PROJECT SUMMARY

From Pegs to Qubits: A Quantum Take on the Classic Galton Board

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Abstract
This report and accompanying code implementation present a structured implementation and empirical evaluation of quantum Galton boards (QGBs) as universal statistical simulators, extending the framework proposed by Carney and Varcoe (2022). We construct scalable QGB circuits that generate Gaussian, exponential and uniform distributions via discrete–time quantum walks, achieving 76 gates for a four-layer board, less than half the depth of reference methods, while preserving output fidelity. To make the algorithm viable on Noisy Intermediate-Scale Quantum (NISQ) hardware, we incorporate hardware-aware transpilation, gate cancellation and zero-noise extrapolation. Distribution fidelity, quantified with Kolmogorov–Smirnov and Wasserstein distances, remains above 80% on current IBM-Q noise models after mitigation, confirming that depth-optimised QGBs withstand realistic error rates. Our findings demonstrate that quantum Galton boards provide a practical framework for quantum-enhanced statistical sampling on current hardware, transforming theoretical quantum advantages into implementable solutions for complex Monte Carlo problems.

Keywords: quantum computing, Galton board, Monte Carlo simulation, NISQ optimization, quantum walks

1. Introduction

Quantum computing promises exponential advantages over classical computation for specific problem classes, with Monte Carlo methods representing a particularly compelling application domain. The ability to efficiently simulate complex statistical distributions has significant implications across fields such as particle transport modeling and financial risk analysis.

The Universal Statistical Simulator framework introduced by Carney and Varcoe provides an intuitive demonstration of quantum speedup through the quantum Galton board (QGB), where quantum circuits compute all 2^n trajectories using only $O(n^2)$ resources. This marks a fundamental improvement over classical Monte Carlo methods, which often require exponential scaling for comparable statistical accuracy.

This work extends the framework by implementing a complete QGB system, addressing both theoretical underpinnings and practical NISQ-era constraints. Our key contributions include:

- Scalable quantum circuits for multi-layer Galton boards
- Tunable bias control for generating non-uniform distributions
- NISQ optimization using custom noise modeling and mitigation
- Statistical validation of quantum advantage under hardware noise
- Circuit depth reduction relative to alternative quantum sampling methods

2. Theoretical Foundation

2.1 Classical Galton Board Mechanics

The classical Galton board, invented by Francis Galton in 1894, demonstrates the central limit theorem through mechanical means. A ball dropped from the top encounters a series of pegs, with each collision producing a binary choice (left or right). After n layers, the probability distribution is given by:

$$P(k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{1}$$

where $p = 0.5$ for an unbiased board, resulting in the characteristic bell-curve distribution.

2.2 Quantum Advantage Through Superposition

The quantum analogue exploits superposition to represent all possible trajectories simultaneously. While a classical simulation must track 2^n distinct paths, the quantum system encodes them into a single quantum state:

$$|\psi\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} |i\rangle \tag{2}$$

This enables parallel evaluation of all paths, forming the basis of quantum advantage in statistical sampling.

2.3 Quantum Fourier Transform Connection

The QGB's efficiency is conceptually linked to the Quantum Fourier Transform (QFT), which provides exponential speedup for certain structured problems. The QGB mirrors QFT-like interference patterns and amplitude modulation, allowing efficient generation of distributions such as Gaussian and exponential, demonstrating quantum advantage in a statistically intuitive context.

3. Implementation and Methodology

3.1 Quantum Circuit Architecture

Our implementation follows a modular "quantum peg" design, where each peg is constructed using Hadamard gates, controlled-SWAP operations, and CNOT gates. The core module includes:

- Hadamard initialization of the control qubit
- Controlled-SWAPs for trajectory branching
- CNOTs for entanglement and probability routing
- Reset operations for control qubit reuse

The complete n-layer circuit requires $2n^2+5n+2$ gates in the worst case- a notable reduction from the 167-gate reference for 5-bin output.

3.2 Distribution Generation Mechanisms

3.2.1 Gaussian Distribution

The unbiased quantum Galton board naturally produces Gaussian distributions through the quantum analogue of the central limit theorem. Each quantum peg maintains equal probability amplitudes for left and right trajectories, resulting in symmetric bell curve outputs.

3.2.2 Exponential Distribution

To produce an exponential distribution, we replace the Hadamard gates with layer-dependent $R_x(\theta)$ gates applied to the control qubit:

$$R_x(\theta) = \begin{pmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ -i\sin(\theta/2) & \cos(\theta/2) \end{pmatrix} \tag{3}$$

In the right-biased configuration, the angle θ starts at $\pi/4$ and decreases with each layer, reducing the probability of leftward branching. This creates an asymmetric scattering pattern, with a higher likelihood of the particle moving right at each stage, effectively generating an exponential decay in the bin population from left to right.

3.2.3 Hadamard Quantum Walk

The distribution arises from a quantum walk using Hadamard coin operators. This maximum entropy configuration demonstrates the QGB's versatility beyond classical distribution types.

3.3 Post-Processing Pipeline

Quantum measurement outputs are bitstrings representing qubit states, which must be processed to extract meaningful statistical distributions. Our post-processing procedure includes:

1. Mapping multi-qubit bitstrings to discrete position bins using a custom Galton board encoding scheme
2. Aggregating measurement counts across repeated circuit executions (shots) to estimate probability distributions
3. Normalizing counts and applying noise-aware corrections where applicable (e.g., readout noise compensation)
4. Comparing observed distributions to target models using statistical distance metrics (fidelity, KL divergence, Wasserstein distance)

4. NISQ Optimization and Hardware Implementation

4.1 Noise Model Validation

NISQ devices introduce several error sources that must be addressed for practical implementation. We validate our approach using realistic noise models derived from IBM quantum backends, incorporating:

- Single-qubit gate errors (0.1% error rate)
- Two-qubit gate errors (1% error rate)
- Readout errors (2% error rate)
- Thermal relaxation and dephasing

4.2 Error Mitigation Strategies

Our noise mitigation approach employs four complementary techniques:

- **Custom Noise Model**: Realistic error rates with single-qubit, two-qubit, three-qubit depolarizing errors, and 2% readout errors
- **Fidelity-Based Early Stopping**: Automatic termination when distribution fidelity drops below 75%, preventing excessive circuit depth
- **Gate-Efficient Design**: 76-gate implementation vs. 167 gates in reference methods, providing passive noise reduction through shorter circuits
- **Multi-Metric Validation**: Wasserstein distance, KL divergence, and Total Variation distance to quantify noise-induced degradation and guide optimization

These strategies collectively maintain distribution fidelities above 80% across Gaussian, exponential, and Hadamard walk implementations under realistic NISQ conditions.

4.3 Scalability Analysis

We investigate the trade-off between circuit depth and statistical accuracy, determining optimal layer counts for different noise levels. Our analysis shows that upto 6-layer implementations maintain statistical fidelity above 75% under realistic NISQ conditions.

5. Results and Analysis

5.1 Performance Metrics

Distribution fidelity is quantified using multiple statistical measures:

- **Total Variation Distance**: Measures probability distribution overlap
- **Wasserstein Distance**: Quantifies distribution shape similarity
- **Kolmogorov-Smirnov Statistic**: Tests distribution equivalence

5.2 Distribution Fidelity Measurements

We evaluate implementation success through statistical fidelity metrics. Under noise, our results demonstrate:

- Gaussian distributions achieve $\approx$ 86% fidelity

- Exponential distributions maintain ≈ 93% fidelity
- Hadamard quantum walks preserve ≈ 95% fidelity

5.3 Circuit Efficiency Comparison

Our QGB implementation achieves significant resource advantages:

- 76 gates maximum (vs. 167 for reference methods)
- Reduced circuit depth improves noise resilience
- Modular architecture enables efficient scaling

6. Applications and Future Directions

The quantum Galton board framework extends naturally to a range of computational domains. In high-dimensional Monte Carlo problems, it can be applied to particle transport simulation in complex geometries, financial risk modeling with multiple correlated variables, climate modeling with stochastic components, and optimization problems requiring extensive statistical sampling.

Beyond traditional simulation, quantum statistical simulators offer potential advantages for machine learning tasks that demand high-quality random sampling. This includes their integration into variational quantum algorithms, quantum generative models, and other data-driven quantum pipelines where sampling efficiency directly impacts performance.

7. Limitations and Challenges

Current implementation faces several constraints:

- **Qubit overhead**: Linear scaling with desired output precision
- **NISQ noise**: Limits practical circuit depth and accuracy
- **Gate synthesis precision**: Small rotation angle errors in $R_x(\theta)$ accumulate over layers
- **Compilation overhead**: Hardware transpilation can increase depth and add non-native gates
- **Measurement overhead**: Multiple runs required for statistical significance
- **Post-processing complexity**: Classical computation required for distribution conversion
- **Statistical distance sensitivity**: High sensitivity of KL and Wasserstein metrics to shot noise

8. Conclusion

This work addresses the critical question: can quantum Galton boards maintain their theoretical advantages when implemented on realistic NISQ hardware?

Our investigation provides a comprehensive empirical evaluation of QGB performance under realistic noise conditions. We implement scalable circuit architectures based on the Carney-Varcoe framework, develop hardware-aware optimization techniques, and quantify distribution fidelity using rigorous statistical metrics. Through systematic NISQ simulation and noise mitigation strategies, we demonstrate that quantum circuits can achieve exponential speedup over classical Monte Carlo methods while maintaining acceptable statistical accuracy.

The intuitive nature of the QGB demonstration makes it particularly valuable for quantum computing education and outreach, providing a clear example of quantum advantage without requiring deep complexity theory background.

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Data Availability Statement The implementation code and analysis complementing this project are provided in two Jupyter notebooks, available at: GitHub-Ansal06

Author Contributions [Ansal]: Conceptualization, Implementation, Analysis, Writing. All quantum circuit implementations, NISQ optimization strategies, and statistical analysis performed by the author. [Divyanshu]: Suggestions, discussions, and feedback.

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