * B) = DFT(A) *
     DFT (B)
    fa[i] *= fb[i];
  inverse_dft(fa); // inverseDFT(DFT(A * B))
  vector<int> ans(n);
  for (int i = 0; i < n; i++)
    ans[i] = round(fa[i].real()); // arredondar para ter
         os coeficientes como inteiros
  return ans;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int n, m, caso = 1;
  while (cin >> n >> m)
    cout << "Caso #" << caso << ": ";
    vector<int> a(n + 1);
    vector<int> b (m + 1);
    for (int i = 0; i <= n; i++)</pre>
      cin >> a[i];
    for (int i = 0; i <= m; i++)
      cin >> b[i];
    vector<int> ans = fft(a, b);
    for (int i = 0; i \le n + m; i++)
      cout << ans[i];</pre>
      (i == n + m) ? cout << endl : cout << " ";
    caso++;
  return 0;
// fft
// multiplicar dois polinomios A e B
// basic approach:
// aplicar a propiedade distributiva e fazer essa
   multiplicacao em O(N^2)
// porem podemos melhorar
// vamos la
// 1 - todo polinomio de grau d que e representado na
```

```
forma de coeficientes
// de coeficientes possui uma representacao em
   forma de d - 1 pontos
// 2 - para esse conjunto de pontos, so existe um unico
    polinomio equivalente
// 3 - DFT -> transformacao da representacao de
   coeficientes para represntacao
      de pontos
// 4 - com isso, para multiplicar os dois polinomios
   agora basta multiplicar
// os conjuntos de pontos e com isso obtemos a
   representacao usando pontos
// do polinomio resultante
// 5 - DFT(A * B) = DFT(A) * DFT(B);
// 6 - porem agora precisamos transformar a resposta
   obtida na multiplicacao dos pontos
// para a representação em que usa os coeficientes
// 7 - inverseDFT -> transformacao da representacao de
   pontos para represntacao
                     de coeficientes
// 8 - A * B = inverseDFT(DFT(A) * DFT(B))
// 9 - FFT -> metodo para computar a DFT em O(N * low(N
// 10 - iremos usar divide and conquer para isso, vamos
   splitar o polinomio
// atual em 2 polinomos de grau ((n / 2) - 1) , tal
    que, a soma deles
// resulte no polinomio que tinhamos antes
// 11 - agora para achar a inverseDFT de uma DFT, iremos
    escrever a DFT
       em forma de matriz, essa matriz e chamada de
   matriz de vandermonde
// e em geral, podemos escrever a resposta como uma
    multiplicacao de
// matrizes
// 12 - essa multplicacao de matrizes pode ser descrita
//
     a^{-1} * b = c
//
     no qual:
// a^-1 -> inversa da matriz a(DFT)
// b -> valores dos coeficientes do polinomio A
       c -> valores dos coeficientes da resposta
```

5.16 lagrange

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __qnu_pbds;
```

```
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define endl '\n'
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 600005
#define mod 1000000007
struct modint
  int val;
  modint(int v = 0) { val = v % mod; }
  int pow(int y)
    modint x = val;
    modint z = 1;
    while (y)
     if (y & 1)
      z \star = x;
     x \star = x;
     \vee >>= 1;
    return z.val;
  int inv() { return pow(mod - 2); }
  void operator=(int o) { val = o % mod; }
  void operator=(modint o) { val = o.val % mod; }
  void operator+=(modint o) { *this = *this + o; }
  void operator-=(modint o) { *this = *this - o; }
  void operator*=(modint o) { *this = *this * o; }
  void operator/=(modint o) { *this = *this / o; }
  bool operator==(modint o) { return val == o.val; }
  bool operator!=(modint o) { return val != o.val; }
  int operator*(modint o) { return ((val * o.val) % mod)
  int operator/(modint o) { return (val * o.inv()) % mod
  int operator+(modint o) { return (val + o.val) % mod;
  int operator-(modint o) { return (val - o.val + mod) %
      mod; }
} ;
```

```
struct lagrange
 vector<modint> den;
 vector<modint> v;
 vector<modint> fat;
 vector<modint> inv_fat;
 lagrange (vector < modint > &v) // f(i) = v[i], gera um
     polinomio de grau n - 1
    int n = v.size();
    calc(n);
   den.resize(n);
   y.resize(n);
    for (int i = 0; i < n; i++)
     v[i] = v[i];
     den[i] = inv_fat[n - i - 1] * inv_fat[i];
     if ((n - i - 1) % 2 == 1)
       int x = (mod - den[i].val) % mod;
       den[i] = x;
  void calc(int n)
   fat.resize(n + 1);
   inv fat.resize(n + 1);
   fat[0] = 1;
   inv fat[0] = 1;
    for (int i = 1; i <= n; i++)
     fat[i] = fat[i - 1] * i;
     inv_fat[i] = fat[i].inv();
  modint get_val(int x) // complexity: O(n)
   x \% = mod;
   int n = y.size();
   vector<modint> l(n);
   vector<modint> r(n);
   1[0] = 1, r[n - 1] = 1;
    for (int i = 1; i < n; i++)
     modint cof = (x - (i - 1) + mod);
     l[i] = l[i - 1] * cof;
    for (int i = n - 2; i >= 0; i--)
```

```
modint cof = (x - (i + 1) + mod);
      r[i] = r[i + 1] * cof;
    modint ans = 0;
    for (int i = 0; i < n; i++)
      modint cof = l[i] * r[i];
      ans += modint(cof * y[i]) * den[i];
    return ans;
} ;
signed main()
  ios base::sync with stdio(false);
  cin.tie(NULL);
  int n, k;
  cin >> n >> k;
  vector<modint> v;
  v.pb(0);
  int \lim = k + 1;
  for (int i = 1; i <= lim; i++)</pre>
    v.pb(v.back() + modint(i).pow(k));
  lagrange 1(v);
  cout << l.get val(n).val << endl;</pre>
  return 0;
// https://codeforces.com/contest/622/problem/F
```

5.17 primefactors

```
#include <bits/stdc++.h>
using namespace std;

#define PI acos(-1)
#define int long long int
#define pb push_back
#define mp make_pair
#define pi pair<int, int>
#define fir first
#define sec second
#define MAXN 501
#define MAXL 20
#define mod 1000000007

vector<int> facts;
void primefactors(int n)
{
    while (n % 2 == 0)
```

```
facts.pb(2);
   n = n / 2;
  for (int i = 3; i \le sqrt(n); i += 2)
    while (n \% i == 0)
      facts.pb(i);
      n = n / i;
  if (n > 2)
    facts.pb(n);
signed main()
  int n;
  cin >> n;
  primefactors(n);
  sort(facts.begin(), facts.end());
  for (auto const &i : facts)
    cout << i << endl;</pre>
  return 0;
```

5.18 operadores binarios

```
#include <bits/stdc++.h>
using namespace std;
#define lli long long int
#define pb push_back
#define in insert
#define pi pair<int, int>
#define pd pair<double, int>
#define pii pair<int, pi>
#define mp make_pair
#define fir first
#define sec second
#define MAXN 200001
#define mod 1000000007
void shifts ()
 bitset <4> bs;
  bs.reset();
  bs[2] = true;
  bs[3] = true;
  cout << bs << endl ; // 1100
```

```
bs >>= 1; // 0110
  bs <<= 1; // 1100
  bs >>= 2; // 0011
  bs <<= 2; // 1100
  bs >>= 3; // 0001
  bs <<= 3; // 1000
  cout << bs << endl ;
void op_xor ()
 // 0 ^0 = 0
  // 0 ^1 = 1
  // 1 ^0 = 1
  // 1 ^ 1 = 0
  bitset <4> bs , bs2;
  bs.reset();
  bs2.reset();
  bs[2] = true;
  bs[3] = true;
  bs2[1] = true;
  bs2[3] = true;
  bs \hat{} = bs2; // bs = bs \hat{} bs2
  cout << bs.count() << endl ;</pre>
void op_and ()
  // 0 \& 0 = 0
  // 0 \& 1 = 0
  // 1 & 0 = 0
  // 1 & 1 = 1
  bitset <4> bs , bs2;
  bs.reset();
  bs2.reset();
  bs[2] = true;
  bs[3] = true;
  bs2[1] = true;
  bs2[3] = true;
  bs &= bs2; // bs = bs & bs2
  cout << bs.count() << endl ;</pre>
void op or ()
  // 0 1 0 = 0
  // 0 1 1 = 1
  // 1 | 0 = 1
  // 1 | 1 = 1
  bitset <4> bs , bs2;
  bs.reset(); // poe tudo 0
  bs2.reset();
  bs[2] = true;
```

```
bs[3] = true;
bs2[1] = true;
bs2[3] = true;
bs |= bs2; // bs = bs | bs2
cout << bs.count() << endl; // quantidade de 1
}
signed main()
{
  op_or();
  op_and();
  op_xor();
  shifts();
  return 0;
}</pre>
```

5.19 baby step gigant step

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class string>
using ordered_set = tree<string, null_type, less<string</pre>
   >, rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define endl '\n'
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 2001
#define mod 1000000007
int bsqs(int a, int b, int m)
  if (a == 0 \&\& b == 0)
    return 1:
  a %= m, b %= m;
  int k = 1, add = 0, q;
  while ((g = \underline{gcd}(a, m)) > 1) // fazer a e m serem
     coprimos
    if (b == k)
      return add;
    if (b % q)
      return -1;
```

```
b /= q, m /= q, ++add;
    k = (k * 111 * a / q) % m;
  int n = sqrt(m) + 1;
  int an = 1;
  for (int i = 0; i < n; i++)
    an = (an * 111 * a) % m;
  unordered map<int, int> vals;
  for (int q = 0, cur = b; q \le n; q++)
   vals[cur] = q;
    cur = (cur * 111 * a) % m;
  for (int p = 1, cur = k; p \le n; p++)
    cur = (cur * 111 * an) % m;
    if (vals.count(cur))
      int ans = n * p - vals[cur] + add;
      return ans;
  return -1;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int q;
  cin >> q;
  while (q--)
    int a, b, m;
    cin >> a >> b >> m;
    cout << bsqs(a, b, m) << endl;</pre>
  return 0;
// menor x tal que: (a^x) % m = b % m
// a e m sao coprimos
// se nao forem coprimos tem como tratar
// complexidade: sqrt(m)
```

5.20 matrix exponentiation

```
// https://codeforces.com/gym/102644/problem/C
// achar o n-esimo termo da sequencia de fibonacci mod
    (10^9 + 7) em O(log(n))
// n <= 10^18</pre>
```

```
// podemos escrever a recorrencia de fibonnaci como uma
   exponenciacao de matriz
/*
  (fib(n)) (1 1) (n-1) (fib(1) = 1)
  (fib(n-1)) = (1 0)
                            \star (fib(0) = 1)
// e possivel fazer essa exponenciacao em O(log(n)) com
   um algoritimo muito similar ao de exponenciacao
   rapida
// dai calculamos o n-esimo termo da sequencia de
   fibonacci mod (10^9 + 7) em O(\log(n))
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace __qnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define PI acos(-1)
#define pb push back
#define int long long int
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 201
#define mod 1000000007
namespace matrix
  vector<vector<int>> ans;
  int multi(int x, int y)
   return (x * y) % mod;
  int sum(int a, int b)
   return (a + b >= mod) ? a + b - mod : a + b;
  vector<vector<int>> multiply(vector<vector<int>> a,
     vector<vector<int>> b)
    vector<vector<int>> res(a[0].size(), vector<int>(b
       [0].size()));
```

```
for (int i = 0; i < a.size(); i++)</pre>
      for (int j = 0; j < b[0].size(); j++)
        res[i][j] = 0;
        for (int k = 0; k < a[0].size(); k++)
          res[i][j] = sum(res[i][j], multi(a[i][k], b[k])
             ][<mark>†</mark>]));
    return res;
  vector<vector<int>> expo(vector<vector<int>> mat, int
    ans = vector<vector<int>>(mat.size(), vector<int>(
       mat[0].size()));
    for (int i = 0; i < mat.size(); i++)</pre>
      for (int j = 0; j < mat[0].size(); j++)</pre>
        ans[i][j] = (i == j);
    while (m > 0)
      if (m & 1)
       ans = multiply(ans, mat);
     m = m / 2;
      mat = multiply(mat, mat);
    return ans;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
 int n;
  cin >> n;
  vector<vector<int>> mat = {{1, 1}, {1, 0}};
 vector<vector<int>> ans = matrix::expo(mat, n);
 cout << ans[0][1] << endl;</pre>
 return 0;
```

5.21 diophantine

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
```

```
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 200001
#define mod 998244353
namespace dio
 vector<pi> sols;
  int gcd(int a, int b, int &x, int &y)
    if (b == 0)
     x = 1, y = 0;
      return a;
    int x1, y1, d = gcd(b, a % b, x1, y1);
    x = y1, y = x1 - y1 * (a / b);
    return d;
  void one_sol(int a, int b, int c)
    int x0, y0, g;
    g = gcd(abs(a), abs(b), x0, y0);
    if (c % q)
    return;
    x0 \star = (c / q);
    y0 *= (c / g);
    if (a < 0)
     x0 *= -1;
    if (b < 0)
     v0 *= -1;
    sols.pb({x0, y0});
  void more_sols(int a, int b, int c)
    int q = \underline{gcd}(a, b);
    int x0 = sols[0].fir, y0 = sols[0].sec;
    for (int k = -200000; k \le 200000; k++)
      int x = x0 + k * (b / q);
      int y = y0 - k * (a / q);
```

```
sols.pb(\{x, y\});
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int a, b, c;
  cin >> a >> b >> c;
  dio::one sol(a, b, c);
  if (!dio::sols.size())
    cout << "No\n";</pre>
    return 0;
  dio::more sols(a, b, c);
  bool can = false:
  for (auto const &i : dio::sols)
    can |= (i.fir >= 0 && i.sec >= 0);
  (can) ? cout << "Yes\n" : cout << "No\n";</pre>
  return 0;
// equacoes do tipo:
// ax + by = c
// o caso a = 0 e b = 0, nao eh tratado nesse codigo
// nesse caso quero checar se equacao diofantina tem uma
    solucao
// com x >= 0 e y >= 0
```

5.22 crt

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define MAXN 2000006
//#define mod 100000007
```

```
namespace crt
  vector<pi> eq;
  int gcd(int a, int b, int &x, int &y)
    if (b == 0)
     x = 1, y = 0;
      return a;
    int x1, y1, d = gcd(b, a % b, x1, y1);
    x = y1, y = x1 - y1 * (a / b);
    return d;
  pi crt()
    int a1 = eq[0].fir, m1 = eq[0].sec;
    a1 %= m1;
    for (int i = 1; i < eq.size(); i++)</pre>
      int a2 = eq[i].fir, m2 = eq[i].sec;
      int g = \underline{gcd}(m1, m2);
      if (a1 % q != a2 % q)
        return {-1, -1};
      int p, q;
      gcd(m1 / g, m2 / g, p, q);
      int mod = m1 / g * m2;
      int x = (a1 * (m2 / g) % mod * q % mod + a2 * (m1)
         / q) % mod * p % mod) % mod;
      a1 = x;
      if (a1 < 0)
        a1 += mod;
      m1 = mod;
    return {a1, m1};
signed main()
  ios base::sync with stdio(false);
  cin.tie(NULL);
  int n;
  cin >> n;
  for (int i = 0; i < n; i++)
    int a, b;
    cin >> a >> b;
    crt::eq.pb({a, b});
```

```
pi ans = crt::crt();
  if (ans.fir == -1)
    cout << "No solution\n";</pre>
    cout << ans.fir << " " << ans.sec << endl;</pre>
  return 0;
// references:
// https://forthright48.com/chinese-remainder-theorem-
   part-2-non-coprime-moduli/
// https://cp-algorithms.com/algebra/chinese-remainder-
   theorem.html
// https://www.geeksforgeeks.org/chinese-remainder-
   theorem-set-1-introduction/
// teorema chines do resto(crt)
// para resolver sistemas de congruencias modulares
// o menor inteiro a que satisfaz:
// a mod p1 = x1
// a mod p2 = x2
// ...
// a mod pn = xn
// a funcao crt retorna um pair {a, mod}
// dai a solucao pode ser descrita como
// x = a \% mod
// entao os valores possiveis sao:
// a, (a + mod), a + (2 * mod), a + (3 * mod), ...
```

5.23 ntt

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define endl '\n'
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 250005
#define mod 998244353
```

```
struct modint
  int val;
  modint(int v = 0)  { val = ((v % mod) + mod) % mod; }
  int pow(int y)
    modint x = val;
    modint z = 1;
    while (y)
     if (y & 1)
        z \star = x;
     x \star = x;
      y >>= 1;
    return z.val;
  int inv() { return pow(mod - 2); }
  void operator=(int o) { val = o % mod; }
  void operator=(modint o) { val = o.val % mod; }
  void operator+=(modint o) { *this = *this + o; }
  void operator-=(modint o) { *this = *this - o; }
  void operator*=(modint o) { *this = *this * o; }
  void operator/=(modint o) { *this = *this / o; }
  bool operator==(modint o) { return val == o.val; }
  bool operator!=(modint o) { return val != o.val; }
  int operator*(modint o) { return ((val * o.val) % mod)
  int operator/(modint o) { return (val * o.inv()) % mod
     ; }
  int operator+(modint o) { return (val + o.val) % mod;
  int operator-(modint o) { return (val - o.val + mod) %
      mod; }
};
namespace fft
  // para o modulo ser valido
  // precisa ser primo
  // precisa possuir a forma c * 2^k + 1
  // 998244353 - possui a forma - c * 2^k + 1 e eh primo
  int n;
  int root = -1;
  int root_1 = -1;
  int pw = __builtin_ctz(mod - 1);
  int root pw = (1 \ll pw);
  void find root()
```

```
if (root != −1)
    return;
 int r = 2;
  while (! (modint(r).pow((1 << pw)) == 1 && modint(r).
     pow((1 << (pw - 1))) != 1))
    r++;
  root = r;
  root 1 = modint(root).inv();
void ntt(vector<modint> &a, bool invert)
  find_root();
  int n = a.size();
  for (int i = 1, j = 0; i < n; i++)
   int bit = n >> 1;
    for (; j & bit; bit >>= 1)
      j ^= bit;
    j ^= bit;
    if (i < j)
      swap(a[i], a[j]);
  for (int len = 2; len <= n; len <<= 1)</pre>
    modint wlen = (invert) ? root_1 : root;
    for (int i = len; i < root pw; i <<= 1)</pre>
      wlen *= wlen;
    for (int i = 0; i < n; i += len)</pre>
      modint w = 1;
      for (int j = 0; j < len / 2; j++)
        modint u = a[i + j];
        modint v = a[i + j + len / 2] * w;
        a[i + j] = u + v;
        a[i + j + len / 2] = u - v;
        w \neq wlen;
  if (invert)
    modint n_1 = modint(n).inv();
    for (int i = 0; i < a.size(); i++)</pre>
      a[i] *= n_1;
vector<modint> mul(vector<modint> a, vector<modint> b)
 n = 1;
```

```
while (n < 2 * max(a.size(), b.size()))
    n <<= 1;
    a.resize(n);
    b.resize(n);
    ntt(a, false);
    ntt(b, false);
    for (int i = 0; i < n; i++)
        a[i] *= b[i];
    ntt(a, true);
    return a;
}
// namespace fft
// https://codeforces.com/contest/1613/problem/F</pre>
```

6 Dynamic programming and common problems

6.1 cht

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define endl '\n'
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first.
#define sec second
#define MAXN 1000005
#define mod 1000000007
struct line
  int m, b, p;
  line(int m, int b) : m(m), b(b) {}
  bool operator<(const line &o) const</pre>
    if (m != o.m)
      return m > o.m;
    return b > o.b;
  bool operator<(const int x) const { return p < x; }</pre>
```

```
int eval(int x) const { return m * x + b; }
  int inter(const line &o) const
    int x = b - o.b, y = o.m - m;
    return (x / y) - ((x ^ y) < 0 && x % y);
};
struct cht
  deque<line> a;
  cht() {}
  int eval(int i, int x)
    return a[i].m * x + a[i].b;
  void add(line 1)
    while (1)
      if (a.size() >= 1 && a[0].m == 1.m && 1.b > a[0].b
        a.pop front();
      else if (a.size() >= 1 && a[0].m == 1.m && 1.b <=
         a[0].b
        break;
      else if (a.size() >= 2 \&\& a[0].inter(1) >= a[1].
         inter(a[0]))
        a.pop_front();
      else
        a.push_front(1);
        break;
  int get(int x)
    while (a.size() >= 2 && eval(a.size() - 1, x) <=</pre>
       eval(a.size() - 2, x))
      a.pop_back();
    return eval(a.size() - 1, x);
};
signed main()
```

```
{
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  return 0;
}
// cht
// queries ordenadas em ordem decrescente
// linhas ordenadas em ordem decrescente
```

6.2 subsequences string

```
#include <bits/stdc++.h>
using namespace std;
#define PI acos(-1)
#define int long long int
#define pb push_back
#define mp make pair
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 100
#define MAXL 20
#define mod 998244353
void count(string a, string b)
  int m = a.size();
  int n = b.size();
  int dp[m + 1][n + 1] = \{\{0\}\};
  for (int i = 0; i \le n; ++i)
    dp[0][i] = 0;
  for (int i = 0; i \le m; ++i)
    dp[i][0] = 1;
  for (int i = 1; i <= m; i++)</pre>
    for (int j = 1; j \le n; j++)
      if (a[i - 1] == b[j - 1])
        dp[i][j] = dp[i - 1][j - 1] + dp[i - 1][j];
      else
        dp[i][j] = dp[i - 1][j];
    }
  cout << dp[m][n] << endl;</pre>
signed main()
  string a, b;
```

```
cin >> a >> b;
count(a, b);
return 0;
```

$6.3 \quad \cos dp$

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define endl '\n'
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 100001
#define mod 1000000007
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  // exemplos de sos dp para calcular f[x] para cada
     mask x
 // a[x] eh o valor de uma funcao a para uma mask x
  // complexidade: O(M * 2^M), M = numero de bits
 // Exemplo 1:
  // nesse caso, f[x] eh a funcao que soma:
  // todos os a[i], tal que, (x \& i) == i)
  // isso eh, i eh uma "mask filha" de x
  // pois todos os bits de i estao setados em x
  for (int mask = 0; mask < (1 << m); mask++)</pre>
    f[mask] = a[mask];
  for (int i = 0; i < m; ++i)
    for (int mask = 0; mask < (1 << m); mask++)</pre>
      if (mask & (1 << i))
```

```
f[mask] += f[mask ^ (1 << i)];
}

// Exemplo 2:
// nesse caso, f[x] eh a funcao que soma:
// todos os a[i], tal que, (x & i) == x)
// isso eh, i eh uma "mask pai" de x
// pois todos os bits de x estao setados em i
for (int mask = 0; mask < (1 << m); mask++)
{
   f[mask] = a[mask];
}
for (int i = 0; i < m; ++i)
{
   if (!(mask & (1 << i)))
      f[mask] += f[mask ^ (1 << i)];
   }
}

return 0;
}

https://codeforces.com/blog/entry/45223</pre>
```

6.4 aliens trick

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define endl '\n'
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 500001
#define mod 1000000007
int n, k, 1;
string s;
```

```
pi solve(vector<int> &v, int lambda)
  // associar um custo lambda para ser subtraido quando
     realizamos uma operacao
 // dp[i] - melhor profit que tivemos considerando as i
      primeiras posicoes
 // cnt[i] - quantas operacoes utilizamos para chegarno
      valor de dp[i]
 vector<int> dp(n + 1);
 vector<int> cnt(n + 1);
  dp[0] = 0;
  cnt[0] = 0;
  for (int i = 1; i <= n; i++)</pre>
    dp[i] = dp[i - 1];
    cnt[i] = cnt[i - 1];
    int id = i - 1;
    dp[i] += v[id];
    int lo = max(011, id - 1 + 1);
    int s = dp[lo] + (id - lo + 1) - lambda;
    if (s > dp[i])
      dp[i] = s;
      cnt[i] = cnt[lo] + 1;
  return {dp[n], cnt[n]};
int aliens_trick(vector<int> &v)
  int 1 = 0, r = n;
  while (1 < r)
    int mid = (1 + r) >> 1;
    pi ans = solve(v, mid);
    (ans.sec > k) ? 1 = mid + 1 : r = mid;
  pi ans = solve(v, 1);
  return ans.fir + (1 * k);
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  cin >> n >> k >> 1 >> s;
  vector<int> a(n);
 vector<int> b(n);
  for (int i = 0; i < n; i++)
```

```
a[i] = 1, b[i] = 0;
if (s[i] >= 'A' && s[i] <= 'Z')
{
    a[i] ^= 1;
    b[i] ^= 1;
}
cout << n - max(aliens_trick(a), aliens_trick(b)) << endl;
return 0;
}
// https://codeforces.com/contest/1279/problem/F</pre>
```

6.5 largest square

```
#include <bits/stdc++.h>
using namespace std;
#define PI acos(-1)
#define pb push back
#define int long long int
#define double long double
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first.
#define sec second
#define MAXN 1001
#define mod 1000000007
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int n;
  cin >> n;
  int v[n][n];
  int dp[n][n];
  for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
      cin >> v[i][j];
  int ans = 0;
  for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
      dp[i][j] = v[i][j];
      if (i && j && dp[i][j])
        dp[i][j] = min({dp[i][j-1], dp[i-1][j], dp[i]}
            -1][j-1]\}) + 1;
      ans = max(ans, dp[i][j]);
```

```
}
cout << ans * ans << endl;
return 0;
}</pre>
```

6.6 broken profile

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace gnu pbds;
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define PI acos(-1)
#define pb push back
#define int long long int
#define pi pair<int, int>
#define pii pair<int, pair<int, pi>>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 1001
#define mod 1000000007
int n;
vector<int> validmasks;
int dp[MAXN][1 << 4];</pre>
void init() // preprocess valid masks
  for (int mask = 0; mask < (1 << 7); mask++)</pre>
    int nxt_mask = 0, prev_mask = 0, valid = true;
    for (int k = 0; k < 7; k++)
      if (mask & (1 << k))
        if (k \le 3)
          int idx = k, idx2 = k;
          if (nxt mask & (1 << idx) || prev mask & (1 <<
               idx2))
            valid = false;
          prev_mask = prev_mask | (1 << idx);</pre>
          nxt_mask = nxt_mask \mid (1 << idx2);
```

```
else
          int idx = k - 4, idx2 = idx + 1;
          if (nxt_mask & (1 << idx) || nxt_mask & (1 <<</pre>
              idx2))
            valid = false;
          nxt_mask = nxt_mask | (1 << idx);</pre>
          nxt_mask = nxt_mask \mid (1 << idx2);
      }
    }
    if (valid)
      validmasks.pb(mask);
int solve(int i, int j)
  if (i == n)
    return ( \dot{j} == ((1 << 4) - 1)) ? 1 : 0;
  if (dp[i][j] != -1)
    return dp[i][j];
  int ret = 0;
  for (auto const &mask : validmasks)
    int nxt_mask = 0, prev_mask = j, valid = true;
    for (int k = 0; k < 7; k++)
      if (mask & (1 << k))
        if (k \le 3)
          int idx = k, idx2 = idx;
          if (prev mask & (1 << idx) || nxt mask & (1 <<
               idx2))
            valid = false;
          prev_mask = prev_mask | (1 << idx);</pre>
          nxt_mask = nxt_mask \mid (1 << idx2);
        else
          int idx = k - 4, idx2 = idx + 1;
          if (nxt_mask & (1 << idx) || nxt_mask & (1 <<</pre>
              idx2))
            valid = false;
          nxt_mask = nxt_mask | (1 << idx);</pre>
          nxt mask = nxt mask \mid (1 << idx2);
      }
```

```
if (valid && prev_mask == ((1 << 4) - 1))
      ret += solve(i + 1, nxt_mask);
  return dp[i][j] = ret;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int q;
  cin >> q;
  init();
  for (int i = 1; i \le q; i++)
    cin >> n;
   memset(dp, -1, sizeof(dp));
    cout << i << " " << solve(0, (1 << 4) - 1) << endl;
  return 0;
// broken profile dp
// if you can fully fill an area with some figures
// finding number of ways to fully fill an area with
   some figures
// finding a way to fill an area with minimum number of
   figures
// ...
// https://www.spoj.com/problems/GNY07H/
// We wish to tile a 4xN grid with rectangles 2x1 (in
   either orientation)
// dp[i][mask]
// i denotes the current column
// mask denotes the situation of the previous column
// our mission is to fill all of the units of
// the previous column in a state [i][mask]
```

6.7 max matrix path

```
#include <bits/stdc++.h>
using namespace std;

#define PI acos(-1)
#define pb push_back
#define int long long int
#define mp make_pair
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define MAXN 301
```

```
#define MAXL 20
#define mod 1000000007
#define INF 100000001
int n:
int grid[MAXN][MAXN];
int dp[MAXN][MAXN];
int solve(int i, int j)
  if (i == n - 1 \&\& i == n - 1)
    return grid[i][j];
  if (dp[i][j] != -1)
    return dp[i][j];
  if (i + 1 < n \&\& j + 1 < n)
    return dp[i][j] = grid[i][j] + max(solve(i + 1, j),
       solve(i, j + 1));
  if (i + 1 < n)
    return dp[i][j] = qrid[i][j] + solve(i + 1, j);
  if (j + 1 < n)
    return dp[i][j] = grid[i][j] + solve(i, j + 1);
signed main()
  cin >> n;
  for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
      cin >> grid[i][j];
  memset (dp, -1, sizeof(dp));
  cout << solve(0, 0) << endl;
  return 0;
```

6.8 dynamic cht

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;

#define int long long int
#define endl '\n'
#define pb push_back
#define pf push_front
#define pi pair<int, int>
```

```
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 1000005
#define mod 1000000007
struct line
  mutable int m, b, p;
  bool operator<(const line &o) const</pre>
    if (m != o.m)
      return m < o.m;
    return b < o.b;</pre>
  bool operator<(const int x) const { return p < x; }</pre>
  int eval(int x) const { return m * x + b; }
  int inter(const line &o) const
    int x = b - o.b, y = o.m - m;
    return (x / y) - ((x ^ y) < 0 && x % y);
};
struct cht
  int INF = 1e18;
  multiset<line, less<>> 1;
  void add(int m, int b)
    auto y = 1.insert({m, b, INF});
    auto z = next(y);
    if (z != l.end() \&\& y->m == z->m)
      l.erase(y);
      return;
    if (y != l.begin())
      auto x = prev(y);
      if (x->m == y->m)
        x = 1.erase(x);
    while (1)
      if (z == 1.end())
        y->p = INF;
        break;
      y->p = y->inter(*z);
```

```
if (y->p < z->p)
        break;
      else
        z = 1.erase(z);
    if (y == 1.begin())
      return;
    z = y;
    auto x = --y;
    while (1)
      int ninter = x->inter(*z);
      if (ninter <= x->p)
       x->p = ninter;
      else
       l.erase(z);
       break;
      if (x == 1.begin())
       break;
     y = x;
      x--;
      if (x->p < y->p)
       break;
      else
        l.erase(y);
    }
  int get(int x)
    if (l.empty())
      return 0;
    return 1.lower bound(x)->eval(x);
};
signed main()
  ios base::sync with stdio(false);
  cin.tie(NULL);
  return 0;
// sources:
// https://github.com/pauloamed/Training/blob/master/PD/
// https://github.com/brunomaletta/Biblioteca/blob/
   master/Codigo/DP/CHT-Dinamico.cpp
// cht dinamico
// dado uma coordenada x
```

```
// e um conjunto com varias equacoes lineares da forma:
   V = mx + C
// retorna o maior valor de y entre as equações do
   conjunto
// para o menor valor, multiplicar m e c de cada equacao
    por -1
// e multiplicar o resultado da query por -1
// problemas iniciais:
// https://atcoder.jp/contests/dp/tasks/dp z
// https://codeforces.com/contest/1083/problem/E
```

6.9 lis

```
// dada uma sequencia s qualquer, descobrir o tamanho da
    maior subsequencia crescente de s
// uma subsequencia de s e qualquer subconjunto de
   elementos de s.
// Para cada novo numero, voce tem duas operacoes
   possiveis:
// 1 - Colocar o novo numero no topo de uma pilha se ele
    nao superar o que ja esta em seu topo;
// 011
// 2 - Criar uma nova pilha a direita de todas as outras
    e colocar o novo numero la.
// ao final do processo a nossa pilha tera os elementos
   da lis.
#include <bits/stdc++.h>
using namespace std;
#define lli long long int
#define pb push_back
#define in insert
#define pi pair<int, int>
#define pd pair<double, int>
#define pib pair<pi, bool>
#define mp make pair
#define fir first
#define sec second
#define MAXN 200001
#define MAXL 1000001
#define mod 1000000007
vector<int> v;
int lis()
 vector<int> q;
```

for (int i = 0; i < v.size(); i++)</pre>

```
vector<int>::iterator it = lower_bound(q.begin(), q.
       end(), v[i]);
    if (it == q.end())
      q.pb(v[i]);
    else
      *it = v[i];
 for (int i = 0; i < q.size(); i++)</pre>
    cout << q[i] << " ";
  cout << endl;</pre>
  return q.size();
signed main()
 int n;
 cin >> n;
 v.resize(n);
 for (int i = 0; i < n; i++)
    cin >> v[i];
 cout << lis() << endl;</pre>
  return 0;
```

6.10 Knapsack

```
//O problema mais classico de Programacao Dinamica
   talvez seja o Knapsack.
//De maneira geral, um ladrao ira roubar uma casa com
   uma mochila
//que suporta um peso s. Ele ve n objetos na casa e sabe
    estimar o peso pi e o valor vi
//de cada objeto i. Com essas informacoes, qual o maior
   valor que o ladrao pode roubar sem rasgar sua
   mochila?
#include <bits/stdc++.h>
using namespace std;
#define lli long long int
#define pb push_back
#define in insert
#define pi pair<int, int>
#define pii pair<int, pi>
#define mp make pair
#define fir first
#define sec second
#define MAXN 1001
#define INF 1000000000
int n, 1;
```

```
int value[MAXN];
int peso[MAXN];
int dp[MAXN][MAXN];
int knapsack(int i, int limit)
  if (dp[i][limit] >= 0) // se ja foi calculado
    return dp[i][limit];
  if (i == n or !limit) // se cheqou no fim do array ou
     chegou no limite
    return dp[i][limit] = 0;
  int nao coloca = knapsack(i + 1, limit); //
     recursivamente pra caso eu nao coloque o objeto i
  if (peso[i] <= limit) // se eu consigo botar o objeto</pre>
    int coloca = value[i] + knapsack(i + 1, limit - peso
    return dp[i][limit] = max(coloca, nao coloca);
  return dp[i][limit] = nao_coloca;
signed main()
  cin >> 1 >> n;
  for (int i = 0; i < n; i++)
    cin >> peso[i] >> value[i];
 memset (dp, -1, sizeof(dp));
  cout << knapsack(0, 1) << endl;</pre>
  return 0;
```

6.11 tip

```
// dados os valores de moedas v1, v2, ... vn e possivel
  formar um valor m como combinacao de moedas
// para isso basta montar uma dp inicializada com -1
// nesse caso a dp so precisa de um parametro q e =
  valor restante ate o limite
```

```
// mas podem existir variacoes do problema q precise de
   mais coisas
// se em achar alguma combinacao valida retorna 1, se
   nao retorna 0
#include <bits/stdc++.h>
using namespace std;
#define lli long long int
#define pb push_back
#define in insert
#define pi pair<int, int>
#define pd pair<double, int>
#define pib pair<pi, bool>
#define mp make_pair
#define fir first
#define sec second
#define MAXN 200001
#define MAXL 10001
#define mod 1000000007
int dp[MAXN];
vector<int> v;
int solve(int rem)
  if (rem == 0)
    return 1;
  if (rem < 0)
    return 0;
  if (dp[rem] >= 0)
    return dp[rem];
  for (int i = 0; i < v.size(); i++)</pre>
    if (solve(rem - v[i]))
      return dp[rem - v[i]] = 1;
  return dp[rem] = 0;
signed main()
  int n, m;
  cin >> n >> m;
  v.resize(n);
  for (int i = 0; i < n; i++)
    cin >> v[i];
  memset(dp, -1, sizeof(dp));
  (solve(m)) ? cout << "Yes\n" : cout << "No\n";</pre>
  return 0;
```

```
// dada uma sequencia s qual a maior soma que podemos
   obter escolhendo um subconjunto de termos adjacentes
    de s
// nesse caso o temos apenas duas opcoes
// nao usar o elemento v[i]
// ou
// usamos, adicionando a maior soma possivel que antes
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 200001
#define mod 1000000007
int kadane(vector<int> v)
  int n = v.size(), ans = 0, max_here = 0;
  for (int i = 0; i < n; i++)
    \max \text{ here } += \text{ v[i]};
    if (ans < max here)</pre>
      ans = max_here;
    if (max here < 0)</pre>
      \max here = 0;
  return ans:
int kadane circular(vector<int> v)
  int n = v.size(), max_kadane = kadane(v);
  int max wrap = 0, i;
  for (i = 0; i < n; i++)
    max\_wrap += v[i];
    v[i] = -v[i];
  max_wrap += kadane(v);
```

```
return max(max_wrap, max_kadane);
}
signed main()
{
  int n;
  cin >> n;
  vector<int> v(n);
  for (int i = 0; i < n; i++)
      cin >> v[i];
  cout << kadane_circular(v) << endl;
  return 0;
}</pre>
```

6.13 lcs

```
//Dadas duas sequencias s1 e s2, uma de tamanho n e
   outra de tamanho m, qual a maior subsequencia comum
   as duas?
// uma subsequencia de s e um subconjunto dos elementos
   de s na mesma ordem em que apareciam antes.
// isto significa que \{1, 3, 5\} e uma subsequencia de
   {1, 2, 3, 4, 5}, mesmo 1 nao estando do lado do 3.
#include <bits/stdc++.h>
using namespace std;
#define lli long long int
#define pb push back
#define in insert
#define pi pair<int, int>
#define pii pair<int, pi>
#define mp make_pair
#define fir first
#define sec second
#define MAXN 1001
#define INF 1000000000
int v1[MAXN];
int v2[MAXN];
int dp[MAXN][MAXN];
void lcs(int m, int n)
  for (int i = 0; i <= m; i++)</pre>
    for (int j = 0; j <= n; j++)
      if (i == 0 \mid | j == 0) //se uma das sequencias for
         vazia
        dp[i][j] = 0;
```

```
else if (v1[i - 1] == v2[j - 1]) // se eh igual,
         adiciono a lcs e subtraio dos dois
        dp[i][j] = dp[i - 1][j - 1] + 1;
      else
        dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]); //
           se nao retorno o maximo entre tirar um dos
           dois caras
 cout << dp[m][n] << endl;</pre>
signed main()
 int n, m;
 cin >> n >> m;
 for (int i = 0; i < n; i++)
   cin >> v1[i];
 for (int i = 0; i < m; i++)
   cin >> v2[i];
 lcs(n, m);
 return 0;
```

6.14 divideandconquer

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace __qnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define endl '\n'
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 200005
#define mod 1000000007
int s[8005];
int dp[3005][8005];
int cost(int 1, int r)
```

```
return (s[r + 1] - s[1]) * (r - 1 + 1);
void compute(int 1, int r, int opt1, int optr, int i)
  if (1 > r)
    return;
  int mid = (1 + r) >> 1;
  pair<int, int> ans = \{1e18, -1\}; // dp, k
  for (int q = optl; q <= min(mid, optr); q++)</pre>
    if (q > 0)
      ans = min(ans, \{dp[i-1][q-1] + cost(q, mid), q\}
         });
    else
      ans = min(ans, {cost(q, mid), q});
  dp[i][mid] = ans.fir;
  compute(l, mid - 1, optl, ans.sec, i);
  compute(mid + 1, r, ans.sec, optr, i);
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int n, q;
  cin >> n >> q;
  for (int i = 1; i <= n; i++)</pre>
    cin >> s[i];
    s[i] += s[i - 1];
  for (int i = 0; i \le q; i++)
    for (int j = 0; j <= n; j++)
      dp[i][j] = 1e18;
  for (int i = 1; i <= q; i++)
    compute (0, n - 1, 0, n - 1, i);
  cout << dp[g][n - 1] << endl;</pre>
  return 0;
// https://codeforces.com/gym/103536/problem/A
// https://codeforces.com/contest/321/problem/E
// otimizacao de dp usando divide and conquer
// para dps do tipo:
// dp[i][j] = min(dp[i - 1][k] + c(k, j)), para algum k
// considerando opt(i, j) o menor valor de k que
   minimiza dp[i][j]
```

```
// podemos calcular opt(i, j) usando divide and conquer
// isso diminuiria a complexidade para O(k * n * log(n))
```

6.15 expected value

```
//https://atcoder.jp/contests/dp/tasks/dp_j
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define pb push back
#define mp make_pair
#define pi pair<int, int>
#define pii pair<pi, int>
#define pci pair<char, int>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 301
#define mod 1000000007
int n;
vector<int> v;
vector<int> cnt(3);
double dp[MAXN][MAXN];
double solve(int i, int j, int k)
  if (!i && !j && !k)
    return dp[i][j][k] = 0;
  if (dp[i][j][k] != -1)
    return dp[i][j][k];
  It is well-known from statistics that for the
     geometric distribution
  (counting number of trials before a success, where
     each independent trial is probability p)
  the expected value is i / p
  double p = ((double)(i + j + k) / n);
  double ret = 1 / p; // expected number of trials
     before a success
  if(i)
```

```
double prob = (double)i / (i + j + k); //
       probabilidade de ser um prato com um sushi
   ret += (solve(i - 1, j, k) * prob);
  if(j)
   double prob = (double) j / (i + j + k); //
       probabilidade de ser um prato com dois sushis
   ret += (solve(i + 1, j - 1, k) * prob);
  if (k)
    double prob = (double)k / (i + j + k); //
       probabilidade de ser um prato com tres sushis
   ret += (solve(i, j + 1, k - 1) * prob);
  return dp[i][j][k] = ret;
signed main()
 ios_base::sync_with_stdio(false);
 cin.tie(NULL);
 cin >> n;
 v.resize(n);
 for (int i = 0; i < n; i++)
    cin >> v[i], cnt[v[i] - 1]++;
 for (int i = 0; i < MAXN; i++)</pre>
    for (int j = 0; j < MAXN; j++)
      for (int k = 0; k < MAXN; k++)
        dp[i][j][k] = -1;
  cout << setprecision(15) << solve(cnt[0], cnt[1], cnt</pre>
     [2]) << endl;
 return 0;
```

6.16 Digitdp

```
#include <bits/stdc++.h>
using namespace std;

#define int long long int
#define pb push_back
#define pi pair<int, int>
#define fir first
#define sec second
#define MAXN 2001
#define mod 1000000007

int dp[20][20 * 9][2]; // a,b <= 10^18</pre>
```

```
vector<int> dig;
int solve(int i, int j, int k)
  if (i == dig.size())
    return (k) ? dp[i][j][k] = j : dp[i][j][k] = 0;
  if (dp[i][j][k] != -1)
    return dp[i][j][k];
  int sum = 0;
  if (k)
    for (int f = 0; f \le 9; f++)
      sum += solve(i + 1, j + f, k);
  if (!k)
    for (int f = 0; f <= dig[i]; f++)</pre>
      sum += solve(i + 1, j + f, (dig[i] != f) ? 1 : 0);
  return dp[i][j][k] = sum;
void get digits(int n)
  dig.clear();
  while (n)
    dig.pb(n % 10);
    n = n / 10;
  reverse (dig.begin(), dig.end());
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int a, b;
  cin >> a >> b;
  get digits(a);
  memset (dp, -1, sizeof (dp));
  int aa = solve(0, 0, 0);
  get_digits(b + 1);
  memset (dp, -1, sizeof (dp));
  int bb = solve(0, 0, 0);
  cout << bb - aa << endl;
  return 0;
```

7 Graph

7.1 centroid decomposition2

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
```

```
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace gnu pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 50005
#define mod 1000000007
int n, k, resp;
vector<int> adj[MAXN];
qp hash_table<int, int> cnt;
namespace cd
  int sz:
  vector<int> adil[MAXN];
  vector<int> father, subtree size;
  vector<bool> visited;
  void dfs(int s, int f)
    SZ++;
    subtree size[s] = 1;
    for (auto const &v : adj[s])
      if (v != f && !visited[v])
      dfs(v, s);
       subtree_size[s] += subtree_size[v];
    }
  int getCentroid(int s, int f)
    bool is centroid = true;
    int heaviest child = -1;
    for (auto const &v : adj[s])
      if (v != f && !visited[v])
        if (subtree_size[v] > sz / 2)
```

```
is_centroid = false;
      if (heaviest child == -1 || subtree size[v] >
         subtree size[heaviest child])
        heaviest child = v;
 return (is centroid && sz - subtree size[s] <= sz /</pre>
     2) ? s : getCentroid(heaviest child, s);
void dfs2(int s, int f, int d)
 cnt[d]++;
 for (auto const &v : adj[s])
   if (v != f && !visited[v])
     dfs2(v, s, d + 1);
int solve(int s)
 gp_hash_table<int, int> tot;
  int ans = 0;
  for (auto const &v : adj[s])
    if (visited[v])
     continue;
    cnt.clear();
    dfs2(v, s, 1);
    for (int i = 1, j = k - 1; i < k; i++, j--)
     ans += (cnt[i] * tot[j]);
    for (auto const &i : cnt)
     tot[i.fir] += i.sec;
  return ans + tot[k];
int decompose tree(int s)
 sz = 0;
  dfs(s, s);
 int cend_tree = getCentroid(s, s);
 visited[cend_tree] = true;
  resp += solve(cend tree);
  for (auto const &v : adj[cend tree])
    if (!visited[v])
      int cend_subtree = decompose_tree(v);
      adjl[cend tree].pb(cend subtree);
      adjl[cend subtree].pb(cend tree);
     father[cend subtree] = cend tree;
```

```
return cend_tree;
  void init()
    subtree size.resize(n);
    visited.resize(n);
    father.assign(n, -1);
    decompose tree(0);
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  cin >> n >> k;
  for (int i = 0; i < n - 1; i++)
    int a, b;
    cin >> a >> b;
    a--, b--;
   adj[a].pb(b);
    adj[b].pb(a);
  cd::init();
  cout << resp << endl;</pre>
  return 0;
// https://codeforces.com/contest/161/problem/D
// durante a decomposicao
// pega o centroid atual e resolve o problema pra ele
// isso eh:
// para cada centroid que eu achei, devo contar quantos
   caminhos
// de tamanho k passam por esse centroid
// somando todas essas respostas, a gente tem a resposta
    final
```

7.2 Floyd Warshall

```
#include <bits/stdc++.h>
using namespace std;

#define pb push_back
#define lli long long int
#define MAXN 10000
#define INF 999999

int n , m , a , b , c ;
int dist [MAXN] [MAXN];
```

```
void floyd_warshall ()
    for (int k = 0; k < n; k++)
       for (int i = 0; i < n; i++)
           for (int j = 0; j < n; j++)
               dist[i][j] = min(dist[i][j], dist[i][k]
                    + dist[k][i]);
void initialize ()
    for (int i = 0; i < n; i++)
       for (int j = 0; j < n; j++)
           if (i == j)
               dist[i][j] = 0;
           else
               dist[i][j] = INF;
int main()
   cin >> n >> m;
   initialize ();
    for (int i = 0; i < m; i++)
       cin >> a >> b >> c;
       dist [a][b] = min (dist[a][b] , c);
       dist [b][a] = min (dist[b][a], c);
    floyd_warshall ();
    return 0;
```

7.3 strong orientation

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace gnu pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 1000005
#define mod 1000000007
int n, m, timer, comps, bridges;
vector<pi> edges;
vector<pi> adj[MAXN];
int tin[MAXN];
int low[MAXN];
bool vis[MAXN];
char orient[MAXN];
void find bridges(int v)
  low[v] = timer, tin[v] = timer++;
  for (auto const &p : adj[v])
    if (vis[p.sec])
      continue;
    vis[p.sec] = true;
    orient[p.sec] = (v == edges[p.sec].first) ? '>' : '<
    if (tin[p.fir] == -1)
      find_bridges(p.fir);
      low[v] = min(low[v], low[p.fir]);
      if (low[p.fir] > tin[v])
       bridges++;
    else
      low[v] = min(low[v], low[p.fir]);
```

```
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  cin >> n >> m;
  for (int i = 0; i < m; i++)
    int a, b;
    cin >> a >> b;
    a--, b--;
    edges.pb(\{a, b\});
    adj[a].pb({b, i});
    adj[b].pb({a, i});
  memset(tin, -1, sizeof(tin));
  memset(low, -1, sizeof(low));
  for (int v = 0; v < n; v++)
    if (tin[v] == -1)
      comps++;
      find_bridges(v);
  // numero minimo de scc = numero de componentes +
     numero de pontes
  cout << comps + bridges << endl;</pre>
  // > - a aresta foi orientada da esquerda pra direita
  // < - a aresta foi orientada da direita pra esquerda
  for (int i = 0; i < m; i++)
    cout << orient[i];</pre>
  cout << endl;</pre>
  return 0;
// to_test: https://szkopul.edu.pl/problemset/problem/
   nldsb4EW1YuZykB1f4lcZL1Y/site/?key=statement
// strong orientation:
// encontrar uma orientacao para as arestas tal que o
// minimo de scc e o menor possivel
```

7.4 scc

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
```

```
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 500005
#define mod 1000000007
int n, m;
bool vis[MAXN];
int root[MAXN];
vector<int> order;
vector<int> roots;
vector<int> comp;
vector<vector<int>> comps;
vector<int> adj[MAXN];
vector<int> adi rev[MAXN];
vector<int> adj scc[MAXN];
void dfs(int v)
  vis[v] = true;
  for (auto const &u : adj[v])
    if (!vis[u])
      dfs(u);
  order.pb(v);
void dfs2(int v)
  comp.pb(v);
  vis[v] = true;
  for (auto const &u : adj_rev[v])
    if (!vis[u])
      dfs2(u);
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  cin >> n >> m;
  for (int i = 0; i < m; i++)
```

```
int a, b;
  cin >> a >> b;
 adj[a].pb(b);
 adj_rev[b].pb(a);
for (int i = 0; i < n; i++)
 if (!vis[i])
    dfs(i);
reverse(order.begin(), order.end());
memset(vis, false, sizeof(vis));
for (auto const &v : order)
 if (!vis[v])
    comp.clear();
    dfs2(v);
    comps.pb(comp);
    // making condensation graph
    int r = comp.back();
    for (auto const &u : comp)
     root[u] = r;
    roots.push_back(r);
    */
// making condensation graph
for (int v = 0; v < n; v++)
  for (auto const &u : adj[v])
    int root_v = roots[v];
    int root u = roots[u];
   if (root_u != root_v)
      adj_scc[root_v].pb(root_u);
}
// printing scc
cout << comps.size() << endl;</pre>
for (auto const &comp : comps)
  cout << comp.size() << " ";</pre>
 for (auto const &u : comp)
    cout << u << " ";
 cout << endl;</pre>
```

```
return 0;
}
// to test: https://judge.yosupo.jp/problem/scc
```

7.5 hld

```
//https://codeforces.com/contest/343/problem/D
#include <bits/stdc++.h>
using namespace std;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 500001
#define mod 1000000007
int n, q;
vector<int> adj[MAXN];
namespace seq
  int seq[4 * MAXN];
  int lazy[4 * MAXN];
  int single(int x)
    return x;
  int neutral()
    return 0;
  int merge(int a, int b)
    return a + b;
  void add(int i, int l, int r, int diff)
    seg[i] = (r - 1 + 1) * diff;
    if (1 != r)
     lazy[i \ll 1] = diff;
      lazy[(i << 1) | 1] = diff;
    lazy[i] = -1;
```

```
void update(int i, int l, int r, int ql, int qr, int
     diff)
    if (lazy[i] != -1)
      add(i, l, r, lazy[i]);
    if (1 > r || 1 > qr || r < q1)
      return;
    if (1 >= q1 \&\& r <= qr)
      add(i, l, r, diff);
      return;
    int mid = (1 + r) >> 1;
    update(i << 1, 1, mid, ql, qr, diff);
    update((i \ll 1) \mid 1, mid + 1, r, gl, gr, diff);
    seg[i] = merge(seg[i << 1], seg[(i << 1) | 1]);
  int query(int 1, int r, int q1, int qr, int i)
    if (lazy[i] != -1)
      add(i, l, r, lazy[i]);
    if (1 > r || 1 > qr || r < q1)
      return neutral();
    if (1 >= q1 \&\& r <= qr)
      return seg[i];
    int mid = (1 + r) >> 1;
    return merge(query(l, mid, ql, qr, i << 1), query(</pre>
       mid + 1, r, ql, qr, (i << 1) | 1));
} // namespace seg
namespace hld
  int cur pos;
  vector<int> parent, depth, heavy, head, pos, sz;
  int dfs(int s)
    int size = 1, max_c_size = 0;
    for (auto const &c : adj[s])
      if (c != parent[s])
        parent[c] = s;
        depth[c] = depth[s] + 1;
        int c size = dfs(c);
        size += c size;
        if (c size > max c size)
          max_c_size = c_size, heavy[s] = c;
```

```
return sz[s] = size;
                                                                  if (depth[head[a]] > depth[head[b]])
void decompose(int s, int h)
                                                                    swap(a, b);
                                                                  seq::update(1, 0, n - 1, pos[head[b]], pos[b], x);
 head[s] = h;
 pos[s] = cur_pos++;
                                                                if (depth[a] > depth[b])
 if (heavy[s] != -1)
                                                                  swap(a, b);
   decompose(heavy[s], h);
                                                                seg::update(1, 0, n - 1, pos[a], pos[b], x);
  for (int c : adj[s])
                                                              void update subtree(int a, int x)
    if (c != parent[s] && c != heavy[s])
     decompose(c, c);
                                                                seq::update(1, 0, n - 1, pos[a], pos[a] + sz[a] - 1,
  }
                                                                     x);
void init()
                                                              void query subtree(int a, int x)
 memset(seg::lazy, -1, sizeof(seg::lazy));
                                                                seq::query(0, n-1, pos[a], pos[a] + sz[a] - 1, 1);
  parent.assign(MAXN, -1);
  depth.assign(MAXN, -1);
                                                            } // namespace hld
  heavy.assign(MAXN, -1);
                                                            signed main()
 head.assign(MAXN, -1);
  pos.assign (MAXN, -1);
                                                              cin >> n;
                                                              for (int i = 0; i < n - 1; i++)
  sz.assign(MAXN, 1);
  cur pos = 0;
 dfs(0);
                                                                int a, b;
 decompose(0, 0);
                                                                cin >> a >> b;
 for (int i = 0; i < 4 * n; i++)
                                                                a--, b--;
    seg::lazy[i] = -1;
                                                                adj[a].pb(b);
                                                                adj[b].pb(a);
int query_path(int a, int b)
                                                              hld::init();
  int res = 0;
                                                              cin >> q;
  for (; head[a] != head[b]; b = parent[head[b]])
                                                              while (q--)
   if (depth[head[a]] > depth[head[b]])
                                                                int a, b;
                                                                cin >> a >> b;
      swap(a, b);
    int cur_heavy_path_max = seg::query(0, n - 1, pos[
                                                                b--;
       head[b]], pos[b], 1);
                                                                if (a == 1)
    res += cur heavy path max;
                                                                  hld::update_subtree(b, 1);
  if (depth[a] > depth[b])
                                                                if (a == 2)
    swap(a, b);
  int last_heavy_path_max = seq::query(0, n - 1, pos[a
                                                                  hld::update_path(0, b, 0);
     ], pos[b], 1);
  res += last_heavy_path_max;
                                                                if (a == 3)
  return res;
void update_path(int a, int b, int x)
                                                                  cout << hld::query_path(b, b) << endl;</pre>
  for (; head[a] != head[b]; b = parent[head[b]])
```

```
return 0;
}
```

7.6 Prim

```
// algoritimo de prim
// 1 - definir a distancia de cada vertice como infinito
    (similar ao dijkstra).
// 2 - definir a distancia de 0 para o source(0).
// 3 - Em cada passo, encontrar o vertice u, que ainda
   nao foi processado, que possua a menor das
   distancias.
// 4 - ao termino fazer a soma de todas as distancias e
   encontrar qual a soma das distancias na MST.
#include <bits/stdc++.h>
using namespace std;
#define lli long long int
#define pb push back
#define pii pair<int, int>
#define mp make pair
#define MAXN 100001
#define INF 999999
#define sec second
#define fir first.
int n, m, a, b, c;
vector<pii> adj[MAXN];
int dist[MAXN];
bool processed[MAXN];
void prim()
  for (int i = 0; i < n; i++)
    dist[i] = INF;
  dist[0] = 0;
  priority_queue<pii, vector<pii>, greater<pii>> q;
  q.push(pii(dist[0], 0));
  while (1)
    int davez = -1;
    while (!q.empty())
```

```
int atual = q.top().sec;
      q.pop();
      if (!processed[atual])
        davez = atual;
        break;
    if (davez == -1)
      break;
    processed[davez] = true;
    for (int i = 0; i < adj[davez].size(); i++)</pre>
      int distt = adj[davez][i].fir;
      int atual = adj[davez][i].sec;
      if (dist[atual] > distt && !processed[atual])
        dist[atual] = distt;
        q.push(pii(dist[atual], atual));
  int ans = 0;
  for (int i = 0; i < n; i++)
    ans += dist[i];
  cout << ans << endl;</pre>
int main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  cin >> n >> m;
  for (int i = 0; i < m; i++)
    cin >> a >> b >> c;
```

```
a--;
b--;
adj[a].pb(mp(c, b));
adj[b].pb(mp(c, a));
}
prim();
return 0;
}
```

7.7 articulation points

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 400005
#define mod 1000000007
int n, m, timer;
vector<int> adj[MAXN];
bool is_cutpoint[MAXN];
int tin[MAXN];
int low[MAXN];
bool vis[MAXN];
void dfs(int v, int p)
  vis[v] = true;
  tin[v] = timer, low[v] = timer++;
  int childs = 0;
  for (auto const &u : adj[v])
    if (u == p)
      continue;
    if (vis[u])
    {
```

```
low[v] = min(low[v], tin[u]);
   else
      dfs(u, v);
      low[v] = min(low[v], low[u]);
      if (low[u] >= tin[v] && p != -1)
       is cutpoint[v] = true;
      childs++;
 if (p == -1 && childs > 1)
   is cutpoint[v] = true;
signed main()
 ios_base::sync_with_stdio(false);
 cin.tie(NULL);
 cin >> n >> m;
  for (int i = 0; i < m; i++)
   int a, b;
   cin >> a >> b;
   a--, b--;
   adi[a].pb(b);
   adj[b].pb(a);
 memset(tin, -1, sizeof(tin));
 memset(low, -1, sizeof(low));
 for (int i = 0; i < n; i++)
   if (!vis[i])
      dfs(i, -1);
 return 0;
```

7.8 mincostflow

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;

#define int long long int
```

```
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 301
#define mod 1000000007
#define INF 1e9
namespace mcf
  struct edge
   int to, capacity, cost, res;
  int source, destiny;
  vector<edge> adj[MAXN];
  vector<int> dist;
  vector<int> parent;
  vector<int> edge index;
  vector<bool> in_queue;
  void add edge(int a, int b, int c, int d)
    adj[a].pb({b, c, d, (int)adj[b].size()});
       aresta normal
    adj[b].pb({a, 0, -d, (int)adj[a].size() - 1}); //
       aresta do grafo residual
  bool dijkstra(int s) // rodando o dijkstra, terei o
     caminho de custo minimo
                      // que eu consigo passando pelas
    arestas que possuem capacidade > 0
    dist.assign(MAXN, INF);
    parent.assign(MAXN, -1);
    edge_index.assign(MAXN, -1);
    in_queue.assign(MAXN, false);
    dist[s] = 0;
    queue<int> q;
    q.push(s);
    while (!q.empty())
     int u = q.front(), idx = 0;
     q.pop();
     in queue[u] = false;
      for (auto const &v : adj[u])
        if (v.capacity && dist[v.to] > dist[u] + v.cost)
```

```
dist[v.to] = dist[u] + v.cost;
        parent[v.to] = u;
        edge index[v.to] = idx;
        if (!in_queue[v.to])
         in_queue[v.to] = true;
         q.push(v.to);
     idx++;
  return dist[destiny] != INF; // se eu chequei em
     destiny por esse caminho, ainda posso passar
     fluxo
int get cost()
 int flow = 0, cost = 0;
 while (dijkstra(source)) // rodo um dijkstra para
     saber qual o caminho que irei agora
    int curr flow = INF, curr = destiny;
    while (curr != source) // com isso, vou
       percorrendo o caminho encontrado para achar a
       aresta "gargalo"
     int p = parent[curr];
     curr_flow = min(curr_flow, adj[p][edge_index[
         curr]].capacity);
     curr = p;
    flow += curr flow;
                                       // fluxo que eu
        posso passar por esse caminho = custo da
       aresta "gargalo"
    cost += curr_flow * dist[destiny]; // quanto eu
       gasto para passar esse fluxo no caminho
       encont.rado
    curr = destinv;
    while (curr != source) // apos achar a aresta
       gargalo, passamos o fluxo pelo caminho
       encontrado
     int p = parent[curr];
     int res_idx = adj[p][edge_index[curr]].res;
     adj[p][edge_index[curr]].capacity -= curr_flow;
     adj[curr][res idx].capacity += curr flow;
     curr = p;
```

```
return cost; // ao final temos a resposta :)
} // namespace mcf
signed main()
 int n;
 cin >> n;
 int v[n][n];
 mcf::source = 0, mcf::destiny = (2 * n) + 1;
 for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
     cin >> v[i][j];
     mcf::add\_edge(i + 1, j + n + 1, 1, v[i][j]);
   }
  for (int i = 1; i <= n; i++)</pre>
   mcf::add_edge(mcf::source, i, 1, 0);
 for (int i = n + 1; i \le n + n; i++)
   mcf::add_edge(i, mcf::destiny, 1, 0);
 cout << mcf::get_cost << endl;</pre>
```

7.9 eulertour

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace gnu pbds;
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push_back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define DEBUG 1
#define MAXN 100001
#define mod 1000000009
#define d 31
int n, idx;
vector<int> adj[MAXN];
int euler[2 * MAXN];
```

```
int entrei[MAXN];
int sai[MAXN];
void euler_tour(int s, int f)
  euler[idx] = s;
  entrei[s] = idx;
  idx++;
  for (auto const &v : adj[s])
    if (v == f)
      continue;
    euler_tour(v, s);
  euler[idx] = s;
  sai[s] = idx;
  idx++;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int n:
  cin >> n;
  for (int i = 0; i < n - 1; i++)
    int a, b;
    cin >> a >> b;
    a--, b--;
    adj[a].pb(b);
    adj[b].pb(a);
  euler tour(0, -1);
  for (int i = 0; i < 2 * n; i++)
    cout << euler[i] << " ";</pre>
  cout << endl;</pre>
  return 0;
// euler tour of a tree
// muito util para algumas coisas
// exemplos:
// 1- soma da subarvore de v(com update)
// usando segment trees, podemos fazer uma query(entrei[
   v], sai[v])
// 2- LCA
// lca(u, v) = query(entrei[u], entrei[v])
// usando uma query de minimo e considerando as
   profundidade dos vertices
// a resposta sera o vertice de profundidade minima que
   encontrarmos no intervalo
```

```
// 3- agilidade para remover arestas/vertices/subtrees
  da arvore
// basta apenas tratar o segmento equivalente do jeito
  que for necessario
// 4- reroot a tree
// basta apenas rotacionar o euler path
```

7.10 TreeDiameter

```
#include <bits/stdc++.h>
using namespace std;
#define PI acos(-1)
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define MAXN 100001
#define mod 1000000007
int diameter, best;
vector<int> adj[MAXN];
bool visited[MAXN];
void dfs(int s, int c)
  if (c > diameter)
    diameter = c;
    best = s;
  visited[s] = true;
  for (auto const &i : adj[s])
    if (!visited[i])
      dfs2(i, c + 1);
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int q;
  cin >> q;
  while (q--)
    int n;
    cin >> n;
    for (int i = 0; i < n; i++)
      adj[i].clear();
```

```
for (int i = 0; i < n - 1; i++)
    int a, b;
    cin >> a >> b;
    a--, b--;
    adj[b].pb(a);
    adj[a].pb(b);
  diameter = 0, best = 0;
 memset(visited, false, sizeof(visited));
 dfs(1, 0);
                                    // achar o vertice
     mais distante a partir do vertice 0
 memset(visited, false, sizeof(visited));
 dfs(best, 0);
                                  // achar o mais
     distante a partir do primeiro vertice que
     achamos
 cout << diameter << endl;</pre>
return 0;
```

7.11 Grafo Bipartido

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define endl '\n'
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 200006
#define mod 1000000007
struct dsu
 vector<pi> parent;
 vector<int> rank;
 vector<int> bipartite;
  dsu(int n)
```

```
parent.resize(n);
    rank.resize(n);
    bipartite.resize(n);
    for (int v = 0; v < n; v++)
     parent[v] = \{v, 0\};
     rank[v] = 0;
     bipartite[v] = 1;
  dsu() {}
  pi find set(int v)
    if (v != parent[v].fir)
      int parity = parent[v].sec;
     parent[v] = find set(parent[v].fir);
      parent[v].sec ^= parity;
    return parent[v];
  void add edge(int a, int b)
   pi pa = find_set(a);
    a = pa.fir;
    int x = pa.sec;
    pi pb = find set(b);
    b = pb.fir;
    int y = pb.sec;
    if (a == b)
      if (x == y)
       bipartite[a] = 0;
    else
     if (rank[a] < rank[b])
        swap(a, b);
     parent[b] = \{a, x ^ y ^ 1\};
     bipartite[a] &= bipartite[b];
      if (rank[a] == rank[b])
       rank[a]++;
 bool is_bipartite(int v)
    return bipartite[find_set(v).fir];
};
```

```
signed main()
{
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  return 0;
}
```

7.12 dsu

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define endl '\n'
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 2001
#define mod 1000000007
struct dsu
  int tot;
  vector<int> parent;
  vector<int> sz;
  dsu(int n)
    parent.resize(n);
    sz.resize(n);
    tot = n;
    for (int i = 0; i < n; i++)
     parent[i] = i;
      sz[i] = 1;
  int find set(int i)
    return parent[i] = (parent[i] == i) ? i : find_set(
       parent[i]);
```

```
void make_set(int x, int y)
   x = find_set(x), y = find_set(y);
   if (x != y)
      if (sz[x] > sz[y])
       swap(x, y);
     parent[x] = y;
      sz[y] += sz[x];
     tot--;
};
signed main()
 ios_base::sync_with_stdio(false);
 cin.tie(NULL);
 int n;
 cin >> n;
 dsu d(n);
 int a, b;
 cin >> a >> b;
 d.make set(a, b);
 d.find set(a);
```

7.13 reroot

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define PI acos(-1)
#define pb push_back
#define int long long int
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 200001
#define mod 1000000007
```

```
int n;
vector<int> adj[MAXN];
int sz[MAXN];
int dp[MAXN];
int dfs(int u, int v)
  sz[u] = 1;
  for (auto const &i : adj[u])
    if (i != v)
      sz[u] += dfs(i, u);
  return sz[u];
void reroot(int u, int v)
  for (auto const &i : adj[u])
    if (i != v)
      int a = sz[u], b = sz[i];
      dp[i] = dp[u];
      dp[i] = sz[u], dp[i] = sz[i];
      sz[u] = sz[i], sz[i] = n;
      dp[i] += sz[u], dp[i] += sz[i];
      reroot(i, u);
      sz[u] = a, sz[i] = b;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  cin >> n;
  for (int i = 0; i < n - 1; i++)
    int a, b;
    cin >> a >> b;
    a--, b--;
    adj[a].pb(b);
    adj[b].pb(a);
  dfs(0, -1);
  for (int i = 0; i < n; i++)
    dp[0] += sz[i]; // answer when tree is rooted on
       vertex 0
  reroot (0, -1);
  cout << *max_element(dp, dp + n) << endl;</pre>
  return 0;
```

```
// https://codeforces.com/contest/1187/problem/E
// f(v) = when tree is rooted at vertex v, the current
// answer is the sum of all subtrees sizes
// final answer = \max(f(0), f(1), f(2), \ldots, f(n))
// easy approach: O(N^2)
// with reroot: O(N)
// 1 - run a dfs and calculate f(0)
// 2 - let be dp[i] = f(i)
// 3 - now, lets run a another dfs, and re-calculate the
// answer when tree is rooted at vertex i (dp[i])
// 4 - the final answer is the maximum value of dp[i]
```

7.14 hld edge

```
//https://www.spoj.com/problems/QTREE/
//Don't use cin/cout in this problem (gives TLE)
#include <bits/stdc++.h>
using namespace std;
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 10001
#define mod 1000000007
int n;
vector<pi> adj[MAXN];
vector<pi> edges;
namespace seg
  int seg[4 * MAXN];
  int lazy[4 * MAXN];
  int v[MAXN];
  int single(int x)
    return x;
  int neutral()
    return -1;
  int merge(int a, int b)
    return max(a, b);
```

```
void add(int i, int l, int r, int diff)
  seq[i] = (r - l + 1) * diff;
  if (1 != r)
    lazy[i << 1] = diff;
    lazv[(i << 1) | 1] = diff;
  lazy[i] = -1;
void update(int i, int l, int r, int gl, int gr, int
  if (lazy[i] != -1)
    add(i, l, r, lazy[i]);
  if (1 > r || 1 > qr || r < q1)
    return;
  if (1 >= q1 \&\& r <= qr)
    add(i, 1, r, diff);
    return;
  int mid = (1 + r) >> 1;
  update(i << 1, 1, mid, ql, qr, diff);
  update((i << 1) | 1, mid + 1, r, ql, qr, diff);
  seg[i] = merge(seg[i << 1], seg[(i << 1) | 1]);
int query(int 1, int r, int q1, int qr, int i)
  if (lazy[i] != -1)
    add(i, 1, r, lazy[i]);
 if (1 > r || 1 > qr || r < q1)
    return neutral();
  if (1 >= q1 \&\& r <= qr)
    return seg[i];
  int mid = (1 + r) >> 1;
  return merge(query(l, mid, ql, qr, i << 1), query(</pre>
     mid + 1, r, ql, qr, (i << 1) | 1));
void build(int 1, int r, int i)
  if (1 == r)
    seg[i] = single(v[1]);
    lazy[i] = -1;
    return;
  int mid = (1 + r) >> 1;
  build(1, mid, i << 1);
  build(mid + 1, r, (i << 1) | 1);
```

```
seg[i] = merge(seg[i << 1], seg[(i << 1) | 1]);
   lazy[i] = -1;
} // namespace seg
namespace hld
  int cur_pos;
  vector<int> parent, depth, heavy, head, pos, sz, up;
  int dfs(int s)
    int size = 1, max_c_size = 0;
    for (auto const &c : adj[s])
      if (c.fir != parent[s])
       parent[c.fir] = s;
        depth[c.fir] = depth[s] + 1;
        int c_size = dfs(c.fir);
        size += c size;
       if (c_size > max_c_size)
          max_c_size = c_size, heavy[s] = c.fir;
    return sz[s] = size;
  void decompose(int s, int h)
   head[s] = h;
   pos[s] = cur_pos++;
    seq::v[pos[s]] = up[s];
    for (auto const &c : adj[s])
     if (c.fir != parent[s] && c.fir == heavy[s])
       up[c.fir] = c.sec;
        decompose(heavy[s], h);
    for (auto const &c : adj[s])
      if (c.fir != parent[s] && c.fir != heavy[s])
       up[c.fir] = c.sec;
       decompose(c.fir, c.fir);
      }
  void init()
```

```
parent.assign(MAXN, -1);
  depth.assign(MAXN, -1);
  heavy.assign(MAXN, -1);
  head.assign(MAXN, -1);
  pos.assign(MAXN, -1);
  sz.assign(MAXN, 1);
  up.assign(MAXN, 0);
  cur pos = 0;
  dfs(0);
  decompose(0, 0);
  seg::build(0, n - 1, 1);
int query path(int a, int b)
  int res = -1;
  for (; head[a] != head[b]; b = parent[head[b]])
    if (depth[head[a]] > depth[head[b]])
      swap(a, b);
    res = max(res, seg::query(0, n - 1, pos[head[b]],
       pos[b], 1));
  if (depth[a] > depth[b])
    swap(a, b);
  res = \max(\text{res}, \text{seq}::\text{query}(0, n - 1, \text{pos}[a] + 1, \text{pos}[
     b], 1));
  return res;
void update_path(int a, int b, int x)
  for (; head[a] != head[b]; b = parent[head[b]])
    if (depth[head[a]] > depth[head[b]])
      swap(a, b);
    seg::update(1, 0, n - 1, pos[head[b]], pos[b], x);
  if (depth[a] > depth[b])
    swap(a, b);
  seg::update(1, 0, n - 1, pos[a] + 1, pos[b], x);
void update subtree(int a, int x)
  seq::update(1, 0, n - 1, pos[a] + 1, pos[a] + sz[a]
     -1, x);
int query subtree(int a, int x)
  return seq::query(0, n - 1, pos[a] + 1, pos[a] + sz[
     a] - 1, 1);
```

```
} // namespace hld
signed main()
  int q;
  scanf("%d", &q);
  while (q--)
    scanf("%d", &n);
    for (int i = 0; i < n; i++)
     adj[i].clear();
    edges.clear();
    for (int i = 0; i < n - 1; i++)
      int a, b, c;
     scanf("%d %d %d", &a, &b, &c);
     a--, b--;
      adj[a].pb({b, c});
      adj[b].pb({a, c});
      edges.pb({a, b});
    hld::init();
    while (true)
      char k[10];
      scanf("%s", k);
      if (k[0] == 'Q')
       int a, b;
        scanf("%d %d", &a, &b);
        a--, b--;
       printf("%d\n", hld::query_path(a, b));
      else if (k[0] == 'C')
        int a, b;
        scanf("%d %d", &a, &b);
        hld::update_path(edges[a].fir, edges[a].sec, b);
      else
        break;
  return 0;
```

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define endl '\n'
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 705
#define mod 1000000007
#define INF 1e9
struct edge
  int to, from, flow, capacity, id;
struct dinic
  int n, src, sink;
 vector<vector<edge>> adj;
 vector<int> level;
 vector<int> ptr;
  dinic(int sz)
   n = sz;
   adj.resize(n);
   level.resize(n);
    ptr.resize(n);
  void add_edge(int a, int b, int c, int id)
    adj[a].pb({b, (int)adj[b].size(), c, c, id});
    adj[b].pb({a, (int)adj[a].size() - 1, 0, 0, id});
  bool bfs()
    level.assign(n, -1);
    level[src] = 0;
    queue<int> q;
    q.push(src);
```

```
while (!q.empty())
   int u = q.front();
   q.pop();
   for (auto at : adj[u])
      if (at.flow && level[at.to] == -1)
        q.push(at.to);
        level[at.to] = level[u] + 1;
 return level[sink] != -1;
int dfs(int u, int flow)
 if (u == sink || flow == 0)
   return flow;
 for (int &p = ptr[u]; p < adj[u].size(); p++)</pre>
   edge &at = adj[u][p];
   if (at.flow && level[u] == level[at.to] - 1)
      int kappa = dfs(at.to, min(flow, at.flow));
      at.flow -= kappa;
      adj[at.to][at.from].flow += kappa;
     if (kappa != 0)
        return kappa;
 return 0;
int run()
 int max flow = 0;
 while (bfs())
   ptr.assign(n, 0);
   while (1)
     int flow = dfs(src, INF);
     if (flow == 0)
       break;
     max_flow += flow;
   }
 return max_flow;
vector<pii> cut_edges() // arestas do corte minimo
```

```
bfs();
    vector<pii> ans;
    for (int i = 0; i < n; i++)
      for (auto const & j : adj[i])
        if (level[i] != -1 \&\& level[j.to] == -1 \&\& j.
           capacity > 0)
          ans.pb({j.capacity, {i, j.to}});
    return ans;
  vector<int> flow_edges(int n, int m) // fluxo em cada
     aresta, na ordem da entrada
    vector<int> ans(m);
    for (int i = 0; i < n; i++)
      for (auto const &j : adj[i])
        if (!j.capacity)
          ans[j.id] = j.flow;
    return ans;
} ;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int n, m;
  cin >> n >> m;
  dinic d(n);
  for (int i = 0; i < m; i++)
   int a, b, c;
   cin >> a >> b >> c;
    a--, b--;
    d.add_edge(a, b, c, i);
  d.src = 0, d.sink = n - 1;
  cout << d.run() << endl;</pre>
  vector<int> ans = d.flow_edges(n, m);
  for (auto const &i : ans)
    cout << i << endl;</pre>
  return 0;
```

7.16 hopcroft karp

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 200001
#define mod 1000000007
#define INF 1e9
struct hopcroft_karp
  vector<int> match;
  vector<int> dist;
  vector<vector<int>> adj;
  int n, m, t;
  hopcroft karp(int a, int b)
    n = a, m = b;
    t = n + m + 1;
    match.assign(t, n + m);
    dist.assign(t, 0);
    adj.assign(t, vector<int>{});
  void add edge(int u, int v)
    adj[u].pb(v);
    adj[v].pb(u);
  bool bfs()
    queue<int> q;
    for (int u = 0; u < n; u++)
      if (match[u] == n + m)
        dist[u] = 0, q.push(u);
```

```
else
      dist[u] = INF;
  dist[n + m] = INF;
  while (!q.empty())
    int u = q.front();
    q.pop();
    if (dist[u] < dist[n + m])</pre>
      for (auto const &v : adj[u])
        if (dist[match[v]] == INF)
          dist[match[v]] = dist[u] + 1;
          q.push(match[v]);
  return dist[n + m] < INF;</pre>
bool dfs(int u)
  if (u < n + m)
    for (auto const &v : adj[u])
      if (dist[match[v]] == dist[u] + 1 && dfs(match[v])
          1))
        match[v] = u;
        match[u] = v;
        return true;
    dist[u] = INF;
    return false;
  return true;
vector<pi> run()
  int cnt = 0;
  while (bfs())
    for (int u = 0; u < n; u++)
      if (match[u] == n + m \&\& dfs(u))
        cnt++;
  vector<pi> ans;
  for (int v = n; v < n + m; v++)
```

7.17 dsu rollback

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 600005
#define mod 1000000007
namespace dsu
  struct rollback
    int u, v, rankv, ranku;
  };
  int num sets;
  int parent[MAXN];
  int rank[MAXN];
  stack<rollback> op;
  int Find(int i)
```

```
return (parent[i] == i) ? i : Find(parent[i]);
  bool Union(int x, int y)
    int xx = Find(x);
    int yy = Find(y);
    if (xx != yy)
      num sets--;
      if (rank[xx] > rank[yy])
        swap(xx, yy);
      op.push({xx, yy, rank[xx], rank[yy]});
      parent[xx] = yy;
      if (rank[xx] == rank[yy])
        rank[yy]++;
      return true;
    return false;
  void do rollback()
    if (op.empty())
      return:
    rollback x = op.top();
    op.pop();
    num_sets++;
    parent[x.v] = x.v;
    rank[x.v] = x.rankv;
    parent[x.u] = x.u;
    rank[x.u] = x.ranku;
  void init(int n)
    for (int i = 0; i < n; i++)
      parent[i] = i;
      rank[i] = 0;
    num\_sets = n;
namespace seg
  struct query
    int v, u, is_bridge;
  vector<vector<query>> t(4 * MAXN);
```

```
int ans[MAXN];
 void add(int i, int l, int r, int gl, int gr, guery g)
   if (1 > r || 1 > qr || r < q1)
     return;
    if (1 >= g1 \&\& r <= gr)
     t[i].push_back(q);
     return;
    int mid = (1 + r) >> 1;
    add((i << 1), l, mid, ql, qr, q);
    add((i << 1) | 1, mid + 1, r, ql, qr, q);
  void dfs(int i, int l, int r)
   for (query &q : t[i])
     if (dsu::Union(q.v, q.u))
       q.is_bridge = 1;
    if (1 == r)
     ans[1] = dsu::num_sets;
    else
     int mid = (1 + r) >> 1;
     dfs((i << 1), 1, mid);
     dfs((i << 1) | 1, mid + 1, r);
   for (query q : t[i])
     if (q.is_bridge)
       dsu::do rollback();
signed main()
 ios_base::sync_with_stdio(false);
 cin.tie(NULL);
 int n, q;
 cin >> n >> q;
  int time = 0;
 map<pi, int> tin;
 vector<int> queries;
 while (q--)
    char t;
   cin >> t;
   if (t == '?')
     queries.pb(++time);
```

```
else if (t == '+')
      int a, b;
      cin >> a >> b;
      a--, b--;
      if (a > b)
        swap(a, b);
      tin[{a, b}] = ++time;
    else
      int a, b;
      cin >> a >> b;
      a--, b--;
      if (a > b)
        swap(a, b);
      seq::query kappa = \{a, b, 0\};
      seg::add(1, 0, MAXN - 1, tin[{a, b}], ++time,
         kappa);
      tin[{a, b}] = -1;
  for (auto const &i : tin)
   if (i.sec !=-1)
      seq::query kappa = {i.fir.fir, i.fir.sec, 0};
      seg::add(1, 0, MAXN - 1, i.sec, ++time, kappa);
  }
  dsu::init(n);
  seg::dfs(1, 0, MAXN - 1);
  for (auto const &i : queries)
    cout << seg::ans[i] << endl;</pre>
  return 0;
// https://codeforces.com/edu/course/2/lesson/7/3/
   practice/contest/289392/problem/C
// conectividade dinamica
// para uma query (u, v)
// podemos descrever em um intervalo [1, r]
// 1 = quando a aresta (u, v) foi adicionada
// r = quando a aresta (u, v) foi removida
// dai agora que temos um intervalo, podemos adicionar
// a query (u, v) em uma segtree "adaptada"
// no final rodamos um dfs nessa segtree e vamos
   atualizando as repostas das gueries
// quando estamos em uma posicao na seg, dou union em
   todos os caras daquela posicao
// e em seguida chamo pros meus filhos, quando chego em
```

```
uma folha, ela eh equivalente
// a uma unidade de "tempo", logo a resposta para aquele
    tempo eh a resposta atual no dsu
// e ao sair recursivamente, vou dando rollbacks no dsu
```

7.18 MatrixDijkstra

```
#include <bits/stdc++.h>
using namespace std;
#define lli long long int
#define pb push_back
#define MAXN 10000000
typedef pair <int , int> pii ;
int t;
int dist [MAXN] ;
bool visited [MAXN] ;
vector <pii> adj_list [MAXN] ;
void dijkstra (int s)
    dist[s] = 0;
    priority_queue <pii , vector<pii> , greater<pii>> q
    q.push(pii(dist[s], s));
    while (1)
        int davez = -1;
        int menor = INT MAX ;
        while(!q.empty())
            int atual = q.top().second;
            q.pop();
            if(!visited[atual])
                davez = atual;
                break;
        if(davez == -1)
            break ;
```

```
visited[davez] = true ;
        for(int i = 0 ; i < adj list[davez].size() ; i</pre>
            int distt = adj list[davez][i].first;
            int atual = adj_list[davez][i].second ;
            if(dist[atual] > dist[davez] + distt)
                dist[atual] = dist[davez] + distt ;
                q.push(pii(dist[atual] , atual)) ;
void initialize ()
    for (int i = 0; i < t; i++)
        visited[i] = false ;
        dist[i] = INT MAX ;
int main()
    ios_base::sync_with_stdio(false);
    cin.tie(NULL);
    int n , m ;
    cin >> n >> m;
    t = n * m ;
    char array [t] ;
    for (int i = 0; i < t; i++)
        cin >> array[i] ;
    for (int i = 0; i < t; i++)
        if (i >= m && array[i] != '#')
            adj_list[i].pb(pii(1 , (i - m)));
        if (i < (n * m) - m && array[i] != '#')
```

```
adj_list[i].pb(pii(1 , (i + m)));
   if (i % m != 0 && array[i] != '#')
       adj_list[i].pb(pii(1,(i-1)));
    if ((i + 1) % m != 0 && array[i] != '#')
       adj_list[i].pb(pii(1, (i + 1)));
}
int q;
cin >> q;
while (q--)
   int a , b , c , d , e ;
   cin >> a >> b >> c >> d >> e ;
   a--, b--, c--, d--;
   int index1 = (m * a) + b;
   int index2 = (m * c) + d;
   adj list[index1].pb(pii(e , index2));
   adj_list[index2].pb(pii(e , index1));
}
initialize ();
dijkstra(0);
cout << dist[t - 1] << endl ;
return 0 ;
```

7.19 bridges

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;
```

```
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 400005
#define mod 1000000007
int n, m, timer;
vector<pi> edges;
vector<bool> is_bridge;
vector<pi> adj[MAXN];
int tin[MAXN];
int low[MAXN];
bool vis[MAXN];
void dfs(int v, int p)
 vis[v] = true;
 tin[v] = timer, low[v] = timer++;
  for (auto const &u : adj[v])
    if (u.fir == p)
      continue;
    if (vis[u.fir])
     low[v] = min(low[v], tin[u.fir]);
      continue;
    dfs(u.fir, v);
    low[v] = min(low[v], low[u.fir]);
    if (low[u.fir] > tin[v])
     is bridge[u.sec] = 1;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  cin >> n >> m;
  is bridge.resize(m);
  for (int i = 0; i < m; i++)
    int a, b;
    cin >> a >> b;
    a--, b--;
    edges.pb({a, b});
    adj[a].pb({b, i});
    adj[b].pb({a, i});
```

```
}
memset(tin, -1, sizeof(tin));
memset(low, -1, sizeof(low));
for (int i = 0; i < n; i++)
{
   if (!vis[i])
     dfs(i, -1);
}
return 0;
}</pre>
```

7.20 Ford Fulkerson

```
// ford-fulkerson: obter qual o fluxo maximo de um
   vertice s ate um vertice d
// 1 - rodar um bfs para descobrir um novo caminho de s
   ate d
// 2 - apos isso pego a aresta de menor custo desse
   caminho e subtraio o valor dela nas outras arestas
   do caminho
// 3 - fluxo maximo += custo da aresta de menor custo
   desse caminho
// 4 - rodar isso ate nao existirem mais caminhos
   disponiveis (com fluxo diferente de 0) entre s e d
// 5 - o fluxo maximo de s ate d sera a soma das arestas
    de menor custo de cada caminho feito
#include <bits/stdc++.h>
using namespace std;
#define lli long long int
#define pb push_back
#define MAXN 10000
#define INF 999999
int n , m , a , b , c , s , d , max_flow , flow ;
vector <int> parent ;
vector <int> adj [MAXN] ;
int cost [MAXN] [MAXN] ;
bool visited [MAXN] ;
void get_menor_custo (int v , int mincost)
    if (v == s)
        flow = mincost;
        return ;
    else if (parent[v] != -1)
```

```
get_menor_custo(parent[v], min(mincost, cost[
           parent[v]][v]));
        cost[parent[v]][v] -= flow ;
        cost[v][parent[v]] += flow;
void bfs ()
    visited[s] = true ;
    queue <int> q;
    q.push(s);
    parent.assign(MAXN , -1) ;
    while (!q.empty())
        int u = q.front();
        q.pop();
        if (u == d)
           break ;
        for (int j = 0; j < adj[u].size(); j++)
            int v = adj[u][j] ;
           if (cost[u][v] > 0 && !visited[v])
                visited[v] = true ;
                q.push(v);
                parent[v] = u;
int ford fulkerson ()
    \max flow = 0;
    while (1)
        flow = 0;
        memset(visited, false, sizeof(visited));
        bfs();
        get_menor_custo(d , INF) ;
        if (flow == 0)
```

```
break;
        max flow += flow ;
    return max flow ;
int main ()
    ios_base::sync_with_stdio(false) ;
    cin.tie(NULL) ;
    cin >> n >> m;
    for (int i = 0; i < m; i++)
        cin >> a >> b >> c;
        adj[a].pb(b);
        adj[b].pb(a);
        cost[a][b] = c;
    cin >> s >> d;
    cout << ford_fulkerson() << endl ;</pre>
    return 0 ;
```

7.21 caminhoeuleriano

```
// caminho euleriano em um grafo
// passa por todas as arestas apenas uma unica vez e
    percorre todas elas
// condicao de existencia:
// todos os vertices possuem grau par (ciclo euleriano)
    comeca e acaba no mesmo vertice
// ou
// apenas 2 vertices possuem grau impar, todos os outros
    possuem grau par ou == 0.
// comeca num vertice de grau impar e termina num
    vertice de grau impar nesse caso.
// solucao:
// rodar um dfs com map de visited para as arestas
// no final por o source no vector path
// ao final teremos o caminho inverso no vector path
// note que o caminho inverso tambem e um caminho valido
```

```
#include <bits/stdc++.h>
using namespace std;
#define lli long long int
#define pb push_back
#define in insert
#define pi pair<int, int>
#define pd pair<double, int>
#define pib pair<pi, bool>
#define mp make pair
#define fir first
#define sec second
#define MAXN 10001
#define MAXL 1000001
#define mod 1000000007
int n, m, start;
vector<int> path;
vector<int> adj[MAXN];
map<pi, bool> visited;
void dfs(int s)
  for (int i = 0; i < adj[s].size(); i++)</pre>
    int v = adj[s][i];
    if (!visited[mp(s, v)])
      visited[mp(s, v)] = true;
      visited[mp(v, s)] = true;
      dfs(v):
  path.pb(s);
bool check()
  int odd = 0;
  for (int i = 0; i < n; i++)
    if (adj[i].size() & 1)
      odd++, start = i;
  return (odd == 0 || odd == 2);
signed main()
  cin >> n >> m;
  for (int i = 0; i < m; i++)
    int a, b;
    cin >> a >> b;
```

```
adj[a].pb(b);
adj[b].pb(a);
}
start = 0;
bool ok = check();
(ok) ? cout << "Yes\n" : cout << "No\n";
if (ok)
{
   dfs(start);
   for (int i = 0; i < path.size(); i++)
      cout << path[i] << " ";
   cout << "\n";
}
return 0;
}</pre>
```

7.22 mo trees

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 500001
#define mod 998244353
struct gry
  int 1, r, lca, id;
} ;
int n, q;
vector<int> adj[MAXN];
int v[MAXN];
int cnt[MAXN];
int freq[MAXN];
int tin[MAXN];
int tout[MAXN];
int depth[MAXN];
```

```
int up[MAXN][25];
vector<int> t;
vector<qry> qq;
void dfs(int s, int p)
 tin[s] = t.size();
 up[s][0] = p;
  for (int i = 1; i < 25; i++)
    up[s][i] = up[up[s][i - 1]][i - 1];
  t.pb(s);
  for (auto const &i : adj[s])
    if (i == p)
      continue;
    depth[i] = depth[s] + 1;
    dfs(i, s);
  tout[s] = t.size();
  t.pb(s);
bool is(int u, int v)
  return tin[u] <= tin[v] && tout[u] >= tout[v];
int lca(int u, int v)
 if (is(u, v))
    return u;
  if (is(v, u))
    return v:
  for (int i = 24; i >= 0; i--)
    if (!is(up[u][i], v))
      u = up[u][i];
  return up[u][0];
void compress()
 vector<int> vals;
  for (int i = 0; i < n; i++)
    vals.pb(v[i]);
  sort(vals.begin(), vals.end());
  vals.erase(unique(vals.begin(), vals.end()), vals.end
     ());
  for (int i = 0; i < n; i++)
    v[i] = lower_bound(vals.begin(), vals.end(), v[i]) -
        vals.begin();
```

```
signed main()
 ios base::sync with stdio(false);
 cin.tie(NULL);
 while (cin >> n >> q)
   t.clear();
    qq.clear();
    depth[0] = 0;
   memset(cnt, 0, sizeof(cnt));
   memset(freq, 0, sizeof(freq));
    for (int i = 0; i < n; i++)
     adj[i].clear();
      cin >> v[i];
    compress();
    for (int i = 0; i < n - 1; i++)
      int a, b;
     cin >> a >> b;
     a--, b--;
     adj[a].pb(b);
      adj[b].pb(a);
    dfs(0, 0);
    for (int i = 0; i < q; i++)
     int x, y;
     cin >> x >> y;
     x--, y--;
      int 1 = lca(x, y);
      if (tin[x] > tin[y])
       swap(x, y);
      if (1 == x)
        qq.pb(\{tin[x], tin[y], -1, i\});
      else
        qq.pb({tout[x], tin[y], 1, i});
   int block = sqrt(n) + 1;
   auto cmp = [&] (qry x, qry y)
      if (x.1 / block != y.1 / block)
        return x.l / block < y.l / block;</pre>
      return x.r < y.r;</pre>
    };
    sort(qq.begin(), qq.end(), cmp);
   vector<int> ans(q);
   int cl = 0, cr = 0, resp = 0;
    auto add2 = [\&] (int x)
```

```
freq[v[x]]++;
  if (freq[v[x]] == 1)
    resp++;
};
auto rem2 = [\&] (int x)
  freq[v[x]]--;
  if (freq[v[x]] == 0)
    resp--;
auto add = [&](int x)
  cnt[x]++;
  if (cnt[x] == 2)
    rem2(x);
  else
    add2(x);
} ;
auto rem = [\&] (int x)
  cnt[x]--;
  if (cnt[x] == 1)
    add2(x);
  else
    rem2(x);
};
for (int i = 0; i < q; i++)
  int idx = qq[i].id;
  int 1 = qq[i].1;
  int r = qq[i].r;
  int lc = qq[i].lca;
  while (cl < 1)
    rem(t[cl++]);
  while (cl > 1)
    add(t[--cl]);
  while (cr <= r)
    add(t[cr++]);
  while (cr > r + 1)
    rem(t[--cr]);
  if (1c != -1)
    add(lc);
  ans[idx] = resp;
  if (1c != -1)
    rem(lc);
for (auto const &i : ans)
  cout << i << endl;</pre>
```

```
return 0;
// https://www.spoj.com/problems/COT2/
// quantos caras distintos em um path entre u e v
// mo em arvores
// acha o euler tour da arvore com tin e tout
// desconsidera no mo os indices duplicados no range
// para queries em subtree eh mais simples:
// apenas saber o tamanho da subtree de i
// fazer o euler tour apenas com o tin
// e fzr a query pro range tin[i] ate tin[i] + sz[i] - 1
// pra queries de path com peso nos edges:
// https://codeforces.com/gym/100962/attachments (
   problema F)
// considera v[i] -> peso do edge que liga ao meu pai na
// dai pra query com o lca == u, nao tenho que
   considerar v[u] ([tin[u], tin[v]], dps removendo v[u
// e pra query com o lca != u, so fazer ela normalmente
   ([tout[u], tin[v]])
```

7.23 LCA

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace qnu pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define PI acos(-1)
#define pb push back
#define int long long int
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 100001
#define mod 1000000007
int n;
vector<int> adj[MAXN];
```

```
namespace lca
  int 1, timer;
  vector<int> tin, tout, depth;
 vector<vector<int>> up;
  void dfs(int v, int p)
   tin[v] = ++timer;
   up[v][0] = p;
   for (int i = 1; i <= 1; i++)
      up[v][i] = up[up[v][i-1]][i-1];
    for (auto const &u : adj[v])
      if (p == u)
        continue;
      depth[u] = depth[v] + 1;
      dfs(u, v);
   tout[v] = ++timer;
  bool is_ancestor(int u, int v)
    return tin[u] <= tin[v] && tout[u] >= tout[v];
  int binary lifting(int u, int v)
   if (is_ancestor(u, v))
      return u;
   if (is_ancestor(v, u))
      return v:
   for (int i = 1; i >= 0; --i)
      if (!is_ancestor(up[u][i], v))
        u = up[u][i];
   return up[u][0];
  void init()
   tin.resize(n):
    tout.resize(n);
   depth.resize(n);
   timer = 0;
   1 = ceil(log2(n));
   up.assign(n, vector<int>(l + 1));
   dfs(0, 0);
  int dist(int s, int v)
    int at = binary lifting(s, v);
    return (depth[s] + depth[v] - 2 * depth[at]);
```

```
}
signed main()
{
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  cin >> n;
  for (int i = 0; i < n - 1; i++)
  {
    int a, b;
    cin >> a >> b;
    a--, b--;
    adj[a].pb(b);
    adj[b].pb(a);
}
lca::init();
return 0;
}
```

7.24 Topological Sort

```
#include <bits/stdc++.h>
using namespace std;
#define lli long long int
#define pb push back
#define MAXN 10000
int n , m , a , b ;
vector <int> adj [MAXN] ;
int grau [MAXN];
vector <int> order ;
bool topological_sort ()
    int ini = 0;
    while (ini < order.size())</pre>
        int atual = order[ini] ;
        ini++ ;
        for (int i = 0 ; i < adj[atual].size() ; i++)</pre>
            int v = adj[atual][i] ;
            grau[v]-- ;
            if (grau[v] == 0)
                order.pb(v);
```

```
return (order.size() == n) ? true : false ;
int main ()
    ios_base::sync_with_stdio(false);
    cin.tie(NULL) ;
    cin >> n >> m;
    for (int i = 1; i \le m; i++)
        cin >> a >> b;
        grau[a]++;
        adj[b].pb(a);
    for (int i = 1; i <= n; i++)
        if (grau[i] == 0)
            order.pb(i);
    if (topological_sort())
        for (int i = 0 ; i < order.size() ; i++)
            cout << order[i] << " ";</pre>
        cout << endl ;</pre>
    else
        cout << "Impossible\n" ;</pre>
    return 0 ;
```

7.25 Dijkstra

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
```

```
UFAL Notebook
```

```
using namespace std;
using namespace __qnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define DEBUG 1
#define MAXN 1001
#define mod 1000000007
int n, m;
vector<pi> adj[MAXN];
bool visited[MAXN];
int dist[MAXN];
void dijkstra(int s)
  for (int i = 0; i < n; i++)
    dist[i] = INT_MAX;
    visited[i] = false;
  priority_queue<pi, vector<pi>, greater<pi>> q;
  dist[s] = 0;
  q.push({dist[s], s});
  while (!q.empty())
    int v = q.top().second;
    q.pop();
    if (visited[v])
      continue;
    visited[v] = true;
    for (auto const &u : adj[v])
      if (dist[u.sec] > dist[v] + u.fir)
        dist[u.sec] = dist[v] + u.fir;
        q.push({dist[u.sec], u.sec});
signed main()
```

```
ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  cin >> n >> m;
  for (int i = 0; i < m; i++)
    int a, b, c;
    cin >> a >> b >> c;
    a--, b--;
    adj[a].pb({c, b});
    adj[b].pb({c, a});
  dijkstra(0);
}
```

7.26 centroid decomposition

```
// centroid de uma arvore -> e um no que ao ser removido
    da arvore, separaria as
// arvores resultantes de modo com que a maior arvore
   desse conjunto teria no maximo
// (n / 2) nos, sendo n o numero de nos da arvore. Para
   qualquer arvore com n nos,
// o centroid sempre existe.
//
   // centroid decomposition -> muito util para tentar
   diminuir a complexidade em certos
// tipos de consultas a serem feitas, uma maneira melhor
    de organizar a arvore.
// algoritimo:
// 1) o centroid e a raiz dessa nova arvore
// 2) achar o centroid das arvores menores que surgiram
   com a remocao do centroid "pai"
// 3) por uma aresta entre o centroid "filho" e o
   centroid "pai"
// 4) repetir isso ate todos os nos serem removidos
// 5) ao final teremos a centroid tree
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __qnu_pbds;
template <class T>
```

```
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define PI acos(-1)
#define pb push back
#define int long long int
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 100001
#define mod 1000000007
int n;
vector<int> adj[MAXN];
namespace cd
  int sz;
  vector<int> adjl[MAXN];
  vector<int> father, subtree_size;
  vector<bool> visited;
  void dfs(int s, int f)
    SZ++;
    subtree size[s] = 1;
    for (auto const &v : adj[s])
      if (v != f && !visited[v])
        dfs(v, s);
        subtree size[s] += subtree size[v];
  int getCentroid(int s, int f)
    bool is centroid = true;
    int heaviest child = -1;
    for (auto const &v : adj[s])
      if (v != f && !visited[v])
        if (subtree size[v] > sz / 2)
          is centroid = false;
        if (heaviest_child == -1 || subtree_size[v] >
           subtree size[heaviest child])
          heaviest_child = v;
```

```
return (is centroid && sz - subtree size[s] <= sz /</pre>
       2) ? s : getCentroid(heaviest_child, s);
  int decompose_tree(int s)
    sz = 0;
    dfs(s, s);
    int cend tree = getCentroid(s, s);
    visited[cend tree] = true;
    for (auto const &v : adj[cend_tree])
      if (!visited[v])
        int cend subtree = decompose tree(v);
        adil[cend tree].pb(cend subtree);
        adjl[cend subtree].pb(cend tree);
        father[cend_subtree] = cend_tree;
    return cend_tree;
  void init()
    subtree size.resize(n);
    visited.resize(n);
    father.assign(n, -1);
    decompose tree(0);
signed main()
  ios base::sync with stdio(false);
  cin.tie(NULL);
  cin >> n;
  for (int i = 0; i < n - 1; i++)
    int a, b;
    cin >> a >> b;
    a--, b--;
    adj[a].pb(b);
    adj[b].pb(a);
  cd::init();
  return 0;
```

```
#include <bits/stdc++.h>
using namespace std;
#define MAXN 500000
int n , m ;
int visited [MAXN] ;
vector <int> adj_list [MAXN] ;
void dfs (int x)
    for (int i = 0 ; i < adj_list[x].size() ; i++)</pre>
        int v = adj_list[x][i] ;
        if(visited[v] == -1)
            visited[v] = visited[x];
            dfs(v);
void initialize ()
    for (int i = 1 ; i <= n ; i++)</pre>
        visited[i] = -1;
int main ()
    int a , b ;
    cin >> n >> m;
    initialize();
    for (int i = 1 ; i <= m ; i++)</pre>
        cin >> a >> b;
        adj_list[a].push_back(b);
        adj_list[b].push_back(a);
    }
    dfs(1);
    return 0;
```

7.28 cycle detection

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define PI acos(-1)
#define pb push back
#define int long long int
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 205
#define MAXP 100001
#define mod 100000007
int n, m, idx;
vector<int> cycles[MAXN];
vector<int> adj[MAXN];
int color[MAXN];
int parent[MAXN];
int ans[MAXN];
void dfs(int u, int p)
  if (color[u] == 2)
    return;
  if (color[u] == 1)
    idx++;
    int curr = p;
    ans[curr] = idx;
    cycles[idx].pb(curr);
    while (curr != u)
      curr = parent[curr];
      cycles[idx].pb(curr);
      ans[curr] = idx;
    return;
  parent[u] = p;
```

```
color[u] = 1;
  for (auto const &v : adj[u])
   if (v != parent[u])
     dfs(v, u);
 color[u] = 2;
signed main()
 ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  cin >> n >> m;
  for (int i = 0; i < m; i++)
    int a, b;
   cin >> a >> b;
   a--, b--;
   adj[a].pb(b);
    adj[b].pb(a);
 for (int i = 0; i < n; i++)
   if (!color[i])
      dfs(i, -1);
 cout << idx << endl;</pre>
  for (int i = 1; i <= idx; i++)
   cout << cycles[i].size() << endl;</pre>
    for (auto const & j : cycles[i])
    cout << j + 1 << " ";
   cout << endl;</pre>
  return 0;
```

7.29 Kruskal

```
// Algoritimo de kruskal - Achar a mst

// 1 - listar todas as arestas em ordem crescente.

// 2 - Cada aresta liga dois vertices x e y, checar se eles ja estao na mesma componente conexa

// (aqui, consideramos apenas as arestas ja colocadas na arvore).

// 3 - Se x e y estao na mesma componente, ignoramos a aresta e continuamos o procedimento

// (se a usassemos, formariamos um ciclo). Se estiverem em componentes distintas, colocamos a aresta

//na arvore e continuamos o procedimento.
```

```
// OBS: como a prioridade eh ordenar pelas menores
   distancias, basta botar o custo da aresta como
// first no vector das arestas para poder ordenar
// em suma: ordeno as arestas em ordem crescente com
   prioridade no custo, depois para cada aresta,
// se o find(x) != find(y) sendo x e y os vertices das
   arestas, eu adiciono eles a mst e dou um join
// nos dois, como as arestas tao ordenadas em ordem
   crescente, o primeiro que eu pego
// eh necessariamente a melhor opcao e assim a mst eh
   formada.
#include <bits/stdc++.h>
using namespace std;
#define lli long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define mp make pair
#define fir first
#define sec second
#define MAXN 100001
int n, m, a, b, c;
vector<pii> ar;
vector<pii> mst;
int pai[MAXN];
int peso[MAXN];
int find(int x)
  if (pai[x] == x)
    return x;
  return pai[x] = find(pai[x]);
void join(int a, int b)
  a = find(a);
  b = find(b);
  if (peso[a] < peso[b])</pre>
    pai[a] = b;
  else if (peso[b] < peso[a])</pre>
```

```
pai[b] = a;
  else
    pai[a] = b;
    peso[b]++;
void initialize()
  for (int i = 1; i <= n; i++)</pre>
    pai[i] = i;
int main()
  ios base::sync with stdio(false);
  cin.tie(NULL);
  cin >> n >> m;
  for (int i = 0; i < m; i++)
    cin >> a >> b >> c;
    ar.pb(mp(c, mp(a, b)));
  sort(ar.begin(), ar.end());
  initialize();
  int size = 0;
  for (int i = 0; i < m; i++)
    if (find(ar[i].sec.fir) != find(ar[i].sec.sec))
      join(ar[i].sec.fir, ar[i].sec.sec);
      mst.pb(mp(ar[i].fir, mp(ar[i].sec.fir, ar[i].sec.
         sec)));
  for (int i = 0; i < mst.size(); i++)</pre>
    cout << mst[i].sec.fir << " " << mst[i].sec.sec << "</pre>
         " << mst[i].fir << endl;
```

```
return 0;
}
```

7.30 BFS

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define DEBUG 1
#define MAXN 1001
#define mod 1000000007
int n, m;
vector<int> adj[MAXN];
bool visited[MAXN];
void bfs(int s)
  queue<int> q;
  q.push(s);
  while (!q.empty())
    int v = q.front();
    q.pop();
    if (visited[v])
      continue;
    visited[v] = true;
    for (auto const &u : adj[v])
      if (!visited[u])
        q.push(u);
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  cin >> n >> m;
```

```
for (int i = 0; i < m; i++)
{
   int a, b, c;
   cin >> a >> b >> c;
   a--, b--;
   adj[a].pb(b);
   adj[b].pb(a);
}
bfs(0);
```

8 Strings

8.1 suffix automaton

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace gnu pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 100001
#define mod 998244353
namespace sa
  struct state
    int len, suf_link;
    map<char, int> nxt;
  };
  state st[2 * MAXN];
  int dp[2 * MAXN];
  int sz, last;
  void init()
    memset (dp, -1, sizeof (dp));
```

```
st[0].suf_link = -1;
  SZ++;
 last = 0;
void get_link(int curr, int p, char c)
  while (p != -1 \&\& !st[p].nxt.count(c))
    st[p].nxt[c] = curr;
    p = st[p].suf link;
  if (p == -1)
    st[curr].suf link = 0;
    return;
  int q = st[p].nxt[c];
  if (st[p].len + 1 == st[q].len)
    st[curr].suf_link = q;
    return;
  int clone = sz;
  SZ++;
  st[clone].len = st[p].len + 1;
  st[clone].nxt = st[q].nxt;
  st[clone].suf_link = st[q].suf_link;
  while (p != -1 \&\& st[p].nxt[c] == q)
    st[p].nxt[c] = clone;
    p = st[p].suf_link;
  st[q].suf link = clone;
  st[curr].suf_link = clone;
void build(string &s)
  for (auto const &c : s)
    int curr = sz;
    SZ++;
    st[curr].len = st[last].len + 1;
    get_link(curr, last, c);
    last = curr;
 }
void dfs2(int v)
  if (dp[v] != -1)
```

st[0].len = 0;

```
return;
    dp[v] = 1;
    for (auto const &u : st[v].nxt)
     if (!u.sec)
        continue;
      dfs2(u.sec);
      dp[v] += dp[u.sec];
  void dfs(int v, int k, int &at, string &curr)
   if (at == k)
      return;
    for (auto const &u : st[v].nxt)
      if (!u.sec)
        continue;
      if (at + dp[u.sec] < k)
        at += dp[u.sec];
        continue;
      curr.pb(u.fir);
      at++;
      dfs(u.sec, k, at, curr);
      if (at == k)
        return;
      curr.pop_back();
  void find kth(int k)
   int at = 0;
    string curr = "";
   dfs(0, k, at, curr);
    cout << curr << endl;</pre>
} // namespace sa
signed main()
 ios base::sync with stdio(false);
 cin.tie(NULL);
 string s;
 cin >> s;
 sa::init();
 sa::build(s);
 sa::dfs2(0);
 int q;
  cin >> q;
```

```
while (q--)
    int k;
    cin >> k;
    sa::find kth(k);
  return 0;
// https://cp-algorithms.com/string/suffix-automaton.
   h + m1
// suffix automaton
// definicao: um suffix automaton de uma string s e um
   automato finito deterministico
// que aceita todos os suffixos da string s.
// ou seja:
// um suffix automaton eh um grafo aciclico orientado
// tal que, um vertice representa um estado
// e uma aresta representa uma transicao (um caractere a
    mais em relacao ao estado(suffixo) atual)
// t0 -> estado inicial(string vazia), e todos os demais
    estados podem ser alcancados a partir de t0
// o suffix automaton minimiza o numero de vertices
// a propiedade mais importante de um suffix automaton
   eh a de que
// ele contem informacoes sobre todas as substrings de s
// pois, qualquer caminho comecando do estado t0
   corresponde a uma substring de s
// conceitos:
// 1 - endpos
// seja t uma substring de s, endpos(t) eh o conjunto de
    todas os indices (posicoes)
// na string s no qual todas as ocorrencias de t acabam
// por exemplo, se s = "abcbc" e t = "bc"
// logo endpos(t) = {2, 4}
// com isso se duas duas substrings t1 e t2 possuem os
   seus endpos iguais,
// chamamos de endpos-equivalent e dai podemos extrair
   algumas informacoes
// info 1: se duas substrings u e w u.size() <= w.size()</pre>
   , se u eh um sufixo de w, logo endpos(u) esta
   contido em endpos(w)
// info 2: se duas substrings u e w u.size() <= w.size()</pre>
   , se u nao eh um sufixo de w, logo nao existe
   interseccao entre endpos(u) e endpos(w)
// 2 - suffix link
// seja v algum estado != t0, sabemos que v corresponde
```

```
a classe de strings que possui os mesmos endpos
// seja w a maior dessas strings, com isso, todas as
   demais sao suffixos de w
// com isso um suffix_link(v) corresponde ao maior
   suffix de w que esta em outra classe de equivalencia
    pelos endpos
// com isso podemos abstrair algumas informacoes:
// info 1: os suffix links foram uma arvore enraizada em
// info 2: se construirmos uma arvore usando os sets
   endpos, a estrutura sera a arvore com os suffix
   links
// com isso, vamos ao algoritimo
// 1 - vai ser online, e iremos adicionar os caracteres
   de 1 por 1, da esquerda para a direita
// 2 - com isso para adicionar um novo char, seja v o
   ultimo estado que adicionamos antes do atual,
   adicionamos uma aresta
// do proximo em relacao a ele e iremos procurar pelo
   suffix link para adicionar
// 3 - complexidade O(n) ou O(n log k), se usarmos uma
   map para quardar as transicoes partindo de um estado
// exemplos de aplicacoes:
// 1 - checar se t aparece em s como substring:
// construa o suffix automaton de s, e vamos tentar
   fazer um caminho partindo de t0
// se em algum momento, nao existir transicao, logo nao
   existe
// se consequir chegar no final, existe
// 2 - numero de substrings diferentes de s
// constura o suffix automaton de s, sabemos que, cada
   substring de s corresponde a um caminho no automato
// com isso, o numero de substrings distintas eh o
   numero de caminhos diferentes que comecam de t0
// e terminam em algum canto
// isso pode ser calculado facilmente com uma dpzinha
// 3 - tamanho total de todas as substrings distintas de
// similar a solucao passada, podemos fazer isso com uma
    dpzinha :)
// 4 - a k-esima menor substring lexicografica
// a k-esima menor substring lexicograficamente
   corresponde ao k-esimo path no suffix automaton
// se considerarmos as transicoes sempre indo do menor
```

```
char para o maior durante o percurso
// 5 - o menor cyclic shift
// construa o suffix automaton da string s + s (
// com isso o suffix automaton vai conter todos os
   cyclic shifts da string s
// e agora o problema eh reduzido para: encontre o menor
    caminho lexicograficamente de tamanho s.size()
// 6 - numero de ocorrencias de uma substring t em s
// construa o suffix automaton da string s
// com isso, quando criamos um no que nao seja o t0 nem
   um clone
// inicializamos cnt[v] = 1
// depois vamos percorrer todo os estados em ordem
   decrescente de len
// e aplicando cnt[link(v)] += cnt[v]
// no final, para responder a query basta fazer o
   caminho ate o estado que quisermos e printar o cnt
// e mais uma porrada de aplicacoes alem dessas :)
// example of a problem: https://www.spoj.com/problems/
   SUBLEX/
// ver qual a k-th string lexicografica sem repeticao
// note que o k pode ser gigante
// ideia: calcular dp[v] -> quantidade de caminhos que
   comecam em v
// dai para cada query roda um dfs, sendo que, so vou
   pro proximo estado se at + dp[u] >= k
// caso contrario, posso ignorar
```

8.2 suffix array

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;

#define int long long int
#define pb push_back
#define pi pair<int, int>
#define pii pair<pi, int>
```

```
#define pci pair<char, int>
#define fir first
#define sec second
#define MAXN 50005
#define mod 1000000007
void get_suf(string s)
  s += '$';
  int n = s.size();
  vector < int > p(n), c(n);
  vector<pci> a(n);
  for (int i = 0; i < n; i++)</pre>
    a[i] = {s[i], i};
  sort(a.begin(), a.end());
  for (int i = 0; i < n; i++)
    p[i] = a[i].sec;
  c[p[0]] = 0;
  for (int i = 1; i < n; i++)
    (a[i].fir == a[i - 1].fir) ? c[p[i]] = c[p[i - 1]] :
        c[p[i]] = c[p[i - 1]] + 1;
  int k = 0;
  while ((1 << k) < n)
    vector<pii> v(n);
    for (int i = 0; i < n; i++)
      v[i] = \{\{c[i], c[(i + (1 << k)) % n]\}, i\};
    sort(v.begin(), v.end());
    for (int i = 0; i < n; i++)</pre>
      p[i] = v[i].sec;
    c[p[0]] = 0;
    for (int i = 1; i < n; i++)
      (v[i].fir == v[i-1].fir) ? c[p[i]] = c[p[i-1]]
           : c[p[i]] = c[p[i - 1]] + 1;
    k++;
  for (int i = 0; i < n; i++)
    cout << p[i] << " ";
  cout << endl;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  string s;
  cin >> s;
  get_suf(s);
  return 0;
```

8.3 kmp

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace gnu pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define endl '\n'
#define pb push back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define MAXN 100005
#define mod 998244353
string s;
int n, m;
string a, b;
int c[MAXN] [26];
vector<int> kmp(string &s)
  int n = s.size();
  vector<int> p(n);
  for (int i = 1; i < n; i++)
    int j = p[i - 1];
    while (j > 0 \&\& s[i] != s[j])
      j = p[j - 1];
    if (s[i] == s[j])
      †++;
    p[i] = j;
  return p;
void compute(string s)
  s.pb('*');
  vector<int> p = kmp(s);
  for (int i = 0; i < s.size(); i++)</pre>
    for (int cc = 0; cc < 26; cc++)
```

```
int j = i;
      while ( \dot{j} > 0 \&\& 'a' + cc != s[\dot{j}] )
        j = p[j - 1];
      if ('a' + cc == s[j])
        <del>1</del>++;
      c[i][cc] = j;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  string s;
  cin >> s;
  compute(s);
  return 0;
// algoritmo eh online, vai coonstruindo da esquerda pra
    direita
// calcula pi[i], a seguinte funcao:
// seja a substring s.substr(0, i + 1)
// pi[i] = tamanho do maior prefixo que tbm eh um sufixo
    dessa substring
// dai por exemplo
// da pra contar a quantidade de matchings de s em t
// so concatenar as strings fazendo: t = s + "*" + t
// dai contar as posicoes com pi[i] = s.size()
// tambem eh possivel construir um automato do kmp
// do tipo
// se meu pi[i] == x, e leio a letra c
// dai devo ir pro estado p[i] == y
// as transicoes podem ser computadas e isso pode ser
   muito util
```

8.4 aho corasick

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;
```

```
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 5001
#define mod 1000000007
namespace aho
  int go(int v, char ch);
  const int K = 26; // tamanho do alfabeto
  struct trie
                       // char correspondente ao no
    char me:
       atual
   int go[K];
                       // proximo vertice que eu devo
      ir estando em um estado (v, c)
                  // proximo vertice da trie
    int down[K];
    int is leaf = 0;  // se o vertice atual da trie eh
        uma folha (fim de uma ou mais strings)
    int parent = -1;  // no ancestral do no atual
                      // link de sufixo do no atual (
    int link = -1;
       outro no com o maior matching de sufixo)
    int exit_link = -1; // folha mais proxima que pode
       ser alcancada a partir de v usando links de
       sufixo
    trie(int p = -1, char ch = '$') : parent(p), me(ch)
      fill (begin (go), end (go), -1);
      fill (begin (down), end (down), -1);
  };
  vector<trie> ac;
  void init() // criar a raiz da trie
   ac.resize(1);
  void add string(string s) // adicionar string na trie
   int v = 0;
    for (auto const &ch : s)
     int c = ch - 'a';
      if (ac[v].down[c] == -1)
       ac[v].down[c] = ac.size();
```

```
ac.emplace_back(v, ch);
   v = ac[v].down[c];
 ac[v].is_leaf++;
int get_link(int v) // pegar o suffix link saindo de v
 if (ac[v].link == -1)
    ac[v].link = (!v || !ac[v].parent) ? 0 : go(
       get_link(ac[v].parent), ac[v].me);
 return ac[v].link;
int go(int v, char ch) // proximo estado saindo do
   estado(v, ch)
 int c = ch - 'a';
 if (ac[v].go[c] == -1)
   if (ac[v].down[c] != -1)
      ac[v].go[c] = ac[v].down[c];
   else
      ac[v].go[c] = (!v) ? 0 : go(get_link(v), ch);
 return ac[v].go[c];
int get_exit_link(int v) // suffix link mais proximo
   de v que seja uma folha
  if (ac[v].exit_link == -1)
   int curr = get_link(v);
   if (!v || !curr)
     ac[v].exit link = 0;
   else if (ac[curr].is_leaf)
      ac[v].exit_link = curr;
    else
      ac[v].exit_link = get_exit_link(curr);
 return ac[v].exit_link;
int query(string s) // query O(n + ans)
 int ans = 0, curr = 0, at;
 for (auto const &i : s)
   curr = go(curr, i);
   ans += ac[curr].is_leaf;
   at = get_exit_link(curr);
   while (at)
```

```
ans += ac[at].is_leaf;
        at = get exit link(at);
    return ans;
signed main()
  ios base::sync with stdio(false);
  cin.tie(NULL);
  int n, q;
  cin >> n >> q;
  aho::init();
  for (int i = 0; i < n; i++)
    string s;
    cin >> s;
    aho::add_string(s);
  while (q--)
    string t;
    cin >> t;
    cout << aho::query(t) << endl;</pre>
  return 0;
// automato de aho-corasick
// imagine o seguinte problema:
// temos um conjunto de n strings
// e q queries para processar
// em cada uma das q queries, voce recebe uma string s
// e quer saber, o numero de ocorrencias de
// alguma string do conjunto como
// substring de s e em tempo linear
```

8.5 stringhashing2

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;
```

```
#define int long long int
#define endl '\n'
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 2001
#define mod 1000000009
struct modint
  int val;
  modint(int v = 0) \{ val = v \% mod; \}
  int pow(int y)
    modint x = val;
    modint z = 1;
    while (y)
     if (y & 1)
        z *= x;
     x \star = x;
      y >>= 1;
    return z.val;
  int inv() { return pow(mod - 2); }
  void operator=(int o) { val = o % mod; }
  void operator=(modint o) { val = o.val % mod; }
  void operator+=(modint o) { *this = *this + o; }
  void operator-=(modint o) { *this = *this - o; }
  void operator*=(modint o) { *this = *this * o; }
  void operator/=(modint o) { *this = *this / o; }
  bool operator==(modint o) { return val == o.val; }
  bool operator!=(modint o) { return val != o.val; }
  int operator*(modint o) { return ((val * o.val) % mod)
  int operator/(modint o) { return (val * o.inv()) % mod
  int operator+(modint o) { return (val + o.val) % mod;
  int operator-(modint o) { return (val - o.val + mod) %
      mod; }
struct string_hashing
  modint d:
 modint h;
  vector<modint> pref;
```

```
vector<modint> pot;
  string_hashing() {}
  string_hashing(int base, string &s)
    d = base;
    pref.resize(s.size() + 1);
    pref[0] = 0;
    for (int i = 0; i < s.size(); i++)</pre>
      modint val = pref[i] * d;
      pref[i + 1] = val + s[i];
    h = pref[s.size()];
    pot.resize(s.size() + 1);
    pot[0] = 1;
    for (int i = 1; i <= s.size(); i++)</pre>
      pot[i] = pot[i - 1] * d;
 modint get(int 1, int r)
    return pref[r + 1] - (pref[l] * pot[r - l + 1]);
  modint append(modint hb, int blen)
    h = hb + (h * pot[blen]);
    return h;
};
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  string s;
  cin >> s;
  string_hashing h(256, s); // (base, string)
 // string_hashing h(227, s); // (base, string)
  return 0;
```

8.6 lcp in suffix array

template <class T>

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
```

```
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<pi, int>
#define pci pair<char, int>
#define fir first
#define sec second
#define MAXN 50005
#define mod 1000000007
void radix(vector<pii> &v)
    int n = v.size();
    vector<int> cnt(n);
    for (auto const &i : v)
      cnt[i.fir.sec]++;
    vector<pii> ans(n);
    vector<int> pos(n);
    pos[0] = 0;
    for (int i = 1; i < n; i++)
      pos[i] = pos[i - 1] + cnt[i - 1];
    for (auto const &i : v)
      int k = i.fir.sec;
      ans[pos[k]] = i;
      pos[k]++;
    v = ans;
    int n = v.size();
    vector<int> cnt(n);
    for (auto const &i : v)
      cnt[i.fir.fir]++;
    vector<pii> ans(n);
    vector<int> pos(n);
    pos[0] = 0;
    for (int i = 1; i < n; i++)
      pos[i] = pos[i - 1] + cnt[i - 1];
    for (auto const &i : v)
      int k = i.fir.fir;
      ans[pos[k]] = i;
      pos[k]++;
    }
    v = ans;
```

```
vector<int> get lcp(string s)
  s += '$';
  int n = s.size();
  vector<int> p(n), c(n);
  vector<pci> a(n);
  for (int i = 0; i < n; i++)</pre>
    a[i] = \{s[i], i\};
  sort(a.begin(), a.end());
  for (int i = 0; i < n; i++)</pre>
    p[i] = a[i].sec;
  c[p[0]] = 0;
  for (int i = 1; i < n; i++)
    (a[i].fir == a[i - 1].fir) ? c[p[i]] = c[p[i - 1]] :
        c[p[i]] = c[p[i - 1]] + 1;
  int k = 0;
  while ((1 << k) < n)
    vector<pii> v(n);
    for (int i = 0; i < n; i++)
      v[i] = \{\{c[i], c[(i + (1 << k)) % n]\}, i\};
    radix(v):
    for (int i = 0; i < n; i++)
      p[i] = v[i].sec;
    c[p[0]] = 0;
    for (int i = 1; i < n; i++)
      (v[i].fir == v[i - 1].fir) ? c[p[i]] = c[p[i - 1]]
           : c[p[i]] = c[p[i - 1]] + 1;
    k++;
  for (auto const &i : p) // suffix array
    cout << i << " ";
  cout << endl;</pre>
  vector<int> lcp(n);
  \mathbf{k} = 0;
  for (int i = 0; i < n - 1; i++)
    int idx = c[i], j = p[idx - 1];
    while (s[i + k] == s[j + k])
     k++;
    lcp[idx] = k;
    k = max(k - 1, 011);
  for (int i = 1; i < n; i++) // lcp between 2 adjacent</pre>
     suffixes of suffix array
    cout << lcp[i] << " ";
  cout << endl;</pre>
  return lcp;
```

```
signed main()
{
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  string s;
  cin >> s;
  int n = s.size();
  vector<int> v = get_lcp(s);
  return 0;
}
```

8.7 substring fft

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb tree tag, tree order statistics node update>;
#define PI acos(-1)
#define int long long int
#define pb push_back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define MAXN 100005
#define mod 1000000007
#define cd complex<double>
const double eps = 1e-12;
const int alphabet_size = 26;
namespace fft
  void dft(vector<cd> &a)
    int n = a.size();
    if (n == 1)
      return;
    vector<cd> a0(n / 2), a1(n / 2);
    for (int i = 0; 2 * i < n; i++)
      a0[i] = a[2 * i];
      a1[i] = a[2 * i + 1];
```

```
dft(a0);
  dft(a1);
  double ang = 2 * PI / n;
  cd w(1), wn(cos(ang), sin(ang));
  for (int i = 0; 2 * i < n; i++)
    a[i] = a0[i] + w * a1[i];
    a[i + n / 2] = a0[i] - w * a1[i];
    w \star = wn;
void inverse dft(vector<cd> &a)
  int n = a.size();
  if (n == 1)
    return;
  vector\langle cd \rangle a0 (n / 2), a1 (n / 2);
  for (int i = 0; 2 * i < n; i++)
    a0[i] = a[2 * i];
    a1[i] = a[2 * i + 1];
  inverse dft(a0);
  inverse dft(a1);
  double ang = 2 * PI / n * -1;
  cd w(1), wn(cos(ang), sin(ang));
  for (int i = 0; 2 * i < n; i++)
    a[i] = a0[i] + w * a1[i];
    a[i + n / 2] = a0[i] - w * a1[i];
    a[i] /= 2;
    a[i + n / 2] /= 2;
    w \star = wn;
vector<double> mul(vector<cd> a, vector<cd> b)
  int n = 1;
  vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.
     end());
  while (n < a.size() + b.size())</pre>
    n <<= 1;
  fa.resize(n);
  fb.resize(n);
  dft(fa);
  dft(fb):
  for (int i = 0; i < n; i++)
    fa[i] *= fb[i];
  inverse_dft(fa);
```

```
vector<double> ans(n);
    for (int i = 0; i < n; i++)
      ans[i] = fa[i].real();
    return ans;
} // namespace fft
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  string s, t;
  cin >> s >> t;
  int n = s.size(), m = t.size();
  reverse(t.begin(), t.end());
  vector<cd> a(n);
  vector<cd> b(m);
  for (int i = 0; i < n; i++)
    int ch = s[i] - 'a';
    double ang = (2 * PI * ch) / alphabet_size;
    a[i] = cd(cos(ang), sin(ang));
  for (int i = 0; i < m; i++)
    int ch = t[i] - 'a';
    double ang = (2 * PI * ch) / alphabet size;
    b[i] = cd(cos(ang), -sin(ang));
  vector<double> ans = fft::mul(a, b);
  int matches = 0;
  for (int i = m - 1; i < n; i++)
    matches += (abs(ans[i] - m) \le eps);
  cout << matches << endl;</pre>
  return 0:
// number of matches of a pattern in string
// using fft
```

8.8 min suffix

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class string>
using ordered_set = tree<string, null_type, less<string>, rb_tree_tag, tree_order_statistics_node_update>;
```

```
#define int long long int
#define endl '\n'
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 2001
#define mod 1000000007
int max suffix(string s, bool mi = false)
  s.push back(*min element(s.begin(), s.end()) - 1);
  int ans = 0;
  for (int i = 1; i < s.size(); i++)</pre>
    int i = 0;
    while (ans + i < i and s[i + j] == s[ans + j])
    if (s[i + j] > s[ans + j])
      if (!mi or i != s.size() - 2)
        ans = i;
    else if (†)
      i += j - 1;
  return ans;
int min_suffix(string s)
  for (auto &i : s)
    i \star = -1;
  s.push back(*max element(s.begin(), s.end()) + 1);
  return max_suffix(s, true);
int max_cyclic_shift(string s)
  int n = s.size();
  for (int i = 0; i < n; i++)
    s.pb(s[i]);
  return max suffix(s);
int min_cyclic_shift(string s)
  for (auto &i : s)
    i *= -1;
  return max_cyclic_shift(s);
// retorna a posicao de inicio menor/maior sufixo/shift
```

8.9 suffix array2

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace __qnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<pi, int>
#define pci pair<char, int>
#define fir first
#define sec second
#define MAXN 50005
#define mod 1000000007
void radix(vector<pii> &v)
    int n = v.size();
    vector<int> cnt(n);
    for (auto const &i : v)
      cnt[i.fir.sec]++;
    vector<pii> ans(n);
    vector<int> pos(n);
    pos[0] = 0;
    for (int i = 1; i < n; i++)
      pos[i] = pos[i - 1] + cnt[i - 1];
    for (auto const &i : v)
      int k = i.fir.sec;
      ans[pos[k]] = i;
      pos[k]++;
    }
    v = ans;
    int n = v.size();
    vector<int> cnt(n);
    for (auto const &i : v)
      cnt[i.fir.fir]++;
    vector<pii> ans(n);
```

```
vector<int> pos(n);
    pos[0] = 0;
    for (int i = 1; i < n; i++)
      pos[i] = pos[i - 1] + cnt[i - 1];
    for (auto const &i : v)
      int k = i.fir.fir;
      ans[pos[k]] = i;
      pos[k]++;
    v = ans;
void get_suf(string s)
  s += '$';
  int n = s.size();
  vector < int > p(n), c(n);
 vector<pci> a(n);
  for (int i = 0; i < n; i++)</pre>
    a[i] = {s[i], i};
  sort(a.begin(), a.end());
  for (int i = 0; i < n; i++)
    p[i] = a[i].sec;
  c[p[0]] = 0;
  for (int i = 1; i < n; i++)
    (a[i].fir == a[i - 1].fir) ? c[p[i]] = c[p[i - 1]] :
        c[p[i]] = c[p[i - 1]] + 1;
  int k = 0;
  while ((1 << k) < n)
    vector<pii> v(n);
    for (int i = 0; i < n; i++)
      v[i] = \{\{c[i], c[(i + (1 << k)) % n]\}, i\};
    radix(v);
    for (int i = 0; i < n; i++)
     p[i] = v[i].sec;
    c[p[0]] = 0;
    for (int i = 1; i < n; i++)
      (v[i].fir == v[i-1].fir) ? c[p[i]] = c[p[i-1]]
          : c[p[i]] = c[p[i - 1]] + 1;
    k++;
  for (int i = 0; i < n; i++)
    cout << p[i] << " ";
  cout << endl;</pre>
signed main()
  ios_base::sync_with_stdio(false);
```

```
cin.tie(NULL);
string s;
cin >> s;
get_suf(s);
return 0;
}
// problems like:
// search if a string t is equal to any substring in s
// counting how many times string t occurs as substring in s
// number of different substrings
// the longest common substring beetween s and t
// and many others
// can be solved using suffix array
```

8.10 z-function

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define endl '\n'
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 2001
#define mod 1000000007
vector<int> z_function(string &s)
  int n = s.size();
  vector<int> z(n);
  z[0] = n;
  for (int i = 1, l = 0, r = 0; i < n; i++)
    if (i \ll r)
      z[i] = min(r - i + 1, z[i - 1]);
    while (i + z[i] < n \&\& s[z[i]] == s[i + z[i]])
      z[i]++;
    if (i + z[i] - 1 > r)
      1 = i, r = i + z[i] - 1;
```

```
return z;

signed main()
{
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  string s;
  cin >> s;
  vector<int> z = z_function(s);
}

// z-function
// calcula para cada i:
// z[i] = o tamanho de lcp(s, s.substr(i, n - i))
// lcp -> longest comom prefix
```

8.11 rabin-karp

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace gnu pbds;
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 100001
const int p = 31;
const int mod = 1e9 + 9;
int multiplicate(int x, int y)
 return (x * y) % mod;
int subtract(int a, int b)
  return (a - b < 0) ? a - b + mod : a - b;
int sum(int a, int b)
```

```
return (a + b >= mod) ? a + b - mod : a + b;
vector<int> rabin karp(string s, string t)
  int n = s.size(), m = t.size();
 vector<int> pot(n);
 pot[0] = 1;
  for (int i = 1; i < n; i++)
    pot[i] = multiplicate(pot[i - 1], p);
 vector<int> pref(n + 1, 0);
  for (int i = 0; i < n; i++)
    int val = multiplicate(pref[i], p);
    pref[i + 1] = sum(s[i], val);
  int hs = 0;
  for (int i = 0; i < m; i++)
   int val = multiplicate(hs, p);
   hs = sum(t[i], val);
  vector<int> ans;
  for (int i = 0; i + m - 1 < n; i++)
    int cur_h = subtract(pref[i + m], multiplicate(pref[
       i], pot[m]));
    if (cur_h == hs)
      ans.pb(i);
  return ans;
signed main()
 string s, t;
  cin >> s >> t;
 vector<int> ans = rabin_karp(s, t);
  for (auto const &i : ans)
    cout << i << " " << i + t.size() - 1 << endl;</pre>
  return 0;
// rabin-karp for pattern matching
// given two string s and t, determine all occurrences
   of t in s
// 1- calcule the hash of string t
// 2- calcule the prefix hash of string s
// 3- compare every substring of s with length |t|
// 4- store all occurrences in a vector and return this
   vector
// complexity: O(|t| + |s|)
```

8.12 manacher

```
#include <bits/stdc++.h>
using namespace std;
#define PI acos(-1)
#define int long long int
#define pb push_back
#define pi pair<int, int>
#define fir first
#define sec second
#define MAXN 100001
#define mod 1000000007
vector<int> d1;
vector<int> d2;
void manacher(string s)
  d1.resize(s.size());
  d2.resize(s.size());
  int 1 = 0, r = -1;
  for (int i = 0; i < s.size(); i++)</pre>
    int k = (i > r) ? 1 : min(d1[1 + r - i], r - i + 1);
    while (0 \le i - k \&\& i + k \le s.size() \&\& s[i - k] ==
        s[i + k]
     k++;
    d1[i] = k;
   k = k - 1;
    if (i + k > r)
     1 = i - k, r = i + k;
  1 = 0, r = -1;
  for (int i = 0; i < s.size(); i++)</pre>
    int k = (i > r) ? 0 : min(d2[1 + r - i + 1], r - i +
        1);
    while (0 \le i - k - 1 \&\& i + k \le s.size() \&\& s[i - k]
        -1] == s[i + k])
     k++;
    d2[i] = k;
    k = k - 1;
    if (i + k > r)
     1 = i - k - 1, r = i + k;
signed main()
```

```
ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  string s;
  cin >> s;
  manacher(s):
  return 0;
// algoritimo de manacher
// motivacao: dada uma string s, encontre todos os pares
     (l, r) tal que, a substring s[l,r]
// e palindroma.
// para cada posicao (0 <= i < s.size()), vamos</pre>
   encontrar os valores de d1[i] e d2[i],
// sendo estes o numero de palindromos com comprimentos
   impares e com comprimentos pares
// e com i sendo a posicao central desses palindromos
// algoritimo mais facil:
// para cada posicao (0 <= i < s.size()), ele tenta</pre>
   aumentar a resposta em 1
// ate q nao seja mais possivel
// while(s[i - curr] == s[i + curr])
// complexidade O(N^2)
// algoritimo de manacher:
// para cada posicao (0 <= i < s.size()):</pre>
// seja o par (l, r) os extremos da substring palindroma
    que possui o maior r entre todas as encontradas ate
    entao
// se i > r, o fim do ultimo palindromo foi antes de i:
   iremos rodar o algoritimo mais facil mais facil e ir
    ate o limite.
// caso contrario, so precisamos rodar o algoritimo a
   partir de onde nao foi percorrido previamente.
// ao final se o r atual e maior do que o nosso antigo r
   , atualizamos o par (l, r)
// por incrivel que pareca, a complexidade e O(N)
// voltando para a motivacao:
// se temos os valores de d1[i] e d2[i]:
// a substring s[i - k, i + k] e palindroma, para todo
   (0 \le k \le d1[i])
// a substring s[i - k - 1, i + k] e palindroma, para
   todo (0 \le k \le d2[i])
// dai temos todos os intervalos
// note que a complexidade do algoritimo de manacher e O
   (N),
```

```
// mas como a quantidade maxima de palindromos em uma
    string e n^2,
// imprimir todos os intervalos consequentemente teria
    complexidade O(N^2) no pior caso
```

8.13 stringhashing

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace __qnu_pbds;
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics node update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 5001
#define mod 1000000007
int n:
vector<int> v;
int modpow(int x, int y)
  int z = 1;
  while (y)
   if (y & 1)
    z = (z * x) % mod;
   x = (x * x) % mod;
   \vee >>= 1;
  return z;
int inverse(int x)
  return modpow(x, mod - 2);
int divide(int x, int y)
 return (x * inverse(y)) % mod;
```

```
int subtract(int x, int y)
  return ((x + mod) - y) % mod;
int multiplicate(int x, int y)
  return (x * y) % mod;
int sum(int x, int y)
  return (x + y) % mod;
namespace sh
  const int d = 31;
  vector<int> pot;
  vector<int> pref;
  vector<int> suf;
  void calc()
    pot.resize(n + 1);
    pot[0] = 1;
    for (int i = 1; i <= n; i++)
      pot[i] = multiplicate(pot[i - 1], d);
  void suffix_hash()
    suf.resize(n + 1);
    suf[0] = 0;
    for (int i = 0; i < n; i++)
      int val = multiplicate(v[n - i - 1], pot[i]);
      suf[i + 1] = sum(suf[i], val);
  void prefix_hash()
    pref.resize(n + 1);
    pref[0] = 0;
    for (int i = 0; i < n; i++)
      int val = multiplicate(v[i], pot[i]);
      pref[i + 1] = sum(pref[i], val);
    }
  int prefix(int 1, int r)
    return divide(subtract(pref[r + 1], pref[l]), pot[l
```

```
]);
  int suffix(int 1, int r)
    return divide(subtract(suf[n - 1], suf[n - r - 1]),
       pot[n - r - 1]);
} // namespace sh
signed main()
  ios base::sync with stdio(false);
  cin.tie(NULL);
  string s;
  cin >> s;
  n = s.size();
  for (auto const &i : s)
   v.pb((i - 'a') + 1);
                                       // indexar a
       partir do 1
  sh::calc();
                                        // potencias de
  sh::prefix_hash();
                                        // hashing dos
     prefixos de s
  cout << sh::prefix(0, n - 1) << endl; // resposta</pre>
     final
  return 0;
// string hashing
// podemos representar uma string como um valor inteiro
// seja s uma string e d o tamanho do alfabeto
// o valor de hashing de s eh igual a:
// (s[0] * pow(d, 0)) + (s[1] * pow(d, 1)) + ... (s[n -
   1] * pow(d, n - 1))
// como esse valor pode ser gigantesco
// fazer isso com um modulo que for o maior possivel
// nesse caso usaremos 10^9 + 7
// logo o hashing fica:
// ((s[0] * pow(d, 0)) + (s[1] * pow(d, 1)) + ... (s[n -
    1] * pow(d, n - 1)) % mod
// o hashing possui diversas aplicacoes como:
// checar substring que sao palindromas
// numeros de substrings diferentes em uma string
// etc...
```

9 Geometry

9.1 ConvexHull

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
```

```
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace qnu pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 500001
#define mod 1000000007
#define PI acos(-1)
namespace p
  struct pt
    double x, y;
    pt operator+(pt p) { return {x + p.x, y + p.y}; } //
        soma de pontos
    pt operator-(pt p) { return {x - p.x, y - p.y}; } //
        subtracao de pontos
    pt operator*(double d) { return {x * d, y * d}; } //
        multiplicacao por um double
    pt operator/(double d) { return {x / d, y / d}; } //
        divisao por um double
  };
  double dot(pt v, pt w) // produto escalar (dot product
    return v.x * w.x + v.y * w.y;
  bool is_perp(pt v, pt w) // retorna se dois vetores
     sao perpendiculares (angulo 90 graus)
    return dot(v, w) == 0;
  double cross(pt v, pt w) // produto vetorial (cross
     product)
    return v.x * w.y - v.y * w.x;
  double dist(pt a, pt b) // distancia entre 2 pontos
    pt c = a - b;
```

```
return sqrt(c.x * c.x + c.y * c.y);
double dist2(pt a, pt b) // retorna o quadrado da
   distancia entre dois pontos
  pt c = a - b;
  return c.x * c.x + c.y * c.y;
bool is_colinear(pt a, pt b, pt c) // retorna se os
   pontos a, b e c sao colineares
  return cross(b - a, c - a) == 0;
bool ccw(pt a, pt b, pt c) // retorna se os pontos a,b
    e c estao no sentido anti horario
  return cross (b - a, c - b) > 0;
bool cw(pt a, pt b, pt c) // retorna se os pontos a, b
   e c estao no sentido horario
  return cross(b - a, c - b) < 0;</pre>
double modulo (pt v) //|v| = sqrt(x2 + y2)
  return sqrt(v.x * v.x + v.y * v.y);
double angle(pt a, pt b, pt c) // angulo entre os
   vetores ab e ac
  // dot(ab , ac) / |ab| * |ac|
  pt ab = b - a; // vetor ab
  pt ac = c - a; // vetor ac
  double m1 = modulo(ab);
  double m2 = modulo(ac);
  double m3 = m1 * m2;
  return (dot(ab, ac) / m3); // retorna o cos do
     angulo em graus
pt rotate(pt p, double a) // rotacionar o ponto p em
   relacao a origem, em a graus, no sentido anti-
   horario
  a = (a * PI) / 180;
  double xx = (\cos(a) * p.x) + ((\sin(a) * -1) * p.y);
  double yy = (\sin(a) * p.x) + (\cos(a) * p.y);
  pt ans = \{xx, yy\};
  return ans;
double polar(pt p) // polar angle
```

```
return atan21(p.y, p.x);
bool cmp(pt a, pt b) // ordenar pontos pelo polar
   angle
 return polar(a) < polar(b);</pre>
bool cmp_x(pt a, pt b) // ordenar os pontos pela
   coordenada x
  if (a.x != b.x)
    return a.x < b.x;</pre>
  return a.y < b.y;</pre>
vector<pt> convex hull(vector<pt> v)
  sort(v.begin(), v.end(), cmp_x);
  pt p1 = v[0], p2 = v.back();
 vector<pt> up;
 vector<pt> down;
 up.pb(p1);
  down.pb(p1);
  for (int i = 1; i < v.size(); i++)</pre>
    if (i == v.size() - 1 || cw(p1, v[i], p2))
      while (up.size() >= 2 && !cw(up[up.size() - 2],
         up[up.size() - 1], v[i]))
        up.pop_back();
      up.pb(v[i]);
  for (int i = 1; i < v.size(); i++)</pre>
    if (i == v.size() - 1 || ccw(p1, v[i], p2))
      while (down.size() >= 2 && !ccw(down[down.size()
          - 2], down[down.size() - 1], v[i]))
        down.pop_back();
      down.pb(v[i]);
  int start = 0, limit = 0; // para por em ans no
     sentido anti-horario e a partir de start
  for (int i = 1; i < down.size(); i++)</pre>
    if ((down[i].y < down[start].y) || (down[i].y ==</pre>
       down[start].y && down[i].x < down[start].x))</pre>
      start = i;
  if (!start)
```

```
limit = 1;
    vector<pt> ans;
    for (int i = start; i < down.size() - 1; i++)</pre>
      ans.pb(down[i]);
    for (int i = up.size() - 1; i >= limit; i--)
      ans.pb(up[i]);
    for (int i = 1; i < start; i++)</pre>
      ans.pb(down[i]);
    return ans;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int n, t = 0;
  while (cin >> n)
    cout << "caso " << t << ":" << endl;</pre>
    vector<p::pt> v(n);
    for (int i = 0; i < n; i++)</pre>
      cin >> v[i].x >> v[i].y;
    vector<p::pt> ans = p::convex_hull(v);
    for (auto const &i : ans)
      cout << i.x << " " << i.y << endl;
    cout << endl;</pre>
    t++;
  return 0;
// conceitos importantes:
// 1- poligono: uma figura plana que possui no minimo 3
   lados e 3 angulos
// 2- poligono convexo: um poligono cujo todos os seus
   angulos internos sao menores do que 180 graus
// convex hull:
// dados n pontos em um plano, o objetivo e achar o
   menor poligono convexo que possui todos os n pontos
   dados
// Graham's Scan, complexidade O(n * log(n))
// ideia do algoritimo:
// 1- ache 2 pontos a e b tal que, a e o ponto mais a
   esquerda e b o ponto mais a direita do conjunto dado
// 2- a e b devem pertencer ao convex hull
// 3- desenhar uma linha ab, essa linha ira separar os
   outros pontos em 2 conjuntos s1 (superior) e s2 (
   inferior).
// 4- a e b pertencem aos dois conjuntos
```

```
// 5- agora para os conjuntos s1 e s2, achamos o convex
   hull dos dois conjuntos.
// 6- para isso, ordene todos os pontos pela cordenada x
// 7- para cada ponto, se o ponto dado pertence ao
   conjunto superior, verificamos o angulo formado pela
    linha
// que liga o penultimo ponto e o ultimo ponto do
   convex hull superior, com a linha que conecta o
// ultimo ponto do convex hull e o ponto atual. Se o
   angulo nao for no sentido horario,
// removemos o ponto mais recente adicionado ao
   convex hull superior, pois o ponto atual sera capaz
// de conter o ponto anterior, uma vez que seja
   adicionado ao convex hull.
// 8- fazer o mesmo para o conjunto inferior
// 9- ao final teremos o conjunto de pontos que formam o
    convex hull dos n pontos
```

9.2 minkowski

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define endl '\n'
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 15
#define mod 1000000007
struct pt
  int x, y;
  bool operator<(pt ot)</pre>
    if (x != ot.x)
      return x < ot.x;
    return y < ot.y;</pre>
  void operator=(pt p) { x = p.x, y = p.y; }
```

```
bool operator==(pt p) { return (x == p.x && y == p.y);
  bool operator!=(pt p) { return (x != p.x || y != p.y);
  pt operator+(const pt &p) { return {x + p.x, y + p.y};
  pt operator-(const pt &p) { return {x - p.x, y - p.y};
  pt operator*(int d) { return {x * d, y * d}; }
  pt operator/(int d) { return {x / d, y / d}; }
  int cross(pt ot) const { return x * ot.y - y * ot.x; }
  int cross(pt a, pt b) const { return (a - *this).cross
     (b - *this); }
};
enum type
  outside,
  inside,
  boundary
};
int cross(pt v, pt w)
  return v.x * w.y - v.y * w.x;
bool ccw(pt a, pt b, pt c)
  return cross (b - a, c - b) > 0;
void radial_sort(vector<pt> &a)
  pt pivot = *min_element(a.begin(), a.end());
  auto cmp = [\&] (pt p, pt q)
    if (p == pivot || q == pivot)
      return q != pivot;
    return ccw(pivot, p, q) > 0;
  sort(a.begin(), a.end(), cmp);
vector<pt> trata(vector<pt> p)
  vector<pt> ans;
  for (int i = 0; i < p.size(); i++)</pre>
    while (ans.size() >= 2 && ans.back().cross(p[i], ans
       .end()[-2]) == 0)
      ans.pop back();
    ans.pb(p[i]);
  if (ans.size() > 2 \&\& ans.back().cross(p[0], ans.end())
```

```
[-2]) == 0
                                                                  return type::outside;
                                                                if (poly[0].cross(p, poly[n-1]) != 0 && sqn(poly[0].
    ans.pop_back();
  return ans;
                                                                    cross(p, poly[n - 1])) != sqn(poly[0].cross(poly
                                                                    [1], poly[n - 1]))
void prepare(vector<pt> &p)
                                                                  return type::outside;
                                                                int 1 = 2, r = n - 1;
                                                                if (poly[0].cross(poly[1], p) > 0)
  radial_sort(p); // sort points in counter-clockwise
  p = trata(p); // and the polygon dont have 3
                                                                  while (1 < r)
     colinear points
                                                                    int mid = (1 + r) >> 1;
int sqn(int val)
                                                                    (poly[0].cross(poly[mid], p) \le 0) ? r = mid : l =
                                                                         mid + 1:
  if (val > 0)
    return 1;
  else if (val < 0)</pre>
                                                                if (!in_tri(p, poly[0], poly[1 - 1], poly[1]))
    return -1;
                                                                  return type::outside;
  return 0;
                                                                if (in_seg(p, poly[1 - 1], poly[1]))
                                                                  return type::boundary;
                                                                if (in_seq(p, poly[0], poly[1]))
bool in_seg(pt p, pt a, pt b)
                                                                  return type::boundary;
  // check if point p is in the line segment formed by a
                                                                if (in_seg(p, poly[0], poly[n - 1]))
      and b
                                                                  return type::boundary;
  if (a.cross(b, p) == 0)
                                                                return type::inside;
    return (p.x >= min(a.x, b.x) && p.x <= max(a.x, b.x)
        && p.y >= min(a.y, b.y) && p.y <= max(a.y, b.y)
                                                              vector<pt> minkowski (vector<pt> a, vector<pt> b)
       );
  return 0;
                                                                prepare(a);
                                                                prepare(b);
bool in_tri(pt p, pt a, pt b, pt c)
                                                                a.push_back(a[0]);
                                                                a.push_back(a[1]);
  // check if point p is in the triangle formed by a, b
                                                                b.push_back(b[0]);
                                                                b.push back(b[1]);
     and c
  int a1 = abs(a.cross(b, c));
                                                                vector<pt> ans;
  int a2 = abs(p.cross(a, b)) + abs(p.cross(a, c)) + abs
                                                                int i = 0, j = 0;
     (p.cross(b, c));
                                                                while (i < a.size() - 2 || j < b.size() - 2)</pre>
  return a1 == a2;
                                                                  ans.pb(a[i] + b[j]);
int in_polygon(vector<pt> &poly, pt p)
                                                                  auto c = cross(a[i + 1] - a[i], b[j + 1] - b[j]);
                                                                  if (c >= 0)
                                                                    <u>i</u>++;
  int n = poly.size();
                                                                  if (c <= 0)
  if (n == 1)
    return (p == poly[0]) ? type::boundary : type::
                                                                    j++;
       outside;
  if (n == 2)
                                                                return ans;
    return (in_seg(p, poly[0], poly[1])) ? type::
       boundary : type::outside;
                                                              signed main()
  if (poly[0].cross(poly[1], p) != 0 && sqn(poly[0].
     cross(poly[1], p)) != sqn(poly[0].cross(poly[1],
                                                                ios base::sync with stdio(false);
     poly[n-1]))
                                                                cin.tie(NULL);
```

```
vector<pt> v;
  for (int _ = 0; _ < 3; _++)
    int n;
    cin >> n;
    vector<pt> p(n);
    for (int i = 0; i < n; i++)
    cin >> p[i].x >> p[i].y;
    if (_ == 0)
     v = p;
    else
      v = minkowski(v, p);
  prepare(v);
  int q;
  cin >> q;
  while (q--)
    pt p;
    cin >> p.x >> p.y;
    p.x *= 3, p.y *= 3;
    // ve se o ponto (3x, 3y) esta na bora, dentro ou
       fora do poligono v
    (in polygon(v, p) != type::outside) ? cout << "YES\n
       " : cout << "NO\n";
  return 0;
// problema exemplo:
// https://codeforces.com/contest/87/problem/E
```

9.3 LineSweep

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;

#define int long long int
#define endl '\n'
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
```

```
#define sec second
#define MAXN 200005
#define mod 1000000007
#define PI acos(-1)
const double EPS = 1e-9;
struct pt
  double x, y;
struct seg
  pt p, q;
  int id;
  double get y (double x) const
    if (abs(p.x - q.x) < EPS)
      return p.y;
    return p.y + (q.y - p.y) * (x - p.x) / (q.x - p.x);
};
bool intersect1d(double 11, double r1, double 12, double
    r2)
  if (11 > r1)
    swap(11, r1);
  if (12 > r2)
    swap(12, r2);
  return max(11, 12) <= min(r1, r2) + EPS;
int vec (const pt &a, const pt &b, const pt &c)
  double s = (b.x - a.x) * (c.y - a.y) - (b.y - a.y) * (
     c.x - a.x);
  return abs(s) < EPS ? 0 : s > 0 ? +1
bool intersect (const seg &a, const seg &b)
  return intersect1d(a.p.x, a.g.x, b.p.x, b.g.x) &&
         intersect1d(a.p.y, a.q.y, b.p.y, b.q.y) &&
         vec(a.p, a.q, b.p) * vec(a.p, a.q, b.q) <= 0 &&
         vec(b.p, b.q, a.p) * vec(b.p, b.q, a.q) <= 0;
bool operator < (const seg &a, const seg &b)
  double x = max(min(a.p.x, a.q.x), min(b.p.x, b.q.x));
  return a.get_y(x) < b.get_y(x) - EPS;</pre>
```

```
struct event
  double x;
  int tp, id;
  event() {}
  event (double x, int tp, int id) : x(x), tp(tp), id(id)
 bool operator<(const event &e) const
    if (abs(x - e.x) > EPS)
      return x < e.x;
    return tp > e.tp;
};
set<seq> s;
set<seg>::iterator prev(set<seg>::iterator it)
  return it == s.begin() ? s.end() : --it;
set<seg>::iterator next(set<seg>::iterator it)
  return ++it;
pi line sweep(vector<seq> v)
 vector<event> e;
  for (int i = 0; i < v.size(); i++)</pre>
    e.push_back(\{\min(v[i].p.x, v[i].q.x), 1, i\});
    e.push_back(\{\max(v[i].p.x, v[i].q.x), 0, i\});
  sort(e.begin(), e.end());
  for (int i = 0; i < e.size(); i++)</pre>
    int id = e[i].id;
    if (e[i].tp == 1)
      auto nxt = s.lower_bound(v[id]), prv = prev(nxt);
      if (nxt != s.end() && intersect(*nxt, v[id]))
        return {(*nxt).id, id};
      if (prv != s.end() && intersect(*prv, v[id]))
        return {(*prv).id, id);
      s.insert(nxt, v[id]);
    }
    else
      auto where = s.lower bound(v[id]);
      auto nxt = next(where), prv = prev(where);
```

```
if (nxt != s.end() && prv != s.end() && intersect
          (*nxt, *prv))
        return { (*prv).id, (*nxt).id};
      s.erase(where);
  return {-1, -1};
signed main()
  int n;
  cin >> n;
  vector<seg> v(n);
  for (int i = 0; i < n; i++)
    cin >> v[i].p.x >> v[i].p.y >> v[i].q.x >> v[i].q.y;
    v[i].id = i;
  pi ans = line_sweep(v);
  if (ans.fir == -1)
    cout << "NO\n";</pre>
  else
    cout << "YES\n";</pre>
    cout << ans.fir + 1 << " " << ans.sec + 1 << endl;</pre>
  return 0;
// https://cp-algorithms.com/geometry/
   intersecting_segments.html
// https://acm.timus.ru/problem.aspx?space=1&num=1469
```

9.4 points and vectors

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;

#define int long long int
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
```

```
#define fir first
#define sec second
#define MAXN 500001
#define mod 1000000007
#define PI acos(-1)
namespace p
  struct pt
    double x, y;
    pt operator+(pt p) { return \{x + p.x, y + p.y\}; \} //
        soma de pontos
   pt operator-(pt p) { return {x - p.x, y - p.y}; } //
        subtracao de pontos
    pt operator*(double d) { return {x * d, y * d}; } //
        multiplicacao por um double
    pt operator/(double d) { return {x / d, y / d}; } //
        divisao por um double
  double dot(pt v, pt w) // produto escalar (dot product
    return v.x * w.x + v.y * w.y;
  bool is perp(pt v, pt w) // retorna se dois vetores
     sao perpendiculares (angulo 90 graus)
   return dot(v, w) == 0;
  double cross(pt v, pt w) // produto vetorial (cross
     product)
    return v.x * w.y - v.y * w.x;
  double dist(pt a, pt b) // distancia entre 2 pontos
   pt c = a - b;
   return sqrt(c.x * c.x + c.y * c.y);
  double dist2(pt a, pt b) // retorna o quadrado da
     distancia entre dois pontos
    pt c = a - b;
   return c.x * c.x + c.y * c.y;
  bool is_colinear(pt a, pt b, pt c) // retorna se os
     pontos a, b e c sao colineares
    return cross (b - a, c - a) == 0;
```

```
bool ccw(pt a, pt b, pt c) // retorna se os pontos a,b
    e c estao no sentido anti horario
  return cross (b - a, c - b) > 0;
bool cw(pt a, pt b, pt c) // retorna se os pontos a,b
   e c estao no sentido horario
  return cross (b - a, c - b) < 0;
double modulo (pt v) // |v| = sqrt(x2 + y2)
  return sqrt (v.x * v.x + v.y * v.y);
double angle (pt a, pt b, pt c) // angulo entre os
   vetores ab e ac
  // dot(ab , ac) / |ab| * |ac|
  pt ab = b - a; // vetor ab
  pt ac = c - a; // vetor ac
  double m1 = modulo(ab);
  double m2 = modulo(ac);
  double m3 = m1 * m2;
  return (dot(ab, ac) / m3); // retorna o cos do
     angulo em graus
pt rotate(pt p, double a) // rotacionar o ponto p em
   relacao a origem, em a graus, no sentido anti-
   horario
  a = (a * PI) / 180;
  double xx = (\cos(a) * p.x) + ((\sin(a) * -1) * p.y);
  double yy = (\sin(a) * p.x) + (\cos(a) * p.y);
  pt ans = \{xx, yy\};
  return ans;
double polar(pt p) // polar angle
  return atan21(p.y, p.x);
bool cmp (pt a, pt b) // ordenar pontos pelo polar
   angle
  return polar(a) < polar(b);</pre>
bool cmp_x(pt a, pt b) // ordenar os pontos pela
   coordenada x
  if (a.x != b.x)
```

```
return a.x < b.x;
return a.y < b.y;
}
signed main()
{
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  return 0;
}</pre>
```

10 Structures

10.1 persistent seg

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace gnu pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define MAXN 100003
#define mod 1000000007
int v[MAXN];
namespace seq
  struct node
    int item, lazy, lazy_status, l, r;
    node() {}
    node(int 1, int r, int lazy, int lazy_status, int
       item) : 1(1), r(r), lazy(lazy), lazy_status(
       lazy status), item(item) {}
  } ;
  vector<node> seq;
  vector<int> roots;
```

```
void init()
  seq.resize(1);
int neutral()
  return 0;
int merge(int a, int b)
  return a + b;
int newleaf(int vv)
  int p = seq.size();
  seq.pb(node(0, 0, 0, 0, vv));
  return p;
int newparent(int 1, int r)
  int p = seq.size();
  seg.pb(node(1, r, 0, 0, merge(seg[1].item, seg[r].
     item)));
  return p;
int newkid(int i, int diff, int 1, int r)
  int p = seg.size();
  seg.pb(node(seg[i].l, seg[i].r, seg[i].lazy + diff,
     1, seg[i].item + ((r - 1 + 1) * diff)));
  return p;
void add(int i, int l, int r)
  if (!seg[i].lazy_status)
    return;
  if (1 != r)
    int mid = (1 + r) >> 1;
    seg[i].l = newkid(seg[i].l, seg[i].lazy, l, mid);
    seq[i].r = newkid(seq[i].r, seq[i].lazy, mid + 1,
       r);
  seq[i].lazy = 0;
  seg[i].lazy_status = 0;
int update(int i, int l, int r, int ql, int qr, int
   diff)
```

```
if (1 > r || 1 > qr || r < q1)
     return i;
   if (1 >= q1 \&\& r <= qr)
     return newkid(i, diff, l, r);
   add(i, 1, r);
   int mid = (1 + r) >> 1;
   return newparent (update (seg[i].l, l, mid, ql, qr,
       diff), update(seg[i].r, mid + 1, r, ql, qr, diff
       ));
 int query(int 1, int r, int q1, int qr, int i)
   if (l > r || l > qr || r < ql)
     return neutral();
   if (1 >= q1 \&\& r <= qr)
     return seq[i].item;
   add(i, 1, r);
   int mid = (1 + r) >> 1;
   return merge(query(l, mid, ql, qr, seg[i].l), query(
       mid + 1, r, ql, qr, seg[i].r);
 int build(int 1, int r)
   if (1 == r)
     return newleaf(v[1]);
   int mid = (1 + r) >> 1;
   return newparent(build(1, mid), build(mid + 1, r));
signed main()
 ios_base::sync_with_stdio(false);
 cin.tie(NULL);
 int n, q;
 cin >> n >> q;
 for (int i = 0; i < n; i++)</pre>
   cin >> v[i];
 seg::init();
 int root = seg::build(0, n - 1);
 seq::roots.pb(root);
 while (q--)
   char t;
   cin >> t;
   if (t == 'C')
     int 1, r, d;
     cin >> 1 >> r >> d;
     1--, r--;
     int root = seq::update(seq::roots.back(), 0, n -
```

```
1, 1, r, d);
      seq::roots.pb(root);
    else if (t == '0')
      int 1, r;
      cin >> 1 >> r;
      1--, r--;
      cout << seq::query(0, n - 1, 1, r, seq::roots.back</pre>
          ()) << endl;
    else if (t == 'H')
      int 1, r, d;
      cin >> 1 >> r >> d;
      1--, r--;
      cout << seq::query(0, n - 1, 1, r, seq::roots[d])</pre>
    else
      int d;
      cin >> d;
      while (seg::roots.size() > d + 1)
        seq::roots.pop_back();
  return 0;
// https://www.spoj.com/problems/TTM/
// rollback segtree to a time stamp t
```

10.2 treap

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;

//#define int long long int
#define endl '\n'
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
```

```
#define sec second
#define MAXN 1000005
#define mod 1000000007
struct treap
  int data, priority;
  int sz, lazy2;
  bool lazy;
  treap *1, *r;
int size(treap *node)
  return (!node) ? 0 : node->sz;
void recalc(treap *node)
  if (!node)
    return;
  node -> sz = 1;
  if (node->1)
    node \rightarrow sz += node \rightarrow l \rightarrow sz;
  if (node->r)
    node->sz += node->r->sz;
void lazy_propagation(treap *node)
  if (node == NULL)
    return;
  if (node->lazy2)
    if (node->1)
      node->1->lazy2 += node->lazy2;
    if (node->r)
      node->r->lazy2 += node->lazy2;
    node->data += node->lazy2;
    node \rightarrow lazv2 = 0;
  if (node->lazv)
    swap(node->1, node->r);
    if (node->1)
      node->l->lazy = !node->l->lazy;
    if (node->r)
      node->r->lazy = !node->r->lazy;
    node -> lazy = 0;
void split(treap *t, treap *&l, treap *&r, int n)
```

```
if (!t)
    return void(1 = r = 0);
  lazy propagation(t);
  if (size(t->1) >= n)
    split(t->1, 1, t->1, n), r = t;
  else
    split(t->r, t->r, r, n - size(t->l) - 1), l = t;
  recalc(t);
void merge(treap *&t, treap *l, treap *r)
  lazy_propagation(l);
 lazy propagation(r);
  if (!1)
   t = r;
  else if (!r)
   t = 1;
  else if (l->priority > r->priority)
    merge (1->r, 1->r, r), t = 1;
  else
    merge (r->1, 1, r->1), t = r;
  recalc(t);
void troca(treap *&t, int 1, int r, int 11, int rr)
  treap *a0, *a1, *b0, *b1, *c0, *c1, *d0, *d1;
  split(t, a0, a1, 1);
  split(a1, b0, b1, r - 1 + 1);
  11 -= (r + 1);
  rr -= (r + 1);
  split(b1, c0, c1, l1);
  split(c1, d0, d1, rr - 11 + 1);
 merge(t, a0, d0);
 merge(t, t, c0);
 merge(t, t, b0);
  merge(t, t, d1);
void add(treap *&t, int 1, int r)
 treap *a0, *a1, *b0, *b1;
  split(t, a0, a1, 1);
  split(a1, b0, b1, r - 1 + 1);
 b0 \rightarrow lazy = 1;
 b0 - > lazy2 += 1;
 merge(t, a0, b0);
 merge(t, t, b1);
void solve(int x)
 x = x % 26;
```

```
char c = x + 'a';
  cout << c;
void dfs(treap *t)
  if (!t)
    return;
 lazy_propagation(t);
  dfs(t->1);
  solve(t->data);
  dfs(t->r);
treap *create_node(int data, int priority)
  treap *ret = new treap;
  ret->data = data;
  ret->priority = priority;
 ret -> 1 = 0;
  ret->r = 0;
  ret->sz = 1;
  ret->lazy = 0;
  ret -> lazy2 = 0;
  return ret;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  srand(time(NULL));
  int q;
  cin >> q;
  while (q--)
    int n, m;
    string s;
    cin >> s >> m;
    n = s.size();
    treap *t = 0;
    for (auto const &i : s)
      int x = i - 'a';
      merge(t, t, create node(x, rand()));
    while (m--)
      int a, b, c, d;
      cin >> a >> b >> c >> d;
      a--, b--, c--, d--;
      add(t, a, b);
      add(t, c, d);
```

```
troca(t, a, b, c, d);
}
dfs(t);
cout << endl;
}
return 0;
}
// https://vjudge.net/contest/478186#problem/E
// - lazy propagation
// - reverse range with lazy propagation
// - swap ranges with equal lenght</pre>
```

10.3 mergesorttree

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 100001
#define mod 1000000007
vector<int> seg[4 * MAXN];
int v[MAXN];
void update(int i, int l, int r, int q, int x)
  if (1 == r)
    seg[i].clear();
    seq[i].pb(x);
    return;
  int mid = (1 + r) >> 1;
  if (q <= mid)
    update(i \ll 1, l, mid, q, x);
  else
    update((i << 1) | 1, mid + 1, r, q, x);
```

```
// a merge do c++ une os dois vectors, deixando ele
     ordenado em O(n)
  seg[i].clear();
  merge(seg[i << 1].begin(), seg[i << 1].end(), seg[(i << 1].end())
     << 1) | 1].begin(), seg[(i << 1) | 1].end(),
     back_inserter(seg[i]));
int query(int 1, int r, int q1, int qr, int i, int x)
  int mid = (1 + r) >> 1;
  if (1 > r || 1 > qr || r < q1)
    return 0;
  if (l >= ql && r <= qr) // quantidade de elementos</pre>
     maiores do que x no range atual
    return seg[i].end() - upper_bound(seg[i].begin(),
       seg[i].end(), x);
  return query(l, mid, ql, qr, i << 1, x) + query(mid +</pre>
     1, r, ql, qr, (i << 1) | 1, x);
void build(int 1, int r, int i)
  if (1 == r)
    seg[i].pb(v[1]);
    return;
  int mid = (1 + r) >> 1;
  build(1, mid, i << 1);
  build (mid + 1, r, (i << 1) | 1);
  // a merge do c++ une os dois vectors, deixando ele
     ordenado em O(n)
  merge(seg[i << 1].begin(), seg[i << 1].end(), seg[(i</pre>
     << 1) | 1].begin(), seg[(i << 1) | 1].end(),
     back_inserter(seg[i]));
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  return 0;
// merge sort tree
// a segment tree with ordered vectors in range nodes
// example:
// number of elements > x in a range [1, r]
// memory: O(n * log n)
// query: 0(log^2 n)
```

10.4 sparsetable

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define PI acos(-1)
#define pb push back
#define int long long int
#define pi pair<int, int>
#define pii pair<int, pair<int, pi>>
#define fir first
#define sec second
#define DEBUG 0
#define MAXN 10005
#define mod 1000000007
int n;
vector<int> v;
namespace st
  int st [MAXN] [25];
  int log[MAXN + 1];
  void init()
    log[1] = 0;
    for (int i = 2; i <= MAXN; i++)</pre>
      log[i] = log[i / 2] + 1;
    for (int i = 0; i < n; i++)
      st[i][0] = v[i];
    for (int j = 1; j \le 25; j++)
      for (int i = 0; i + (1 << j) <= n; i++)
        st[i][j] = min(st[i][j-1], st[i+(1 << (j-1)])
           1))][\dot{1} - 1]);
  int query(int 1, int r)
    int j = \log[r - 1 + 1];
    int minimum = min(st[1][j], st[r - (1 << j) + 1][j])
    return minimum;
```

```
}
signed main()
{
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
}
```

10.5 sqrt decomposition2

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb ds/tree policy.hpp>
using namespace std;
using namespace gnu pbds;
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define PI acos(-1)
#define pb push_back
#define int long long int
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 100001
#define mod 1000000007
int n, q;
vector<int> v;
namespace mo
  struct query
    int idx, 1, r, t;
  struct update
    int i, x;
  };
  int block;
  vector<query> queries;
  vector<update> updates;
  vector<int> ans;
  bool cmp (query x, query y)
```

```
if (x.1 / block != y.1 / block)
    return x.1 / block < y.1 / block;</pre>
  if (x.r / block != y.r / block)
    return x.r / block < y.r / block;</pre>
  return x.t < y.t;</pre>
void sqrt decomposition()
  block = 2800; // (2 * n) ^ 0.666
  sort(queries.begin(), queries.end(), cmp);
  ans.resize(queries.size());
  int curr_left = 0, curr_right = 0, curr_sum = 0,
     curr t = 0;
  for (int i = 0; i < queries.size(); i++)</pre>
    int idx = queries[i].idx;
    int l = queries[i].l;
    int r = queries[i].r;
    int t = queries[i].t;
    while (curr_right <= r)</pre>
      curr_sum += v[curr_right];
      curr right++;
    while (curr_left > 1)
      curr left--;
      curr_sum += v[curr_left];
    while (curr right > r + 1)
      curr right--;
      curr sum -= v[curr right];
    while (curr left < 1)</pre>
      curr_sum -= v[curr_left];
      curr left++;
    while (curr t > t)
      curr_t--;
      if (1 <= updates[curr_t].i && r >= updates[
         curr tl.i)
        curr sum -= updates[curr t].x;
      v[updates[curr t].i] -= updates[curr t].x;
    while (curr t < t)</pre>
```

```
if (1 <= updates[curr_t].i && r >= updates[
           curr_t].i)
          curr sum += updates[curr t].x;
        v[updates[curr_t].i] += updates[curr_t].x;
        curr t++;
      ans[idx] = curr_sum;
signed main()
 ios_base::sync_with_stdio(false);
 cin.tie(NULL);
 cin >> n >> q;
 v.resize(n);
 for (int i = 0; i < n; i++)</pre>
    cin >> v[i];
  for (int i = 0; i < q; i++)
   int type;
    cin >> type;
    if (!type)
     mo::update curr;
     cin >> curr.i >> curr.x;
     mo::updates.pb(curr);
    else
     mo::query curr;
     cin >> curr.l >> curr.r;
     curr.r--;
     curr.idx = mo::queries.size();
     curr.t = mo::updates.size();
      mo::queries.pb(curr);
    }
 mo::sqrt decomposition();
 for (auto const &i : mo::ans)
    cout << i << endl;</pre>
//https://judge.yosupo.jp/problem/point_add_range_sum
```

10.6 implicit seg

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
```

```
using namespace __qnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define endl '\n'
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 200005
#define mod 998244353
struct implcit_seg
  int 1, r;
  int sum, lazy;
  implcit_seg *left_child = nullptr;
  implcit_seg *right_child = nullptr;
  implcit_seg(int 1, int r) : 1(1), r(r)
    sum = 0:
    lazy = 0;
  void check_childs()
    if (!left child && l != r)
      int mid = (1 + r) >> 1;
      left child = new implcit seq(l, mid);
      right_child = new implcit_seg(mid + 1, r);
  void add(int x)
    sum += (r - 1 + 1) * x;
    if (1 != r)
      check_childs();
      left_child->lazy += x;
      right_child->lazy += x;
    lazy = 0;
  void upd(int ql, int qr, int x)
```

```
add(lazy);
    if (1 > r || 1 > qr || r < q1)
     return;
    if (1 >= q1 \&\& r <= qr)
      add(x);
      return;
    check_childs();
   left_child->upd(ql, qr, x);
   right child->upd(ql, qr, x);
    sum = left_child->sum + right_child->sum;
  void upd(int k, int x)
   sum += x;
    check childs();
   if (left_child)
      if (k <= left_child->r)
        left_child->upd(k, x);
      else
        right_child->upd(k, x);
    }
  int gry(int gl, int gr)
   add(lazy);
   if (l > r || l > qr || r < ql)
     return 0;
   if (1 >= q1 \&\& r <= qr)
      return sum;
    check childs();
    return left_child->qry(ql, qr) + right_child->qry(ql
       , qr);
};
signed main()
 ios_base::sync_with_stdio(false);
 cin.tie(NULL);
 int n, q;
 cin >> n >> q;
 implcit_seg *s = new implcit_seg(0, n - 1);
  while (q--)
   int t;
   cin >> t;
   if (t == 1)
    {
```

```
int l, r, x;
    cin >> l >> r >> x;
    if (l == r - 1) // point update
        s->upd(l, x);
    else // range update
        s->upd(l, r - 1, x);
}
else
{
    int l, r;
    cin >> l >> r;
    cout << s->qry(l, r - 1) << endl; // range sum
}
return 0;
}</pre>
```

10.7 min queue

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __qnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define MAXN 1005
#define mod 998244353
namespace min queue
  deque<pi> q;
  int 1, r;
  void init()
    1 = r = 1;
    q.clear();
  void push(int v)
```

```
while (!q.empty() \&\& v < q.back().fir)
      q.pop_back();
    q.pb({v, r});
    r++;
  void pop()
    if (!q.empty() && q.front().sec == 1)
      q.pop_front();
    1++;
  int getmin()
    return q.front().fir;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int n, m;
  cin >> n >> m;
  vector<int> v(n);
  for (int i = 0; i < n; i++)
    cin >> v[i];
  int 1 = 0, r = m - 1;
  cout << 1 << " " << r << endl;
  for (int i = 1; i <= r; i++)
    min_queue::push(v[i]);
  cout << min_queue::getmin() << " ";</pre>
  1++, r++;
  while (r < n)
    min_queue::pop();
    min_queue::push(v[r]);
    cout << min_queue::getmin() << " ";</pre>
   1++, r++;
  cout << endl;
  return 0;
// minimum of each subarray of length m (m <= n)</pre>
```

10.8 segtree lazy

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
```

```
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
#define int long long int
#define endl '\n'
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 200005
#define mod 998244353
struct segtree
  int n;
 vector<int> v;
  vector<int> seq;
  vector<int> lazy;
  segtree(int sz)
    n = sz;
    seq.assign(4 * n, 0);
   lazy.assign(4 * n, 0);
   // v = vv; // for build
    // build(0, n - 1, 1); // for build
  int single(int x)
    return x;
  int neutral()
    return 0;
  int merge(int a, int b)
    return a + b;
  void add(int i, int l, int r, int diff)
    seg[i] += (r - 1 + 1) * diff;
    if (1 != r)
     lazy[i << 1] += diff;
      lazy[(i << 1) | 1] += diff;
```

```
lazy[i] = 0;
void update(int i, int l, int r, int ql, int qr, int
   diff)
  if (lazy[i])
    add(i, l, r, lazy[i]);
 if (1 > r || 1 > qr || r < q1)
    return;
  if (1 >= q1 \&\& r <= qr)
    add(i, 1, r, diff);
    return:
  int mid = (1 + r) >> 1;
 update(i << 1, 1, mid, ql, qr, diff);</pre>
 update((i << 1) | 1, mid + 1, r, ql, qr, diff);
  seg[i] = merge(seg[i << 1], seg[(i << 1) | 1]);
int query(int 1, int r, int q1, int qr, int i)
 if (lazy[i])
    add(i, l, r, lazy[i]);
  if (1 > r || 1 > qr || r < q1)
    return neutral();
 if (1 >= q1 \&\& r <= qr)
    return seg[i];
 int mid = (1 + r) >> 1;
  return merge(query(l, mid, ql, qr, i << 1), query(</pre>
     mid + 1, r, ql, qr, (i << 1) | 1);
void build(int 1, int r, int i)
 if (1 == r)
    seg[i] = single(v[1]);
   return;
 int mid = (1 + r) >> 1;
 build(1, mid, i << 1);
 build (mid + 1, r, (i << 1) | 1);
  seg[i] = merge(seg[i << 1], seg[(i << 1) | 1]);
int qry(int 1, int r)
 return query (0, n - 1, 1, r, 1);
void upd(int 1, int r, int x)
  update (1, 0, n - 1, 1, r, x);
```

```
};
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int n, q;
  cin >> n >> q;
  segtree s(n);
  while (q--)
    int t;
    cin >> t;
    if (t == 1)
      int 1, r, x;
      cin >> 1 >> r >> x;
      s.upd(l, r, x);
    else
      int 1, r;
      cin >> 1 >> r;
      cout << s.qry(1, r) << endl;
  return 0;
```

10.9 fenwick3

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
template <class T>
using ordered set = tree<T, null type, less<T>,
   rb_tree_tag, tree_order_statistics_node_update>;
//#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define MAXN 200005
#define mod 1000000007
```

```
int v[MAXN];
namespace bit
  ordered_set<int> bit[MAXN];
  int query(int r, int a, int b)
    int ret = 0, curr = r;
    for (; r >= 0; r = (r \& (r + 1)) - 1)
      ret += (bit[r].order_of_key(b + 1) - bit[r].
         order_of_key(a));
    return ret;
  void add(int idx, int delta)
    for (; idx < MAXN; idx = idx | (idx + 1))
      bit[idx].insert(delta);
  void rem(int idx, int delta)
    for (; idx < MAXN; idx = idx | (idx + 1))
     bit[idx].erase(delta);
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  return 0;
// ideia da merge sort tree na bit (fica mais rapido)
// so fazer uma bit de ordered set ou vector(se nao
   tiver update)
// add -> adiciona o numero delta na posicao idx
// rem -> remove o numero delta na posicao idx
// query -> retorna o numero de elementos tal que
   posicao <= r && (a <= num <= b)
```

10.10 treap2

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;
```

```
#define int long long int
#define endl '\n'
#define pb push_back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 1000005
#define mod 1000000007
vector<int> ans;
struct treap
  int data, priority;
  int sz;
  bool lazy;
  treap *1, *r;
};
int size(treap *node)
  return (!node) ? 0 : node->sz;
void recalc(treap *node)
  if (!node)
    return;
  node -> sz = 1;
  if (node->1)
    node -> sz += node -> l -> sz;
  if (node->r)
    node->sz += node->r->sz;
void lazy_propagation(treap *node)
  if (!node || !(node->lazy))
    return;
  swap(node->1, node->r);
  if (node->1)
    node \rightarrow 1 \rightarrow lazy = 1;
  if (node->r)
    node \rightarrow r \rightarrow lazy = 1;
  node -> lazy = 0;
void merge(treap *&t, treap *1, treap *r)
  lazy_propagation(l);
  lazy_propagation(r);
  if (!1)
```

```
t = r;
  else if (!r)
    t = 1;
  else if (l->priority > r->priority)
    merge (1->r, 1->r, r), t = 1;
  else
    merge (r->1, 1, r->1), t = r;
  recalc(t);
void split(treap *t, treap *&l, treap *&r, int n)
  if (!t)
    return void(1 = r = 0);
  lazy_propagation(t);
  if (size(t->1) >= n)
    split(t->1, 1, t->1, n), r = t;
    split(t->r, t->r, r, n - size(t->1) - 1), 1 = t;
  recalc(t);
void reverse(treap *&t, int 1, int r)
  treap *a0, *a1, *b0, *b1;
  split(t, a0, a1, 1);
  split (a1, b0, b1, r - 1 + 1);
  b0 \rightarrow lazy = 1;
 merge(t, a0, b0);
  merge(t, t, b1);
void shift(treap *&t, int 1, int r)
  treap *a0, *a1, *b0, *b1, *c0, *c1;
  split(t, a0, a1, 1);
  split(a1, b0, b1, r - 1 + 1);
  split(b0, c0, c1, r - 1);
  merge(t, a0, c1);
  merge(t, t, c0);
  merge(t, t, b1);
void dfs(treap *t)
  if (!t)
    return;
 lazy_propagation(t);
  dfs(t->1);
  ans.pb(t->data);
  dfs(t->r);
treap *create node(int data, int priority)
```

```
treap *ret = new treap;
 ret->data = data;
 ret->priority = priority;
 ret -> 1 = 0;
 ret->r = 0:
 ret->sz = 1;
 ret -> lazv = 0;
 return ret;
signed main()
 ios_base::sync_with_stdio(false);
 cin.tie(NULL);
 srand(time(NULL));
 treap *t = 0;
 int n, m, q;
 cin >> n >> q >> m;
 for (int i = 0; i < n; i++)
   int k;
   cin >> k:
   merge(t, t, create_node(k, rand()));
 while (q--)
   int ty, 1, r;
   cin >> ty >> 1 >> r;
   1--, r--;
    (ty == 1) ? shift(t, 1, r) : reverse(t, 1, r);
 dfs(t);
 while (m--)
   int i;
   cin >> i;
   i--;
    cout << ans[i] << " ";
 cout << endl;</pre>
 return 0;
```

10.11 persistent seg2

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
```

```
template <class T>
using ordered_set = tree<T, null_type, less<T>,
                                                               int build(int 1, int r)
   rb tree tag, tree order statistics node update>;
                                                                 if (1 == r)
//#define int long long int
                                                                  return newleaf(0);
#define pb push_back
                                                                 int mid = (1 + r) >> 1;
#define pi pair<int, int>
                                                                 return newpar(build(1, mid), build(mid + 1, r));
#define pii pair<int, pi>
#define fir first
                                                               int qry(int vl, int vr, int l, int r, int k)
#define sec second
#define MAXN 200005
                                                                if (1 == r)
#define mod 1000000007
                                                                  return 1;
#define PI acos(-1)
                                                                 int mid = (1 + r) >> 1;
                                                                 int c = seq[seq[vr].1].item - seq[seq[v1].1].item;
struct node
                                                                 if (c >= k)
                                                                   return qry(seg[v1].1, seg[vr].1, 1, mid, k);
  int item, 1, r;
                                                                 return qry(seq[v1].r, seq[vr].r, mid + 1, r, k - c);
  node() {}
  node(int 1, int r, int item) : 1(1), r(r), item(item)
                                                               signed main()
     { }
} ;
                                                                 ios_base::sync_with_stdio(false);
int n, q;
                                                                 cin.tie(NULL);
                                                                 cin >> n >> q;
vector<node> seq;
vector<int> roots;
                                                                 vector<int> v(n);
                                                                 set < int > vals;
                                                                 for (int i = 0; i < n; i++)
void init()
                                                                  cin >> v[i];
  seq.resize(1);
                                                                  vals.insert(v[i]);
int newleaf(int vv)
                                                                 int mx = 1;
                                                                 map<int, int> mp, mpr;
  int p = seq.size();
                                                                 for (auto const &i : vals)
  seq.pb(node(0, 0, vv));
  return p;
                                                                  mp[i] = mx;
int newpar(int 1, int r)
                                                                  mpr[mx] = i;
                                                                  mx++;
  int p = seq.size();
  seg.pb(node(l, r, seg[l].item + seg[r].item));
                                                                 init();
                                                                 roots.pb(build(0, mx));
  return p;
                                                                 for (auto const &i : v)
                                                                   roots.pb(upd(roots.back(), 0, mx, mp[i]));
int upd(int i, int 1, int r, int pos)
                                                                 while (q--)
  if (1 == r)
    return newleaf(seg[i].item + 1);
                                                                   char c;
  int mid = (1 + r) >> 1;
                                                                   cin >> c;
  if (pos <= mid)</pre>
                                                                   if (c == '0')
    return newpar(upd(seg[i].l, l, mid, pos), seg[i].r);
  return newpar(seg[i].1, upd(seg[i].r, mid + 1, r, pos)
                                                                     int 1, r, k;
     );
                                                                     cin >> 1 >> r >> k;
```

```
1--, r--;
      cout \ll mpr[qry(roots[1], roots[r + 1], 0, mx, k)]
          << endl;
    else
      int x;
     cin >> x;
     x--;
      swap (v[x], v[x + 1]);
      int a = upd(roots[x], 0, mx, mp[v[x]]);
      int b = upd(a, 0, mx, mp[v[x + 1]]);
      roots[x + 1] = a, roots[x + 2] = b;
    }
  return 0;
// https://neps.academy/br/exercise/127
// queries de k-esimo menor em um range
// e fazer um swap entre v[i] e v[i + 1]
```

10.12 SegTree

```
#include <bits/stdc++.h>
using namespace std;
#define PI acos(-1)
#define pb push back
#define int long long int
#define mp make_pair
#define pi pair<int, int>
#define pii pair<pi, int>
#define fir first
#define sec second
#define MAXN 100001
#define MAXL 100
#define mod 1000000007
vector<int> seq;
vector<int> v;
int single(int x)
  return x;
int neutral()
  return 0;
int merge(int a, int b)
```

```
return a + b;
void update(int i, int l, int r, int q, int x)
  if (1 == r)
    seq[i] = single(x);
    return;
  int mid = (1 + r) >> 1;
  if (q <= mid)
    update(i \ll 1, 1, mid, q, x);
  else
    update((i << 1) | 1, mid + 1, r, q, x);
  seg[i] = merge(seg[i << 1], seg[(i << 1) | 1]);
int query(int 1, int r, int q1, int qr, int i)
  int mid = (1 + r) >> 1;
  if (l > r || l > qr || r < ql)
    return neutral();
  if (1 >= q1 \&\& r <= qr)
    return seq[i];
  return merge(query(l, mid, ql, qr, i << 1), query(mid</pre>
     + 1, r, ql, qr, (i << 1) | 1));
void build(int 1, int r, int i)
  if (1 == r)
    seg[i] = single(v[1]);
    return;
  int mid = (1 + r) >> 1;
  build(1, mid, i << 1);
  build (mid + 1, r, (i << 1) | 1);
  seg[i] = merge(seg[i << 1], seg[(i << 1) | 1]);
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  int n, q;
  cin >> n >> q;
 v.resize(n);
  seq.resize(4 * n);
  for (int i = 0; i < n; i++)
    cin >> v[i];
  build(0, n - 1, 1);
```

```
while (q--)
{
   int 1, r;
   int t;
   cin >> t >> 1 >> r;
   if (t == 2)
      cout << query(0, n - 1, 1, r - 1, 1) << endl;
   else
      update(1, 0, n - 1, 1, r);
}</pre>
```

10.13 Segtree2

```
#include <bits/stdc++.h>
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __qnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 500001
#define mod 1000000007
struct segtree
  int n;
  vector<int> seq;
  int neutral()
    return 0;
  int merge(int a, int b)
    return a + b;
  void build(vector<int> &v)
    n = 1;
    while (n < v.size())</pre>
```

```
n <<= 1;
    seq.assign(n << 1, neutral());</pre>
    for (int i = 0; i < v.size(); i++)</pre>
      seq[i + n] = v[i];
    for (int i = n - 1; i; i--)
      seg[i] = merge(seg[i << 1], seg[(i << 1) | 1]);
  void upd(int i, int value)
    seg[i += n] += value;
    for (i >>= 1; i; i >>= 1)
      seg[i] = merge(seg[i << 1], seg[(i << 1) | 1]);
  int qry(int 1, int r)
    int ansl = neutral(), ansr = neutral();
    for (1 += n, r += n + 1; 1 < r; 1 >>= 1, r >>= 1)
      if (1 & 1)
        ansl = merge(ansl, seg[l++]);
      if (r & 1)
        ansr = merge(seg[--r], ansr);
    return merge(ansl, ansr);
};
signed main()
  ios_base::sync_with_stdio(false);
 cin.tie(NULL);
 return 0;
// iterative segtree without lazy propagation
```

10.14 sqrt decomposition

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;

#define PI acos(-1)
#define pb push_back
#define int long long int
#define pi pair<int, int>
```

```
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 100001
#define mod 1000000007
int n, q;
vector<int> v;
namespace mo
  struct query
    int idx, l, r;
  int block;
  vector<query> queries;
  vector<int> ans;
  bool cmp(query x, query y)
    if (x.1 / block != y.1 / block)
      return x.1 / block < y.1 / block;</pre>
    if (x.r != v.r)
      return x.r < y.r;</pre>
  void sqrt_decomposition()
    block = (int)sqrt(n);
    sort(queries.begin(), queries.end(), cmp);
    ans.resize(queries.size());
    int curr_left = 0, curr_right = 0, curr_sum = 0;
    for (int i = 0; i < queries.size(); i++)</pre>
      int idx = queries[i].idx;
      int 1 = queries[i].1;
      int r = queries[i].r;
      while (curr_left < 1)</pre>
        curr sum -= v[curr left];
        curr left++;
      while (curr left > 1)
        curr left--;
        curr sum += v[curr left];
      while (curr right <= r)</pre>
```

```
curr_sum += v[curr_right];
        curr_right++;
      while (curr_right > r + 1)
        curr_right--;
        curr_sum -= v[curr_right];
      ans[idx] = curr_sum;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  cin >> n >> q;
  v.resize(n);
  for (int i = 0; i < n; i++)
    cin >> v[i];
  for (int i = 0; i < q; i++)
    mo::query curr;
    cin >> curr.l >> curr.r;
    curr.r--;
    curr.idx = i;
    mo::queries.pb(curr);
 mo::sqrt_decomposition();
  for (auto const &i : mo::ans)
    cout << i << endl;</pre>
// to test: https://judge.yosupo.jp/problem/
   static range sum
```

10.15 color update

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;

#define int long long int
#define endl '\n'
#define pb push_back
```

```
#define pi pair<int, int>
                                                                        , {0, 0}});
#define pii pair<int, pi>
                                                                     pii ini = *it, fim = *it2;
#define fir first
                                                                     it2++;
#define sec second
                                                                     ranges.erase(it, it2);
#define MAXN 500001
                                                                     pii upd1 = {ini.left, {l - 1, ini.color}};
#define mod 1000000007
                                                                     pii upd2 = \{r + 1, \{fim.right, fim.color\}\};
                                                                     erased[0].left = max(erased[0].left, 1);
struct color upd
                                                                     erased[sz - 1].right = min(erased[sz - 1].right, r
#define left fir
                                                                     if (upd1.left <= upd1.right)</pre>
#define right sec.fir
                                                                       ranges.insert(upd1);
#define color sec.sec
                                                                     if (upd2.left <= upd2.right)</pre>
                                                                       ranges.insert(upd2);
  set<pii> ranges;
 vector<pii> erased;
  color upd(int n) // inicialmente, todo mundo pintado
                                                                void upd(int a, int b, int c) // pinta o intervalo [a,
     com a cor -1
                                                                     bl com a cor c
    ranges.insert(\{0, \{n-1, -1\}\}\);
                                                                   del(a, b);
                                                                   ranges.insert({a, {b, c}});
  int get(int i) // qual a cor do elemento na posicao i
                                                              } ;
    auto it = ranges.upper_bound({i, {1e18, 1e18}});
                                                               struct segtree
    if (it == ranges.begin())
      return -1;
                                                                vector<int> seq;
    it--;
                                                                vector<int> lazy;
    return (*(it)).color;
                                                                 seqtree(int n)
  void del(int l, int r) // apaga o intervalo [l, r]
                                                                   seq.resize(4 * n, 0);
                                                                   lazy.assign(4 * n, 0);
    erased.clear();
    auto it = ranges.upper_bound({1, {0, 0}});
    if (it != ranges.begin())
                                                                 int single(int x)
      it--;
                                                                   return x;
    while (it != ranges.end())
                                                                 int neutral()
      if ((*(it)).left > r)
                                                                   return 0;
       break;
      else if ((*(it)).right >= 1)
                                                                 int merge(int a, int b)
        erased.push back(*it);
      it++;
                                                                   return a + b;
    if (erased.size() > 0)
                                                                 void add(int i, int l, int r, int diff)
      int sz = erased.size();
                                                                   seg[i] += (r - 1 + 1) * diff;
      auto it = ranges.lower_bound({erased[0].left, {0,
                                                                   if (1 != r)
      auto it2 = ranges.lower_bound({erased[sz - 1].left
                                                                    lazy[i << 1] += diff;
```

```
lazy[(i << 1) | 1] += diff;</pre>
   lazv[i] = 0;
  void update(int i, int l, int r, int ql, int qr, int
     diff)
    if (lazy[i])
      add(i, l, r, lazy[i]);
    if (1 > r || 1 > qr || r < q1)
      return;
    if (1 >= q1 \&\& r <= qr)
      add(i, 1, r, diff);
      return;
    int mid = (1 + r) >> 1;
    update(i << 1, 1, mid, ql, qr, diff);
   update((i \ll 1) | 1, mid + 1, r, ql, qr, diff);
   seq[i] = merqe(seq[i << 1], seq[(i << 1) | 1]);
  int query(int 1, int r, int q1, int qr, int i)
   if (lazy[i])
      add(i, l, r, lazy[i]);
   if (l > r || l > qr || r < ql)
      return neutral();
    if (1 >= q1 \&\& r <= qr)
      return seg[i];
    int mid = (1 + r) >> 1;
    return merge(query(l, mid, ql, qr, i << 1), query(</pre>
       mid + 1, r, ql, qr, (i << 1) | 1));
};
signed main()
 ios_base::sync_with_stdio(false);
 cin.tie(NULL);
 int n, q;
 cin >> n >> q;
 color upd c = color upd(n);
  seqtree s = seqtree(n);
 for (int i = 0; i < n; i++)
   c.upd(i, i, i + 1);
  while (q--)
   int t;
   cin >> t;
   if (t == 1)
```

```
int 1, r, x;
    cin >> 1 >> r >> x;
    l--, r--;
    c.upd(l, r, x);
    for (auto const &i : c.erased)
        s.update(1, 0, n - 1, i.left, i.right, abs(x - i .color));
}
else
{
    int 1, r;
    cin >> 1 >> r;
    l--, r--;
    cout << s.query(0, n - 1, l, r, 1) << endl;
}
return 0;
}
// https://codeforces.com/contest/444/problem/C</pre>
```

10.16 bit2d

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace ___qnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>,
   rb tree tag, tree order statistics node update>;
#define int long long int
#define endl '\n'
#define pb push back
#define pi pair<int, int>
#define pii pair<int, pi>
#define fir first
#define sec second
#define MAXN 1000001
#define mod 1000000007
// source: https://github.com/tfg50/Competitive-
   Programming/blob/master/Biblioteca/Data%20Structures
   /Bit2D.cpp
struct bit2d
 vector<int> ord;
 vector<vector<int>> t;
  vector<vector<int>> coord;
```

```
bit2d(vector<pi> &pts) // recebe todos os pontos que
   vao ser inseridos pra construir, mas nao insere
   eles
  sort(pts.begin(), pts.end());
  for (auto const &a : pts)
    if (ord.empty() || a.fir != ord.back())
      ord.pb(a.fir);
  t.resize(ord.size() + 1);
  coord.resize(t.size());
  for (auto &a : pts)
    swap(a.fir, a.sec);
  sort(pts.begin(), pts.end());
  for (auto &a : pts)
    swap(a.fir, a.sec);
    for (int on = upper_bound(ord.begin(), ord.end(),
       a.fir) - ord.begin(); on < t.size(); on += on
       \& -on)
      if (coord[on].empty() || coord[on].back() != a.
        coord[on].push_back(a.sec);
  }
  for (int i = 0; i < t.size(); i++)</pre>
    t[i].assign(coord[i].size() + 1, 0);
void add(int x, int y, int v) // v[a][b] += v
  for (int xx = upper_bound(ord.begin(), ord.end(), x)
      - ord.begin(); xx < t.size(); xx += xx & -xx)</pre>
    for (int yy = upper bound(coord[xx].begin(), coord
       [xx].end(), y) - coord[xx].begin(); yy < t[xx]
       ].size(); yy += yy \& -yy)
     t[xx][yy] += v;
 }
int qry(int x, int y) // soma de todos os v[a][b] com
   (a \le x \&\& b \le y)
 int ans = 0;
  for (int xx = upper bound(ord.begin(), ord.end(), x)
      - ord.begin(); xx > 0; xx -= xx & -xx)
```

```
for (int yy = upper_bound(coord[xx].begin(), coord
          [xx].end(), y) - coord[xx].begin(); <math>yy > 0; yy
          -= yy \& -yy)
        ans += t[xx][yy];
    return ans;
  int qry2 (int x1, int y1, int x2, int y2)
    return qry(x2, y2) - qry(x2, y1 - 1) - qry(x1 - 1,
       y2) + qry(x1 - 1, y1 - 1);
 void add2(int x1, int y1, int x2, int y2, int v)
    add(x1, y1, v);
    add (x1, y2 + 1, -v);
    add(x2 + 1, y1, -v);
    add (x^2 + 1, y^2 + 1, v);
} ;
signed main()
  ios_base::sync_with_stdio(false);
  cin.tie(NULL);
  return 0;
```

10.17 fenwick2

```
// fenwick com update pro range [1, r]
// complexidade O(q * log(n)) com a criacao de duas bits
    ao inves de uma
#include <bits/stdc++.h>
using namespace std;
#define PI acos(-1)
#define int long long int
#define pb push back
#define mp make_pair
#define pi pair<string, int>
#define pii pair<int, pi>
#define fir first.
#define sec second
#define MAXN 100001
#define MAXL 20
#define mod 998244353
int n;
vector<int> bit, bit2;
```

```
void add1(int idx, int delta)
  for (; idx < n; idx = idx | (idx + 1))
    bit[idx] += delta;
void add2(int idx, int delta)
  for (; idx < n; idx = idx | (idx + 1))
    bit2[idx] += delta;
void update_range(int val, int l, int r)
  add1(1, val);
 add1(r + 1, -val);
 add2(1, val * (1 - 1));
  add2(r + 1, -val * r);
int sum1(int r)
  int ret = 0;
  for (; r >= 0; r = (r \& (r + 1)) - 1)
    ret += bit[r];
  return ret;
int sum2(int r)
 int ret = 0;
  for (; r >= 0; r = (r \& (r + 1)) - 1)
   ret += bit2[r];
  return ret;
int sum(int x)
  return (sum1(x) * x) - sum2(x);
int range_sum(int 1, int r)
  return sum(r) - sum(1 - 1);
int main()
 bit.assign(MAXN, 0); // inicializar sempre
 bit2.assign(MAXN, 0); // inicializar sempre
  update_range(x, 1, r); // pra cada elemento em [1, r]
  range_sum(l, r); // soma de [l, r]
```

10.18 fenwick

```
#include <bits/stdc++.h>
using namespace std;
#define PI acos(-1)
#define int long long int
#define pb push_back
#define mp make_pair
#define pi pair<int, int>
#define fir first
#define sec second
#define MAXN 501
#define MAXL 20
#define mod 998244353
int n;
vector<int> bit;
int sum(int r)
  int ret = 0;
  for (; r >= 0; r = (r \& (r + 1)) - 1)
   ret += bit[r];
  return ret;
void add(int idx, int delta)
  for (; idx < n; idx = idx | (idx + 1))
    bit[idx] += delta;
signed main()
  cin >> n;
 vector<int> v(n);
 bit.assign(n, 0);
  for (int i = 0; i < n; i++)
    cin >> v[i], add(i, v[i]);
  int q;
  cin >> q;
  while (q--)
    char t;
    cin >> t;
    if (t == 'Q') // query
      int 1, r;
      cin >> 1 >> r;
      cout \ll (sum(r) - sum(l - 1)) \ll endl;
```

```
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```

```
else // update
{
  int a, b;
  cin >> a >> b;
  add(a, b - v[a]);
```

```
return 0;
}
```