DETERMINANTS

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Chapter - 4

Definition (Determinant)

For $n \geq 2$ the determinant of an $n \times n$ matrix $A = [a_{ij}]$ of n terms of the form.

$$\det(A) = \sum_{j=1}^{n} (-1)^{1+j} a_{1j} \det(A_{1j}).$$

Theorem 1: The determinant of an $n \times n$ matrix A can be computed by a cofactor expansion across the i^{th} row as,

$$\det(A) = a_{i1} C_{11} + a_{i2} C_{12} + \cdots + a_{in} C_{in}$$

where,

$$C_{ij} = (-1)^{i+j} \det(A_{ij})$$

The Cofactor expansion across the j^{th} column is

$$\det(A) = a_{1j} C_{1j} + a_{2j} C_{2j} + \cdots + a_{nj} C_{nj}$$

where C_{ij} is defined above

Theorem 2: If A is a triangular matrix, then $\det(A)$ is the product of the entries on the main diagonal of A. that is, if

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{1n} \\ 0 & a_{22} & a_{2n} \\ \cdots & \cdots & \cdots \\ 0 & 0 & a_{nn} \end{bmatrix}$$

Then,

$$\det(A) = (a_{11})(a_{22})(a_{33}) \cdots (a_{nn})$$

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Exercise :- 4.1

Compute the determinants Using ~~a~~ cofactor expansion.

1. $\begin{vmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{vmatrix}$

Sol^n: Here,

Let $A = \begin{bmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{bmatrix}$

$\det(A) = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

$= 3\begin{vmatrix} 3 & 2 \\ 5 & -1 \end{vmatrix} - 0 \, \cancel{\det(A_i} \begin{vmatrix} 2 & 2 \\ 0 & -1 \end{vmatrix} + 4\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix}$

$= 3(-3-10) - 0 + 4(10-0)$

$= 3(-13) + 40$

$= -39 + 40$

$= 1$

2. $\begin{vmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{vmatrix}$

Sol^n: Here,

let $A = \begin{vmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{vmatrix}$

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$$\det(A) = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$
$$= a_{11}(-1)^{1+1}\det(A_{11}) + a_{12}(-1)^{1+2}\det(A_{12}) + a_{13}(-1)^{1+3}\det(A_{13})$$

$$= 0\begin{vmatrix} -3 & 0 \\ 4 & 1 \end{vmatrix} - 5\begin{vmatrix} 4 & 0 \\ 2 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & -3 \\ 2 & 4 \end{vmatrix}$$

$$= 0 - 5(4-0) + 1(16+6)$$
$$= -20 + 22$$
$$= 2$$

(3) $\begin{vmatrix} 2 & -4 & 3 \\ 3 & 1 & 2 \\ 1 & 4 & -1 \end{vmatrix}$

Sol^n: Here

let $A = \begin{vmatrix} 2 & -4 & 3 \\ 3 & 1 & 2 \\ 1 & 4 & -1 \end{vmatrix}$

$$\det(A) = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$
$$= a_{11}(-1)^{1+1}\det(A_{11}) + a_{12}(-1)^{1+2}\det(A_{12}) + a_{13}(-1)^{1+3}\det(A_{13})$$
$$= 2\begin{vmatrix} 1 & 2 \\ 4 & -1 \end{vmatrix} + 4\begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix} + 3\begin{vmatrix} 3 & 1 \\ 1 & 4 \end{vmatrix}$$
$$= 2(-1-8) + 4(-3-2) + 3(12-1)$$
$$= -18 + 4(-5) + 33$$
$$= -18 - 20 + 33$$
$$= -5 \quad \text{Ans}$$

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5. $\begin{vmatrix} 2 & 3 & -4 \\ 4 & 0 & 5 \\ 5 & 1 & 6 \end{vmatrix}$

Soln: Here,

let $A = \begin{vmatrix} 2 & 3 & -4 \\ 4 & 0 & 5 \\ 5 & 1 & 6 \end{vmatrix}$

$\det(A) = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

$= a_{11}(-1)^{1+1}\det(A_{11}) + a_{12}(-1)^{1+2}\det(A_{12}) + a_{13}(-1)^{1+3}\det(A_{13})$

$= 2\begin{vmatrix} 0 & 5 \\ 1 & 6 \end{vmatrix} - 3\begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} - 4\begin{vmatrix} 4 & 0 \\ 5 & 1 \end{vmatrix}$

$= 2(0-5) - 3(24-25) - 4(4-0)$

$= -10 + 3 - 16$

$= -23$ Ans

7. $\begin{vmatrix} 4 & 3 & 0 \\ 6 & 5 & 2 \\ 9 & 7 & 3 \end{vmatrix}$

Soln: Here

let $A = \begin{vmatrix} 4 & 3 & 0 \\ 6 & 5 & 2 \\ 9 & 7 & 3 \end{vmatrix}$

~~det(A) = a11~~ Here the first row has most zero so by co-factor expansion,

$\det(A) = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

$= a_{11}(-1)^{1+1}\det(A_{11}) + a_{12}(-1)^{1+2}\det(A_{12}) + a_{13}(-1)^{1+3}\det(A_{13})$

$$= 4\begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} - 3\begin{vmatrix} 6 & 2 \\ 9 & 3 \end{vmatrix} + 0\begin{vmatrix} 6 & 5 \\ 9 & 7 \end{vmatrix}$$

$$= 4(15-14) - 3(18-18) + 0$$

$$= 4.1 - 3.0$$

$$= 4 \text{ Ans}$$

(9) $\begin{vmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{vmatrix}$

Sol^n: Here,

the ~~first~~ 3rd row has most zero so

let $A = \begin{vmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{vmatrix}$

By Co-factor expansion

$$\det(A) = a_{31} C_{13} + a_{32} C_{32} + a_{33} C_{33} + a_{34} C_{34}$$

$$= a_{31}(-1)^{3+1}\det(A_{31}) + a_{32}(-1)^{3+2}\det(A_{32})$$

$$+ a_{33}(-1)^{3+3}\det(A_{33}) + a_{34}(-1)^{3+4}\det(A_{34})$$

$$= 2\begin{vmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 8 & 1 & 8 \end{vmatrix} - 0 + 0 - 0$$

$$= 2\left| 5 \begin{bmatrix} 7 & 2 \\ 3 & 1 \end{bmatrix} \right| = 2\left[5\,(7-6)\right]$$

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$$= \cancel{2[5(16+5)]}$$

$$= \cancel{2 \cdot 5 \cdot 21}$$

$$=$$

(10)
$$\begin{bmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 5 & 0 & 4 & 4 \end{bmatrix}$$

Sol^n: Here,

let $A = \begin{bmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 5 & 0 & 4 & 4 \end{bmatrix}$

The second Row has most zero so by co-factor expansion,

$$\det(A) = a_{21} C_{21} + a_{22} C_{22} + a_{23} C_{23} + C_{24} \cdot a_{24}$$

$$= a_{21} (-1)^{2+1} \det(A_{21}) + a_{22} (-1)^{2+2} \det(A_{22}) + a_{23} (-1)^{2+3} \det(A_{23}) + a_{24} (-1)^{2+4} \det(A_{24})$$

$$\cancel{= 0+0+3 \begin{vmatrix} 0 & 0 & 0 \\ 2 & -6 & 5 \\ 5 & & \end{vmatrix}}$$

$$= 0+0+3 \begin{vmatrix} 1 & -2 & 2 \\ 2 & -6 & 5 \\ 5 & 0 & 4 \end{vmatrix}$$

$$= -3 \left[5 \begin{bmatrix} -2 & 2 \\ -6 & 5 \end{bmatrix} + 0 + 4 \begin{bmatrix} 1 & -2 \\ 2 & -6 \end{bmatrix} \right]$$

$$= -3 \left[5(-10+12) + 0 + 4(-6+4) \right]$$

$$= -3 [5 \times 2 + 4 \times (-2)]$$

$$= -3 [10 - 8]$$

$$= -3 \times 2$$

$$= -6 \text{ Ans}$$

(11) $\begin{bmatrix} 3 & 5 & -8 & 4 \\ 0 & -2 & 3 & -7 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

Sol^n: Here

let $A = \begin{bmatrix} 3 & 5 & -8 & 4 \\ 0 & -2 & 3 & -7 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

Here fourth ~~ro~~ row has most zero so By co-factor expansion

$\det(A) = 2 \begin{vmatrix} 3 & 5 & -8 \\ 0 & -2 & 3 \\ 0 & 0 & 1 \end{vmatrix}$

$= 2 \left[3 \begin{bmatrix} -2 & 3 \\ 0 & 1 \end{bmatrix} \right]$

$= 2 \left[3 (-2) \right]$

$= 2 \cdot (-6)$

$= -12$ Ans

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(12) $\begin{bmatrix} 4 & 0 & 0 & 0 \\ 7 & -1 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 5 & 8 & 4 & -3 \end{bmatrix}$

Solⁿ: Here,

let $A = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 7 & -1 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 5 & 8 & 4 & -3 \end{bmatrix}$

Here the first Row has most zero so by co-factor expansion

$\det(A) = 4 \begin{vmatrix} -1 & 0 & 0 \\ 6 & 3 & 0 \\ 8 & 4 & -3 \end{vmatrix} + 0 + 0 + 0$

$= 4 \left[-1 \begin{vmatrix} 3 & 0 \\ 4 & -3 \end{vmatrix} \right]$

$= 4 \left[-1 (-9 - 0) \right]$

$= 4 \cdot 9$

$= 36$ Ans

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(13) $\begin{vmatrix} 4 & 0 & -7 & 3 & -5 \\ 0 & 0 & 2 & 0 & 0 \\ 7 & 3 & -6 & 4 & -8 \\ 5 & 0 & 5 & 2 & -3 \\ 0 & 0 & 9 & -1 & 2 \end{vmatrix}$

Soln: Here.

let $A = \begin{vmatrix} 4 & 0 & -7 & 3 & -5 \\ 0 & 0 & 2 & 0 & 0 \\ 7 & 3 & -6 & 4 & -8 \\ 5 & 0 & 5 & 2 & -3 \\ 0 & 0 & 9 & -1 & 2 \end{vmatrix}$

Here the second row has most zero. so by co-factor expansion,

$$\det(A) = -2 \begin{vmatrix} 4 & 0 & 3 & -5 \\ 7 & 3 & 4 & -8 \\ 5 & 0 & 2 & -3 \\ 0 & 0 & -1 & 2 \end{vmatrix}$$

$$= -2 \left[3 \begin{vmatrix} 4 & 3 & -5 \\ 5 & 2 & -3 \\ 0 & -1 & 2 \end{vmatrix} \right]$$

$$= -2 \left[3 \left[0 + 1 \begin{bmatrix} 4 & -5 \\ 5 & -3 \end{bmatrix} + 2 \begin{vmatrix} 4 & 3 \\ 5 & 2 \end{vmatrix} \right] \right]$$

$$= -2 \left[3 \left[(-12 + 25) + 2(5) \right] \right] - 2[3[$$

$$= -2 \left[3(13 - 10) \right]$$

$$= -2 \left[3 \times -12 \right]$$

$$= -2 \cdot -39$$

$$= 78$$

$$-2\left[3\left[+1\begin{vmatrix}4 & -5\\5 & -3\end{vmatrix}+2\begin{vmatrix}4 & 3\\5 & 2\end{vmatrix}\right]\right]$$

$$-2\left[3\left[(-12+25)+2(8-15)\right]\right]$$

$$-2\left[3\left[(13)+2(-7)\right]\right]$$

$$=-2\left[3(13-14)\right]$$

$$=-2.\left[-3\right]$$

$$=6 \text{ Ans}$$

(14)
$$\begin{vmatrix}6 & 3 & 2 & 4 & 0\\9 & 0 & -4 & 1 & 0\\8 & -5 & 6 & 7 & 1\\3 & 0 & 0 & 0 & 0\\4 & 2 & 3 & 2 & 0\end{vmatrix}$$

Solⁿ: Here

let $A = \begin{vmatrix}6 & 3 & 2 & 4 & 0\\9 & 0 & -4 & 1 & 0\\8 & -5 & 6 & 7 & 1\\3 & 0 & 0 & 0 & 0\\4 & 2 & 3 & 2 & 0\end{vmatrix}$

Here the fourth row has most zero.

$$\det(A) = -3\begin{vmatrix}3 & 2 & 4 & 0\\0 & -4 & 1 & 0\\-5 & 6 & 7 & 1\\2 & 3 & 2 & 0\end{vmatrix}$$

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$$= -3 \left[-1 \begin{bmatrix} 3 & 2 & 4 \\ 0 & -4 & 1 \\ 2 & 3 & 2 \end{bmatrix} \right.$$

$$= -3 \left[-1 \left[0 - 4 \begin{bmatrix} 3 & 4 \\ 2 & 2 \end{bmatrix} - 1 \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} \right] \right.$$

$$= -3 \left[-1 \left[-4 (6-8) - 1 (9-4) \right] \right.$$

$$= -3 \left[-1 \left[8 - 5 \right] \right]$$

$$= -3 \left[-1 \cdot 3 \right]$$

$$= 9 \text{ Ans}$$

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compute the determinants.

15. $\begin{vmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{vmatrix}$

Sol^n: Here

let $A = \begin{vmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{vmatrix}$

$\cancel{R_1 \to R_1 + 4R_3}$

$A = \begin{vmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{vmatrix}$

$\det(A) = 3 \begin{vmatrix} 3 & 2 \\ 5 & -1 \end{vmatrix} - 0 + 4 \begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix}$

$\det(A) = 3(-3-10) + 4(10-0)$

$\det(A) = -39 + 40$

$\det(A) = 1$ Ans

16) $\begin{vmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{vmatrix}$

Sol^n: Here

let $A = \begin{vmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{vmatrix}$

By co-factor expansion

$$\det(A) = 0 - 5\begin{vmatrix} 4 & 0 \\ 2 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & -3 \\ 2 & 4 \end{vmatrix}$$

$$= 0 - 5.(4-0) + 1(16+6)$$

$$= \cancel{45+22} = -20+22$$

$$= \cancel{37} = 2 \text{ Ans}$$

17) $\begin{vmatrix} 2 & -4 & 3 \\ 3 & 1 & 2 \\ 1 & 4 & -1 \end{vmatrix}$

Sol^n: Here

let $A = \begin{vmatrix} 2 & -4 & 3 \\ 3 & 1 & 2 \\ 1 & 4 & -1 \end{vmatrix}$

$R_1 \to R_1 + R_3$

$A = \begin{vmatrix} 3 & 0 & 2 \\ 3 & 1 & 2 \\ 1 & 4 & -1 \end{vmatrix}$

$R_1 \to R_1 - R_2$

$A = \begin{vmatrix} 0 & -1 & 0 \\ 3 & 1 & 2 \\ 1 & 4 & -1 \end{vmatrix}$

$$\det(A) = 1\begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix} = 1(-3-2)$$

$$= -5 \text{ Ans}$$

(18) $\begin{vmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{vmatrix}$

Sol^n! Here

Let $A = \begin{vmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{vmatrix}$

$\cancel{R_1 \to 2R_1 - R_2}$

$A = \begin{vmatrix} 0 & 5 & 9 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{vmatrix}$

$\det(A) = -5 \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 9 \begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix}$

$= -5(4-3) + 9(8-3)$

$= -5 + 45$

$=$

* ——— * ——— * ——— *

$A = \begin{vmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{vmatrix}$

$\det(A) = 1 \begin{vmatrix} 1 & 1 \\ 4 & 2 \end{vmatrix} - 3 \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 5 \begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix}$

$= 1(2-4) - 3(4-3) + 5(8-3)$

$= -2 - 3 + 5 \cdot 5$

$= -5 + 25$

$= 20$ Ans

19 Explore the effect of an elementary row operation on the determinant of a matrix. In each case, state the row operation and describe how it affects the determinant.

(19) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} c & d \\ a & b \end{bmatrix}$

Sol^n: Here,

let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

& $\det(A) = ad - bc$

For matrix $\begin{bmatrix} c & d \\ a & b \end{bmatrix}$

let $B = \begin{vmatrix} c & d \\ a & b \end{vmatrix}$

$\det(B) = bc - ad$

& $\det(B) = -(ad - bc)$

∴ Two rows are interchanged, The determinant changes sign.

*

(20) $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ 5+3K & 6+4K \end{bmatrix}$

Sol^n: Here

let $A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

$\det(A) = 18 - 20 = -2$ &

let $B = \begin{vmatrix} 3 & 4 \\ 5+3K & 6+4K \end{vmatrix}$

$\det(B) = 3(6+4K) - 4(5+3K)$

$= 18 + \cancel{12K} - 20 - \cancel{12K}$

$= -2$

One row times K is added to another row.
The determinant doesnot change

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21 $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\begin{bmatrix} a+Kc & b+Kd \\ c & d \end{bmatrix}$

Sol^n: Here,

let $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$\det(A) = ad - bc$

let $B = \begin{vmatrix} a+Kc & b+Kd \\ c & d \end{vmatrix}$

$= d(a+Kc) - c(b+Kd)$

$= ad + \cancel{Kcd} - bc \cancel{+ Kcd}$

$= ad - bc$

$\det(A) = \det(B)$

$\therefore$ One row times K is added to another row The determinant does not change.

(22) $\begin{bmatrix} 1 & 1 & 1 \\ -3 & 8 & -4 \\ 2 & -3 & 2 \end{bmatrix}$, $\begin{bmatrix} K & K & K \\ -3 & 8 & -4 \\ 2 & -3 & 2 \end{bmatrix}$

Sol^n: Here

let $A = \begin{vmatrix} 1 & 1 & 1 \\ -3 & 8 & -4 \\ 2 & -3 & 2 \end{vmatrix}$

$B = \begin{vmatrix} K & K & K \\ -3 & 8 & -4 \\ 2 & -3 & 2 \end{vmatrix}$

The $\det(A) = 1 \begin{vmatrix} 8 & -4 \\ -3 & 2 \end{vmatrix} - 1 \begin{vmatrix} -3 & -4 \\ 2 & 2 \end{vmatrix} + 1 \begin{vmatrix} -3 & 8 \\ 2 & -3 \end{vmatrix}$

$= (16-12) - 1(-6+8) + 1(9-16)$

$= 4 - 2 - 7$

$= 4 - 9$

$= -5$

$$\det(B) = K\begin{vmatrix} 8 & -4 \\ -3 & 2 \end{vmatrix} - K\begin{vmatrix} -3 & -4 \\ 2 & 2 \end{vmatrix} + K\begin{vmatrix} -3 & 8 \\ 2 & -3 \end{vmatrix}$$

$$= K(16-12) - K(-6+8) + K(9-16)$$

$$= 4K - 2K - 7K$$

$$= -5K$$

$\therefore \det B = K \det(A)$

First row of matrix was multiplied with K and resulting determinant was multiplied with K

(Q3) $\begin{vmatrix} a & b & c \\ 3 & 2 & 2 \\ 6 & 5 & 6 \end{vmatrix}$, $\begin{vmatrix} 3 & 2 & 2 \\ a & b & c \\ 6 & 5 & 6 \end{vmatrix}$

Solⁿ: Here,

let $A = \begin{vmatrix} a & b & c \\ 3 & 2 & 2 \\ 6 & 5 & 6 \end{vmatrix}$

$$\det(A) = a\begin{vmatrix} 2 & 2 \\ 5 & 6 \end{vmatrix} - b\begin{vmatrix} 3 & 2 \\ 6 & 6 \end{vmatrix} + c\begin{vmatrix} 3 & 2 \\ 6 & 5 \end{vmatrix}$$

$$= a(12-10) - b(18-12) + c(15-12)$$

$$= 2a - 6b + 3c$$

let $B = \begin{vmatrix} 3 & 2 & 2 \\ a & b & c \\ 6 & 5 & 6 \end{vmatrix} \Rightarrow 3\begin{vmatrix} b & c \\ 5 & 6 \end{vmatrix} - 2\begin{vmatrix} a & c \\ 6 & 6 \end{vmatrix} + 2\begin{vmatrix} a & b \\ 6 & 5 \end{vmatrix}$

$$= 3(6b-5c) - 2(6a-6c) + 2(5a-6b)$$

$B = \begin{vmatrix} a & b & c \\ 3 & 2 & 2 \\ 6 & 5 & 6 \end{vmatrix}$ $\quad R_1 \longleftrightarrow R_2$

$$= 18b - 15c - 12a + 12c + 10a - 12b$$

$$= -2a - 3c + 6b = -(2a-6b+3c)$$

$$B = a\begin{vmatrix} 2 & 2 \\ 5 & 6 \end{vmatrix} - b\begin{vmatrix} 3 & 2 \\ 6 & 6 \end{vmatrix} + c\begin{vmatrix} 3 & 2 \\ 6 & 5 \end{vmatrix}$$

$$= 2a - 6b + 3c$$

$\therefore$ Two rows were swapped in given matrix which resulted in multiplying determinant with -1

compute the determinants of the elementary matrices

(24) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & K & 1 \end{bmatrix}$

Sol^n: Here

let $A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & K & 1 \end{vmatrix}$

Here the given matrix is Lower triangular matrix so that the determinant of A is obtained by multiplying its diagonal elements Such that

$\det(A) = a_{11} \cdot a_{22} \cdot a_{33} = 1 \times 1 \times 1$

$\det(A) = 1$

(25) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ K & 0 & 1 \end{bmatrix}$

Sol^n: Here,

let $A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ K & 0 & 1 \end{vmatrix}$

Same as above,

$\det(A) = 1 \times 1 \times 1$

$= 1$ Ans

(26) $\begin{bmatrix} K & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Sol^n: Here

let $A = \begin{vmatrix} K & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = K$

classmate

$\det(A) = K \cdot 1 \cdot 1 = K$

$= K$ Ans

27. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Soln: Here,

let $A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$\det(A) = 1 \cdot K \cdot 1$

$= K$ Ans

28. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Soln: Here,

let $A = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$R_1 \leftrightarrow R_2$

$A = -\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$\det(A) = -1\,(1 \cdot 1 \cdot 1)$

$= -1$ Ans

29 $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Sol^n: Here

let $A = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$

$A = -\begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}$ $\quad R_3 \leftrightarrow R_2$

$A = -\left[- \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \right]$ $\quad R_1 \leftrightarrow R_2$

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$A = -\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ $\quad R_2 \leftrightarrow R_3$

$\det(A) = -1\,(1.1.1)$

$= -1$ Ans

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(31) ~~What is the~~

verify that $\det(BA) = \det(B).\det(A)$, where B is the elementary matrix shown ~~and~~ $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Sol^n: Here

let $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ & $B = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

$\det(A) = ad - bc$ & $\det(B) = (0-1)$

$= ad - bc$ $\quad = -1$

$\det(A) \cdot \det(B) = -1(ad-bc) = bc - ad$

Now

$AB = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \cdot \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

$= \begin{vmatrix} b & a \\ d & c \end{vmatrix}$

then,

$\det(AB) = bc - ad$

$\therefore \det(AB) = \det(B) \cdot \det(A)$. verified.

Sol^n:-

33 $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

let $B = \begin{vmatrix} 1 & K \\ 0 & 1 \end{vmatrix}$

$\det(A) = ad - bc$

$\det(B) = (1-0) = 1$

$BA = \begin{vmatrix} 1 & K \\ 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$= \begin{vmatrix} a+Kc & b+dK \\ c & d \end{vmatrix}$

$\det(BA) = ad + \cancel{Kcd} - bc - \cancel{cdK}$

$= ad - bc$

$\det(A) \cdot \det(B) = 1(\cancel{ah}\ ad - bc)$

$\therefore \det(BA) = \det(A) \cdot \det(B)$ verified.

(34) let $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$. write $5A$. Is $\det(5A) = 5\det(A)$?

Sol^n: Here

$A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

$5A = 5\begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix}$

$= \begin{vmatrix} 15 & 5 \\ 20 & 10 \end{vmatrix}$

$\det(5A) = 150 - 100$

$= 50.$

$\det(A) = (6-4)$

$= 2$

$5\det(A) = 5 \times 2 = 10$

 $\det(5A) \neq 5\det(A)$

95. let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and let K be a Scalar. find a formula that relates $\det(KA)$ to K and $\det(A)$

Soln: Here,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$KA = \begin{bmatrix} Ka & Kb \\ Kc & Kd \end{bmatrix}$$

$$\det(KA) = K^2 ad - K^2 bc$$
$$= K^2 (ad - bc)$$

$$\det(A) = (ad - bc)$$

$$\therefore \det(KA) = K^2 \det(A).$$

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Exercise :- 4.2

Properties of determinant

Theorem :- (Row operation)

let A be a square matrix

(a) If a multiple of one row of A is added to another row to produce a matrix B, then det(A) = det(B)

(b) If two rows of A interchanged to produce of a matrix B, then det(A) = - det(B)

(c) If one row of A is multiplied by K (scalar) to produce a matrix B, then K det(A) = det(B).

*. Using row operation, compute det(A) where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & -4 \\ 3 & 7 & 4 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & -4 \\ 3 & 7 & 4 \end{vmatrix}$$

$R_3 \to R_3 - 3R_1$

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & -4 \\ 0 & 1 & -5 \end{vmatrix}$$

$R_3 \to R_3 - \frac{1}{5} R_2$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & -4 \\ 0 & 0 & -21/5 \end{vmatrix}$$

$\det(A) = 1 \times 5 \times -21/5 = -21$

then By expansion,

$$\det(A) = 1\begin{vmatrix} 5 & -4 \\ 7 & 4 \end{vmatrix} - 2\begin{vmatrix} 0 & -4 \\ 3 & 4 \end{vmatrix} + 3\begin{vmatrix} 0 & 5 \\ 3 & 7 \end{vmatrix}$$

$$= (20+28) - 2(0+12) + 3(-15)$$

$$= 48 - 24 - \cancel{30}\,45$$

$$= -21$$

$\therefore$,

applying case ⓑ

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & -4 \\ 3 & 7 & 4 \end{vmatrix}$$

$\det(A) = -21$

$R_1 \longleftrightarrow R_2$

$$A = \begin{vmatrix} 0 & 5 & -4 \\ 1 & 2 & 3 \\ 3 & 7 & 4 \end{vmatrix}$$

$$\det A = -5\begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix} - 4\begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix}$$

$$= -5(4-9) - 4(7-6)$$

$$= 25 - 4$$

$$= 21.$$

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Theorem :- A square matrix A is invertible if and only if $\det(A) \neq 0$

NOTE : A set of vectors $\{v_1, v_2, v_3 \ldots v_n\}$ where $v \in \mathbb{R}^n$, are linearly dependent if $\det\{v_1, v_2, v_3 \ldots v_n\} = 0$ and linearly independent if $\det\{v_1, v_2, v_3, \ldots v_n\} \neq 0$

Ex. Use determinant to decide v_1, v_2, v_3, v_4 are linearly independent or not where

$$v_1 = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 5 \\ 3 \\ -5 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ -7 \\ 6 \\ 4 \end{bmatrix}, v_4 = \begin{bmatrix} -1 \\ 3 \\ 2 \\ -2 \end{bmatrix}$$

$$\det(v_1, v_2, v_3, v_4) = \begin{vmatrix} 0 & 1 & 2 & -1 \\ 2 & 5 & -7 & 3 \\ 0 & 3 & 6 & 2 \\ -2 & -5 & 4 & -2 \end{vmatrix}$$

$R_4 \to R_4 + R_2$

$$= \begin{vmatrix} 0 & 1 & 2 & -1 \\ 2 & 5 & -7 & 3 \\ 0 & 3 & 6 & 2 \\ 0 & 0 & -3 & 1 \end{vmatrix}$$

$$\det(v_1, v_2, v_3, v_4) = 0 - 2 \begin{vmatrix} 1 & 2 & -1 \\ 3 & 6 & 2 \\ 0 & -3 & 1 \end{vmatrix} + 0 - 0$$

$= R_2 \to R_2 - 3R_1$

$$= -2 \begin{vmatrix} 1 & 2 & -1 \\ 0 & 0 & 5 \\ 0 & -3 & 1 \end{vmatrix}$$

$$= -2 \left| 1 \begin{bmatrix} 0 & 5 \\ -3 & 1 \end{bmatrix} \right|$$

$$= -2 \cdot (15)$$

$$= -30 \neq 0$$

$\therefore$ the set of vector $\{V_1, V_2, V_3, V_4\}$ are linearly independent.

Exercise:- 4.2

Using properties of determinant, show that

1. $\begin{vmatrix} 0 & 5 & -2 \\ 1 & -3 & 6 \\ 4 & -1 & 8 \end{vmatrix} = - \begin{vmatrix} 1 & -3 & 6 \\ 0 & 5 & -2 \\ 4 & -1 & 8 \end{vmatrix}$

Sol^n: Here,

let $A = \begin{vmatrix} 0 & 5 & -2 \\ 1 & -3 & 6 \\ 4 & -1 & 8 \end{vmatrix}$

$\det A = 0 - 5 \begin{vmatrix} 1 & 6 \\ 4 & 8 \end{vmatrix} - 2 \begin{vmatrix} 1 & -3 \\ 4 & -1 \end{vmatrix}$

$\det(A) = 0 - 5(8-24) - 2(-1+12)$

$= -5 \cdot -16 - 2 \cdot 11$

$= 80 - 22$

$= \cancel{-102}\ 58$

let $B = \begin{vmatrix} 1 & -3 & 6 \\ 0 & 5 & -2 \\ 4 & -1 & 8 \end{vmatrix}$ $R_1 \leftrightarrow R_2$ $\begin{vmatrix} 0 & 5 & -2 \\ 1 & -3 & 6 \\ 4 & -1 & 8 \end{vmatrix}$

$\det(B) = 1 \begin{vmatrix} 5 & -2 \\ -1 & 8 \end{vmatrix} + 3 \begin{vmatrix} 0 & -2 \\ 4 & 8 \end{vmatrix} + 6 \begin{vmatrix} 0 & 5 \\ 4 & -1 \end{vmatrix}$

$= 1(40-2) + 3(0+8) + 6(0-20)$

$= 38 + 24 - 120$

$= -58$

$\det(B) = - \begin{vmatrix} 1 & -3 & 6 \\ 0 & 5 & -2 \\ 4 & -1 & 8 \end{vmatrix}$

$\therefore \det(A) = -\det(B)$

$\therefore \begin{vmatrix} 0 & 5 & -2 \\ 1 & -3 & 6 \\ 4 & -1 & 8 \end{vmatrix} = - \begin{vmatrix} 1 & -3 & 6 \\ 0 & 5 & -2 \\ 4 & -1 & 8 \end{vmatrix}$

2. $\begin{vmatrix} 2 & -6 & 4 \\ 3 & 5 & -2 \\ 1 & 6 & 3 \end{vmatrix} = 2 \begin{vmatrix} 1 & -3 & 2 \\ 3 & 5 & -2 \\ 1 & 6 & 3 \end{vmatrix}$

Soln: Here

let $A = \begin{vmatrix} 2 & -6 & 4 \\ 3 & 5 & -2 \\ 1 & 6 & 3 \end{vmatrix}$

$\det(A) = 2 \begin{vmatrix} 5 & -2 \\ 6 & 3 \end{vmatrix} + 6 \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} + 4 \begin{vmatrix} 3 & 5 \\ 1 & 6 \end{vmatrix}$

$= 2(15+12) + 6(9+2) + 4(18-5)$

$= 2(27) + 6(11) + 4(13)$

$= 54 + 66 + 52$

$= 172$

let $B = \begin{vmatrix} 1 & -3 & 2 \\ 3 & 5 & -2 \\ 1 & 6 & 3 \end{vmatrix}$

$\det(B) = 1 \begin{vmatrix} 5 & -2 \\ 6 & 3 \end{vmatrix} + 3 \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} + 2 \begin{vmatrix} 3 & 5 \\ 1 & 6 \end{vmatrix}$

$= (15+12) + 3(9+2) + 2(18-5)$

$= 27 + 3\cdot 11 + 2\cdot 13$

$= 27 + 33 + 26$

$= 86$

$2\det(B) = 2 \times 86$

$= 72$

$\because \det(A) = 2\det(B)$

$\therefore \begin{vmatrix} 2 & -6 & 4 \\ 3 & 5 & -2 \\ 1 & 6 & 3 \end{vmatrix} = 2 \begin{vmatrix} 1 & -3 & 2 \\ 3 & 5 & -2 \\ 1 & 6 & 3 \end{vmatrix}$

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3. $\begin{vmatrix} 1 & 3 & -4 \\ 2 & 0 & -3 \\ 5 & -4 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 3 & -4 \\ 0 & -6 & 5 \\ 5 & -4 & 7 \end{vmatrix}$

sol: Here

let $A = \begin{vmatrix} 1 & 3 & -4 \\ 2 & 0 & -3 \\ 5 & -4 & 7 \end{vmatrix}$

$\det(A) = 1 \begin{vmatrix} 0 & -3 \\ -4 & 7 \end{vmatrix} - 3 \begin{vmatrix} 2 & -3 \\ 5 & 7 \end{vmatrix} - 4 \begin{vmatrix} 2 & 0 \\ 5 & -4 \end{vmatrix}$

$= (0-12) - 3(14+15) - 4(-8-0)$

$= -12 - 3 \cdot 29 + 32$

$= \cancel{-12}\, 20 - 87$

$= -67$

let $B = \begin{vmatrix} 1 & 3 & -4 \\ 0 & -6 & 5 \\ 5 & -4 & 7 \end{vmatrix}$

$\det(B) = 1 \begin{vmatrix} -6 & 5 \\ -4 & 7 \end{vmatrix} - 3 \begin{vmatrix} 0 & 5 \\ 5 & 7 \end{vmatrix} - 4 \begin{vmatrix} 0 & -6 \\ 5 & -4 \end{vmatrix}$

$= (-42+20) - 3(0-25) - 4(0+30)$

$= -22 + 75 - 120$

$= -67$

$\therefore \det(A) = \det(B)$

$\therefore \begin{vmatrix} 1 & 3 & -4 \\ 2 & 0 & -3 \\ 5 & -4 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 3 & -4 \\ 0 & -6 & 5 \\ 5 & -4 & 7 \end{vmatrix}$

(4) $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & -4 \\ 3 & 7 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & -4 \\ 0 & 1 & -5 \end{vmatrix}$

Solⁿ: Here

let $A = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & -4 \\ 3 & 7 & 4 \end{vmatrix}$

$\det(A) = 1 \begin{vmatrix} 5 & -4 \\ 7 & 4 \end{vmatrix} - 2 \begin{vmatrix} 0 & -4 \\ 3 & 4 \end{vmatrix} + 3 \begin{vmatrix} 0 & 5 \\ 3 & 7 \end{vmatrix}$

$= (20+28) - 2(0+12) + 3(0-15)$

$= 48 - 24 - 45$

$= -21$

let $B = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & -4 \\ 0 & 1 & -5 \end{vmatrix}$

$\det B = 1 \begin{vmatrix} 5 & -4 \\ 1 & -5 \end{vmatrix}$

$= (-25+4)$

$= -21$

$\therefore \det(A) = \det(B)$

$\therefore \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & -4 \\ 3 & 7 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & -4 \\ 0 & 1 & -5 \end{vmatrix}$ Ans

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Find the determinants by row reduction to echelon form.

5. $\begin{vmatrix} 1 & 5 & -6 \\ -1 & -4 & 4 \\ -2 & -7 & 9 \end{vmatrix}$

Sol^n: Here,

let $A = \begin{vmatrix} 1 & 5 & -6 \\ -1 & -4 & 4 \\ -2 & -7 & 9 \end{vmatrix}$

$R_2 \to R_2 + R_1$ and $R_3 \to R_3 + 2R_1$

$A = \begin{vmatrix} 1 & 5 & -6 \\ 0 & 1 & -2 \\ 0 & 3 & -3 \end{vmatrix}$

$R_3 \to R_3 - 3R_2$

$= \begin{vmatrix} 1 & 5 & -6 \\ 0 & 1 & -2 \\ 0 & 0 & 3 \end{vmatrix}$

$\det(A) = 1 \cdot 1 \cdot 3$

$= 3$ Ans

6. $\begin{vmatrix} 1 & 5 & -3 \\ 3 & -3 & 3 \\ 2 & 13 & -7 \end{vmatrix}$

Sol^n: Here,

let $A = \begin{vmatrix} 1 & 5 & -3 \\ 3 & -3 & 3 \\ 2 & 13 & -7 \end{vmatrix}$

$R_2 \to R_2 - 3R_1$ and $R_3 \to R_3 - 2R_1$

$A = \begin{vmatrix} 1 & 5 & -3 \\ 0 & -18 & 12 \\ 0 & 3 & -1 \end{vmatrix}$

$R_3 \to R_3 + \frac{1}{6} R_2$

$A = \begin{vmatrix} 1 & 5 & -3 \\ 0 & -18 & 12 \\ 0 & 0 & 1 \end{vmatrix}$

$\det(A) = 1 \cdot -18 \cdot 1$

$= -18$ Ans

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7 $\begin{vmatrix} 1 & 3 & 0 & 2 \\ -2 & -5 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & -3 \end{vmatrix}$

Solⁿ: Here

let $A = \begin{vmatrix} 1 & 3 & 0 & 2 \\ -2 & -5 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & -3 \end{vmatrix}$

$R_2 \to R_2 + 2R_1,\quad R_3 \to R_3 - 3R_1,\quad R_4 \to R_4 - R_1$

$A = \begin{vmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 0 & 8 \\ 0 & -4 & 0 & -5 \\ 0 & -4 & 2 & -5 \end{vmatrix}$

$R_3 \to R_3 + 4R_2$ and $R_4 \to R_4 + 4R_2$

$A = \begin{vmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 0 & 27 \\ 0 & 0 & 0 & 27 \end{vmatrix}$

$\det(A) = 1 \cdot 1 \cdot 0 \cdot 27 = 0$

$= 0$ Ans

8. $\begin{vmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 5 & 4 & -3 \\ -3 & -7 & -5 & 2 \end{vmatrix}$

Soln: Here

let $A = \begin{vmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 5 & 4 & -3 \\ -3 & -7 & -5 & 2 \end{vmatrix}$

$R_3 \rightarrow R_3 - 2R_1$ and $R_4 \rightarrow R_4 + 3R_1$

$A = \begin{vmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & -1 & -2 & 5 \\ 0 & 2 & 4 & -10 \end{vmatrix}$

$R_3 \rightarrow R_3 + R_2$ and $R_4 \rightarrow R_4 - 2R_2$

$A = \begin{vmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$

$\det(A) = 1 \cdot 1 \cdot 0 \cdot 0$

$= 0$ Ans

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(9) $\begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ -1 & 2 & 8 & 5 \\ 3 & -1 & -2 & 3 \end{vmatrix}$

Solⁿ: Here

$R_3 \to R_3 + R_1$ and $R_4 \to R_4 - 3R_1$

$$A = \begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 0 & 1 & 5 & 5 \\ 0 & 2 & 7 & 3 \end{vmatrix}$$

$R_3 \to R_3 - R_2$ and $R_4 \to R_4 - 2R_2$

$$A = \begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -3 & -5 \end{vmatrix}$$

~~det A =~~

$$\det(A) = \begin{vmatrix} 1 & 5 & 4 \\ 0 & 0 & 1 \\ 0 & -3 & -5 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 0 & 1 \\ -3 & -5 \end{vmatrix}$$

$$= 1.(0+3)$$

$$= 3 \quad \text{Ans}$$

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(10)
$$\begin{vmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ -2 & -6 & 2 & 3 & 9 \\ 3 & 7 & -3 & 8 & -7 \\ 3 & 5 & 5 & 2 & 7 \end{vmatrix}$$

Sol^n: Here

let $A = \begin{vmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ -2 & -6 & 2 & 3 & 9 \\ 3 & 7 & -3 & 8 & -7 \\ 3 & 5 & 5 & 2 & 7 \end{vmatrix}$

$R_3 \to R_3 + 2R_1$, $R_4 \to R_4 - 3R_1$, $R_5 \to R_4 - 3R_1$

$A = \begin{vmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ 0 & 0 & 0 & 3 & 5 \\ 0 & -2 & 0 & 0 & -1 \\ 0 & -4 & 8 & 2 & 13 \end{vmatrix}$

$R_4 \to R_4 + R_2$, $R_5 \to R_5 + 2R_2$

$A = \begin{vmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ 0 & 0 & 0 & 3 & 5 \\ 0 & 0 & -4 & -1 & -7 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$

$\det(A) = 1 \begin{vmatrix} 1 & 3 & -1 & 0 \\ 0 & 2 & -4 & -1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & -4 & -1 \end{vmatrix}$

$= -3 \begin{vmatrix} 1 & 3 & -1 \\ 0 & 2 & -4 \\ 0 & 0 & -4 \end{vmatrix}$ $\Rightarrow -3 \cdot (-4 \cdot 2)$

$\Rightarrow 24$ Ans

$= -3 \cdot \left[-4 \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}\right]$

Combine the methods of row reduction and co-factor expansion to compute the determinants.

(11) $\begin{vmatrix} 2 & 5 & -3 & -1 \\ 3 & 0 & 1 & -3 \\ -6 & 0 & -4 & 9 \\ 4 & 10 & -4 & -1 \end{vmatrix}$

Sol^n: Here,

let $A = \begin{vmatrix} 2 & 5 & -3 & -1 \\ 3 & 0 & 1 & -3 \\ -6 & 0 & -4 & 9 \\ 4 & 10 & -4 & -1 \end{vmatrix}$

$= R_3 \to R_3 + 2R_2, \quad R_4 \to R_4 - 2R_1$

$A = \begin{vmatrix} 2 & 5 & -3 & -1 \\ 3 & 0 & 1 & -3 \\ 0 & 0 & -2 & 3 \\ 0 & 0 & 2 & 1 \end{vmatrix}$

$R_4 \to R_4 + R_3$

$A = \begin{vmatrix} 2 & 5 & -3 & -1 \\ 3 & 0 & 1 & -3 \\ 0 & 0 & -2 & 3 \\ 0 & 0 & 0 & 4 \end{vmatrix}$

$R_2 \to R_2 - 3/2 \, R_1$

$A = \begin{vmatrix} 2 & 5 & -3 & -1 \\ 0 & -15/2 & 11/2 & -3/2 \\ 0 & 0 & -2 & 3 \\ 0 & 0 & 0 & 4 \end{vmatrix}$

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factor

$\det(A) = 2 \cdot -15/2 \cdot -2 \cdot 4$

$= 15 \cdot 2 \cdot 4$

$= 120$ **Ans**

(12)
$$\begin{vmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 5 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{vmatrix}$$

Sol^n^: let $A = \begin{vmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 5 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{vmatrix}$

$R_2 \to R_2 + 3R_1,\ R_3 \to R_3 + 5R_1,\ R_4 \to R_4 + 4R_1$

$$= \begin{vmatrix} -1 & 2 & 3 & 0 \\ 0 & 10 & 12 & 0 \\ 0 & 14 & 21 & 6 \\ 0 & 10 & 16 & 3 \end{vmatrix}$$

~~$R_2 \to \frac{1}{2} R_2,\ R_3 \to \frac{1}{7} R_3$, R_4~~

$$= -1 \begin{vmatrix} 10 & 12 & 0 \\ 14 & 21 & 6 \\ 10 & 16 & 3 \end{vmatrix}$$

$$\cancel{= \begin{vmatrix} -1 & 2 & 3 & 0 \\ 0 & 10 & 12 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & 10 & 16 & 3 \end{vmatrix}}$$

$$\cancel{-1 \begin{vmatrix} 5 & 6 & 0 \\ 2 & 3 & 0 \\ 10 & 16 & 3 \end{vmatrix}}$$

$\cancel{-1 \left[3 \begin{bmatrix} 5 & 6 \\ 2 & 3 \end{bmatrix} \right]}$

$\cancel{-1 \left[3 (15-12) \right]}$

$\cancel{-1 \cdot 3 \cdot 3}$

$\cancel{-9}$ ~~Ans~~

$$= -1 \left| 3 \begin{vmatrix} 10 & 12 \\ 14 & 21 \end{vmatrix} - 6 \begin{vmatrix} 10 & 12 \\ 10 & 16 \end{vmatrix} \right|$$

$= -1 \left| 3(210-168) - 6(160-120) \right|$

$= -1(3 \cdot (-42)) - 240$

$= (126 - 240)$

$= 114$ **Ans**