Chapter → 6
Probability distribution

❀ Binomial distribution:

Probability mass function of random variable X is $P(X=x)$
$$= nC_x p^x q^{n-x} \text{ OR } C(n,x)p^x q^{n-x}$$
where, $x = 0,1,2,\ldots,n$.

↳ This can be written as $X \sim B(n,p)$ and read as X follows the binomial distribution with parameters n and p.

Properties:

i) Binomial distribution is a discrete probability distribution because random variable X takes values $0,1,2,\ldots n$.

ii) Binomial distribution is also known as biparametric distribution because it has two parameters n and p.

iii) The mean of binomial distribution is np and variance is npq.

iv) Mean > Variance OR $np > npq$.

v) Additive property → Let X and Y be two independent variables such that $X \sim B(n,p)$, $Y \sim B(n,p)$ then $(X+Y) \sim B(n_1+n_2, p)$.

❀ Poisson distribution:

It is used if:-

i) Probability of success is very small i.e, $p \to 0$

ii) If the no. of trial is very large i.e, $n \to \infty$

iii) If $\lambda = np$
$$\Rightarrow \boxed{p = \frac{\lambda}{n}}$$

Probability mass function of random variable X with parameter λ is $P(X=x) = f(x)$
$$= \frac{e^{-\lambda} \cdot \lambda^x}{x!} \quad \text{where, } x = 0,1,2,3,\ldots,\infty.$$

This can be written as $X \sim P(\lambda)$ and read as: X follows poisson distribution with parameter λ.

Properties

i) Poisson distribution is a discrete probability distribution because the random variable X takes the values $0, 1, 2, \ldots \infty$.

ii) Poisson distribution is also known as uniparametric distribution because it has only one parameter λ. If we know the value of λ then we can easily determine the probability.

iii) The mean and variance of poisson distribution is λ.

iv) Let X and Y be two independent random variables such that $X \sim P(\lambda_1)$ and $Y \sim P(\lambda_2)$ then $(X+Y) \sim P(\lambda_1 + \lambda_2)$.

Numericals related to Binomial Distribution (From kec book exercise)

Q20. In a binomial distribution with 6 independent trials the probability of 3 and 4 sucess is found to be 0.2457 and 0.08189 respectively. Find the parameters, mean, variance.

Soln Given,

no. of trial $(n) = 6$.

$P(3 \text{ sucess}) = 0.2457 = P(X=3)$

$P(4 \text{ success}) = 0.08189 = P(X=4)$

Now,

$$P(X=3) = 6C_3 p^3 (1-p)^3 = 0.2457 \quad \text{--- (i)}$$

$$P(X=4) = 6C_4 p^4 (1-p)^2 = 0.08189 \quad \text{--- (ii)}$$

$\left(\because P(X=x) = nC_x p^x q^{n-x}\right)$

dividing (i) by (ii).

$$\frac{6C_3 p^3 (1-p)^3}{6C_4 p^4 (1-p)^2} = \frac{0.2457}{0.08189} = \frac{20(1-p)}{15p}$$

$$\text{or, } 3 = \frac{20(1-p)}{15p}$$

$$\Rightarrow 20 - 20p = 45p$$

$$\Rightarrow 65p = 20$$

$$\Rightarrow p = 20/65$$

$$\Rightarrow p = 4/13$$

Now, mean $= np$

$= 6 \times 4/13$

$= 24/13$

& Variance $= npq$

$= 6 \times 4/13 \times 9/13$

$= 1.27$

$\left(\because q = 1-p = 1 - 4/13 = 9/13\right)$

Important Note:

① Question मा at most भए $\leq$ use गर्ने र at least भए $\geq$.

② 1−(One minus) गर्ने बेला sign यसरि change हुन्छ $\geq \leftrightarrow <$
$\leq \leftrightarrow >$

For e.g. $P(X \geq 1) \geq 0.99$

or, $1 - P(X < 1) \geq 0.99$

③ Question Binomial distribution को कि Poisson को कसरि छुट्याउने,

i) Parameter question मा average(λ) दिएको छ भने Poisson हुन्छ $\lambda = np$ र $P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$ formula use गर्ने

होइन parameter n and p दिएको भए Binomial distribution हुन्छ $P(X=x) = {}^nC_x p^x q^{n-x}$ OR $C(n,x)p^x q^{n-x}$ formula use गर्ने

ii) Number(N) को value Question मा धेरै बढि र probability of occurance धेरै कम दिएको छ भने Poisson हुन्छ नत्र Binomial.

For e.g. $p = 0.0005$, $N = 1000$

iii) धेरै number बाट single को probability निकाल्नु परे Poisson नत्र Binomial.

Q.No.29. A programmer succeeds twice as often as it fails while developing a specific program. Find the chance that the next six attempt, there will be at least four successes.

Soln

According to the question, the ratio of success and faliure is 2:1.

$$\text{So, } P(\text{success}) = \frac{2}{2+1} = \frac{2}{3}$$

$$\Rightarrow p = 2/3$$

$$P(\text{faliure}) = q = 1 - 2/3 = 1/3$$

$$\text{Now, } P(\text{at least 4 success}) = P(X \geq 4)$$

$$= P(X=4) + P(X=5) + P(X=6)$$

$$= 6C_4 \left(\frac{2}{3}\right)^4 \cdot \left(\frac{1}{3}\right)^2 + 6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^1 + 6C_6 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^0$$

$$= 15 \times 0.197 \times 0.111 + 6 \times 0.132 \times 0.333 + 1 \times 0.087 \times 1$$

$$= 0.328 + 0.263 + 0.087$$

$$= 0.678$$

Q.No.32. A discrete random variable X has mean equal to 6 and variance equal to 2. If it is assumed that the underlying distribution of X is binomial, what is the probability that $5 < X < 7$?

Soln

Given, Mean $(np) = 6$

Variance $(npq) = 2$

$$\Rightarrow 6q = 2$$

$$\text{or, } q = 2/6 = 1/3$$

$$\Rightarrow p = 1 - 1/3 = 2/3 \quad (\text{We know that } p = 1 - q)$$

Now, from $np = 6$

$$\Rightarrow n \times 2/3 = 6$$

$$\Rightarrow n = 18/2 = 9$$

Then, $P(5 < X < 7)$

$$= P(X = 6)$$

$$= 9C_6 (2/3)^6 \cdot (1/3)^3$$

$$= 84 \times 0.087 \times 0.037$$

$$= 0.27$$

Q. Example 22. It is believed that 80% of Neplease do not have health insurance. Suppose this is true and let x be the number of health insurance in a random sample of 12 neplease.

i) What is the probability of no health insurance?

ii) Find the mean and variance of x.

iii) Find $P(X>2)$.

Solution.-

Since x = number of health insurance.

i) Probability of no. health insurance $(p) = 80\%$
$= 0.8$

$q = 1-p = 1-0.8 = 0.2$

No. of Neplease people $(n) = 12$

ii) Mean $E(X) = np$
$= 12 \times 0.8$
$= 9.6$

& Variance $V(x) = npq$
$= 12 \times 0.8 \times 0.2$
$= 1.92$

iii) $P(X>2) = 1-P(X\leq 2)$

$= 1-\{P(X=0)+P(X=1)+P(X=2)\}$

$= 1-\{c(12,0)\cdot(0.8)^0\cdot(0.2)^{12}+c(12,2)(0.8)^2(0.2)^{10}\}$

$= 1-0$

$= 1$.

Hence the propability of no health insurance is 0.8 Mean is 9.6, variance is 1.92 and $P(X>2)$ is 1.

Numericals related to Poisson Distribution:

Q.No.35 (Model Question). A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%

ⓐ. The inspector randomly pics 20 items from a shipment. What is the probability that there will be at least one defective item ~~alor~~ among these 20?

ⓑ Suppose that the retailer receives 10 shipments in a month and the inspector randomly tests 20 devices per shipment. What is the probability that there will be exactly 3 shipments each containing at least one defective device among the 20 that are selected and tested from the shipment?

Imp Q.No.45 A manufacturer of pen drives knows that 3% of his product is defective. If he sells in boxes of 200 and guarantees that not more than 2 pen drives will be defective, what is the probability that a box will fail to meet the guaranteed quality?

Soln

P (defective) = 3% = 0.03

no. of box (n) = 200

then $\lambda = np$

$= 200 \times 0.03$

$= 6.$

question मा average भए त्यो भनेको λ हुन्छ।

Now,

P(Box will fail to meet the guaranteed quality)

$= P(X > 2)$

$= 1 - P(X \leq 2)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \left[\dfrac{e^{-6} \cdot 6^0}{0!} + \dfrac{e^{-6} \cdot 6^1}{1!} + \dfrac{e^{-6} \cdot 6^2}{2!}\right]$

$= 1 - e^{-6}(1 + 8 + 18)$

$= 1 - e^{-6} \times 25$

$= 0.93$

Again,

P(Box will meet guarenteed quality)

$= 1 -$ P (Box will not meet)

$= 1 - P(X > 2)$

$= 1 - 0.93$

$= 0.07$

Q.No.43. If a random variable X follows poisson distribution such that $P(X=1) = P(X=2)$. Find mean & variance.

Soln Given, $P(X=1) = P(X=2)$

or, $\dfrac{e^{-\lambda} \lambda^1}{1!} = \dfrac{e^{-\lambda} \lambda^2}{2!}$

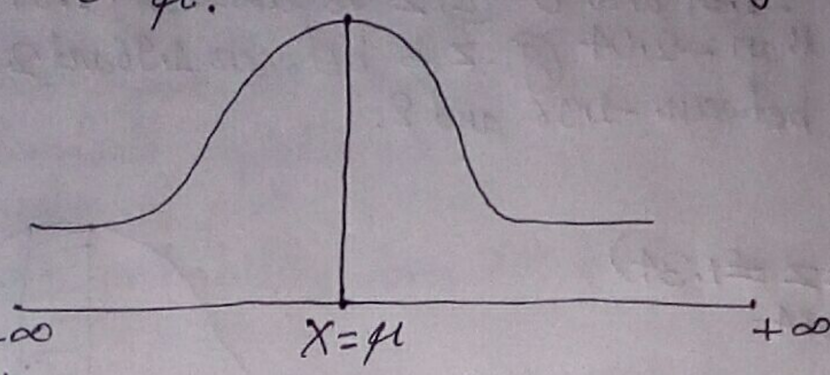
$\Rightarrow \lambda = 2.$

So, mean = variance = $\lambda = 2$.

Properties of normal distribution:

i> Normal distribution is a continious probability distribution having parameters μ and σ^2.

ii> Normal curve is bell shaped curve and symmetrical about mean μ.



iii> Since the curve is symmetrical, so, mean = median = mode.

iv> The distribution of the curve is maximum when $X=\mu$.

$$\text{i.e., } f(x)=\frac{1}{\sigma\sqrt{2\pi}}$$

v> The coefficient of skewness for normal curve is zero.

$$\text{i.e., } \beta_1=0$$
$$\Rightarrow \gamma_1=0.$$

vi> The coefficient of kurtosis $\beta_2=3$ or $\gamma_2=0$.

vii> The area property of Normal Curve:

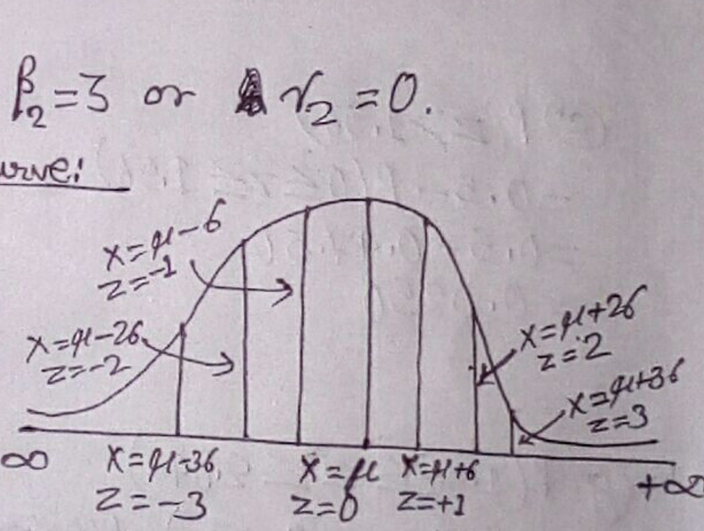
Let $Z=\frac{X-\mu}{\sigma}$

when $X=\mu+6$

Then $Z=\frac{\mu+6-\mu}{\sigma}=1$

when $X=\mu-6$

Then $Z=\frac{\mu-\sigma-\mu}{\sigma}=-1$



Now,

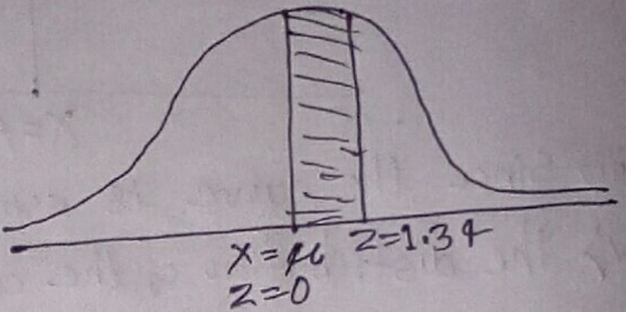
$$P(\mu-6\le X\le\mu+\sigma)$$
$$=P(-1\le Z\le+1)$$
$$=P(-1\le Z\le 0)+P(0\le Z\le 1)$$
$$=P(0\le Z\le 1)+P(0\le Z\le 1) \quad (\because \text{by Symmetry})$$
$$=2P(0\le Z\le 1)$$
$$=2\times 0.3413$$
$$=0.6826$$
$$=68.26\%$$

Numericals:

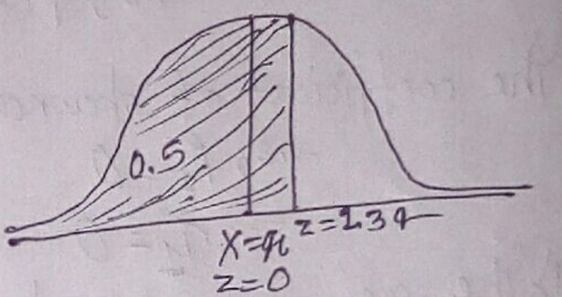
Q.No.53 If Z is the standard normal variable then calculate the following probabilities: (a) Z is between 0 and 1.34 (b) Z is less than 1.34 (c) Z is more than 1.96 (d) Z is between −1.34 and 0 (e) Z is less than −1.34 (f) Z is greater than −2.04 (g). Z is between 1.96 and 2.84 (h). Z is between −1.96 and 2.

Soln

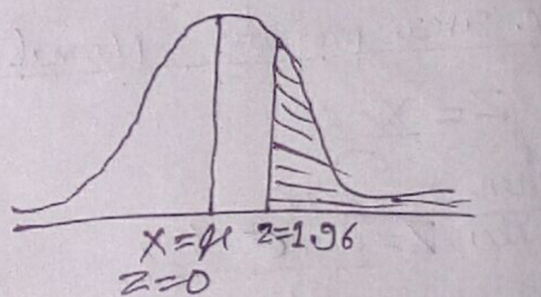
(a) $P(0 \le z \le 1.34)$
$= 0.4034$
≈ 0.4



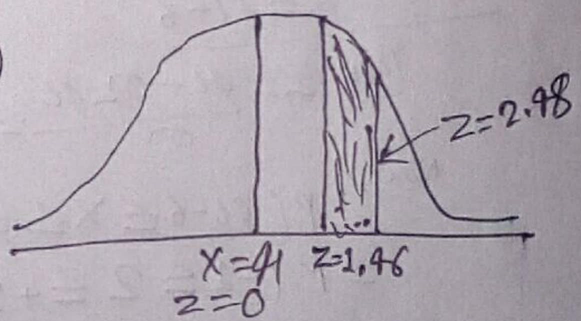
(b) $P(z < 1.34)$
$= 0.5 + P(0 \le z \le 1.34)$
$= 0.5 + 0.4$
$= 0.9$



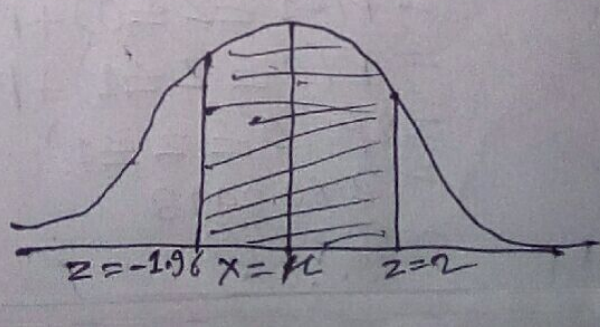
(c) $P(z > 1.96)$
$= 0.5 - P(0 \le z \le 1.96)$
$= 0.5 - 0.4750$
$= 0.0250$



(g) $P(1.96 \le z \le 2.84)$
$= P(0 \le z \le 2.84) - P(0 \le z \le 1.96)$
$= 0.4977 - 0.4750$
$= 0.0227$



(h) $P(-1.96 \le z \le 2)$
$= P(-1.96 \le z \le 0) + P(0 \le z \le 2)$
$= P(0 \le z \le 1.96) + P(0 \le z \le 2)$
$= 0.4750 + 0.4772$
$= 0.4522$



Q.No.58 The mean yeild for one acre plot is 662 kilos with standard deviation 32 kilos. Assuming normal distribution how many one-acre plots in a batch of 1000 plots would you expect to have yeild (i) over 700 kilos (ii) below 650 kilos and (iii) What is the lowest yeild of the best 100 plots?

Soln

Given,

mean yeild $(\mu) = 662$

standard deviation $= 32$

Number of plot $(N) = 1000$

(i) For the probability (over 700 kilo)

$= P(X > 700)$

when x is 700 then $z = \frac{700-662}{32} = 1.18$

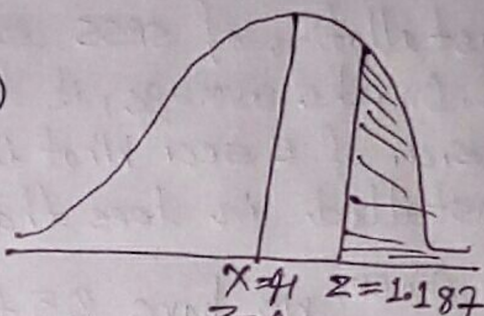
Now, $P(X > 700)$

$= P(z > 1.187)$

$= 0.5 - P(0 < z < 1.187)$

$= 0.5 - 0.3810$

$= 0.1190$



So, the no. of plots when yeild over 700 kilos is

$N \times P(X > 700)$

$= 1000 \times 0.1190$

$= 119.$

(ii) For $P(X < 650$ kilo$)$

When $X = 650$, $z = \frac{650-662}{32} = \frac{-12}{32} = -0.375.$

So, $P(X < 650)$

$= P(z < -0.375)$

$= 0.5 - P(-0.375 < z < 0)$

$= 0.5 - P(0 < z < 0.375)$

$= 0.5 - 0.144$

$= 0.356$

So, the number of plots which yeilds less than 650 kilos

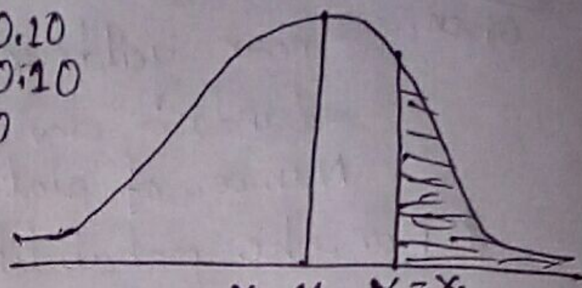
is $N \times P(X < 650) = 1000 \times 0.356$

$= 356$

(iii) Let x_1 be the least yeild, then $P(X>X_1)=\frac{100}{1000}\times 100$
$=10\%$
$=0.10$

When $X=X_1$ then, $Z=\frac{X_1-662}{32}=z_1$ (say) —— (i)

So, $P(X>X_1)=P(Z>z_1)=0.10$
$\Rightarrow 0.5-P(0<z<z_1)=0.10$
$\Rightarrow P(0<z<z_1)=0.40$
$\Rightarrow z_1=1.29$



Putting value of z_1 in eq^n (i)

$$\frac{X_1-662}{32}=1.29$$
$$X_1=703.28.$$

Q.No.59 Installation of SPSS software package requires downloading 95 files. On the average, it takes 16 secs to download one file with s.d. of 5 sec. What is the probability that the software is installed in less than 25 minutes?

Sol^n

Given; we have 95 files.

So, mean $(\mu)=95\times 16$
$=1520$

& Variance $(\sigma^2)=95\times 25$
$=2375$

or, $\sigma=\sqrt{2375}=48.73.$

Now, $P(X<25\text{ min})=P(X<25\times 60\text{ sec})$
$=P(X<1500)$

when $X=1500$, $Z=\frac{1500-1520}{48.73}=\frac{-20}{48.75}=-0.41$

So, $P(X<1500)=P(z<-0.41)$
$=0.5-P(-0.41<z<0)$
$=0.5-P(0<z<0.41)$
$=0.5-0.1591$
$=0.3419$

Q.No.65 Incomes of a group of 10,000 computer operators were found to be normally distributed with mean Rs 15,20 and standard deviation Rs 1600 find (i) highest income of poorest 2000 computer operators (ii) lowest income of richest 1000 computer operators.

Soln

Given, Total no. of computer operators = 10,000.

mean (μ) = 1520

standard deviation (σ) = 160.

Let x_1 be the highest income of poorest 2000 computer operators, then, according to the question.

$$P(X \le x_1) = \frac{2000}{10000} = 0.2$$

~~Let x_1 be the highest income of poorest 20~~

Let $X = x_1$ then $z = \frac{x_1 - 1520}{160} = -z_1$ — (i) (say).

Now,

$P(X \le x_1) = 0.20$

or, $P(Z \le -z_1) = 0.20$

or, $0.5 - P(-z_1 \le z \le 0) = 0.20$

or, $P(0 \le z < z_1) = 0.30$

$\Rightarrow z_1 = 0.84$

Putting value of z in eqn (i).

$$\frac{x_1 - 1520}{160} = -0.84$$

or, $x_1 = 1385.6$

(b). Let x_2 be the lowest income of richest 1000 computer operator.

Then $P(X > x_2) = \frac{1000}{10,000} = 0.10.$

When $X = x_2$, then $z = \frac{x_2 - 1520}{160} = z_2$ — (ii) (say).

Now,

$P(X > x_2) = 0.10$

or, $P(z > x_2) = 0.10$

or, $0.5 - P(0 < z < z_2) = 0.10$

or, $P(0 < z < z_2) = 0.40$

$\Rightarrow z_2 = 1.29.$

Putting value of z_2 in eqn (ii).

$$\frac{x_2 - 1520}{160} = 1.28$$

or, $x_2 = 1724.8$

Hence the highest income of poorest 2000 computer operator is 1385.6 and richest computer operators is 1724.8.

Q.No.66. It is known that the life time of a calculator manufactured by Casio Company Ltd. has a normal distribution with mean of 54 months and a standard deviation of 8 months. If the company wants to replace just 22% calculator which starts not working properly. What would be warranty period?

Soln

Given, mean $(\mu) = 54$

standard deviation $(\sigma) = 8$

Let x_1 be the warranty period of calculators.

If fail is also asked then > is used and percentage is 78%

So, $P(X \leq x_1) = 22\% = 0.22$

when $X = x_1$ then $z = \frac{x_1 - 54}{8} = -z_1$ (say) (i)

Now,

$P(X \leq x_1) = 0.22$

$\Rightarrow P(z \leq -z_1) = 0.22$

$\Rightarrow 0.5 - P(-z_1 \leq z \leq 0) = 0.22$

$\Rightarrow P(0 \leq z \leq z_1) = 0.28$

$\Rightarrow z_1 = 0.77$

From eqn (i)

$$\frac{x_1 - 54}{8} = 0.77$$

or, $x_1 = 54 - 0.77 \times 8$

or, $x_1 = 47.84$.

Note → Binomial र Poission Distribution को यति Question ले मात्र पुग्दैन धेरै भन्दा धेरै question practice गर्नै important छ धेरै।

Exponential Probability Distribution:

Let X be a continious random variable assuming non-negative values is said to follow exponential distibution with parameter θ if its probability density function

$$f(x) = f(X=x) = \begin{cases} \theta . e^{-\theta . x} & , x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Moments of exponential distribution:

$$\boxed{\mu_r' = E(X^r) = \int_0^\infty x^r f(x) . dx}$$

i) $\mu_1' = \frac{1}{\theta}$

ii) $\mu_2' = \frac{2}{\theta^2}$

iii) $\mu_3' = \frac{6}{\theta^3}$

iv) $\mu_4' = \frac{24}{\theta^4}$

So, In General $\boxed{\mu_r' = \frac{r!}{\theta^r}}$

Variance $= E(X^2) - [E(X)]^2$

$= \frac{2}{\theta^2} - \frac{1}{\theta^2}$

$= \frac{1}{\theta^2}$

Q.No.76 Suppose that waiting time (hrs) for bus in a bus station has a negative exponential distribution with parameters $\theta = 5$ hours. What is the probability that a men has to wait at least 15 minutes? Also find expected waiting time for bus.

Solⁿ Given,

Parameter $(\theta) = 5$ hours.

Now, $P(X \geq 15 \text{ min}) = P(X \geq 15/60) = P(X \geq 0.25)$

$$= \int_{0.25}^{\infty} \theta . e^{-\theta x} dx$$

$$= \int_{0.25}^{\infty} 5 . e^{-5x} dx$$

$$= 5 \left[\frac{e^{-5x}}{-5} \right]_{0.25}^{\infty}$$

$$= -\left[e^{-5x} \right]_{0.25}^{\infty}$$

$$= -0 + e^{-5 \times 0.25}$$

$$= e^{-1.25}$$

$$= 0.286$$

and mean of negative exponential distribution is $\frac{1}{\theta}$

$$= \frac{1}{5}$$

Q.No.77 The daily consumption of electricity in a city has an exponential distribution with mean 2000 kilowatt. Find the probability that electricity consumption on a particular day is (i) at least 1500 kilowatt (ii) at most 2500 kilowatt.

Solⁿ

Given, mean $\left(\frac{1}{\theta}\right) = 2000$

$\Rightarrow \theta = \frac{1}{2000} = 0.0005.$

Now, $(X \geq 1500) = \int_{1500}^{\infty} \theta . e^{-\theta x} dx.$

$$= \int_{1500}^{\infty} 0.0005 . e^{-0.0005x} dx$$

$$= 0.0005 \left[\frac{e^{-0.0005x}}{-0.0005} \right]_{1500}^{\infty}$$

$$= -\left[e^{-0.0005x}\right]_{1500}^{\infty}$$

$\boxed{\text{Note: } e^{-\infty} = 1}$

$$= 0 + e^{-0.0005 \times 1500}$$
$$= e^{-0.75}$$
$$= 0.47$$

(b). Now, P(at most 2500)

$$= P(X \leq 2500)$$

$$= \int_0^{2500} \theta . e^{-\theta . x}\, dx.$$

$$= \int_0^{2500} 0.0005 . e^{-0.0005x} . dx$$

$$= 0.0005 \left[\frac{e^{-0.0005x}}{-0.0005}\right]_0^{2500}$$

$$= -e^{-0.0005 \times 2000} + e^{-0}$$

$$= -0.286 + 1$$

$$= 0.714$$

Normal approximation to Binomial.

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{X_{adj} - np}{\sqrt{npq}}$$

X_{adj} means X adjusted.
i.e. $X \pm 0.5$
eg $P(X \geq 20)$
$= P(X_{adj} \geq 19.5)$ ← taking -ve
OR $P(X_{adj} \geq 20.5)$
↑ taking +ve

Q For overseas flight an airlines has three different choices on its desert menu, icecream, apple pie and chocolate cakes. Based on past expeience the airline feels that each desert is equally choosen, of the random sample of 90 passangers is selected what is the probability that

(a) At least 20 will choose ice cream for desert.

(b). Exactly 20 will choose ~~icec~~ icecream for desert.

(c) Less than 20 will choose icecream for desert.

Soln

Given,

No. of passanger$(n) = 90$.

Probability of choosing any desert $(P) = 1/3 = 0.33$

$\Rightarrow q = 1 - P$
$= 1 - 0.33$
$= 0.67$

(a) P(at least 20 will choose icecream as a desert)

$= P(X \geq 20)$

$= P(X_{adj} \geq 19.5)$

When $X_{adj} = 19.5$ then, $Z = \dfrac{X_{adj} - np}{\sqrt{npq}}$

$= \dfrac{19.5 - 90 \times 0.33}{\sqrt{90 \times 0.33 \times 0.67}}$

$= \cancel{4.46}\ -2.28$

Now,

$P(X \geq 20)$

$= P(Z \geq -2.28)$

$= 0.5 + P(-2.28 < Z \leq 0)$

$= 0.5 + P(0 \leq Z \leq 2.28)$

$= 0.5 + 0.4887$

$= 0.9587$

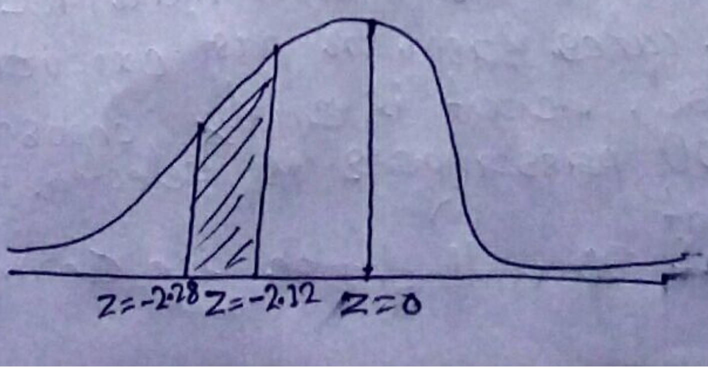
(b) P(exactly 20 will choose icecream).

$= P(X = 20)$

$= P(19.5) < X_{adj} < 20.5)$

When $X_{adj} = 19.5$, then $Z = \dfrac{20.5 - 30}{\sqrt{90 \times 0.33 \times 0.67}}$

$= -2.12$



So, $P(19.5 < X_{adj} < 20.5)$

$= P(-2.28 < z < -2.12)$

$= P(-2.28 < z < 0) - P(-2.12 < z < 0)$

$=$

(c). P (less than 20)

$= P(X < 20)$

$= P(X_{adj} < 20.5)$

When $X_{adj} = 20.5$ then $z = -2.12$

Now,

$P(X_{adj} < 20.5)$

$= P(z < -2.12)$

$= 0.5 - P(-2.12 < z < 0)$

$= 0.5 - P(0 < z < 2.12)$

$= 0.5 - 0.4830$

$= 0.0170$

Q.No.78 A printer can print 15 pages per minute on average. Using normal distribution as approximation of Poisson distribution find probability that printer can print more than 25 pages per minute.

Soln

Given, parameter(λ) = mean = 15.

Now, P(more than 25 pages)

$= P(X > 25)$

$= P(X_{adj} > 24.5)$

When X_{adj} 24.5, $z = \frac{X_{adj} - \lambda}{\sqrt{\lambda}}$

$= \frac{24.5 - 15}{\sqrt{15}}$

$= 2.32$

Now, $P(X_{adj} > 24.5)$

$= P(z > 2.32)$

$= 0.5 - P(0 < z < 2.32)$

$= 0.5 - 0.4898$

$= 0.0102$

Q.No.79 The life time of Lenevo cell phone has gamma distribution with parameter 2. Find the probability that cell phone has life (i) more than 2 years (ii) Between 3 years to 5 years.

Soln

Given, Parameter(α) = 2

Now, P(Cell phone life more than 2 years).

$= P(X>2)$

$$= \int_2^\infty \frac{e^{-x} \cdot x^{\alpha-1}}{\Gamma\alpha} \cdot dx$$

$\because f(x) = \frac{e^{-x} \cdot x^{\alpha-1}}{\Gamma\alpha}$ $0<x<\infty$

mean is λ and σ is $\sqrt{\lambda}$

$\frac{X-\mu}{\sigma}$

$$= \int_2^\infty \frac{e^{-x} \cdot x}{\Gamma 2} dx$$

$$= \int_2^\infty x \cdot e^{-x} dx$$

$$= \left[x \int e^{-x} dx - \int \left[\frac{dx}{dx} \int e^{-x} dx\right] dx\right]_2^\infty$$

$$= \left[x \left[\frac{e^{-x}}{-1}\right] - \int \left(\frac{e^{-x}}{-1}\right) dx\right]_2^\infty$$

$$= \left[-x \cdot e^{-x} - e^{-x}\right]_2^\infty$$

$= 0 - e^{-\infty} + 2e^{-2} + e^{-2}$

$= 3e^{-2}$

$= 0.406$

Now, $P(3<X<5)$

$$= \int_3^5 \frac{e^{-x} \cdot x^{\alpha-1}}{\Gamma\alpha} \cdot dx$$

$$= \int_3^5 e^{-x} \cdot x \cdot dx$$

$$= \left[-x \cdot e^{-x} - e^{-x}\right]_3^5$$

$= -5e^{-5} - e^{-5} + 3e^{-3} + e^{-3}$

$= -6e^{-5} + 4e^{-3}$

$= 0.158$

Exercise Q.No.1 to 17 (related)

Joint Probability density function:

Let x and y be the continious random variable $(-\infty \leq x \leq \infty, -\infty \leq y \leq +\infty)$ then the function $f(x,y)$ is said to be a joint probability density function of random variable x and y if it satisfy the following condition:

i) If $f(x,y) \geq 0$.

ii) $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx.dy = 1, \quad \begin{matrix} -\infty \leq x \leq \infty \\ -\infty \leq Y \leq \infty \end{matrix}$$

Marginal Probability density function:

$$f(x) = \int_{-\infty}^{\infty} f(x,y).dy \quad \text{— (i)} \quad \text{where, } -\infty \leq Y \leq \infty$$

$$f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx \quad \text{— (ii)} \quad \text{where, } -\infty \leq X \leq \infty.$$

Let $f(x,y)$ be the joint probability density function of continious random variable X and Y then, marginal probability density function of X or probability density function of X is $f(x)$ and obtained as in (i).

$f(x)$ is probability density function if it saties following conditions:

i) if $f(x) \geq 0$.

ii) if $\int_{-\infty}^{\infty} f(x).dx = 1$.

Similarly $f(y)$ is obtained as in (ii).

$f(y)$ is probability density function (pdf) if it satisfies following conditions.

i) $f(y) \geq 0$.

ii) $\int_{-\infty}^{\infty} f(y).dy = 1$.

Conditional Probability density function:

Let $f(x,y)$ be the joint probability density function of continious random variable X and Y and $f(x)$ and $f(y)$ be the normal probability density function of X and Y respectively. Then conditional probability density function of X given Y is.

$$f(x/y) = \frac{f(x,y)}{f(y)}$$

Conditions

i) if $f(x,y) \geq 0$

ii) if $\int_{-\infty}^{\infty} f(x).dx = 1$.

Similarly $f(y/x) = \frac{f(x,y)}{f(x)}$.

Note: If X and Y are independent random variable then joint probability is $f(x,y) = f(x).f(y)$.

Numerical Questions

1. If two random variables have the joint probability density function

$$f(x,y)=\begin{cases} 6\dfrac{(x+y^2)}{5} & \text{for } 0< x < 1,\ 0<y<1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability that (i) $0.2<x<0.5$ and $0.4< y<0.6$ (ii) $x > 0.4$ and $y< 0.5$.

0.093

[Ans: 0.43, 0.28]

2. Suppose that x & y have joint density function

$$f(x,y)=\begin{cases} (x + y), & \text{if } \cancel{0<x, y<1}\ 0<x<1,\ 0<y<1 \\ 0 & \text{elsewhere} \end{cases}$$

Find: (i) Marginal density function of x & y (ii) Covariance between x & y. $x+1/2$

[Ans: ~~(x+1)/2~~, (y+1)/2, -0.006]

$\frac{1}{2}+y$

3. If two random variables X_1 and X_2 have the joint probability density function

$$f(x_1,x_2) = \begin{cases} \dfrac{2}{3}(x_1 + 2x_2) & \text{for } 0 < x_1< 1,\ 0 < x_2< 1 \\ 0 & \text{elesewhere} \end{cases}$$

Find the conditional density of X_1 given X_2 and X_2 given X_1

$\left(\text{Ans: } \frac{2(x_1+2x_2)}{1+4x_2}, \frac{x_1+2x_2}{x_1+1}\right)$

4. Let (X,Y) be two dimensional random variable with joint pmf $P(x,y)=\frac{x-y+3}{48}$; x=0,1,2,3; y=0,1,2,3. Find: (i) marginal pmf of X and Y (ii) conditional distribution of X for given Y=1 (iii) conditional distribution of Y for given X=2.

$\left(\text{Ans: } \frac{2x+3}{24}, \frac{9y-2}{24}, \frac{x+2}{14}, \frac{5-y}{14}\right)$

5. Given the following bivariate probability distribution of X & Y.

Y \ X	-1	0	1
0	1/15	2/15	1/15
1	3/15	2/15	1/15
2	2/15	1/15	2/15

Find:(i) P(X=1,y≤1) (ii)P (Y≤1) (iii) P(X=-1)

[Ans: 2/15 , 2/3 , 2/5]

6. A company has 3 hardware engineering consultants and 4 software engineering consultants. From these 2 engineering consultants are selected at random. Let X denote the number of hardware engineering consultant and Y denotes the number of software engineering consultant. Find the (i)Joint probability distribution of X & Y. (ii)Marginal distribution of X and that of Y (iii)Conditional distribution of X for given Y.

7. A two-dimensional discrete random variable (XY) has the joint probability distribution as given below:

Y \ X	0	1	2	3
-1	0.03	0.02	0.05	0
0	0.01	0.25	0.45	0.01
1	0	0.03	0.05	0.10

Determine:(i) marginal distribution of X & Y (ii)Conditional distribution of X for given Y and that of Y for given X (iii) P(X=0,Y≤2)(iv) P(X≥0,Y<3) (v)P(X≤0/Y=1)

8. Suppose two-dimensional random variable (XY) has pdf given by

$$f(x,y)=\begin{cases} x^2+kxy; & 0\le x\le 1;\ 0\le y\le 2 \\ 0 & \text{elsewhere} \end{cases}$$

Determine: (i) constant k (ii) marginal pdf of X and that of Y(iii) conditional pdf of X given Y and that of Y given X (iv)Are X and Y independent.

$\frac{f(x,y)}{f(x)}$; $\frac{f(x,y)}{f(y)}$

$f(x,y)=f(x)\cdot f(y)$; independent,

$\text{Ans: }\left(\frac{1}{3}, \frac{6x^2+2x}{3}, \frac{2+y}{6}, \frac{2(3x^2+xy)}{2+y}, \frac{3x^2+xy}{6x^2+2x}, \text{dependent}\right)$

9. The joint pdf of two dimensional random variable (X,Y) is $f(x,y)=\begin{cases} 2x; & 0\le x\le 1,\ 0\le y\le 1 \\ 0 & \text{elsewhere} \end{cases}$ (i)Find marginal pdf of X & Y (ii) Check independence of X & Y.

[Ans: 2x, 1, independent]

10. Let two dimensional random variable (X,Y) have joint pdf $f(x,y)=\begin{cases} k(6-x-y); & 0\le x\le 2,\ 2\le y\le 4 \\ 0 & \text{elsewhere} \end{cases}$

Determine: (i) Constant k (ii) P(X≤1∩Y<3) (iii)P (Y≤3) (iv) P(X≤1/Y≤3)

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Numerical Questions:

Q.No.1 Solⁿ

Given, Joint probability density function $f(x,y) = \frac{6(x+y^2)}{5}$ $\quad 0<x<1, 0<y<1$

$= 0$ otherwise.

Now,

$P(0.2<x<0.5 \text{ and } 0.4<y<0.6)$

$$= \int_{0.2}^{0.5}\int_{0.4}^{0.6} f(x,y)\, dx.dy$$

$$= \int_{0.2}^{0.5}\left[\int_{0.4}^{0.6} \frac{6(x+y^2)}{5} dy\right].dx$$

$$= \int_{0.2}^{0.5} \frac{6}{5}\left[xy + \frac{y^3}{3}\right]_{0.4}^{0.6} dx$$

$$= \frac{6}{5}\int_{0.2}^{0.5} 0.6x + \frac{0.216}{3} - 0.4x - \frac{0.064}{3}\ .dx$$

$$= \frac{6}{5}\int_{0.2}^{0.5} (\overset{0.2x + 0.047x}{\cancel{0.2x+0.05}})\, dx$$

$$= \frac{6}{5}\left[\frac{0.2x^2}{2} + \overset{0.047x}{\cancel{0.05x}}\right]_{0.2}^{0.5}$$

$$= \frac{6}{5}\left[\frac{0.2(0.5)^2}{2} + \overset{0.047x}{\cancel{0.05x}}\right]_{0.2}^{0.5}$$

$$= \frac{6}{5}\left[\frac{0.2\times 0.5^2}{2} + \overset{0.047}{\cancel{0.05}}\times 0.5 - \frac{0.2\times 0.1^2}{2} - \overset{0.047}{\cancel{0.05}}\times 0.2\right]$$

$$= 0.043$$

(iii) $P(x > 0.4 \text{ and } y < 0.5)$

$$= \int_{0.4}^{1}\int_{0}^{0.5} f(x,y)\,dx.dy$$

$$= \int_{0.4}^{1}\int_{0}^{0.5} \frac{6}{5}(x+y^2).dx.dy.$$

$$= \frac{6}{5}\int_{0.4}^{1}\left[\int_{0}^{0.5}(x+y^2).dy\right]dx,$$

$$= \frac{6}{5}\int_{0.4}^{1}\left[xy+\frac{y^3}{3}\right]_{0}^{0.5}dx.$$

$$= \frac{6}{5}\int_{0.4}^{1} 0.5x + \left(\frac{0.125}{3}\right).dx$$

$$= \frac{6}{5}\left[\frac{0.5x^2}{2} + 0.0416x\right]_{0.4}^{1}$$

$$= \frac{6}{5}\left[\frac{0.5\times 1}{2} + 0.0416. - \frac{0.5\times(0.4)^2}{2} - 0.0416\times 0.4\right]$$

$$= 0.28$$

reads as

$f(x_1/x_2) = f(x_1)$ given $f(x_2)$

Q.No.3 Soln

Given, $f(x_1, x_2) = \frac{2}{3}(x_1 + 2x_2) \quad 0 < x_1 < 1, 0 < x_2 < 1$

$= 0$ otherwise.

We know,

Conditional probability density function of x_1 and x_2 is

$$f(x_1/x_2) = \frac{f(x_1, x_2)}{f(x_2)}$$

and $$f(x_2/x_1) = \frac{f(x_1, x_2)}{f(x_1)}$$

Now,

Marginal probability density function of x_1 is $f(x_1)$.

and $$f(x_1) = \int_0^1 f(x_1, x_2)\, dx_2.$$

$$= \frac{2}{3}\int_0^1 (x_1 + 2x_2).\, dx_2.$$

$$= \frac{2}{3}\left[x_1 x_2 + x_2^2\right]_0^1$$

$$= \frac{2}{3}\left[1 + x_1\right]$$

Since value of x_2 is 1 $x_1.1+1$

$$f(x_2) = \int_0^1 f(x_1, x_2).\, dx_1.$$

$$= \frac{2}{3}\int_0^1 (x_1 + 2x_2)\, dx_1,$$

$$= \frac{2}{3}\left[\frac{x_1^2}{2} + 2.x_2.x_1\right]_0^1$$

$$= \frac{2}{3}\left[\frac{1}{2} + 2x_2\right]$$

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Now, $f(x_1/x_2) = \frac{f(x_1, x_2)}{f(x_2)} = \frac{2/3\,(x_1+2x_2)}{2/3\,(1/2+2x_2)}$

$= \frac{2\,(x_1+2x_2)}{1+4x_2}$

Similarly. $f(x_2/x_1) = \frac{f(x_1, x_2)}{f(x_1)} = \frac{2/3\,(x_1+2x_2)}{2/3\,(1+x_1)}$

$= \frac{x_1+2x_2}{1+x_1}$

Q.No.8 Sol^n

given, $f(x,y) = x^2+kxy \;;\; 0 \le x \le 1 \;;\; 0 \le y \le 2.$

$= 0$ elsewhere.

(i) For finding the value of 'k'.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx.\,dy = 1.$$

or, $\int_0^1\int_0^2 f(x^2+kxy).dx.dy = 1,$

~~$\int_0^1\int_0^2 (x^2+kxy).dx.$~~

or, $\int_0^1\left[\int_0^2 (x^2+kxy)\,dy\right]dx = 1$

or, $\int_0^1\left[x^2y + kxy^2/2\right]_0^2 .dx = 1.$

or, $\int_0^1 (2x^2+2kx)\,dx = 1.$

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or, $\left[\frac{2x^3}{3} + kx^2\right]_0^1 = 1$

or, $\frac{2}{3} + k = 1$

or, $k = 1 - \frac{2}{3}$

or $k = \frac{1}{3}$.

(ii). Here,

$$f(x) = \int_0^2 f(x,y).dy$$

$$= \int_0^2 (x^2 + \tfrac{1}{3}xy)dy.$$

$$= \left[x^2y + \frac{xy^2}{6}\right]_0^2$$

$$= 2x^2 + \frac{4x}{6}$$

$$= \frac{6x^2 + 2x}{3}.$$

(iii) Here, $f(y) = \int_0^1 f(x,y).dx.$

$$= \int_0^1 (x^2 + \tfrac{1}{3}xy)dx.$$

$$= \left[\frac{x^3}{3} + \frac{x^2y}{6}\right]_0^1$$

$$= \frac{1}{3} + \frac{y}{6}$$

$$= \frac{2+y}{6}$$

~~(iii) f(x)~~

(iii) र (iv) आगे चलें.

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Q. No. 10 Sol^n

Given, $f(x,y) = \cancel{2x} \; k(6-x-y)$; $0 \le x \le 2$, $2 \le y \le 4$.
$= 0$ elsewhere.

(i) Sol^n For finding the value of 'k'.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) \cdot dx \cdot dy = 1.$$

or, $\int_0^2 \int_2^4 k(6-x-y) \cdot dx \cdot dy = 1$

or, $k \int_0^2 \left[\int_2^4 (6-x-y)\, dy \right] \cdot dx = 1.$

or, $k \int_0^2 \left[6y - xy - \frac{y^2}{2} \right]_2^4 dx = 1$

or, $k \int_0^2 (24 - 4x - 8 - 12 + 2x + 2)\, dx = 1.$

or, $k \int_0^2 [6 - 2x)\, dx = 1.$

or, $k \left[6x - x^2 \right]_0^2 = 1$

or, $k [12 - 4] = 1$

or, $k = \frac{1}{8}$

(ii). Now, $f(x,y) = \frac{1}{8}(6 \cdot xy)$.
So, the probability $(x \le 1 \cap Y < 3)$

$$= \int_0^1 \int_2^3 f(x,y) \cdot dx \cdot dy.$$

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$$= \int_0^1 \left[\int_2^3 \frac{1}{8} (6-x-y)\, dy \right] dx$$

$$= \frac{1}{8} \int_0^1 \left[6y - xy - y^2/2 \right]_2^3 dx$$

$$= \frac{1}{8} \int_0^1 (18 - 3x - 9/2 - 12 + 2x + 2)\, dx.$$

$$= \frac{1}{8} \int_0^1 (3.5 - x)\, dx$$

$$= \frac{1}{8} \left[3.5x - x^2/2 \right]_0^1$$

$$= \frac{1}{8} \left[3.5 - \frac{1}{2} \right]$$

$$= \frac{3}{8}$$

(iii) $P(Y \leq 3)$. Since <3 otherwise ?

$$= \int_2^3 f(y)\, dy.$$

where, $f(y) = \int_0^2 f(x,y)\, dx$

$$= \int_0^2 \frac{1}{8} (6-x-y)\, dx$$

$$= \frac{1}{8} \left[6x - x^2/2 - xy \right]_0^2$$

$$= \frac{1}{8} (10 - 2y)$$

$$= \left(\frac{5-y}{4} \right)$$

Now, $P(Y \leq 3)$

$$= \int_2^3 \left(\frac{5-\cancel{2y}\, y}{4}\right) dy$$

$$= \frac{1}{4}\left[5y - \frac{y^2}{2}\right]_2^3$$

$$= \frac{1}{4}\left[15 - \frac{9}{2} - 10 + 2\right]$$

$$= \frac{1}{4}\,[2.5]$$

$$= \frac{2.5}{4} \quad \text{or, } \frac{2.5}{4} \times \frac{2}{2} = \frac{5}{8}.$$

(iv). $P(X \leq 1 / Y \leq 3)$

$$= \frac{P(X \leq 1, Y \leq 3)}{P(Y \leq 3)}$$

$$= \frac{3/8}{5/8}$$

$$= 3/5.$$