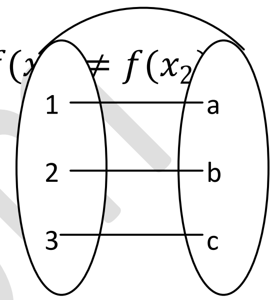
Question collection (2067) 4th batch

Q.N.1)Define one to one and onto function with suitable examples.

Solution:

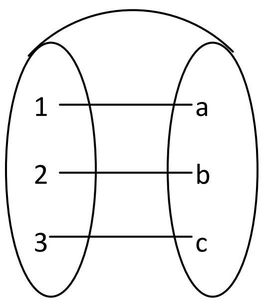
A function f , A->B is called one to one if (A->x_1,x_2) , $x_1 \neq x_2 => f(x_1) \neq f(x_2)$



$or; [f(x_1) = f(x_2) => x_1 = x_2]$

If different elements in A have different images.

Onto : A function f; A->B is called onto function if B=f(A)



Q.N.2) Show the series by integral test

$$\sum_{n=1}^{\infty} \frac{1}{x^p}, \; converges\; if\; p > 1.$$

Solution

If p>1

Let, $f(x) = \frac{1}{x^p}$

Now, $\int_1^{\infty} f(x)dx = \int_1^{\infty} \frac{1}{x^p} dx$

$$= \lim_{a\to\infty} \int_1^a \frac{1}{x^p} dx$$

$$= \lim_{a\to\infty}\left[\frac{x^{-p+1}}{-p+1}\right]_1^a$$

$$= \lim_{a\to\infty}\left[a^{-p+1} - \frac{1^{-p+1}}{1-p}\right]$$

$$= \frac{1}{1-p}\lim_{a\to\infty}\left[\frac{1}{a^{p-1}} - 1\right]$$

Since p>1

$$= \frac{1}{1-p}\ \lim_{a\to\infty}\left[\frac{1}{a^{p-1}} - 1\ \right]$$

$$= \frac{1}{1-p}[0-1]$$

$$= \frac{1}{p-1}\ which\ is\ finite\ , converges.$$

Q.N.3) Test the convergence of the integrals.

$$\sum_{\mathrm{n}=1}^{\infty}(-1)^{x+1}\ \frac{1}{x^2}$$

Solution;

$$\sum_{n=1}^{\infty}\left|(-1)^{x+1}\ \frac{1}{x^2}\right|$$

$$= 1 + \frac{1}{4} + \frac{1}{9} + \cdots \ldots \ldots \ldots . \frac{1}{x^2}$$

The given series is convergent as p>1 by p series test & it is absolute convergent by alternating series test.

$= 1, \frac{1}{4}, \frac{1}{9} \dots \dots \dots .. are\ all\ positive$

$1 > \frac{1}{4} > \frac{1}{9} \dots .$

$\lim_{x\to\infty} \frac{1}{x^2}$=0

So, it is absolutely convergent.

Q.N.4)Find the focus and the directrix of the parabola y^2=10x.

Here , $y^2 = 10x$

Comparing with $y^2 = 4ax$

$a = \frac{5}{2}$

$focus = (a, 0) = \left(\frac{5}{2}, 0\right)$

eq^n of directrix, $x + a = 0 \quad i.e; x + \frac{5}{2} = 0$

$or; 2x + 5 = 0$

$\therefore directrix\ of\ the\ parabolais\ 2x + 5 = 0.$

5. Find the point where the line $\boldsymbol{x = \frac{8}{3} + 2t, y = -2t}$ & $z = 1 + t$ intersects the plane $\boldsymbol{3x + 2y + 6z = 6}$.

The point $\left(\frac{8}{3} + 2t, -2t, 1 + t\right)$ lies in the plane if its coordinates satisfy the equation of the plane; that is; if

$$3\left(\frac{8}{3} + 2t\right) + 2(-2t) + 6(1 + t) = 6$$

or, $\quad 8 + 6t - 4t + 6 + 6t = 6$

or, $t = -1$

The point of intersection is

$$(x, y, z)|_{t=-1} = \left(\frac{8}{3} - 2, 2, 1 - 1\right) = \left(\frac{2}{3}, 2, 0\right)$$

Q.N.6) Find a spherical coordinate equation for the sphere $\boldsymbol{x^2 + y^2 + (z-1)^2 = 1}$

Solution

Given, $x^2 + y^2 + (z-1)^2 = 1$

$(\rho sin\varphi\ cos\theta)^2 + (\rho sin\varphi sin\theta)^2 + (z-1)^2 = 1$

$\rho^2 \sin^2\varphi(\sin^2\theta + \cos^2\theta) + (z-1)^2 = 1$

$\rho^2 \sin^2\varphi + (\rho cos\varphi - 1)^2 = 1$

$\rho^2 \sin^2\varphi + \rho^2\cos^2\varphi - 2\rho cos\varphi + 1 = 1$

$\rho^2(\sin^2\varphi + \cos^2\varphi) = 2\rho cos\varphi$

$\rho^2 - 2\rho cos\varphi = 0$

$\rho(\rho - 2cos\varphi) = 0$

Either,
$\boldsymbol{\rho = 0}$

Or,
$\boldsymbol{\rho = 2cos\emptyset}$

Q.N.7) Find the area of the region R bounded by y=x and y=x^2 in the first quadrant by using double integrals.

Solution:

$y = x \ldots\ldots\ldots\ldots\ldots (i)$

$y = x^2 \ldots \ldots \ldots \ldots ..(ii)$

Imagine a vertical line which enters at $y = x^2$ & exists at $y = x$

$\therefore x^2 \leq y \leq x$

& $x = 0 \; to \; x = 1$

If $o \leq x \leq 1$

Now,

Area(A)= $\int \int_R dA$

$\int_0^1 \int_x^x dydx\_$

$$\int_0^1 [y]_{x^2}^{x} dx$$

$$\int_0^1 [x - x^2]dx = \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{2} - \frac{1}{3}$$

$$= \frac{3-2}{6} = \frac{1}{6}$$

$$\therefore Area = \frac{1}{6}$$

Q.N.8) Define jacobian determinant for X=g(u,v,w), y=h(u,v,w), z=k(u,v,w).

Solution

Jacobian determinant for X=g(u,v,w), y=h(u,v,w), z=k(u,v,w) is

$J(u, v, w) = \frac{\partial(x,y,z}{\partial(u,v,w)}$

$$= \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial u} & \frac{\partial z}{\partial u} \end{vmatrix}$$

Q.N.9) Find the extreme values of f(x,y)=x^2+y^2

Solution:

$f(x, y) = x^2 + y^2$

Now, $fx = 2x$

$fy = 2y$

Now,

$fxx = 2$

$fyy = 2$

$fxy = 0$

Since$fxx > o$ & $fxx.fxy - (fxy)^2$

$or; 2X2 - 0$

2>0

$\therefore f(x, y) has\ the\ minimum\ value.$

Q.N.10) Define partial differential equation of the second order with suitable examples.

Partial differential equation

If a dependent variable is a function of two or more than two independent variable then on equation involving with partial differential coefficient it is known as partial differential equation. This is the relation of dependent variable independent variable and partial differential coefficient.

$\frac{d^2z}{dx^2}+\frac{d^2}{dxdy}+\frac{2d^2}{dy^2}=0$ is a second order partial differential equation

<u>Group B(5X4=20)</u>

Q.N.11)State Rolle's Theorem for a differential function. Support with examples that the hypothesis of theorem are essential to hold the theorem.

(Rolle's Theorem) Suppose f (x) is continuous on [a, b] and dif- ferentiable on (a, b). If f (a) = 0 = f (b), then there exists a point c ∈ (a, b) such that f ′(c) = 0.

Here the given function is $f(x)=\frac{x^3}{3}-3x$

The given interval is [-3,3]

According to Rolle's theorem,

$$f(-3)=\frac{(-3)^3}{3}-3X(-3)=0$$

$$f(3)=\frac{(3)^3}{3}-3X(3)=0$$

Now, $\mathrm{f}'(\mathrm{x})=\frac{3\mathrm{x}^2}{3}-3=x^2-3$

We have, $\mathrm{f}'(\mathrm{c})=0$

or, $c^2-3=0$

or, $c=\sqrt{3}$

Here, c falls between [-3,3]. Thus, Rolle's theorem is verified.

Q.N.12) Test if the following series converges

a) $\sum_{x=1}^{\infty} \frac{x^2}{2^x}$

b)$\sum_{x=1}^{\infty} \frac{2^x}{x^2}$

soluition:

a) $\sum_{x=1}^{\infty} \frac{x^2}{2^x}$

solution.

$$a_x = \frac{x^2}{2^x}$$

Now, $\lim_{x\to\infty} \left(\frac{x^2}{2^x}\right)^{\frac{1}{x}}$

$$\lim_{x\to\infty} \frac{x^{\frac{2}{x}}}{2}$$

$$\lim_{x\to\infty} \frac{\left(x^{\frac{1}{x}}\right)^2}{2}$$

$$= \frac{1}{2} < 1$$

Hence, the series converges.

b) **b)**$\sum_{x=1}^{\infty} \frac{2^x}{x^2}$

solution:

$$a_x = \frac{2^x}{x^2}$$

Now,

$$\lim_{x\to\infty}\left(\frac{2^x}{x^2}\right)^{\frac{1}{x}}$$

$$= \lim_{x\to\infty}\frac{2}{\left(x^{\frac{1}{x}}\right)^2} = 2 > 1$$

Hence , the series diverges.

Q.N.13) Obtain the polar equations for circles through the origin centered on the x- and y-axis and radius a.

Solution:

From the question

Equation of circle through the origin centered on the x and y axis and radius a is given by

$x^2 + y^2 = a^2 \ldots\ldots\ldots\ldots..(i)$

Changing it to polar form we have,

$x = rcos\theta \quad y = rsin\theta$

Now, putting the value of x and y in equation (i) we have,

$r^2\cos^2\theta + r^2\sin^2\theta = a^2$
$r^2(\cos^2\theta + \sin^2\theta) = a^2$
$\boldsymbol{r \pm a = 0}$

Which is the required equation

Q.N.14) Show that the function $f(x)=\begin{cases}\frac{2xy}{x^2+y^2}, & (x,y)=(0,0)\\ 0, & (x,y)=0\end{cases}$

is continuous at every point except the origin.

Solution

$$f(x)=\begin{cases}\frac{2xy}{x^2+y^2}, & (x,y)=(0,0)\\ 0, & (x,y)=0\end{cases}$$

The functional value of above function is 0 at (0,0)

Now,

$$\lim_{(x,y)\to(0,0)} f(x,y)=\lim_{(x,y)\to(0,0)}\frac{2xy}{x^2+y^2}$$

Now, we take path $y=mx$

$$=\lim_{(x,y)\to(0,0)}\frac{2mx^2}{x^2+m^2x^2}$$

$$=\lim_{(x,y)\to(0,0)}\frac{2m}{1+m^2}$$

$$=\frac{2m}{1+m^2}$$

i.e; limit doesn't exist as the m has not fixed value. So , function is discontinuous.

Q.N.15) Find the solution of the equation $\frac{\partial^2 y}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}$**=x-y.**

Q.N.16)Find the area of the region enclosed by the parabola y=2-x^2 and the line y=-x.

Solution

$y=2-x^2 \ldots\ldots\ldots.(i)$

$y=-x \ldots\ldots\ldots(ii)$

X	-1	0	2
Y	1	2	-2

X	-1	0	2
Y	1	0	-2

We find limits of integratin from eq^n (i)&(ii)

$2 - x^2 = -x$

$or; x^2 - x - 2 = 0$

$or; (x^2 - 2x + x - 2)$=0

$or; (x + 1)(x - 2) = 0$

$x = -1, x = 2$

Since, the region is bounded by upper curve $f(x) = 2 - x^2$ & lower line $g(x) = x$

$-1 \le x \le 2$

Area(A)=$\int_{-1}^{2}[f(x) - g(x)]dx$

$$\int_{-1}^{2}[2 - x^2 + x]dx$$

$$= \left[2 - \frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{2}$$

$$= \left[4 - \frac{8}{3} + 2\right] - \left[-2 + \frac{1}{3} - \frac{1}{2}\right]$$

$$= \frac{\mathbf{9}}{\mathbf{2}}\ \boldsymbol{sq.units}.$$

OR

Q.N.16) Evaluate the integrals.

a) $\int_0^3 \frac{dx}{(x-1)^{\frac{2}{3}}}$

Solution

The integral $f(x) = \int_0^1 \frac{1}{1-x}$ is continuous on[0,1] but becomes infinite at x=1 so we evaluate the integral as $b \rightarrow 1^-$

$$\therefore I = \lim_{b\rightarrow 1^-} \int_a^b \frac{1}{1-x} dx$$

$$\lim_{b\rightarrow 1^-} \left[\frac{log(1-x)}{1}\right]_0^b$$

$$= \lim_{b\rightarrow 1^-} -log(1-b) + log(1)$$

$$= \lim_{b\rightarrow 1^-} log(b-1) + 0$$

$$= \lim_{b\rightarrow 1^-} log\left(\frac{1}{1-b}\right)$$

$$= log\left(\frac{1}{1-1}\right)$$

$= log\infty$ **which is infinite, so the given integral diverges**

Q.N.17) Define a curvature of a space curve. Find the curvature for the helix

$\boldsymbol{r(t) = (acost)i + (asin\ t)j + btk(a, b \geq 0, a^2 + b^2 \neq 0)}$

$r(t) = (\mathrm{a}\cos t\ )i + (\mathrm{a}\sin t)j + btk$

$\frac{dr}{dt} = \vec{v} = -\mathrm{a}\sin t\ \vec{\imath} + acost\ \vec{\jmath}\ +\overrightarrow{bk}$……………….(i)

$|\vec{v}|$=$\sqrt{(-asint)^2 + (acost)^2 + b^2}$……………….(ii)

$= \sqrt{a^2 + b^2}$

$\therefore T = \frac{\vec{v}}{|\vec{v}|}$

$$\frac{dT}{dt} = \frac{-asint\vec{\imath} + acost\vec{\jmath} + b\vec{k}}{(\sqrt{a^2 + b^2})}$$

$$\left|\frac{dT}{dt}\right| = \sqrt{\left(\frac{acost}{\sqrt{a^2 + b^2}}\right)^2 + \left(-\frac{asint}{\sqrt{a^2 + b^2}}\right)}$$

Diff equation (i) w.r.t ‘t’ we get,

$$\vec{a} = \frac{dv}{dt} = -acost\vec{\imath} - asint\vec{\jmath}$$

$$\vec{v} \times \vec{a} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ -asint & acost & b \\ -acost & -asint & 0 \end{vmatrix}$$

$$= (0 + absint)\vec{\imath} - (0 + abcost)\vec{\jmath} + (\mathrm{a}^2\sin^2\mathrm{t} + \mathrm{a}^2\cos^2\mathrm{t})\ \vec{k}$$
$$= absint\vec{\imath} - abcost\vec{\jmath} + a^2\vec{k}$$
$$|\vec{v} \times \vec{a}| = \sqrt{a^2b^2\sin^2 t + a^2b^2\cos^2 t + a^4}$$
$$= \sqrt{a^2b^2 + a^4}$$
$$= a\sqrt{a^2 + b^2}$$

Now,

$$(k) = \frac{|\vec{v} + \vec{a}|}{|\vec{v}|^3}$$

$$= \frac{a}{a^2 + b^2}$$

$$curvature(\rho) = \frac{1}{k} = \frac{a^2 + b^2}{a}$$

Q.N.18)Find the volume of the region D enclosed by the surfaces z=x^2+3$y^2$and z=8-x^2-y^2.

Solution:

$z = x^2 + 3y^2 \ldots\ldots\ldots\ldots\ldots\ldots (i)$

$z = 8 - x^2 - y^2 \ldots\ldots\ldots\ldots..(ii)$

From(i)&(ii)

$x^2 + 3y^2 = 8 - x^2 - y^2$

$or; 2x^2 + 4y^2 - 8$

$or; 2y^2 = 4 - x^2$

$or; x^2 + 2y^2 - 4 \ldots\ldots\ldots\ldots\ldots (iii)$

$$or; y = \pm\sqrt{\frac{(4 - x^2)}{2}}$$

$i.e; -\sqrt{\frac{4-x^2}{2}} \leq \sqrt{\frac{4-x^2}{2}}$

Now , to find the points for x-axis.

$y = 0$

$$\sqrt{\frac{4 - x^2}{2}} = 0$$

$\therefore x = \pm 2. \quad i.e;\ -2 \leq x \leq 2.$

The points of x-axis are(2,0,0) &(-2,0,0)

Then x^2 plane will be (2,0,4) &(-2,0,4)

$$voume(v) = \int_{-2}^{2}\int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}}\int_{x^2+3y^2}^{8-x^2-y^2} dzdydx$$

$$\int_{-2}^{2}\int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}} 8-2x^2-4y^2 dydx$$

$$\int_{-2}^{2}\left[8-2x^2-y-\frac{4y^3}{3}\right]_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}} dx$$

$$\left[\int_{-2}^{2} 8\sqrt{\frac{(4-x^2)}{2}} - 2x^2\sqrt{\frac{(4-x^2)}{2}} - 4\left(\sqrt{\frac{(4-x^2)}{2}}\right)^3 + 8\sqrt{\frac{(4-x^2)}{2}} - 2x^2\sqrt{\frac{(4-x^2)}{2}} - \frac{4}{3}\left(\sqrt{\frac{(4-x^2)}{2}}\right)^3\right] dx$$

$$= \left[\int_{-2}^{2}[16\sqrt{\frac{(4-x^2)}{2}} - 4x^2\sqrt{\frac{(4-x^2)}{2}} - \frac{8}{3}\left(\sqrt{\frac{(4-x^2)}{2}}\right)^3\right] dx$$

$$= \int_{0}^{2}\frac{8}{3\sqrt{2}}(4-x^2)^{\frac{3}{2}} dx$$

Put $x = 2sin\theta$

$dx = 2cos\theta d\theta$

$$= \frac{16}{3\sqrt{2}}\int_{0}^{\frac{\pi}{2}}(4-4\sin^2\theta)^{\frac{3}{2}} 2cos\theta d\theta$$

$$\frac{32}{3\sqrt{2}}\int_0^{\frac{\pi}{2}}(4\cos^2\theta)^{\frac{3}{2}}\quad cos\theta d\theta$$

$$=\frac{32}{3\sqrt{2}}\int_0^{\frac{\pi}{2}}8\cos^4\theta d\theta$$

$$=\frac{32}{3\sqrt{2}}\int_0^{\frac{\pi}{2}}2(2\cos^2\theta)^2\ d\theta$$

$$\frac{64}{3\sqrt{2}}\int_0^{\frac{\pi}{2}}(1+\cos\theta)^2\ d\theta$$

Therefore volume= $8\sqrt{2\pi}$

Q.19.)find the maximum and minimum values of the function f(x,y)= $3x+4y$ on the circle $x^2+y^2=1$

Solution

We have to find the values of x, y, γ which satisfies the condition

$\nabla f=\lambda\nabla g\ and\ g(x,y)=0$

Now,

$fx\vec{\imath}+fy\vec{\jmath}=\lambda(gx\vec{\imath}+gy\vec{\jmath})$
$3\vec{\imath}+4\vec{\jmath}=\lambda(2x\vec{\imath}+2y\vec{\jmath})$
$3\vec{\imath}+4\vec{\jmath}=2\,\lambda x\vec{\imath}+\ 2\,\lambda y\vec{\jmath}$

From which we obtain

$3=2\,\lambda x$ $\qquad$ $4=2\,\lambda y$

$x=\frac{3}{2\,\lambda}$ $\qquad$ $y=\frac{2}{\lambda}$

Substituting those values in $g(x,y)=0\ we\ get,$

$x^2 + y^2 - 1 = 0$

$\frac{9}{4\,\lambda^2} + \frac{4}{\lambda^2} - 1 = 0$

$\frac{9 + 16 - 9}{4\,\lambda^2} = 0$

$25 = 4\,\lambda^2$

$\lambda = \pm \frac{5}{2}$

$Since\ x = \frac{3}{2\,\lambda}\ and\ y = \frac{2}{\lambda}, x\ and\ y\ have\ the\ sign\ x = \frac{3}{2} \pm \frac{5}{2} = \pm \frac{3}{5}\ y$

$= \pm \frac{4}{5}\ and\ f(x,y) = 3x + 4y\ has\ the\ extreme\ value\ at\ (x,y)$

$= \left(\pm \frac{3}{5}, \pm \frac{4}{5}\right)$

$By\ calculating\ the\ values\ of\ 3x + 4y\ at\ the\ point$
$\pm \left(\frac{3}{5}, \frac{4}{5}\right)\ we\ see\ that\ its\ maximum\ and\ minimum\ values\ on\ the\ circle\ x^2$
$+ y^2 = 1\ are$:

$3 \times \frac{3}{5} + 4 \times \frac{4}{5} = 5$

$and\ 3 \times \left(-\frac{3}{5}\right) + 4 \times \left(-\frac{4}{5}\right) = -5$

OR

State the condition of second derivative test for local extreme values. Find the local extreme values of the function $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$

Solution

Suppose $f(x,y)$ and its first and second derivative are continuous throughout a disk centered at (a,b) and that $f_x(a,b) = 0, f_y(a,b) = 0$. Then

a) F has a local maximum at (a,b)
   If $f_{xx} < 0\ and\ f_{xx}.f_{yy} - f_{xy^2} > 0\ at\ (a,b)$

b) If
$f_{xx} > 0\ and\ f_{xx} f_{yy} - f_{xy^2} >$
$0\ at\ (a,b) then\ f\ has\ local\ minimum\ at\ (a,b)$

c) If $f_{xx} f_{yy} - f_{xy^2} = 0\ at\ (a,b) then\ f\ has\ a\ saddle\ point\ at\ (a,b)$

Given

$f_x = 2x + y + 3$ $\qquad$ $f_{xx} = 2$
$f_y = x + 2y - 3$ $\qquad$ $fyy = 2$
$fx = 0$
$=> 2x + y + 3 = 0 - - - - - - - - - -(1)$

$y = -2x - 3$

$f_y = x + 2y - 3 = 0$
Putting the values of y in f_y we get

$x + 2 \times (-2x - 3) - 3 = 0$
$x + 4x - 6 - 3 = 0$
$-3x - 9 = 0$
$x + 3 = 0$
$x = -3$

Again putting the value of x in (1) we get

$2(-3) + y + 3 = 0$

$y = 3$

Now,

$f_{xx} fyy - f_{xy^2} > 0$
$2 \times 2 - 1 > 0$

3>0

Since $f_{xx} is\ alsogreater\ than\ 0\ f\ has\ local\ minimum\ value$

i.e.

$f(-3,3) = 9 - 9 + 9 - 9 + 9 + 4 = -5$

Which is the required minimum value

Q.N.20) Define one-dimensional wave equation and one-dimensional heat equations with initial conditions. Derive solution of any of them.

Solution

The wave equation $\frac{d^2u}{dt^2} = \frac{C^2 d^2 u}{dx^2}$

$where\ C^2 = \frac{T}{\rho}$

The solution of equation(i) can be obtained by introducing thwo independent variables v and z defined by $v = x + ct\ and\ z = x - ct$

Differentiating v w.r.t x

$$\frac{dv}{dx} = \frac{du}{dv}.\frac{dv}{dx} + \frac{du}{dz}.\frac{dz}{dx}$$

$$= \frac{du}{dv} + \frac{du}{dz}$$

Again differentiate w.r. to x

$$\frac{d^2}{dv}\left(\frac{du}{dv}.\frac{du}{dz}\right)\frac{dv}{dx} + \frac{d}{dz}\left(\frac{du}{dv} + \frac{du}{dz}\right)\frac{dz}{dx}$$

$$= \frac{d^2u}{dvz} + \frac{d^{2u}}{dv^2z} + \frac{d^{2u}}{dzdv} + \frac{d^2u}{dz^2}$$

$$\frac{d^{2u}}{dvz} + \frac{zd^2u}{dvdz} + \frac{d^2u}{dz^2}$$

Again diff u w.r.to t.

$$\frac{du}{dt} = \frac{du}{dv}.\frac{dv}{dt} + \frac{du}{dz}.\frac{dz}{dt}$$

$$= \frac{du}{dv}.c + \frac{du}{dz}(-c)$$

$$= \frac{cdu}{dv} - \frac{cdu}{dz}$$

$$\frac{d^2u}{dt^2} = c\left[\frac{du}{dv}\left(\frac{du}{dv} - \frac{du}{dz}\right)\frac{dv}{dt} + \frac{du}{dz}\left(\frac{du}{dv} - \frac{du}{dz}\right)\frac{dz}{dt}\ \right]$$

$$c^2\left[\frac{d^2u}{dv^2} - \frac{d^2u}{dv^2z} - \frac{d^2u}{dzdv} + \frac{d^2u}{dz^2}\right]$$

$$= C^2\left[\frac{d^2u}{dv^2} - \frac{2d^2u}{dvdz} + \frac{d^2u}{dz^2}\right]$$

Inserting these value in eqn (1)

We get,

$$C^2\left[\frac{d^2u}{dv^2} - \frac{2d^2u}{dvdz} + \frac{d^2u}{dz^2}\right] = C^2\left(\frac{d^2u}{dv^2} + \frac{2d^2u}{dvdz} + \frac{d^2u}{dz^2}\right)$$

$$\frac{4d^2u}{dvdz} = 0$$

$$or; \frac{d^2u}{dvdz} = 0 \dots\dots..(2)$$

To get solution integrating partially w.r. to z .

$$\frac{du}{dv} = \gamma(v)$$

Integrating w.r to v

$$u = \int \gamma(v)dv + \varphi(2)$$

$$u(x,t) = \emptyset(v) + \ \varphi(2)$$

$= \emptyset(\boldsymbol{x} + \boldsymbol{ct})$
$+ \boldsymbol{\varphi}(\boldsymbol{x}$
$- \boldsymbol{ct}) \ldots \ldots (\mathbf{3})\boldsymbol{is\ known\ as\ D'\ Alembart'\ ssoltion\ of\ wave\ equation}.$

D'Alembert's Solution satisfies initial condition

$u(x, a) = f(x) \ldots \ldots \ldots \ldots \ldots (4)$

$u_t(x, c) = g(x) \ldots \ldots \ldots \ldots . . (5)$

Differentiating equation (iii) with respect to t

$u_t(x, t) = c\emptyset'(x + ct) - c\varphi(x - ct) - - - - - (6)$

$Now,$
$u(x, 0) = \emptyset(x) + \varphi(x) = f(x) \ldots . . (7)$
$u_t(x, 0) = c\emptyset'^{(x)} - c\varphi'(x) = g(x) \ldots \ldots \ldots \ldots . (8)$

Equation (8) can be written as

$$\emptyset(x) - \varphi(x) = \frac{1}{c}\int_{x_0}^{x} g(s)ds + k(x_0) \ldots \ldots \ldots \ldots (9)$$

Integrating (9)
$\emptyset(x) - \varphi(x) = \frac{1}{c}\int_{x_0}^{x} g(s)ds + k(x_0)$ where $k(x_0) = \emptyset(x_0) - \varphi(x_0) \ldots \ldots \ldots (10)$

Adding (7)and (10), we get

$$2\emptyset(x) = f(x) + \frac{1}{c}\int_{x_0}^{x} g(s)ds + k(x_0)$$

$$\emptyset(x) = \frac{1}{2}f(x) + \frac{1}{2c}\int_{x_0}^{x} g(s)ds + \frac{1}{2}k(x_0) \ldots \ldots \ldots \ldots . . (11)$$

Substituting (7) and (11)

$$\varphi(x) = \frac{1}{2}f(x) - \frac{1}{2c}\int_{x_0}^{x} g(s)ds - \frac{1}{2}k(x_0) \ldots \ldots \ldots \ldots \ldots \ldots . . (12)$$

Replacing x by(x+ct) in (ii) and x by (x-ct) in equation (12)

$$\emptyset(x+ct)=\frac{1}{2}f(x+ct)+\frac{1}{2c}\int_{x_0}^{x+ct} g(s)ds-\frac{1}{2}(kx_0)$$

$$\varphi(x-ct)=\frac{1}{2}f(x-ct)-\frac{1}{2c}\int_{x_0}^{x-ct} g(s)ds-\frac{1}{2}k(x_0)$$

Placing the values in wave equation

$$u(x,t)=\emptyset(x+ct)+\varphi(x-ct)$$

$$\frac{1}{2}f(x+ct)+\frac{1}{2}c\int_{x_0}^{x+ct} g(s)ds+\frac{1}{2}k(x_0)+\frac{1}{2}f(x-ct)$$

$$-\frac{1}{2c}\int_{x_o}^{x-ct} g(s)ds-\left(\frac{1}{2}\right)k(x_0)$$

$$=\frac{1}{2}f(x+ct)+\frac{1}{2}f(x-ct)+\frac{1}{2}c\int_{x_0}^{x+ct} g(s)ds-\frac{1}{2}c\int_{x_0}^{x-ct} g(s)ds$$

$$\left[\because \int_b^a f(x)dx=-\int_a^b f(x)dx\right]$$

$$\left[\because \int_a^b f(x)dx+\int_b^c f(x)dx=\int_a^c f(x)dx\right]$$

$$=\frac{1}{2}f(x+ct)+\frac{1}{2}f(x-ct)+\frac{1}{2}c\int_{x_0}^{x-ct} g(s)+\frac{1}{2}\int_{x-ct}^{x_0} g(s)ds$$

$$=\frac{1}{2}f(x+ct)+\frac{1}{2}f(x-ct)+\frac{1}{2}\int_{x-ct}^{x_0} g(s)ds+\frac{1}{2}c\int_{x_o}^{x-ct} g(s)$$

$$u(x.t)=\frac{1}{2}f(x+ct)+\frac{1}{2}f(x-ct)+\frac{1}{2c}\int_{x-ct}^{x+ct} g(s)ds$$

When initial velocity is zero. The above solution will reduce to

$$\boldsymbol{u(x,t)=\frac{1}{2}f(x+ct)+\frac{1}{2}f(x-ct)}$$