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(1) ~~If~~ f has local maximum at (a,b) if $f_{xx} < 0$ and D > 0 and max. value = f(a,b)

(2) f has local minimum at (a,b) if $f_{xx} > 0$ & D > 0 & min. value = f(a,b)

(3) f has saddle point at (a,b) if D < 0

(4) The test is inclusive at (a,b) if D = 0

1. Find the local maxima, local minima & saddle point if (possible) of the functions.

(i) $x^2 + xy + y^2 + 3x - 3y + 4$

Sol^n: Here,

$$f = x^2 + xy + y^2 + 3x - 3y + 4$$
$$f_x = 2x + y + 3$$
$$f_y = x + 2y - 3$$

$f_{xx} = 2$, $f_{xy} = 1$

$f_{yy} = 2$, $f_{yx} = 1$

For critical point

$f(x)$ & $f_y = 0$,

$2x + y + 3 = 0$ — (i) & $x + 2y - 3 = 0$ — (ii)

Subtracting 2× (ii) from (i).

$$\begin{array}{r} 2x + y + 3 = 0 \\ 2x + 4y - 6 = 0 \\ - \quad - \quad + \\ \hline -3y + 9 = 0 \end{array}$$

$y = 3$

when $y = 3$ then, $x = -3$,

critical point is $(-3, 3)$

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At $(1,1)$

$f(xx) = 2$ $\quad$ $f_{xy} = 1$

$f_{yy} = 2$ $\quad$ $f_{yx} = 1$

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$= 2.2 - 1^2$

$= 4 - 1$

$= 3 > 0$

& $f_{xx} = 2 > 0$

$\therefore f(x,y)$ has minimum value at $(-3,3)$

& min value $= f(3,3) = $

$9 - 9 + 9 - 9 - 9 + 4$

-5 Ans

(ii) $x^2 - 4xy + y^2 + 6y + 2$

Solⁿ. Here,

let,

$f(x,y) = x^2 - 4xy + y^2 + 6y + 2$

$f_x = 2x - 4y$

$f_y = -4x + 2y + 6$

$f_{xx} = 2$ $\quad$ $f_{xy} = -4$

$f_{yy} = 2$ $\quad$ $f_{yx} = -4$

For critical point,

$f(x) = 0$ & $f_y = 0$

$2x - 4y = 0$ $\quad$ $-4x + 2y + 6 = 0$

$x - 2y = 0$ $\quad$ $-2x + y + 3 = 0$

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Adding eqⁿ (i) & (ii) we get

$-4x + 2y + 6 = 0 \quad \times 2$

$+x - 2y = 0$

$-3x + 6 = 0$

$-x = -6/3$

$x = 2$

when $x = 2$ then $y = 1$

$\therefore$ The critical point is $(2,1)$.

At $(3,3)$

$f_{xx} = 2$,

$f_{yy} = 2$

$f_{xy} = -4$

$D = f_{xx} \cdot f_{yy} - [f(xy)]^2$

$D = 2.2 - (-4)^2$

$D = 4 - 16$

$D = -8 < 0$

Now, $f_{xx} = 2 > 0$,

The function $f(x,y)$ is minimum at point $(3,3)$ and min. value is -8

The saddle point is $(2,1)$

(iii) $2x^2+3xy+4y^2-5x+2y$

Sol^n: Here,

let, $f(x,y) = 2x^2+3xy+4y^2-5x+2y$

$f_x = 4x+3y-5$

$f_y = 3x+8y+2$

$f_{xx} = 4$ $\qquad$ $f_{xy} = 3$

$f_{yy} = 8$ $\qquad$ $f_{yx} = 3$

For critical point,

$f_x = 0$ & $f_y = 0$

$4x+3y-5=0$ and $3x+8y+2=0$

$4x+3y-5=0$ $\quad \times 3$

$3x+8y+2=0$ $\quad \times 4$

$\cancel{12x}+9y-15=0$

$\underline{\cancel{12x}+32y+8=0}$

$-23y-23=0$

$y=-1$ &,

when $y=-1$ then $4x-3-5=0$

$4x=8$

$x=2$

The critical point is $(2,-1)$

Now,

At point $(2,-1)$

$f(xx) = 4$ $\qquad$ $f_{xy}=3$

$f_{yy}=8$

$D = f_{xx}\cdot f_{yy} - [f(xy)]^2$

$D = 4\cdot 8 - (3)^2$

$D = 32-9$

$D = 23>0$, & $f_{xx} = 4>0$

$\therefore$ the function $f(x,y)$ is max at point $(2,-1)$

& min. value is $= f(2,-1) = 2\times4 - 6+4-10-2$

$= 8-6+4-10-2$

$= 12-18$

$= -8$ Ans

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(iv) $5xy+3x-6y+2-7x$

Sol^n:

$f(x,y) = 5xy+3x-6y+2-7x$

$f_x = 5y+3-7$

$f_y = 5x-6$

$f_{xx} = 0$ $\quad$ $f_{xy} = 5$

$f_{yy} = 0$ $\quad$ $f_{yx} = 5$

For critical point,

$5y+3-7=0$ $\quad$ & $5x-6=0$,

$5y-4=0$

$y = 4/5$ $\quad$ and $x = 6/5$

critical point is $(6/5, 4/5)$

and,

$f_{xx}=0$ $\quad$ $f_{xy}=5$

$f_{yy}=0$ $\quad$ $f_{yx}=5$,

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$D = 0 \cdot 0 - (5)^2$

$D = -25 < 0$,

So that the shaddle point is $(6/5, 4/5)$

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(v) $x^2+xy+3x+2y+5$

sol^n: here,

let $f(x,y) = x^2+xy+3x+2y+5$

$f_x = 2x+y+3$

$f_y = \cancel{y+2}\ x+2$

$f_{xx} = 2$ $\quad$ $f_{xy} = 1$

$f_{yy} = 0$ $\quad$ $f_{yx} = 1$

For critical point

$2x+y+3=0$ —(i) & $\quad$ $x+2=0$ —(ii)

solving eq^n (i) and (ii) we get,

$2x-2+y+3=0$

$\cancel{\Rightarrow}\ -4+y+3=0$

$y=1$

and when $y=1$ then $x=-2$

critical point is $(-2,1)$,

At $(-2,1)$

$f_{xx}=2$

$f_{xy}=1$

$f_{yy}=0$

$D = f_{xx} \cdot f_{yy} - [f(xy)]^2$

$D = 1 \cdot 0 - (2)^2$

$D = -4 < 0$,

The saddle point is $(-2,1)$.

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(vi) $6x^2-2x^3+3y^2+6xy$

Sol^n: Here,

$f(x,y) = 6x^2-2x^3+3y^2+6xy$

$f_x = 12x-6x^2+6y$

$f_y = 6y+6x$

$f_{xx} = 12-12x$ & $f_{xy} = 6$

$f_{yy} = 6$ & $f_{yx} = 6$

For critical point,

$12x-6x^2+6y = 0$ & $6y+6x=0$

Solving eq^n ① and ⑪ we get,

$12x-6x^2+6y = 0$

$+6x+6y = 0$

$6x-6x^2 = 0$

$6x(1-x)=0$

$x=0, x=1,$

If $x=0$ then $y=0$, i.e. $(0,0)$

If $x=1$ then $y=-1$

Now: critical points are $(1,-1)$ and $(0,0)$

At point $(0,0)$

$f_{xx} = 12$

$f_{xy} = 6$

$f_{yy} = 6,$

$D = f_{xx}\cdot f_{yy} - [f_{xy}]^2$

$= 12\cdot 6 - [6]^2$

$= 72-36$

$= 36>0$ where $f_{xx}>0$

The function (x,y) is min. at point $(0,0)$

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At point $(1,-1)$

$f_{xx} = 0$

$f_{xy} = 6$

$f_{yy} = 6$

$D = f_{xx}\cdot f_{yy} - (f_{xy})^2$

$D = 0\cdot 6 - (6)^2$

$D = -36<0$

The saddle point is $(1,-1)$

(vii) $e^{4y-x^2-y^2}$

Sol^n: Here,

$f_x = e^{4y-x^2-y^2}\cdot -2x = -2x\, e^{4y-x^2-y^2}$

$f(y) = e^{4y-x^2-y^2}\cdot(4-2y) = (4-2y)\,(e^{4y-x^2-y^2})$

$f_{xx} = e^{4y-x^2-y^2}\cdot -2 + (-2x)\cdot e^{4y-x^2-y^2}\cdot(-2x)$

$= -e^{4y-x^2-y^2} + 4x^2 e^{4y-x^2-y^2}$

$f_{xy} = e^{4y-x^2-y^2}\cdot 0 + (-2x)\, e^{4y-x^2-y^2}\cdot(4-2y)$

$f_{xy} = (4xy-8x)\, e^{4y-x^2-y^2}$

$f_{yy} = e^{4y-x^2-y^2}\cdot(-2) + (4-2y)\,(e^{4y-x^2-y^2})\cdot(4-2y)$

$f_{yy} = -2e^{4y-x^2-y^2} + (4-2y)^2\,(e^{4y-x^2-y^2})$

For critical point,

$f_x = 0$ & $f_y = 0$

$-2x\, e^{4y-x^2-y^2} = 0$ & $4-2y\,(e^{4y-x^2-y^2}) = 0$

$-2x=0$, $4-2y=0$ | $e^{4y-x^2-y^2} = e^0$

$x=0$ & $y=2$ | $4y-x^2-y^2=0$

The critical point is $(0,2)$

$f_{xx} = -e^{\ 1 6x^2}$

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At point (0,2) $= e^{8-0-4} = -e^4$ which is < 0

$f_{xx} = -e^{8-0-4} = -e^4$

$f_{yy} = 2e^4 + 0 = 2e^4$

$f_{xy} = 0$.

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$D = -e^4 \cdot 2e^4 - 0^2$

$D = e2e^8 > 0$

So that the function $f(x, y)$ is max. at point (0,2).

(viii) $y\sin x$

Sol^n: Here,

$f(x,y) = y\sin x$

$f_x = y \cdot \cos x$

$f_y = \sin x$

$f_{xx} = -y\sin x$ $\quad f_{xy} = \cos x$

$f_{xy} = \cos x$ $\quad f_{yy} = 0$, $f_{yx} = \cos x$

$f_{xx} = -y\sin$ $\quad f_{xy} = \cos x$

$f_{yy} = 0$ $\quad f_{yx} = \cos x$

For critical point,

$y\cos x = 0$ & $\sin x = 0$

$y = 0$ & $x = n\pi$

Critical point is $\cancel{(n\pi, 0)}$ $(n\pi, 0)$

$f_{xx} = 0$ & $f_{xy} = -1$

$f_{yy} = 0$ $\quad f_{yx} = -1$

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$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$D = 0 \cdot 0 - (-1)^2$

$D = -1 < 0$

The saddle point is $(n\pi, 0)$ Ans.

2. Find the absolute maxima & minima of the functions on the given domains.

(i) $f(x,y) = 4x - 5y + 1$ on the closed triangular region having vertices (0,0), (2,0) and (0,3)

Sol^n: Here,

$f_x = 4$

$f_y = -5$

$f_{xx} = 0$ & $f_{xy} = 0$

$f_{yy} = 0$ $\quad f_{yx} = 0$