UNIT- 1.0.1.

Partial Derivative and Multiple Integral.

limit :- let f be a function of two variables whose domain includes the point close to (a,b). Then we say that the limit of $f(x,y)$ as (x,y) approaches to (a,b) is L & we write.

$$\lim_{(x,y)\to(0,0)} f(x,y) = 1$$

Ex. evaluate.

(a) $\lim_{(x,y)\to(0,1)} \frac{x-xy+3}{x^2y+(xy-y^3)}$.

$= \frac{0-0+3}{0+0-1^3}$

$= \frac{3}{-1}$

$= -3$ Ans.

(b) $\lim_{(x,y)\to(3,4)} \sqrt{x^2+y^2} =$

(c) $\lim_{(x,y)\to(0,0)} \left(\frac{x^2-xy}{\sqrt{x}-\sqrt{y}}\right) \times \frac{(\sqrt{x}+\sqrt{y})}{(\sqrt{x}+\sqrt{y})}$

$\lim_{(x,y)\to(0,0)} \frac{(x^2-xy)(\sqrt{x}+\sqrt{y})}{(x-y)}$

$\lim_{(x,y)\to(0,0)} = \frac{x\cancel{(x-y)}(\sqrt{x}+\sqrt{y})}{\cancel{(x-y)}}$

$= 0$ Ans.

Continuity of a function of two variable.

A function $f(x,y)$ is continuous at a point (x_0,y_0)

if (i) $f(x_0,y_0)$ i.e. exist [functional value]

(ii) $\lim_{(x,y)\to(x_0,y_0)} f(x,y)$ is exist [~~function~~ limiting value].

(iii) $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0,y_0)$ [equality].

The two-path test for non-existence of a limit.

If a function $f(x,y)$ has different value along two different path as (x,y) approaches to (x_0,y_0) then

$\lim_{(x,y)\to} f(x,y)$ doesnot exist.

Ex. show that $f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$

is continuous at every point except the origin.

Sol^n:- here

Since the function is rational function It is continuous at every point except $(x,y)\neq(0,0)$

At the origin $(0,0)$

let $y=mx$, $x\neq 0$ & m is some finite value.

$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \left(\frac{2xy}{x^2+y^2}\right.$

2, b - 2i, c = 3,

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$$\lim_{(x,y)\to(0,0)} \frac{2mx^2}{x^2(1+m^2)}$$

$$\lim_{(x,y)\to(0,0)} \frac{2m}{1+m^2}$$

Since the limiting value is not equal for different value of m so the function $f(x,y)$ is discontinuous at origin.

Ex. Examine whether $f(x,y)$ is continuous at $(0,0)$ for the function

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Sol^n. let $y = mx$, $x \neq 0$ and m is finite value,

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to} \left(\frac{xy}{x^2+y^2}\right)$$

$$= \lim_{(x,y)\to(0,0)} \left(\frac{x.mx}{x^2+m^2x^2}\right)$$

$$= \lim_{(x,y)\to(0,0)} \left(\frac{mx^2}{x(1+m^2x)}\right)$$

$$\lim_{(x,y)\to(0,0)} \left(\frac{mx}{(1+m^2x)}\right)$$

$= 0$, for any value of m.

Since, the limiting value are zero for any value of m i.e. the limit is exist.

and $f(0,0) = 0$.

$\therefore \lim_{(x,y)\to(0,0)} f(x,y) = f(0,0)$.

Hence the function $f(x,y)$ is continuous at $(0,0)$

(Ex) Show that the function $f(x,y)$ is discontinuous at $(0,0)$

$$\text{if } f(x,y) = \begin{cases} \frac{xy(x-2y)}{x^3+y^3}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

(2) Show that the function $f(x,y)$ is continuous at $(x,y) = (0,0)$ if $f(x,y) = \begin{cases} \frac{xy(x-2y)}{x^2+y^2}, & (x,y) \neq 0 \\ 0 & (x,y) = 0 \end{cases}$

Partial Derivative:

The partial derivative of $z = f(x,y)$ with respect to x, keeping y as constant at point (x_0, y_0) is

$$\frac{df}{dx} = \lim_{h\to 0} \frac{f(x_0+h, y_0) - f(x_0, y_0)}{h}$$

and with respect y, keeping x as constant.

$$\frac{df}{dy} = \lim_{h\to 0} \frac{f(x_0, y_0+h) - f(x_0, y_0)}{h}$$

$3x^2$
$3\frac{dz}{dx}$
$6x$

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Notation $z = f(x,y)$

$\frac{\partial f}{\partial x} = \frac{\partial z}{\partial x}, f_x, z_x$

$\frac{\partial f}{\partial y}, \frac{\partial z}{\partial y}, \& f_y, z_y.$

Ex. If $z = x^3+y^3+3x^2y$. find $\frac{dz}{dx}$ & $\frac{dz}{dy}$,

Sol^n:

$z = x^3 + y^3 + 3x^2y.$

$\frac{dz}{dx} = 3x^2+6xy$

$\frac{dz}{dy} = 3y^2+3x^2$

Ex find $\frac{dz}{dx}$ & $\frac{dz}{dy}$ if z is defined as a function of x and y by the ~~funct~~ equation $x^3+y^3+z^3+6xyz=1$:-

Sol^n: Diff ① partially w.r.to. x.

$3x^2+3z^2\frac{dz}{dx} + 6xy\frac{dz}{dx} + 6yz.1 = 0$

$3\frac{dz}{dx}(z^2+2xy) = -3x^2-6yz$

$\frac{dz}{dx} = \frac{-3(x^2+2yz)}{3(z^2+2xy)}$

$\frac{1}{z}\frac{dz}{dy}$

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Diff ① partially w.r.t. y.

$3y^2+3z^2\frac{dz}{dy} + 6xy\frac{dz}{dy} + 6xz.1 = 0.$

$3\frac{dz}{dy}(z^2+2xy) = -3y^2-6xz = -3(y^2+2xz)$

$\frac{dz}{dy} = \frac{-3(y^2+2xz)}{3(z^2+2xy)} = \frac{-(y^2+2xz)}{(z^2+2xy)}$ Ans.

Ex!. If $yz - \log z = x+y$, $z = f(x,y)$ find $\frac{dz}{dx}$ & $\frac{dz}{dy}$.

Sol^n:- Diff ① partially w.r.to x.

$y.\frac{dz}{dx} - \frac{1}{z}\frac{dz}{dx} = 1.$

$\frac{dz}{dx}\left(y-\frac{1}{z}\right) = 1$

$\frac{dz}{dx} = \frac{z}{yz-1}$ Ans.

Diff(1) partially w.r.to y.

$y.\frac{dz}{dy} + z.1 - \frac{1}{z}\frac{dz}{dy} = 1.$

$\frac{dz}{dy}\left(y-\frac{1}{z}\right) = 1-z$

$\frac{dz}{dy} = \frac{(1-z)z}{(yz-1)}$

For exercise 1.1.

(2) Find the limits,

(i) $\lim_{(x,y)\to(0,0)} \left(\frac{3x^2-y^2+5}{x^2+y^2+2}\right)$

Soln: Here,

$\lim_{(x,y)\to(0,0)} \left(\frac{3x^2-y^2+5}{x^2+y^2+2}\right)$

$\frac{3\times0-0+5}{0+0+2}$

$5/2$ Ans

(ii) $\lim_{(x,y)\to(0,\ln(2))} e^{x-y}$

Soln: Here

$\lim_{(x,y)\to(0,\ln(2))}$. $e^{x-y} = e^{x}.e^{-y}$

$e^{0}.e^{\ln(2)^{-1}}$

$1.\ e^{\ln(1/2)}$

$= \frac{1}{2}$ Ans

(iii) $\lim_{(x,y)\to(3,4)} \left(\sqrt{x^2+y^2-1}\right)$

Soln: Here,

$\lim_{(x,y)\to(3,4)} \left(\sqrt{x^2+y^2-1}\right)$

$\sqrt{9+16-1}$

$\sqrt{24}$

$2\sqrt{6}$ Ans

(iv) $\lim_{(x,y)\to(1,0)} \left(\frac{e^{y}\sin x}{x}\right)$

Soln: here,

$\lim_{(x,y)\to(1,0)} \left(\frac{e^{y}\sin x}{x}\right)$

$\frac{e^{0}\sin 1}{1}$

$\sin 1$ Ans

(v) $\lim_{(x,y,z)\to(1,3,4)} \left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

Soln: here

$\left(\frac{1}{1}+\frac{1}{3}+\frac{1}{4}\right)$

$\frac{12+4+3}{12} = \frac{19}{12}$ Ans

(vi) $\lim_{p \to (0,0,3)} \left(ze^{-2y}\cos 2x\right)$ when $p = x, y, z$

Sol^n: Here,

$\lim_{(x,y,z) \to (0,0,3)} \left(ze^{-2y}.\cos 2x\right)$

$3e^0 \cos 0$

3 Ans //

(3) Show that the limit of the following functn doesnot exist

(i) $\lim_{(x,y) \to (0,0)} \left(\frac{x^2+\sin^2 y}{2x^2+y^2}\right)$ $\left[\frac{0}{0}\right]$

Sol^n: Here,

while applying limit value i.e $\lim_{(x,y) \to (0,0)}$ the function is in ($0/0$ form).

so that,

let $y = mx$, $x \neq 0$ and m is finite

$\lim_{(x,y) \to (0,0)} \left(\frac{x^2+\sin^2 mx}{2x^2+m^2x^2}\right)$

$\left(\frac{0+\sin 20}{0+0}\right) = 0/0$

Since the limiting value are $0/0$ for many value of m i.e. the limit ~~exist~~ doesnot exist.

(ii) $\lim_{(x,y) \to (0,0)} \left(\frac{xy\cos y}{3x^2+y^2}\right)$

Sol^n: here,

$\lim_{(x,y) \to (0,0)} \left(\frac{xy\cos y}{3x^2+y^2}\right)$

let $y = mx$, $x \neq 0$ & m is finite,

$\lim_{(x,y) \to (0,0)} \left(\frac{x.mx\cos mx}{3x^2+m^2x^2}\right)$

$\lim_{(x,y) \to (0,0)} \left(\frac{x^2 m\cos x}{x^2(3+m^2)}\right)$

$\lim_{(x,y) \to (0,0)} \left(\frac{m\cos x}{3+m^2}\right)$

$\left(\frac{m}{3+m^2}\right)$

Since the limiting value are different for ~~many~~ different value of m i.e. the limit doesnot exist

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(iii) $\lim_{(x,y)\to(0,0)} \left(\frac{6x^3y}{2x^4+y^4}\right)$

Sol^n: Here,

$\lim_{(x,y)\to(0,0)} \left(\frac{6x^3y}{2x^4+y^4}\right)$

let $y = mx$, $x \neq 0$, & m is finite

$\lim_{(x,y)\to(0,0)} \left(\frac{6x^3 mx}{2x^4+m^4x^4}\right)$

$\lim_{(x,y)\to(0,0)} \left(\frac{6x^4m}{2x^4+x^4m^4}\right)$

$\lim_{(x,y)\to(0,0)} \left(\frac{6m}{2+m^4}\right)$

Since ~~for~~ any ~~of~~ limiting value ~~of~~ are different for different value of m. i.e. the limit doesnot exist.

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(4) Examine the continuity of the following functions:

(i) $f(x,y) = \begin{cases} \frac{xy(x-2y)}{x^3+y^3} & \text{at } (x,y) \neq (0,0) \\ 0 & \text{at } (x,y) = (0,0) \end{cases}$

Sol^n: Here,

$\lim_{(x,y)\to(0,0)} \frac{xy(x-2y)}{x^3+y^3}$ i.e. $\left[\frac{0}{0} \text{ form}\right]$

let $y = mx$; $x \neq 0$, & m is finite.

$\lim_{(x,y)\to(0,0)} \frac{x\,mx(x-2mx)}{x^3+m^3x^3}$

$\lim_{(x,y)\to(0,0)} \frac{mx^2(x-2mx)}{x^3+m^3x^3}$

$\lim_{(x,y)\to(0,0)} \frac{\cancel{x^3}(m-2m^2)}{\cancel{x^3}(1+m^3)}$

$\frac{m-2m^2}{1+m^3}$

Since ~~for~~ limiting ~~all~~ value are different for different value of m i.e. the limit doesnot exist therefore the function is discontinuous.

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(ii) $f(x,y) = \begin{cases} \dfrac{xy(x-2y)}{x+y} & \text{at } (x,y) \neq (0,0) \\ 0 & \text{at } (x,y) = (0,0) \end{cases}$

Sol: Here

$\lim_{(x,y)\to(0,0)} f(x,y) = \dfrac{xy(x-2y)}{x+y}$

Let $y = mx$; $x \neq 0$ and m is finite.

$\lim_{(x,y)\to(0,0)} \dfrac{xmx(x-2mx)}{x+mx}$

$= \dfrac{x^2(mx - 2m^2x)}{x(1+m)}$

$= \dfrac{x^{\cancel{3}}(m-2m^2)}{x(1+m)}$

$\lim_{(x,y)\to(0,0)} \dfrac{x^2(m-2m^2)}{1+m}$

$0.$

Since the limiting value are same for any value of m i.e. the limit is exist

& $f(0,0) = 0$

$\lim_{(x,y)\to(0,0)} = f(x,y) = f(0,0)$

Hence the function is continuous

(iii) $f(x,y) = \begin{cases} \dfrac{xy(x-2y)}{x^2-y^2} & \text{at } (x,y) \neq (0,0) \\ 0 & \text{at } (x,y) = (0,0) \end{cases}$

Sol: Here,

$\lim_{(x,y)\to(0,0)} \dfrac{xy(x-2y)}{x^2-y^2}$

Let $y = mx$; $x \neq 0$, & m is the finite

$\lim_{(x,y)\to(0,0)} \dfrac{xmx(x-2mx)}{x^2-m^2x^2}$

$\lim_{(x,y)\to(0,0)} \dfrac{m\cancel{x^2}(x-2mx)}{\cancel{x^2}(1-m^2)}$

$\lim_{(x,y)\to(0,0)} \dfrac{x(1-2m)}{1-m^2}$

0

Since the limiting value are same or equal for any value of m i.e. the limit exist.

and $f(x,y) = f(0,0) = 0$

$\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0) = 0.$

Hence the function $f(x,y)$ is continuous.

Ans

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Exercise :- 10.2

1. Calculate the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

(i) $f(x,y) = 2x^2-3y-4$.

Sol^n: Here,

(i) $f(x,y) = 2x^2-3y-4 \quad \text{—(i)}$

diff. w.r.t x of eq^n (i) we get,

$$\frac{\partial f}{\partial x} = 4x-0-0$$

$$\frac{\partial f}{\partial x} = 4x$$

& again diff. w.r.t y of eq^n (i) we get,

$$\frac{\partial f}{\partial y} = \frac{\partial(2x^2-3y-4)}{\partial y} = 0-3-0$$

$$= -3 \text{ Ans}$$

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(ii) $f(x,y) = (x^2-1)(y+1)$

Sol^n: Here,

$$f(x,y) = x^2y+x^2-1\cdot y$$

$$f(x,y) = x^2y+x^2-y-1 \quad \text{—(i)}$$

diff w.r.to. x of eq^n (i) we get,

$$\frac{\partial f}{\partial x} = \frac{\partial(x^2y+x^2-y-1)}{\partial x}$$

$$\frac{\partial f}{\partial x} = y\cdot 2x+2x$$

$$\frac{\partial f}{\partial x} = 2xy+2x$$

$$2x(y+2)$$

diff w.r.to y of eq^n (i) we get,

$$\frac{\partial f}{\partial y} = \frac{\partial(x^2y+x^2-y-1)}{\partial y} = x^2+0-1-0$$

$$= x^2-1 \text{ Ans}$$

(iii) $f(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$

diff. w.r. to x of eq^n (i) we get

$$\frac{\partial f}{\partial x} = \frac{\partial\left((x^2+y^2+z^2)^{-1/2}\right)}{\partial x}$$

$$\frac{\partial f}{\partial x} = \cancel{2x+0+0} \; -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}\cdot 2x$$

$$\frac{\partial f}{\partial x} = -x(x^2+y^2+z^2)^{-3/2} = \frac{-x}{(x^2+y^2+z^2)^{3/2}}$$

diff. w.r.to. y ① we get

$$\frac{\partial f}{\partial y} = \frac{\partial (x^2+y^2+z^2)^{-1/2}}{dy}$$

$$\frac{\partial f}{\partial y} = \cancel{\partial 0+2y+0} \quad -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}\cdot(0+2y+0)$$

$$-\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2y)$$

$$\frac{-y}{(x^2+y^2+z^2)^{3/2}} \text{ Ans}$$

(iv) $f(x,y) = e^{(x+y+1)}$ at $(3,4)$

Sol^n: Here,

$f(x,y) = e^{(x+y+1)}$. at $(3,4)$ — ①

diff. w.r.to. x eq ① we get,

$$\frac{\partial f}{\partial x} = \frac{\partial (e^x.e^y.e)}{\partial x}$$

$$\frac{\partial f}{\partial x} = e^y.e \; e^x.$$

$$\frac{df}{\partial x} = e^4.e.e^3$$

$$= e^8 \text{ Ans}$$

diff. w.r. to. y on both side we get,

$$\frac{\partial f}{\partial y} = \frac{\partial (e^x.e^y.e)}{\partial y}$$

$$= e^x.e \frac{\partial (e^y)}{dy}$$

$$= e^x.e\, e^y$$

$$= e^3.e.e^4$$

$$= e^8 \text{ Ans}$$

(v) $f(x,y) = \dfrac{x+y}{xy-1}$ —①

Sol^n: Here,

diff. w.r.t. x of eq^n ① we get,

$$\frac{\partial(f)}{\partial x} = \frac{\partial(x+y)}{\partial(xy-1)} \Big/ \partial x = \frac{d(x+y).(xy-1) - (x+y)\,d(xy-1)}{(xy-1)^2}$$

$$\frac{\partial f}{\partial x} = \frac{(1+0)\,(xy-1) - (x+y).\,y}{(xy-1)^2}$$

$$\frac{xy-1-xy-y^2}{(xy-1)^2}$$

$$\frac{-1-y^2}{(xy-1)^2}$$

diff. eq^n ① w.r.to y we get

$$\frac{\partial f}{\partial y} = \partial\left(\frac{x+y}{xy-1}\right)$$

$$\frac{\partial f}{\partial x} = \frac{(xy-1)\frac{d(x+y)}{dx} - (x+y)\frac{d(xy-1)}{dx}}{(xy-1)^2}$$

$$\frac{\partial f}{\partial x} = \frac{(xy-1)(0+1) - (x+y)(x)}{(xy-1)^2}$$

$$\frac{\partial f}{\partial x} = \frac{xy-1-x^2-xy}{(xy-1)^2}$$

$$\frac{-1-x^2}{(xy-1)^2} \text{ Ans}$$

(vi) $f(x,y) = e^{xy} \ln y$ at $(2,1)$

Sol^n! Here

diff. w.r.to x we get,

$$\frac{\partial f}{\partial x} = \frac{\partial(e^{xy}.\ln y)}{\partial x} = \ln y \frac{d(e^{xy})}{d(xy)} \times \frac{d(xy)}{dx}$$

$$= \ln y\,(e^{xy}) \cdot y$$

$$= \ln 1\,(e^2) \cdot 1$$

$$= 0 \text{ Ans}$$

diff. w.r.to y we get,

(vi) ~~f(x,y)~~

$$\frac{\partial f}{\partial y} = \frac{\partial(e^{xy}.\ln y)}{\partial y}$$

$$\frac{\partial f}{\partial y} = e^{xy}.\frac{1}{y} + \ln y \frac{d(e^{xy})}{dy}$$

$$\frac{\partial f}{\partial y} = \frac{e^{xy}}{y} + \ln y \cdot e^{xy}.x$$

$$\frac{\partial f}{\partial y} = e^2 + 0$$

$$e^2 \text{ Ans}$$

2. calculate the partial derivatives f_x, f_y, f_z.

(i) $f(x,y,z) = 1+xy^2-2z^2$

Sol^n: Here,

(x) diff. w.r. to x.

$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(1+xy^2-2z^2)$

$\frac{\partial f}{\partial x} = 0+y^2-0 = y^2$

diff. w.r. to. y

$\frac{\partial f}{\partial y} = \frac{\partial(1+xy^2-2z^2)}{\partial y} = (0+2yx-0)$

$= 2xy.$

(iii) diff. w.r. to. z

$\frac{\partial f}{\partial z} = \frac{\partial(1+xy^2-2z^2)}{dz} = 0+0-4z$

$= -4z$ Ans

(ii) $f(x,y,z) = x-\sqrt{y^2+z^2}$

Sol^n: here,

$f(x,y,z) = x-\sqrt{y^2+z^2}$

diff. w.r. to x we get,

$\frac{\partial f}{\partial x} = 1-0 = 1$

diff w.r. to y. we get

$\frac{\partial f}{\partial y} = \frac{\partial(x-\sqrt{y^2+z^2})}{\partial y} = (0-\frac{1}{2}(y^2+z^2)^{-1/2}\cdot 2y.$

$= \frac{-y}{(y^2+z^2)^{1/2}}$

diff. w.r. to z we get,

$\frac{\partial f}{\partial z} = \frac{\partial(x-\sqrt{y^2+z^2})}{\partial z}$

$= 0-\frac{1}{2}(y^2+z^2)^{-1/2}\cdot(0+2z)$

$= \frac{-z}{(y^2+z^2)^{1/2}}$ Ans

(iii) $f(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$

Sol^n: here,

diff. w.r. to x we get,

$\frac{\partial f}{\partial x} = \frac{\partial(x^2+y^2+z^2)^{-1/2}}{dx} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}\cdot 2x$

$= \frac{-x}{(x^2+y^2+z^2)^{3/2}}$

Diff w.r. to y we get

$\frac{\partial f}{\partial y} = \frac{\partial(x^2+y^2+z^2)^{-1/2}}{\partial y} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}\cdot 2y$

$= \frac{-y}{(x^2+y^2+z^2)^{3/2}}$

diff. w.r. to z we get,

$\frac{\partial f}{\partial z} = \frac{-z}{(x^2+y^2+z^2)^{3/2}}$ Ans

(iv) $f(x,y,z) = yz\ln(xy)$ at $(3,1,-1)$

Sol^n: Here,

$f(x,y,z) = yz\ln(xy)$ at $(3,1,-1)$ —①

diff. eq^n ① w.r.to x we get

$$\frac{\partial f}{\partial x} = \frac{\partial(yz\ln(xy))}{\partial x} = yz\frac{\ln(xy)}{\partial(xy)} \times \frac{\partial(xy)}{dx}$$

$$yz.\frac{1}{xy}\cdot y$$

$$yz/x$$

$$\frac{1.-1}{3}$$

$$-1/3$$

diff. w.r.to y eq^n ① we get,

$$\frac{\partial f}{\partial y} = \frac{\partial(yz\ln(xy))}{\partial y} = \ln(xy)\frac{d(yz)}{dy} + yz\frac{d(\ln xy)}{dy}$$

$$= \ln xy\, z + \cancel{y}z\frac{1}{x\cancel{y}} \times \cancel{x}$$

$$= \ln xy.z + z$$

$$= -\ln 3 - 1 \text{ Ans}$$

diff. w.r. to.z we get,

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}(yz\ln(xy))$$

$$= \ln(xy)\,y.1$$

$$= y\ln(xy)$$

$$= 1\ln(3)$$

$$\ln 3 \text{ Ans}$$

(v) $f(x,y,z) = e^{-(x^2+y^2+z^2)}$ at $(2,4,5)$

Sol^n: Here,

$$f(x,y,z) = e^{-(x^2+y^2+z^2)}$$

Diff. w.r. to x we get

$$\frac{\partial f}{\partial x} = \frac{d(e^{-(x^2+y^2+z^2)})}{dx}$$

$$= \frac{d(e^{-(x^2+y^2+z^2)})}{d[-(x^2+y^2+z^2)]} \cdot \frac{d(-[x^2+y^2+z^2])}{dx}$$

$$= e^{-(x^2+y^2+z^2)}.-2x$$

$$= -2x\,e^{-(x^2+y^2+z^2)}$$

$$= -2\times 2\,e^{-(4+16+25)}$$

$$= -4e^{-45} \text{ Ans}$$

diff. w.r. to y we get,

$$\frac{\partial f}{\partial y} = \frac{\partial(e^{-(x^2+y^2+z^2)})}{dy} = e^{-(x^2+y^2+z^2)}.-2y.$$

$$= e^{-(4+16+25)} - 2\times 4$$

$$= -8e^{-45} \text{ Ans}$$