

Chapter - 5

RANDOM VARIABLE AND MATHEMATICAL EXPECTATION

Random Variable \rightarrow There are two types of random variable

a) Discrete Random variable \rightarrow A Random variable which takes the integer values for e.g. If we toss a coin then head or tail appear and X be the discrete random variable which represent the number of heads as shown below:

Outcomes - H T

$$X=x : 1 \quad 0$$

q) Cont Probability mass function \rightarrow Let "X" be a discrete random variable takes the values x_1, x_2, \dots, x_n with their probabilities p_1, p_2, \dots, p_n respectively. Then, ~~also~~ $p_i = P(X=x_i)$ is a probability mass function if it satisfies the following condition;

$$\text{q) } p_p = P(X=x_p) \geq 0.$$

(probability negative \Rightarrow not)

$$\text{p) } \sum_{i=1}^n p_i = 1 \quad \text{(i.e. total probability is 1)}$$

For e.g.

X	1	2	3	4	Sum of these gives total probability 1.
P_i	k	$2k$	$3k$	$4k$	

find the value of k ?

Here,

Since $\sum p_i = 1$.

$$\text{or, } k + 2k + 3k + 4k = 1$$

$$\text{or, } 10k = 1$$

$$\text{or, } k = 0.1$$

Expectation of Discrete random variables:

Expectation of random variable X is the sum of the product of values taken by random variables and their corresponding probabilities. Let random variable x takes the values x_1, x_2, \dots, x_n with corresponding probabilities p_1, p_2, \dots, p_n then,

$$E(x) = x_1 p_1 + x_2 p_2 + \dots + x_n p_n \\ = \sum_{i=1}^n x_i p_i$$

Similarly

$$E(x^2) = \sum_{i=1}^n x_i^2 p_i$$

Properties of Expectation

(1) $E(\text{constant}) = \text{constant}$.

for e.g. $E(4) = 4$

(2) $E(ax+b) = aE(x) + b$.

(3) $E(x+y) = E(x) + E(y)$.

(4) $E(xy) = E(x) \cdot E(y)$ (if x and y are independent random variables)

Variance of Random variable

Here, Variance of $X = \sum (x_i^2) - [E(x)]^2$

$$= \sum_{i=1}^n x_i^2 p_i - \left[\sum_{i=1}^n x_i p_i \right]^2$$

Properties:

i) Variance (constant) = 0

ii) Variance ($c \cdot x$) = $c^2 \cdot V(x)$ variance

iii) $V(ax+b) = a^2 V(x)$

Numerical Questions:

Q1 A random variable x has the following probability distribution.

x	0	1	2	3	4	5	6	7	8
$P(x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

→ Determine value of a

or Find $P(x < 3)$, $P(x \geq 3)$, $P(0 \leq x \leq 5)$.

Soln:

x	$P(x)$
0	$a = \frac{1}{81}$
1	$3a = \frac{3}{81}$
2	$5a = \frac{5}{81}$
3	$7a = \frac{7}{81}$
4	$9a = \frac{9}{81}$
5	$11a = \frac{11}{81}$
6	$13a = \frac{13}{81}$
7	$15a = \frac{15}{81}$
8	$17a = \frac{17}{81}$
	$\sum P(x) = 81a$

Since,

$$\textcircled{1} \quad \sum P(x) = 1$$

$$\text{or, } 81a = 1$$

$$\text{or, } a = \frac{1}{81}$$

\textcircled{2} Now, $P(x < 3)$

$$= P(x=0) + P(x=1) + P(x=2)$$

$$= \frac{1}{81} + \frac{3}{81} + \frac{5}{81}$$

$$= \frac{9}{81}$$

$$= \frac{1}{9}$$

(Q1P) $P(X \geq 3)$
 $= 1 - P(X < 3)$
 $= 1 - \frac{1}{9}$
 $= \frac{8}{9}$

(Q2) $P(2 \leq X \leq 5)$
 $= P(X=1) + P(X=2) + P(X=3) + P(X=4)$
 $= \frac{3}{81} + \frac{5}{81} + \frac{7}{81} + \frac{9}{81}$
 $= \frac{24}{81}$
 $= \frac{8}{27}$

Q3. Two unbiased dice are thrown. Find the expected values of the sum of numbers of points on them.
Soln

When we throw two dice, then, the possible outcomes are as follows:

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

For the calculation of expected and variance of the sum of the number of points on the dice.

~~Sum of all numbers = 216
Sum of squares = 324~~

X	X^2	$P(X=x)$ or $P(x)$	$X \cdot P(x)$
2	4	$\frac{2}{36}$	$\frac{2 \times 2}{36} = \frac{4}{36}$
3	9	$\frac{3}{36}$	$\frac{9}{36}$
4	16	$\frac{4}{36}$	$\frac{32}{36}$
5	25	$\frac{5}{36}$	$\frac{25}{36}$
6	36	$\frac{6}{36}$	$\frac{36}{36}$
7	49	$\frac{7}{36}$	$\frac{42}{36}$
8	64	$\frac{8}{36}$	$\frac{40}{36}$
9	81	$\frac{9}{36}$	$\frac{36}{36}$
10	100	$\frac{10}{36}$	$\frac{30}{36}$
11	121	$\frac{11}{36}$	$\frac{22}{36}$
12	144	$\frac{12}{36}$	$\frac{32}{36}$
$\sum P(x=x) = 1$		$\sum x \cdot P(x) = \frac{252}{36} = 7$	

Now,

$$E(X) = \sum x \cdot P(x)$$

$$= 7$$

and $E(X^2) \geq \sum x^2 \cdot P(x)$

$$= 4 \times \frac{4}{36} + 9 \times \frac{6}{36} + 16 \times \frac{10}{36} + 25 \times \frac{20}{36}$$

~~$$+ 36 \times \frac{30}{36} + 49 \times \frac{42}{36} + 64 \times \frac{40}{36} + 81 \times \frac{36}{36}$$~~

~~$$+ 100 \times \frac{30}{36} + 121 \times \frac{22}{36} + 144 \times \frac{12}{36}$$~~

and $E(X^2) = \sum x^2 \cdot P(x)$.

$$= 4 \times \frac{4}{36} + 9 \times \frac{6}{36} + 16 \times \frac{10}{36} + 25 \times \frac{20}{36} + 36 \times \frac{30}{36} + 49 \times \frac{42}{36}$$

$$+ 64 \times \frac{40}{36} + 81 \times \frac{36}{36} + 100 \times \frac{30}{36} + 121 \times \frac{22}{36} + 144 \times \frac{12}{36}$$

$$= 54.83$$

$$\begin{aligned}
 \text{Now, } V(x) &= E(x^2) - [E(x)]^2 \\
 &= 54.83 - (7)^2 \\
 &= 54.83 - 49 \\
 &= 5.83 //
 \end{aligned}$$

Ex. 2

A coin is tossed until a tail appears. What is the expected number of toss required?

Soln.

Outcomes	No. of toss (x)	$P(x=x)$
T	1	$\frac{1}{2}$
HT	2	$\frac{1}{4}$
HHT	3	$\frac{1}{8}$
HHHT	4	$\frac{1}{16}$
HHHHT	5	$\frac{1}{32}$
:	:	:

Now,

Expectation of no. of tosses i.e. $E(x) = 1 \times \frac{1}{2} + 2 \times \frac{1}{4} + 3 \times \frac{1}{8} + 4 \times \frac{1}{16} + 5 \times \frac{1}{32} + \dots$

$$S = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \dots$$

$$\text{and } \frac{S}{2} = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \dots$$

$$\text{then, } S - \frac{S}{2} = \frac{1}{2} + \left(\frac{2}{4} - \frac{1}{4}\right) + \left(\frac{3}{8} - \frac{2}{8}\right) + \left(\frac{4}{16} - \frac{3}{16}\right) + \dots$$

$$\frac{S}{2} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

$$\frac{S}{2} = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = \frac{1}{2} \times \frac{2}{1} = 1$$

$$\therefore S = \frac{a}{1-a}$$

in G.S

$$\therefore S = 2.$$

So, Expected No. of tosses is 2.

Q.No.7 An importer is offered a shipment of machine tools for Rs 700, 900, 1100, 1300, 1500 and 1700 with probabilities $\frac{1}{12}, \frac{1}{12}, \frac{1}{12}$, Rs 140,000 and the probability that he will be able to sell them for Rs. 180,000, Rs 170,000 or Rs 150,000 are 0.32, 0.55 and 0.13 respectively. What is the importers expected gross profit?

Soln

Sell ($x=x$)	Probability (P_x)
180,000	0.32
170,000	0.55
150,000	0.13

Now,

$$E(x) = \sum x_p P_x$$

$$= 280000 \times 0.32 + 170000 \times 0.55 + 150,000 \times 0.13$$

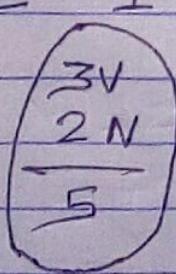
$$= 170600$$

$$\therefore \text{Gross profit} = 170,000 - 140,000$$

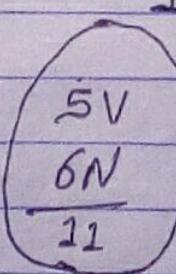
$$= 30600$$

Q9 Three bags contain 3 video cards and 2 network cards, 5 video cards and 6 network cards, 2 video cards and 4 network cards respectively. One card is drawn from each urn. Find the expected number of video cards.

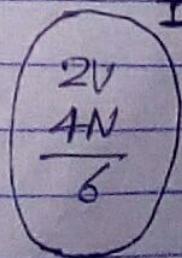
Soln



II



III



For selecting one card from each urn the network card and for the calculation of expected number

of network card we have the following possible outcomes.

Now, for Expected no. of network card

$X=x$	$P(X=x)$	$X \cdot P(x)$	$P(V_1 V_2 V_3)$
0	$\frac{30}{330}$	0	
1	$\frac{116}{330}$	$\frac{116}{330}$	$= \frac{3}{5} \times \frac{5}{11} \times \frac{2}{6}$
2	$\frac{136}{330}$	$\frac{272}{330}$	
3	$\frac{48}{330}$	$\frac{144}{330}$	$= 330$
$\sum X \cdot P(x) = \frac{532}{330}$			$P(N_1 N_2 V_3 \text{ or } N_1 V_2 N_3 \text{ or } V_1 N_2 N_3)$

$$P(N_1 \cdot N_2 V_3 \text{ or } N_1 V_2 N_3 \text{ or } V_1 N_2 N_3)$$

$$= P(N_1 N_2 V_3) + P(N_1 V_2 N_3) + P(V_1 N_2 N_3)$$

$$= \frac{2}{5} \times \frac{6}{11} \times \frac{2}{6} + \frac{2}{5} \times \frac{5}{11} \times \frac{4}{6} + \frac{3}{5} \times \frac{6}{11} \times \frac{4}{6}$$

$$= \frac{136}{330}$$

$$P(N_1 N_2 N_3)$$

$$= P(N_1) \cdot P(N_2) \cdot P(N_3)$$

$$= \frac{2}{5} \times \frac{6}{11} \times \frac{4}{6}$$

$$= \frac{48}{330}$$

~~Calculation~~

$$P(V_1 V_2 V_3)$$

$$= \frac{3}{5} \times \frac{5}{11} \times \frac{2}{6}$$

$$= 330$$

$$P(N_1 N_2 V_3 \text{ or } V_1 N_2 N_3 \text{ or } V_1 V_2 N_3)$$

$$= \frac{2}{5} \times \frac{5}{11} \times \frac{2}{6}$$

$$= \frac{3}{5} \times \frac{6}{11} \times \frac{3}{6}$$

$$+ \frac{3}{5} \times \frac{5}{11} \times \frac{4}{6}$$

$$= \frac{116}{330}$$

$$\int x \cdot dx = \frac{x^2}{2}$$

$$\int x^n \cdot dx = \frac{x^{n+1}}{n+1}$$

$$\int_0^\infty x^n e^{-x} \cdot dx = n! = (n-1)!$$

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Continuous Random Variable:

Let x be a continuous random variable and $f(x)$ is the continuous probability function of x and it is also known as probability density function.

The probability of x lies in the interval $\cancel{[a, b]}$ is given by $P(a \leq x \leq b) = \int_a^b f(x) \cdot dx$

The function $f(x)$ is called probability density function if it satisfies the following conditions

1) $f(x) \geq 0$ (probability cannot be -ve)

2) $\int_{-\infty}^{\infty} f(x) \cdot dx = 1$ (total probability is 1)

Q11 Suppose a continuous random variable x has the density function $f(x) = \begin{cases} k(1-x)^2 & \text{for } 0 < x < 1 \\ 0; & \text{elsewhere.} \end{cases}$

Sol'n Find the value of ① k ② $P(0 < x < 0.5)$ and ③ $E(x)$.

Given, $f(x) = \begin{cases} k(1-x)^2 & \text{for } 0 < x < 1 \\ 0; & \text{elsewhere.} \end{cases}$

④ Since, $\int_{-\infty}^{\infty} f(x) \cdot dx = 1$

So, $\int_0^1 f(x) \cdot dx = 1$

or, $\int_0^1 k(1-x)^2 dx$

$$\text{or, } k \left[\frac{(1-x)^3}{3 \times (-2)} \right]_0^1 = 1.$$

$$\text{or, } k \left[0 + \frac{1}{3} \right] = 1$$

$$\text{or, } \frac{k}{3} = 1$$

$$\text{or, } k = 3$$

Now,

$$\textcircled{P} \quad f(x) = 3(1-x)^2$$

$$\text{So, } P(0 < x < 0.5)$$

$$= \int_0^{0.5} f(x) dx$$

$$= \int_0^{0.5} 3(1-x)^2 dx$$

$$= 3 \left[\frac{(1-x)^3}{3(-2)} \right]_0^{0.5}$$

$$= 3 \left[\frac{-0.125}{3} + \frac{1}{3} \right]$$

$$= 0.875.$$

$$\textcircled{Q} \quad E(x)$$

$$= \int_0^1 x \cdot f(x) dx$$

$$= \int_0^1 3x(1-x)^2 dx.$$

~~Ans~~

$$= \int_0^1 3x(1-2x+x^2) dx$$

$$= \int_0^1 3x - 6x^2 + 3x^3 dx$$

$$= \left[\frac{3x^2}{2} - \frac{6x^3}{3} + \frac{3x^4}{4} \right]_0^1$$

$$= \frac{3}{2} \times 1 - 6 \cdot \frac{1}{3} + \frac{3}{4}$$

$$= \frac{3}{2} - 6 \cdot \frac{1}{3} + \frac{3}{4}$$

$$= \frac{18 - 24 + 9}{12}$$

$$= \frac{27 - 24}{12}$$

$$= \frac{3}{12}$$

$$= \frac{1}{4}$$

Q.14 Find the mean of the random variable having probability density function $f(x) = xe^{-x}$, $x \geq 0$.

Soln

Given, $f(x) = xe^{-x}$, $x \geq 0$

Here,

$$\text{Mean } E(x) = \int_0^\infty xc \cdot f(x) dx$$

$$= \int_0^\infty x \cdot xe^{-x} dx$$

$$\begin{aligned}
 &= \int_0^\infty x^2 \cdot e^{-x} dx \\
 &= \int_0^\infty e^{-x} \cdot x^{3-1} dx \\
 &= (3-1)! \\
 &= 2! \\
 &= 2 \quad \underline{\text{Ans}}
 \end{aligned}$$

By gamma distribution

$$\begin{aligned}
 &\int_0^\infty e^{-x} \cdot x^{n-1} dx \\
 &= \sqrt{n} \\
 &= (n-1)!
 \end{aligned}$$

Q.15. The pdf of a continuous random variable is given by

$$f(x) = \begin{cases} 12x(1-x)^2, & 0 \leq x \leq 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find $E(x)$, $E(x^2)$.

Solⁿ

Given, $f(x) = \begin{cases} 12x(1-x)^2, & 0 \leq x \leq 1 \\ 0, & \text{elsewhere.} \end{cases}$

Now,

$$E(x) = \int_0^1 x \cdot f(x) \cdot dx.$$

$$= \int_0^1 x \cdot 12x(1-x)^2 dx$$

$$= \int_0^1 12x^2(1+x^2-2x) dx.$$

$$= 12 \left[\int_0^1 x^2 dx + \int_0^1 x^4 dx - \int_0^1 2x^3 dx \right]$$

$$= 12 \left\{ \left[\frac{x^3}{3} \right]_0^1 + \left[\frac{x^5}{5} \right]_0^1 - 2 \left[\frac{x^4}{4} \right]_0^1 \right\}$$

$$= 12 \left[\frac{1}{3} + \frac{1}{5} - \frac{2}{5} \right]$$

$$= 12 \left[\frac{20+12-30}{6} \right]$$

$$= \frac{12 \times 2}{60}$$

$$= \frac{2}{5} \quad \underline{\text{Ans.}}$$

$$\& E(x^2) = \int_0^1 x^2 \cdot f(x) \cdot dx$$

$$= \int_0^1 x^2 \cdot 12x(1-x)^2 dx$$

$$= \int_0^1 12x^3(1+x^2-2x) dx$$

$$= 12 \left\{ \int_0^1 x^3 dx + \int_0^1 x^5 dx - 2 \int_0^1 x^4 dx \right\}$$

$$= 12 \left\{ \left[\frac{x^4}{4} \right]_0^1 + \left[\frac{x^6}{6} \right]_0^1 - 2 \left[\frac{x^5}{5} \right]_0^1 \right\}$$

$$= 12 \left\{ \frac{1}{4} + \frac{1}{6} - \frac{2}{5} \right\}$$

$$= 12 \left(\frac{15+10-24}{60} \right)$$

$$= \frac{12 \times 1}{60}$$

$$= \frac{1}{5} \quad \underline{\text{Ans.}}$$

- Q.17 A continuous random variable X has a pdf $f(x) = 3x^2$, $0 \leq x \leq 1$. Find a and b such that
 Q. $P(X \leq a) = P(X > a)$ Q. $P(X > b) = 0.05$.

Soln

Given, Pdf $f(x) = 3x^2$, $0 \leq x \leq 1$ according to question

Q. $P(X \leq a) = P(X > a)$

or, $\int_0^a f(x) dx = \int_a^1 f(x) dx$.

or, $\int_0^a 3x^2 dx = \int_a^1 3x^2 dx$.

or, $3 \left[\frac{x^3}{3} \right]_0^a = 3 \left[\frac{x^3}{3} \right]_a^1$

or, $\frac{a^3 - 0}{3} = \frac{1 - a^3}{3}$

or, $a^3 = 1 - a^3$

or, $2a^3 = 1$

or, $a^3 = \frac{1}{2}$

or, $a = \sqrt[3]{\frac{1}{2}}$

or, $a = 0.79$ Ans

Q. $P(X > b) = 0.05$

Soln

$\int_b^1 f(x) dx = 0.05$

or, $\int_b^1 3x^2 dx = 0.05$

$$\text{or, } 3 \left[\frac{x^3}{3} \right]_0^b = 0.05$$

$$\text{or, } 1 - b^3 = 0.05$$

$$\text{or, } b^3 = 1 - 0.05$$

$$\text{or, } b = \sqrt[3]{0.95}$$

$$\text{or, } b = 0.98 \text{ Ans}$$

Q.18 Find mean and S.D. of a distribution having pdf $f(x) = ke^{-x/\sigma}$, $0 < x < \infty$ and $\sigma > 0$.

Soln

Given, pdf is $f(x) = k \cdot e^{-x/\sigma}$, $0 < x < \infty$, $\sigma > 0$
we have,

$$\int_{-\infty}^{\infty} f(x) \cdot dx = 1.$$

$$\text{or, } \int_{-\infty}^{\infty} k \cdot e^{-x/\sigma} \cdot dx = 1.$$

$$\text{Let, } \frac{x}{\sigma} = y.$$

$$\text{or, } \frac{1}{\sigma} = \frac{dy}{dx}$$

$$\text{or, } dx = \sigma \cdot dy.$$

$$\text{when, } x=0, y=0$$

$$\text{at } x=\infty, y=0$$

$$\therefore \int_0^{\infty} k e^{-x/\sigma} \cdot dx = \int_0^{\infty} k \cdot e^{-y/\sigma} \cdot \sigma \cdot dy = 1$$

$$\Rightarrow k \cdot \sigma \int_0^{\infty} e^{-y} \cdot dy = 1.$$

$$\Rightarrow k \cdot \sigma \left[\frac{e^{-y}}{-1} \right]_0^{\infty} = 1$$

$$\Rightarrow k \cdot \sigma = 1$$

$$\Rightarrow k = \frac{1}{\sigma}$$

Now,

$$\text{Mean } E(x) = \int_0^\infty x \cdot f(x) \cdot dx.$$

$$= \int_0^\infty x \cdot \frac{1}{\sigma} \cdot e^{-x/\sigma} \cdot dx$$

$$= \frac{1}{\sigma} \int_0^\infty x \cdot e^{-x/\sigma} \cdot dx.$$

$$\text{Let, } \frac{x}{\sigma} = y \Rightarrow x = \sigma y.$$

$$\frac{1}{\sigma} = \frac{dy}{dx}$$

$$\text{or, } dx = \sigma \cdot dy.$$

$$\text{Mean or } E(x) = \frac{1}{\sigma} \int_0^\infty \sigma \cdot y \cdot e^{-y} \cdot \sigma \cdot dy$$

$$= \sigma \int_0^\infty y^{2-1} \cdot e^{-y} \cdot dy.$$

$$= \sigma \cdot (2-1)!$$

$$= \sigma \cdot 1!$$

$$= \sigma \quad \underline{\text{Ans.}}$$

~~$$E(x^2) = \int_0^\infty x^2 f(x) \cdot dx.$$~~

$$= \int_0^\infty x^2 \cdot \frac{1}{\sigma} \cdot e^{-x/\sigma} \cdot dx$$

$$= \int_0^\infty \sigma^2 y^2 \cdot \frac{1}{\sigma} \cdot e^{-y} \cdot \sigma \cdot dy$$

$$= \sigma^{-2} \int_0^\infty y^2 \cdot e^{-y} \cdot dy.$$

$$= \sigma^2 \int_0^\infty y^{3-1} \cdot e^{-y} \cdot dy = \sigma^2 (3-1)! = \sigma^2 (2)! \\ = \sigma^2 \cdot 2$$

$$\text{Variance} = E(x^2) - \{E(x)\}^2$$

$$= 2\sigma^2 - \sigma$$

$$= \sigma^2$$

$$\therefore S.D = \sqrt{\text{Variance}}$$

$$= \sqrt{\sigma^2}$$

$$= \sigma \quad \underline{\text{Ans.}}$$

Q.19. X is a random variable taking values $-3, -2, -1, 0, 1, 2, 3$ with following probability:

x	-3	-2	-1	0	1	2	3
$P(x)$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

Find (i) First four moments about origin (ii) first four moments about mean. (iii) Measure of central tendency, dispersion, skewness and kurtosis.

Solⁿ

For the calculation of first four moments about origin.

$$\mu'_1 = E(x) = \sum x_i P(x=x_i)$$

$$\mu'_2 = E(x^2) = \sum x_i^2 P(x=x_i)$$

$$\mu'_3 = E(x^3) = \sum x_i^3 P(x=x_i)$$

$$\mu'_4 = E(x^4) = \sum x_i^4 P(x=x_i).$$

$x = x_i$	$P(x=x_i) = P_i$	x^2	x^3	x^4	$x_i P(x=x_i)$	$x_i^2 P(x=x_i)$	$x_i^3 P(x=x_i)$	$x_i^4 P(x=x_i)$
-3	$\frac{1}{9}$	9	27	81	$-\frac{3}{9}$	$\frac{9}{9}$	$\frac{27}{9}$	$\frac{81}{9}$
-2	$\frac{1}{9}$	4	8	16	$-\frac{2}{9}$	$\frac{4}{9}$	$\frac{8}{9}$	$\frac{16}{9}$
-1	$\frac{1}{9}$	1	1	1	$-\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
0	$\frac{1}{3}$	0	0	0	0	0	0	0
1	$\frac{1}{9}$	1	1	1	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
2	$\frac{1}{9}$	4	8	16	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{8}{9}$	$\frac{16}{9}$
3	$\frac{1}{9}$	9	27	81	$\frac{3}{9}$	$\frac{9}{9}$	$\frac{27}{9}$	$\frac{81}{9}$
$\sum P_i = 1$					$\sum x_i P(x=x_i) = 0$	$\sum x_i^2 P(x=x_i) = 3.11$	$\sum x_i^3 P(x=x_i) = 0$	$\sum x_i^4 P(x=x_i) = 21.77$

$$\therefore \mu'_1 = E(x) = \sum x_i P(X=x_i) = 0.$$

$$\mu'_2 = E(x^2) = \sum x_i^2 \cdot P(X=x_i) = 3.11$$

$$\mu'_3 = E(x^3) = \sum x_i^3 \cdot P(X=x_i) = 0.$$

$$\mu'_4 = E(x^4) = \sum x_i^4 \cdot P(X=x_i) = 21.77$$

Measure of Central tendency ~~$\mu'_1 = 0$~~

$$\text{dispersion} = \mu'_2 = \text{variance} \\ = \sqrt{3.11} \\ = 1.76$$

$$\text{skewness} \del{\mu'_3} \cdot (\mu'_3) = 0.$$

~~Kurtosis:~~

For Kurtosis

$$\beta_1 = \frac{\mu'_3^2}{\mu'_2^3}$$

$$\beta_2 = \frac{\mu'_4}{\mu'_2^2}$$

Q2 Let $P(x,y) = \frac{2x+y}{28}$, $x=1,3$ and $y=2,4$; $P(y) = \frac{y+4}{14}$,

find $E(X/Y=2)$ and $V(X/Y=2)$.

Soln

Given, $P(x,y) = \frac{2x+y}{28}$, and $x=1,3$

and $P(y) = \frac{y+4}{14}$, $y=2,4$.

Now,

$$\begin{aligned}
 E(X/Y=2) &= \sum_{x=1,3} x \cdot P(X/Y=2) \\
 &= \sum_{x=1,3} x \cdot \frac{P(X,Y=2)}{P(Y=2)} \\
 &= \sum_{x=1,3} x \cdot \frac{\frac{2x+2}{28}}{\frac{6}{14}} \\
 &= \sum_{x=1,3} x \cdot \frac{(2x+2)}{28} \times \frac{14}{6} \\
 &= \sum_{x=1,3} x \cdot \frac{(x+1)}{6} \\
 &= \frac{2}{6} + \frac{12}{6} \\
 &= \frac{2+12}{6} \\
 &= \frac{14}{6} \\
 &= \frac{7}{3}
 \end{aligned}$$

$$E(X^2/Y=2) = \sum_{x=1,3} x^2 \cdot P(X/Y=2)$$

$$= \sum_{x=1,3} x^2 \cdot \frac{P(X,Y=2)}{P(Y=2)}$$

$$= \sum_{x=1,3} x^2 \cdot \frac{(x+1)}{6} = \frac{2^2+6^2}{6} = \frac{2+6}{6} = \frac{38}{6} = 6.33$$

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Now,

$$\begin{aligned} V(x/y=2) &= E(x^2/y=2) \{E(x/y=2)\}^2 \\ &= 6.33 - (7/3)^2 \\ &= 8/9. \end{aligned}$$