(Q) Find the LU factorization of the following matrices:

(a) $A = \begin{bmatrix} 2 & 5 \\ -3 & 4 \end{bmatrix}_{2\times 2}$

Sol^n//

Since A has 2x2 size so L should be 2x2 size

$A = \begin{bmatrix} 2 & 5 \\ -3 & 4 \end{bmatrix}$

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$R_2 \to 2R_2 + 3R_1$

$$\sim \begin{bmatrix} 2 & 5 \\ 0 & 23 \end{bmatrix} = U$$

Now

at each highlighted column is divided by the corresponding pivot and get L.

$$\begin{bmatrix} 2 \\ -3 \end{bmatrix} \quad [23]$$

$$\Rightarrow \begin{bmatrix} 1 \\ -3/2 \end{bmatrix} \quad [1]$$

$$\therefore L = \begin{bmatrix} 1 & 0 \\ -3/2 & 1 \end{bmatrix}$$

$$\therefore A = LU$$
$$= \begin{bmatrix} 1 & 0 \\ -3/2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 0 & 23 \end{bmatrix} //$$

(b) $\begin{bmatrix} 3 & -1 & 2 \\ -3 & -2 & 10 \\ 9 & -5 & 6 \end{bmatrix}_{3\times3}$

$\Rightarrow$ Soln,

Since A has 3x3 size, so L should be 3x3 size

$$\therefore A = \begin{bmatrix} 3 & -1 & 2 \\ -3 & -2 & 10 \\ 9 & -5 & 6 \end{bmatrix}$$

$R_2 \to R_2 + R_1$ and $R_3 \to R_3 - 3R_1$

$$\sim \begin{bmatrix} 3 & -1 & 2 \\ 0 & -3 & 12 \\ 0 & -2 & 0 \end{bmatrix}$$

$R_3 \to 3R_3 - 2R_2$

$$\sim \begin{bmatrix} 3 & -1 & 2 \\ 0 & -3 & 12 \\ 0 & 0 & -24 \end{bmatrix} = U$$

Now,

at each highlighted column is divided by the corresponding pivot and get L,

$$\begin{bmatrix} 3 \\ -3 \\ 9 \end{bmatrix} \quad \begin{matrix} 0 \\ \begin{bmatrix} -3 \\ -2 \end{bmatrix} \end{matrix} \quad \begin{matrix} 0 \\ 0 \\ \begin{bmatrix} -24 \end{bmatrix} \end{matrix}$$

$$\Rightarrow \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} \quad \begin{matrix} 0 \\ \begin{bmatrix} 1 \\ 2/3 \end{bmatrix} \end{matrix} \quad \begin{bmatrix} 1 \end{bmatrix}$$

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 2/3 & 1 \end{bmatrix}$$

$$\therefore A = LU = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 2/3 & 1 \end{bmatrix} \begin{bmatrix} 3 & -1 & 2 \\ 0 & -3 & 12 \\ 0 & 0 & -24 \end{bmatrix}$$

//

(c) Let, $A = \begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}$

⇒ Soln,

Let,

$$A = \begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$ and $R_3 \to 3R_3 + R_1$

$$\sim \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 15 & 3 \end{bmatrix}$$

$R_3 \to R_3 - 3R_2$

$$\sim \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 0 & 3 \end{bmatrix} = U$$

Now,

at each highlighted column is divided by the corresponding pivot and get L,

$$\begin{bmatrix} 3 \\ 6 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 5 \\ 15 \end{bmatrix} \quad \begin{bmatrix} 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 \\ 2 \\ -1/3 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad \begin{bmatrix} 1 \end{bmatrix}$$

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$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1/3 & 3 & 1 \end{bmatrix}$$

$$\therefore A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1/3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 0 & 3 \end{bmatrix}$$ //

(d) $\begin{bmatrix} 1 & 3 & -5 & -3 \\ -1 & -5 & 8 & 4 \\ 4 & 2 & -5 & -7 \\ -2 & -4 & 7 & 5 \end{bmatrix}$

$\Rightarrow$ Solⁿ//

Let,

$$A = \begin{bmatrix} 1 & 3 & -5 & -3 \\ -1 & -5 & 8 & 4 \\ 4 & 2 & -5 & -7 \\ -2 & -4 & 7 & 5 \end{bmatrix}$$

$R_2 \to R_2 + R_1$, $R_3 \to R_3 - 4R_1$ and $R_4 \to R_4 + 2R_1$

$$\sim \begin{bmatrix} 1 & 3 & -5 & -3 \\ 0 & -2 & 3 & 1 \\ 0 & -10 & 15 & 5 \\ 0 & 2 & -3 & -1 \end{bmatrix}$$

$R_3 \to R_3 - 5R_2$ and $R_1 \to R_1 + R_2$

$$\sim \begin{bmatrix} 1 & 3 & -5 & -3 \\ 0 & -2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = U$$

Now

at each highlighted column is divided by the corresponding pivot and get L,

$$\begin{bmatrix} 1 \\ -1 \\ 4 \\ -2 \end{bmatrix} \quad \begin{bmatrix} -2 \\ -10 \\ 2 \end{bmatrix} \quad \cancel{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} \quad \cancel{\begin{bmatrix} 1 \end{bmatrix}}$$

$$\Rightarrow \begin{bmatrix} 1 \\ -1 \\ 4 \\ -2 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$$

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 4 & 5 & 1 & 0 \\ -2 & -1 & 0 & 1 \end{bmatrix}$$

$$\therefore A = LU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 4 & 5 & 1 & 0 \\ -2 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & -5 & -3 \\ 0 & -2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ //

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(e)
$$A=\begin{bmatrix} 2 & -4 & 4 & -2 \\ 6 & -9 & 7 & -3 \\ -1 & -4 & 8 & 0 \end{bmatrix}_{3\times4}$$

Ans Sol^n,

Since A has 3×4 size so L should be 3×3 ~~mat~~ size.

$$A=\begin{bmatrix} 2 & -4 & 4 & -2 \\ 6 & -9 & 7 & -3 \\ -1 & -4 & 8 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$ and $R_3 \to 2R_3 + R_1$

$$\sim A \begin{bmatrix} 2 & -4 & 4 & -2 \\ 0 & 3 & -5 & 3 \\ 0 & -12 & 20 & -2 \end{bmatrix}$$

$R_3 \to R_3 + 4R_2$

$$\sim A \begin{bmatrix} 2 & -4 & 4 & -2 \\ 0 & 3 & -5 & 3 \\ 0 & 0 & 0 & \boxed{10} \end{bmatrix} = U$$

now,

at each highlighted column is divided by the corresponding pivot and get L,

$$\begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix} \begin{bmatrix} 3 \\ -12 \end{bmatrix} \begin{bmatrix} 10 \end{bmatrix}$$

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$$\Rightarrow \begin{bmatrix} 1 \\ 3 \\ -1/2 \end{bmatrix} \begin{bmatrix} 1 \\ -4 \end{bmatrix} \boxed{1}$$

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1/2 & -4 & 1 \end{bmatrix}$$

Hence

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1/2 & -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -4 & 4 & -2 \\ 0 & 3 & -5 & 3 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$ //

(f) $A = \begin{bmatrix} 2 & 5 \\ 6 & 7 \end{bmatrix}$

Sol^n/ Given

$$A = \begin{bmatrix} 2 & 5 \\ 6 & 7 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$$\sim A \begin{bmatrix} 2 & 5 \\ 0 & \boxed{-8} \end{bmatrix} = U$$

$R_2 \to -\frac{1}{8} R_2$

$$\sim A \begin{bmatrix} 2 & 5 \\ 0 & \end{bmatrix}$$

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Now,

at each highlighted column is divided by the corresponding pivot and get L,

$$\begin{bmatrix} 2 \\ 6 \end{bmatrix} \begin{bmatrix} -8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix}$$

$$\therefore L = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

Hence,

$$A = LU = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 0 & -8 \end{bmatrix} \text{ //}$$

(Q)

$$A = \begin{bmatrix} 2 & -4 & -2 & 3 \\ 6 & -9 & -5 & 8 \\ 2 & -7 & -3 & 9 \\ 4 & -2 & -2 & -1 \\ -6 & 3 & 3 & 4 \end{bmatrix}_{5\times4}$$

Soln, Since,

A has 5x4 size so L should be 5x5 size.

$$\therefore A = \begin{bmatrix} 2 & -4 & -2 & 3 \\ 6 & -9 & -5 & 8 \\ 2 & -7 & -3 & 9 \\ 4 & -2 & -2 & -1 \\ -6 & 3 & 3 & 4 \end{bmatrix}$$

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$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - R_1$, $R_4 \to R_4 - 2R_1$, $R_5 \to R_5 + 3R_1$

$$\sim A \begin{bmatrix} 2 & -4 & -2 & 3 \\ 0 & 3 & 1 & -1 \\ 0 & -3 & -1 & 6 \\ 0 & 6 & 2 & -7 \\ 0 & -9 & -3 & 13 \end{bmatrix}$$

$R_3 \to R_3 + R_2$, $R_4 \to R_4 - 2R_2$, $R_5 \to R_5 + 3R_2$

$$\sim A \begin{bmatrix} 2 & -4 & -2 & 3 \\ 0 & 3 & 1 & -1 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$R_4 \to R_4 + R_3$ and $R_4 \to R_4 - 2R_3$

$$\sim A \begin{bmatrix} 2 & -4 & -2 & 3 \\ 0 & 3 & 1 & -1 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = U$$

Now,

Gt each highlighted column is divided by the corresponding pivot and get L,

$$\begin{bmatrix} 2 \\ 6 \\ 2 \\ 4 \\ -6 \end{bmatrix} \quad \begin{bmatrix} 3 \\ -3 \\ 6 \\ -9 \end{bmatrix} \quad \begin{bmatrix} 5 \\ -5 \\ 10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 \\ 3 \\ 1 \\ 2 \\ -3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \\ -3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & 2 & -1 \\ -3 & -3 & 2 \end{bmatrix}$$

Hence

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & 2 & -1 \\ -3 & -3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -4 & -2 & 3 \\ 0 & 3 & 1 & -1 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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