Question solution 2066(2nd batch)

Group A(2X10=20)

Q1)Find the length of curve $y = x^{3/2}$ from x=0 to x=4.

Solution:

$y = x^{3/2}$ ----------------------------- $0\le x\le 4$

$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$

$\frac{dy}{dx} = \frac{9}{4}x$

$L=\int_0^4 \sqrt{1+\frac{9}{4}x}\,dx$

$= \int_4^0 (1+\frac{9}{4}x)^{1/2}dx$

$=\left[\frac{\left(1+\frac{9}{4}x\right)^{\frac{3}{2}}}{\frac{3}{2}X9/4}\right]$

$=\frac{8}{27}\left[(1+9)^{\frac{3}{2}}-1\right]$

$=\frac{8}{27}\left[(10)^{\frac{3}{2}}-1\right]$Ans

Q.2) Find critical points of the function $f(x)= x^{3/2}(x-4)$

Solution:

$F(x)=x^{3/2}(x-4)$

$=x^{3/2+1}-4x^{3/2}$

$=x^{5/2}-4x^{3/2}$

$f'(X)=\frac{5}{2}x^{\frac{3}{2}}-4*\frac{3}{2}x^{\frac{1}{2}}$

$=\frac{5}{2}x^{\frac{3}{2}}-\frac{12}{2}x^{\frac{1}{2}}$

$=\frac{5}{2}x^{\frac{3}{2}}-6x^{\frac{1}{2}}$

$f'(x)=0$

or; $\frac{5}{2}x^{\frac{3}{2}}-6x^{\frac{1}{2}}=0$

or; $\frac{5}{2}x^{\frac{1}{2}}-x^{1}-6x^{\frac{1}{2}}=0$

or; $x^{\frac{1}{2}}\left[\frac{5}{2}x-6\right]=0$

or; $\frac{5}{2}x-6=0$

or; $x=\frac{12}{5}$

therefore critical point=(12,0)

Q.N.3) Does the following series Converge?

$$\sum_{n=1}^{\infty}\frac{1}{n^2}=1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots.$$

Solution: we have the given series

$$\sum_{n=1}^{\infty}\frac{1}{n^2}=1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots.$$

Let $f(n)=1/n^2$

$$\sum_{n=1}^{\infty}\frac{1}{n^2} = .f(n) = f(0) + f(1) + f(2) + f(3) + \cdots f(n)$$

= $f(1) + \int_1^{\infty} f(n)$

= $f(1) + \lim_{b\to\infty} \int_1^{b} f(n)$

= $f(1) + \lim_{b\to\infty} \int_1^{b} \frac{1}{n^2} dx$

=$f(1) + \lim_{b\to\infty} \left[-\frac{1}{n}\right]_1^b$

$1 + \lim_{b\to\infty} \left[-\frac{1}{b} + \frac{1}{1}\right]$ =1+(0+1)

$$\sum_{n=1}^{\infty}\frac{1}{n^2} < 2\ , since\ the\ sum\ is\ less\ than\ 2, the\ series\ converges.$$

Q.N.4) Find the polar equation of the circle $(x+2)^2+y^2=4$

Solution:

=$(x+2)^2+y^2=4$

=$x^2+4x+4+y^2$

$r^2+4r\cos\theta=0$

$r^2=-4r\cos\theta$

r=-4cosθ(Ans)

Q.N.5) Find the area of the parallelogram where vertices are A(0,0), B(7,3),C(9,8) and D(2,5).

Solution:

$|\overrightarrow{AB}| = -\bar{i} + \bar{j}$

$|\overrightarrow{AD}| = \bar{i} + \bar{j}$

=>$|\overrightarrow{AB}|$ X$|\overrightarrow{AD}|$=$\begin{vmatrix} i & j \\ -1 & 1 \\ 1 & 1 \end{vmatrix}$

=(-i+j)X(-i+j)

= $|1 + 1| = |2|$=2

Q.N.6)Evaluate the integral

$$= \int_t^{2t} \int_0^t (sinx + cosx)dxdy$$
$$= \int_t^{2t} [-cosx + sinx]_0^t dy$$
$$= \int_t^{2t} [-cost + cos0 + sint - sin0]dy$$
$$= \int_t^{2t} [-cost + sint + 1]dy$$
$$= [-ycost + ysint + y]_t^{2t}$$
$$= -tcost + tsint + t$$

Which is the required equation

Q.N.7) Evaluate the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$$

Solution:

$= \lim_{(x,y)\to(0,0)} \frac{x(x-y)}{\sqrt{x}-\sqrt{y}}$

$= \lim_{(x,y)\to(0,0)} \frac{x(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})}{(\sqrt{x}-\sqrt{y})}$

$= \lim_{(x,y)\to(0,0)} x(\sqrt{x} + \sqrt{y})$

=0

Q.N.8) Find $\left(\frac{\partial\omega}{\partial x}\right)y, z$ if ω=x^2+y-z+sin t and x+y=t.

Solution:

$$\frac{\partial\omega}{\partial x} = 2x + \frac{\partial \sin t}{\partial t} X \frac{\partial t}{\partial x}$$

$$\mathbf{2x + \cos t\, X1 = 2x + \cos(x + y)}$$

Q.N.9) Solve the partial differential equation p+q=x

Solution:

p+q=x

$$\frac{dx}{1} = \frac{dy}{1} = \frac{dz}{1}$$

$$=> \frac{dx}{1} = \frac{dy}{1}$$

$$=> x = y - C1$$

$=> C1 = x - y$……………………………………(i)

$=> \frac{dx}{1} = \frac{dz}{x}$

=>$x.dx = dz$

=>$\frac{x^2}{2}$ =z+c2

C2=$\frac{x^2}{2} - z$...(ii)

(C1,C2)=($x - y, \frac{x^2}{2} - z$)

Q.N.10) Find the general integral of the linear partial differential equation z($xp - yq) = y^2 - x^2$

Solution:

z($xp - yq) = y^2 - x^2$

comparing to the Pp+Qq=R

P=zx, Q=-y, R=$y^2 - x^2$

$\frac{dx}{zx} = \frac{dy}{-zy} = \frac{dz}{y^2 - x^2}$

$\frac{dx}{x} = \frac{dy}{-xy}$

$\log x = -\log y + C1$

C1=$\log x + \log y$

$\log C1 = \log xy$

C1= xy...................................(i)

Now, $\frac{dx+dy}{zx-zy}$=$\frac{dz}{y^2-x^2}$

$\frac{dx+dy}{z(x-y)}$=$\frac{dz}{(x-y)(x+y)}$

csitnepal.com

$$\frac{d(x+y)}{z} = -\frac{dz}{x+y}$$

$$(x+y)d(x+y) = -zdz$$

$$\frac{(x+y)^2}{2} = -\frac{z^2}{2} + C2$$

(C1,C2)=0

$$\left(\boldsymbol{xy}, \frac{(\boldsymbol{x}+\boldsymbol{y})^{\mathbf{2}}}{\mathbf{2}} + \frac{\boldsymbol{z}^{\mathbf{2}}}{\mathbf{2}}\right) = \mathbf{0}$$

Group B(5X4=20)

Q.N 11)State and proove Rolle's Theorem.

(Rolle's Theorem) Suppose f (x) is continuous on [a, b] and dif- ferentiable on (a, b). If f (a) = 0 = f (b), then there exists a point c ∈ (a, b) such that f 0 (c) = 0.

Proof. Suppose f (x) is continuous on [a, b], differentiable on (a, b) and f (a) =

0 = f (b). We will prove the theorem using two cases. First, suppose that f (x) > 0 for some x ∈ (a, b). Since f (x) is continuous on [a, b], there exists a point c ∈ [a, b] for which f (c) is the maximum value of f on [a, b]. Furthermore, f (c) > 0 implies c = a and c = b, so c ∈ (a, b) and so f 0 (c) = 0 because f (x) is differentiable on (a, b).

Now suppose f (x) ≤ 0 for all x ∈ (a, b). Then either f (x) = 0 for all x ∈ (a, b) in which case f 0 (x) = 0 for all x ∈ (a, b), or else f (x) < 0 for some x ∈ (a, b). Since f (x) is continuous on [a, b], we know that there is a point c ∈ [a, b] for which f (c) is the minimum value of f (x) on [a, b]. Since f (c) is the minimum on [a, b] and f (x) < 0 for some x ∈ (a, b), f (c) < 0. Consequently, c = a and c = b, so c ∈ (a, b) and therefore f 0 (c) = 0 because f (x) is differentiable on (a, b). This proves the theorem.

Q.N.12) Find the length of the cardoid r=1+cos$\boldsymbol{\theta}$.

Q.N.13)Define unit tangent vector of a differentiable curve.Find the unit tangent vector of the curve

r(t)=(cos t + t sin t)I+(sint-tcos t) t>0

Unit tangent vector of a differentiable curve r (t) is

$$T = \frac{d\vec{r}}{ds} = \frac{\frac{d\vec{r}^2}{dt}}{\frac{ds}{dt}} = \frac{v}{|v|}$$

Solution:

here,

(cost + tsint)i+(sint-tcost),t>0

we have,

$$V = \frac{d\vec{r}}{dt} = (-sint + sint + tcost)\vec{\imath} + (cost + tsint - cost)\vec{\jmath}$$

|V|=$\sqrt{(tcost)^2 + (tsint)^2}$ =t

$$T = \frac{v}{|v|}$$

$= (cost)\vec{\imath}$+(sint)$\vec{\jmath}$

csitnepal.com

Q.N.14)what do you mean by critical point of a function f(x,y)in a region?Find local extreme values of the function f(x,y)=xy-x^2-y^2-2x-2y+4.

Solution:

An interior point of the domain of a function f where f' is 0 or undefined is called point of function f.

$f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$

Here, $fx = 4 - 2x - 2$

$fy = x - 2y - 2$

For critical points

$fx = 0$

$fy = 0$

ie; $-2x + y = 0$

or;$x - 2y - 2 = 0$

$or; x = -2, y = -2$

Again,

$fxx = -2 < 0$

$fyy = -2 < 0$

$fxy = 1$

$\therefore fxx.fyy - fxy^2$

$= -2X - 2 - (1)^2$

$= 3, which\ is > 0$

$\therefore the\ function\ f(x, y) having\ local\ maximum\ is\ point(-2. -2)$

csitnepal.com

Fmax=-2X-2-(-2)^2-(-2)^2-2X-2-2X-2

=8

$critcial\ point\ f(x) = 0$

$f(y) = 0$

$f(x, y) = xy$

$f(x) = y$

$f(y) = x$

For critical points $f(x) = 0, f(y) = 0$

$y = 0$

$x = 0$

$\therefore critical\ point\ is (0,0)$

Again

$fxx = 0$

$fyy = 0$

$\therefore fxx.fyy - (fxy)^2 = 0.0 - (1)^2 = -1$

$\boldsymbol{\therefore f\ has\ local\ minimum\ at\ points\ (0, 0)}$

Q.N.15)Find a particular integral of the equation:

$$\boldsymbol{\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 2y - x^2}$$

Solution:

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 2y - x^2$$

ie; $(D^2 - D'^2)z = 2y - x^2$

$$PI = \frac{1}{D^2 - D'^2} 2y - x^2$$

$$= \frac{\left[1 - \left(\frac{D'}{D}\right)^2\right]^{-1}}{D^2} (2y - x^2)$$

$$= \frac{1}{D^2} [2y - x^2 + \frac{D'^2}{D^2} (2y - x^2]$$

$$= \frac{1}{D^2} \left[2y - x^2 + \frac{2yD'}{D^2} - \frac{D'}{D^2} x^2\right]$$

$$= \frac{2y}{D^2} - \frac{x^2}{D^2} + \frac{2yD'}{D^4} - \frac{D'}{D^4} x^2$$

$$= \frac{2x^2 y}{2} - \frac{x^4}{12} + \frac{2x^4}{24} - 0$$

$$= x^2 y - \frac{x^4}{12} + \frac{x^4}{12}$$

$$= \boldsymbol{x^2 y}$$

Group C

Q.N.16) Graph the function y=x$^{4/3}$-4x$^{1/3}$

Solution

Given equation is $y = x^{\frac{4}{3}} - 4x^{\frac{1}{3}}$

The first derivative

$$f'(x) = \frac{d}{dx}\left(x^{\frac{4}{3}} - 4x^{\frac{1}{3}}\right) 4x^{\frac{1}{3}}$$

$= \frac{4}{3}x^{\frac{1}{3}} - \frac{4}{3}x^{-\frac{2}{3}}$

$= \frac{4}{3}x^{-\frac{2}{3}}(x-1)$

Solving $f^{'}(x) = 0$ we get

$x = 0 \; and \; x = 1$ which is critical point

Now

$f^{''}(x) = \frac{4}{9}x^{-\frac{2}{3}} + \frac{8}{9}x^{-\frac{5}{2}}$

$= \frac{4}{9}x^{-\frac{5}{2}}(x+2)$

Solving $f^{''}(x) = 0$ we get

X=0 and x=-2 which is the required point of inflection

The sign pattern of $f'$is:

Sign of $\frac{4}{3}x^{\frac{2}{3}}$	+	+	+
Sign of (x-1)	-	-	-
Sign of $f^{'}(x) = \frac{4}{3}x^{\frac{2}{3}}$	- 0	- 1	+
Change in f	↘	↘	↗

For concativity

Sign of $\frac{4}{9}x^{\frac{5}{3}}$:	+	+	+
Sign of $(x+2)$:	-	+	+
Sign of $\frac{4}{9}x^{\frac{5}{3}}$	-	+	+
	Concave down -2	Concave up 0	Concave up

Summary

csitnepal.com

concave down	Concave up	Concave up	concaveup

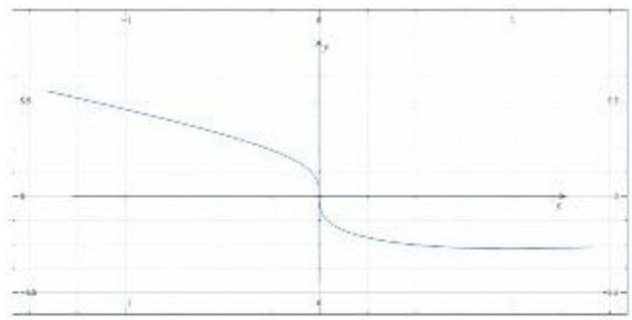

Fig:- Graph (Q.N.16)

Q.N.17)What do you mean by Taylor's Polynomial of order n?Obtain taylor's polynomial and Taylor's series generated by the funcion f(x)=cosx at x=0.

Solution:

The functions & its derivatives are

$f(x) = cosx$ $\qquad f(0) = 1$

$f^{'}(x) = -sinx$ $\qquad f^{'}(0) = 0$

$f^{''}(x) = -cosx$ $\qquad f^{''}(0) = -1$

$f^{'''}(x) = sinx$ $\qquad f^{'''}(0) = 0$

$f^{2k}(0) = (-1^k)$

The series has only even power term for n=2k, taylor's representation is given as:

$$cosx = f(0) + f^{'}(0)(x-0) + \frac{f^{''}(0)}{2!}(x-0)^2 + \frac{f^{'''}(0)}{3!}(x-0)^3 + \frac{f^4(0)}{4!}(x-0)^4 + \cdots + \frac{f^{2k}(0)(x-0)^{2k}}{(2k)!} + R_{2k}(x)$$

csitnepal.com

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \ldots \ldots \ldots \ldots .. + \frac{(-1)^k (x)^{2k}}{(2k)!} + R_{2k}(x)$$

[Note: even term are 0 since every sinx=sin(0)=0]

Because the derivative of the cosine have absolute values less than or equal to 1, the remainder estimation theorem with M=1 gives

$$|R_{2k}(x)| \leq 1.\frac{|x|^{2k+1}}{(2k+1)!}$$

For every value of x, $R_{2k} \to 0\ as\ k \to \infty$. Therefore, the series converges to $cosx$ for every value of x. Thus,

$$cosx = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \ldots \ldots \ldots \ldots \ldots ..$$

csitnepal.com

Q.N.18)Find the volume of the region D enclosed by the surfaces z=x^2+3$y^2$and z=8-x^2-$y^{2.}$

Solution:

$z = x^2 + 3y^2 \ldots \ldots \ldots \ldots \ldots \ldots (i)$

$z = 8 - x^2 - y^2 \ldots \ldots \ldots \ldots .. (ii)$

From(i)&(ii)

$x^2 + 3y^2 = 8 - x^2 - y^2$

$or; 2x^2 + 4y^2 - 8$

$or; 2y^2 = 4 - x^2$

$or; x^2 + 2y^2 - 4 \ldots\ldots\ldots\ldots\ldots (iii)$

$$or; y = \pm\sqrt{\frac{(4 - x^2)}{2}}$$

$i.e; -\sqrt{\frac{4-x^2}{2}} \leq \sqrt{\frac{4-x^2}{2}}$

Now , to find the points for x-axis.

$y = 0$

$$\sqrt{\frac{4 - x^2}{2}} = 0$$

$\therefore x = \pm 2. \quad i.e;\ -2 \leq x \leq 2.$

The points of x-axis are(2,0,0) &(-2,0,0)

Then x^2 plane will be (2,0,4) &(-2,0,4)

$$voume(v) = \int_{-2}^{2}\int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}}\int_{x^2+3y^2}^{8-x^2-y^2} dzdydx$$

$$\int_{-2}^{2}\int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}} 8 - 2x^2 - 4y^2 dydx$$

$$\int_{-2}^{2}\left[8 - 2x^2 - y - \frac{4y^3}{3}\right]_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}} dx$$

$$\left[\int_{-2}^{2} 8\sqrt{\frac{(4-x^2)}{2}} - 2x^2\sqrt{\frac{(4-x^2)}{2}} - 4\left(\sqrt{\frac{(4-x^2)}{2}}\right)^3 + 8\sqrt{\frac{(4-x^2)}{2}} - 2x^2\sqrt{\frac{(4-x^2)}{2}} - \frac{4}{3}\left(\sqrt{\frac{(4-x^2)}{2}}\right)^3\right]dx$$

$$= \left[\int_{-2}^{2}[16\sqrt{\frac{(4-x^2)}{2}} - 4x^2\sqrt{\frac{(4-x^2)}{2}} - \frac{8}{3}\left(\sqrt{\frac{(4-x^2)}{2}}\right)^3\right]dx$$

$$= \int_{0}^{2} \frac{8}{3\sqrt{2}}(4-x^2)^{\frac{3}{2}}\,dx$$

Put $x = 2sin\theta$

$dx = 2cos\theta d\theta$

$$= \frac{16}{3\sqrt{2}}\int_{0}^{\frac{\pi}{2}}(4-4\sin^2\theta)^{\frac{3}{2}}\;2cos\theta d\theta$$

$$\frac{32}{3\sqrt{2}}\int_{0}^{\frac{\pi}{2}}(4\cos^2\theta)^{\frac{3}{2}}\;cos\theta d\theta$$

$$= \frac{32}{3\sqrt{2}}\int_{0}^{\frac{\pi}{2}} 8\cos^4\theta d\theta$$

$$= \frac{32}{3\sqrt{2}}\int_{0}^{\frac{\pi}{2}} 2(2\cos^2\theta)^2\;d\theta$$

$$\frac{64}{3\sqrt{2}}\int_{0}^{\frac{\pi}{2}}(1+\cos\theta)^2\;d\theta$$

Therefore volume$=$ $\boldsymbol{8\sqrt{2\pi}}$

$\therefore \boldsymbol{volume(v) = 8\sqrt{2}}$

Q.N.19) Obtain the absolute maximum and minimum values of the function $f(x,y)=2+2x+2y-x^2-y^2$ on the triangular plate in the first quadrant bounded by lines x=0,y=0,y=9-x.

Solution: Since f is differentiable, the only places where can assume these values are points inside the triangle where $f_x=f_y=0$ and points on the boundary .

a) **Interior points .** For these we have

$f_x=2-2x=0, f_y=2-2y=0,$

yielding the single point(x,y)=(1,1). The valuable of f there is f(1,1)=4

b) **Boundary points**. We take the triangle one side at a time.
i)On the segment OA,y=0. The function
$f(x,y)=f(x,0)=2+2x-x^2$
may now be regarded as function of x defined on the closed interval $0\leq x\leq 9$.Its extreme values may occur at the endpoints
x=0 where f(0,0)=2
x=9 where f(9,0)=2+18-81= -61
and at the interior points where f′(x,0)=2-2x=0. The only interior point where f′(x,0)=0 is x=1, where
f(x,0)=f(1,0)=3
ii)On the segment OB,x=0 and
$f(x,y0=f(0,y)=2+2y-y^2$
we know from the symmetry of f in x and y and from the analysis we just carried out that the candidates on this segment are
f(0,0)=2, f(0,9)= -61, f(0,1)=3.
iii) we have already acounted for the values of f at the endpoints of AB, so we need only
$f(x,y)=2+2x+2(9-x)-x^2-(9-x)^2= -61 + 18x – 2x^2$

setting f′(x,9-x)=18- 4x =0 gives

$x = \frac{18}{4} = \frac{9}{2}.$

At this value of x,

$y = 9 - \frac{9}{2} = \frac{9}{2}\ and\ f(x, y) = f\left(\frac{9}{2}, \frac{9}{2}\right) = -\frac{41}{2}.$

Summary, We list all the candidates: 4,2,-61,3,-(41/2). The maximum is 4, which f assumes at(1,1). The minimum is -61 , which f assumes at (0,9) and (9,0).

OR

Q.N.19) Evaluate the integral

$$\int_0^1 \int_0^{3-3x} \int_0^{3-3x-y} dzdydx$$

Solution:

$$= \int_0^1 \int_0^{3-3x} 3 - 3x - y\ dydx$$

$$= \int_0^1 \left[3y - 3xy - \frac{y^2}{2}\right]_o^{3-3x} dx$$

$$= \int_0^1 3(3 - 3x) - 3x(3 - 3x) - \frac{(3 - 3x)^2}{2} dx$$

$$= \int_0^1 9 - 9x - 9x + 9x^2 - \frac{9 - 18x + 9x^2}{2} dx$$

$$= \frac{1}{2}\int_0^1 18 - 18x - 18x + 18x^2 - 9 + 18x - 9x^2 dx$$

$$= \frac{1}{2}\int_0^1 9x^2 - 18x + 9\ dx$$

$$= \frac{1}{2}[3x^3 - 9x^2 + 9x]_0^1$$

$$= \frac{1}{2}[3X1 - 9X1^2 + 9X1\ ]$$

$$= \mathbf{\frac{3}{2}}$$

Q.N.20) Show the solution of the wave equation.

Solution:

The wave equation $\frac{d^2u}{dt^2} = \frac{C^2d^2u}{dx^2}$

$$where\ C^2 = \frac{T}{\rho}$$

The solution of equation(i) can be obtained by introducing thwo independent variables v and z defined by $v = x + ct\ and\ z = x - ct$

Differentiating v w.r.t x

$$\frac{dv}{dx} = \frac{du}{dv}.\frac{dv}{dx} + \frac{du}{dz}.\frac{dz}{dx}$$

$$= \frac{du}{dv} + \frac{du}{dz}$$

Again differentiate w.r. to x

$$\frac{d^2}{dv}\left(\frac{du}{dv}.\frac{du}{dz}\right)\frac{dv}{dx} + \frac{d}{dz}\left(\frac{du}{dv} + \frac{du}{dz}\right)\frac{dz}{dx}$$

$$= \frac{d^2u}{dvz} + \frac{d^{2u}}{dv^2z} + \frac{d^{2u}}{dzdv} + \frac{d^2u}{dz^2}$$

$$\frac{d^{2u}}{dvz} + \frac{zd^2u}{dvdz} + \frac{d^2u}{dz^2}$$

Again diff u w.r.to t.

$$\frac{du}{dt} = \frac{du}{dv}.\frac{dv}{dt} + \frac{du}{dz}.\frac{dz}{dt}$$

$$= \frac{du}{dv}.c + \frac{du}{dz}(-c)$$

$$= \frac{cdu}{dv} - \frac{cdu}{dz}$$

$$\frac{d^2u}{dt^2} = c\left[\frac{du}{dv}\left(\frac{du}{dv} - \frac{du}{dz}\right)\frac{dv}{dt} + \frac{du}{dz}\left(\frac{du}{dv} - \frac{du}{dz}\right)\frac{dz}{dt}\ \right]$$

$$c^2\left[\frac{d^2u}{dv^2} - \frac{d^2u}{dv^2z} - \frac{d^2u}{dzdv} + \frac{d^2u}{dz^2}\right]$$

$$= C^2\left[\frac{d^2u}{dv^2} - \frac{2d^2u}{dvdz} + \frac{d^2u}{dz^2}\right]$$

Inserting these value in eqn (i)

We get,

$$C^2\left[\frac{d^2u}{dv^2} - \frac{2d^2u}{dvdz} + \frac{d^2u}{dz^2}\right] = C^2\left(\frac{d^2u}{dv^2} + \frac{2d^2u}{dvdz} + \frac{d^2u}{dz^2}\right)$$

$$\frac{4d^2u}{dvdz} = 0$$

$$or; \frac{d^2u}{dvdz} = 0 \ldots \ldots . . (ii)$$

To get solution integrating partially w.r. to z .

$$\frac{du}{dv} = \gamma(v)$$

Integrating w.r to v

$$u = \int \gamma(v)dv + \varphi(2)$$

$$u(x,t) = \emptyset(v) + \varphi(2)$$

$$= \emptyset(\boldsymbol{x}+\boldsymbol{ct}) + \boldsymbol{\varphi}(\boldsymbol{x} - \boldsymbol{ct})\boldsymbol{is\ known\ as\ D^{'}Alembart^{'}ssoltion\ of\ wave\ equation}.$$

OR

Find a particular integral of the equation(D^2-d')z=Acos(lx+my) where A,l,m are constants.

solution;

given equation is $(D^2 - D^{'})z = Acos(lx+my)$

Given equation is $(D^2 - D^{'})z = Acos(lx+my) \dots \dots \dots (i)$

Let

$z = C_1\cos(lx+my) + c_2\sin(lx+my) \dots \dots .(ii)$be the solution of (i) where $C_1\ and\ C_2$ are constant to be determined . then from equation (i) and (ii)

$$(D^2 - D^{'})\ [C_1cos(lx+my) + C_2\sin(lx+my)] = Acos(lx+my)$$

$$or; D^2[C_1cos(lx+my) + C_2\sin(lx+my)] - D^{'}(C_1\cos(lx+my) + C_2\sin(lx+my)] = Acos(lx+my)$$

$$A_1 = D^2[C_1\cos(lx+my) + C_2\sin(lx+my)]$$

$$= D[-C_1\sin(lx+my).l + C_2\cos(lx+my).l]$$

$$-C_1l^2\cos(lx+my) - C_2l^2\sin(lx+my)$$

$$B_1 = D^{'}[C_1\cos(lx+my) + C_2\sin(lx+my)]$$

$$= -C_1m\sin(lx+my) + C_2m\cos(lx+my)$$

$$= C_2m\cos(lx+my) - C_1m\sin(lx+my)$$

solution equation (iii) becomes

$$A_1 - B_1 = Acos(lx+my)$$

$$=> -C_1 l^2 \cos(lx+my) - C_2 l^2 \sin(lx+my) - C_2 m \cos(lx+my) + C_1 m \sin(lx+my) = Acos(lx+my)$$

$$=> -C_1 l^2 - C_2 m) \cos(lx+my) + (C_1 m - C_2 l^2) \sin(lx+my) = Acos(lx+my)$$

Comparing the coefficient of like terms on the both side

$-C_1 l^2 - C_2 m = A$……..(iv)

$C_1 m - C_2 l^2 = 0 \ldots\ldots\ldots..(v)$

From equation(iv)

$$C_2 = -\frac{C_1 l^2 - A}{m}$$

putting C_2 $on\ equatin\ (v)$

$C_1\ m + C1\ l^4 + Al^2 = 0$

$$=> C_1 = -\frac{Al^2}{m^2 + l^4}$$

Now, C_2 becomes $C_2 = -\frac{Am}{m^2+l^4}$

Now, the solution on equation (iii) becomes

$$\boldsymbol{z = -\frac{Al^2}{m^2+l^4}\ cos(lx+my) - \frac{Am}{l^4+m^2}\ \sin(lx+my)}$$