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Entropic stochastic resonance without external force in oscillatory confined space

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We have studied the dynamics of Brownian particles in a confined geometry of dumbbell-shape with periodically oscillating walls. Entropic stochastic resonance (ESR) behavior, characterizing by a maximum value of the coherent factor Q at some optimal level of noise, is observed even without external periodic force in the horizontal direction, which is necessary for conventional ESR where the wall is static and the particle is subjected to the force. Interestingly, the ESR can be remarkably enhanced by the particle gravity G, in contrast to the conventional case. In addition, Q decreases (increases) with G in the small (large) noise limit, respectively, while it non-monotonically changes with G for moderate noise levels. We have applied an effective 1D coarsening description to illustrate such a nontrivial dependence on G, by investigating the property of the 1D effective potential of entropic nature and paying special attention to the excess part resulting from the boundary oscillation. Dependences of the ESR strength with other related parameters are also discussed. © *2015 AIP Publishing LLC.* [http://dx.doi.org/10.1063/1.4921372]

I. INTRODUCTION

In the last two decades, stochastic resonance (SR)-like phenomenon has gained extensive study in a great variety of fields including physics, chemistry, engineering, and biological science.[1–5] SR originally describes a fascinating phenomenon that, in a bistable energy profile, an appropriate amount of noise induced hopping between the potential wells can become synchronized with a small periodic signal.[6] Primarily, SR involves a system with energy potential. However, for soft condensed matter and biological systems down to mesoscopic level and in small confined space, entropic barrier[6–13] resulted from the uneven boundaries[14] could play a prominent role in such processes as motion of particles through an ion channel[15] or in the interior of a living cell.[16] It has been found that such entropic barriers could affect significantly the transport properties of Brownian particles in confined spaces,[7,13,17–21] including rectification,[9,22,23] entropic trapping,[24–26] and particle separation,[27,28] to list just a few.

More interestingly, it is found that such entropic barrier can also lead to SR-like phenomenon, known as entropic stochastic resonance (ESR), which has attracted great attention in recent years. At the very first time, Burada *et al.*[8] reported ESR of a Brownian particle subject to periodic external force in the horizontal direction in a confined geometry of dumbbell shape. The phenomenon is characterized by the presence of one peak in the spectral amplification at the corresponding optimal values of noise strength. Since no energy potential is involved and the phenomenon was shown to result from the effective entropic potential of bistable shape, it is thus called ESR. They also found that a Brownian particle driven by a constant bias along the longitudinal direction could exhibit double ESR.[29] Brownian particles confined to two distinct regions divided by a porous membrane could show a type of ESR completely dependent on a geometric effect.[30,31] Recent study showed that Brownian particles confined to a periodic channel could be trapped by ESR.[26] Note that among most of these studies, gravity force in the vertical direction is of key importance for the Brownian particle to sample the entropic barrier and thus the occurrence of ESR.

While most of previous studies have considered static boundaries,[12,17,19,29,31–34] recently dynamic behavior in confined space with fluctuating or time-dependent boundary has gained growing attention. Actually, in real systems, time-changing boundary could be more important. For instance, the elastic boundary such as microvasculature channel,[35] cell wall[36] could enhance the microswimmers motility.[37] Regular changing of boundary can execute some biological functions, like oscillating boundary which is induced by Ca^{2+} signal transduction in gastrointestinal smooth muscle[38] or airway smooth muscle[39] control cells contraction and relaxation by time. The cell membranes support a variety of wavelike and oscillatory phenomena that involve shape deformations and swimming strokes at low Reynolds number.[40–42] There are also some recent studies investigating dynamics of Brownian particles in confined space with fluctuating or oscillating boundary. It was reported that these oscillating boundaries can give rise to uncustomary behaviors such as dynamic hysteresis[43] and entropic resonant activation.[44] Therefore, it is interesting to ask how an oscillating boundary may influence ESR in a confined geometry. The result to this question may rise new understanding on the dynamics of substance biological systems. To the best of our knowledge, this issue has not been studied yet.

Motivated by this, in the present paper, we have studied the dynamics of Brownian particles in a dumbbell-shaped confined

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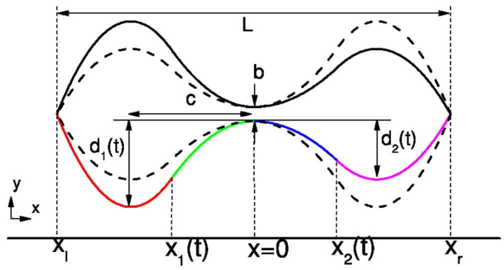

FIG. 1. Schematic illustration of the time-changing dumbbell space. The solid and dashed lines denote the boundary at different time. The boundary line is constructed by four connected quadratic functions. The locations $x_1(t)$ and $x_2(t)$ change with time to make sure the boundary is smooth everywhere expect for the two endpoints. b, c, L are constants fixed during the simulation. $d_1(t)$ and $d_2(t)$ are periodically controlled with frequency Ω.

space as depicted in Fig. 1 by overdamped Langevin equation (LE). The particle is subjected to gravity force in the vertical direction, but no periodic force in the horizontal direction. We assume that the walls are periodically changing with possible phase lags between the left side and the right side. We find that ESR-like behavior can occur, characterizing by a maximum in the coherent factor Q at an optimal noise level. Since no external force is applied, this new behavior may be called coherent-ESR (c-ESR), in accordance with coherent SR which has been widely used in the literature. Interestingly, we find that c-ESR shows rather nontrivial dependence on the particle gravity G. Overall, c-ESR is enhanced by the gravity force, i.e., the maximum Q shifts to larger value with increasing G, in contrast to the conventional ESR where the maximum Q decreases with G. In addition, Q decreases/increases with G in the small/large noise limit, while it non-monotonically depends on G for moderate noise levels. We have tried to apply an effective 1D coarsening description to illustrate such a nontrivial dependence on G, by investigating the property of the 1D effective potential of entropic nature and paying special attention to the excess part resulting from the boundary oscillation. Numerical solution of the 1D kinetic equation also reproduces the simulation results of the Langevin equation.

The paper is organized as follows. In Sec. II, we describe the model with particular attention paid to the setting of the oscillating boundary. Results are presented in Sec. III followed by conclusions in Sec. IV.

II. MODEL

We consider a Brownian particle moving in a confined geometry of dumbbell-shape as depicted in Fig. 1. The overdamped motion of the particle is described through

$$\gamma \frac{d\overrightarrow{r}}{dt} = -G\vec{e}_y + \sqrt{2\gamma k_B T}\,\overrightarrow{\xi}(t). \tag{1}$$

Herein, $\overrightarrow{r} = (x, y)$ is position vector of the particle, $\vec{e}_y$ is the unit vector along the transversal direction, G is a constant denoting the gravitational force, γ denotes the frictional coefficient, and k_B and T are the Boltzmann constant and temperature of the system, respectively. $\vec{\xi}(t)$ is Gaussian white noise with zero mean and unit variance $\langle \xi_i(t)\xi_j(t')\rangle = \delta_{ij}\delta(t-t')$ for $i, j = x, y$.

In contrast to most previous studies, here we consider that the boundary of the dumbbell space is not static but changing with time periodically. There might be many ways to realize an oscillatory boundary, but here we want to fix the bottleneck width b, the dumbbell length L, as well as the locations of points of maximal width as depicted in Fig. 1. Also, we require that the boundary to be smooth during the periodic change. To this end, the lower boundary of the dumbbell is constructed by four smoothly connected quadratic functions. Choosing the bottleneck position to be $x = 0$, it may be described explicitly by

$$w_l(x,t) = \begin{cases} a_1(t)(x+c)^2 - [d_1(t) + \dfrac{b}{2}], & x_l \le x < x_1(t) \\ -a_2(t)x^2 - \dfrac{b}{2}, & x_1(t) \le x < 0 \\ -a_3(t)x^2 - \dfrac{b}{2}, & 0 \le x < x_2(t) \\ a_4(t)(x-c)^2 - [d_2(t) + \dfrac{b}{2}], & x_2(t) \le x < x_r \end{cases}. \tag{2}$$

Herein, $d_1(t) + \frac{b}{2}$ and $d_2(t) + \frac{b}{2}$ denote half the maximum width in the transverse direction of the left and right cell, respectively. The time-dependent coefficients $a_1(t)$ and $a_4(t)$ can be determined $d_{1,2}(t)$ and the condition $w_l(x,t) = 0$ for $x = \mp L/2$, i.e., $a_{(1,4)}(t) = \frac{d_{(1,2)}(t)+\frac{b}{2}}{(L/2-c)^2}$. The smooth condition at $x_1(t)$ and $x_2(t)$ is used to calculate the coefficients $a_2(t)$ and $a_3(t)$, as well as the time-dependent locations of $x_{1,2}(t)$ themselves. The boundary oscillation is realized through periodic modulation of $d_1(t)$ and $d_2(t)$ via $d_1(t) = d_0 + \delta d \sin(\Omega t)$ and $d_2(t) = d_0 + \delta d \sin(\Omega t + \phi)$, respectively, where d_0, δd, and the frequency Ω are constant parameters. In particular, we have considered here a phase lag ϕ between $d_1(t)$ and $d_2(t)$, which characterize synchronization of the "breathing" behaviors between the two cells. If $\phi = 0$, both sides enlarge and shrink simultaneously, and thus, the total volume of the space is not constant but changes periodically. If one expects that the volume of the confined space remains nearly unchanged during the periodic modulation, ϕ would be nearly π, and the two cells breathe in an almost anti-phase way. In the present paper, we will consider mainly the case for $\phi = \pi$ which is reasonable in real systems where the media inside the space may be incompressible, but other values of ϕ will also be investigated for comparison.

To proceed further, we use the dimensionless description of the dynamics, by scaling length by c and time by $\tau = \gamma c^2/k_B T_R$, respectively, where T_R is some reference temperature. Accordingly, the force G is scaled by $k_B T/c$ and temperature is scaled by T_R, and the boundary function is also rescaled in time and length. The Langevin equation is written in a form involving the dimensionless variables $\tilde{\overrightarrow{r}} = \vec{r}/c$, $\tilde{t} = t/(\frac{\gamma c^2}{k_B T_R})$, $\tilde{G} = G/(\frac{k_B T}{c})$, etc. After removing the tilde symbols, the reduced LE reads

$$\frac{d\vec{r}}{dt} = -G\vec{e}_y + \sqrt{2D}\vec{\xi}(t), \tag{3}$$

where $D = T/T_R$ represents a reduced temperature characteristic of the strength of thermal noise. We use Euler method to simulate the LE with a time step $\Delta t = 10^{-4}$. The dimensionless parameters are $b = 0.02$, $L = 3.2$, $\delta d = 0.1$, $d_0 = 0.25$, $\Omega = 0.01$, $G = 5.0$, and $\phi = \pi$ if not otherwise stated, while the noise intensity D is changed.

III. RESULTS

In Fig. 2, we show typical time series of $x(t)$ at a few values of D. Also shown is the function $d_1(t)$ characterizing the boundary oscillation during the time evolution. Clearly, only rare transitions through the bottleneck are observed when D is small, and many random transitions occur when D is large. While for a moderate value of D, the transition seems to be the most coherent with the boundary change, characteristic of the occurrence of SR-like behavior. We note here that no periodic external force exists compared to the well known ESR behavior.[8] To quantitatively measure the resonance intensity, here we use the coherent factor Q defined as follows:[12,45,46]

$$Q = \frac{1}{T_0}\sqrt{\{\int_0^{T_0} 2x(t')\sin(\Omega t')dt'\}^2 + \{\int_0^{T_0} 2x(t')\cos(\Omega t')dt'\}^2}, \quad (4)$$

where T_0 is a long-enough time interval for averaging. In accordance with Fig. 2, the dependence of Q on noise intensity D for $G = 5.0$ is depicted in Fig. 3. A clear-cut maximum appears at $D \simeq 0.3$ indicating ESR *without periodic force*. Note in the literature, SR without external force is often termed as *coherent* SR. Therefore, for convenience of notation, the phenomenon here may be termed as c-ESR. Fig. 3 shows the dependence of Q on D for different driving frequencies Ω with fixed G and oscillating amplitude δd. Clearly, the resonance peak is more pronounced with decreasing frequency Ω, indicating that c-ESR only makes sense when the boundary changes slowly, which is consistent with Ref. 8.

Very interestingly, Figs. 4 and 5 demonstrate rather nontrivial features when one takes a look at the dependence on G. First, we find that c-ESR is apparently enhanced with increasing G, namely, the maximum value of Q increases with G. This is quite in contrast to the case of the ESR reported in Ref. 8, where the particle is subjected to a periodic force in the horizontal direction. Therein, the maximum Q monotonically decreased with increment of G. Second, the dependence of Q on G may show different tendencies for different noise intensities. If D is very small, Q decreases with increasing G. For some intermediate value of D, for instance $D = 0.2$, Q reaches a maximum for $G = 3.0$ as compared to $G = 2.0$ or 5.0. If D is large enough, however, Q becomes a monotonic increasing function of G. These features can be seen more clearly from Fig. 6, where the contour curves of Q in the $G - D$ plane are depicted. The red area indicates the maximum values of Q corresponding to c-ESR, which moves to larger noise intensity D with increasing G. These are also quite different from the case of ESR, where Q is always a decreasing function of G no matter what value D is.

To understand such a nontrivial dependence on G, here we try to use the coarsening description as that used in Refs. 8, 20, and 21, by reducing the Langevin equation to an effective 1D Fokker-Plank equation, reading in dimensionless form

$$\frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial x}\{D\frac{\partial P}{\partial x} + V'(x,D,t)P\}, \quad (5)$$

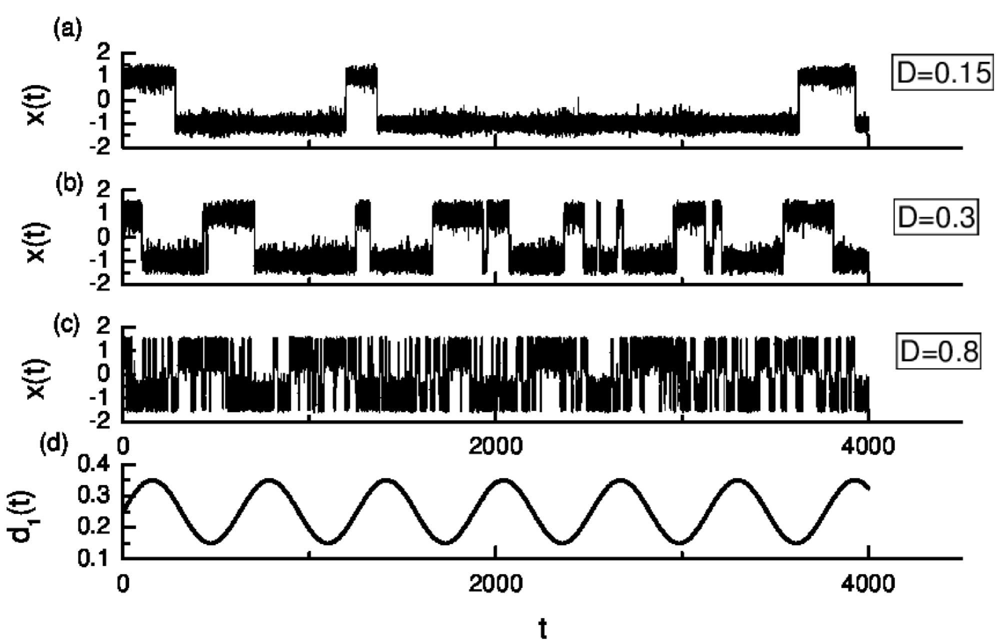

FIG. 2. Typical time series $x(t)$ of the Brownian particle for different noise strengths (a) $D = 0.15$, (b) $D = 0.3$, and (c) $D = 0.8$, respectively. The function $d_1(t)$ related to the boundary oscillation is also shown in (d). The dimensionless parameters are $b = 0.02$, $L = 3.2$, $\delta d = 0.1$, $d_0 = 0.25$, $\Omega = 0.01$, $G = 5.0$, and $\phi = \pi$.

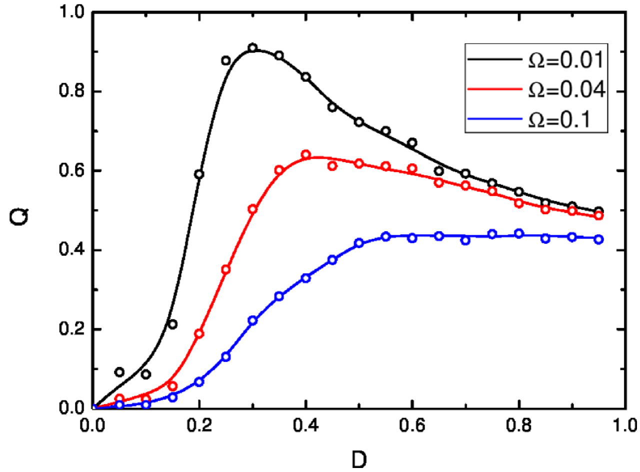

FIG. 3. Q as a function of the noise strength D for various values of oscillating frequency Ω. The empty circles are obtained by simulation and the solid curves are drawn to guide the eyes. Other parameters are the same as in Fig. 2.

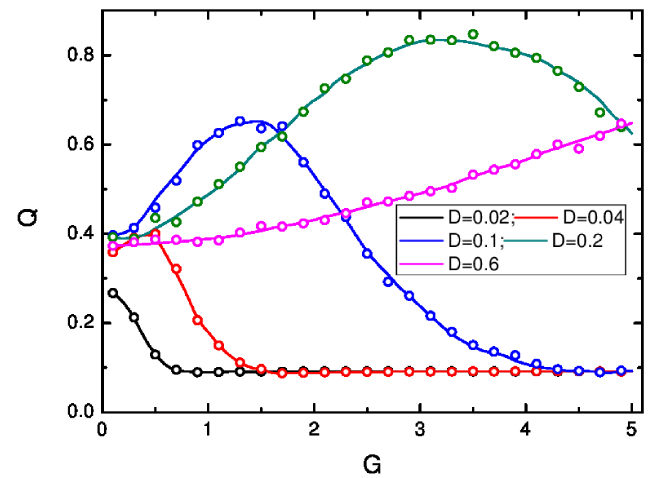

FIG. 5. Q as a function of the gravity G for various values of noise strength D. Circles are obtained by simulation and solid curves are drawn to guide the eyes. Other parameters are the same as in Fig. 2.

where the prime refers to derivative with respect to x and

$$V(x,D,t) = -D\ln[\frac{2D}{G}\sinh(\frac{Gw(x,t)}{D})], \tag{6}$$

with $w(x,t) = -w_l(x,t)$. This equation describes the motion of a Brownian particle in a time-dependent bistable potential of entropic nature. To reach Eqs. (5) and (6), we have adopted the approximation that the boundary changes slowly and the dependence of effective diffusion coefficient on coordinate x is ignored.

It is instructive to compare the present work (with oscillatory boundary but no force) to the conventional case where a periodic force $F(t) = F_0\sin(\Omega t)$ is added in the x-direction.[8] In this latter case, the effective 1D potential reads

$$V(x,D,t) = V_0(x,D) - F_0\sin(\Omega t)\cdot x, \tag{7}$$

where $V_0(x,D) = -D\ln[\frac{2D}{G}\sinh(\frac{Gw_0(x)}{D})]$, where $w_0(x)$ denotes the static boundary function for $\delta d = 0$. Comparing Eq. (6) with Eq. (7), one can see that the oscillatory boundary leads to a type of effective entropic force, but now being dependent on the position x, as well as noise intensity D and particle gravity G. We thus split $V(x,D,T)$ into a static part and an excess part, i.e., $V(x,D,t) = V_0(x,D) + V_{ex}(x,D,t)$, where the excess potential is given by

$$V_{ex}(x,D,t) = -D\ln\left\{\frac{\sinh[Gw(x,t)/D]}{\sinh[Gw_0(x)/D]}\right\}. \tag{8}$$

Now, we may consider two limiting cases. If D is very small, one has $Gw(x,t) \gg D$ and the system is so-called energy-dominated. In the opposite case where $Gw(x,t) \ll D$, the system is entropy-dominated. In Fig. 7, we have plotted the profiles of the total potential $V(x,D,t)$ and the excess part $V_{ex}(x,D,t)$ for $\Omega t = \pi/2$, where $D = 0.01$ for the left panels (a) and (c) and $D = 0.8$ for the right panels (b) and (d), corresponding to the energy- or entropy-dominated case, respectively. Clearly, in both cases, the depth of the excess potential increases with the gravity G, indicating that a heavier particle corresponds to a larger average entropic force. In the former case, the dynamics is controlled by the rare

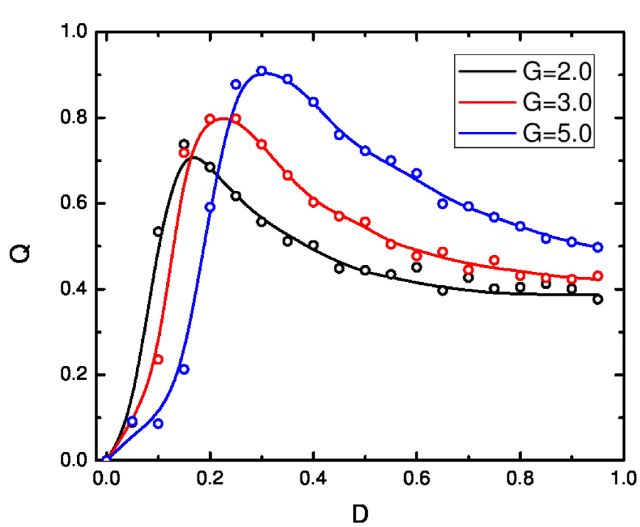

FIG. 4. Q as a function of the noise strength D for various values of gravity G. Circles are obtained by simulation and solid curves are drawn to guide the eyes. Other parameters are the same as in Fig. 2.

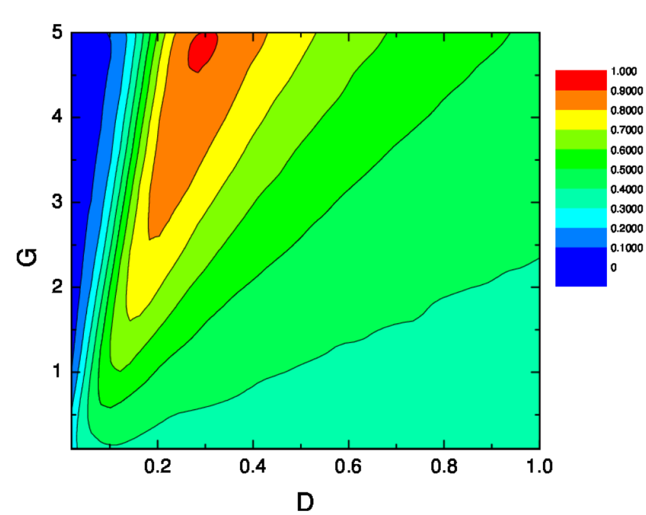

FIG. 6. Contour plot of Q in the $G-D$ plane. Other parameters are the same as in Fig. 2.

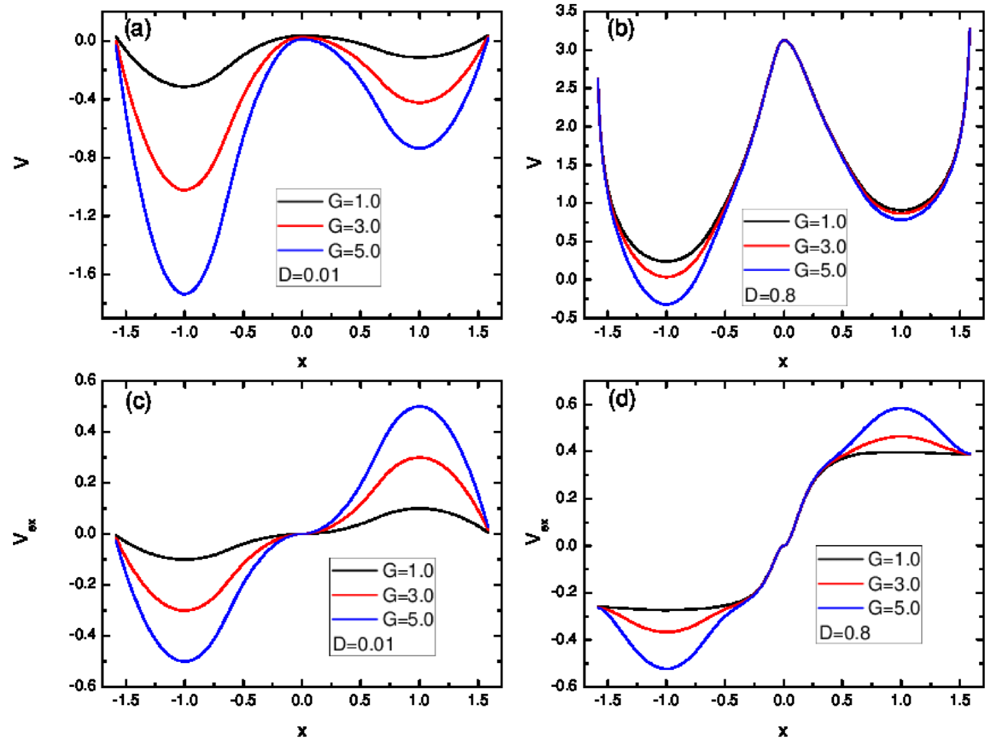

FIG. 7. 1D effective potential $V(x,D,t)$ (upper panels) and the excess part $V_{ex}(x,D,t)$ (lower panels) for $D=0.01$ and 0.8. The shown snapshots are at time $\Omega t=\pi/2$.

barrier-crossing events. Since the barrier height increases with G, it gets harder for the particle to translocate through the bottleneck, leading to a decrease of the Q-factor. In the latter case when the noise is very large, however, the particle randomly jumps between the two cells if the boundary is static. With the increment of G, the particle might become more trapped into the two valley regions around $x=\pm 1$ with increasing entropic force, such that the coherent factor Q can be enhanced. Since Q decreases with G when D is small and it increases with G when D is large, it is reasonable that Q will show non-monotonic dependence on G for intermediate values of D. Note that if the particle is subjected to a constant external force without any G-dependence, such an effect would be absent.

Above discussions provide a possible understanding of the dependence of Q on G. The central point is that oscillatory boundary results in an effective entropic force which is dependent on G itself. Surely, these pictures rely on the validity of 1D Eq. (5), which only works under certain circumstances. In the present work, we have also performed numerical integration of 1D kinetic equation (5) by using a Chebyshev spectral collocation method.[47,48] The time-dependent mean value is defined as $\langle x(t)\rangle = \int xP(x,t)dx$. The dependence of Q on D for different values of the gravity G was shown in Fig. 8. Clearly, it shows very similar features with Fig. 4, although quantitative agreements are not good. On one hand, the overall c-ESR strength is enhanced with increasing gravity force G.

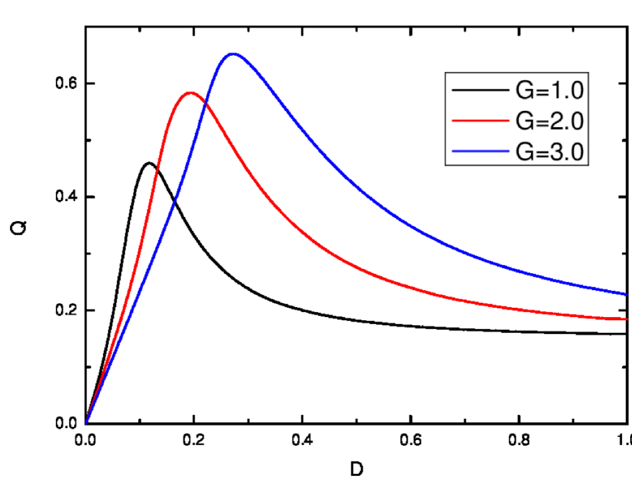

FIG. 8. Dependence of the factor Q on noise level D for different gravities G obtained by numerical solution of 1D kinetic equation (5). Qualitative agreements with Fig. 4 can be observed.

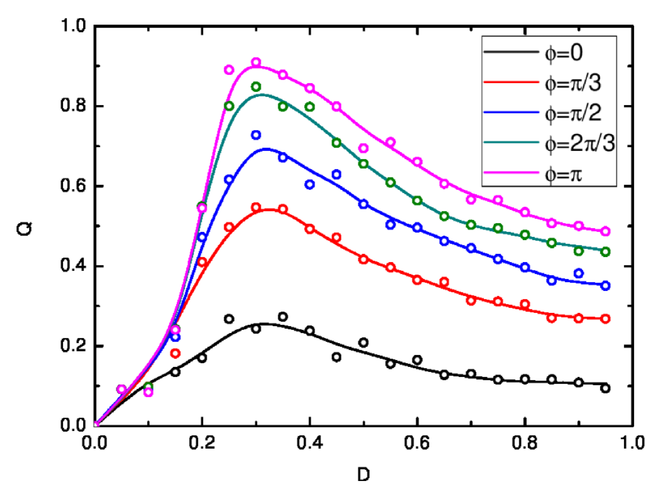

FIG. 9. Q as a function of the noise strength for various values of the phase lag ϕ. Circles are obtained by simulation and solid curves are drawn to guide the eyes. Other parameters are the same as in Fig. 2.

On the other hand, Q decreases with G for small D, increases with G for large D, and depends on G non-monotonically for a moderate level of D.

Finally, we would like to investigate how the phase lag ϕ (all above results are for $\phi = \pi$) would influence c-ESR. This is demonstrated in Fig. 9, where Q as functions of D for different ϕ are shown. Clearly, c-ESR is considerably suppressed when ϕ changes from π to 0, while the optimal value of D keeps nearly unchanged. Therefore, anti-phase oscillation of the two cells of the dumbbell shape is most favorable for c-ESR, but not necessary.

IV. CONCLUSIONS

In summary, we have investigated the dynamics of a Brownian particle in a confined space of dumbbell-shape with oscillatory walls. The boundary oscillation may generate effective entropic force, which causes the particle to move back and forth through the bottleneck and leads to possible coherent motion. The coherent factor can show maximum at an optimal noise level, indicating the occurrence of stochastic-resonance-like behavior. Since no external periodic force is present and the effective potential is of entropic nature, we may call this phenomenon c-ESR. The most nontrivial feature of c-ESR, compared to the conventional ESR where the particle is subjected to periodic force and the wall is static, is that it can be enhanced with the increase of particle gravity. We have tried to use a 1D coarsening description of the kinetics to illustrate such an interesting feature, by investigating how the effective potential and the excess part resulting from the boundary oscillation depend on the gravity. For small noise, the dynamics is energy-dominated and the increase of particle gravity also increases the potential barrier, thus reducing the spectral amplification. If noise is large, on the other hand, the dynamics is entropy-dominated and increasing gravity helps to trap the particle around the two valleys, leading to enhancement of the coherence. Such a picture can help to understand the simulation results that the coherent factor is a monotonic function of gravity in the small or large noise limit, while it may show a maximum at some moderate noise levels. These results will be useful for many practical systems where the boundary effects can play a constructive role, for instance, in rectifying motion of natural microswimmers and Janus particles where boundary effects enhance ratcheting efficiency by an order of magnitude[18] and in escaping dynamics of Janus particles from cavity interactions with boundary can play a dictating role.[49] Moreover, our work may shed new light on the understanding of substance delivery as well as its optimization and control in confined geometry, which may be of great significance in real biological systems.

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