Exercise:- 4.1

(5) $y = \frac{5}{2}x^{2/3} - x^{5/3}$

Sol^n: Here,

$y = \frac{5}{2}x^{2/3} - x^{5/3}$

(1) Domain :- For all value of x, y exist
$\therefore$ domain $= (-\infty, \infty)$

(11) Intercept :-
(1) put $x=0$, $y=0$ so, that the curve passes through the origin $(0,0)$

(11) put $y=0$

$\frac{5}{2}x^{2/3} - x^{5/3} = 0$

$\frac{5}{2}x^{2/3} - x^{2/3}.x^{1} = 0$

$x^{2/3}\left(\frac{5}{2} - x\right) = 0$

$x=0$ and $x = \frac{5}{2}$

so that the curve passes through the point $(\frac{5}{2}, 0)$

(3) Symmetry:
$f(-x) = \frac{5}{2}(-x)^{2/3} - (-x)^{5/3}$

$f(-x) = \frac{5}{2}x^{2/3} + x^{5/3}$

$\therefore$ No symmetry

(4) Asymptote
$\therefore$ No asymptote

(5) Increasing and decreasing

$$f'(x) = \frac{5}{2} \times \frac{2}{3} x^{2/3-1} - \frac{5}{3} x^{5/3-1}$$

$$f'(x) = \frac{5}{3} x^{-1/3} - \frac{5}{3} x^{2/3}$$

(i) setting $f'(x) = 0$

$$\frac{5}{3}(x^{-1/3} - x^{2/3}) = 0$$

$$x^{-1/3} = x^{2/3}$$

$$\frac{1}{x^{1/3}} = x^{2/3}$$

$$1 = x^{3/3}$$

$$x = 1$$

(ii) Setting $f'(x) = \infty$

$$\frac{5}{3}(x^{-1/3} - x^{2/3}) = \infty$$

$$\frac{5}{3}\left(\frac{1}{x^{1/3}} - x^{2/3}\right) = \infty$$

$$\frac{5}{3}\left(\frac{1-x}{x^{1/3}}\right) = \frac{1}{0}$$

$$x^{1/3} = 0$$

$$x = 0$$

Interval	sign	nature
$(-\infty, 0)$	'−'	decreasing
$(0, 1)$	'+'	increasing
$(1, \infty)$	'+'	in ~~decreasing~~

(6) Maxima and Minima

$f'(x)$ change the sign from '−ve' to '+ve'

∴ $f(x)$ is minimum at $x=0$

minimum value $f(0) = 0$

" point $= (0,0)$

$f'(x)$ change the sign from '+ve' to '−ve'

$f(x)$ is maximum at $x=1$

max. value $f(1) = \frac{3}{2}$

∴ $(1, 3/2)$

(7) Concavity

$$f''(x) = \frac{5}{3} \times -\frac{1}{3} x^{-1/3-1} - \frac{5}{3} \cdot \frac{2}{3} \times x^{2/3-1}$$

$$f''(x) = \frac{-5}{9} x^{-4/3} \, \frac{-10}{9} x^{-1/3}$$

$$f''(x) = \frac{-5}{9}\left(x^{-4/3} + 2x^{-1/3}\right)$$

Setting $f''(x) = 0$

$$-\frac{5}{9}\left(x^{-4/3} + 2x^{-1/3}\right) = 0$$

$$x^{-4/3} + 2x^{-1/3} = 0$$

$$x^{-1/3} \cdot x^{-1} + 2 \cdot x^{-1/3}) = 0$$

$$x^{-1/3}\left(\frac{1}{x} + 2\right) = 0$$

$$x = 0, \text{ and } \frac{1}{x} = -2$$

$$x = -\frac{1}{2} = (-0.5)$$

Setting $f''(x) = \infty$

$$\frac{-5}{9} x^{-1/3}\left(\frac{1}{x} + 2\right) = \infty$$

$$-\frac{5}{9} x^{-1/3}\left(\frac{1+2x}{x}\right) = \frac{1}{0}$$

$$9x = 0$$

$$x = 0$$

∴ point of inflection $(0,0)$ and $(-0.5, 1.25)$

(8) Graphical representation.

(6) $y = x^{5/3} - 5x^{2/3}$

Sol^n: Here,

$y = x^{5/3} - 5x^{2/3}$

(1) Domain: For all real value of x, y exist

$\therefore$ domain $= (-\infty, \infty)$

(2) Intercept:

(i) put $x = 0$ then $y = 0$, so that the curve passes through the origin $(0,0)$

(ii) put $y = 0$

$x^{5/3} - 5x^{2/3} = 0$

$x^{2/3} \cdot x - 5x^{2/3} = 0$

$x^{2/3}(x-5) = 0$

$x = 0, 5$

$\therefore$ the curve passes through $(5, 0)$

③ Symmetric :-

$$f(-x) = (-x)^{5/3} - 5(-x)^{2/3}$$
$$= -x^{5/3} + 5x^{2/3}$$
$$= -(x^{5/3} + 5x^{2/3})$$

$\therefore$ No symmetric

④ Asymptote

$\therefore$ No asymptote.

⑤ Increasing and decreasing.

$$f'(x) = 5/3\, x^{5/3 - 1} - 5 \times 2/3\, x^{2/3 - 1}$$

$$f'(x) = 5/3\, x^{2/3} - \frac{10}{3} x^{-1/3}$$

Setting $f'(x) = 0$

$$5/3\, x^{2/3} - \frac{10}{3} x^{-1/3} = 0$$

$$5/3\, x^{1} \cdot x^{-1/3} - 10/3\, x^{-1/3} = 0$$

$$5/3\, x^{-1/3} (x - 2) = 0$$

$$x = 0, \text{ and } 2$$

Setting $f'(x) = \infty$

$$\frac{5}{3x^{1/3}} (x-2) = \frac{1}{0}$$

$$3x^{1/3} = 0$$
$$x^{1/3} = 0$$
$$x = 0$$

Interval	Sign	Nature
$(-\infty, 0)$	'+'	increasing
$(0, 2)$	'−'	decreasing
$(2, \infty)$	'+'	increasing

(6) Maxima and minima:-

The $f'(x)$ changes the sign from '+ve' to '-ve'

i.e. $f(x)$ is maximum at $x = 0$

max. value $f(0) = 0$

max. point $= (0, 0)$

The $f'(x)$ changes the sign from '-ve' to '+ve' i.e. $f(x)$ is minimum at $x = 2$

min val. $f(2) = (1.01)$

min. point $= (2, 1.01)$

(7) Concavity

$$f''(x) = \frac{5}{3} \times \frac{2}{3} \cdot x^{2/3 - 1} - \frac{10}{3} \times \frac{-1}{3} x^{-1/3 - 1}$$

$$f''(x) = \frac{10}{9} x^{-1/3} + \frac{10}{9} x^{-1/3} \cdot x^{-1}$$

$$f''(x) = \frac{10}{9} x^{-1/3} (1 + x^{-1})$$

$\therefore$ setting $f''(x) = 0$

$$\frac{10}{9} x^{-1/3} (1 + x^{-1}) = 0$$

$$(1 + x^{-1}) = 0$$

$$1 + \frac{1}{x} = 0$$

$$x + 1 = 0$$

$$x = -1$$

$f(-1) = -6$

$\therefore$ Point of inf. $(-1, -6)$

setting $f''(x) = \infty$ | point of inflection

$\frac{10}{9} x^{-1/3} \left(\frac{x+1}{x} \right) = \frac{1}{0}$ | $(0,0)$

$9x = 0$

$x = 0$

Interval	sign	nature
$(-\infty,-1)$	+	upword
$(-1,0)$	−	downward
$(0,\infty)$	−	downward

(7) $y = \frac{x}{x-1}$

Sol^n: here

$y = \frac{x}{x-1}$

(i) Domain:- For all value of x except 0, Y ~~det~~ exist

domain :- $(-\infty, 1) \cup (1, \infty)$

(ii) Intercept:

(i) put $x = 0$, $y = 0$ so that the curve passes through the point origin $(0,0)$

(ii) put $y = 0$

$\frac{x}{x-1} = 0$

$x = 0$ the curve passes through the origin.

(iii) symmetry

$f(-x) = \frac{-x}{-x-1} = -\left(\frac{x}{x+1}\right)$

No symmetry

(iv) asymptote

$\lim_{x \to \infty} \frac{x}{x-1} = \frac{1}{1-1/x} = \frac{1}{1-1/\infty} = 1$

~~$\lim_{x\to\infty}$~~ there is ~~no~~ a ~~any~~ horizontal asymptote at $x=1$

$\lim_{x \to 1^+} \frac{x}{x-1} = \frac{1}{1-1/x} = \frac{1}{0} = \infty$

There is existence of vertical asymptote at $(1,0)$

(v) Increasing and decreasing.

$$f'(x) = \frac{(x-1)\cdot 1 - x\cdot(1-0)}{(x-1)^2}$$

$$f'(x) = \frac{(x-1)-x}{(x-1)^2} = \frac{x-1-x}{(x-1)^2} = \frac{-1}{(x-1)^2}$$

Setting $f'(x)=0$

$$\frac{-1}{(x-1)^2} = 0$$

(ii) Setting $f'(x) = \infty$

$$\frac{-1}{(x-1)^2} = \frac{1}{0}$$

$$0 = (x-1)^2$$

$$x-1 = 0$$

$$x = 1$$

Interval	sign	nature
$(-\infty,1)$	~~+~~ $-$	~~in~~decreasing
$(1,\infty)$	~~+~~ $-$	~~in~~decreasing

(vi) Maxima and Minima

There donot exist any maxima or minima point as shown in above table

(vii) concavity

$$f''(x) = -1\times -2\,(x-1)^{-3}\cdot 1$$

$$= 2(x-1)^{3}$$

$$= \cancel{2(1)}\ \frac{2}{(x-1)^3}$$

Setting $f''(x) = 0$

$$2/(x-1)^3 = 0$$

Setting $f''(x) = \infty$

$$\frac{2}{(x-1)^3} = \frac{1}{0}$$

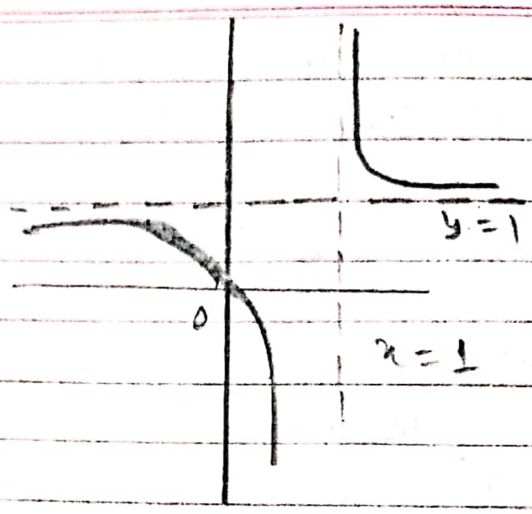
$$(x-1)^3 = 0$$

$x = 1$ point of inf.

$$f(1) = \infty$$

Date: $(-\infty, \frac{1}{1})$
Page: 109

$(-\infty, 1)$
decrea



(8) $y = \frac{x^2}{x^2+9}$

Sol^n: Here

$y = \frac{x^2}{x^2+9}$

(i) Domain:- ~~put~~ for all value of x ~~except ±3~~ y exist so,

domain = ~~$(-\infty,-3)\cup(-3,3)\cup(3,\infty)$~~ $(-\infty,\infty)$

(ii) Intercept :-

(i) put $x=0$, $y = 0/9 = 0$

put $y=0$, $x=0$ it means the curve passes through the origin,

(iii) Symmetry

$f(-x) = \frac{x^2}{x^2+9}$ = $f(x)$ even function

(iv) Asymptote

(i) $\lim_{x\to\infty}$ then $f(x)$ ~~desired value is ∞~~

~~so No horizontal asymptote exist~~

(ii) $\frac{x^2}{x^2+9}$

$= \frac{1}{1+9/x^2} = \frac{1}{1+9/\infty} = 1$,

then there is a horizontal asymptote at $x = 1$ i.e. $(0,1)$

(iii) No vertical asymptote

Increasing and decreasing

$$f'(x) = \frac{(x^2+9)\times 2x - x^2(2x+0)}{(x^2+9)^2}$$

$$f'(x) = \frac{2x(x^2+9) - 2x^3}{(x^2+9)^2}$$

$$f'(x) = \frac{\cancel{2x^3} + 18x - \cancel{2x^3}}{(x^2+9)^2}$$

$$f'(x) = \frac{18x}{(x^2+9)^2}$$

setting $f'(x) = 0$

$$\frac{18x}{(x^2+9)^2} = 0$$

$$x = 0$$

setting $f'(x) = \infty$

$$\frac{18x}{(x^2+9)^2} = \frac{1}{0}$$

$$(x^2+9)^2 = 0$$

$$x^2 = -9$$

$$x = \sqrt{-9} = \text{Img}$$

Interval	sign	nature
$(-\infty, 0)$	$(-)$	decreasing
$(0, \infty)$	$(+)$	increasing

Maxima and Minima

$f'(x)$ change the sign from -ve' to +ve'

$f(x)$ is minimum at $x = 0$

minimum value $f(0) = 0$

" point $= (0, 0)$

poin concavity

$$f''(x) = \frac{(x^2+9)^2 \times 18 - 18x(x^2+9).2 \quad 2x}{(x^2+9)^4}$$

$$f''(x) = \frac{18(x^2+9)^2 - 18 \times 4x^2 (x^2+9)}{(x^2+9)^4}$$

$$f''(x) = \frac{18(x^2+9)(x^2+9-4x^2)}{(x^2+9)^4}$$

$$f''(x) = \frac{18(x^2+9)(9-3x^2)}{(x^2+9)^4}$$

setting $f''(x) = 0$

$$\frac{18(x^2+9)(9-3x^2)}{(x^2+9)^4} = 0$$

$18(x^2+9)(9-3x^2) = 0$

$\cancel{18}(x^2+9)(9-3x^2) = 0$

$x^2+9 \neq 0$ | $9-3x^2 = 0$ $\quad x_1 = -1.73$

$x^2 = -9$ | $0 - 3x^2 = -9$ $\quad x = 1.73$

$3x^2 = 9$

$x^2 = 3$

$x = \sqrt{3}$

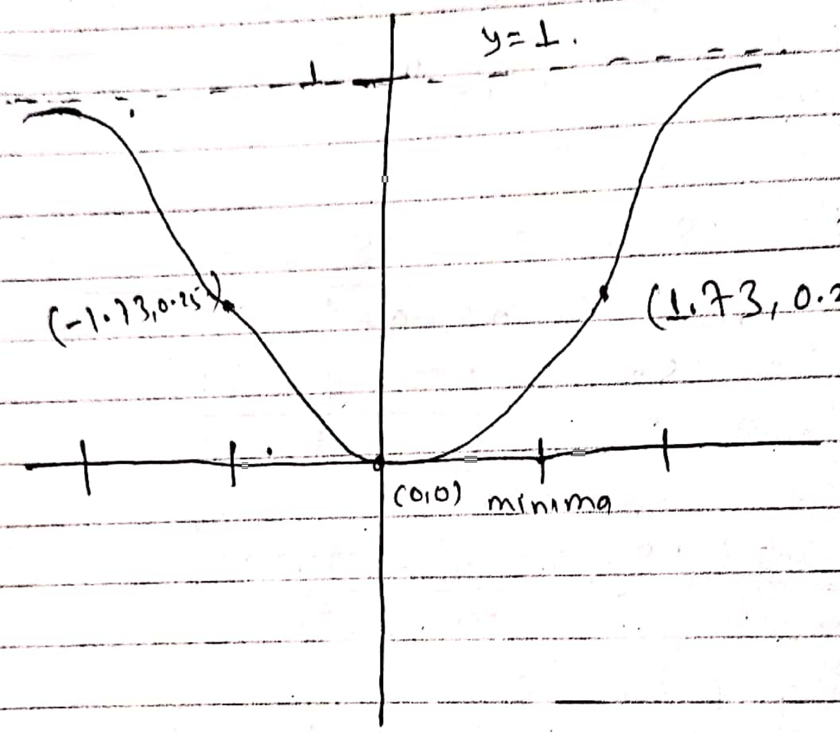
Interval	Sign	Nature
$(-\infty, -1.73)$	$-$	downward
~~$(-3, 3)$~~ $(-1.73, 1.73)$	$+$	upward
~~$(3, \infty)$~~ $(1.73, \infty)$	$-$	downward

point of inflection

$f(1.73) = \frac{9}{9+9} = \frac{9}{18} = 0.25 = $ ~~$(-3, 0.5)$~~ $(1.73, 0.25)$

$f(1.73) = \frac{9}{18} = .025 = $ ~~$(-3, 0.5)$~~ $(-1.73, 0.25)$

Graph Representation



Increasing and decreasing

$$f'(x) = \frac{(x^2+3)\times 2x - x^2(2x+0)}{(x^2+3)^2}$$

$$f'(x) = \frac{2x^3+6x - 2x^3}{(x^2+3)^2}$$

$$f'(x) = \frac{6x}{(x^2+3)^2}$$

Setting $f'(x) = 0$

$$\frac{6x}{(x^2+3)^2} = 0$$

$$6x = 0$$

$$x = 0$$

Setting $f'(x) = \infty$

$$\frac{6x}{(x^2+3)^2} = \frac{1}{0}$$

$$(x^2+3)^2 = 0$$

$$x^2+3 = 0$$

$$x = \sqrt{-3}$$ which is imaginary

Interval	Sign	nature
$(-\infty, 0)$	$(-)$	decreasing
$(0, \infty)$	$+$	increasing

Maxima and minima

$f'(x)$ change the sign from -ve to +ve the $f(x)$ minimum at $x = 0$

min. value $f(0) = 0$

" point $= (0, 0)$

P. concavity

$$f''(x) = \frac{(x^2+3)^2 \times 6 - 6x \cdot 2(x^2+3) \cdot 2x}{(x^2+3)^4}$$

$$f''(x) = \frac{6(x^2+3)^2 - 6 \times 4x^2(x^2+3)}{(x^2+3)^4}$$

$$f''(x) = \frac{6(x^2+3)(x^2+3-4x^2)}{(x^2+3)^4}$$

$$f''(x) = \frac{6(x^2+3)(3-3x^2)}{(x^2+3)^4}$$

$$f''(x) = \frac{18(x^2+3)(1-x^2)}{(x^2+3)^4}$$

setting $f''(x) = 0$

$$(x^2+3)(1-x^2) = 0$$

$$x^2 - x^4 + 3 - 3x^2 = 0$$

$$-x^4 - 2x^2 + 3 = 0$$

$$x = -1, 1$$

Setting $f''(x) = \infty$

$$\frac{18(x^2+3)(1-x^2)}{(x^2+3)^4} = \frac{1}{0}$$

$$(x^2+3)^4 = 0$$

$$x^2+3 = 0$$

$x = \sqrt{-3}$ which is imaginary

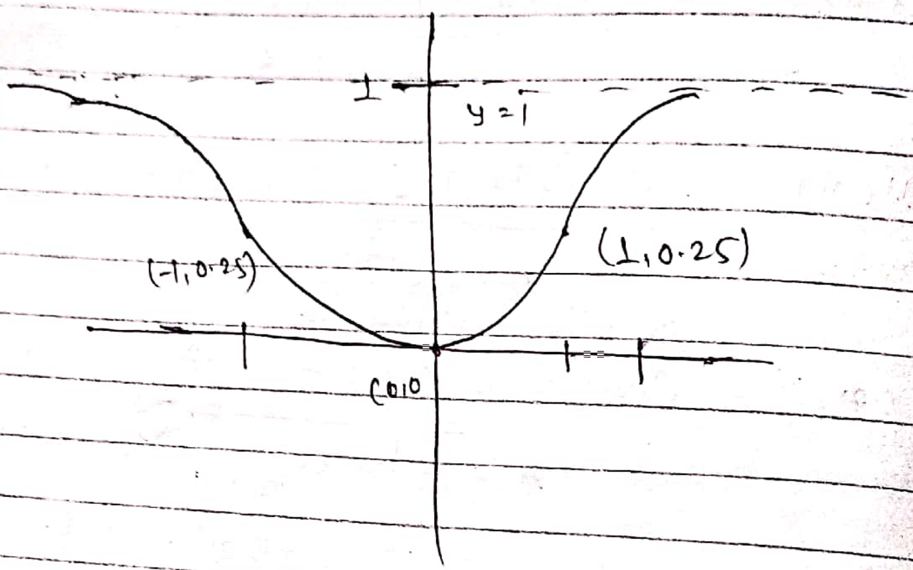
~~Poin~~ point of inflection

$f(1) = \frac{1}{4} = 0.25 = (1, 0.25)$

$f(-1) \quad \frac{1}{4} = 0.25 = (-1, 0.25)$

Interval	Sign	nature	
$(-\infty,-1)$	$-$	~~decreasing~~	downward
$(-1,1)$	$+$	increasing	upward
$(1,\infty)$	$-$	decreasing	downward

Graph Representation



(11) $y = \sqrt{x^2+x-2}$

Sol^n : Here

$y = \sqrt{x^2+x-2}$

put ~~y = 0 x =~~

(i) domain :- $x^2+x-2 \geq 0$

$x^2+(2-1)x-2 \geq 0$

$x^2+2x-x-2 > 0$

$x(x+2)-1(x+2) > 0$

$(x-1)(x+2) \geq 0$

$x \geq 1, -2$

$-2 \leq x < 1$

domain $= (\cancel{-\infty, \infty})$ $(-\infty, -2] \cup \cancel{(-2, \infty}$ $\cancel{(-2, 1)}$ $[1, \infty)$

(ii) Intercept put $x = 0$

$y = \sqrt{-2}$ which is imaginary

put $y = 0$

$x^2+x-2 = 0$

$x = -2, 1$

$\therefore$ $(-2, 0)$ and $(1, 0)$

(iii) symmetry :-

$f(-x) = \sqrt{x^2-x-2}$

No symmetry

(iv) Asymptote :- No asymptote

(5) Increasing and decreasing

(v) $f'(x) = \dfrac{1}{2\sqrt{x^2+x-2}} \times (2x+1)$

$f'(x) = \dfrac{2x+1}{2\sqrt{x^2+x-2}}$

$\frac{-0.5}{2\sqrt{9}}$

Setting $f'(x) = 0$

$$\frac{2x+1}{2\sqrt{x^2+x-2}} = 0$$

$$2x+1 = 0$$
$$2x = -1$$
$$x = -\frac{1}{2} = -0.5$$

Interval	Sign	nature
$(-\infty, -2)$	'+'	~~Imaginary~~ increasing
$(\cancel{\tfrac{1}{2}}, \infty)$ $[1, \infty)$	'+'	increasing

Maxima and minima
(i) No Maxima
(ii) No- minima

concavity

$$f''(x) = \frac{2\sqrt{x^2+x-2}\cdot 2 - \left[(2x+1)\frac{2(2x+1)}{2\sqrt{x^2+x-2}}\right]}{4(x^2+x-2)}$$

~~$f''(x)$~~

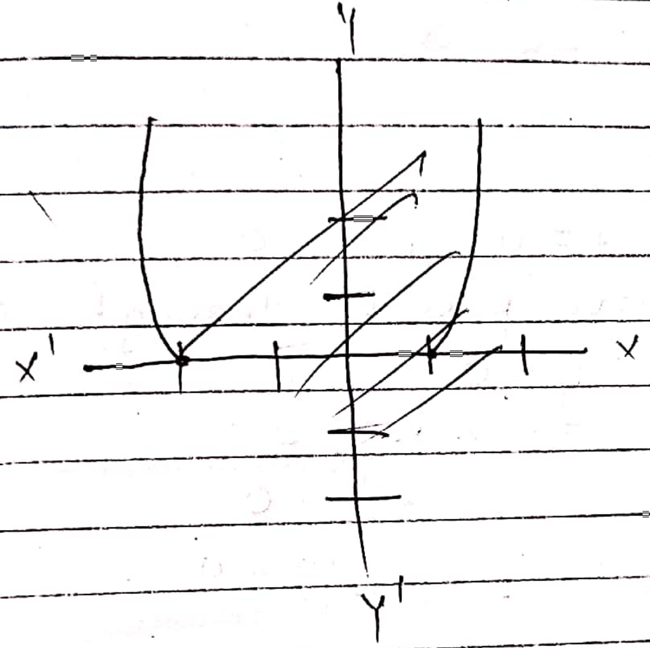
$$f''(x) = \frac{8\cdot(\sqrt{x^2+x-2})^2 - [2(2x+1)^2]}{8(\sqrt{x^2+x-2})(x^2+x-2)}$$

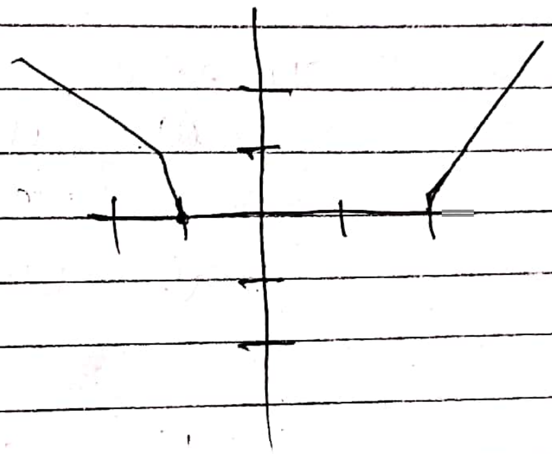
$$= \frac{8(x^2+x-2) - 2(4x^2+4x+1)}{8(\sqrt{x^2+x-2})(x^2+x-2)}$$

$$= \frac{\cancel{8x^2+8}\ \cancel{4x^2}+\cancel{4x}-8-\cancel{4x^2}-\cancel{4x}-1}{4(\sqrt{x^2+x-2})(x^2+x-2)}$$

$$\frac{-9}{4(\sqrt{x^2+x-2})(x^2+x-2)} = 0$$

ϕ setting g $f''(x) = 0$
No point of inflection





(12) $y = xe^x$

Sol^n: here

$y = x.e^x$

(I) Domain : for all value of x y exist

domain: $(-\infty, \infty)$

(II) Intercept

(i) put $x = 0$, $y = 0$

curve passes through $(0,0)$

(ii) put $y = 0$

$x.e^x = 0$

$e^x = 0$

$x = 0$

(III) Symmetric

$f(-x) = -x e^{-x} =$ No symmetry

(iv) Asymptote

(i) $\lim_{x \to \infty} xe^x = \infty e^\infty = \infty$

No Horizontal asymptote.

$\lim_{x \to 0^+} \frac{e^x}{x^{-1}}$ (ii) $\lim_{x \to 1}$ (+)

$= \frac{e^x}{1/x} = \frac{e^0}{1/0} = \frac{e^0}{\infty} = 0$, ~~No~~ vertical asymptote at $x=0$

(v) Increasing and decreasing

$f'(x) = e^x.1 + x.e^x$

$= e^x + x.e^x$

$= e^x(1+x)$

setting

$f'(x) = 0$

$e^x \cdot (1+x) = 0$

$x = -1, 0$

Interval	Sign	Nature
$(-\infty,-1)$	-ve	~~increasing~~ decreasing
$(-1,0)$	+ve	increasing
$(0,\infty)$	+ve	increasing

(vi) Maxima and Minima

No maxima

~~No minima~~ $f'(x)$ change the sign from '-ve' to +ve $f(x)$ is minimum at $x=-1$

min. val $f(-1) = -0.37$

point $= (-1, -0.37)$

(vii) concavity

$f''(x) = (1+x).e^x + e^x(1)$

$= e^x (1+x+1)$

$= e^x (x+2)$

setting $f''(x) = 0$

$e^x (x+2) = 0$

$x = 0$

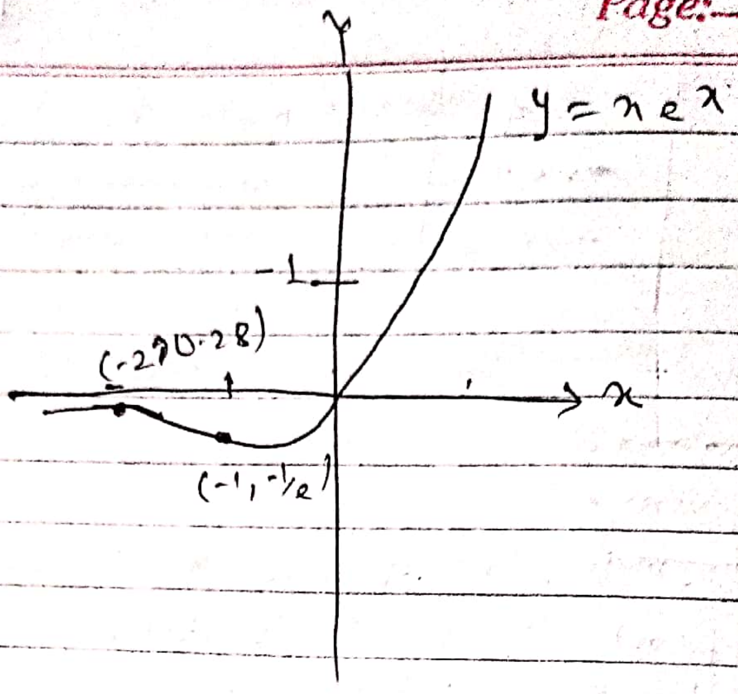
$x = -2$

Interval	Sign	Nature
$(-\infty,-2)$	$-$	downward
$(-2,0)$	$-$	downward
$(0,\infty)$	'+'	upward

point of inflection at $f(0) = 0 = (0,0)$

$f(-2) = -2 \times e^{-2} = -0.28$

$(-2, -0.28)$


y
y = xe^x
1
(-2, -0.28)
x
(-1, -1/e)

Exercise: 4.2

① Find two numbers whose difference is 100 and whose product is a minimum.

Soln: here

let x and y be any two numbers

given

$$x - y = 100$$

$$x = 100 + y \quad \text{--- ①}$$

product of two numbers is

$$p = xy$$

$$p = (100 + y)\,y$$

$$p = 100y + y^2$$

$$p' = 100 + 2y$$

$$p'' = 2$$

for critical point

$$p' = 0$$

$$100 + 2y = 0$$

$$2y = -100$$

$$y = -50$$

put $y = -50$ in eqn ①

$$x = 100 - 50$$

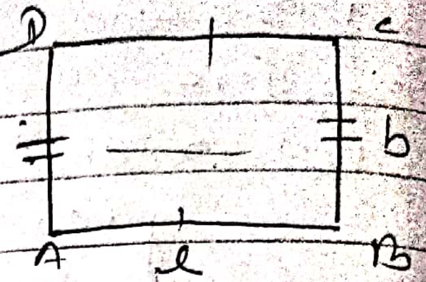
$$x = 50$$

At $y = -50$

$$p'' = 2 > 0$$

The product is minimum when $x = 50$, $y = -50$

(2) Find the dimension of a rectangle with perimeter 100m whose area is as large as possible



Sol^n: here.
let l be the length and b be the breadth of the rectangle ABCD

perimeter of rectange = 100m

$$2(l+b) = 100m$$
$$l+b = 50$$
$$l = 50-b$$

$$\text{Area} = l \times b$$
$$A = (50-b)\times b$$
$$A = 50b - b^2$$
$$A' = 50-2b$$
$$A'' = -2$$

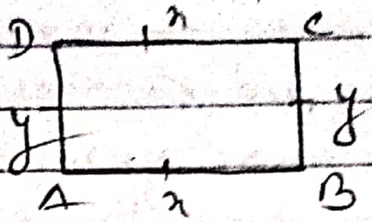
For critical point
put $A' = 0$
$$50-2b = 0$$
$$2b = 50$$
$$b = 25$$

$$l = 50-25 = 25$$

At $b = 25$
$A'' = -2 < 0$

the area is a large/maximum when $l = 25$ and $b = 25$m.

(3) Show that ~~of~~ all the rectangles with given area, the one with smallest perimeter is a square.



Sol^n:- here
let x, y be the length and breadth of a rectangle ABCD.
Area of rectangle

$$x \times y = A \text{ (where A is constant)}$$

$$x \times y = A$$

$$y = A/x$$

perimeter of a rectangle

$$p = 2(x+y)$$

$$p = 2x + 2y$$

$$p = 2x + 2 \times A/x$$

$$p' = 2 - \frac{2A}{x^2} \qquad p'' = \frac{4A}{x^3}$$

For a critical point

$$p' = 0$$

$$2 - \frac{2A}{x^2} = 0$$

$$2 = \frac{2A}{x^2}$$

$$2x^2 = 2A$$

$$x^2 = A$$

$$x = \sqrt{A}.$$

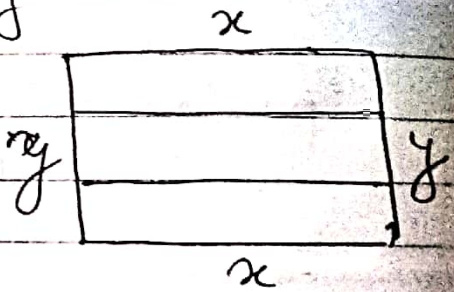
put $x = \sqrt{A}$ in eq^n (1)

$$y = \frac{A}{\sqrt{A}} = \sqrt{A}$$

$$p''(x) = \frac{4A}{x^3}$$ If $p > 0$ therefor, $\frac{4A}{\sqrt{A}}$ gives p.m.

(4) The highway department is planning to built a picnic park of motorist along a major highway. the park is to be rectangular with an area 5000 Square yards and is to be fenced off on the three sides not adjascent to the highway. what is the least amount of fencing required for this job? How long and wide should the park be for the fencing to be maximized?

Sol: here.



Area of ~~square~~ rectangle

$$x \times y = 5000$$

$$y = \frac{5000}{x} \quad -(1)$$

perimeter of of fencing on the three sides are,

$$p = 4x + 2y$$

$$p = 4x + 2 \times \frac{5000}{x}$$

$$p = 4x + 10{,}000\, x^{-1}$$

$$p' = 4 - \frac{10{,}000}{x^2}$$

$$p'' = 0 + \frac{2 \times 10{,}000}{x^3}$$

$$p'' = \frac{20{,}000}{x^{-3}}$$

For critical point

$p' = 0$

$$4 - \frac{10,000}{x^2} = 0$$

$$\frac{10,000}{x^2} = 4$$

$$\frac{x^2}{10,000} = \frac{1}{4}$$

$$x^2 = 2500$$

$$x = 50$$

then

$$y = \frac{5000}{50} = 100$$

$$y = 100$$

$$p'' = \frac{20000}{x^3} \quad \text{where } p'' > 0$$

the At $x = 50$,

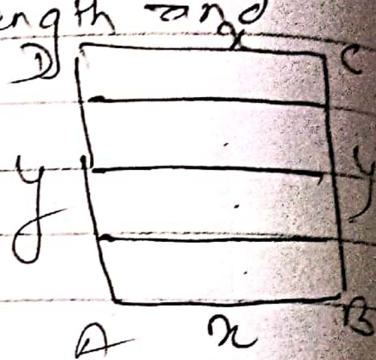
$$p'' = \frac{20000}{(50)^3} > 0$$

The perimeter is minimum when length $l = 50$ and $b = 100$

5 A farmer with 750 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of rectangle. What is the largest possible total area of the four pens?

Sol^n: here

let x & y be the length and breadth of rectangle ABCD



~~Area of~~

perimeter of rectangle when it is divided into four pan

$$750 = 2y + 5x$$

$$5x + 2y = 750$$

$$y = \frac{750 - 5x}{2} \quad - (1)$$

Area of rectangle

$$A = x \times y$$

$$A = x\left(\frac{750 - 5x}{2}\right)$$

$$A = \frac{750x - 5x^2}{2}$$

$$A' = \frac{750 - 10x}{2}$$

$$A'' = \frac{0 - 10}{2} = -5$$

point of inflection

$$A' = 0$$

$$\frac{750 - 10x}{2} = 0$$

$10x = 750$

$x = 75$

$y = \dfrac{750 - 375}{2}$

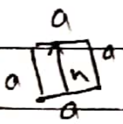
$y = 187.5$

then the area will be maximum when $x = 75$ and $y = 187.5$

Area $= 75 \times 187.5 = 14062.5 \text{ ft}^2$

(7) A box with a square base and open top must have a volm of 32000 cm3. Find the dimension of the box that minimize the amount of material?

Soln: here,



Volm of base = volm of open top.

$l \times b \times h$ = volm of open top (V)

$V_T = x \times x \times h = x^2 h$

$x^2 h = 32000 \text{ cm}^3$

then Area of ~~of~~ the box is

Area of base + Area of 4 wall's

$A = x^2 + 4hx = x^2 + 4hx$

$A = x^2 + 4hx, \quad = x^2 + 4hx$

the mimimizing the amount of material is depend upon the area of box so,

minimizing the area of a box

$A = \cancel{a^2 + 4ha} \quad x^2 + 4hx$

$A = x^2 + 4x \times \dfrac{32000}{x^2}$

$A = x^2 + 128000\, x^{-1}$

$$A' = 2x - 1 \cdot \frac{128000}{x^2}$$

point 0' for critical point,

$$A' = 0$$

$$2x - \frac{128000}{x^2} = 0$$

$$2x = \frac{128000}{x^2}$$

$$2x^3 = 128000$$

$$x = 40$$

$$h = \frac{32000}{1600}$$

$$h = 20$$

then Area $= x^2 + 4hx$

$= 40^2 + 4 \times 20 \times 40$

$= 1600 + 3200$

$= 4800$ Ans

(8) If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volm of the box.

Soln : here

Area of a box with open top $= x^2 + 4xh$

$$A = x^2 + 4xh \quad -(1)$$

$$1200 = x^2 + 4xh \quad , \quad h = \frac{1200 - x^2}{4x}$$

Volm of open top $= x^2 h$

then

$$V = \frac{(1200 - x^2)}{4x} \times x^2$$

$$V = \frac{(1200 - x^2)x}{4}$$

$$V = \frac{1200x - x^3}{4}$$

$$V' = \frac{1200 - 3x^2}{4}$$

From critical point

$$V' = 0$$

$$1200 - 3x^2 = 0$$

$$3x^2 = 1200$$

$$x^2 = 400$$

$$x^2 = 400$$

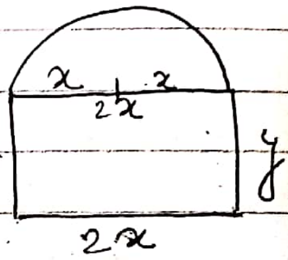
$$x = 20$$

$V' =$ ~~1200 - 3 × 400~~ $\quad \frac{1200 \times 20 - 20^3}{4}$

$V' = 4000 \, cm^3$ Ans

(6) A Normal window has the shape of a rectangle surmounted by a semicircle (Thus the diameter of the semicircle is equal to the width of the rectangle. If the perimeter of the window is 30ft find the dimension of a window so that the greatest possible amount of light is admitted.?

Sol^n: here



The whole perimeter of the window

$= 2x+2y+\pi x$

$2x+2y+\pi x = 30$

$2y = 30-2x-\pi x$

$y = \frac{30-2x-\pi x}{2}$ —(1)

the area of the whole window is

$A = 2x \times y + \frac{1}{2}\pi x^2$

$A = 2xy + \frac{1}{2}\pi x^2$

$A = \cancel{2}.x\frac{1}{\cancel{2}}(30-2x-\pi x) + \frac{1}{2}\pi x^2$

$A = (30x-2x^2-\pi x^2) + \frac{1}{2}\pi x^2$

$A' = (30-4x-2\pi x) + \frac{1}{\cancel{2}}\cancel{2}\pi x$

$A' = \frac{1}{2}(30-4x-\cancel{2}\pi x)+\pi x$

$a' = 0$

$30-4x-2\pi x+2\pi x$

$A' = 30 - 4x - 2\pi x + \pi x$

$A' = 30 - 4x - \pi x$

~~1st~~ For critical point

$A' = 0$

$30 - 4x - \pi x = 0$

$30 = 4x + \pi x$

$x = \frac{30}{(4+\pi)}$

$y = \frac{30 - 2\left(\frac{30}{4+\pi}\right) - \pi \times \frac{30}{(4+\pi)}}{2}$

$y = \cancel{\frac{30 - 47.12 - 13.197}{2}}$

$y = \frac{30 - 8.4 - 13.197}{2}$

$y = \frac{8.403}{2}$

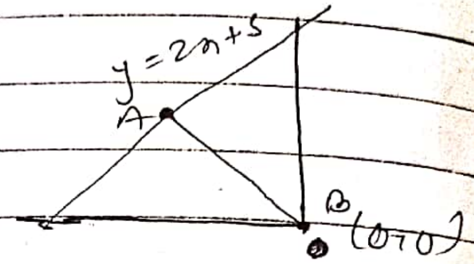
$y = 4.2$ ft

$y = 2\left(\frac{30}{4+\pi}\right)$

$y = 8.4$ ft. Ans

(11) Find the point on the line $y=2x+3$ that is closest to the origin.

Sol^n: here,



eq^n of the line

$$y=2x+3 \quad -(1)$$

distance between two point $A(x_1,y_1)$ and $B(x_2,y_2)$

$$d^2=(x_2-x_1)^2+(y_2-y_1)^2$$

$$d^2=(x-0)^2+(y-0)^2$$

$$d^2=x^2+y^2$$

$$d^2=x^2+(2x+3)^2$$

$$d^2=x^2+4x^2+12x+9$$

$$D=x^2+4x^2+12x+9$$

$$5x^2+12x+9$$

$$D'=\cancel{2x+8x+12} \quad D'=10x+12$$

$$\cancel{D'=4x+16x+24}$$

$$\cancel{D'=20x+124}$$

$$\cancel{\text{let } D'=0}$$

$$\cancel{20x+124=0}$$

$$\cancel{20x=-124}$$

$$\cancel{x=-\frac{124}{20}}$$

For critical point

$$D'=0$$

$$10x+12=0$$

$$10x=-12$$

$$x=-1.2$$

$$y=2\times-1.2+3$$

$$y=-2.4+3$$

$$y=0.6$$

the line is closest point $(-1.2, 0.6)$ to the origin at

(12) Find the point on the curve $y = \sqrt{x}$ that is ~~cot~~ closet to the ~~ori~~ point $(3,0)$

sol^n: here

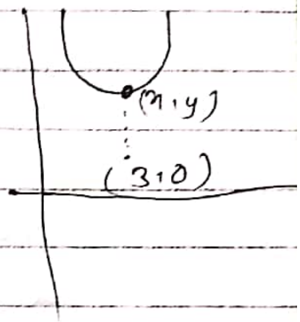
$$y = \sqrt{x} \quad -(1)$$

The distance betwn any two points is

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d^2 = (x - 3)^2 + (y - 0)^2$$

$$d^2 = x^2 - 6x + 9 + y^2$$



↓

$$D = x^2 - 6x + 9 + (\sqrt{x})^2$$

$$D = x^2 - 6x + 9 + x$$

$$D = x^2 - 5x + 9$$

$$D' = 2x - 5 \qquad \text{where } D'' = 2 > 0$$

which is mini.

For critical point

$$2x - 5 = 0$$

$$2x = 5$$

$$x = 2.5$$

$$y = \sqrt{2.5}$$

$$y = 1.6$$

the point is closet to the curve when it drawn on the curve at point $(2.5, 1.6)$

(13) Find the point on the ellipse $4x^2+y^2=4$ that are farthest away from the point (1,0)

Sol^n: here,

P (1,0)   S (x,y)

$$4x^2+y^2=4$$
$$y^2=4-4x^2$$
$$y=\sqrt{4-4x^2}$$
$$y=2\sqrt{1-x^2} \quad -(1)$$

The distance betw^n two point P and S are

$$d=\sqrt{(x-1)^2+(y-0)^2}$$
$$d=\sqrt{x^2-2x+1+y^2}$$

$$d^2=x^2-2x+1+y^2$$

$$d^2=x^2-2x+1+4(1-x^2)$$
$$d^2=x^2-2x+1+4-4x^2$$
$$d^2=-3x^2-2x+5$$
$$\downarrow$$
$$D=-3x^2-2x+5$$
$$D'=-6x-2$$
$$D''=-6<0 \quad \text{which is maximum}$$

For stationary point

$$D'=0$$
$$-6x-2=0$$
$$6x=-2$$
$$x=-2/6=-1/3$$

$$y=2\sqrt{1-1/9}$$
$$y=2\sqrt{8/9}$$
$$y=4/3\sqrt{2}$$

: (-1/3, t

(-1/3, t

(11) At which points on the curve $y = 1+40x^3-3x^5$ does the tangent line have the largest slope?

Sol^n: here,

The eq^n of the curve is

$$y = 1+40x^3-3x^5$$
$$y' = 0+120x^2-15x^4$$
$$y'' = 120\times 2x - 60x^3$$
$$y'' = 240x - 60x^3$$

~~set~~ setting $y''=0$

$$240x-60x^3=0$$
$$60x(\cancel{4x} \ 4-x^2)=0$$
$$(2-x)(2+x)\cdot x=0$$
$$x = 2, -2, 0,$$

To findout the greatest value ~~while~~ put ~~x~~, the value of x in first derivative so

$$y(0) = (120\times 0 - 15\times 0) = 0.$$
$$y(2) = 120\times 4 - 15\times 16$$
$$= 480-240$$
$$= 240$$
$$y(-2) = 120\times(-2)^2 - 15(-2)^4$$
$$= 120\times 4 - 15\times 16$$
$$= 240,$$

So, 2 and -2 gives greatest value so,

$$y(2) = 1+40\times 8 - 3\times 32 = 1+320-96 = 225$$
$$y(-2) = 1+40\times(-8) - 3\times(-32) = 1-320+96 = -223$$

Hence the tangent line have largest slope when at point (2, 225) and (-2, -223) are drawn.

Exercise 4.3

① Suppose the tangent line to the curve $y = f(x)$ at the point $(2, 5)$ has the equation $y = 9 - 2x$. If the Newton's method is used to locate a root of the equation $f(x) = 0$ and the initial approximation is $x_1 = 2$ find the second approximation x_2

Sol^n: here

$y = 9 - 2x$

$x_1 = 2$

$y = f(x) = 9 - 2x$

$f'(x) = -2$

From Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{1+1} = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 2 - \frac{5}{-2}$$

$$x_2 = 2 + 5/2 = \frac{4+5}{2}$$

$\boxed{x_2 = 4.5}$ which is the second approximation. (x_2).

2. Use Newton's method with the Specified initial approximation x_1 to find x_3, the 3rd ~~initial~~ approximation to the root of the given equation. (Give your answer to four decimal places).

(i) $\frac{1}{3}x^3 + \frac{1}{2}x^2 + 3 = 0$, $x_1 = -3$

Sol^n : Here

$f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + 3$, $f'(x) = x^2 + x$

$x_1 = -3$

From Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{1+1} = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = -3 - \frac{(-9 + 4.5 + 3)}{9 - 3}$$

$$x_2 = -3 - \frac{(-9 + 7.5)}{6}$$

$$x_2 = -3 - \frac{(-1.5)}{6}$$

$$x_2 = -3 + \frac{1.5}{6}$$

$$x_2 = \frac{-18 + 1.5}{6}$$

$$x_2 = -2.75$$

For x_3

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{2+1} = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = -2.75 - \frac{\left[\frac{(-2.75)^3}{3} + \frac{1}{2}(2.75)^2 + 3\right]}{(-2.75)^2 - 2.75}$$

$$x_3 = -2.75 - \frac{[-6.9322 + 3.78 + 3]}{(7.56 - 2.75)}$$

$$x_3 = -2.75 + \frac{0.1522}{4.81}$$

$$x_3 = -2.75 + 0.0316$$

$$x_3 = -2.7184 \text{ Ans}$$

(11) $x^7 + 4 = 0$, $x_1 = -1$

Soln: here,

$$f(x) = x^7 + 4$$

$$f'(x) = 7x^6$$

$$x_1 = -1$$

From Newton's method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{1+1} = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = -1 - \frac{(-1+4)}{7}$$

$$x_2 = -1 - 3/7$$

$$x_2 = \frac{-7-3}{7} = -10/7 = -1.4286$$

$x_2 = -1.4286$

Similarly for x_3

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{2+1} = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = -1.4286 - \frac{(-8.1444)}{59.5062}$$

$$x_3 = \frac{-85.0106 + 8.1444}{59.5062}$$

$$x_3 = \frac{-76.8662}{59.5062}$$

$$x_3 = -1.29173 \quad \text{Ans}$$

(#3) Starting with $x_1 = 2$, find the third approximation x_3 to the root of the eq^n $x^3 - 2x - 5 = 0$

Sol^n: here,

$$f(x) = x^3 - 2x - 5$$

$$f'(x) = 3x^2 - 2$$

$$x_1 = 2$$

$$x_3 = ?$$

From Newton's method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{1+1} = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 2 - \frac{(8-4-5)}{10}$$

$$x_2 = \frac{20 - (-1)}{10}$$

$$x_2 = \frac{21}{10} = 2.1$$

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TARGET

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Page: 144 LATE

$x_3 = ?$

Simplarly for x_3

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{2+1} = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 2.1 - \frac{[(2.1)^3 - 2 \times 2.1 - 5]}{3.(2.1)^2 - 2}$$

$$x_3 = 2.1 - \frac{[9.261 - 4.2 - 5]}{(13.23 - 2)}$$

$$x_3 = 2.1 - \frac{[0.061]}{11.23}$$

$$x_3 = \frac{23.583 - 0.061}{11.23}$$

$$x_3 = \frac{23.522}{11.23}$$

$x_3 = \cancel{2.095}$ 2.0946 Ans

4. Use Newton's method with initial approximation $x_1 = 1$ to find x_2, the second approximation to the root of the equation $x^4 - x - 1 = 0$. Explain how the method works by first graphing the function and its tangent line at $(1, -1)$

Sol^n: Here

$f(x) = x^4 - x - 1$
$f'(x) = 4x^3 - 1$
$x_1 = 1$

From Newton's method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{1+1} = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 1 - \frac{(-1)}{3}$$

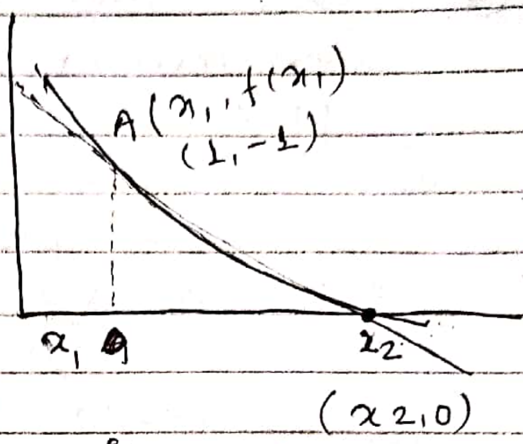
$$x_2 = \frac{3+1}{3} = 4/3$$

$$x_2 = 4/3$$

$\frac{dy}{dx} = f(x)$

$\left(\frac{dy}{dx}\right)_1 = f'(x)_1 = f'(x_1)$

Working method



eq^n of tangent at point A

$y - y_1 = \frac{dy}{dx}(x - x_1)$

$y - f(x_1) = \left(\frac{dy}{dx}\right)_1 (x - x_1)$

$y - f(x_1) = f'(x_1)(x - x_1)$

Since the ~~tin~~ point $(x_2, 0)$ is on the tangent line so,

$0 - f(x_1) = f'(x_1)(x_2 - x_1)$

$\frac{-f(x_1)}{f'(x_1)} = x_2 - x_1$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 1 - \frac{(-1)}{3}$

$x_2 = 1 + \frac{1}{3}$

$\boxed{x_2 = 4/3}$

(5) Use Newton's Method to approximate the given number correct to eight decimal places.

(a) $\sqrt[5]{20}$

Sol^n: here put

$x = \sqrt[5]{20} = (20)^{1/5}$

$x^5 = 20$

$x^5 - 20 = 0$

From Newton's method,

$x_{n+1} = \left[x_n - \frac{f(x_n)}{f'(x_n)}\right]$

$f(x) = x^5 - 20$

$f(x) = x^5 - 20$

$f'(x) = 5x^4$

checking the root value,

$x^5 - 20$

$f(0) = 0 - 20 = -20$

$f(1) = 1 - 20 = -19$

$f(2) = 32 - 20 = 12$

in f(1) and f(2) sign changing so that the f(x) exist in between 1 & 2

From Newton's method,

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

put $x_1 = 1$,

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{(-19)}{-15} = 1 - \frac{19}{15} = -0.26666667$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -0.26666667 -$

put $x_2 = 2$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{2+1} = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 2 - \frac{(12)}{80} = 1.85$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.85 - \frac{(1.66998656)}{(58.56753125)}$$

$$= \cancel{1.85} = 1.82148614$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 1.82148614 - \frac{(0.05069149)}{55.03937431}$$

$$1.82056514$$

$$x_6 = x_5 - \frac{f(x_5)}{f'(x_5)}$$

$$x_6 = 1.82056514 - \frac{(0.00005142)}{54.9281.4024}$$

$$x_6 = 1.82056420$$

$$x_7 = x_6 - \frac{f(x_6)}{f'(x_6)}$$

$$x_7 = 1.82056420 - \frac{(-0.00000017)}{54.92802680}$$

$$x_7 = 1.82056420$$

$$\sqrt[5]{20} = 1.82056420 \quad \text{Ans}$$

6 (b) $\sqrt[100]{100}$

Sol^n: here,

put

$x = (100)^{1/100}$

$x^{100} - 100 = 0$

$f(x) = x^{100} - 100$

$f'(x) = 100x^{99}$

checking.

$f(x) = x^{100} - 100$

$f(0) = -100$

$f(1) = -99$

$f(2) = +ve$

From Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

put $x_1 = 1$,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 1 - \left(\frac{-99}{100}\right) = 1.99$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.99 - \left(\frac{76.79052574 \times 10^{28}}{3858.82038904 \times 10^{28}}\right)$$

$x_3 = 1.9701$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.9701 - ($$

do in radian
fixed calc. in rad.

6\) Use Newton's method to find all root of the equation $3\cos x = x+1$ correct to six decimal place with initial approximation $x_1 = -3.5$

Sol^n: here,

$$3\cos x = x+1$$

$$3\cos x - x - 1 = 0$$

$$f(x) = 3\cos x - x - 1$$

$$f'(x) = -3\sin x - 1$$

where $x_1 = -3.5$

From Newton's method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = -3.5 - \frac{(3\cos(-3.5) + 3.5 - 1)}{-3\sin(-3.5) - 1} = -3.5 - \frac{(-0.309370}{-2.052350}$$

$$x_2 = -3.650739$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -3.650739 - \frac{\cancel{2.999498}\,(0.031256)}{-2.462296}$$

$$x_3 = -3.638045$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$x_4 = -3.638045 - \frac{0.000212}{(-2.428928)} = \boxed{-3.637958}$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = -3.637958 - \frac{(-0.000001)}{-2.428607)}$$

$$\boxed{x_5 = -3.637958}$$

$$\boxed{3\cos x - x - 1 = -3.637958}$$ Ans

(7) Use Newtons method to estimate the solutions of the equation $x^2+x-1=0$ start with $x_1=-1$ for the left hand sol^n and with $x=1$ for the sol^n on the right then each case find x_3

Sol^n: here

$$f(x) = x^2+x-1$$
$$f'(x) = 2x+1$$

where $x_1=-1$ for the left hand sol^n, So, finding the sol^n from left hand up to x_3. therefore from Newton's method,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -1 - \frac{(-1)}{-1} = -1-1 = -2$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -2 - \left(\frac{1}{-3}\right) = -2 + \frac{1}{3} = \frac{-6+1}{3}$$

$$\boxed{x_3 = -5/3}$$

Similarly put $x_1 = 1$ for the right handside sol^n, so,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = \frac{2}{3} - \frac{1/9}{7/3} = \frac{2}{3} - \frac{1}{7\times 3}$$

$$x_3 = \frac{2}{3} - \frac{1}{21} = \frac{14-1}{21} = \frac{13}{21}$$ Ans

(8) Use Newton's method to estimate the two zeroes of the function $f(x) = 2x - x^2 + 1$ starts with $x_1 = 0$ for the left hand zero & with $x = 2$ for the zero on the right then each case find x_3

Sol^n here,

$$f(x) = 2x - x^2 + 1$$
$$f'(x) = 2 - 2x$$
$$f'(x) = 2(1 - x)$$

where $x_1 = 0$ for the left hand sol^n we have findout up to x_3 in each case so,

from Newton's method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0 - \frac{1}{2} = 0 - 0.5 = -0.5$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -\frac{1}{2} - \left(\frac{-1/4}{3}\right) = -\frac{1}{2} + \frac{1}{12}$$

$$= \frac{-6+1}{12} = \frac{-5}{12}$$

Similarly,

For $x_1 = 2$ for the right hand sol^n, so,

from Newton's method.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{1}{-2} = 2 + \frac{1}{2} = \frac{5}{2}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = \frac{5}{2} - \frac{-1/4}{-3}$$

$$x_3 = \frac{5}{2} - \frac{1}{4 \times 3} = \frac{5}{2} - \frac{1}{12} = \frac{30-1}{12}$$

$$\boxed{x_3 = \frac{29}{12}}$$

Ans//

Chapter-5

CHAPTER-5

ANTIDERIVATIVE

(1) $\int c\,f(x)\,dx = c\int f(x)\,dx$

(2) $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx.$

(3) $\int K\,dx = Kx + c$

(4) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$

(5) $\int \frac{1}{x}\,dx = \log x + c$

(6) $\int e^{ax}\,dx = \frac{e^{ax}}{a} + c$

(7) $\int a^x\,dx = \frac{a^x}{\log a} + c$

(8) $\int \sin x\,dx = -\cos x + c$

(9) $\int \cos x\,dx = \sin x + c$

(10) $\int \sec^2 x\,dx = \tan x + c$

(11) $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$

(12) $\int \sec x \cdot \tan x\,dx = \sec x + c$

(13) $\int \operatorname{cosec} x \cdot \cot x\,dx = -\operatorname{cosec} x + c$

(14) $\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + c$

(15) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + c$

LATE

$\frac{\sin x}{\cos x}$

$-\frac{1}{3}-\frac{2}{3}+1$

$\frac{-1-2+3}{3}$ x^0

$d(\log\sqrt[3]{x}) = \frac{1}{3\sqrt[3]{x}} \times \frac{1}{2\sqrt{6}}$

$\frac{1}{6x}$

(16) $\int \sinh x\, dx = \cosh x + c$

(17) $\int \cosh x\, dx = \sinh x + c.$

(18) Integral of product function.

$$\int f(x)\cdot g(x)\,dx$$

$$f(x)\cdot\int g(x)\,dx - \int\left[\frac{d f(x)}{dx}\cdot\int g(x)\,dx\right]dx$$

(19) $\int \tan x\, dx = \log \sec x + c = -\log\cos x + c$. (20) $\int \cot x\, dx = \log \sin x$

Exercise 5.5

Evaluate.

(1) $\int \log \sqrt[3]{x}\, dx.$

Sol^n: here $\int \log\sqrt[3]{x}\,dx$

$$\int \log(x)^{1/3}\,dx$$

$\frac{d(x^{1/3})}{dx} = \frac{1}{3}x^{1/3-1}$

$\frac{1}{3}x^{-2/3}$

$\frac{1}{3x^{2/3}}$

$$\log x^{1/3}\int dx - \int\left[\frac{d\log(x)^{1/3}}{dx}\int dx\right]dx$$

$$\log x^{1/3}\cdot x - \int\left[\frac{1}{x^{1/3}}\,\frac{1}{3}x^{-2/3}\cdot x\right]dx$$

$$\log x^{1/3}\cdot x - \frac{1}{3}\int x^{-1/3}\cdot x^{-2/3}\cdot x\,dx$$

$$\log x^{1/3}\cdot x - \frac{1}{3}\int x^{-1/3-2/3+1}\,dx$$

$$\log x^{1/3}\cdot x - \frac{1}{3}\int dx$$

$$x\log x^{1/3} - \frac{1}{3}x + c$$

$$x\log\sqrt[3]{x} - \frac{x}{3} + c \quad \text{Ans.}$$

$d(\sin^{-1}x)$

$\log_e^x \quad \ln_{10}^x$

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(h) $\int \sin^{-1}x \cdot dx$

Sol^n: Here $\int \sin^{-1}x \cdot dx$

$\sin^{-1}x \int dx - \int \left[\frac{d(\sin^{-1}x)}{dx} \int dx\right] \cdot dx$

$\sin^{-1}x \cdot x - \int \frac{1}{\sqrt{1-x^2}} \times \cdot dx$

$x \sin^{-1}x - \int \frac{1}{\sqrt{1-x^2}} \cdot dx$

$x \sin^{-1}x - \left[x \int \frac{1}{\sqrt{1-x^2}} \cdot dx - \int \left[\frac{d(x)}{dx} \int \frac{1}{\sqrt{1-x^2}} \cdot dx\right] \cdot dx\right.$

$x \sin^{-1}x - \left[x \times \sin^{-1}x - \int \left[1 \times \sin^{-1} dx\right.\right.$

$x \sin^{-1}x - \int \frac{1}{\sqrt{1-x^2}} \cdot dx$

$x \sin^{-1}x - \sin^{-1}x$

$\int \sin^{-1}x \cdot dx$

$\sin^{-1}x \int dx - \int \left[\frac{d(\sin^{-1}x)}{dx} \int dx\right] \cdot dx$

$x \cdot \sin^{-1}x - \int \frac{1}{\sqrt{1-x^2}} \cdot x \cdot dx$

$x \sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}} \cdot dx$

$x \sin^{-1}x + \frac{1}{2} \int \frac{-2x}{\sqrt{t}} \cdot dx$

$x \sin^{-1}x + \frac{1}{2} \int dt \cdot t^{-1/2}$

let $t = 1-x^2$

$\frac{dt}{dx} = -2x$

$dt = -2x \cdot dx$

$$x\sin^{-1}x + \frac{1}{2}\int t^{-1/2}.dt$$

$$x\sin^{-1}x + \frac{1}{2}\left(\frac{t^{-1/2+1}}{-1/2+1}\right)$$

$$x\sin^{-1}x + \frac{1}{2}\left(\frac{t^{1/2}}{1/2}\right)$$

$$x\sin^{-1}x + \sqrt{t}$$

$$x\sin^{-1}x + \sqrt{1-x^2} + c \quad \text{Ans}$$

II^nd method

(b) $\int \sin^{-1}x.dx$

sol^n: here

$$\int \sin^{-1}x.dx$$

let $t = \sin^{-1}x$

$$x = \sin t$$

$$\frac{dx}{dt} = \cos t$$

$$dx = \cos t.dt$$

$$\int t.\cos t.dt$$

$$t\int \cos t.dt - \int\left[\frac{dt}{dt}\int \cos t\,dt\right].dt$$

$$t\sin t - \int 1.\sin t.dt$$

$$t\sin t + \cos t + c$$

$$\sin^{-1}x\,\sin(\sin^{-1}x) + \sqrt{1-\sin^2 t} + c$$

$$\sin^{-1}x.x + \sqrt{1-\sin^2(\sin^{-1}x)} + c$$

$$x\sin^{-1}x + \sqrt{1-x^2} + c \quad \text{Ans}$$

$$\int s \cdot 2^s \, ds$$

let $s = x$

$$\int x \cdot 2^x \cdot dx$$

$$x \int 2^x \cdot dx - \int \left[\frac{dx}{dx} \cdot \int 2^x \cdot dx \right] \cdot dx$$

$$x \times \frac{2^x}{\log 2} - \int \frac{2^x}{\log 2} \cdot dx$$

$$\frac{x\,2^x}{\log 2} - \frac{1}{\log 2} \int 2^x \cdot dx$$

$$x \cdot \frac{2^x}{\log 2} - \frac{1}{\log 2} \times \frac{2^x}{\log 2}$$

$$\frac{x \cdot 2^x}{\log 2} - \frac{2^x}{(\log 2)^2} + c$$

$$\frac{s \cdot 2^s}{\log 2} - \frac{2^s}{(\log 2)^2} + c$$ Ans//

(d) $\int z^3 e^z \, dz$

LATE

Sol^n: here.
let $z = x$

$\int x^3 . e^x . dx$

$x^3 \int e^x - \int \left[\frac{dx^3}{dx} . \int e^x . dx \right] . dx$

$x^3 . e^x - \int 3x^2 . e^x . dx$

$x^3 . e^x - 3 \int x^2 . e^x$

$x^3 . e^x - 3 \left[x^2 \int e^x . dx - \int \left[\frac{dx^2}{dx} \int \frac{e^x . dx}{dx} \right] . dx \right]$

$x^3 . e^x - 3 \left[x^2 . e^x - \int 2x . e^x . dx \right.$

$x^3 . e^x - 3 \left[x^2 e^x - 2 \left[\int x . e^x . dx \right] \right.$

$x^3 . e^x - 3 \left[x^2 e^x - 2 \left\{ \left[x \int e^x - \int \left[\frac{dx}{dx} . \int e^x \, dx \right] . dx \right] \right\} \right.$

$x^3 . e^x - 3x^2 e^x - 6 x e^x - 6 \int 1 . e^x . dx$

$x^3 . e^x - 3x^2 e^x - 6 x e^x - 6 e^z + c$

$\therefore z^3 e^z - 3z^2 e^z - 6 z e^z - 6 e^z + c$ Ans

(e) $\int x \cdot \tan^2 x \cdot dx$

$\int x \cdot (\sec^2 x - 1) \cdot dx$

$\int x \sec^2 x \cdot dx - \int x \cdot dx$

$x \int \sec^2 x \cdot dx - \int \left[\frac{dx}{dx} \int \sec^2 x \cdot dx\right] \cdot dx - \frac{x^2}{2} + c$

$x \times \tan x - \int 1 \cdot \tan x \, dx - x^2/2 + c$

$x \tan x - \int \frac{\sin x \, dx}{\cos x} - x^2/2 + c$

$x \tan x - (-) \int \frac{-\sin x}{\cos x} \cdot dx - x^2/2 + c$

$x \tan x + \log(\cos x) - x^2/2 + c$ Ans

(f) $\int (\sin^{-1} x)^2 \cdot dx$

Sol^n: here

let $t = \sin^{-1} x$

$x = \sin t$

$\frac{dx}{dt} = \cos t$

$dx = \cos t \cdot dt$

$\int t^2 \cdot \cos t \cdot dt$

$t^2 \int \cos t \cdot dx - \int \left[\frac{dt^2}{dt} \int \cos t \cdot dt\right] \cdot dt$

$t^2 \sin t \cdot - \int 2t \sin t \cdot dt$

$t^2 \sin t - 2 \int t \sin t \cdot dt$

$t^2 \sin t - 2\int t \sin t \cdot dt$

$t^2 \sin t - 2\left[t\int \sin t - \int\left[\frac{dt}{dt}\int \sin t \cdot dt\right] \cdot dt\right.$

$t^2 \sin t - 2\left[-t\cos t - \int 1 \cdot -\cos t \cdot dt\right.$

$t^2 \sin t - 2\left[-t\cos t + \int \cos t \cdot dt\right]$

$t^2 \sin t - 2 \cdot \left[-t\cos t + \sin t\right] + c$

$t^2 \sin t + 2t\cos t - 2\sin t + c$

$(\sin^{-1}x)^2 \sin(\sin^{-1}x) + 2\sin^{-1}x \cdot \sqrt{1-\sin^2 t} - 2\sin(\sin^{-1}x) + c$

$(\sin^{-1}x)^2 \cdot x + 2\sin^{-1}x\sqrt{1-\sin^2(\sin^{-1}x)} - 2x + c$

$x \cdot (\sin^{-1}x)^2 + 2\sin^{-1}x\sqrt{1-x^2} - 2x + c$ Ans//

(g) $\int_0^{2\pi} t^2 \sin 2t \, dt$

Sol^n: here.

$\int_0^{2\pi} t^2 \sin 2t \, dt$

$t^2\int_0^{2\pi} \sin 2t \cdot dt - \int_0^{2\pi}\left[\frac{dt^2}{dt}\int_0^{2\pi} \sin 2t \cdot dt\right] \cdot dt$

$-t^2\left[\frac{\cos 2t}{2}\right]_0^{2\pi} - \int_0^{2\pi} \cancel{2}t \, \frac{-\cos 2t}{\cancel{2}} \cdot dt$

$-t^2\left[\frac{\cos 2t}{2}\right]_0^{2\pi} - \int_0^{2\pi} -t\cos 2t$

$$-t^2\left[\frac{\cos 2t}{2}\right]_0^{2\pi} + \int_0^{2\pi} t\cos 2t \,.dt$$

$$-t^2\left[\frac{\cos 2t}{2}\right]_0^{2\pi} + t\int_0^{2\pi}\cos 2t - \int\left[\frac{dt}{dt}\int \cos 2t \,.dt\right].dt$$

$$-\frac{t^2}{2}\left[\cos 4\pi - \cos 0\right] + \frac{t}{2}\left[\sin 2t\right]_0^{2\pi} - \int_0^{2\pi}\frac{\sin 2t}{2}\,.dt$$

$$\frac{-t^2}{2}\left[\cos 4\pi - \cos 0^\circ\right] + \frac{t}{2}\left[\sin 2\times 2\pi - \sin 0\right] + \left[\frac{\cos 2t}{4}\right]_0^{2\pi}$$

$$\frac{-t^2}{2}\left[1-1\right] + \frac{t}{2}\left[0-0\right] + \frac{1}{4}\left[\cos 4\pi - \cos 0\right]$$

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(8) $\int_0^{2\pi} t^2 \sin 2t$

Sol^n: here.

$$\int_0^{2\pi} t^2 \sin 2t$$

$$t^2 \int \sin 2t \cdot dt - \int \left[\frac{dt^2}{dt} \int \sin 2t \cdot dt\right] \cdot dt$$

$$-t^2 \cdot \frac{\cos 2t}{2} - \int \cancel{2}t \cdot \frac{-\cos 2t}{\cancel{2}} \cdot dt$$

$$-t^2 \frac{\cos 2t}{2} + \int t \cos 2t \cdot dt$$

$$\frac{-t^2}{2} \cos 2t + t \int \cos 2t - \int \left[\frac{dt}{dt} \cdot \int \cos 2t \cdot dt\right] \cdot dt$$

$$-t^2/_2 \cos 2t + t/_2 \sin 2t - \int \cdot \frac{\sin 2t}{2} \cdot dt$$

$$-t^2/_2 \cos 2t + t/_2 \sin 2t - 1/_2 \int \sin 2t \cdot dt$$

$$-t^2/_2 \cos 2t + t/_2 \sin 2t + 1/_2 \cdot 1/_2 \cos 2t + c$$

$$\left[-t^2/_2 \cos 2t + t/_2 \sin 2t + 1/_4 \cos 2t + c\right]_0^{2\pi}$$

$$-\frac{4\pi^2}{2} \cos 4\pi + \frac{2\pi}{2} \sin 4\pi + \frac{1}{4} \cos 4\pi - 0 + 0 - 1/_4 \cos 0$$

$$-2\pi^2 \cdot 1 + \pi \times 0 + \frac{1}{4} - 1/_4$$

$$-2\pi^2$$ Ans

(h) $\int_1^2 x^4 (\ln x)^2 . dx$

Sol^n: Here

$$\int_1^2 x^4 (\ln x)^2 . dx$$

$$\ln x^2 \int x^4 . dx - \int \left[\frac{d(\ln x)}{dx} \int x^4 . dx \right] . dx$$

$$\ln x^2 \times \frac{x^5}{5} - \int 2 \ln x \times \frac{1}{x} \times \frac{x^5}{5} . dx$$

$$\ln x^2 \times \frac{x^5}{5} - \frac{2}{5} \int \ln x . x^4 . dx$$

$$\ln x^2 \times \frac{x^5}{5} - \frac{2}{5} \left[\ln x \int x^4 . dx - \int \left[\frac{d \ln x}{dx} \int x^4 . dx \right] . dx \right]$$

$$\frac{x^5}{5} . \ln x^2 - \frac{2}{5} \left[\ln x \times \frac{x^5}{5} - \int \frac{1}{x} \frac{x^5}{5} . dx \right]$$

$$\frac{x^5}{5} \ln x^2 - \frac{2}{5} \left[\ln x \frac{x^5}{5} - \frac{1}{5} \int x^4 . dx \right]$$

$$\frac{x^5}{5} \ln x^2 - \frac{2}{5} \left[\frac{x^5}{5} . \ln x - \frac{1}{5} \times \frac{x^5}{5} \right] + c$$

$$\left[\frac{x^5}{5} \ln x^2 - \frac{2x^5}{25} \ln x + \frac{2}{125} x^5 + c \right]_1^2$$

$$\frac{32}{5} \ln 2^2 - \frac{64}{25} \ln 2 + \frac{64}{125} - \frac{2}{125}$$

$$\frac{32}{5} (\ln 2)^2 - \frac{64}{125} \ln 2 + \frac{62}{125}$$

2. Evaluate

(a) $\int \sin^2 x \cdot \cos^3 x \, dx$

$t = \sin^2 x$

$\frac{dt}{dx} = 2\sin x \cdot \cos x$

$dt = 2\sin x \cos x \cdot dx$

$t = \cos^3 x$

$t = -3\cos^2 x \times \sin x$

Sol^n: Here,

$$\int \sin^2 x \cdot \cos^3 x \cdot dx$$

$$\cancel{\int (1-\cos^2 x) \cos^3 x \cdot dx}$$

$$\int \sin^2 x \cos^3 x \cdot dx$$

$t = \sin x$

$\frac{dt}{dx} = \cos x$

$dt = \cos x \cdot dx$

$$\cancel{\int t^2 \cdot \cos^2 x \cdot \cos x \cdot dx}$$

$$\int t^2 (1-\sin^2 x) \cdot dt$$

$$\int t^2 (1-t^2) \cdot dt$$

$$\int (t^2 - t^4) \cdot dt$$

$$\int t^2 \cdot dt - \int t^4 dt$$

$$\frac{t^3}{3} - \frac{t^5}{3} + c$$

$$\boxed{\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c}$$ Ans

$$\int_0^{\pi} \sin^2 t \cos^4 t \, dt$$

$x = \sin t$

~~$t = \sin^{-1} x$~~

$\frac{dx}{dt} = \cos t$

$dx = \cos t . dt$

$$\int_0^{\pi} x^2 . \cos t . dt \cos^3 t$$

$$\int_0^{\pi} x^2 . dx \times \cos^2 t \left(\sqrt{1-\sin^2 t}\right)$$

$$\int_0^{\pi} x^2 . dx \times (1-\sin^2 t)\left(\sqrt{1-\sin^2 t}\right)$$

$$\int x^2 . dx (1-x^2) \sqrt{1-x2}$$

$$\int (x^2 - x^4) \left(\sqrt{1-x2}\right) . dx$$

$$\int \left(\sqrt{(x^2-x^4)}\right)^2 \left(\sqrt{1-x2}\right) . dx$$

$t = \cos t$

$\frac{dt}{dx} = \frac{-\sin t)}{dt}$

$$\int_0^{\pi} \sin 2 t \times \cos^2 t . \quad \cos^2 t . dt$$

$$\int_0^{\pi} \cos^2 t . dt$$

$$\int_0^{\pi} \sin^2 t \cos^4 t \, dt$$

Sol^n: Here,

$$\int_0^{\pi}$$

3. Evaluate

a. $\int \frac{dx}{x^2\sqrt{4-x^2}}$

Sol^n: Here.

$= \frac{P}{h}$ $\frac{x}{2} = \sin\theta$

$b = \sqrt{4-x^2}$

$$\int \frac{dx}{x^2\sqrt{4-x^2}}$$

$x = 2\sin\theta$

$\frac{dx}{d\theta} = 2\cos\theta$

$dx = 2\cos\theta \cdot d\theta$

$$\int \frac{2\cos\theta \cdot d\theta}{4\sin^2\theta\sqrt{4-4\sin^2\theta}}$$

$$\frac{1}{2}\int \frac{\cos\theta \cdot d\theta}{\sin^2\theta \times 2 \times \cos\theta}$$

$\frac{P}{b}$

$$\frac{1}{2\times 2}\int \operatorname{cosec}^2\theta \times d\theta$$

$$\frac{1}{4} \times -\cot\theta$$

$$\frac{-\cot\theta}{4} = \frac{-b}{P\times 4} = \frac{-\sqrt{4-x^2}}{4x}$$

(c) $\int \frac{\sqrt{x^2-4}}{x} \cdot dx$

$x = 2\sec\theta$

$\frac{dx}{d\theta} = 2\sec\theta \cdot \tan\theta$

Sol^n: Here.

$\int \frac{\sqrt{4\sec^2\theta - 4}}{2\sec\theta} \cdot 2\cancel{\sec\theta} \cdot \tan\theta \cdot d\theta$, $dx = 2\sec\theta\tan\theta d\theta$

$\int 2\sqrt{\sec^2\theta - 1} \cdot \tan\theta \cdot d\theta$

$\int 2 \times \tan^2\theta \cdot d\theta$

$2\int (\sec^2\theta - 1) \cdot d\theta$

$2\left[\int \sec^2\theta \cdot d\theta - \int d\theta\right] + c$

$2 \; \cancel{\sec} \tan\theta - 2\theta$

$2\tan\theta - 2\sec^{-1}(x/_2)$

$\cancel{2}\frac{\sqrt{x^2-4}}{\cancel{2}} - 2\sec^{-1} x/_2 + C$

$\sqrt{x^2-4} - 2\sec^{-1} x/_2 + c$

$\sqrt{x^2-4} - 2\sec^{-1}(x/_2) + c$

Exercise:- 5.3.

Definite Integral (Net change- Theorem)

$$\int_a^b f(x)\,dx = \left[f(x)\right]_a^b = f(b)-f(a)$$

Fundamental Theorem of Integral calculus is

If f is continuous on $[a,b]$ then $F(x)=\int_a^b f(t)\,dt$ is continuous on $[a,b]$, then its derivative is $f(x)$

i.e. $F'(x) = \frac{d(F(x))}{dx} = \frac{d}{dx}\int_a^b f(t)\,dt = f(x)$

Exercise 5.3

(2) Use part 1 of the fundamental theorem of calculus to find the derivative of the function

(a) $g(x) = \int_1^x \frac{1}{t^3+1}\,dt$

Sol^n: here.

$g(x) = \int_1^x \frac{1}{t^3+1}\,dt$

$f(t) = \frac{1}{t^3+1}$

$\frac{d}{dt}\int_1^x f(t).dt - f(x)$

$f(x) - \frac{1}{x^3+1}$ Ans

(b) $g(x) = \int_3^x e^{t^2-t}.dt$

Sol^n: here

$g(x) = \int_3^x e^{t^2-t}.dt$

$f(t) = e^{t^2-t}$

$\frac{d}{dx}\int_3^x f(t).dt = f(x)$

$f(x) = e^{x^2-x}$ Ans

(c) $g(s) = \int_5^{s} (t-t^2)^8\,dt$

Sol^n:- here

$g(s) = \int_5^s (t-t^2)^8\,dt$

$f(t) = (t-t^2)^8$

$\frac{d}{ds}\int f(t).dt = f(s)$

$f(s) = (s-s^2)^8$ Ans

(d) $g(r) = \int_0^r \sqrt{x^2+4}\cdot dx$

Sol^n: here,

$g(r) = \int_0^r \sqrt{x^2+4}\cdot dx$

$\frac{d}{dr}\int^r f(x)\cdot dx = f(r)$

$f(r) = \sqrt{r^2+4}$ Ans

(e) $F(x) = \int_x^{\pi} \sqrt{1+\sec t}\cdot dt$

Sol^n: here

$F(x) = \int_x^{\pi} \sqrt{1+\sec t}\cdot dt$, $f(t) = \sqrt{1+\sec t}\cdot dt$

$\cancel{F(x) = \int_0^{\pi} \sqrt{1+\sec t}\cdot dt + \int_{\pi}^{x} \sqrt{1+\sec t}\cdot dt}$

$\cancel{= f(\pi) + f(x)}$

$\cancel{= \sqrt{1+\sec\pi} + \sqrt{1+\sec x}}$

$F(x) = -\int_{\pi}^{x} \sqrt{1+\sec t}\cdot dt$

$= -\int_{\pi}^{x} f(t)\cdot dt$

$-\frac{d}{dx}\int_{\pi}^{x} f(t)\cdot dt = -f(x)$

$-f(x) = -\sqrt{1+\sec x}$ Ans

(f) $G(x) = \int_x^1 \cos\sqrt{t}\cdot dt$

Sol^n: here

$G(x) = \int_x^1 \cos\sqrt{t}\cdot dt$

$f(x) = -\int_1^x \cos\sqrt{t}\cdot dt$

$f(t) = \cos\sqrt{t}$

$-\frac{d}{dx}\int_1^x f(t)\cdot dt = -f(x)$

$-f(x) = -\cos\sqrt{x}$ Ans

(g) $h(x) = \int_1^{e^x} \ln t \cdot dt$

Sol^n: here
$h(x) = \int_1^{e^x} \ln t \cdot dt$

let $u = e^x$, $\frac{du}{dx} = e^x$

and $f(t) = \ln t$.

$\frac{d}{du}\int_1^u \ln t \cdot dt \cdot \frac{du}{dx}$

$\frac{d}{du}\int_1^u f(t) \cdot dt \cdot \frac{du}{dx} = f(u) \cdot \frac{du}{dx}$

$f(u) \cdot \frac{du}{dx} = \ln e^x \cdot e^x = e^x \ln e^x$

$e^x \cdot x(\ln_e e)$

$x \cdot e^x$ Ans

(h) $h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1} \cdot dz$

let $u = \sqrt{x}$, $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$f(z) = \frac{z^2}{z^4+1}$

$\frac{d}{du}\int_1^u \frac{z^2}{z^4+1} dz \frac{du}{dx} = \frac{d}{du}\int_1^u f(z) \cdot dz \frac{du}{dx}$

$= f(u) \cdot \frac{1}{2\sqrt{x}}$

$= \frac{u^2}{u^4+1} \cdot \frac{1}{2\sqrt{x}}$

$= \frac{x}{x^2+1} \times \frac{1}{2\sqrt{x}}$

$= \frac{\sqrt{x}}{2(x^2+1)}$ Ans

(i) $y = \int_{1}^{x^4} \cos^2\theta \cdot d\theta$

Sol^n: here

let $u = x^4 \Rightarrow \frac{du}{dx} = 4x^3$

$f(\theta) = \cos^2\theta$.

$\frac{d}{du}\int_{1}^{u} f(\theta)\cdot d\theta \cdot \frac{du}{dx} = f(u)\cdot\frac{du}{dx}$

$f(u)\cdot\frac{du}{dx} = \cos^2 u \cdot 4x^3 = 4x^3\cos^2 x^4$ Ans

$= 4x^3(\cos x^4)^2$ Ans

(j) $y = \int_{\sin x}^{1} \sqrt{1+t^2}\cdot dt$

Sol^n: here.

$y = -\int_{1}^{\sin x} \sqrt{1+t^2}\cdot dt$

let $u = \sin x \Rightarrow \frac{du}{dx} = \cos x$

$f(t) = \sqrt{1+t^2}$

$-\frac{d}{du}\int_{1}^{u} f(t)\cdot dt \,\frac{du}{dx} = -f(u)\cdot\frac{du}{dx}$

$-\sqrt{1+u^2}\cdot\cos x$

$-\sqrt{1+\sin^2 x}\cdot\cos x$

$-\cos x\sqrt{1+\sin^2 x}$ Ans

(k) $g(x) = \int_{1-2x}^{1+2x} t\sin t \cdot dt$

Sol^n: here

$g(x) = \int_{1-2x}^{1+2x} t\sin t\cdot dt$

$g(x) = \int_{1-2x}^{0} t(\sin t)\cdot dt + \int_{0}^{1+2x} t(\sin t)\cdot dt$

$g(x) = -\int_{0}^{1-2x} t(\sin t)\cdot dt + \int_{0}^{1+2x} t(\sin t)\cdot dt$

let $u = 1-2x \Rightarrow \frac{du}{dx} = -2$

let $v = (1+2x) \Rightarrow \frac{dv}{dx} = 2$

$f(t) = t\sin t$

$-\frac{d}{du}\int_{0}^{u} f(t)\cdot dt \frac{du}{dx} + \frac{d}{dv}\int_{0}^{v} f(t)\cdot dt \cdot \frac{dv}{dx}$

$-f(u)\cdot\frac{du}{dx} + f(v)\cdot\frac{dv}{dx}$

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$-(u \sin u) \cdot -2 + (v \sin v) \cdot 2$

$2[(1-2x)\sin(1-2x)] + 2[(1+2x)\sin(1+2x)]$

$2[(1-2x)\sin(1-2x)] + 2[(1+2x)\sin(1+2x)]$

$(2-4x)\sin(1-2x) + (2+4x)\sin(1+2x)$

Ans

(2) $y = \int_{\cos x}^{\sin x} \ln(1+2v) \cdot dv$

Sol^n: Here

$y = \int_{\cos x}^{\sin x} \ln(1+2v) \cdot dv$

$y = \int_{\cos x}^{0} \ln(1+2v) \cdot dv + \int_{0}^{\sin x} (1+2v) \cdot dv$

$y = -\int_{0}^{\cos x} \ln(1+2v) \cdot dv + \int_{0}^{\sin x} (1+2v) \cdot dv$

let $u = \cos x \Rightarrow \frac{du}{dx} = -\sin x$

let $W = \sin x \Rightarrow \frac{dv}{dx} = \cos x$

$f(v) = \ln(1+2v)$

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$-\frac{d}{du}\int_0^u f(v) \cdot dv \cdot \frac{du}{dx} + \frac{d}{dw}\int_0^w f(v) \cdot dv \cdot \frac{dw}{dx}$

$-f(u) \cdot \frac{du}{dx} + f(w) \cdot \frac{dw}{dx}$

$-\ln(1+2u) \cdot -\sin x + \ln(1+2w) \cos x$

$-\ln(1+2u) \cdot -\sin x + \cos x \ln(1+2w)$

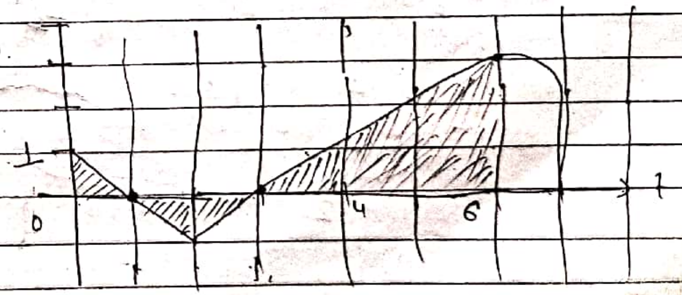
$\sin x \ln(1+2u) + \cos x \ln(1+2w)$

$\sin x \ln(1+2\cos x) + \cos x \ln(1+2\sin x)$ Ans

1. let $\int_0^x f(t)\,dt$, where f is the function whose graph is shown in following figure.

(a) Evaluate $g(x)$ for $x = 0, 1, 2, 3, 4, 5$ and 6

graph.



$g(x) = \int_0^x f(t)\,dt$

$g(0) = \int_0^0 f(t) \cdot dt = 0 =$ from figure

$g(1) = \int_0^1 f(t)\cdot dt = $ Area of $\Delta = \frac{1}{2}(a \times b)$

$= \frac{1}{2}(1 \times 1)$

Area of triangle Δ $= \frac{1}{2}$

Again,

$f(2) = \int_0^1 f(t)\cdot dt + \int_{-1}^{1} f t\cdot dt$

$= \frac{1}{2} + \left(\frac{1}{2}\right)(1 \times -1)$

$= \frac{1}{2} - \frac{1}{2} = 0.$

$f(3) = \int_0^1 f(t)\cdot dt + \int_{-1}^{1} f(t)\cdot dt + \int_{-1}^{1} f(t)\cdot dt$

$= \frac{1}{2} - \frac{1}{2} + \frac{1}{2}(1 \cdot -1)$

$= 0 - \frac{1}{2}$

$= -\frac{1}{2}$

$f(4) = f(0) + f(1) + f(2) + f(3) + \int_1^1 f(t)\cdot dt$

$= 0 + \frac{1}{2} + \frac{0}{2} + \frac{-1}{2} + \frac{1}{2}$

$= \frac{1}{2} - \frac{1}{2} - \frac{1}{2} + \frac{1}{2} = 0$

$f(5) = f(0) + f(1) + f(2) + f(3) + \frac{1}{2}(2 \times 2) - f(4)$

$f(5) = 0 + \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} + 2$

$= \frac{-2}{2} + 2$

$= -1 + 2$

③ Evaluate as a Riemann sum:

$\lim_{n \to \infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \sqrt{\frac{3}{n}} + \dots + \sqrt{\frac{n}{n}}\right)$

Sol^n: Here

$\lim_{n \to \infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \sqrt{\frac{3}{n}} + \dots + \sqrt{\frac{n}{n}}\right)$

$\lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{n} \left(\sqrt{\frac{r}{n}}\right)$

$\lim_{n \to \infty} \sum \Rightarrow \int_0^1$

$\frac{1}{n} \Rightarrow dx$

$\frac{r}{n} = x$

then, $\int_0^1 \sqrt{x}\cdot dx = \left[\frac{x^{\frac{1}{2}+1}}{\frac{3}{2}}\right]_0^1 = \left[\frac{x^{3/2}}{3/2}\right]_0^1$

$\frac{2}{3}$ Ans

Improper Integral.

The definite integral $\int_a^b f(x).dx$ is said to be improper integral if

(i) a or b or both are infinite.
(ii) f(x) become infinite at some interior point in the interval [a,b].

(iii) Improper Integral of type I
The integral

(i) $\int_a^\infty f(x).dx = \lim_{h\to\infty} \int_a^h f(x).dx$

(ii) $\int_{-\infty}^b f(x).dx = \lim_{h\to\infty} \int_{-h}^b f(x).dx$

(iii) $\int_{-\infty}^\infty f(x).dx = \int_{-\infty}^a f(x).dx + \int_a^\infty f(x).dx$

$\lim_{h\to\infty} \int_{-h}^a f(x).dx + \lim_{h\to\infty} \int_a^h f(x).dx$

$\Rightarrow I_1 + I_2$

Exercise 5.6

1. Determine whether each integral is convergent or divergent. Evaluate for convergent.

(a) $\int_3^\infty \frac{1}{(x-2)^{3/2}}.dx$

Sol^n, here

$\lim_{h\to\infty} \int_3^h (x-2)^{-3/2}.dx$

$\lim_{h\to\infty} \left[\frac{(x-2)^{-3/2+1}}{-3/2+1}\right]_3^h$

$\lim_{h\to\infty} \left[\frac{(x-2)^{-1/2}}{-1/2}\right]_3^h$

$\lim_{h\to\infty} -2\left[\frac{1}{\sqrt{x-2}} - \frac{1}{1}\right]$

$-2\left[\frac{1}{\infty} - 1\right]$

$-2[0-1]$

2

converges 2 Ans.

(b) $\int_{-\infty}^{0} \frac{1}{(3-4x)} \cdot dx$

Sol: here.

$\lim_{h\to\infty} \int_{-h}^{0} \frac{1}{(3-4x)} \cdot dx$

$\lim_{h\to\infty} \int_{-h}^{0} (3-4x)^{-1} \cdot dx$

$\cancel{\int_{-h}^{0} \frac{1}{(3-4x)} \cdot dx}$

$\cancel{\left[\log(3-4x)\right]_{-h}^{0}}$

$\lim_{h\to\infty} \int_{-h}^{0} \frac{1}{(3-4x)} \cdot dx$

$t = 3-4x$

$\frac{dt}{dx} = 0-4 = -4$

$dt = -4 \cdot dx$

$\lim_{h\to\infty} \frac{-1}{4} \int_{-h}^{0} \frac{-4 \cdot dx}{(3-4x)}$

$\lim_{h\to\infty} \frac{-1}{4} \left[\log(3-4x)\right]_{-h}^{0}$

$\lim_{h\to\infty} \frac{-1}{4} \left[\log 3 - \log(3+4h)\right]$

$\lim_{h\to\infty} \frac{-1}{4} \left[\log 3 - \log(3+4h)\right]$

$\frac{-1}{4} \left[\log 3 - \log(3+\infty)\right]$

$\frac{-1}{4} \left[\log\left(\frac{3}{\infty}\right)\right]$

$\frac{-1}{4} \left[\log(0)\right]$

$\frac{-1}{4} \times \infty$

∞

diverge

(c) $\int_2^{\infty} e^{-5x}.dx$

$\frac{d(e^{-5x})}{d(-5x)} \times \frac{d(-5x)}{dx}$

$e^{-5x} \int e^{-5x}. -5.$

Sol^n: Here

$\lim_{h \to \infty} \int_2^h e^{-5x}.dx$

$\lim_{h \to \infty} \frac{-1}{5} \int_2^h -5.e^{-5x}.dx$

$\lim_{h \to \infty} \frac{-1}{5} \left[e^{-5x} \right]_2^h$

$\lim_{h \to \infty} \frac{-1}{5} \left[e^{-5h} - e^{-10} \right]$

$\lim_{h \to \infty} \frac{-1}{5} \left[\frac{1}{e^{5h}} - \frac{1}{e^{10}} \right]$

$\frac{-1}{5} \left[\frac{1}{e^{\infty}} - \frac{1}{e^{10}} \right]$

$\frac{-1}{5} \left[0 - \frac{1}{e^{10}} \right]$

$\frac{1}{5e^{10}}$

$= \frac{e^{-10}}{5}$ converges.

Ans

(d) $\int_0^{\infty} \frac{x^2}{\sqrt{1+x^3}}.dx$

Sol^n: Here

$\lim_{h \to \infty} \int_0^h \frac{x^2}{\sqrt{1+x^3}}.dx$

let $t = 1+x^3$

$\frac{dt}{dx} = 3x^2$

$dt = 3x^2.dx$

$\lim_{h \to \infty} \frac{1}{3} \int_0^h \frac{3x^2}{\sqrt{1+x^3}}.dx$

$\lim_{h \to \infty} \frac{1}{3} \int_0^h \frac{dt}{\sqrt{t}}$

$\lim_{h \to \infty} \frac{1}{3} \int_0^h (t)^{-1/2}.dt.$

$\lim_{h \to \infty} \frac{1}{3} \left[\frac{t^{-1/2+1}}{-\frac{1}{2}+1} \right]_0^h$

$\lim_{h \to \infty} \frac{1}{3} \left[\frac{t^{1/2}}{1/2} \right]_0^h$

$\lim_{h \to \infty} \frac{2}{3} \left[t^{1/2} \right]_0^h$

$\lim_{h \to \infty} \frac{2}{3} \left[\sqrt{1+x^3} \right]_0^h$

$$\lim_{h\to\infty} \frac{2}{3}\left[\sqrt{1+h^3} - \sqrt{1+0^3}\right]$$

$$\lim_{h\to\infty} \frac{2}{3}\left[\sqrt{1+h^3} - 1\right]$$

$$\frac{2}{3}\left[\sqrt{1+\infty} - 1\right]$$

∞ diverge

(e) $$\int_{-\infty}^{\infty} x e^{-x^2}\, dx$$

$$\frac{d(e^{-x^2})}{d(-x^2)} \times \frac{d(-x^2)}{dx}$$

$$e^{-x^2} \cdot -2x$$

$$\int -2x e^{-x^2}$$

sol^n here.

$$\int_{-\infty}^{\infty} x \cdot e^{-x^2}\, dx$$

$$\lim_{h\to\infty} \int_{-\infty}^{0} x \cdot e^{-x^2}\, dx + \int_{0}^{\infty} x \cdot e^{-x^2}\, dx$$

$$\lim_{h\to\infty} \int_{-h}^{0} x \cdot e^{-x^2}\, dx + \lim_{h\to\infty} \int_{0}^{h} x \cdot e^{-x^2}\, dx$$

$$\lim_{h\to\infty} \frac{-1}{2} \int_{-h}^{0} -2x e^{-x^2} + \lim_{h\to\infty} -\frac{1}{2}\int_{0}^{h} -2x e^{-x^2}\, dx$$

$$\lim_{h\to\infty} \frac{-1}{2}\left[e^{-x^2}\right]_{-h} + \lim_{h\to\infty} -\frac{1}{2}\left[e^{-x^2}\right]^{h}$$

$$\lim_{h\to\infty} \frac{-1}{2}\left[\frac{1}{e^{x^2}}\right]_{-h}^{0} + \lim_{h\to\infty} \frac{1}{2}\left[\frac{1}{e^{x^2}}\right]_{0}^{h}$$

$$\lim_{h\to\infty} \frac{-1}{2}\left[\frac{1}{e^0} - \frac{1}{e^{-h^2}}\right] - \frac{1}{2}\left[\frac{1}{e^{h^2}} - \frac{1}{e^0}\right]$$

$$\lim_{h\to\infty} \frac{-1}{2}\left[1 - \frac{1}{e^{\infty}}\right] \frac{-1}{2}\left[\frac{1}{e^{\infty}} - 1\right]$$

$$\frac{-1}{2}\left[1 - 0\right] \frac{-1}{2}\left[0 - 1\right]$$

$$\frac{-1}{2} + \frac{1}{2}$$

$$0$$

converge.

(f) $\int_0^\infty \sin^2 x \cdot dx$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 1 - \sin^2 x - \sin^2 x$

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1-\cos 2x}{2}$

Sol^n: here

$\lim_{h\to\infty} \int_0^h \sin^2 x \cdot dx$

$\lim_{h\to\infty} \int_0^h \frac{1-\cos 2x}{2} \cdot dx$

$\lim_{h\to\infty} \frac{1}{2}\int_0^h 1-\cos 2x \cdot dx$

$\lim_{h\to\infty} \frac{1}{2}\int_0^h 1\, dx - \frac{1}{2}\int_0^h \cos 2x \cdot dx$

$\lim_{h\to\infty} \left[\frac{1}{2}\times x\right]_0^h - \frac{1}{2}\left[\frac{\sin 2x}{2}\right]_0^h$

$\lim_{h\to\infty} \frac{1}{2}\left[x\right]_0^h - \frac{1}{4}\left[\sin 2x\right]_0^h$

$\lim_{h\to\infty} \frac{1}{2}[h-0] - \frac{1}{4}[\sin 2h - 0]$

$\lim_{h\to\infty} \frac{h}{2} - \frac{1}{4}\sin 2h$

$\frac{\infty}{2} - \frac{1}{4}\sin\infty = \infty$

diverge

$\frac{1}{x-1} + \frac{1}{x+3}$ $\quad -3-1 \quad \left(\frac{-1}{4}\right) \quad \frac{1}{4}$

(g) $\int_2^\infty \frac{dv}{v^2+2v-3}$ put $v = x$.

$\int_2^\infty \frac{dx}{x^2+2x-3}$

$\int_2^\infty \frac{dx}{x^2+(3-1)x-3}$

$\int_2^\infty \frac{dx}{(x^2+3x-x-3)} = \int_2^\infty \frac{dx}{x(x+3)-1(x+3)}$

$\int_2^\infty \frac{dx}{(x-1)(x+3)}$

$\int_2^\infty \left(\frac{4}{x-1} + \frac{-4}{x+3}\right) \cdot dx$

$\int_2^\infty \frac{4}{x-1} \cdot dx - \int_2^\infty \frac{4}{x+3} \cdot dx$

$\lim_{h\to\infty} \int_2^h \frac{4}{x-1}\, dx - \int_2^h \frac{4}{(x+3)} \cdot dx$

$\lim_{h\to\infty} 4\left[\log(x-1)\right]_2^h - 4\left[\log(x+3)\right]_2^h$

$\lim_{h\to\infty} 4\left[\log(h-1)\right] - 4\left\{\log(h+3) - \log 5\right\}$

$4\left[\log(\infty-1)\right] - 4\left\{\log(\infty+3) - \log 5\right\}$

∞ diverge

(g) $\int_2^{\infty} \frac{dx}{x^2+2x-3}$

Sol^n: here

$\int_2^{\infty} \frac{dx}{(x-1)(x+3)} = \frac{1}{4(x-1)} + \frac{-1}{4(x+3)}$

$= \int_2^h \frac{1}{4(x-1)} \cdot dx - \int_2^h \frac{1}{4(x+3)} dx.$

$= \frac{1}{4}\int_2^h \frac{1}{x-1} \cdot dx - \frac{1}{4}\int_2^h \frac{1}{x+3} \cdot dx$

$\lim_{h\to\infty} \left[\frac{1}{4}\left[\log(x-1)\right]_2^h - \frac{1}{4}\left[\log(x+3)\right]_2^h\right]$

$\lim_{h\to\infty}$

$\cancel{\left[\frac{1}{4}\log(h-1) - \frac{1}{4}\log\right]_2^h}$

$\frac{1}{4}\cancel{\left[\log(h-1)\right.}$

$\frac{1}{4}\left\{(\log(h-1) - \log(2-1)) - (\log(h+3) - \log 5)\right\}$

$\lim_{h\to\infty} \frac{1}{4}\left\{(\log(h-1) - 0) - (\log(h+3) - \log 5)\right\}$

$\frac{1}{4}\left\{\log(h-1) - \log(h+3) + \log 5\right\}$

$\frac{1}{4}\left\{\log\left(\frac{h-1}{h+3}\right) + \log 5\right\}$

$\frac{1}{4}\left\{\log\left(\frac{h+3-4}{h+3}\right) + \log 5\right\}$

$\frac{1}{4}\left\{\log\left(1 - \frac{4}{h+3}\right) + \log 5\right\}$

$\frac{1}{4}\left\{\log\left(1 - \frac{4}{\infty}\right) + \log 5\right\}$

$\frac{1}{4}\left\{\log 1 + \log 5\right\}$

$\frac{\log 5}{4}$ Ans

converge

(h) $\int_1^{\infty} \frac{\ln x}{x} \cdot dx = \int_1^{\infty} \ln x \frac{1}{x} \cdot dx$

let $y = \ln x$

$\frac{dy}{dx} = \frac{1}{x}$

$dy = \frac{1}{x} \cdot dx$

$\int_1^{\infty} dy \cdot y$

$\int_1^{\infty} y \cdot dy$

$\lim_{h\to\infty} \int_1^h y \cdot dy$

$\lim_{h\to\infty} \left[\frac{y^2}{2}\right]_1^h$

$\lim_{h\to\infty} \frac{1}{2}\left[(\ln x)^2\right]_1^h$

$\lim_{h\to\infty} \frac{1}{2}\left[(\ln h)^2 - (\ln 1)^2\right]$

$\frac{1}{2}\left[(\ln \infty)^2 - 0\right]$

$\frac{1}{2}\infty = \infty$ diverge.

(K) $\int_{-2}^{14} \frac{dx}{(x+2)^{1/4}}$

$\int_{-2}^{14} (x+2)^{-1/4}.dx$

$\left[\frac{(x+2)^{-1/4+1}}{-1/4+1}\right]_{-2}^{14}$

$\frac{4}{3}\left[(x+2)^{3/4}\right]_{-2}^{14}$

$\frac{4}{3}\left[(14+2)^{3/4} - (-2+2)^{3/4}\right]$

$\frac{4}{3}\left[\sqrt[4]{16^3}\right]$

$\frac{4}{3}\left[\sqrt[4]{16\times16\times16}\right]$

$\frac{4}{3}\left[\sqrt{4\times4\times4\times4\times4\times4}\right]$

$\frac{4}{3}\left[4\times2\right]$

$32/3$ Ans

(l) $\int_{-2}^{3} \frac{1}{x^4}.dx$

Sol^n: here.

$\int_{-2}^{3} x^{-4}.dx$

$\left[\frac{x^{-4+1}}{-4+1}\right]_{-2}^{3}$

$\left[\frac{x^{-3}}{-3}\right]_{-2}^{3}$

$-\frac{1}{3}\left[\frac{1}{x^3}\right]_{-2}^{3}$

$-\frac{1}{3}\left[\frac{1}{27}+\frac{1}{8}\right]$

$-\frac{1}{3}\left[\frac{8+27}{27\times8}\right]$

$-\frac{1}{3}\left[\frac{1}{x^3}\right]_{-2}^{3}$

$\lim_{x\to0} -\frac{1}{3}\left[\frac{1}{0}\right]$

∞. diverge.

(m) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

$\int_0^1 \frac{1}{\sqrt{1-x^2}} \cdot dx$

$\left[\frac{\sin^{-1}x}{1} \right]_0^1 + c$

$(\sin^{-1} 1 - \sin^{-1} 0)$.

$\pi/2 - 0 + c$

$\pi/2$ converge.

(n). $\int_0^9 \frac{1}{(x-1)^{1/3}} \cdot dx$

Solⁿ: Here

$\int_0^9 (x-1)^{-1/3} \cdot dx$

$\left[\frac{(x-1)^{-1/3+1}}{-1/3+1} \right]_0^9$

$3/2 \left[(x-1)^{2/3} \right]_0^9$

$3/2 \left[(9-1)^{2/3} - (0-1)^{2/3} \right]$

$3/2 \left[\sqrt[3]{8 \times 8} - \sqrt[3]{-1^2} \right]$

$3/2 \left[2 \times 2 - 1 \right]$

$\frac{3}{2} \times 3$

$9/2$ converge.

(p) $\int_{-1}^0 \frac{e^{1/x}}{x^3} \cdot dx$

$\int_{-1}^0 e^{1/x} \cdot 1/x \times 1/x^2 \cdot dx$

let $y = 1/x = x^{-1}$

$dy/dx = -1x^{-2}$

$dy/dx = -1/x^2$

$-dy = \frac{dx}{x^2}$

$\int_{-1}^0 e^y \cdot y \cdot -dy$

$-1 \int_{-1}^0 e^y \cdot y \cdot dy$

$-1 \left[y \int e^y \cdot dy - \int \left[\frac{dy}{dy} \int e^y \cdot dy \right] \cdot dy \right]_{-1}^0$

$-1 \left[y \cdot e^y - \int 1 \cdot e^y \cdot dy \right]_{-1}^0$

$-1 \left[y \cdot e^y - e^y \right]_{-1}^0$

$\left[e^y - y \cdot e^y \right]_{-1}^0$

$1 - 0 \cdot e^0 - 1 e^{-1} + 1 e^{-1}$

$1 - e^{-1} - e^{-1}$

$1 - 2e^{-1}$

$\left[\cancel{e^{1/x} - 1/x \, e^{1/x}} \right]_{-1}^0$

$e^{1/x}\left[\cancel{1}-\cancel{x}_{/x}\right]_{-1}^{0}$

$\left[e^{y}-y.e^{y}\right]_{-1}^{0}$

$\left[e^{0}-0.e^{0}\right]-\left[e^{-1}+1.e^{-1}\right]$

$1-0-(2e^{-1})$

$1-2e^{-1}$

$1-2/e.$ converge.

(q) $\int_{0}^{1} \ln x.dx.$ LATE.

$\ln x\int dx-\int\left[\frac{d\ln x}{dx}\int dx\right].dx$

$\ln x \times x-\int \frac{1}{x}\cdot x.dx$

$\left[x.\ln x - x.\right]_{0}^{1}$

$x\ln x - x$

$1\ln 1-1-(0.\ln 0-0)$

$\ln 1-1-0$

$0-1-0$

-1 converges.

(o) $\int_{\pi/2}^{\pi} \operatorname{cosec} x.dx.$

$\int_{\pi/2}^{\pi} \frac{1}{\sin x}.dx$

$\int_{\pi/2}^{\pi} \frac{\sin x}{\sin^2 x}.dx.$

$\int_{\pi/2}^{\pi} \frac{\sin x}{1-\cos^2 x}.dx$

$y=\cos x$

$\frac{dy}{dx}=-\sin x.$

$-dy=\sin x.dx.$

$\int_{\pi/2}^{\pi} \frac{-dy}{1-y^2}$

$\int_{\pi/2}^{\pi} \frac{dy}{y^2-1} = \frac{1}{2}\int_{\pi/2}^{\pi} \frac{2}{y^2-1}.dy.$

$$\frac{1}{2}\int_{\pi/2}^{\pi} \frac{2}{y^2-1} \cdot dy$$

$$\frac{1}{2}\int_{\pi/2}^{\pi} \frac{y+1-(y-1)}{(y^2-1)} \cdot dy$$

$$\frac{1}{2}\int_{\pi/2}^{\pi} \frac{(y+1)-(y-1)}{(y^2-1)} \cdot dy$$

$$\frac{1}{2}\left\{\int_{\pi/2}^{\pi} \frac{y+1}{(y^2-1)} \cdot dy - \int_{\pi/2}^{\pi} \frac{(y-1)}{(y^2-1)} \cdot dy\right.$$

$$\frac{1}{2}\left[\int_{\pi/2}^{\pi} \frac{1}{y-1} \cdot dy - \int_{\pi/2}^{\pi} \frac{dy}{y+1}\right].$$

$$\frac{1}{2}\Big[\log(y-1) - \log(y+1)\Big]_{\pi/2}^{\pi}$$

$$\frac{1}{2}\Big[\log(\cos x-1) - \log(\cos x+1)\Big]_{\pi/2}^{\pi}$$

$$\frac{1}{2}\left\{\Big[\log(\cos\pi-1) - \log(\cos\pi+1)\Big] - \Big[\log(\cos\pi/2-1) - \log(\cos\pi/2+1)\Big]\right\}.$$

$\cos 0 = 1$

$$\frac{1}{2}\Big\{\Big[\log(-1-1) - \log 0\Big] - \Big[\log(0-1) - \log 1\Big]\Big\}$$

$$\frac{1}{2}\Big[0.3-\infty\Big] - \Big[\log -1 - 0\Big].$$

∞ diverge

$$\frac{1}{2}\Big[\cancel{\log}(-2)$$

(1) $$\int_0^\infty \frac{x \arctan x}{(1+x^2)^2}\,dx$$

Sol^n: Here,

$$\int_0^\infty \frac{x \tan^{-1}x \, .dx}{(1+x^2)^2}$$

$$\lim_{h \to \infty} \int_0^h \frac{x \tan^{-1}x . dx}{(1+x^2)^2}$$

Put $\theta = \tan^{-1}x \Rightarrow x = \tan\theta$

$$\frac{d\theta}{dx} = \frac{1}{1+x^2}$$

$$d\theta = \left(\frac{dx}{1+x^2}\right)$$

where limit value,

$h = \tan^{-1}h$

$0 = \tan^{-1}0 = 0,$

We have

$$\int_0^{\tan^{-1}h} \tan\theta . \theta . \left(\frac{dx}{1+x^2}\right) . \frac{1}{(1+x^2)}$$

$$\int_0^{\tan^{-1}h} \tan\theta . \theta . d\theta \; \frac{1}{1+\tan^2\theta}$$

$$\int_0^{\tan^{-1}h} \frac{\tan\theta . \theta . d\theta}{\sec^2\theta}$$

$$\int_0^{\tan^{-1}h} \frac{\sin\theta}{\cos\theta} \times \cos^2\theta \times \theta \times d\theta$$

$$\int_0^{\tan^{-1}h} \theta . \sin\theta \cos\theta . d\theta$$

$$\frac{1}{2}\int_0^{\tan^{-1}h} \sin 2\theta . \theta . d\theta$$

Integration by parts,

$$\theta . \int \sin 2\theta . d\theta - \int \left[\frac{d\theta}{d\theta} \int \sin 2\theta . d\theta\right] d\theta$$

$$\theta . \frac{-\cos 2\theta}{2} - \int 1 . \frac{-\cos 2\theta}{2} . d\theta$$

$$-\theta . \frac{\cos 2\theta}{2} + \frac{1}{2}\int \cos 2\theta . d\theta$$

$$-\theta \frac{\cos 2\theta}{2} + \frac{1}{2} \frac{\sin 2\theta}{2}$$

$$-\frac{\theta \cos 2\theta}{2} + \frac{1}{4} \sin 2\theta$$

$$\therefore \frac{1}{2}\left[-\frac{\theta \cos 2\theta}{2} + \frac{1}{4} \sin 2\theta\right]_0^{\tan^{-1}h}$$

$$\frac{1}{2}\left[\frac{1}{2} - \tan^{-1}h \cos 2(\tan^{-1}h) + \frac{1}{4} \sin 2(\tan^{-1}h) - 0 + \frac{1}{4}\sin 0\right]$$

$$\frac{1}{2}\left[\frac{1}{2} - \tan^{-1}h \cos 2(\tan^{-1}h) + \frac{1}{4} \sin 2(\tan^{-1}h)\right]$$

$$\frac{1}{2}\left[-\frac{1}{2}\tan^{-1}(\infty) . \cos 2 \times (\tan^{-1}\infty) + \frac{1}{4} \sin 2 (\tan^{-1}\infty)\right]$$

$$\frac{1}{2}\left[\frac{1}{2}\cdot -\frac{\pi}{2}\cdot \cos 2\times\frac{\pi}{2} + \frac{1}{4}\sin 2\times\frac{\pi}{2}\right]$$

$$\frac{1}{2}\left[\frac{1}{2}\cdot-\frac{\pi}{2}\cdot -1 + \frac{1}{4}\sin\pi\right]$$

$$\frac{1}{2}\left[\frac{\pi}{4}+0\right]$$

$\frac{\pi}{8}$ converge.

(K) $$\int_{-2}^{14}\frac{dx}{\sqrt[4]{x+2}}$$

sol^n: here

$$\int_{-2}^{14}\frac{dx}{(x+2)^{1/4}}$$

$$\Big[\quad\Big]_{-2}^{14}$$