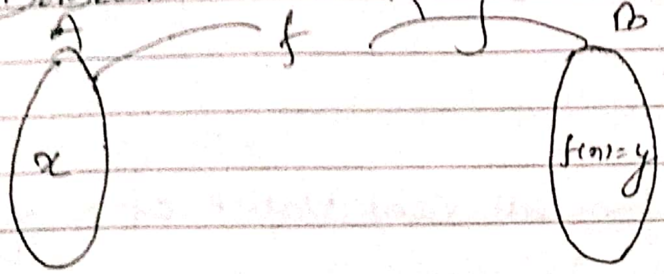
Number System.

i) Natural number (N) → $\{1, 2, 3, \ldots\ldots\}$
ii) Integer (Z or I) → $\{\ldots -3, -2, -1, 0, 1, 2, 3, 4 \ldots\}$
iii) Rational number (Q) → $\{x : x = p/q,\ q \neq 0\ \ p, q \in Z\}$
iv) Irrational Number

Domain and Range.



$x \rightarrow$ Domain
$y \rightarrow$ co-domain
the value of y is called Range.

- Domain
for domain, for all real value of x, y is exist.
so, it provide then after we get absolute Domain value.

- Range
for Range, for all real value of y, x is exist.
then ~~after~~ we get absolute Range.

while finding Domain, we let $y = f(x)$, e.g x^2, $y = x^2$ → y gives domain such as $(-\infty, \infty)$	while finding range we let. $y = x^2$ $x = \sqrt{y}$. this gives Range with respect to y. $[0, \infty)$

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In case, of Root.
Incase the expr eqn is in the Root the domain may give negative interval but the range gives ~~psi~~ only positive value.

example-1.
Identify the domain and range function
① $y=x^2$.
Soln. Here
$y=x^2$
For domain, for all real value of x, y is exist so,
$y=x^2$,
It's domain is $(-\infty,\infty)$

For range,
$y=x^2$
$x=\sqrt{y}$
For range, for $y\geq 0$ x is defined so,
the range of the given function is $[0,\infty)$

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(ii) Gr $y=1/x$
Soln. Here,
$y=1/x$
For domain, $y=1/x$, when $x=0$ the value of y doesnot exist so, the domain is $R-\{0\}$ or $(-\infty,0)\cup(0,\infty)$

For range,
$y=1/x$ when $y=0$,
$x=1/y$, the value of x is doesnot exist so, the range of the given function is $R-\{0\}$ or $(-\infty,0)\cup(0,\infty)$

(iii) Given $y=\sqrt{4-x}$
For domain
$y=\sqrt{4-x}$
$4-x\geq 0$
$x-4\leq 0$
$x\leq 4$
When $x\leq 4$ then the value of y exist
So, the domain is $(-\infty,4]$
For range
$y^2=4-x$ or, $x=4-y^2$
~~$4-y^2\geq 0$~~ $4-x\geq 0$
$x\leq 4$. For all real value of y
x is exist/defined so that the value of y is non-negative so the range of the function is $[0,\infty)$

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(R) $y = \sqrt{9-x^2}$

For Domain, y is exist for $y \geq 0$

$9-x^2 \geq 0$

$x^2-9 \leq 0$

$(x+3)(x-3) \leq 0$

$-3 \leq x \leq 3$

so the domain is $[-3,3]$

For Range,

$y^2 = 9-x^2$

$x^2 = 9-y^2$

$x = \sqrt{9-y^2}$

$x \geq 0$ or, $9-y^2 \geq 0$ then the value of x is defined so,

$9-y^2 \geq 0$

$y^2-9 \geq 0$

$-3 \leq y \leq 3$

the range of the function is $-3 \leq y \leq 3$ but the y is non-negative hence range of the function is $[0,3]$

Exercise: 1.1

1. Evaluate the difference quotient for given function:

(i) $f(x) = 4-3x$; $\frac{f(3+h)-f(3)}{h}$

Soln: Here.

$= \frac{f(3+h)-f(3)}{h}$

$= \frac{4-3(3+h)-(4-3\times3)}{h}$

$= \frac{4-9-3h-(4-9)}{h}$

$= \frac{-5-3h+5}{h}$

$= -3$ Ans//

(ii) $f(x) = \frac{x+3}{x+1}$; $\frac{f(x)-f(1)}{x-1}$

Soln: Here.

$= \frac{f(x)-f(1)}{x-1}$

$= \frac{\frac{x+3}{x+1} - \frac{1+3}{1+1}}{x-1}$

$= \frac{\frac{x+3}{x+1} - 4/2}{x-1}$

$= \frac{x+3-2(x+1)}{(x+1)(x-1)}$

$= \frac{x+3-2x-2}{(x+1)(x-1)}$

$= \frac{1-x}{(x+1)(x-1)} = \frac{-1}{x+1}$

Ans

(2) Find the domain of the function

(i) $f(x) = \frac{x+4}{x^2-9}$.

For domain let

$y = \frac{x+4}{x^2-9}$ if $x=3$ then the value of y doesnot exist hence

$x^2-9 \geq 0$, the value of y exist.

or $x^2-9 \geq 0$

or, $(x+3)(x-3) \geq 0$

$-3 \geq x \geq 3$

So, the domain of the given function is $(-\infty,3) \cup (-3,3) \cup (3,\infty)$

(ii) $f(x) = \frac{2x^3-5}{x^2+x-6}$.

Sol^n: Here,

For domain,

$x^2+x-6 \geq 0$

$x^2+(3-2)x-6 \geq 0$

$x^2+3x-2x-6 \geq 0$

$x(x+3)-2(x+3) \geq 0$

$(x-2)(x+3) \geq 0$

$x \geq 2$

$x \geq -3$

$-3 \geq x \geq 2$, the value of y doesnot exist if $x = 3$, and 2 So, the domain of the given function is $(-\infty,-3) \cup (-3,2) \cup (2,\infty)$.

$(-\sqrt[3]{7})^3$ $(-3)^{3 \times 1/3}$ -3. 28 8

13 02 -1,1,0 -1.

(vi) $G(x) = \frac{3x+|x|}{x}$

Sol^n:

For domain

$y = \frac{3x+|x|}{x}$

for all the real value of x Y is defined except 0, so the Domain is $R-\{0\}$

$(-\infty,0) \cup (0,\infty)$

(iv) $\sqrt{3-t} - \sqrt{2+t}$

Sol^n: here

For domain.

$y = \sqrt{3-t} - \sqrt{2+t}$.

the value of y exist if only $t = 3$ and -2

So, the domain is $[3,-2]$

(v) $f(p) = \sqrt{2-\sqrt{p}}$.

Sol^n: here,

For domain

$y = \sqrt{2-\sqrt{p}}$.

$p \leq 4$.

the value of y exist only when $p \leq 4$

$0-\sqrt{5}$

(iii) $f(t) = \sqrt[3]{2t-1}$

For domain $f(t)$ exist

$f(t) = \sqrt[3]{2t-1} = (2t-1)^{1/3}$

Since for all values of t, Domain $(-\infty,\infty)$

cube root doesnot give complex number and it is negative function, so it's range may be in negative.

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3. Find domain and range of the function

(i) $h(x) = \sqrt{4-x^2}$

Sol^n: Here,

let $y = \sqrt{4-x^2}$

For domain,

$4-x^2 \geq 0$

$x^2-4 \leq 0$

$(x+2)(x-2) \leq 0$

$-2 \leq x \leq 2$

the value of y ~~doesnot~~ exist if $x = \pm 2$ so, that the domain of the given function is defined between -2 and 2

So, domain is $[-2, 2]$

For range,

$y = \sqrt{4-x^2}$

$y^2 = 4-x^2$

$x^2 = 4-y^2$ or $x = \sqrt{4-y^2}$

$4-y^2 \geq 0$

$y^2-4 \leq 0$

$(y+2)(y-2) \leq 0$

$-2 \leq y \leq 2$

the value of x is defined at -2 and 2 but the value of y is non-negative so, the range of the given function is $[0, 2]$

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$x-3 \geq 0$

$x \geq 3$

$(-\infty, 3)$

(ii) $f(x) = \sqrt{x-5}$

For domain

let $y = \sqrt{x-5}$

$x-5 \geq 0$

$x \geq 5$

the value of y exist if $x \geq 5$ so, the domain of the given function is $[5, \infty)$

For range,

$y^2 = x-5$

$y^2 = x-5$

$x = y^2+5$

~~the value~~ for all real value of y x is defined so the range of the given function is ~~$(-\infty, \infty)$~~ non-negative i.e $[0, \infty)$

(iii) $g(x) = \dfrac{2x+1}{x-3}$

Sol^n: Here let $y = \dfrac{2x+1}{x-3}$ $\quad R - \{3\}$

For domain,

$x-3 \geq 0$

$x \geq 3$, the ~~value of y~~ for all real value of x, y is defined except 3 so, the domain is $(-\infty, 3) \cup (3, \infty)$

For Range,

$y = \dfrac{2x+1}{x-3}$

$xy - 3y = 2x+1$

$xy - 2x = 1+3y$

$x(y-2) = 1+3y$

$x = \dfrac{1+3y}{y-2}$

$y-2 \geq 0$

$y \geq 2$

For all real value of ~~the~~ x is defined except 2 so, the range is $(-\infty, 2) \cup (2, \infty)$

Graph function → A curve in the xy plane is the graph of a function of x if and only if no vertical line intersect the curve more than once

G.F Not G.F.

Q. Identify which one is graph of functions

(i) $y=x+2$

Given eqn

$y=x+2$

put $x=0$

$y=2$

Here vertical line $x=0$ meet the curve at only one points $y=2$

Thus $y=x+2$ is a graph of function.

(ii) $x=y^2$

Soln:

Given eqn

$x=y^2$

put $x=0$

$y=0$

Here vertical line $x=0$ meet the curves at only one poin is not a graph of function because the resulting value of y is 0

(iii) $y=x^2$

Given eqn

$y=x^2$

put $x=0$

vertical line $x=0$ meet the curve at only one point thus $y=x^2$ is the graph of function

(iv) $y=-\sqrt{x+2}$

Soln:

Given eqn.

$y=-\sqrt{x+2}$

put $x=0$ then $y=-\sqrt{2}$

the vertical line $x=0$ meet the curve at only one point thus $y=-\sqrt{x+2}$ is a graph of function

(v) $x^2+y=5$

Soln Here

Given eqn

$x^2+y=5$

put $x=0$ then $y=5$

vertical line $x=0$ meet the curve at only one point thus $x^2+y=5$ is graph of function

(vi) $x=y^2-2$

Soln:

Given eqn

$x=y^2-2$

put $x=0$

$0=y^2-2$

$y^2=2$

$y=\pm\sqrt{2}$

$y=\pm 2$

verticle line at $x=0$ meet the curve at two point thus $x=y^2-2$ is not a graph of function.

5. Determine whether the following function even or odd or neither

If $f(x) = f(-x) = f(x)$ is even function
but $f(x) = f(-x) = -f(x)$ is odd "

(i) $f(x) = \frac{x^2}{x^4+1}$

Sol^n: Here
$f(x) = \frac{x^2}{x^4+1}$

So, $f(-x) = \frac{x^2}{x^4+1} \therefore f(x)$

So, the $f(x) = \frac{x^2}{x^4+1}$ is even function.

(ii) $g(x) = x|x|$

Sol^n: here
$g(x) = x|x|$

So, $g(-x) = -x|-x| = -(x|x|) = -g(x)$

Since $g(x) = x|x|$ is a odd function.

(iii) $h(x) = 1+x^3-x^5$

Sol^n: here
$h(x) = 1+x^3-x^5$

so $h(-x) = 1-x^3+x^5 = -(-1+x^3-x^5) \neq h(x)$

Since $h(x) = 1+x^3-x^5$ is neither even or nor odd function.

(iv) $f(x) = 2|x|+1$

Sol^n: Here
$f(x) = 2|x|+1$
So, $f(-x) = 2|-x|+1$
$f(-x) = 2|x|+1 = f(x)$

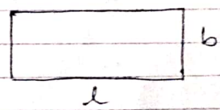
Since $f(x) = 2|x|+1$ is a even function.

(v) $g(x) = 3$

Sol^n: $g(x) = 3$
So $g(-x) = 3 = g(x)$

Since $g(x) = 3$ is a even function.

6. A rectangular has perimeter 20 m. Express the area of the rectangles as a function of the length of one its side



Sol^n: Here,
perimeter of a rectangle
i.e. $2(l+b) = 20$
or, $l+b = 10$
$b = 10-l$.

then, the ~~long~~ Area of rectangles ~~also~~ $= l \times b$

So, the area of the rectangles as a function of the length of one its side is

$= l \times b$

$= l \times (10-l)$

$\therefore l(10-l)$ is the correct Ans.

(7) A rectangles has area $16m^2$. Express the perimeter of the rectangles as a function of the length of one of its sides.

Area of rectangle i.e. $A = l \times b$.

$16 = l \times b$

$\frac{16}{l} = b$

Now:

the perimeter of the rectangle is $2(l+b)$

So Now

perimeter $= 2(l+b)$

$p = 2\left(l + \frac{16}{l}\right)$

$p = \left(\frac{2l^2 + 32}{l}\right)$

or $p =$ [illegible] $\frac{32}{l} + 2l$ Ans

(8) example 8: A function defined by

$$f(x) = \begin{cases} 1-x & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$$

Evaluate $f(-3)$, $f(-1)$ and $f(1)$ and sketch the graph.

So,

$f(-3) = 1-(-3) = 4 = (-3, 4)$

$f(-1) = 1-(-1) = 2 = (-1, 2)$

$f(1) = 1^2 = 1 = (1, 1)$

Required points are $(-3,4)$, $(-1,2)$; $(1,1)$

For curve,

$y = x^2$

put $x = 1$

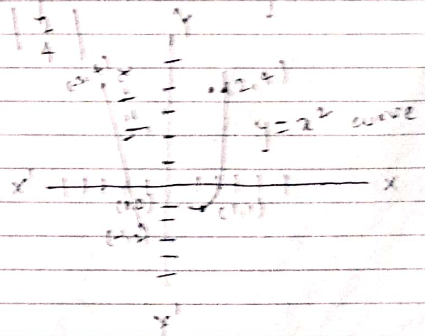
$y = 1$

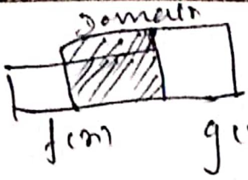
$y = x^2$

put $x = 2$

$y = 4$

x	1	2
y	1	4





[0,2)

[0,2]

Combination of functions:

Let f and g are two functions then $f+g$, $f-g$, $f \cdot g$ and f/g are new function which are defined by:

Sum: $(f+g)(x) = f(x) + g(x)$

Diff.: $(f-g)x = f(x) - g(x)$

product $(f \cdot g)(x) = f(x) \cdot g(x)$. [That is not composi

Quotient: $\left(f/g\right)x = \dfrac{f(x)}{g(x)}$ as long as $g(x) \neq 0$

Example 1: If functions are defined by the formulas

$f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$

Sol^n: Here

$f(x) = \sqrt{x}$, $g(x) = \sqrt{2-x}$

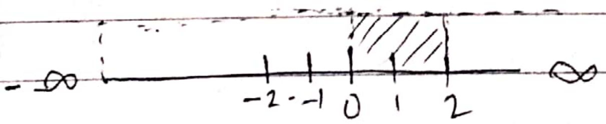
$f+g = \sqrt{x} + \sqrt{2-x}$

Domain for $f(x) = \sqrt{x}$ is $x \geq 0 = [0, \infty)$

Domain for $g(x) = \sqrt{2-x}$ is $2-x \geq 0$ $(-\infty, 2]$

$x \leq 0$

Domain for $(f+g).(x)$ is $A \cap B$.



[0,2]

domain for $(f-g).(x)$: $(\sqrt{x} - \sqrt{2-x}) = [0,2]$

Again

$f \cdot g\,(x) = \sqrt{x}\,\sqrt{(x-2)}$

$= \sqrt{x^2 - 2x}$

domain is $[0,2]$

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Again

$f/g\,(x) = \dfrac{\sqrt{x}}{\sqrt{2-x}}$

domain for f/g is $[0,2)$

~~domain~~ for $g/f = \dfrac{\sqrt{2-x}}{\sqrt{x}}$

domain for $g/f = (0,2]$

1. Find $f+g$, $f-g$, $f\cdot g$ and f/g & state their domain.

i) $f(x) = x^3+2x^2$; $g(x) = 3x^2-1$

Sol^n: Here.

$(f+g)(x) =$

~~domain for $f(x) = x^3+2x^2$ is $x^3+2x^2 \geq 0$~~

~~$x^2(x+2) \geq 0$~~

~~$x+2 \geq 0$~~

~~$x \geq -2, 0$~~

~~domain for $g(x) = 3x^2-1$ is $3x^2-1 \geq 0$~~

~~$3x^2 \geq 1$~~

~~$x^2 \geq \frac{1}{3}$~~

~~$x \geq \frac{1}{\sqrt{3}}$~~

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(i) $f(x) = x^3+2x^2$, $g(x) = 3x^2-1$

Sol^n: Here

$f+g\,(x) = f(x)+g(x)$

$= x^3+2x^2+3x^2-1$

$= x^3+5x^2-1$

$f-g\,(x) = x^3+2x^2-3x^2+1$

$= x^3-x^2+1$

$f\cdot g\,(x) = f(x)\times g(x)$

$= (x^3+2x^2)(3x^2-1)$

$= 3x^5-x^3+6x^4-2x^2$

$= 3x^5+6x^4-x^3-2x^2$

$f/g\,(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^3+2x^2}{3x^2-1}$

domain for $f(x) = x^3+2x^2$ is $(-\infty,\infty)$

domain for $g(x) = 3x^2-1$ is $(-\infty,\infty)$

So, the domain for $f(x)$ and $g(x)$ is $A\cap B$ i.e. $(-\infty,\infty)$

domain for $(f+g)(x)$ is $(-\infty,\infty)$

domain for $(f-g)(x)$ is $(-\infty,\infty)$

domain for $f\cdot g(x)$ is $(-\infty,\infty)$

domain for $f/g\,(x) = R - \{\pm\frac{1}{\sqrt{3}}\}$ i.e.

$(-\infty, -\frac{1}{\sqrt{3}}) \cup (-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}) \cup (\frac{1}{\sqrt{3}}, \infty)$

(ii) $f(x) = \sqrt{3-x}$, $g(x) = \sqrt{x^2-1}$

Sol: here

$(f+g)(x) = f(x)+g(x) = \sqrt{3-x} + \sqrt{x^2-1}$

$(f-g)(x) = f(x)-g(x) = \sqrt{3-x} - \sqrt{x^2-1}$

$(f \cdot g)(x) = f(x) \times g(x) = (\sqrt{3-x}) \times (\sqrt{x^2-1})$

$= \sqrt{(3x^2-3)-x^3+x}$

$= \sqrt{(3x^2-3-x^3+x)}$

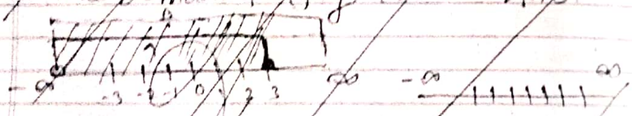
$= \sqrt{3x^2-x^3+x-3}$

$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{3-x}}{\sqrt{x^2-1}}$

~~domain for $f(x) = \sqrt{3-x}$ is $(-\infty, 3]$~~

~~domain for $g(x) = \sqrt{x^2-1}$ is $(-\infty, \infty)$~~

~~So, the domain for $(f+g)(x)$ is $A \cap B$~~



~~$[-\infty, 3]$~~

~~domain for $(f/g)(x)$ is also $[-\infty, 3]$~~

domain for $f \cdot g(x) = \sqrt{3x^2-x^3+x-3}$

$f(x) = \sqrt{3-x}$ and $g(x) = \sqrt{x^2-1}$

~~f(x)~~ domain for $f(x) = \sqrt{3-x}$: $3-x \geq 0$

$x \leq 3$

$(-\infty, 3]$

domain for $g(x) = \sqrt{x^2-1}$: $x^2-1 \geq 0$

$x^2 \geq 1$

$x^2 \geq \pm 1$

$(-\infty, \pm 1)$

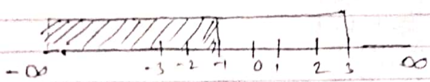
when putting $x = 0$ in $(f+g)(x)$ function then

$(f+g)(x) = \sqrt{3-0} + \sqrt{0-1}$

$= \sqrt{3} + \sqrt{-1} \Rightarrow$ function invalid doesnot exits

So, the domain for $g(x)$ $(-\infty, -1]$

domain for $(f+g)$ is $A \cap B$



$(-\infty, -1]$

So the domain for $(f+g)(x)$, $(f-g)(x)$ and $(f \cdot g)(x)$ is $(-\infty, -1]$

Now : domain $f/g(x) = \frac{\sqrt{3-x}}{\sqrt{x^2-1}}$

$(-\infty, -1)$

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(iii) $f(x) = \sqrt{x}$; $g(x) = \sqrt{1-x}$

Sol^n: Here

$(f+g)(x) = f(x) + g(x) = \sqrt{x} + \sqrt{1-x}$

$(f-g)(x) = f(x) - g(x) = \sqrt{x} - \sqrt{1-x}$

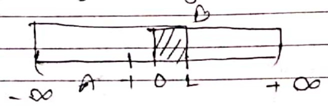
$(f \cdot g)(x) = f(x) \times g(x) = \sqrt{(x-x^2)}$

$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{\sqrt{1-x}}$

domain for $f(x) = \sqrt{x}$ is $[0, \infty)$

domain for $g(x) = \sqrt{1-x}$ is $[-\infty, 1]$

domain for $(f+g)(x)$ is $A \cap B$.



$[0, 1]$

The domain for $(f+g)(x)$, $f-g(x)$, $f \cdot g(x)$ is $[0, 1]$

The domain for $f/g\,(x)$ is $= \frac{\sqrt{x}}{\sqrt{1-x}}$

$1 - x \geq 0$

$x \leq 1$

$[0, 1)$

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(iv) $f(x) = x$, $g(x) = \sqrt{x-1}$

Sol^n: Here.

$(f+g)(x) = f(x) + g(x) = x + \sqrt{x-1}$

$(f-g)(x) = f(x) - g(x) = x - \sqrt{x-1}$

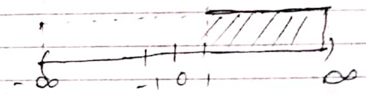
$(f \cdot g)(x) = \sqrt{x^2 - x}$

$\left(\frac{f}{g}\right)(x) = \frac{x}{\sqrt{x-1}}$

domain for $f(x) = x$ is $(-\infty, \infty)$

domain for $g(x) = \sqrt{x-1}$ is $(\infty, 1]$

the Domain of $(f+g)(x)$ is $A \cap B$



$[1, \infty)$

The domain for $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$ is $[1, \infty)$

The Domain for $\frac{x}{\sqrt{x-1}}$ is

$x - 1 \geq 0$

$x > 1$

~~$(\infty, 1)$~~ $(1, \infty)$

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(v) $f(x) = \sqrt{x+1}$, $g(x) = \sqrt{x-1}$

Sol: Here,

$(f+g)(x) = f(x) + g(x) = \sqrt{x+1} + \sqrt{x-1}$

$(f-g)(x) = f(x) - g(x) = \sqrt{x+1} - \sqrt{x-1}$

$(f \cdot g)(x) = f(x) \cdot \times g(x) = \sqrt{x^2-1}$

$(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x+1}}{\sqrt{x-1}}$

domain for $f(x) = \sqrt{x+1}$ is $[-1, \infty)$

$x+1 \geq 0$

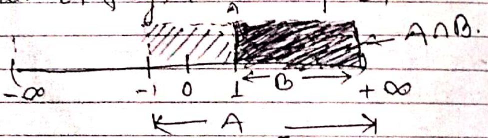
$x \geq -1$

domain for $g(x) = \sqrt{x-1}$ is

$x-1 \geq 0$

$x \geq 1$ $[1, \infty)$

domain of $(f+g)(x)$ is $A \cap B$.



$[-1, 1]$ $[1, \infty)$

So, the domain for $(f+g)x$, $(f-g)(x)$ and $f \cdot g(x)$ is $[-1, 1]$

domain for $f/g(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x+1}}{\sqrt{x-1}}$

$x-1 > 0$

$\cancel{(1, 1)}$ $(1, \infty)$

$fog(x)$ $f[g(x)]$ $fog(x)$. $fog(x)$ $f(g(x))$

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2. Find fog, gof, fof and gog and state their domain.

i) $f(x) = \sqrt{x}$; $g(x) = x+1$

Sol: Here

$fog = f[g(x)] = f[x+1] = \sqrt{x+1}$

$x+1 \geq 0$

$x \geq -1$

$[-1, \infty)$

$gof = g[f(x)] = g[\sqrt{x}] = \sqrt{x}+1$

$x \geq 0$

$[0, \infty)$

$fof = f[f(x)] = f[\sqrt{x}] = \sqrt{\sqrt{x}} = [0, \infty)$

$x \geq 0$

$gog = g[g(x)] = g[x+1] = x+1+1 = x+2$

$(-\infty, \infty)$

(ii) $f(x) = x^2-1$; $g(x) = 2x+1$

$fog = f[g(x)] = f[2x+1] = (2x+1)^2 - 1$

$= 4x^2 + 4x + 1 - 1$

$= 4x^2 + 4x$

domain $= (-\infty, \infty)$

$gof = g[f(x)] = g[x^2-1] = 2(x^2-1)+1$

$= 2x^2 - 2 + 1$

$= 2x^2 - 1$

domain $\rightarrow (-\infty, \infty)$

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$fof = f[f(x)] = f[x^2-1] = (x^2-1)^2 - 1$
$= x^4 - 2x^2 + 1 - 1$
$= x^4 - 2x^2$
$= (-\infty, \infty)$

$gog = go[g(x)] = g[2x+1]$
$2(2x+1)+1 = 4x+3 = (-\infty, \infty)$

(iii) $f(x) = \sqrt{x}$; $g(x) = \sqrt[3]{1-x}$

$fog = f[g(x)] = f[\sqrt[3]{1-x}] = \sqrt{1-\sqrt[3]{1-x}}$ $\quad \sqrt{\sqrt[3]{1-x}}$

domain = $1-x \geq 0$ $\quad \sqrt[6]{1-x}$
$x \leq 1$
$(-\infty, 1]$

$gof = g[f(x)] = g[\sqrt{x}]$
$= g \sqrt[3]{1-\sqrt{x}}$
$x \geq 0$
$[0, \infty)$

$fof = f[f(x)] = f(\sqrt{x})$
$= \sqrt{\sqrt{x}}$ $\quad (x)^{1/4}$ $\quad \sqrt[4]{x}$
$x \geq 0$
$[0, \infty)$

$gog = g[\sqrt[3]{1-x}] =$
$\sqrt[3]{1-\sqrt[3]{x}}$
$[0, 1]$

In cube root $\sqrt[3]{-3}$ this will be ...
so, $\sqrt[3]{x} = (-\infty, \infty)$

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$(-\infty, 1)$
$1-x \geq 0$
$x-1 \leq 0$
$x \leq 0$
$(-\infty$

$gog = g(\sqrt[3]{1-x}) = \sqrt[3]{1-\sqrt[3]{1-x}}$

$(-\infty, \infty)$
$(-\infty, \infty)$

(iv) $f(x) = x + \frac{1}{x}$; $g(x) = \frac{x+1}{x+2}$

Solⁿ: Here
$fog(x) = f[g(x)] = f\left[\frac{x+1}{x+2}\right]$

$= \frac{x+1}{x+2} + \frac{1}{\frac{x+1}{x+2}}$

$= \frac{x+1}{x+2} + \frac{x+2}{x+1}$

$= \frac{(x+1)^2 + (x+2)^2}{(x^2+x+2x+2)}$

$= \frac{(x+1)^2 + (x+2)^2}{(x^2+3x+2)}$

The Domain the $fog(x)$ is $(-\infty, -1) \cup \boxed{(-1, -2)}$ $\quad (-1, -2)$ $\quad (-2, \infty)$ $\quad R - \{-1, -2\}$

$x^2+3x+2 \geq 0$
$x^2 + (2+1)x + 2 \geq 0$
$x^2 + 2x + 1x + 2 \geq 0$
$x(x+2) + 1(x+2) \geq 0$
$(x+1)(x+2) \geq 0$
$x+1 \geq 0$
$x \geq -1$
$x \geq -2$

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$gof = g[f(x)] = g\left[x + \frac{1}{x}\right]$

$$= \frac{x + \frac{1}{x} + 1}{x + \frac{1}{x} + 2}$$

$$= \frac{\frac{x^2+1+x}{x}}{\frac{x^2+1+2x}{x}}$$

$$= \frac{x^2+1+x}{x^2+2x+1}$$

$x^2+2x+1 \geq 0$

$x^2+x+x+1 \geq 0$

$x(x+1)+1(x+1) \geq 0$

$(x+1)(x+1) \geq 0$

$x \geq -1$

The domain of $gof(x)$ is $R - \{-1\}$

$fof = f[f(x)] = f\left(x + \frac{1}{x}\right) = x + \frac{1}{x} + \frac{1}{x + \frac{1}{x}}$

domain $R - \{0\}$

$$= \frac{x^2+1}{x} + \frac{1 \times x}{x^2+1}$$

$$= \frac{x^2+1}{x} + \frac{x}{x^2+1}$$

$$= \frac{(x^2+1)^2 + x^2}{x^3+x}$$

$x^3+x \geq 0$

$x(x^2+1) \geq 0$

$x \geq 0$

$x \geq \pm 1$

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(v) $f(x) = \sqrt{x+1}$, $g(x) = \frac{1}{x}$

$fog = f\,g[x] = f\left[\frac{1}{x}\right] = \sqrt{\frac{1}{x} + 1}$

the domain is $R - \{0\}$

$\frac{1}{x} \geq 0$

$\frac{1}{x} + 1 \geq 0$

$\frac{1}{x} \geq -1$

$gof(x) = go[f(x)] = g\left[\sqrt{x+1}\right]$

$$= \frac{1}{\sqrt{x+1}}$$

$x+1 \geq 0$

$x \geq -1$

$(-1, \infty]$

domain is $(-1, \infty)$

$fof = f\left[\sqrt{x+1}\right] = \sqrt{\sqrt{x+1} + 1}$

$x+1 \geq 0$

$x \geq -1$

$(-1, \infty]$

the domain is $[-1, \infty)$

$gog = g[g(x)] = g\left[\frac{1}{x}\right] = \frac{1}{\frac{1}{x}} = x$

the domain is $(-\infty, \infty)$ Ans

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(vi) $f(x) = x^2$, $g(x) = 1-\sqrt{x}$

$fog = f[g(x)] = f[1-\sqrt{x}] = (1-\sqrt{x})^2$

domain is $[0,\infty)$

$gof = g[x^2] = 1-\sqrt{x^2}$
$= 1-x$
$= (-\infty,\infty)$

$fof = f[f(x)] = f[x^2] = [x^2]^2 = x^4$
domain is $(-\infty,\infty)$

$gog = g[g(x)] = g[1-\sqrt{x}] = 1-\sqrt{1-\sqrt{x}}$

$x \geq 0$

Domain is $\rightarrow [0,1]$

3. Find $fogoh$

(i) $f(x) = 3x-2$, $g(x) = \sin x$, $h(x) = x^2$

$fogo[h(x)] = fogo[x^2] = fo[\sin x^2]$
$= 3\sin x^2 - 2$ Ans

(ii) $f(x) = |x-4|$, $g(x) = 2^x$ $h(x) = \sqrt{x}$

sol^n:-

$fogoh = fogoh(x) = fogo[\sqrt{x}] = fo[2^{\sqrt{x}}]$
$= |2^{\sqrt{x}} - 4|$

Ans

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4 Express the function in form of fog is

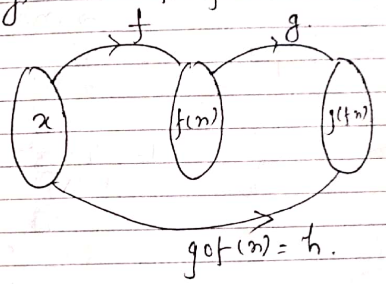
(i) $F(x) = (2x+x^2)^4$
$= \cancel{\sqrt{x+x(x+2)}}$

$f(x) = \sqrt[2]{x}$

Composite function :-

If f and g are functions, then composite function fog is defined by

$(fog)(x) = f[g(x)]$



$gof = (x)$

fog: fogoh(x) : $f[g(h x)]$

fog[hx]

4. Express the ~~domai~~ function in form fog:

① $Fx = (2x+x^2)^4$

$F(x) =$

$f(x) = x^2$, $g(x) = 1-\sqrt{x}$

① $fog(x) = f[g(x)]$

Domain for $g[x] = 1-\sqrt{x}$

$\sqrt{x} \geq 0$

$x \geq 0$.

$[0,\infty)$

Domain for $fog(x) = (1-\sqrt{x})^2$

$[0,\infty)$

$\therefore$ $[0,\infty)$

$fog(x)$

$f[g(x)]$

$1-2\sqrt{x}+x$

$= 1-2\sqrt{x}+x$

$\sqrt{x} \geq 0$

$x \geq 0$

$[0,\infty)$

(4) ① $F(x) = (2x+x^2)^4$

$F(x) = fog(x) = f[g(x)]$

$f(x) = x^4$

$g(x) = 2x+x^2$

$F(x) = fog(x) = f[g(x)] = f(2x+x^2) = (2x+x^2)^4$

(ii) $F(x) = \cos^2 x$

$F(x) = (\cos x)^2$

$F(x) = fog(x) = f g(x)$

$f(x) = x^2$

$g(x) = \cos x$.

$\therefore F(x) = fg[x] = f[(\cos x)] = (\cos x)^2 = \cos^2 x$

(5)

5① $R(x) = \sqrt{\sqrt{x}-1}$

Fogoh $=$ $fog\circ[h(x)]$

$f(x) = \sqrt{x}$

$g(x) = (x-1)$

$h(x) = \sqrt{x}$.

$fogoh = fogo[h(x)] = fog[\sqrt{x}]$

$= fo[\sqrt{x}-1]$

$= \sqrt{\sqrt{x}-1}$ Ans

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(i) $H(x) = \sqrt[8]{2+|x|}$

$fogoh(x) = \sqrt[8]{2+|x|}$

$f(x) = (2+|x|)^{1/8}$

$f(x) = (x)^{1/8}$

$g(x) = (2+x)$

$h(x) = |x|$

$fogoh = fog[h(x)]$

$= fog[|x|]$

$= fo(2+|x|)$

$= (2+|x|)^{1/8}$

(ii) $H(x) = \sec^4(\sqrt{x})$

$fogoh = \sec^4(\sqrt{x})$

$f(x) = (x)^4$

$g(x) = \cancel{\sqrt{x}}$

$h(x) = \cancel{\sec x}$

$fogoh(x) = fog[h(x)]$

$= \cancel{fog[\sec x]}$

$= fog[\sec\sqrt{x}]$

$f(\sec\sqrt{x})$

$\sec^4\sqrt{x}$ Ans

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$f(x) = (x)^4$

$g(x) = \sec x$

$h(x) = \sqrt{x}$

$fogoh(x) = fog[h(x)]$

$= fog[\sqrt{x}]$

$fo \sec\sqrt{x}$

$= (\sec\sqrt{x})^4 = \sec^4\sqrt{x}$

fogoh

$F(x) = \cot^2(x+5)$

$fogoh = \cot^2(x+5)$

$f(x) = x^2$

$g(x) = \cot x$

$h(x) = x+5$

$fogoh(x) = fog[\cancel{\cot x}][(x+5)]$

$= fog[x+5]$

$= fo[\cot(x+5)]$

$= \cot^2(x+5)$ Ans

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4. express the function in form of $f \circ g$

i) $F(x) = (2x+x^2)^4$

$f \circ g(x) = F(x)$

$f \circ g(x) = (2x+x^2)^4 = f[g(x)]$

$f(x) = x^4$

$g(x) = 2x+x^2$

$f \circ g = f[g(x)] = f[2x+x^2] = (2x+x^2)^4$ Ans

(ii) $F(x) = \cos^2 x$

$f \circ g(x) = \cos^2 x$

$f(x) = x^2$

$g(x) = \cos x$

$f \circ g(x) = f[g(x)] = f[\cos x] = \cos^2 x$

(iii) let $v(t) = (\sec t^2)\ \tan(t^2)$

let $t = x$ and $v = F = f \circ g(x)$

$F(x) = (\sec x^2) \cdot (\tan x^2)$

$f(x) = \sec x \cdot \tan x$

$g(x) = \cancel{\sec x \cdot \tan x}\ x^2$

$f \circ g(x) = f[g(x)] = \cancel{f[\sec(x) \cdot \tan(x)]}$

$f \circ g(x) = f[g(x)]$

$f \circ g(x) = f[x^2]$

$= \sec x^2 \cdot \tan x^2$

$\therefore v(t) = \sec t^2 \cdot \tan t^2$

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(iv) $F(x) = \dfrac{\sqrt[3]{x}}{1+\sqrt[3]{x}}$

Sol: Here

$F(x) = f \circ g(x)$

$f(x) = \cancel{(x)^{1/3}}\ \dfrac{x}{1+x}$

$g(x) = \cancel{\dfrac{x}{1+x}}\ (x)^{1/3}$

$f \circ g(x) = f[g(x)] = \cancel{f\left[\dfrac{x}{1+x}\right]}$

$= f[x^{1/3}]$

$= \dfrac{(x)^{1/3}}{1+(x)^{1/3}}$

$= \dfrac{\sqrt[3]{x}}{1+\sqrt[3]{x}}$ Ans

(v) $G(x) = \sqrt[3]{\dfrac{x}{1+x}}$

$G(x) = f \circ g(x)$

$f(x) = (x)^{1/3}$

$g(x) = \dfrac{x}{1+x}$

$\therefore f \circ g(x) = f[g(x)] = f\left[\dfrac{x}{1+x}\right] = \sqrt[3]{\dfrac{x}{1+x}}$ Ans (vi). $U(t) = \frac{\tan t}{1+\tan t}$

Sol^n: Here,
let $t = x$ and $U = f$

$f(x) = \frac{\tan x}{1+\tan x}$

$f(x) = fog(x)$

$f(x) = \cancel{\tan x} \quad \frac{x}{1+x} = \frac{t}{1+t}$

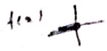
$g(x) = \cancel{\frac{x}{1+x}} \quad \tan x = \tan t$

$fog(x) = f[g(x)]$

$= \cancel{f[\frac{x}{1+x}]} \quad f[\tan x]$

$= \frac{\tan x}{1+\tan x}$

$\therefore U(t) = \frac{\tan t}{1+\tan t}$ Ans.



Transformations of functions:

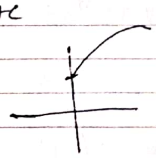
(i) $y = f(x)+c$, shift the graph of $y = f(x)$ at distance c units upward

(ii) $y = f(x)-c$ shift the graph of $y = f(x)$ at distance c units downward

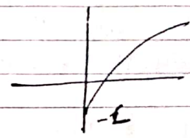
(iii) $y = f(x-c)$, shift the graph of $y = f(x)$ at distance c units to the right.

(iv) $y = f(x+c)$, shift the graph of $y = f(x)$ at distance c units to the left.

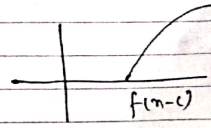
for (i) $y = f(x)+c$



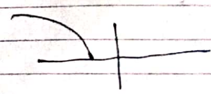
for (ii) $y = f(x)-c$



for (iii) $y = f(x-c)$



for $y = f(x+c)$



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⑥ Suppose the grap of f is given. write equation for the graphs that are obtained from the graph of f as follows

(i) shift 3 unit upward

Sol: Here

∴ $y = f(x)+3$

(ii) Shift 2 unit to the righ

$y = f(x-2)$

(iii) Reflem about y-axis

$y = f(-x)$

(iv) Stretched vertically by a factor of 3

Sol: here

$y = 3f(x)$

(v) Compress horizontally by a factor of 1

$y = f(x)$.

7 ~~Explain~~

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Stretching ~~and~~ Reflecting Transformation

$y = c f(x)$ stretched the graph of $y = f(x)$ vertically by a factor of c

$y = \frac{1}{c} f(x)$ (compress) shrink the graph of $y = f(x)$ vertically by a factor of c

$y = f(cx)$ shrink (compress) the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{c}$

$y = f(\frac{x}{c})$ stretch the graph of the line ~~by~~ $y = f(x)$ horizontally by a factor of c

$y = -f(x)$ reflect the graph of $y = f(x)$ about x-axis

$y = f(-x)$ reflect the graph of $y = f(x)$ about y-axis

④ Explain how each graph is obtained from the graph of $y = f(x)$

i) $y = f(x)+8$.

∴ Shifted the graph $y = f(x)$ ~~by~~ upward by 8 units.

(ii) $y = f(x+8)$

Shifted the graph $y = f(x)$ to the left side by 8 units.

(iii) $y = f(8x)$

compress horizontally the graph of $y = f(x)$ by a factor of 8

(iv) $f = -f(x) - 1$

$f = -f(x) - c$

Reflect ~~shifted~~ the grap $y = f(x)$ ~~to~~ at x-axis ~~by~~ & lifting downward by 1 units

(v) $y = 8f(\frac{1}{8}x)$

$y = cf(\frac{1}{c}x)$

Sol^n: here ~~Compress the function $y = f(x)$ vertically with 8 unit by following stretching with 8 units~~

Stretched horizontally by a factor of 8 followed by stretch vertically by a factor of 8.

(8)(a) Find the new function by using given transaction on given ~~facts~~ functions

(i) $f(x) = -\sqrt{x}$ shifted right by 3

Sol^n: Here:

~~f(x)~~

According to Given condition

$y = -f(x - c)$

$y = -f(x - c)$

$y = -\sqrt{x - 3}$

8(ii) $y = 2x - 7$ shifted up by 7

Sol^n: Here:

$y = 2x - 7$

$y = f(x) + c$

$y = 2x - 7 + 7$

$y = 2x$

(iii) $y = x^2 - 1$ stretched vertically by a factor of 3

Sol^n: $y = 3f(x)$

$= 3(x^2 - 1)$

(iv) $y = \sqrt{x + 1}$ compressed horizontally by a factor 4

Sol^n: $y = f(x/c)$

$y = \sqrt{4x + 1}$ Ans

(v) $y = \frac{1}{2}(x + 1) + 5$ shifted down by 5 folled by right 1.

$y = f(x + 1) - c$

$y = \cancel{\{\frac{1}{2}(x+1)\} - 1 + 5 - 5}$ $\quad f\left(\frac{1}{2}(x+1-1)\right) + 5 - 5$

$y = \frac{1}{2}(x + 1) - 1$ $\quad f\left(\frac{x}{2}\right)$ Ans

$= \cancel{\frac{x+1-2}{2} = \frac{x-1}{2}}$ Ans

(vi) $f(x) = 1/x^2$ shifted left by 2 followed by 1 down !

$f(x+c)-1$

$\cancel{1/x^2+2}-1$

$\frac{1+x^2}{x^2}$

$f(x+c)-1$

$f\ \frac{1}{(x+2)^2}-1$

(vii) $f(x) = x^3 - 4x^2 - 10$ compress vertically by 2 followed by reflection about x-axis

Soln: Here $= -\frac{1}{c} \cdot f(x)$

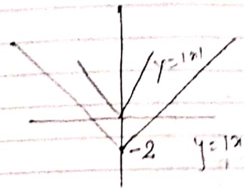
$= -\frac{1}{2} \cdot (x^3 - 4x^2 - 10)$

$= \frac{-x^3}{2} + 2x^2 + 5$

#9. Find the appropriate transformation used thus obtained new function as below, and graph the function by hand not by plotting point.

(i) $y = |x| - 2$

Sol: here $y = f(x) - 2$.



For $y = |x|$

x	0	1	-1	-2	-3	3
y	0	1	1	2	3	3

For $y = |x| - 2$

x	0	1	-1	-2
y	-2	-1	-1	0

original function $f(x) = |x|$, shifts the distance 2 unit downward

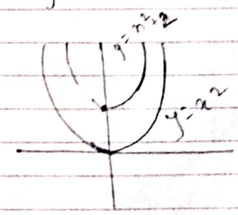
(ii) $y = x^2 + 2$

$y = f(x) + 2$

Soln: here,

$y = x^2$ → is a parabola

Original function $y = x^2$ shifted up by 2 units Upward.

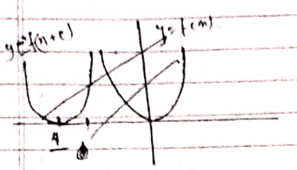


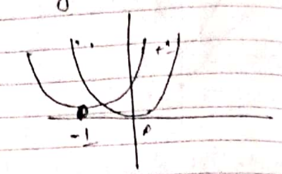
(iii) $y = (x+1)^2$

$y = f(x+c)^2$

$y = x^2$ is a parabola shape

Original function $y = f(x) = x^2$ shifted ~~about~~ to the of left side by 1 unit



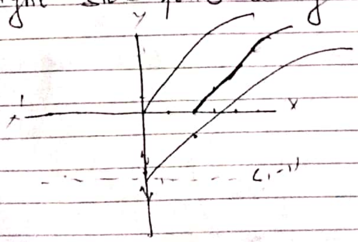


(iv) $y = \sqrt{x-2} - 1$

$y = f(x-c) - 1$

distance 2 unit

Original function $y = \sqrt{x}$ shifted to the right side followed by 1 unit downward



(v) $1 - 2\sqrt{x+3}$

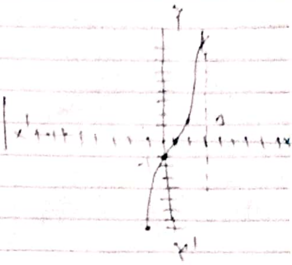
Sol^n: here

$y = -2\sqrt{x+3} + 1$

$y = cf - cf(x+b) + 1$

$y = (x-1)^3$

x	0	1	2	3	-1
y	-1	0	1	8	-8



(10) If f is even and g is odd functions and both are defined on $\mathbb{R}$ then find which of the following combined function even odd or neither?

(i) fg

Sol^n: Here

$f(x)$ = even function, $g(x)$ = odd function

then,

$f[g(x)]$

$f[g(-x)]$

$f[-g(x)]$

$f[-z]$

$f(z) = fg(x)$ is even function

f^2

$g(x) = z$

$f(x) \times g(-x)$

$f(x) - g(x)$

$-fg(x)$ odd function

(ii) $f/g(x) = \frac{f(x)}{g(-x)} = \frac{f(x)}{-g(x)} = -f/g(x)$ is odd function

(iii) $f^2 = ff$

Soln: here

$ff(x) = f(x) \times f(x) = ff(x)$ is even function

(iv) $g^2 = gg$

$gg(x) = g(x) \times g(x) = -g(x) \times -g(x)$

$= gg(x) = gg$ is even function

(v) $fog = f[g(-x)]$ $f[-g(x)]$ let $g(x) = z$

$= f(-z) = f(z) = fg(x) =$ is even function

(vi) $gof = gof(x) = g[f(x)] = [g(-z)]$ a

$= -g(z) = -gf(x)$ is odd function

(vii) $gog = g[g(x)] = g[g(-x)] = g[-g(x)]$

(vii) fof

Soln: here

$fo[f(x)]$ let $f(x) = z$

$fo(z) = f(z) = ff(x) =$ is even function

(viii) $gog = go[g(x)] = go[-g(x)]$ let $g(x) = z$

$go[z]$ $-gg(x)$

$-g(z)$

$g(- -gg(x)$

is odd function

Exercise 1.4

- A function is form of $y = a^x$ where a is positive real number special exponential function

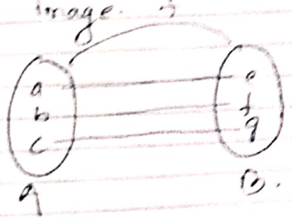
put $a = e$

$y = e^x$

$y = 2^x$

Logarithm function: The inverse of exponential function is called logarithmic function

One to one function: A function is said to be one to one function If distinct element has distinct image.



Inverse function

let f be a one to one function with domain A and range B. then its inverse function f^{-1} has domain B and range A and is defined by $f^{-1}(y) = x \Leftrightarrow f(x) = y$

Ex: (1.2)

$y = \frac{e^x}{1+2e^x}$

$y = 1 + \sqrt{2+3x}$

$y - 1 = \sqrt{2+3x}$

$(y-1)^2 = 2+3x$

$\frac{(y-1)^2 - 2}{3} = x$

$f^{-1}(y) = y - 1)^2$

$f^{-1}(y)$

$y(1+2e^x) = e^x$

$y + 2ye^x = e^x$

$y = e^x(1-2y)$

$e^x = \frac{y}{1-2y}$

$x = \log\left(\frac{y}{1-2y}\right)$

$\cancel{f^{-1}(y) =} \log\left(\frac{x}{1-2x}\right)$

$y = \frac{1}{x}$

4. <u>Inverse function</u>

find the inverse function of $f(x) = x^3 + 2$ how f^{-1} is related to f and what is relation of f and f^{-1}?

<u>Soln</u>: here

$y = x^3 + 2$

$x^3 = y - 2$

$x = (y-2)^{1/3}$

$f^{-1}(y) = (y-2)^{1/3}$

$f^{-1}(x) = (x-2)^{1/3}$ is the required formula for inverse function.

① $f(x) = 1 + \sqrt{2+3x}$

<u>Soln</u>: Here,

$y = 1 + \sqrt{2+3x}$

$y - 1 = \sqrt{2+3x}$

$(y-1)^2 = 2+3x$

$(y-1)^2 - 2 = 3x$

$x = \frac{(y-1)^2 - 2}{3}$

$f^{-1}(y) = \frac{(y-1)^2 - 2}{3}$

$f^{-1}(x) = \frac{(\cancel{y-1})^2 - 2}{3} = \frac{(x-1)^2 - 2}{3}$

$= \frac{x^2 - 2x + 1 - 2}{3}$

$= \frac{x^2 - 2x - 1}{3}$

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(ii) $f(x) = \frac{4x-1}{2x} + 3$

$y = \frac{4x-1}{2x} + 3$

$y - 3 = \frac{4x-1}{2x}$

$2x(y-3) = 4x-1$

$2xy - 6x = 4x-1$

$2xy = 10x-1$

$2xy - 10x = -1$

$x(2y-10) = -1$

$x = \frac{-1}{2y-10}$

$f^{-1}(y) = \left(\frac{-1}{2y-10}\right)$

$f^{-1}(x) = \left(\frac{1}{10-2x}\right)$ Ans

(iii) $f(x) = e^{2x-1}$

Solⁿ: Here

$y = e^{2x-1}$

$\log y = 2x-1$

$2x = 1 + \log y$

$x = \frac{1+\log y}{2}$

$f^{-1}(y) = \frac{1+\log y}{2}$

$f^{-1}(x) = \frac{1+\log x}{2}$

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(iv) $y = \frac{e^x}{1+2e^x}$

$y + 2e^x y = e^x$

$y = e^x - 2e^x . y$

$y = e^x(1-2y)$

$\frac{y}{1-2y} = e^x$

$x = \log\left(\frac{y}{1-2y}\right)$

$f^{-1}(y) = \log\left(\frac{y}{1-2y}\right)$

$f^{-1}(x) = \log\left(\frac{x}{1-2x}\right)$

(1) Starting with the graph of $y = e^x$, write the equation of the graph that result from

(i) shifting 2 units downward.

Solⁿ: Here

$y = e^x - 2$

(ii) Shifting 2 unit to the right

$y = f(x-2)$

$y = e^{x-2}$

(iii) Reflecting about the x-axis

$y = -f(x) = -f(x) = -e^x$

(iv) Reflecting about the y-axis

$y = f(-x) = e^{-x}$ Ans//

(v) reflecting about x-axis and the ~~about~~ y-axis

$y = -f(-x)$

$y = -e^{-x}$

One to one function

A function is said to be one to one if no two different elements of domain has same image.

example: Is function $f(x) = x^2$ one to one?

This function is not one to one function because

$1 \neq -1$ but $f(1) = f(-1) = 1$

(4) A function is given by formula. Determine whether it is one to one.

(i) $f(x) = x^2 - 2x$

$f(1) = 1 - 2\times1 = -1$

$f(-1) = 1 + 2 = 3$

$\therefore$ This function is one to one because

$1 \neq -1$ so, $f(-1) = 3$

$f(1) = -1$

Hence this is one to one function

(ii) $f(x) = 10 - 3x$

Soln: Here

examine whether the function is one to one or not so,

$f(1) = 10 - 3\times1 = 7$

$f(-1) = 10 - 3\times-1 = 13$

$f(0) = 10 - 0 = 10$

Hence $-1 \neq 1$ So, the function $f(x)$ is one to one.

(iii) $g(x) = 1/x$

Soln: Here

examine whether the function is one to one or not so,

$g(x) = 1/x$

$g(-1) = \frac{1}{-1} = -1$

$g(1) = \frac{1}{1} = 1$

So, $-1 \neq 1$ Hence the function $f(x)$ is one to one.

(iv) $h(x) = 2 + |x|$

Soln: Here examine whether the function is one to one or not

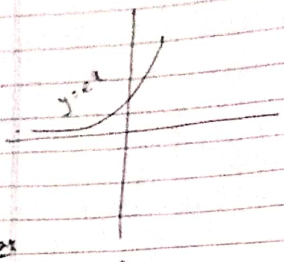
$h(x) = 2 + |x|$

$h(1) = 2 + 1 = 3$

$h(1) = 2 + |-1| = 2 + 1 = 3$

So, $-1 \neq 1$ Hence, the given function is one to one.

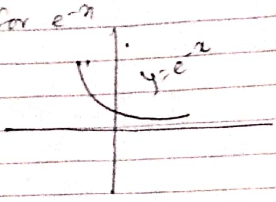
Graph of function in to exponential form



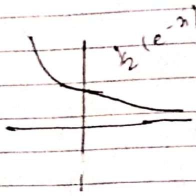
$y = e^x$

x	0	
y	1	

for e^{-x}



For $\frac{1}{2}(e^{-x})$



$y = e^x$ - f(x)

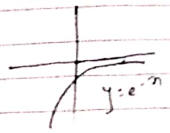
(2) Using the transaction, make rough sketch of graph of function

(i) $y = -2^{-x}$

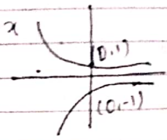
$y = -e^x$

$y = e^{-x}$

For $y = -e^x$



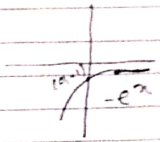
For $y = e^{-x}$



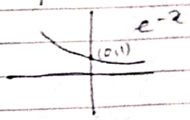
(i) $y = -2^{-x}$

Soln, Here

$y = -e^x$



$y = e^{-x}$

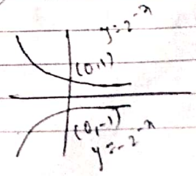


$y = 2^{-x}$ — (1)

$y = 2^{-0} = 1$

$y = -2^{-x}$ (ii) $= -2^{-0} = -1$

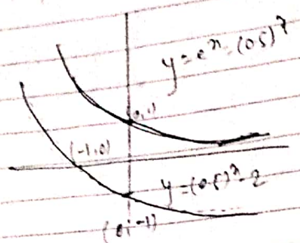
then,



(ii) $y=\left(\frac{1}{2}\right)^x-2$

$y = e^x = (0.5)^x$

y put $x=0$ $=(0.5)^0 = 1$



$y = e^x - 2 = (0.5)^x - 2 =$

$= (0.5)^{-1} - 2 = 0$

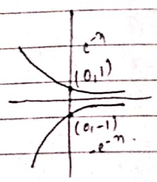
$\left(\frac{1}{2}\right)^{-1} - 2 = 0$

$= 2-2=0 \quad (-1,0)$

$= (0.5)^0 - 2 = -1$

(iii) $y = 1 - \frac{1}{2}e^{-x}$

$y = -\frac{1}{c}[e f(-x)]$



$y = e^{-x} = (0,1)$

$-\frac{1}{2}(e^{-x})$

First graph for e^{-x} and $-e^{-x}$

$y = e^{-x} = e^{-0} = 1$

$y = -e^{-x} = -e^{-0} = -1$

Second graph for

$y = 1-\frac{1}{2}e^{-x}$ and

$y = -\frac{1}{2}e^{-x}$

$y = 1 - \frac{1}{2}e^{-x}$

if $x=0$ $y = 1 - \frac{1}{2}(e^{0})$

(3)

$f(x) = ca^x$

$y = ca^x$

$b = ca$

$a = b/c$

$24 = ca^3$ — (ii)

$24 = c\left(b/c\right)^3$

$24 = c\left(\frac{6\times6\times6}{c^3}\right)$

$c^2 = \frac{36\times6}{24}$

$c^2 = 9$

$c = 3$

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put $c = 3$ in eq^n 1

$a = 6/3 = 2$

$y = 3 \cdot 2^x$

Chapter – 3

DERIVATIVES

① First principle

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Basic Derivatives Rules

1) Constant Rule $\frac{d(c)}{dx} = 0$

2) constant Multiple Rule $= \frac{d[c\,f(x)]}{dx} = c\,f'(x)$

3 power Rules -

$$\frac{d(x^n)}{dx} = nx^{n-1}$$

3 Sum Rule $\frac{d[f(x) + g(x)]}{dx} = f'(x) + g'(x)$

4 Differentiable Rule $\frac{d[f(x) - g(x)]}{dx} = f'(x) - g'(x)$

5. product Rule $\frac{d[f(x) \cdot g(x)]}{dx} = f(x)\,g'(x) + g(x)\,f'(x)$

6 Quotient Rule $\frac{d\left(\frac{f(x)}{g(x)}\right)}{d(x)} = \frac{g(x)\,f'(x) - f(x)\,g'(x)}{[g(x)]^2}$

⑦ Chain Rule $\frac{d[f(x)[g(x)]]}{dx} = f'(x)\,g'$

$f'(g(x))\,g'(x)$

Derivatives of exponential functions

$$\frac{d(e^x)}{dx} = e^x$$

$$\frac{d(e^u)}{d(x)} = e^u \cdot \frac{du}{dx}$$

$$\frac{d(a^x)}{d(x)} = a^x \log a$$

$$\frac{d(a^u)}{d(x)} = \frac{d(a^u)}{d(u)} \times \frac{d(u)}{dx} = a^u \cdot \frac{d(u)}{dx} \cdot \log u$$

$$\therefore a^u \log u \frac{du}{dx}$$

Derivative of logarithmic functions

$$\frac{d[\log(x)]}{dx} = 1/x$$

$$\frac{d \log[f(x)]}{dx} = \frac{1}{f(x)} \cdot f'(x)$$

$$\frac{d \log_a(x)}{dx} \quad \frac{1}{x \times \log a}$$

$$\frac{d}{dx} \log_a[f(x)] = \frac{1}{f(x) \log a} \times f'(x)$$

Derivative of Trignometric function

$$\frac{d(\sin x)}{dx} = \cos x \qquad \frac{d(\cos x)}{d(x)} = -\sin x$$

$$\frac{d(\tan x)}{d(x)} = \sec^2 x \qquad \frac{d(\sec x)}{dx} = \sec x \cdot \tan x$$

$$\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cot x \qquad \frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$$

Derivative of Trignometric functions

$$\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d(\cos^{-1} x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

$$d(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d(\cot^{-1} x)}{dx} = \frac{-1}{1+x^2}$$

$$\frac{d(\sec^{-1} x)}{dx} = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d(\operatorname{cosec}^{-1} x)}{dx} = \frac{-1}{x\sqrt{x^2-1}}$$

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Example:
Show that $f(x) = |x|$ is not differentiable at $x = 0$

$$|x| = \begin{cases} -x & \text{for } x < 0 \\ x & \text{for } x > 0 \\ 0 & \text{for } x = 0 \end{cases}$$

$$Rf'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$\lim_{h \to 0} \frac{0+h-0}{h}$$

$$\lim_{h \to 0} \frac{h}{h} = 1.$$

$$Lf'(0) = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h}$$

$$\lim_{h \to 0} \frac{-(0-h)-0}{-h}$$

$$\lim_{h \to 0} \frac{h}{-h} = -1$$

$$\therefore Rf'(0) \neq Lf'(0)$$

Hence $f(x)$ is not differentiable at a point $x = 0$

Theorem: If f is differentiable at a point 'a' then f is continuous at a point 'a'

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \text{ exist}$$

We have to show that f is continuous at a i.e. to show

$$\lim_{x \to a} f(x) = f(a)$$

here, $f(x) - f(a) = \dfrac{f(x) - f(a)}{(x-a)} \times (x-a)$

$$\lim_{x \to a} [f(x) - f(a)] = \lim_{x \to a} \frac{f(x) - f(a)}{(x-a)} \times (x-a)$$

or, $\lim_{x \to a} [f(x) - f(a)] = f'(a) \cdot \lim_{x \to a} (x-a)$

or, $\lim_{x \to a} [f(x) - f(a)] = f'(a) \times (a-a)$

or, $\lim_{x \to a} [f(x) - f(a)] = f'(a) \times 0$

$\lim_{x \to a} [f(x) - f(a)] = 0.$

$\lim_{x \to a} f(x) = f(a).$

What is differentiable function?

If LHL = RHL are same and equal by putting the limiting value then the it's called differentiable function. Modulus gives us two value that + and - value so,

example 3: show that $f(x) = |x|$ is not differentiable at $x = 0$.

Sol: Here,
By the help of first principle of derivative

Given
$f(x) = |x|$ ~~differen~~ at $x = 0^+$

taking R.H.L. at $x > 0$

$f(x) = |x|$

$f(x+h) = |x+h|$

By first principle

$$\lim_{h \to 0} \frac{|x+h| - |x|}{h} \quad \text{at } x = 0^+$$

$$\lim_{h \to 0} = \frac{0+h-0}{h}$$

$$\frac{h}{h}$$

$$= 1$$

L.H.L at $x < 0$ i.e $x = 0^-$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

and $f(x+h) = -(x+h)$

$f(x) = -(x)$

$$\frac{-(x+h) - (-x)}{h}$$

$$\frac{-(0+h) - (0)}{h}$$

$$-1 = -h/h$$

$\therefore$ LHL $\neq$ R.H.L So the given function is not differentiable.

Theorem: If f is differentiable at a point 'a' then f is continuous at that point 'a'

condition to be continuous and differentiable at point a

if

$$\lim_{x \to a} f(x) = f(a)$$

$$\therefore \lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = f(a)$$

so, taking,

$$f(x) - f(a) = f(x) - f(a)$$

$$f(x) - f(a) = \frac{f(x) - f(a)}{(x-a)} \times (x-a)$$

taking limit on both side so,

$$\lim_{x \to a} f(x) - f(a) = \lim_{x \to a} \frac{f(x) - f(a)}{(x-a)} \times (x-a)$$

$$\lim_{x \to a} f(x) - f(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a} \lim_{x \to a} (x-a)$$

$$\lim_{x \to a} f(x) - f(a) = f'(x) \times \lim_{x \to a} (x-a)$$

$$\lim_{x \to a} f(x) - f(a) = f'(x) \times (a-a) = f'(x) \times 0$$

$$\lim_{x \to a} f(x) - f(a) = 0$$

$\lim_{x \to a} f(x) = f(a)$ Hence the theorem is proved.

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Exercise 3.3

Find the derivative of the function using the definition of derivatives. State the domain of the function and the domain of its derivative.

1. $f(x) = \frac{x}{2} - \frac{1}{3}$

Soln: here

$f(x) = \frac{x}{2} - \frac{1}{3}$

$f(x+h) = \frac{x+h}{2} - \frac{1}{3}$

By the first principle of derivation

or, $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

or, $\lim_{h \to 0} \frac{\frac{x+h}{2} - \frac{1}{3} - \frac{x}{2} + \frac{1}{3}}{h}$

$\lim_{h \to 0} \frac{\frac{x+h}{2} - \frac{x}{2}}{h}$

$\frac{x+h-x}{2h} = \frac{1}{2}$

$\therefore$ the domain of its derivative is $(-\infty, \infty)$ which is R

the domain of its function ~~is~~ $f(x)$ is exist for real value of x. so, the domain of given function is R

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(2) $f(x) = mx + b$

Soln: here

$f(x) = mx + b$

$f(x+h) = m(x+h) + b$

By the 1st principle of derivative

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\lim_{h \to 0} \frac{m(x+h) + b - mx - b}{h}$

$\lim_{h \to 0} \frac{\cancel{mx} + mh + \cancel{b} - \cancel{mx} - \cancel{b}}{h}$

$\frac{mh}{h} = m$

$f'(x) = m$

So the domain of ~~the~~ its derivative is $(-\infty, \infty)$ which is R.

Again

The domain of the given $f(x)$ is exist for real value of x i.e. $(-\infty, \infty)$ i.e. R

$f(x) = x^2 - 2x^3$

Sol^n: here

$f(x) = x^2 - 2x^3$

$f(x+h) = (x+h)^2 - 2(x+h)^3$

By the I^{st} principle of derivative.

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\lim_{h \to 0} \frac{(x+h)^2 - 2(x+h)^3 - x^2 + 2x^3}{h}$

$\lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 2(x^3 + 3x^2h + 3xh^2 + h^3) - x^2 + 2x^3}{h}$

$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{2x^3} - 6x^2h - 6xh^2 - 2h^3 - \cancel{x^2} + \cancel{2x^3}}{h}$

$\frac{2xh + h^2 - 6x^2h - 6xh^2 - 2h^3}{h}$

$\lim_{h \to 0} \frac{\cancel{h}(2x + h - 6x^2 - 6xh - 2h^2)}{\cancel{h}}$

$2x + 0 - 6x^2 - 0 - 0$

$2x - 6x^2$

the domain of its derivative is $(-\infty, \infty)$ which is R and ~~its~~ the ~~fu~~ ~~its~~ domain of its function is $(-\infty, \infty)$ which is also R

4 $g(t) = 1/\sqrt{t}$

Sol^n: here.

$g(t) = 1/\sqrt{t}$

$g(t+h) = \frac{1}{\sqrt{t+h}}$

By the I^{st} principle of derivative

$\lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$

$\lim_{h \to 0} \frac{\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}}{h}$

$\lim_{h \to 0} \frac{\sqrt{t} - \sqrt{t+h}}{h(\sqrt{t}) \times (\sqrt{t+h})}$

$\lim_{h \to 0}$ 1-

$\lim_{h \to 0} \frac{\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}}{h(\sqrt{t}) \times (\sqrt{t+h})}$

$\lim_{h \to 0} \frac{\sqrt{t} - \sqrt{t+h}}{h(\sqrt{t} \times \sqrt{t+h})}$

$\lim_{h \to 0} = \frac{t - t+h}{h \times (\sqrt{t} + \sqrt{t+h})(\sqrt{t} \times \sqrt{t+h})}$

$\lim_{h \to 0} = \frac{-\cancel{h}}{\cancel{h}(\sqrt{t} + \sqrt{t+h}) \times (\sqrt{t} \times \sqrt{t+h})}$

$= \frac{-1}{(\sqrt{t} + \sqrt{t})} \times \frac{1}{(\sqrt{t} \times \sqrt{t})}$

$$\frac{-1}{2\sqrt{t}\sqrt{t}} = \frac{-1}{2\sqrt{t}\cdot t} = \frac{-1}{2(t)^{3/2}}$$

So the domain of its derivative is

$t > 0$

~~(0, ∞~~ $(0, \infty)$

again the domain of given function domain is $\frac{1}{\sqrt{t}}$ i.e. $t \geq 0$

$(0, \infty)$ Ans

(5) $g(x) = \sqrt{9-x}$

Sol. Given

$g(x) = \sqrt{9-x}$

$g(x+h) = \sqrt{9-(x+h)}$

By the first principle of derivative

$$\lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$\lim_{h \to 0} \frac{\sqrt{9-(x+h)} - \sqrt{9-x}}{h}$$

$$\lim_{h \to 0} \frac{9-(x+h) - 9 + x}{h(\sqrt{9-(x+h)} + \sqrt{9-x})}$$

$$\lim_{h \to 0} \frac{\cancel{9}-\cancel{x}-h-\cancel{9}+\cancel{x}}{h(\sqrt{9-x-h} + \sqrt{9-x})}$$

$$\lim_{h \to 0} \frac{-h}{h(\sqrt{9-x-h}) + \sqrt{9-x}}$$

$$\frac{-1}{(\sqrt{9-x} + \sqrt{9-x})}$$

$$\frac{-1}{2\sqrt{9-x}}$$ Ans.

The domain of its derivative is

$9 - x \geq 0$

$x - 9 \leq 0$

$x \leq 9$

~~(-∞~~ $(-\infty, 9)$

The domain of the function is

$g(x) = \sqrt{9-x}$

$9 - x \geq 0$

$x - 9 \leq 0$

$x \leq 9$

$(-\infty, 9]$

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⑥ $f(x) = \frac{x^2-1}{2x-3}$

Sol: Here

$f(x) = \frac{x^2-1}{2x-3}$

$f(x+h) = \frac{(x+h)^2-1}{2(x+h)-3}$

By the 1st principle of derivatives

$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$\lim_{h\to 0} \frac{1}{h}\left[\frac{(x+h)^2-1}{2(x+h)-3} - \frac{x^2-1}{2x-3}\right]$

$\frac{1}{h}\left[\frac{x^2+2xh+h^2-1}{2x+2h-3} - \frac{x^2-1}{2x-3}\right]$

$\lim_{h\to 0} \frac{1}{h}\left[\frac{(2x-3)(x^2+2xh+h^2-1) - \{(x^2-1)(2x+2h-3)\}}{(2x-3)\ (2x+2h-3)}\right]$

$\lim_{h\to 0} \frac{1}{h}\left[\frac{2x^3+4x^2h+2xh^2-2x-3x^2-6xh-3h^2+3-\{2x^3+2x^2h-3x^2-2x-2h+3\}}{(2x-3)\ (2x+2h-3)}\right]$

$\lim_{h\to 0} \frac{1}{h}\left[\frac{\cancel{2x^3}+4x^2h+2xh^2-\cancel{2x}-\cancel{3x^2}-6xh-3h^2+\cancel{3}-\cancel{2x^3}-2x^2h+\cancel{3x^2}+\cancel{2x}+2h-\cancel{3}}{(2x-3)\ (2x+2h-3)}\right]$

$\lim_{h\to 0} \frac{1}{h}\left[\frac{2x^2h+2xh^2-6xh-3h^2+2h}{(2x-3)\ (2x+2h-3)}\right]$

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$\frac{2x^2+2xh-6x-3h+2}{(2x-3)\ (2x+2h-3)}$

$\frac{2x^2-6x+2}{(2x-3)^2}$

domain of its derivative is

$2x-3 > 0$

$2x > 3$

$x = 3/2$

$(-\infty, 3/2) \cup (3/2, \infty)$

$\therefore R - \{3/2\}$

and the domain of the function is

$2x-3 > 0$

$2x > 3$

$x > 3/2$

$R - \{3/2\}$ Ans

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7) $f(x) = x^{3/2}$

Sol: Here

$f(x) = x^{3/2} = \sqrt{x^3}$

$f(x+h) = (x+h)^{3/2} = \sqrt{(x+h)^3}$

By the 1st principle of derivative.

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\lim_{h \to 0} \frac{\sqrt{(x+h)^3} - \sqrt{x^3}}{h}$

$\lim_{h \to 0} \frac{(x+h)^3 - x^3}{(\sqrt{(x+h)^3} + \sqrt{x^3}) \times h}$

$\lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h(\sqrt{(x+h)^3} + \sqrt{x^3})}$

$\lim_{h \to 0} \frac{3x^2 + 3xh + h^2}{(\sqrt{(x+h)^3} + \sqrt{x^3})}$

$\frac{3x^2}{2\sqrt{x^3}} = \frac{3x^2}{2x\sqrt{x}} = \frac{3x}{2\sqrt{x}}$

$= \frac{3}{2}\sqrt{x}$ Ans

The domain of its derivative is

$x \geq 0$

$[0, \infty)$

The domain of the given function is

$\sqrt{x^3}$

$x \geq 0$

$[0, \infty)$ Ans

8) $f(x) = x^4$

Sol: Here

$f(x) = x^4$

$f(x+h) = (x+h)^4$

By the 1st principle of derivatives.

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\frac{(x+h)^4 - x^4}{h}$

$\lim_{h \to 0} \frac{[(x+h)^2]^2 - (x^2)^2}{h}$

$\lim_{h \to 0} \frac{[(x+h)^2 - (x^2)] \times [(x+h)^2 + x^2]}{h}$

$\lim_{h \to 0} \frac{(x^2 + 2xh + h^2 - x^2) \times (x^2 + 2xh + h^2 + x^2)}{h}$

$\lim_{h \to 0} \frac{\cancel{h}(2x+h) \times (2x^2 + 2xh + h^2)}{\cancel{h}}$

$(2x) \times (2x^2) = 4x^3$

The domain of its derivation and given function is same i.e. $(-\infty, \infty)$

R.

Exercise : 3.4

1. Differentiate the function

(a) $f(x) = 2^{40}$

sol^n: here

$$\frac{d f(x)}{dx} = \frac{d(2^{40})}{dx} = 0 \text{ Ans}$$

(b) $f(x) = e^5$

$$\frac{d(f(x))}{dx} = \frac{d(e^5)}{dx} = e^5 \times 0 = 0$$

(c) $F(x) = \frac{3}{4} x^8$

sol^n: here

$$\frac{d(F(x))}{dx} = \frac{d(\frac{3}{4} x^8)}{dx} = \frac{3}{4} \times 8 x^7$$

$$= 6x^7 \text{ Ans}$$

(d) $f(t) = 1.4t^5 - 2.5t^2 + 6.7$

Sol^n: here

$$\frac{d(f(t))}{dt} = 5 \times 1.4 \times t^4 - 5t + 0$$

$$= 7t^4 - 5t \text{ Ans}$$

(e) $h(x) = (x-2)(2x+3)$

sol^n: here

$h(x) = 2x^2 + 3x - 4x - 6$

$= 2x^2 - x - 6$

$$\frac{d[(h(x)]}{dx} = 4x - 1 - 0$$

$$= 4x - 1 \text{ Ans}$$

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(1) $y = x^{5/3} - x^{2/3}$

Sol^n: Here

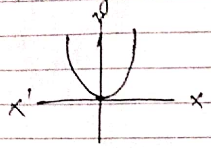
$$\frac{dy}{dx} = \frac{d(x^{5/3})}{dx} - \frac{d(x^{2/3})}{dx}$$

$$= \frac{5}{3}x^{2/3} - \frac{2}{3}x^{-1/3}$$

$$= \frac{1}{3}\left(5x^{2/3} - 2x^{-1/3}\right) \quad \text{Ans.}$$

(2) Find the points on the curve $y = 2x^3+3x^2-12x+1$ where the tangent is horizontal.

Sol^n: Here



At the origin point the tangent is horizontal so, it's cordinate is (0,0)

$\therefore \frac{dy}{dx} = 0$

Given eq^n of the curve $y = 2x^3+3x^2-12x+1$

$$\frac{dy}{dx} = 6x^2+6x-12$$

$$0 = 6x^2+6x-12$$

$$6(x^2+x-2)=0$$

$$x^2+x-2=0$$

$$x^2+(2-1)x-2=0$$

$$x^2+2x-x-2=0$$

$$x(x+2)-1(x+2)=0$$

$$(x-1)(x+2)=0$$

$$x=1$$

$$x=-2$$

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The point on the given curve become the tangent exist

at $x_1 = 1$

$y = 2\times1^3 + 3\times1^2 - 12\times1 + 1$

$= 2 + 3 - 12 + 1$

$= \cancel{8}\,6$

i.e. (-1,6)

at $x = -2$

$y = 2\times(-2)^3 + 3\times(-2)^2 - 12\times(-2) + 1$

$y = -16+12+24+1$

$= 21$

$\therefore (-2,21)$

Ans

(3) Show that the curve $y = 2e^x+3x+5x^3$ has no tangent line with slope 2

Sol^n: Here The given eq^n of the curve,

$$y = 2e^x + 3x + 5x^3$$

$$\frac{dy}{dx} = 2xe^x + 3 + 15x^2$$

$$\frac{dy}{dx} = 2xe^x + 15x^2 + 3$$

By using the quadratic formula of to determine the slope of curve

$$m_1 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$m_1 = -2 \pm \sqrt{4-4\times15\times3}$$

$$= -2 \pm \sqrt{4-180}$$

Which doesnot exist

so, no tangent line with slope 2 lies on the given curve

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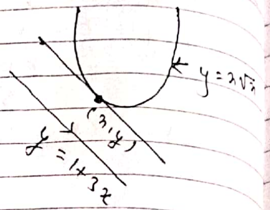
(4) Find an equation of the tangent line to the curve $y = x\sqrt{x}$ that is parallel to the line $y = 1+3x$.

Sol^n: here.



Given eq^n of line which is parallel to the tangent of curve is

$y = 1+3x$

$y = 3x+1$

$y = mx+c$

$\boxed{m=3}$

the $\frac{dy}{dx}$ of given curve is

$\frac{dy}{dx} = \frac{d(x\sqrt{x})}{dx}$

$\frac{dy}{dx} = \frac{d(\sqrt{x^3})}{dx} = \frac{3}{2} x^{\frac{3}{2}-1} = \frac{3}{2} x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{3}{2} x^{\frac{1}{2}}$

At $\frac{dy}{dx} = m = 3$

$3 = \frac{3}{2} x^{\frac{1}{2}}$

$2 = x^{\frac{1}{2}}$

$4 = x$

then $y = 4\sqrt{4} = 8$.

the eq^n of the tangent at point $(4, 8)$ is

$y - 8 = 3(x-4)$

$y - 8 = 3x - 12$

$y = 3x - 4$ Ans//

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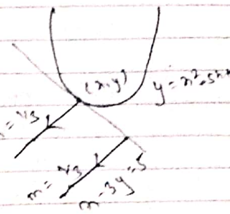
(5) Find an equation of the normal line to the parabola $y = x^2 - 5x + 4$ that is parallel to the line $x - 3y = 5$

Sol^n: here



Given eq^n of the parabola

$y = x^2 - 5x + 4$

Slope(m) $\frac{dy}{dx} = 2x - 5$

$\frac{dy}{dx} = 2x - 5$ —(1)

Slope of the ~~normal~~ parallel line which is parallel to the tangent of parabola

$m_1 = -\frac{x}{y}$

$x - 3y = 5$

$x - 5 = 3y$

$y = \frac{x}{3} - \frac{5}{3}$

Slope of normal (m) $= \frac{1}{3}$

~~eq^n of~~ slope of tangent

$m_1 \times m_2 = -1$

$\frac{1}{3} \times m_2 = -1$

$m_2 = -3$

then the slope of tangent = slope of parabola

$-3 = 2x - 5$

$-3 + 5 = 2x$

$x = \frac{2}{2} = 1$

$y = 1 - 5 \times 1 + 4 = 5 - 5 = 0$

The co-ordinate point of the curve where the tangent is drawn i.e. $(1, 0)$

then slope of normal is $\frac{1}{3}$

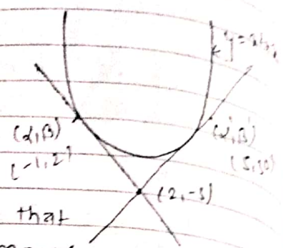
$y - 0 = \frac{1}{3}(x - 1)$

$3y = x - 1$

$\boxed{y = \frac{x}{3} - \frac{1}{3}}$ Ans

(6) (6) Find the equation of both lines through the point (2,−3) that are tangent to the parabola $y = x^2 + x$

Sol^n: Here



Given equation of parabola

$y = x^2 + x$

If (α,β) be any points that lies on the curve that means these ~~vete~~ point satisfied in the eq^n of curve

$\beta = \alpha^2 + \alpha$ — (1)

Slope of the curve $y = x^2 + x$ is

$\frac{dy}{dx} = 2x + 1$

the eq^n of straight line which become tangent of the given curve is

$(y+3) = m(x-2)$ (slope of curve = slope of tangent)

$2x+1 = m$

$(\alpha+\beta) = (2x+1)(x-2)$ — (2)

$(\alpha+3) = (2\beta+1)(\beta-2)$

$\beta+3 = (2x+1)(x-2)$

$(y+3) = (2x+1)(x-2)$

Satisfying the (α,β) in the eq^n of tangent

$\beta+3 = (2\alpha+1)(\alpha-2)$

$\alpha^2 + \cancel{\alpha} + 3 = 2\alpha^2 - 4\alpha + \cancel{\alpha} - 2$

$3 = \alpha^2 - 4\alpha - 2$

$\alpha^2 - 4\alpha - 5 = 0$

$\alpha^2 - (5-1)\alpha - 5 = 0$

$\alpha^2 - 5\alpha + \alpha - 5 = 0$

$\alpha(\alpha-5) + 1(\alpha-5) = 0$

$(\alpha+1)(\alpha-5) = 0$

$\alpha = -1$ and $\alpha = 5$.

putting $\alpha = -1$ in eq^n of curve we get

$\beta = -1^2 + 1 = 0$

$\beta = 2$

$\therefore (-1,2)$

putting $\alpha = 5$ in eq^n — (1)

$\beta = 25 + 5$

$\beta = 30$

$\frac{dy}{dx} = 2x+1$ for point $(-1,2)$ is

$m_1 = -2+1$

$= -1$

$\frac{dy}{dx} = 2x+1$ for point $(5,30)$

$m_2 = 2 \times 5 + 1 = 11$

eq^n of tangent at point $(-1,2)$ is

$x+1 = -1(y-2)$

$x+1 = -y+2$

$x+y = 1$

$y-2 = -1(x+1)$

$y-2 = -x-1$

$x+y = 2-1$

$x+y-1 = 0$

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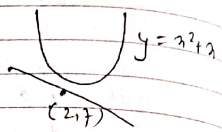
eqⁿ of tangent at point (5,30)

$$y-30 = 11(x-5)$$
$$y-30 = 11x-55$$
$$y = 11x-55+30$$
$$y = 11x-25 \quad \text{Ans}$$

(b) Show that there is no line through the point (2,7) that is tangent to the parabola $y=x^2+x$.

Solⁿ: here



Slope of curve

$$y = x^2+x$$
$$\frac{dy}{dx} = 2x+1$$

$$y-y_1 = m(x-x_1)$$
$$y-7 = (2x+1)(x-2)$$
$$y-7 = 2x^2-4x+x-2$$
$$y-7 = 2x^2-3x-2$$
$$y = 2x^2-3x+5$$

comparing eqⁿ with ax^2+bx+c

$a=2, b=-3, c=5$

then,

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$\frac{-(-3) \pm \sqrt{9-4\times2\times5}}{4}$$
$$\frac{3 \pm \sqrt{9-40}}{4} = \frac{3 \pm \sqrt{-31}}{4}$$ which can not be exist

So, this verifying that (2,7) is the not point of the Given curve.

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(7) Differentiable

(a) $g(x) = \sqrt{x}\, e^x$

Solⁿ: here

$$\frac{g(x)}{d(x)} = \frac{d(\sqrt{x}\cdot e^x)}{dx}$$
$$= \sqrt{x}\cdot\frac{d(e^x)}{dx} + e^x\cdot\frac{d(x^{1/2})}{dx}$$
$$= e^x\cdot\sqrt{x} + e^x\cdot\frac{1}{2}x^{-1/2}$$
$$= e^x\left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)$$
$$= e^x\left[\frac{2x+1}{2\sqrt{x}}\right] \quad \text{Ans//}$$

(b) $y = \dfrac{e^x}{1-e^x}$

Solⁿ: here

$$\frac{dy}{dx} = \frac{d\left[\frac{(e^x)}{1-e^x}\right]}{dx}$$
$$\frac{dy}{dx} = \frac{(1-e^x)\frac{d(e^x)}{dx} - e^x\frac{d(1-e^x)}{dx}}{(1-e^x)^2}$$
$$= \frac{(1-e^x)\times e^x - e^x(0-e^x)}{(1-e^x)^2}$$
$$= \frac{e^x - \cancel{e^x\cdot e^x} + \cancel{e^x\cdot e^x}}{(1-e^x)^2}$$
$$= \frac{e^x}{(1-e^x)^2}$$

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(c) $G(x) = \frac{x^2-2}{2x+1}$

Sol^n: here

$$\frac{d(G(x))}{dx} = \frac{d\left[\frac{x^2-2}{2x+1}\right]}{dx}$$

$$= \frac{(2x+1)\frac{d(x^2-2)}{dx} - \left[(x^2-2)\frac{d(2x+1)}{dx}\right]}{(2x+1)^2}$$

$$= \frac{(2x+1)\ (2x-0) - \left[(x^2-2)\ (2+0)\right]}{(2x+1)^2}$$

$$= \frac{4x^2+2x - 2x^2+4}{(2x+1)^2}$$

$$= \frac{2x^2+2x+4}{(2x+1)^2}$$

(d) $y = \frac{x+1}{x^3+x-2}$

Sol^n: here.

$$\frac{dy}{dx} = \frac{d\left[\frac{x+1}{x^3+x-2}\right]}{dx}$$

$$= \frac{(x^3+x-2)\frac{d(x+1)}{dx} - \left[(x+1)\frac{d(x^3+x-2)}{dx}\right]}{(x^3+x-2)^2}$$

$$= \frac{(x^3+x-2)\ (1+0) - \left[(x+1)(3x^2+1-0)\right]}{(x^3+x-2)^2}$$

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$$\frac{(x^3+x-2) - \left[(x+1)\ (3x^2+1)\right]}{(x^3+x-2)^2}$$

$$\frac{x^3+x-2 - \left[3x^3+x+3x^2+1\right]}{(x^3+x-2)^2}$$

$$\frac{x^3+\cancel{x}-2 - 3x^3-\cancel{x}-3x^2-1}{(x^3+x-2)^2}$$

$$\frac{-2x^3-3x^2-3}{(x^3+x-2)^2}$$ Ans

(e) $f(t) = \frac{2t}{2+\sqrt{t}}$

Sol^n: here

$$\frac{d[f(t)]}{dt} = \frac{d\left[\frac{2t}{2+\sqrt{t}}\right]}{dt}$$

~~f'(t)~~.

$$f'(t) = \frac{(2+\sqrt{t})\frac{d(2t)}{dt} - \left[2t\frac{d(2+\sqrt{t})}{dt}\right]}{(2+\sqrt{t})^2}$$

$$f'(t) = \frac{(2+\sqrt{t})\times 2 - \left[2t\left(0+\frac{1}{2}\times t^{-1/2}\right)\right]}{(2+\sqrt{t})^2}$$ $1-\frac{1}{2}$

$$= \frac{(4+2\sqrt{t}) - \left[2t\times\frac{1}{2}t^{-1/2}\right]}{(2+\sqrt{t})^2}$$

$$= \frac{4+2\sqrt{t}-\sqrt{t}}{(2+\sqrt{t})^2} = \frac{4+\sqrt{t}}{(2+\sqrt{t})^2}$$

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(f) $f(x) = \dfrac{1 - x \cdot e^x}{x + e^x}$

Sol^n: here

$$\frac{d[f(x)]}{dx} = \frac{(x+e^x)\dfrac{d(1-x \cdot e^x)}{dx} - (1-x \cdot e^x)\dfrac{d(x+e^x)}{dx}}{(x+e^x)^2}$$

$$\frac{d[f(x)]}{dx} = \frac{(x+e^x)\left[(0) - \dfrac{d(x \cdot e^x)}{dx}\right] - \left[(1-x \cdot e^x)(1+e^x)\right]}{(x+e^x)^2}$$

$$= \frac{(x+e^x)\left[-[x \cdot e^x + e^x \cdot 1]\right] - \left[(1-x \cdot e^x)(1+e^x)\right]}{(x+e^x)^2}$$

$$= \frac{(x+e^x)(-x \cdot e^x - e^x \cdot 1) - \left[1 + e^x - x \cdot e^x - x \cdot e^x \cdot e^x\right]}{(x+e^x)^2}$$

$$= \frac{-x^2 \cdot e^x - x \cdot e^x - \cancel{x \cdot e^x \cdot e^x} - e^x \cdot e^x - 1 - e^x + x \cdot e^x + \cancel{x \cdot e^x \cdot e^x}}{(x+e^x)^2}$$

$$= \frac{-x^2 e^x - \cancel{x \cdot e^x} - e^x \cdot e^x - 1 - e^x + \cancel{x e^x}}{(x+e^x)^2}$$

$$\frac{-x^2 e^x - \cancel{e^2} e^{2x} - 1 - e^x}{(x+e^x)^2} \quad \text{Ans}$$

$$-e^x(x^2 + e^x + 1) - 1$$

$$\frac{-e^x(x^2+1) - e^{2x} - 1}{(x+e^x)^2} \quad \text{Ans}$$

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(g) $f(x) = \dfrac{x^2}{1+2x}$

$$\frac{d[f(x)]}{dx} = \frac{(1+2x)\,d(x^2) - \left[x^2 \dfrac{d(1+2x)}{dx}\right]}{(1+2x)^2}$$

$$= \frac{(1+2x) \times 2x - \left[x^2(0+2)\right]}{(1+2x)^2}$$

$$= \frac{2x + 4x^2 - 2x^2}{(1+2x)^2}$$

$$= \frac{(2x^2+2x)}{(1+2x)^2} \quad \text{Ans}$$

(8) Differentiate:

(a) $f(x) = 3x^2 - 2\cos x$

$$\frac{d[f(x)]}{dx} = \frac{d(3x^2 - 2\cos x)}{dx}$$

$$= 6x - 2(-\sin x)$$

$$= 6x + 2\sin x \quad \text{Ans}$$

(b) $g(\theta) = e^\theta(\tan\theta - \theta)$

Sol^n: here

$$\frac{d[g(\theta)]}{d\theta} = e^\theta \frac{d(\tan\theta - \theta)}{d\theta} + (\tan\theta - \theta)\frac{d(e^\theta)}{d\theta}$$

$$= e^\theta(\sec^2\theta - 1) + (\tan\theta - \theta)e^\theta$$

$$= e^\theta(\sec^2\theta - 1 + \tan\theta - \theta)$$

$$e^\theta(\tan^2\theta + \tan\theta - \theta)$$

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(c) $y = \frac{x}{2-\tan x}$

Sol^n: here

$y = \frac{x}{2-\tan x}$

$\frac{dy}{dx} = \frac{(2-\tan x)\frac{d(x)}{dx} - \left[x \cdot \frac{d(2-\tan x)}{dx}\right]}{(2-\tan x)^2}$

$= \frac{(2-\tan x)\cdot 1 - [x\cdot(0-\sec^2 x)]}{(2-\tan x)^2}$

$= \frac{2-\tan x + x\sec^2 x}{(2-\tan x)^2}$ Ans

(d) $y = \frac{\cos x}{1-\sin x}$

Sol^n: here,

$\frac{dy}{dx} = \frac{d\left[\frac{\cos x}{1-\sin x}\right]}{dx}$

$= \frac{(1-\sin x)\frac{d(\cos x)}{dx} - \left[\cos x \cdot \frac{d(1-\sin x)}{dx}\right]}{(1-\sin x)^2}$

$= \frac{(1-\sin x)(-\sin x) - [\cos x\cdot(0-\cos x)]}{(1-\sin x)^2}$

$= \frac{-\sin x + (\sin^2 x + \cos^2 x)}{(1-\sin x)^2} = \frac{1-\sin x}{(1-\sin x)^2}$

$= \frac{1}{(1-\sin x)}$ Ans

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(e) $y = \frac{1-\sec x}{\tan x}$

Sol^n: here

$y = \frac{1-\sec x}{\tan x}$

$\frac{dy}{dx} = \frac{d\left[\frac{1-\sec x}{\tan x}\right]}{dx}$

$= \frac{\tan x \frac{d(1-\sec x)}{dx} - \left[(1-\sec x)\cdot\frac{d(\tan x)}{dx}\right]}{\tan^2 x}$

$= \frac{\tan x(0-\sec x.\tan x) - [(1-\sec x)\times\sec^2 x]}{\tan^2 x}$

$= \frac{-\sec x.\tan^2 x - \sec^2 x + \sec^3 x}{\tan^2 x}$

$\frac{-\sec x(\tan^2 x - \sec^2 x) - \sec^2 x}{\tan^2 x}$

$\frac{\sec x(\sec^2 x - \tan^2 x) - \sec^2 x}{\tan^2 x}$

$\frac{\sec x - \sec^2 x}{\tan^2 x}$

$\frac{\sec x(1-\sec x)}{\tan^2 x}$

$\frac{1}{\cos x \times \frac{\sin^2 x}{\cos^2 x}}(1-\sec x)$

$\frac{\cos x(1-\sec x)}{\sin^2 x}$

$\frac{\cos x - \sec x.\cos x}{\sin^2 x} = \frac{\cos x - 1}{\sin^2 x}$

$\dfrac{\cos x - 1}{\sin^2 x}$

$$= \frac{-(1-\cos x)}{(1-\cos^2 x)} = -\frac{(1-\cos x)}{(1+\cos x)(1-\cos x)}$$

$$= \frac{-1}{1+\cos x} \quad \text{Ans}$$

9. Find the eqⁿ of the tangent line to the curve $y = 3x + 6\cos x$ at the point $(\pi/3, \pi+3)$

$\cancel{y = 3x + 6\cos x}$

$\cancel{\dfrac{dy}{dx} = 3 + 6 - \sin x}$

$\cancel{\dfrac{dy}{dx} = 3 - 6\sin x}$

$\cancel{y - (\pi+3) = (3 - 6\sin x)(x - \pi/3)}$

$\cancel{y = 3\pi/3 + 6\cos ...}$

$\cancel{y = \pi + 6\cos(\pi+3)}$

Solⁿ: here

$y = 3x + 6\cos x$

$y = 3 \times \pi/3 + 6\cos 60^\circ$

$= (\pi + 3)$

eqⁿ of tangent become

$(y - \pi - 3) = m(x - \pi/3)$

11. An object at the end of a vertical spring is stretched 4 cm beyond its rest position and released at time $t = 0$. Its position at time t is $s = f(t) = 4\cos t$

Solⁿ: here,

$S = f(t) = 4\cos t$

$\dfrac{d[f(t)]}{dt} = \dfrac{d(4\cos t)}{dt}$

$f'(t) = -4\sin t$

This is the velocity by which the motion of the object is changed.

The acceleration of motion of object is

$f'(t) = -4\sin t$

$f''(t) = -4\cos t.$

$\therefore a = -4\cos t$

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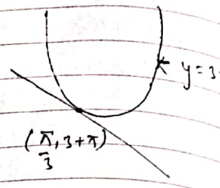
(9) Find an eq^n of the tangent line to the curve $y = 3x + 6\cos x$ at the point $(\pi/3, \pi+3)$

Sol^n: here



Sol^n: Here,
Given eq^n of the curve
$y = 3x + 6\cos x$

point $(\pi/3, \pi+3)$ lies on the curve so, this line passes through the curve i.e.

$y = 3 \times \pi/3 + 6\cos 60$

$y = \pi + 3$

slope of the curve is

$dy/dx = 3 - 6\sin x$

$dy/dx = m = 3 - 6\sin \pi/3$

$= 3 - 6 \times \sqrt{3}/2$

$= \cancel{3\sqrt{3}}\ 3 - 3\sqrt{3}$

eq^n of tangent is

$(y - 3 - \pi) = (3 - 3\sqrt{3})(x - \pi/3)$

$y - 3 - \cancel{\pi} = 3x - \cancel{\pi} - 3x\sqrt{3} + \pi\sqrt{3}$

$y - 3 = 3x - 3x\sqrt{3} + \pi\sqrt{3}$

$y = x(3 - 3\sqrt{3}) + (3 + \pi\sqrt{3})$

Ans//

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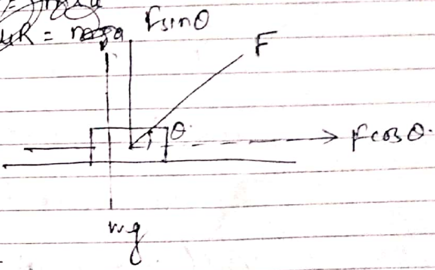
(10) An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$F = \dfrac{\mu W}{\mu\sin\theta + \cos\theta}$

F_f $F - f_r = ma$
$F - \mu R = ma$



Sol^n: here

We know that,

$F\sin\theta + R = mg$

$R = mg - F\sin\theta$ —(1)

If the object starts to move if only

$F_a \geq F_f$

$F\cos\theta = \mu R$

$F\cos\theta = \mu(mg - F\sin\theta)$

$F\cos\theta = \mu mg - \mu F\sin\theta$

$F(\cos\theta + \mu\sin\theta) = \mu mg$

$F = \dfrac{\mu mg}{(\cos\theta + \mu\sin\theta)} = \dfrac{\mu W}{(\cos\theta + \mu\sin\theta)}$

(a) Find the rate of change of force with respect to θ.

$$\frac{d(F)}{d\theta} = \frac{d[\mu W]}{\mu\sin\theta+\cos\theta}$$

$$F'(\theta) = \frac{(\mu\sin\theta+\cos\theta)\frac{d(\mu W)}{dx} + \left[\mu W \frac{d(\mu\sin\theta+\cos\theta)}{dx}\right]}{}$$

$$F'x = \frac{(\mu\sin\theta+\cos\theta)\times 0 - \left[\mu W \frac{d[\mu\sin\theta+\cos\theta]}{dx}\right]}{(\mu\sin\theta+\cos\theta)^2}$$

$$= \frac{-[\mu W\{\cos\theta + (-\sin\theta)\}]}{(\mu\sin\theta+\cos\theta)^2}$$

$$- \frac{[\mu W(\mu\cos\theta-\sin\theta)]}{(\mu\sin\theta+\cos\theta)^2}$$

$$\frac{\mu W(\sin\theta-\mu\cos\theta)}{(\mu\sin\theta+\cos\theta)^2}$$

Exercise:- 3.5

Required formulae

$$\frac{d(\log_a x)}{dx} = \frac{1}{x \log a}$$

$$\frac{d(\log_a f(x))}{dx} = \frac{1}{f(x)\log a} \times f'(x)$$

1. Find the first derivative of the function

a) $y = \sqrt{4+3x} = (4+3x)^{1/2}$

$$\frac{dy}{dx} = \frac{d(\sqrt{4+3x})^{1/2}}{d(4+3x)} \times \frac{d(4+3x)}{dx}$$

$$= \frac{1}{2}(4+3x)^{-1/2} \times (0+3)$$

$$= \frac{3}{2}(4+3x)^{-1/2} \text{ Ans.}$$

Exercise:- 3.6

Roll's Theorem

Statement:- If a function $f(x)$ is

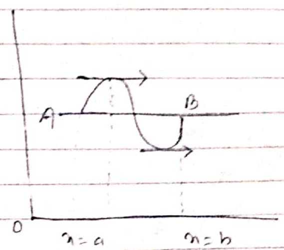
(i) continuous on closed interval $[a,b]$

(ii) differentiable on open interval (a,b)

(iii) $f(a) = f(b)$

then there exist at least one point $c \in (a,b)$ such that $f'(c) = 0$

Geometrical Interpretation



Exercise

verify Rolle's theorem for $f(x) = x^3 - x^2 - 6x + 2$ $[0,3]$

Sol: here

(i) since the function $f(x)$ is polynomial exp^n, so it is continuous on close interval $[0,3]$

(ii) $f'(x) = 3x^2 - 2x - 6$ it exist in $(0,3)$ so the $f(x)$ is a differentiable on ~~clos~~ open interval $(0,3)$

(iii) $f(0) = 0-0-0+2 = 2$

$f(3) = 27-9-18+2 = 2$

$\therefore f(0) = f(3)$

Since all the condition of Rolle's theorem are satisfied, then there exist at least one point $c \in (0,3)$ such that

$f'(c) = 0$

$3c^2 - 2c - 6 = 0$

$c = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$c = \frac{-2 \pm \sqrt{4 - 4\times3(-6)}}{2\times3}$

$c = \frac{-2 \pm \sqrt{76}}{6}$

$c = \frac{2 \pm 8.72}{6}$

$c = \frac{10.72}{6}, -\frac{6.72}{2}$

$= 1.79$ and -1.12

$c = 1.79 \in (0,3)$

Hence verified.

(a) $f(x) = 5-12x+3x^2$, in $[1,3]$

Sol: here,

$f(x) = 5-12x+3x^2$ in $[1,3]$

(i) The function $f(x)$ is a polynomial eqn, so, $f(x)$ is continuous on closed interval $[1,3]$

(ii) $f'(x) = 0-12+6x$

$= 6x-12$

The $f'(x)$ is differentiable on open interval $(1,3)$

(iii) $f(1) = 5-12+3 = -4$

$f(3) = 5-36+27 = -4$

$\therefore f(1) = f(3)$

Since, all the condition of the Rolle's theorem are satisfied so, there exist at least one point i.e. $c \in (1,3)$ such that

$f'(c) = 0$

$5-12x+3x^2 = 0$

$3x^2-12x+5 = 0$

$\therefore \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\therefore \frac{12 \pm \sqrt{144 - 4\times3\times5}}{2\times3}$

$= \frac{12 \pm \sqrt{144-60}}{6}$

$= \frac{12 \pm \sqrt{84}}{6}$

$= \frac{12+9.17}{6}, \frac{12-9.17}{6}$

$\frac{21.17}{6}, 2.83/6$

$\boxed{3.53}$ $\boxed{0.47}$

$6c - 12 = 0$

$6c = 12$

$c = 2$

$\therefore 2 \in (1,3)$

(c) $f(x) = \sqrt{x} - x/3$ in $[0, 9]$

Sol: here

$f(x) = \sqrt{x} - x/3$

$\lim_{x \to 0^+} f(x) = 0 - 0/3 = 0$

$\lim_{x \to 9^-} f(x) = \sqrt{9} - 9/3 = 3 - 3 = 0$

(i) Since the function $f(x)$ is continuous on closed interval $[0, 9]$

(ii) $f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{3}$

The function $f(x)$ is differentiable on open interval $(0, 9)$

(iii) $f(0) = 0$

(iv) $f(9) = 0$

$\therefore f(0) = f(9)$

Since all the condition of Rolle's Theorem are satisfied so, there exist at least one point i.e. $c \in (0, 9)$ Such that

$f'(c) = 0$

$\frac{1}{2\sqrt{c}} - \frac{1}{3} = 0$

$3 - 2\sqrt{c} = 0$

$2\sqrt{c} = 3$

$\sqrt{c} = 3/2$

$c = 9/4$

$c = 2.25$

$\therefore$

(d) $f(x) = \cos 2x$, in $[\pi/8, 7\pi/8]$

Sol: here

$f(x) = \cos 2x$

(i) $\lim_{x \to \pi/8^+} f(x) = \cos 2 \times \pi/8 = \cos 45 = 1/\sqrt{2}$

$\lim_{x \to 7\pi/8^-} f(x) = \cos 2 \times \frac{7\pi}{8} = \cos 14\pi/8 = 1/\sqrt{2}$

$\therefore$ The function $f(x)$ is continuous on closed interval i.e $[\pi/8, 7\pi/8]$

(ii) $f'(x) = -\sin 2x \times 2 = -2\sin 2x$

The function $f'(x)$ is differentiable on open interval i.e $(\pi/8, 7\pi/8)$

(iii) $f(\pi/8) = 1/\sqrt{2}$, $f(7\pi/8) = 1/\sqrt{2}$

Since all the condition of Rolle's theorem is satisfied so there exist at least one point i.e. $c \in (\pi/8, 7\pi/8)$ Such that

$f'(c) = 0$

$-2\sin 2c = 0$

$\sin 2c = 0$

$\cancel{\sin}\ 2c = \sin^{-1}(0)$

$c = 0$

$-2\sin 2c$

$-2 \times 2 \sin c \cdot \cos c = 0$

$\cos c = 0$

$c = \pi/2$

$\therefore \frac{\pi}{2} \in (\pi/8, 7\pi/8)$

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(2) Show that the equation $x^3-15x+c=0$ has at most one root in the interval $[-2,2]$

Solⁿ: Here

(i) $f(x)$ is continuous on closed interval $[-2,2]$ because $f(x)$ is a polynomial eqⁿ

(ii) $f'(x) = 3x^2-15$ it exist in $(-2,2)$ so, it is differentiable function. so if it possible

(iii) $f(-2) = f(2)$. according supposition,

So, there exist at least one point i.e. $c \in (-2,2)$ Such that

$f'(c) = 0$

$3c^2-15=0$

$3c^2 = 15$

$c^2 = 5$

$c = \pm 5$

Hence $+5$ and -5 are the root that most lies in the given interval

(3) Let $f(x) = 1-x^{2/3}$. show that $f(-1)=f(1)$ but there is no number c in $(-1,1)$, Such that $f'(c)=0$ why does this not contradict Rolle's theorem

Solⁿ: Here

$f(x) = 1-x^{2/3}$, in $[-1,1]$

$\lim_{x \to -1} f(x) = 1-1 = 0$

$\lim_{x \to 1} f(x) = 1-1 = 0$

(i) ∴ $f(x)$ is continuous on closed interval $[-1,1]$

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(ii) $f'(x) = 0 - \frac{2}{3} x^{2/3-1}$

$= 0 - \frac{2}{3} x^{-1/3}$

$= \frac{-2}{3(x)^{1/3}}$

If $x = 0$

then $f'(0) = \frac{-2}{3\times 0}$ = the function become undefine so, that this cannot satisfy the condition of Rolle's theorem

(A) let $f(x) = \tan x$ Show that $f(0) = f(\pi)$ but there is no number c in $(0,\pi)$ Such that $f'(c) = 0$. why does this not contradict Rolle's theorem

Solⁿ: Here,

$f(x) = \tan x$, in $[0,\pi]$

Given//

$f(x) = \tan x$

$\lim_{x \to 0} f(x) = \tan 0 = 0$

$\lim_{x \to \pi} f(x) = \tan\pi = 0$

(i) ∴ The function $f(x)$ is continuous on close interval i.e. $[0,\pi]$

(ii) $f'(x) = \sec^2 x = \frac{1}{\cos^2 x} = \frac{1}{\cos^2 \pi/2} = \frac{1}{0} = \infty$

the function $f(x)$ became undefine on open interval $(0,\pi)$ so that this cannot satisfy the condition of Rolle's theorem.

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Mean Value Theorem:

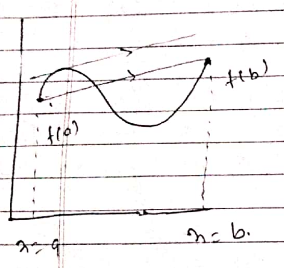
let f be a function that satisfies the following hypothesis

(i) f is continuous on the closed interval [a, b]

(ii) f is differentiable on the open " (a, b)

then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



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(5) verify that the function satisfies the three hypothesis the mean value Theorem on the given interval then find all the numbers c that satisfy the conclusion of the Mean value theorem.

(a) $f(x) = 2x^2 - 3x + 1$, in [0, 2]

sol^n: here

(i) The function f(x) is a polynomial eq^n so, the function is continuous on closed interval i.e. [0, 2]

(ii) $f'(x) = 4x - 3$ is a differentiable function on open interval i.e. (0, 2)

(iii) $f(0) = 0 - 0 + 1 = 1$

$f(2) = 8 - 6 + 1$

∴ so there is a number c in (0, 2) such that

$$f'(c) = \frac{f(b) - f(a)}{b - c} =$$

L.h.s

$4c - \cancel{3} = 0$ and R.h.s. $\frac{f(2) - f(0)}{2 - 0}$

$4c = 3$

$c = 3/4$

$= \frac{8 - 6 + 1 - (1)}{2}$

$4c - 3 = \frac{8 - 6 + 1 - 1}{2}$ ∴ $2/2 = 1$

$4c - 3 = 1$

$4c = 4$

$\boxed{c = 1}$ Hence all the condition of the ~~Rot~~ Mean's theorem is satisfied so, ~~this~~ ~~the~~

(b) $f(x) = x^3 + x - 1$, in $[0, 2]$

Sol: here,

$f(x) = x^3 + x - 1$

(i) The function $f(x)$ is a polynomial so, $f(x)$ is continuous on the closed interval $[0, 2]$

(ii) $f'(x) = 3x^2 + 1$ it exist ~~for~~ $f'(x)$ in open interval $(0, 2)$ hence it is differentiable.

(iii) since there is a number c in $(0, 2)$ such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$3c^2 + 1 = \frac{f(2) - f(0)}{2 - 0}$$

$$3c^2 + 1 = \frac{(8 + 2 - 1) - (-1)}{2}$$

$$3c^2 + 1 = \frac{10 - 1 + 1}{2} = 5$$

$$3c^2 + 1 = 5$$

$$3c^2 = 4$$

$$c^2 = 4/3$$

$$c = \pm 2/\sqrt{3}$$

$\therefore$ hence Mean value theorem is satisfied

(c) $f(x) = e^{-2x}$, in $[0, 3]$

Sol: here

$f(x) = e^{-2x}$

(i) $f(x)$ is a exponential function so, $f(x)$ is continuous on closed interval i.e. $[0, 3]$

(ii) $f'(x) = \frac{d(e^{-2x})}{d(-2x)} \times \frac{d(-2x)}{dx} = -2e^{-2x} = -2e^{-2x}$

$\therefore$ $f'(x)$ is a differentiable function ~~to~~ on open interval i.e. $(0, 3)$

(iii) then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$-2e^{-2c} = \frac{f(3) - f(0)}{3 - 0}$$

$$-2e^{-2c} = \frac{e^{-6} - 1}{3}$$

$$-6e^{-2c} = e^{-6} - 1$$

$$\cancel{\log(-6e^{-2c}) = \log(e^{-6} - 1)}$$

$$\cancel{\log -6 + -2c \log 2 = -6 \log_e e \div \log_e 1}$$

$$e^{-2c} = \frac{e^{-6} - 1}{-6}$$

$$e^{-2c} = \frac{0.00245 - 1}{-6}$$

$$e^{-2c} = \frac{0.998}{6} = 0.166$$

$$-2c = \log(0.166)$$

$$c = 0.89$$

$\therefore c \in (0, 3)$. Ans

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(d) $f(x) = \frac{x}{x+2}$ in $[1,4]$

Sol: here,

(i) $f(x)$ is continuous in closed interval i.e. $[1,4]$

(ii) $f'(x) = \frac{(x+2)\, d(x) - x\, d(x+2)}{(x+2)^2}$

$= \frac{(x+2) - x \cdot 1}{(x+2)^2} = \frac{x+2-x}{(x+2)^2}$

$= \frac{2}{(x+2)^2}$

$\therefore$ $f(x)$ is it exist in $(1,4)$ so it is differentiable on $(1,4)$

(iii) then there is a number c in $(1,4)$ such that

$f'(c) = \frac{f(b)-f(a)}{b-a}$

$f'(c) = \frac{2}{(c+2)^2}$

$f(4) = \frac{4}{4+2} = \frac{4}{6} = \frac{2}{3}$

$f(1) = \frac{1}{1+2} = \frac{1}{3}$

$\therefore \frac{2}{(c+2)^2} = \frac{\frac{2}{3}-\frac{1}{3}}{4-1} = \frac{\frac{1}{3}}{3} = \frac{1}{9}$

$\frac{2}{(c+2)^2} = \frac{1}{9}$

$18 = (c+2)^2$

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$c+2 = \sqrt{18}$

$c+2 = 4.24$

$c = 2.24$

$\therefore$ $c \in (1,4)$ Hence means value theorem is satisfied.

(6) Let $f(x) = (x-3)^{-2}$ show that there is no value c in $(1,4)$ such that $f(4)-f(1) = f'(c)(4-1)$ why does this not contradict the mean value theorem?

Sol: here

$f(x) = (x-3)^{-2}$, in $[1,4]$

(i) $f(x)$ is continuous function on closed interval $(1,4)$

(ii) $f'(x) = \frac{d(x-3)^{-2}}{d(x-3)} \times \frac{d(x-3)}{dx}$

$= -2(x-3)^{-3} \times 1$

$= -2(x-3)^{-3}$

$= \frac{-2}{(x-3)^3}$

$f'(x)$ is not differentiate in open interval i.e. $(1,4)$ such that

$f'(3) = \frac{-2}{(3-3)^3} = \frac{-2}{0} = \infty$.

therefore all the condition of mean value theorem is not satisfied Hence this is not a mean value function. Ans

(1) If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$ show small can $f(4)$ possibly be

$f(1) = 10$

$f'(c) = 2$

Soln: Here

$f(a) = f(1) = 10$

$f(b) = f(4) = f(4)$

$f'(c) = 2$

By using,

Third condition of Mean value theorem to find possible value of (4) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$2 = \frac{f(4) - 10}{4 - 1}$$

$$2 = \frac{f(4) - 10}{3}$$

$$6 = f(4) - 10$$

$f(4) = 16$. which is required value of of $f(4)$

Ans.

(2) Suppose that $3 \leq f'(x) \leq 5$ for all value of x, show that $18 \leq f(8) - f(2) \leq 30$

Soln: Here

According to the Given condition,

If we take least number value for $f(8) - f(2)$ then it is i.e. 18 it will be we have $f'(c)$ must be equal to 3 and if we take greater number i.e. $f(8) - f(2)$ i.e 30 then we have $f'(c)$ must be equal to 5, such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f'(c) = \frac{f(8) - f(2)}{8 - 2}$$

$$f'(c) = \frac{18}{6} = 3$$

$\therefore 3 \leq f'(x) \rightarrow$ satisfied

Again

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{f(8) - f(2)}{8 - 2}$$

$$f'(c) = \frac{30}{6}$$

$$f'(c) = 5$$

$\therefore f'(x) \leq 5$.

Hence $f'(x)$ for all value of x show that the given term of condition.

Does there exist a function f such that $f(0) = -1$, $f(2) = 4$ and $f'(x) \le 2$ for all x?

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(9) ~~Sol~~^n: here

$f(0) = -1$

$f(2) = 4$

$f'(x) \le 2$

$\therefore$ at $f'(x) = 2$

$$f'(x) = \frac{f(2) - f(0)}{2 - 0}$$

$$f'(x) = \frac{4+1}{2} = \frac{5}{2} = 2.5$$

$\therefore$ $2.5 \notin (0, 2)$ so, ~~there~~ this can not satisfy ~~all the~~ condition of Mean value ~~form~~ theorem.

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(11) ~~Sol~~^n: Use the method of example 9 to prove the identity $2\sin^{-1}x = \cos^{-1}(1-2x^2)$ for $x \ge 0$

Sol^n: here

$\cancel{f(x)}$ $2\sin^{-1}(x) - \cos^{-1}(1-2x^2) = 0.$

$\therefore f(x) = 2\sin^{-1}(x) - \cos^{-1}(1-2x^2)$

Then,

$$f'(x) = \frac{2}{\sqrt{1-x^2}} - \frac{-1}{\sqrt{1-(1-2x^2)^2}} \cdot (0 - 2 \cdot 2x)$$

$$= \frac{2}{\sqrt{1-x^2}} - \frac{4x}{\sqrt{1-(1-2x^2)^2}}$$

$$= \frac{2}{\sqrt{1-x^2}} - \frac{4x}{\sqrt{1-(1-4x^2+4x^4)}}$$

$$= \frac{2}{\sqrt{1-x^2}} - \frac{4x}{\sqrt{1-1+4x^2-4x^4}}$$

$$= \frac{2}{\sqrt{1-x^2}} - \frac{4x}{\sqrt{4x^2-4x^4}}$$

$$= \frac{2}{\sqrt{1-x^2}} - \frac{4x}{2x\sqrt{1-x^2}}$$

$$= \frac{2}{\sqrt{1-x^2}} - \frac{2}{\sqrt{1-x^2}}$$

$$= 0$$

Therefore, $f(x) = c$ (constant value)

$\cancel{f(0)}$ set $x = 0$ then,

$f(0) = 2\sin^{-1}(0) - \cos^{-1}(1 - 2\times 0)$

$= 2 \times 0 - \cos^{-1}(1)$

$= 0 - 0$

$= 0.$ and $f(0) = c$

Thus $c = f(x) = 0.$

Thus $2\sin^{-1}(x) = \cos^{-1}x(1-2x^2)$ for $x \ge 0$ proved

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(10) $|\sin a - \sin b| \le |a-b|$.

let $f(x) = \sin x$

$f'(x) = \cos x$

Mean value theorem

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$\cos c = \frac{\sin b - \sin a}{b-a}$$

Since $-1 \le \cos c \le 1$.

$|\cos c| \le 1$.

$$\left|\frac{\sin a - \sin b}{a-b}\right| \le 1$$

$$|\sin a - \sin b| \le |a-b|.$$

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Exercise : 3.7.

Indeterminate form
If the limit of the function

$\lim_{x \to a} \frac{f(x)}{g(x)}$ take the form $0/0$

then the function is called indeterminate form.

Another Indeterminate form:
∞/∞, $\infty - \infty$, $0 \times \infty$, $\infty + \infty$

0^0, ∞^0, 1^∞.

L-Hospital's Rule:
If the function $f(x)$ and $g(x)$ are continuou and differentrable at $x = a$ and $g'(a) \neq 0$ and $f(a) = g(a) = 0$, then.

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} = \frac{f'(a)}{g'(a)}.$$

Exercise 3.7.

$$\lim_{x \to 1} \frac{\log x}{x-1} \quad [0/0 \text{ form}]$$

$$\lim_{x \to 1} \frac{\frac{1}{x}}{1-0} = \frac{1}{x}$$

$\frac{1}{1} = 1$ Ans.

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Ex $\lim_{x \to 0} \frac{\tan x - x}{x^3}$

Sol: here

$\lim_{x \to 0} \frac{\sec^2 x - 1}{3x^2}$

$\lim_{x \to 0} \cancel{2\sec x} \frac{\tan^2 x}{3x^2}$

$\lim_{x \to 0} \frac{1}{3} \times \left(\frac{\tan^2 x}{x^2}\right)$

$\frac{1}{3}$ Ans

1. Find the limit. Use L' Hospital Rule where appropriate. If there is a more elementary method, consider using it. If L' Hospital Rule does not apply explain why.

(a) $\lim_{x \to 1} \left(\frac{x^2 - 1}{x^2 - x}\right) = \left[\frac{0}{0}\right]$ form

using L' Hospital rule,

$\lim_{x \to 1} \left(\frac{2x - 0}{2x - 1}\right) = \left(\frac{2x}{2x-1}\right)$

$\frac{2 \times 1}{2 \times 1 - 1} = \frac{2}{2-1} = 2$ Ans,

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(b) $\lim_{x \to \frac{1}{2}} \left(\frac{6x^2 + 5x - 4}{4x^2 + 16x - 9}\right)$

Sol: here

$\lim_{x \to \frac{1}{2}} \left[\frac{6x^2 + 5x - 4}{4x^2 + 16x - 9}\right] = \left[\frac{0}{0} \text{ form}\right]$

Using L hospital rule

$\lim_{x \to \frac{1}{2}} \left[\frac{12x + 5}{8x + 16}\right]$

$\frac{12 \times \frac{1}{2} + 5}{8 \times \frac{1}{2} + 16}$

$\frac{6 + 5}{4 + 16} = \frac{11}{20}$ Ans,

(c) $\lim_{x \to (\pi/2)^+} \left(\frac{\cos x}{1 - \sin x}\right)$

Sol: here

$\lim_{x \to \pi/2^+} \left(\frac{\cos x}{1 - \sin x}\right) = \left[\frac{0}{0} \text{ form}\right]$

$\lim_{x \to \pi/2} \left[\frac{-\sin x}{\cancel{1} - \cos x}\right]$

$\frac{-\sin 90}{0 - \cos 90} = \frac{-1}{0} = \infty$ Az

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(d) $\lim_{\theta \to \pi/2} \left(\frac{1-\sin\theta}{1+\cos 2\theta}\right)$

Sol^n: here

$\lim_{\theta \to \pi/2} \left(\frac{1-\sin\theta}{1+\cos 2\theta}\right)$ $\left[\frac{0}{0} \text{ form}\right]$

Using L Hospital Rule

$\lim_{\theta \to \pi/2} \left(\frac{0-\cos\theta}{0-\sin 2\theta \times 2}\right)$

$\lim_{\theta \to \pi/2} \left(\frac{-\cos\theta}{-2\sin 2\theta}\right) = \frac{\cos\theta}{2\sin 2\theta}$

$= \frac{\cos 90}{2\sin 180} = \frac{0}{0}$

$\lim_{x \to \pi/2} = \frac{\cancel{\cos\theta}}{2\times 2\sin\theta\times\cancel{\cos\theta}}$

$= \frac{1}{4\sin\theta}$

$\lim_{x \to \pi/2} = \frac{1}{4\sin\theta}$

$= \frac{1}{4\sin 90}$

$= \frac{1}{4}$ Ans

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(e) $\lim_{x \to \infty} \left(\frac{\ln x}{\sqrt{x}}\right)$

Sol^n: here

$\lim_{x \to \infty} \left(\frac{\ln x}{\sqrt{x}}\right)$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

Using L Hospital Rule

$\lim_{x \to \infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \frac{1}{x} \times 2\sqrt{x}$

$\lim_{x \to \infty} = \frac{1}{x} \times 2\sqrt{x}$

$= \frac{2\infty}{\infty} = \infty$

$= \frac{2\sqrt{x}}{(\sqrt{x})^2}$

$= \frac{2}{\sqrt{x}}$

$= \frac{2}{\infty} = 0$ Ans

(f) $\lim_{x \to 0^+} \left(\frac{\ln x}{\cot x}\right)$

Sol^n: here

$\lim_{x \to 0^+} \left(\frac{\ln x}{\cot x}\right) = \left[\frac{\infty}{\infty}\right]$ form

Using L Hospital Rules

$\lim_{x \to 0^+} \left(\frac{1/x}{-\operatorname{cosec}^2 x}\right)$ $\frac{1}{x} \times \frac{-1}{\operatorname{cosec}^2 x}$

$\lim_{x \to 0^+} = \frac{-\sin^2 x}{x}$

$\lim_{x \to 0^+} = \frac{\sin x}{x} \times -\frac{\sin x}{1}$

$\lim_{x \to 0^+} = -\sin x$

$= -\sin 0 = 0$ Ans

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(g) $\lim_{x \to \infty} \left(\frac{\ln\sqrt{x}}{x^2}\right)$

Sol^n: here
$\lim_{x \to \infty} \left(\frac{\ln\sqrt{x}}{x^2}\right)$ $\left[\frac{\infty}{\infty} \text{ form}\right]$

Using L hospital Rule

$\lim_{x \to \infty} \left(\frac{\ln\sqrt{x}}{x^2}\right)$

$\lim_{x \to \infty} \frac{\frac{1}{2x}}{2x}$

$\lim_{x \to \infty} \frac{1}{2x} \times \frac{1}{2x} = \frac{1}{4x}$

0 Ans

Rough
$\frac{d(\ln\sqrt{x})}{d(\sqrt{x})} \times \frac{d(\sqrt{x})}{dx}$
$\frac{1}{\sqrt{x}} \times \frac{1}{2\sqrt{x}} = \frac{1}{2x}$

(h) $\lim_{x \to 0} \left(\frac{\tan hx}{\tan x}\right)$

Sol^n: here,
$\lim_{x \to 0} \left(\frac{\tan hx}{\tan x}\right)$ $\left[\frac{0}{0} \text{ form}\right]$

Using L hospital Rule.

$\lim_{x \to 0} \frac{\sec^2 hx \times h}{\sec^2 x} = \frac{\cos^2 x \times h}{\cos^2 hx}$

$\lim_{x \to 0} \frac{\cos^2 x \times h}{\cos^2 hx}$

$\frac{\cos^2 0 \times h}{\cos^2 0} = 1 \times h = h$

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(i) $\lim_{x \to 0} \frac{(\sin^{-1} x)}{x}$ $\left[\frac{0}{0} \text{ form}\right]$

Sol^n: here
$\lim_{x \to 0} \left(\frac{\sin^{-1} x}{x}\right)$ $\left(\frac{0}{0} \text{ form}\right)$

Using L hospital Rule.

$\lim_{x \to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{1} = \frac{1}{\sqrt{1-x^2}} = \frac{1}{1-0} = 1$ Ans

(j) $\lim_{x \to \infty} [x \sin(\pi/x)]$

Sol^n: here
$\left[\frac{\sin(\pi/x)}{1/x}\right]$ $\left[\frac{0}{0} \text{ form}\right]$

Using L hospital Rule.

$\lim_{x \to \infty} \left[\frac{\sin(\pi/x)}{1/x}\right]$

$\lim_{x \to \infty} \frac{\frac{\cos \pi/x \times \pi}{x^2}}{-1 \cdot x^{-2}}$

$\lim_{x \to \infty} \cos \pi/x \times \frac{\pi}{x^2} \times \frac{-1}{x^{-2}}$

$\lim_{x \to \infty} \cos \pi/x \times \pi$

$(\cos \pi/\infty) \times \pi$

$\cos 0 \times \pi$

$1 \times \pi$

π Ans

Rough
$\frac{d(\sin \pi/x)}{d(\pi/x)} \times \frac{d(\pi/x)}{dx}$

$\frac{d(x/2)}{dx}$ $2.\frac{1}{4}$ DATE: / / $\frac{2}{4} = \frac{1}{2}$
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(k) $\lim_{x \to \infty} [\sqrt{x}\, e^{-x/2}]$

Sol^n: here
$\lim_{x \to \infty} [\sqrt{x}\, e^{-x/2}]$

$\lim_{x \to \infty} \left[\frac{\sqrt{x}}{e^{x/2}}\right]$ $\left[\frac{\infty}{\infty} \text{ form}\right]$

Using L Hospital Rule.

$\lim_{x \to \infty} \dfrac{\frac{1}{2\sqrt{x}}}{e^{x/2} \times \frac{1}{2}}$

$\lim_{x \to \infty} \frac{1}{2\sqrt{x}} \times \frac{2}{e^{x/2}}$

$\lim_{x \to \infty} \frac{1}{\sqrt{x}} \times \frac{1}{e^{x/2}}$

$\lim$ $\frac{1}{\infty} \times \frac{1}{e^{\infty/2}}$

0×0

0 Ans

(l) $\lim_{x \to 0} [\cot 2x \sin 6x]$

Sol^n: here
$\lim_{x \to 0} \left(\frac{\sin 6x}{\tan 2x}\right)$ $\left[0/0 \text{ form}\right]$

Using L Hospital Rule.

$\lim_{x \to 0} \dfrac{\frac{\sin 6x}{6x} \times \frac{6x}{1}}{\frac{\tan 2x}{2x} \times \frac{2x}{1}}$

$\lim_{x \to 0} 1 \times \frac{6x}{1} \times \frac{1}{2x}$ $= 3$

$= 3$ Ans

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(m) $\lim_{x \to 0^+} [\sin x \ln x]$

Sol^n: here
$\lim_{x \to 0^+} [\sin x \ln x]$

$\lim_{x \to 0^+} \left[\frac{\ln x}{\operatorname{cosec} x}\right] = \left(\infty/\infty \text{ form}\right)$

Using L Hospital Rule.

$\lim_{x \to 0^+} \dfrac{\frac{1}{x}}{-\operatorname{cosec} x \cdot \cot x}$

$\lim_{x \to 0^+} \frac{1}{x} \times \frac{-1}{\operatorname{cosec} x \cdot \cot x}$

$\lim_{x \to 0^+} \frac{\sin x}{x}$ $\frac{-1}{\cot x}$

$\lim_{x \to 0^+} 1 \cdot \frac{-1}{\cot x}$

$\lim_{x \to 0} \frac{-1}{\cot x}$

$\frac{-1}{\cot 0} = \frac{-1}{\infty} = 0$ Ans

$\frac{d(-x^2)}{dx} = -2x$

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(b) $\lim_{x\to\infty} [x^3 e^{-x^2}]$

Soln: here

$$\lim_{x\to\infty} \left[\frac{e^{-x^2}}{x^{-3}}\right] \left[\frac{x^3}{e^{x^2}}\right] \left[\frac{\infty}{\infty} \text{ form}\right]$$

Using L Hospital Rule

$$\lim_{x\to\infty} \frac{e^{-x^2}\cdot x - 2x}{-3x^{-4}} \qquad \left[\frac{x^3}{e^{x^2}}\right]$$

$$\lim_{x\to\infty} \frac{e^{-x^2} x - 2}{-3 \times x^{-5}} \qquad \left[\frac{3x^2}{e^{x^2} \times 2x}\right]$$

$$\lim_{x\to\infty} \frac{-2e^{-x^2}}{-3 \times x^{-5}} \qquad \left[\frac{3}{2}\,\frac{x}{e^{x^2}}\right]$$

$$\lim_{x\to\infty} \left[\frac{3}{2}\,\frac{x}{2\cdot x e^{x^2}}\right]$$

$$\lim_{x\to\infty} \left[\frac{3}{4e^{x^2}}\right]$$

$$\left[\frac{3}{4e^{\infty}}\right]$$

$$\left[\frac{3}{\infty}\right]$$

0 Ans

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(c) $\lim_{x\to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right]$

Sol^n: here

$$\left(\frac{x\cdot \ln x - (x-1)}{\ln x\cdot(x-1)}\right) \left[\frac{0}{0} \text{ form}\right]$$

Using L Hospital Rule.

$$\lim_{x\to 1} \frac{\ln x\cdot 1 + x \times \frac{1}{x} - (1-0)}{(x-1)\times\frac{1}{x} + \ln x\,(1-0)}$$

$$\lim_{x\to 1} \frac{\ln x + 1 - 1}{(x-1)\times\frac{1}{x} + \ln x}$$

$$\lim_{x\to 1} \frac{x\cdot \ln x}{(x-1) + x\cdot \ln x} = \frac{x\cdot \ln x}{x \ln x + (x-1)}$$

$$\lim_{x\to 1} \frac{\ln x\cdot 1 + x\cdot \frac{1}{x}}{\ln x\cdot 1 + x \times \frac{1}{x} + (1-0)}$$

$$\lim_{x\to 1} \frac{\ln x + 1}{\ln x + 1 + 1} = \frac{\ln x + 1}{\ln x + 2}$$

$$\frac{\ln 1 + 1}{\ln 1 + 2} = \frac{0+1}{0+2}$$

$\frac{1}{2}$ Ans

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(p) $\lim_{x \to 0} (\operatorname{cosec} x - \cot x)$

Sol^n: here.

$\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)$

$\left(\frac{1-\cos x}{\sin x} \right)$ $\left[0/_0 \text{ form} \right]$

Using L'Hospitals Rule,

$\lim_{x \to 0} \frac{0+\sin x}{\cos x} = \frac{\sin x}{\cos x}$

$\lim_{x \to 0} \tan x$

$\tan 0$

0 Ans

(q) $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{e^x-1} \right)$

Sol^n: here

$\lim_{x \to 0} \left(\frac{e^x-1-x}{x \cdot (e^x-1)} \right)$ $\left[\frac{\infty}{\infty} \text{ form} \right]$

Using L'Hospital's Rule.

$\lim_{x \to 0^+} \left(\frac{e^x-0-1}{(e^x-1) \cdot 1 + x \cdot (e^x-0)} \right.$

$\lim_{x \to 0^+} \frac{(e^x-1)}{(e^x-1)+x \cdot (e^x)}$

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$\lim_{x \to 0^+} \frac{e^x}{e^x + e^x \cdot 1 + x \cdot e^x}$

$\lim_{x \to 0^+} \frac{e^0}{e^0+e^0 \cdot 1+0 \cdot e^0}$

$\frac{1}{1+1+0}$

$\frac{1}{2}$ Ans

(r) $\lim_{x \to 0} \left(\cot x - 1/_x \right)$

Sol^n: here.

$\lim_{x \to 0} \left(\frac{x \cdot \cot x - 1}{x} \right)$ $\left[\frac{0}{0} \text{ form} \right]$

$\cancel{\lim_{x \to 0} \frac{\cot x \cdot 1 + x \cdot (-\operatorname{cosec}^2 x) - 0}{1}}$

$\cancel{\lim_{x \to 0} \frac{\cot x - x/\operatorname{cosec}^2 x}{1}}$

$\cancel{\lim_{x \to 0}}$

$\lim_{x \to 0} \left(\frac{x \cdot \frac{\cos x}{\sin x} - 1}{x} \right)$

$\lim_{x \to 0} \left(\frac{\frac{\cos x}{\sin x/_x} - 1}{x} \right)$

$\lim_{x \to 0} \left(\frac{\cos x - 1}{x} \right)$

$\lim_{x \to 0} \left(\frac{-\sin x - 0}{1} \right) = -\sin x$

$= -\sin 0 = 0$ Ans

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(5) $\lim_{x \to \infty} (x - \ln x)$

Sol^n: Here

$\lim_{x \to \infty} (x - \ln x)$

$\frac{x(x - \ln x)}{x}$

$\frac{x^2 - \ln x \cdot x}{x}$ $\left[\frac{\infty}{\infty} \text{ form}\right]$

Using L Hospital Rule

$\lim_{x \to \infty} \frac{2x - (\ln x \cdot 1 + \frac{1}{x} \times x)}{1}$

$\lim_{x \to \infty} 2x - \ln x + 1$

$\lim_{x \to \infty} 2 - \frac{1}{x} + 0$

$\lim_{x \to \infty} 2 - \frac{1}{x}$

$2 - \frac{1}{\infty}$

$2 - 0$

2 Ans

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(6) $\lim_{x \to 0^+} (x^{\sqrt{x}})$

Sol^n: Here

let $y = (x^{\sqrt{x}})$

Taking log on both side we get,

$\log y = \log(x^{\sqrt{x}})$

$\log y = \sqrt{x} \log(x)$

$\log y = \frac{\log x}{\frac{1}{\sqrt{x}}}$

Taking R.H.S at once

$\lim_{x \to 0^+} \frac{\log x}{\frac{1}{\sqrt{x}}}$ $\left[\frac{\infty}{\infty} \text{ form}\right]$

Using L Hospital Rule,

$\lim_{x \to 0^+} = \frac{\frac{1}{x}}{\frac{-1}{2} x^{-3/2}}$

$\lim_{x \to 0^+} = \frac{1}{x} \times \frac{2}{x^{-3/2}}$

$\lim_{x \to 0^+} = \frac{2}{x^{1-3/2}}$

$\lim_{x \to 0^+} = \frac{2}{x^{-1/2}}$

$\lim_{x \to 0^+} = 2\sqrt{x}$

$= 2\sqrt{0} =$ ~~∞~~ Ans

$= 0$

$\log y = 0$

$y = e^0 = 1$ Ans

$\log(\lim_{x \to 0} y) = 0$

$\lim_{x \to 0} y = e^0$

$y = e^0 = 1$ Ans

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(4) $\lim_{x \to 0^+} (\tan 2x)^x$

Sol: here,

$\lim_{x \to 0^+} (\tan 2x)^x$

let

$y = (\tan 2x)^x$

taking log on both side

$\log y = \log(\tan 2x)^x$

$\log y = x \log(\tan 2x)$

$\lim_{x \to 0^+} x \log(\tan 2x)$

$\lim_{x \to 0^+} \log(\tan 2x) \cdot 1 + x \cdot \frac{1}{\tan 2x} \times \sec^2 2x \times 2$

$\lim_{x \to 0^+} \log(\tan 2x) + 2x \cdot \cot 2x \cdot \sec^2 2x$

$\lim_{x \to 0^+} \log(\tan 2x) + 2x \cdot \frac{\cos 2x}{\sin 2x} \times \sec^2 2x$

$\log(\tan 2x) + \frac{\cos 2x}{\cos^2 2x} \times \frac{1}{\frac{\sin 2x}{2x}}$

$\log(\tan 2x) + \frac{1}{\cos 2x} \cdot 1$

$\log(\tan 2x) + \sec 2x$

$\frac{1}{\tan 2x} \times \sec^2 2x \cdot 2 + \sec 2x \cdot \tan 2x$

$2\cot 2x \times \sec^2 2x + 2\sec 2x \tan 2x$

$2 \frac{\cos 2x}{\sin 2x} \times \frac{1}{\cos^2 2x} + 2 \times \frac{1}{\cos 2x} \times \frac{\sin 2x}{\cos 2x}$

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$2 \frac{1}{\sin 2x \cdot \cos 2x} + \frac{2}{\cos^2 2x} \cdot \sin 2x$

$\frac{2}{\sin 2x \cdot \cos 2x} + \frac{2\sin 2x}{\cos^2 2x}$

$\frac{2\cos 2x + 2\sin^2 2x}{\sin 2x \, \cos^2 2x}$

$\frac{2\cos 2x + 2\sin^2 2x}{\sin 2x (1-}$

$x \log(\tan 2x)$

$\frac{\log(\tan 2x)}{1/x}$ [∞/∞ form]

$\lim_{x \to 0^+} \frac{\frac{1}{\tan 2x} \times \sec^2 2x \times 2}{-1x^{-2}}$

$\lim_{x \to 0^+} \frac{\cos 2x}{\sin 2x} \times \frac{1}{\cos^2 2x} \times 2 \quad x - x^2$

$\lim_{x \to 0^+} \frac{-x}{\frac{\sin 2x}{2x}}$

$\lim_{x \to 0^+} = -x$

$= 0$

taking

$\log \lim_{x \to 0} y = 0$

$y = e^0$

$y = 1$

$\therefore (\tan 2x)^x = 1$ Ans

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(v) $\lim_{x \to 0} (1-2x)^{1/x}$

Sol: here

$\lim_{x \to 0} (1-2x)^{1/x}$

let $y = (1-2x)^{1/x}$

taking log on both side

$\log y = \log(1-2x)^{1/x}$

$\log y = \frac{1}{x} \log(1-2x)$

$\log y = \frac{1}{x} \log(1-2x) = \frac{\log(1-2x)}{x}$

$\lim_{x \to 0} \frac{\log(1-2x)}{x}$

$\lim_{x \to 0} \frac{\frac{1}{(1-2x)} (0-2)}{1}$

$\lim_{x \to 0} \frac{-2}{1-2x}$

$\frac{-2}{1-0} = -2$

then,

$\log \lim_{x \to 0} y = -2$

$\lim_{x \to 0} y = e^{-2}$

$\therefore (1-2x)^{1/x} = e^{-2}$ Ans

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(vi) $\lim_{x \to \infty} (x^{1/x})$

let $y = x^{1/x}$

taking log on both side

$\log y = \log(x^{1/x})$

$\log y = \frac{1}{x} \log x$

$\lim_{x \to \infty} \frac{1}{x} \log x$

$\lim_{x \to \infty} \frac{\log x}{x}$

$\lim_{x \to \infty} \frac{\frac{1}{x}}{1}$

$\lim_{x \to \infty} \frac{1}{x}$

$\frac{1}{\infty}$

0

$\log \lim_{x \to \infty} y = 0$

$\lim_{x \to \infty} y = e^{0}$

$y = 1$

$\therefore x^{1/x} = 1$ Ans

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(x) $\lim_{x\to\infty}(e^x+x)^{1/x}$

Sol^n: here,

let $y = (e^x+x)^{1/x}$

taking log on both side we get

$\log y = \log(e^x+x)^{1/x}$

$\log y = \frac{1}{x}\log(e^x+x)$

$\lim_{x\to\infty} \frac{\log(e^x+x)}{x}$

$\lim_{x\to\infty} \frac{\frac{1}{e^x+x}\times(e^x+1)}{1}$

$\lim_{x\to\infty} \frac{1}{e^x+x}(e^x+1) = \frac{e^x+1}{e^x+x}$

$\lim_{x\to\infty} \frac{e^x+0}{e^x+1}$

$\lim_{x\to\infty} \frac{e^x+0}{e^x+1}$

$\lim_{x\to\infty} \frac{e^x}{e^x+0}$

$\lim_{x\to\infty} \frac{e^x}{e^x}$

1.

$\log \lim_{x\to\infty} y = 1$

$\lim_{x\to\infty} y = e^1,\ y = e$ $\therefore (e^x+x)^{1/x} = e$

$\frac{d(a^x)}{dx}$ DATE: / /
PAGE NO. $a^x \log a$

(y) $\lim_{t\to 0} \frac{(3^t-5^t)}{t}$

let $t = x$

we have,

$\lim_{x\to 0} \frac{(3^x-5^x)}{x}$ $\left[\frac{0}{0}\text{ form}\right]$

Using L hospital rule

$\lim_{x\to 0} \frac{3^x\log 3 - 5^x\log 5}{1}$

$3^x\log 3 - 5^x\log 5$

$3^0\log 3 - 5^0\log 5$

$1.\log 3 - 1.\log 5$

$\log(3/5)$

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(2) $\lim_{x\to\infty} \left[\frac{(\ln x)^2}{x}\right]$

Sol^n: here

$\lim_{x\to\infty} \left(\frac{(\ln x)^2}{x}\right)$ $\left[\frac{\infty}{\infty} \text{ form}\right]$

Using L hospital Rule.

$\lim_{x\to\infty} \dfrac{\frac{d(\ln x)^2}{d(\ln x)} \times \frac{d(\ln x)}{dx}}{1}$

$\lim_{x\to\infty} \dfrac{2\ln x \cdot 1/x}{1}$

$\lim_{x\to\infty} \frac{2\ln x}{x}$

$\lim_{x\to\infty} \dfrac{2\times\frac{1}{x}}{1}$

$\lim_{x\to\infty} 2/x$

$2/\infty$

0 Ans

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([illegible]) $\lim_{x\to 0} \left(\frac{x\cdot 3^x}{3^x-1}\right)$

Sol^n: here.

$\lim_{x\to 0} \left(\frac{x\cdot 3^x}{3^x-1}\right)$ $\left[0/0 \text{ form}\right]$

$\lim_{x\to 0} \left(\frac{3^x\cdot 1 + x\cdot 3^x\log 3}{3^x\log 3 - 0}\right)$

$\lim_{x\to 0} \left(\frac{3^x + x\cdot 3^x\cdot\log 3}{3^x\log 3}\right)$

$\lim_{x\to 0} \frac{\cancel{3^x}}{\cancel{3^x}}\left(\frac{1+x\log 3}{\log 3}\right)$

$\frac{1}{\log 3}$ Ans

([illegible]) $\lim_{x\to\infty} \left(\frac{\ln(\ln x)}{x}\right)$

Sol^n: here.

$\lim_{x\to\infty} \left(\frac{\ln(\ln x)}{x}\right)$ $\left[0/0 \text{ form}\right]$

$\lim_{x\to\infty} \frac{d(\ln(\ln x))}{d(\ln x)} \times \frac{d(\ln x)}{dx}$

$\lim_{x\to\infty} \frac{1}{\ln x} \times \frac{1}{x} = \frac{1}{x\ln x}$

$\frac{1}{\infty \ln\infty}$

0 Ans

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(5) Find the limit of the following function by using L Hospital Rule,

(a) $\lim_{x\to\infty} \left(1+a/x\right)^{bx}$

Sol: here

let $y = \left(1+a/x\right)^{bx}$

taking log on both side

$\log y = bx \log\left(1+a/x\right)$

$\lim_{x\to\infty} \quad bx \log\left(1+a/x\right)$

$b\left[x \log\left(1+a/x\right)\right]$

$b\left[x \log \frac{(x+a)}{x}\right]$

$b\left[x\left[\log(x+a) - \log(x)\right]\right]$

$b[$

$x \log = \frac{1}{x} = x^{-1}$

$\lim_{x\to\infty} \left(1+a/x\right)^{bx}$

$\left[\left(1+\frac{a}{x}\right)^{x}\right]^{b}$

e^{a}

$(e^{a})^{b}$

e^{ab}

(c) $\lim_{x\to\infty} \quad \frac{e^x}{x^n} = \infty$

diff n

---

(2) prove that $\lim_{x\to\infty} \left(\frac{e^x}{x^n}\right) = \infty$ for any positive integer n

Sol: here

$\lim_{x\to\infty} \left(\frac{e^x}{x^n}\right) = \left[\frac{\infty}{\infty} \text{ form}\right]$

Using L-Hospital Rule

$\lim_{x\to\infty} \left(\frac{e^x}{x^n}\right)$

$\frac{e^x}{nx^{n-1}}$ (derivate n times it provide us constant i.e. n

$\lim_{x\to\infty} \frac{e^x}{n}$

$\frac{e^\infty}{n}$

$\frac{\infty}{n}$

$= \infty$ As proved

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(3) Prove that $\lim_{x\to\infty}\left(\frac{\ln x}{x^p}\right) = 0$ for any ~~pa~~ number $p>0$

Sol^n: Here

$$\lim_{x\to\infty}\left(\frac{\ln x}{x^p}\right) = \left[\frac{\infty}{\infty}\right]$$

Using L hospital Rule,

$$\lim_{x\to\infty}\left(\frac{\ln x}{x^p}\right)$$

$$\frac{\frac{1}{x}}{p\cdot x^{p-1}}$$

$$\left(\frac{x^{-1}}{p\cdot x^{p-1}}\right)$$

$$\frac{\frac{d^n(x^{-1})}{dx}}{p\frac{d^n(x^{p-1})}{dx}}$$

$$\frac{\cancel{x^0}}{p} \quad \frac{x^0}{p}.$$

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(A) What happened if you try to use L'hospital Rule to find the limit? Evaluate the limit using another method

(a) $\lim_{x\to\infty}\left(\frac{x}{\sqrt{x^2+1}}\right)$

Sol^n: here

$$\lim_{x\to\infty}\left(\frac{x}{\sqrt{x^2+1}}\right) = \left[\frac{\infty}{\infty}\right]$$

Using L hospital Rule

$$\lim_{x\to\infty}\left(\frac{1}{\frac{1}{\sqrt{x^2+1}}\times 2x}\right)$$

$$\lim_{x\to\infty}\left(\frac{\sqrt{x^2+1}}{2x}\right)$$

It gives us, indefinite value after doing derivative of it's. Repeat So, another method is used.

$$\lim_{x\to\infty}\left(\frac{x}{\sqrt{x^2+1}}\right)$$

$$\lim_{x\to\infty}\frac{x}{x\left(\sqrt{1+\frac{1}{x^2}}\right)}$$

$$\frac{1}{\sqrt{1+1/\infty}}$$

$$\frac{1}{\sqrt{1+0}} = 1 \text{ Ans}$$

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(b) $\lim_{x \to (\pi/2)^-} \left(\frac{\sec x}{\tan x} \right)$

Sol^n: here,

$\lim_{x \to (\pi/2)^{-1}} = (\sec x \times \cot x)$

$\lim_{x \to (\pi/2)^-}$ ~~sec x × cos x~~

$\lim_{x \to \pi/2} \left(\frac{1}{\cos x} \times \frac{\cos x}{\sin x} \right)$

$\lim_{x \to \pi/2} = \frac{1}{\sin x}$

$= \frac{1}{\sin 90}$

$= 1$ Ans

(c) $\lim_{x \to \infty} \left(\frac{\sqrt{3x+4}}{\sqrt{2x+1}} \right)$

Sol^n: here,

$\lim_{x \to \infty} \left(\frac{\sqrt{3x+4}}{\sqrt{2x+1}} \right)$

$x \left(\sqrt{3/x + 4/x} \right)$

$\lim_{x \to \infty} \left(\sqrt{\frac{3x+4}{2x+1}} \right)$

$\lim_{x \to \infty} \left(\sqrt{\frac{\frac{3x}{x} + 4/x}{\frac{2x}{x} + 1/x}} \right)$

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$\lim_{x \to \infty} \left(\sqrt{\frac{3 + 4/x}{2 + 1/x}} \right)$

$\left(\sqrt{\frac{3+0}{2+0}} \right)$

$\sqrt{3/2}$ Ans

(5) (a) $\lim_{x \to \infty} \left(1 + a/x \right)^{bx}$

Sol^n: here

$\lim_{x \to \infty} \left(1 + a/x \right)^{bx}$

$\lim_{x \to \infty} \left[\left(1 + a/x \right)^{x} \right]^{b}$ (Formula $\lim_{x \to \infty} \left(1 + a/x \right)^{x} = e^{a}$)

$(e^{a})^{b}$

e^{ab}

(b) $\lim_{x \to \infty} \left(1 - 3/x \right)^{x}$

Sol^n: here

$\lim_{x \to \infty} \left(1 - 3/x \right)^{x}$

$\lim_{x \to \infty} \left(1 + \frac{(-3)}{x} \right)^{x}$

e^{-3} Ans