Entropic orientational bistability in a magnetically confined colloidal gyroscope

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We report the novel orientational behaviour of a colloidal ferromagnetic rod in a viscous medium in the presence of an external magnetic field. At equilibrium, the rod tends to lie horizontally in the plane and undergoes azimuthal thermal reorientation, while gravity suppresses large thermal deviations of its polar angle from an energy minimum at $\pi/2$. A static external magnetic field acts to trap the rod, confining its azimuthal angle to maximise alignment of the magnetic moment with the field. However, we observe a significant emergence of a tendency for the rod to reorient vertically despite a corresponding increase in total energy of $\sim 6\ \mathrm{k}_B T$. We show that this can be explained theoretically as an entropic effect, and derive analytically the Boltzmann distribution across rod orientations which shows a 2 order of magnitude increase in the probability for a rod to be found in a vertical state when trapped in a static magnetic field. We study the dynamic behaviour of the rods when driven by a rotating magnetic field and observe three distinct regimes of alignment and rod rotation. We hypothesise that these regimes arise from the anisotropic friction of the rod and employ a minimal-model Langevin simulation with no free parameters to corroborate this theory with the experimental data and explore the phase space of the system.

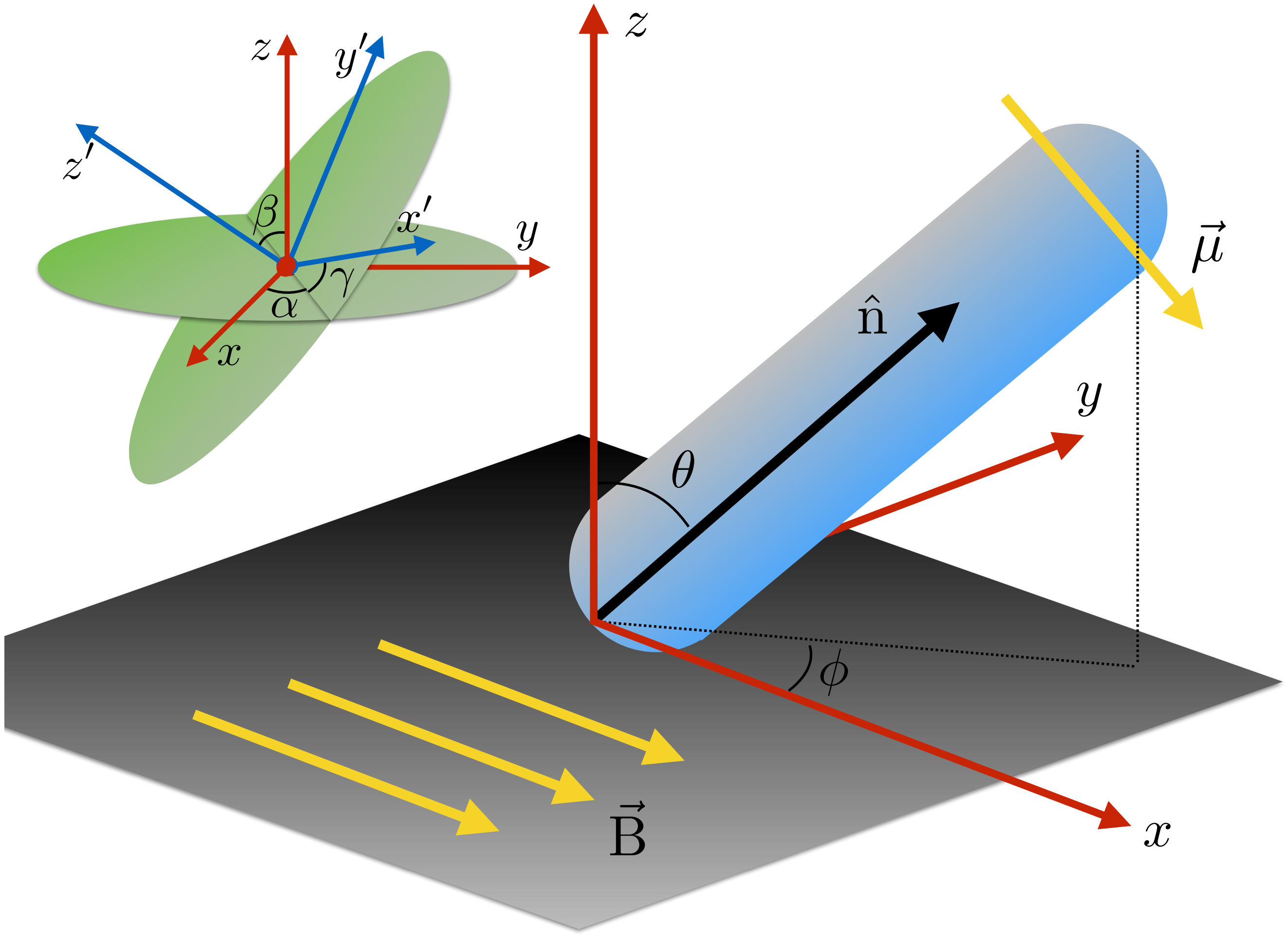
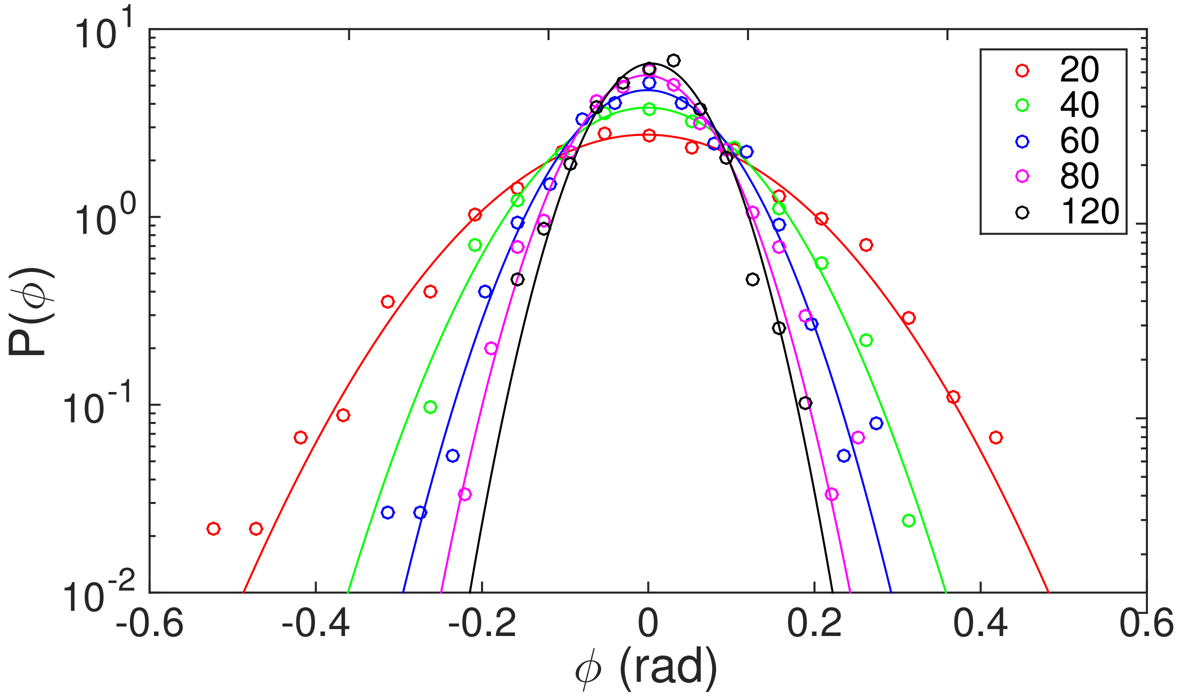
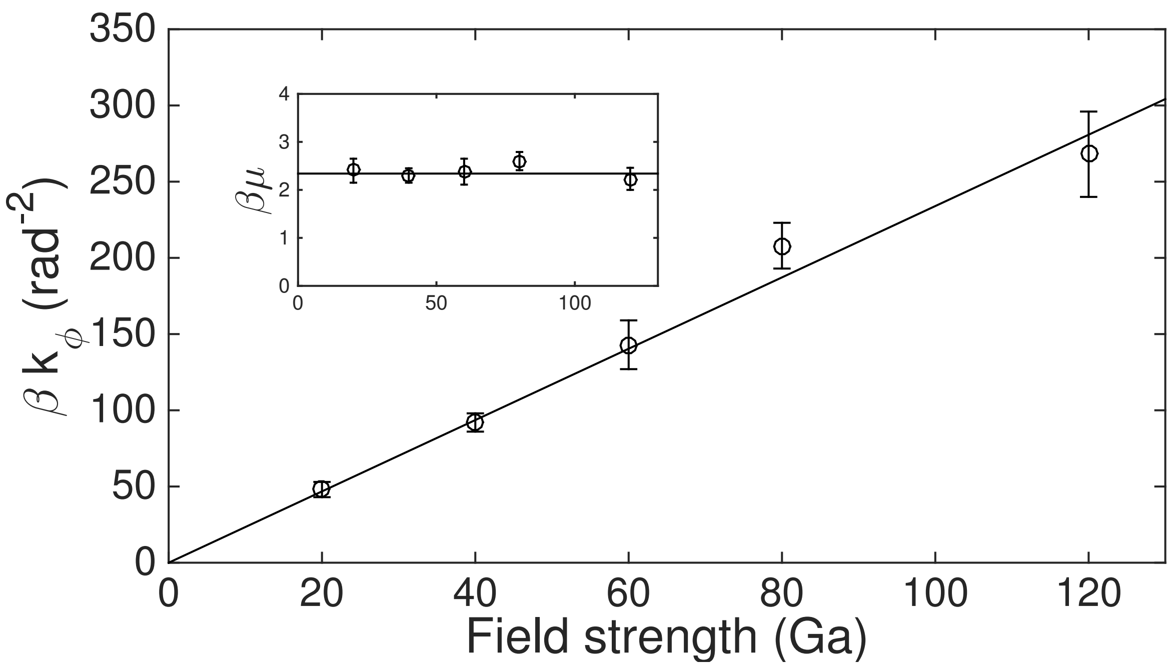


Figure 1: Geometry and notation used throughout. A rod's shape is modelled as a cylinder of length l capped by two hemispheres of diameter d giving it a total length of $L = l + d$. The orientation of the rod, $\hat{\mathbf{n}}$ is defined as the unit vector pointing along the long axis of the rod in the +ve z-direction. This makes an angle θ with the z-axis and its projection in the xy-plane subtends an angle ϕ with the x-axis. A perpendicular permanent magnetic moment $\boldsymbol{\mu}$ is embedded in one of the caps and rotates rigidly with the rod. An external magnetic field $\mathbf{B}$ is applied in the x-direction and interacts with $\boldsymbol{\mu}$, while a gravitational force $m^*(d + L\cos\theta)\mathbf{g}/2$ in the $-$ve z-direction interacts with $\hat{\mathbf{n}}$. Inset: Euler angles are useful for describing the fixed-body rotation of the rod, where $\hat{\mathbf{n}} = \hat{\mathbf{z}}'$, $\boldsymbol{\mu} = \mu\hat{\mathbf{x}}'$, $\beta = \theta$, and $\alpha = \phi + \frac{\pi}{2}$.



(a)



(b)

Figure 2: (a) Probability distribution of the azimuthal angle of deviation ϕ of the rod from alignment with a static magnetic field. The data appear to be distributed normally with a spread $\langle\phi^2\rangle$ obtainable by applying a Gaussian fit. (b) The equipartition theorem states $\frac{1}{2}k_\phi\langle\phi^2\rangle = \frac{1}{2}\mathrm{k}_BT$ where $k_\phi = \mu B$ is the stiffness of the torsional trap. We make use of this to show that $\mathrm{k}_BT/\langle\phi^2\rangle$ increases linearly with B, in other words, μ remains constant across the range of field strengths used. Inset: We estimate the magnetic moment to have a permanent magnitude $\mu = 2.3 \pm 0.2\ \mathrm{k}_BT\mathrm{Ga}^{-1}$.

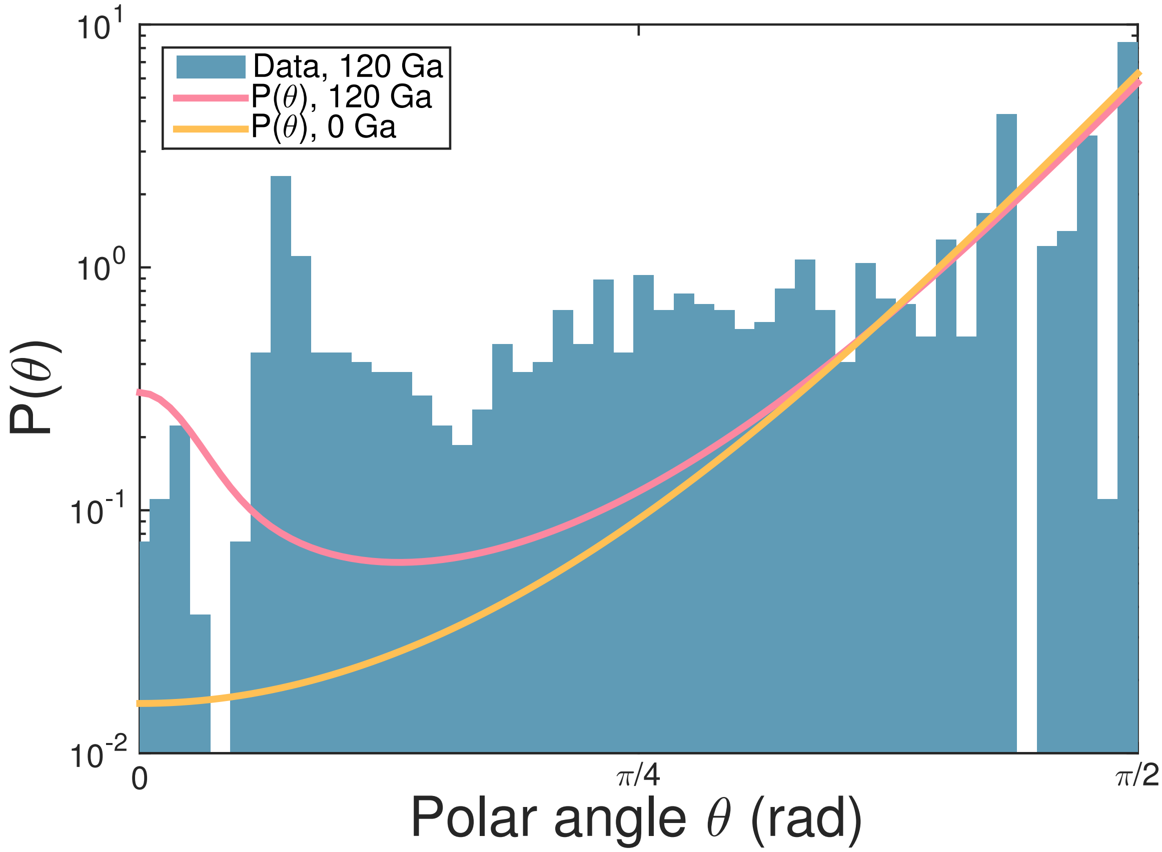


Figure 3: At equilibrium, the presence of a magnetic field has the effect of altering the distribution over polar angle θ resulting in the emergence of a peak at $\theta = 0$. The vertical and horizontal alignments are thus bistable, separated by an entropic barrier. *[Will write more here once we have properly converted the data to distribution over theta.]*

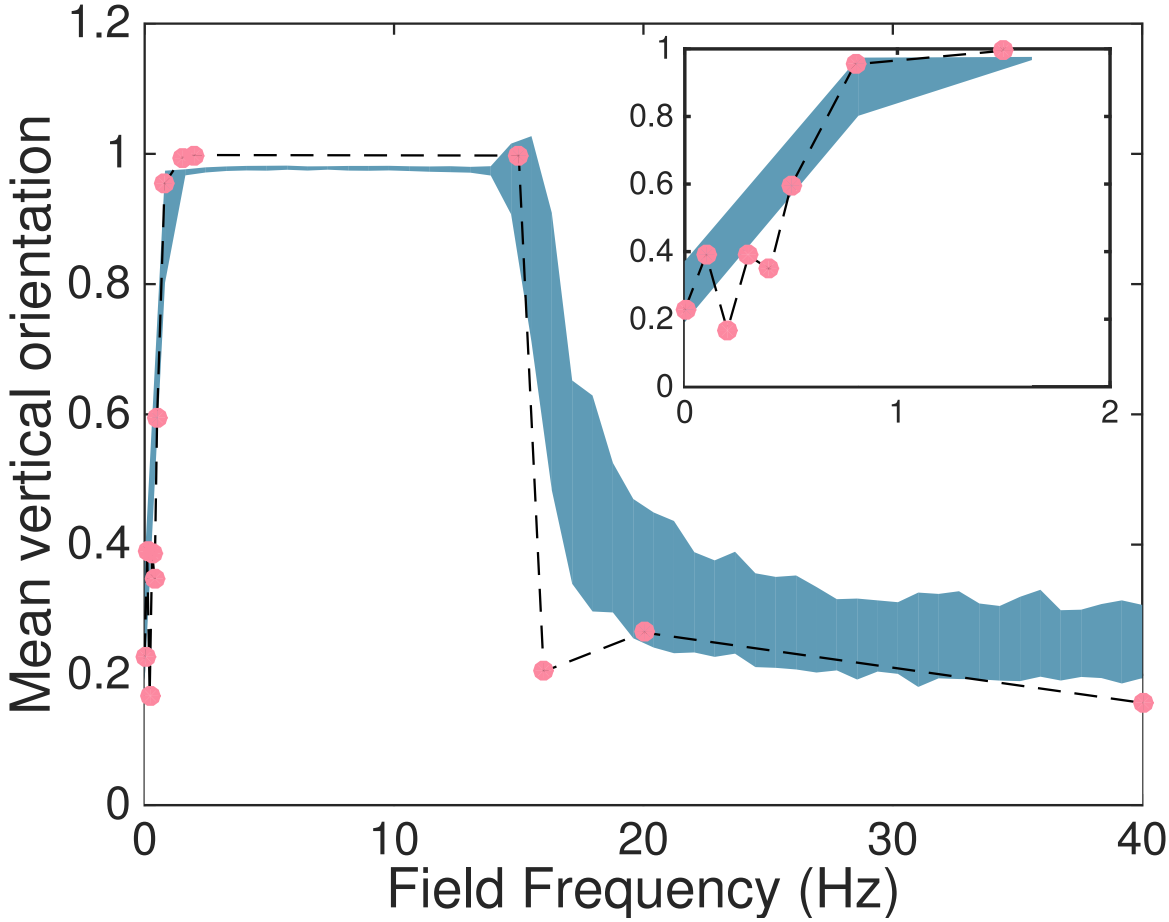


Figure 4: The time-averaged alignment of rod with z-axis, $\langle \hat{\mathbf{n}} \cdot \hat{\mathbf{z}} \rangle$, for varying external field frequency at $B = 50$ Ga. The shaded region represents the mean and standard deviation of results from 100 realisations of a Langevin simulation with no variable parameters, and shows good agreement with the data. This provides evidence to the hypothesis that the frictional anisotropy of the rod gives rise to two threshold frequencies, between which only vertical synchronous rotation of the rod is stable. Inset: Low frequency detail of main figure.