Iterative Phase Retrieval Using the Hybrid Input-Output (HIO) Algorithm

Ansal Kanu, Hansraj Saxena, Yash Raj Sood

March 2025

Introduction

Phase retrieval is a critical problem in imaging where phase information lost during measurement, is reconstructed using specialized methods. The **HIO algorithm** is an iterative technique used to reconstruct images by estimating the missing phase computationally.

Now one asks: Why is phase important? What difference does it make if it is lost during measurement?

To answer this, we talk about what an image is, and what role phase and intensity play in shaping the perception and formation of an image. But, first, we take a look at a general sensor structure and image formation through it.

Sensor Structure and Image Formation

Can be a **CCD** (Charge-Coupled Device) or a **CMOS** (Complementary Metal-Oxide-Semiconductor) sensor. The basic working includes:

- **Microlenses**: Focus incoming light onto photodiodes.
- **Color Filter Array (CFA)**: Separates light into RGB components.
- **Oxide Layer**: Reduces crosstalk, enhancing clarity.
- **Metal Interconnects**: Carry signals from pixels.
- **Photodiode**: Converts photons into electrical signals.
- **Image Processing**: Digital conversion, noise reduction.

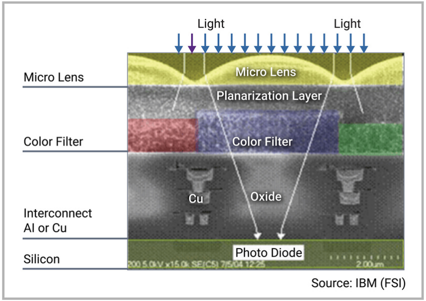


Figure: CMOS Image Sensor Cross-section (IBM)

A Standard Image

Figure: The Lena image.

The Mathematics

- An image is formed when light from a source interacts with an optical system (lens, pinhole, diffraction grating, etc.) and is propagated, focused, or diffracted onto a detector (such as the retina, a camera sensor or a digital photodetector).
- The light wave can be represented as:

$$U(x,y) = A(x,y)e^{i\phi(x,y)}$$

  where, $A(x,y)$ is the amplitude of the wave (related to intensity) and $\phi(x,y)$ is the phase.
- Standard imaging sensors capture only intensity and reject the phase:

$$I(x,y) = |A(x,y)|^2$$

- Phase $\phi(x,y)$ is lost and must be retrieved since it carries necesary information, to form the accurate image.

Importance of Phase Information in Imaging

Phase Encodes Critical Information:

- **Structural Details:** Phase contains most image structure, while magnitude alone leads to loss of clarity.
- **Position Edge Definition:** Determines feature placement and enhances contrast in phase contrast microscopy.
- **Depth 3D Information:** Used in holography, interferometry (LIGO, Michelson), and wavefront sensing.
- Phase is often **more important than magnitude** for accurate image representation. It carries information about depth, transparency, and wavefront distortions.

An example in application

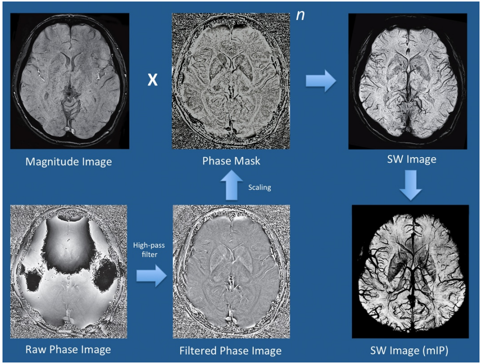


Figure: Phase carries structural information; magnitude alone leads to loss in clarity.

Applications of Phase Retrieval

Phase retrieval has thus become an inseparable need in many fields:

- **Astronomy:** Reconstructing wavefronts for adaptive optics.
- **Life sciences:** Phase contrast imaging in biological studies, MRI, electron microscopy.
- **X-ray Crystallography:** Determining molecular structures.
- **Holography:** 3D imaging and information storage.

Challenge: Recovering lost phase from intensity-only measurements.

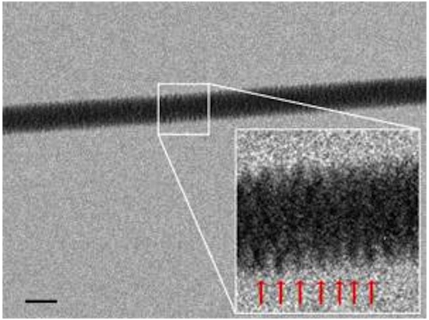
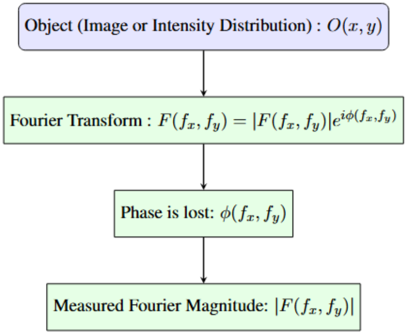


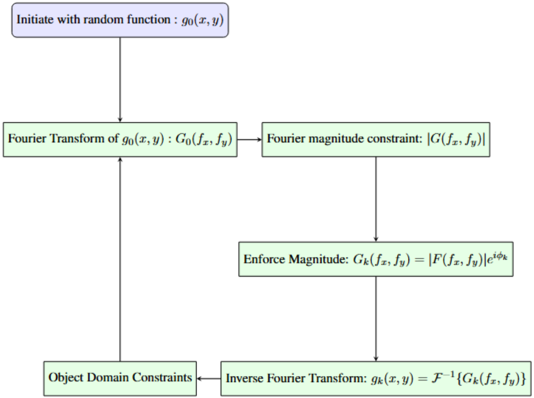
Figure: DNA imaged with an electron microscope. (Credit: Enzo di Fabrizio)

The HIO Algorithm

- Iterative computational approach to estimate missing phase information.
- Alternates between real and Fourier space while applying constraints like **non-negativity** for pixel values and maintaining the **definition of the support** region.
- Overcomes stagnation issues in traditional phase retrieval methods like the **Gerchberg-Saxton** Algorithm.
- **Illumination methods** can be played around with to get better results as suited.
- Current studies implement this with added constraints and variables from different application points of view like **adaptive optics**, **space imaging** and **machine learning**.

Phase Retrieval Process





Mathematical Formulation

Hybrid Input-Output (HIO) Update Rule:

- Given Fourier magnitudes $|F(f_x, f_y)|$, retrieve the lost phase $\phi(f_x, f_y)$.
- The iterative update in the spatial domain:

$$O^{(k+1)}(x,y) = \begin{cases} O'^{(k)}(x,y) & \text{if } (x,y) \in \text{Support}, \\ O^{(k)}(x,y) - \beta\left(O^{(k)}(x,y) - O'^{(k)}(x,y)\right) & \text{otherwise.} \end{cases}$$

- $O'^{(k)}(x,y)$ is the updated solution after enforcing Fourier constraints.
- The parameter β (typically $0.5 \leq \beta \leq 1$) prevents stagnation. We used $\beta = 0.8$.

Implementation Steps

1. Initialize with a random phase distribution.
2. Compute the inverse Fourier transform to real space.
3. Apply support and non-negativity constraints and update using the HIO rule.
4. Transform back to Fourier space and enforce measured magnitudes.
5. Iterate until convergence.

Iterative Phase Retrieval Using HIO Algorithm

Project Overview:

- Our project focuses on implementing the **Hybrid Input-Output (HIO)** algorithm to recover phase from intensity-only measurements, and hence reconstructing an object from the modulus of its Fourier transform.
- We investigate phase retrieval under two different illumination conditions:
  - **Plane wave illumination:** Standard coherent illumination, commonly used in traditional imaging setups.
  - **Charge-1 Vortex beam illumination:** Specialized wavefronts proposing enhanced phase contrast and resolution, in addition to resolving twin-image stagnation.
- A review of relevant existing literature, focusing on the uses of the algorithm and its co-dependents on **Coherent Diffractive Imaging**, **Astronomy** and **Machine Learning**.

Plane wave illumination

Plane wave illumination: 500 iterations were performed to retrieve the grayscale Lena image.

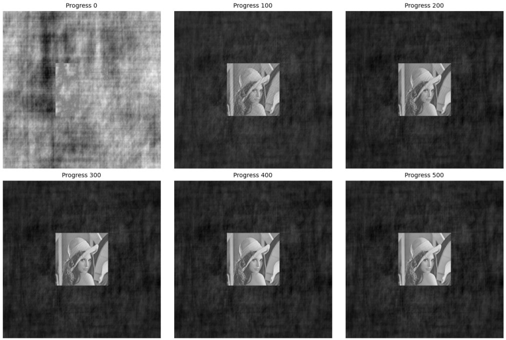


Figure: Progress through increasing iterations.

Original v/s Reconstructed

Charge-1 Vortex Beam Illumination

Why Vortex Beams?

- Standard plane wave illumination can suffer from **twin-image** problems, where the reconstructed object appears with an unwanted **mirror-like duplicate**. Vortex beams help **break this symmetry**, leading to unique reconstructions.
- Enhance phase contrast and improve **retrieval efficiency** in complex objects.
- The helical phase structure of vortex beams provides a more **stable reconstruction** in noisy conditions by reducing artifacts.

What We Did:

- Simulated phase retrieval under vortex beam illumination.
- Implemented the **Hybrid Input-Output (HIO)** algorithm for reconstruction.
- Analyzed improvements in phase recovery compared to plane wave illumination.

Key Observation:

- **Twin-image problem** (ambiguous solutions due to lost phase) is mitigated using vortex illumination. Better reconstruction in fewer iterations.

Reconstruction

Vortex illumination: 400 iterations were performed to retrieve the grayscale Lena image.

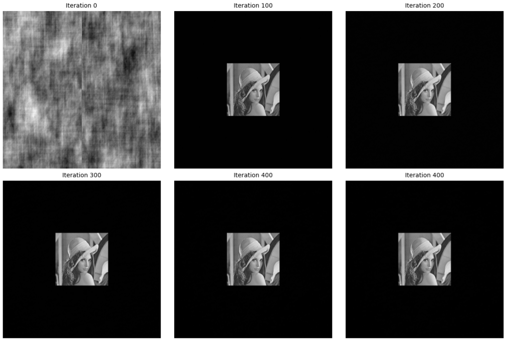


Figure: Progress through increasing iterations.

Original v/s Reconstructed

Original Image

Reconstructed Image (Cropped)

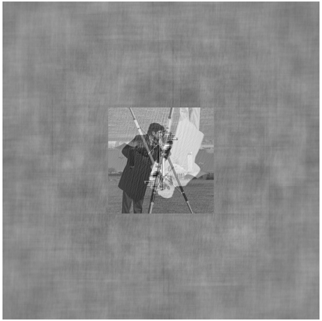
A Comparison

Plane Illumination Reconstruction

Vortex Illumination Reconstruction

Problems encountered: Twin Image Stagnation

Inverted Cameraman Image during reconstruction



Difference: Original - Reconstructed

Application: Coherent Diffractive Imaging

Coherent Diffractive Imaging (CDI) is a technique that enables high-resolution imaging of nanoscale structures by utilizing the principles of diffraction and phase retrieval.

- When the object is sufficiently small and the detector is positioned in the far field, the measured intensity is proportional to the magnitude of the Fourier transform.
- One of the key reasons phase retrieval algorithms work well in optics is due to the intrinsic coherence of electromagnetic waves, which provides a link between the scattered wavefront and the object's structure.

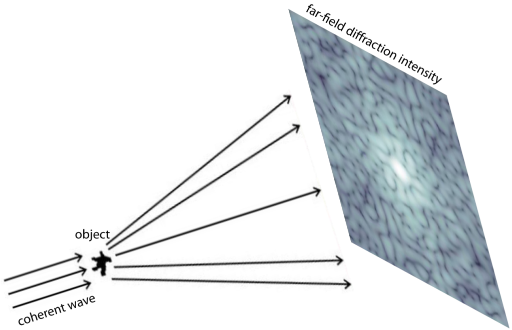


Figure: CDI Setup

Application: Astronomical Imaging

In astronomy, images are often degraded due to optical aberrations, atmospheric turbulence, and instrumental limitations. Recovering phase information allows the correction of distortions caused by atmospheric turbulence and instrumental aberrations, ultimately leading to sharper and more accurate images of celestial objects.

- The **Hubble Space Telescope (HST)** initially suffered from spherical aberration due to a manufacturing flaw in its primary mirror. This was later corrected using the **Corrective Optics Space Telescope Axial Replacement (COSTAR)**, which employed phase retrieval techniques to diagnose and compensate for the aberration, restoring HST's intended imaging capabilities.
- The primary applications of phase retrieval is in wavefront sensing, where phase distortions at the entrance pupil of an imaging system are reconstructed from intensity measurements in the image plane.
- In **exoplanet imaging**, accurate wavefront sensing corrects residual aberrations, improving contrast and enabling the direct detection of faint exoplanets orbiting bright stars.

Alternative Approach (Machine Learning Technique)

Phase retrieval is a longstanding computational problem that is simple to define yet difficult to solve. It consists in the d-dimensional search for $x^* \in \mathbb{C}^d$.

- $y = |Ax^*|^2$
- There, one assumes that the measurements $y \in R^n$ and the matrix $A \in C^{n \times d}$ are known, while $|.|$ is the elementwise modulus operator.

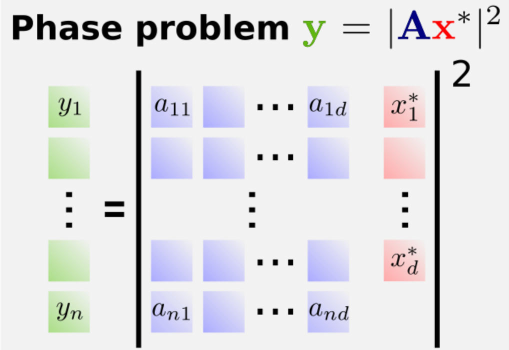


Figure: Phase Equivalent Problem

Conclusion

- Future research will likely focus on reducing the computational complexity of phase retrieval algorithms.
- Hybrid approaches combining machine learning will lead to faster and more accurate solutions.
- The development of provably optimal phase retrieval algorithms that can work under highly noisy and incomplete data conditions is a key challenge.

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Thank You!

(Questions?)