GOOD MORNING PAGE NO.: DATE: / /

UNIT- 06.

The dimension of vector space.

let V be a vector space over the field. let $\{v_1, v_2, \ldots v_n\}$ is a basis of V, then the number of basis element is denoted by dim V and called dimension of the vector space.

Theorem :- If a vector space V has a basis of n vectors, then every basis of V must consist of exactly n
$B = \{b_1, b_2, \ldots b_n\}$, then any set in V containing more them n vectors must be linearly dependent.

Theorem :- If a vector space V has a basis of n vectors, then every basis of V must consist of exactly n vectors.

Finite dimensional vector space.

If a vector space V is spanned by a finite set, then V is said to be finite dimensional vector space and dimensional of V is the number of vectors in a basis for V.
If V is not spanned by a finite set, then V is said to be infinite-dimensional vector space.

Theorem :- let H be a subspace finite dimensional vector space V. Any linearly independent set in H can be expanded, if necessary to a basis for H. Also H is finite- dimensional and dim H ≤ dim V.

Spanning Set theorem:-

let $S = \{v_1, v_2, \dots v_n\}$ be a set in V and $H = \{v_1, v_2, \dots v_n\}$.

(a) If one of the vectors in S say v_1, is a linear combination of the remaining vector in S, then the set formed from S by removing v_1 still span H.

(b) If $H \neq \{0\}$, some subset of S is a basis of H.

The Basis theorem:-

let V be a p-dimensional vector space $p \geq 1$
Any linearly independent set of exactly p-element in V is automatically a basis for V.
Any set of exactly p element that span V is automatically a basis for V.

The dimension of NUL A.

The dimension of NUL A is the number of free variable in the eqn $Ax = 0$.

The dimension of col A.

The dimension of col A is the number of pivot column in A

GOOD MORNING PAGE NO.: DATE: / /

Ex. The find the dimension of Null space and column space of

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

Sol^n: Here

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ -3 & 6 & -1 & 1 & -7 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

$R_2 \to R_2 + 3R_1$, $R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & 5 & 10 & -10 \\ 0 & 0 & 1 & 2 & -2 \end{bmatrix}$$

$R_2 \to \frac{1}{5} R_2$

$$\begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 1 & 2 & -2 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Here x_2 and x_4 and x_5 are free variable

so $\dim \mathrm{Nul} A = 3$

and 1st and 3rd column has pivot column

so $\dim \mathrm{col} A = 2$

Find the dimension of NUL A and col A for the matrices.

(i) $A = \begin{bmatrix} 1 & -6 & 9 & 0 & -2 \\ 0 & 1 & 2 & -4 & 5 \\ 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Sol^n: Here,

$$A = \begin{bmatrix} 1 & -6 & 9 & 0 & -2 \\ 0 & 1 & 2 & -4 & 5 \\ 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Here x_3 and x_5 are free variable and x_1, x_2 and x_4 are basic variable so the dimension of NUL A = 2 and the dimension of col A = 3

Ans

(ii) $A = \begin{bmatrix} 1 & 0 & 9 & 5 \\ 0 & 0 & 1 & -4 \end{bmatrix}$

Sol^n: Here

$$A = \begin{bmatrix} 1 & 0 & 9 & 5 \\ 0 & 0 & 1 & -4 \end{bmatrix}$$

Here x_2 and x_3 are free basic variable so dimension of col A = 2.
Here x_1 and x_4 are free variable so dimension of NUL A = 2

Ans

GOOD MORNING
PAGE NO.:
DATE: / /

4 (iii) $A = \begin{bmatrix} 3 & 2 \\ -6 & 5 \end{bmatrix}$

Soln: Here.

$$A = \begin{bmatrix} 3 & 2 \\ -6 & 5 \end{bmatrix}$$

$R_2 \rightarrow R_2 + 2R_1$

$$A = \begin{bmatrix} 3 & 2 \\ 0 & 9 \end{bmatrix}$$

Here x_1 and x_2 are basic for variable
So the dimension of col A = 2

(iv) $A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}$

Soln: Here,

$$A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}$$

$R_1 \rightarrow -\frac{1}{2} R_1$

$$A = \begin{bmatrix} 1 & -2 & 1 & 2 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}$$

$R_2 \rightarrow R_2 + 2R_1$ and $R_3 \rightarrow R_3 + 3R_1$

$$\begin{bmatrix} 1 & -2 & 1 & 2 \\ 0 & -2 & -5 & -3 \\ 0 & 2 & 5 & 3 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & -2 & 1 & 2 \\ 0 & -2 & -5 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to -1R_2$

$$\begin{bmatrix} 1 & -2 & 1 & 2 \\ 0 & 2 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Here x_1 and x_2 are basic variable so dimension of col A = 2

and x_3 and x_4 are free variable so dimension of NUL A = 2

(v) $A = \begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix}$

Soln: Here,

$$A = \begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix}$$

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19 - 33

GOOD MORNING
PAGE NO. -2 + 9
DATE / /

$$A = \begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - R_1,\ R_4 \to R_4 - 3R_1.$

$$\begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 0 & 0. & 5. & -7 & 8 \\ 0 & 0 & 5 & -7 & 8 \\ 0 & 0 & 10 & -14 & 7 \end{bmatrix}$$

$R_4 \to R_4 - 2R_3$

$$\begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 0 & 0 & 5 & -7 & 8 \\ 0 & 0 & 5 & -7 & 8 \\ 0 & 0 & 0 & 0 & -9 \end{bmatrix}$$

2-16

$R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & -7 & 8 \\ 0 & 0 & 0 & 0 & -9 \end{bmatrix}$$

Here x_1, x_3 and x_5 are basic varíable
so the dimension of $\text{col}A = 3$
and x_2, and x_4 are free variable
so the dimension of $\text{Nul}A = 2$

Ans

Find the basis and dimension for each subspace.

1.

(i) $\left\{ \begin{bmatrix} 2a \\ -4b \\ -2a \end{bmatrix}, a, b \in R \right\}$

Soln: Here,

Let $H = \left\{ \begin{bmatrix} 2a \\ -4b \\ -2a \end{bmatrix}, a, b, \in R \right\}$

$$H = \begin{bmatrix} 2a \\ -4b \\ -2a \end{bmatrix} = \begin{bmatrix} 2a + 0.b \\ 0.a + (-4)b \\ -2.a + 0.b \end{bmatrix} = a \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} + b \begin{bmatrix} 0 \\ -4 \\ 0 \end{bmatrix}$$

which shows that H is a linear combination of the vector V_1 & V_2 ~~and V_3~~. clearly $v_1 \neq 0$ V_2 is not multiple of V_1 and ~~V_3~~ so by $\{V_1, V_2\}$ span H and is linearly independent so it is basis for H and dim $H = 2$

(11) $\left\{ \begin{bmatrix} 2c \\ a-b \\ b-3c \\ a+2b \end{bmatrix}, a, b, c \in R \right\}$

Sol^n: Here.

let $H = \begin{bmatrix} 2c \\ a-b \\ b-3c \\ a+2b \end{bmatrix}, \quad a, b, c \in R$.

$$H = \begin{bmatrix} 0.a + 0.b + 2c \\ a + -1.b + 0.c \\ 0.a + b + (-3).c \\ a + 2b + 0.c \end{bmatrix} = a\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} + b\begin{bmatrix} 0 \\ -1 \\ 1 \\ 2 \end{bmatrix} + c\begin{bmatrix} 2 \\ 0 \\ -3 \\ 0 \end{bmatrix}$$

which shows that H is a linear combin- of V_1, V_2 and V_3. and clearly $V_1 \neq 0$ and V_2 is not multiple of V_1 and V_2 So by $\{V_1, V_2, V_3\}$ span H and is linearly independent So it is basis for H and dim. H = 3 Ans

(iii) $\left\{ \begin{bmatrix} p+2q \\ -p \\ 3p-q \\ p+q \end{bmatrix}, p, q \in R \right\}$.

Sol^n: Here,

let $H = \left\{ \begin{bmatrix} p+2q \\ -p \\ 3p-q \\ p+q \end{bmatrix}, p, q \in R \right\}$

$$H = \begin{bmatrix} p+2q \\ -p \\ 3p-q \\ p+q \end{bmatrix} = \begin{bmatrix} p+2.q \\ -1.p+0.q \\ 3.p+-1.q \\ 1.p+1.q \end{bmatrix} = p\begin{bmatrix} 1 \\ -1 \\ 3 \\ 1 \end{bmatrix} + q\begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

which shows that H is a linear combination of the vector V_1 & V_2 and clearly $V_1 \neq 0$ and V_2 is not the no. multiple of V_1. So by $\{V_1, V_2\}$ span the H and is linearly independent so it is basis for H and $\dim H = 2$

(iv) $\left\{ \begin{bmatrix} p-2q \\ 2p+5r \\ -2q+2r \\ -3p+6r \end{bmatrix}, p, q, r \in R \right\}$

Soln: Here,

$$\text{let } H = \left\{ \begin{bmatrix} p-2q \\ 2p+5r \\ -2q+2r \\ -3p+6r \end{bmatrix}, p, q, r \in R \right\}$$

$$H = \begin{bmatrix} p + (-2)\cdot q + 0\cdot r \\ 2\cdot p + 0\cdot q + 5\cdot r \\ 0\cdot p + (-2)\cdot q + 2r \\ -3\cdot p + 0\cdot q + 6\cdot r \end{bmatrix} = p\begin{bmatrix} 1 \\ 2 \\ 0 \\ -3 \end{bmatrix} + q\begin{bmatrix} -2 \\ 0 \\ -2 \\ 0 \end{bmatrix} + r\begin{bmatrix} 0 \\ 5 \\ 2 \\ 6 \end{bmatrix}$$

which shows that H is a linear combination of vector v_1, v_2 and v_3 and clearly $v_1 \neq 0$ and v_2 is not multiple of v_1 and v_3 so by $\{v_1, v_2, v_3\}$ span H and is linearly independent and it is basis for H.

dim. of H = 3.

Example Find a basis and dimension of the Subspace.

$$H = \left\{ \begin{bmatrix} 3a+6b-c \\ 6a-2b-2c \\ -9a+5b+3c \\ -3a+b+c \end{bmatrix} ; \; a, b, c \in R \right\}$$

Soln: Here

$$\begin{bmatrix} 3a+6b-c \\ 6a-2b-2c \\ -9a+5b+3c \\ -3a+b+c \end{bmatrix} = a \begin{bmatrix} 3 \\ 6 \\ -9 \\ -3 \end{bmatrix} + b \begin{bmatrix} 6 \\ -2 \\ 5 \\ 1 \end{bmatrix} + c \begin{bmatrix} -1 \\ -2 \\ 3 \\ 1 \end{bmatrix}$$

which show that H is a linear combin. of the vector v_1, v_2 and v_3 clearly $v_1 \neq 0$ v_2 is not multiple of v_1 and v_3 is a multiple of v_1. so by $\{v_1, v_2\}$ span H and is linearly independent so it is basis for H

and $\dim H = 2$ Ans

8. Find the dimension of the Subspace of all vectors in R^3. whose first and third entries are equal.

Sol: Here,

According question, Here there are 3 entries of given the vector V and whose first and third entries are equal such that

$$H = \begin{bmatrix} a \\ b \\ a \end{bmatrix}, a, b \in R.$$

$$\therefore \begin{bmatrix} a \\ b \\ a \end{bmatrix} = \begin{bmatrix} a+0.b \\ 0.a+1.b \\ a+0.b \end{bmatrix} = a\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

which shows that H is a linear combinat of v_1 and v_2. and clearly $v_1 \neq 0$ and v_2 is not multiple of v_1. and is $\{v_1, v_2\}$ span H and is linearly independent. So it is basis for H and dim $H = 2$.

Q. Find the dimension of the Subspace spanned by the given vectors.

(1) $\begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}3\\1\\1\end{bmatrix}, \begin{bmatrix}-2\\-1\\1\end{bmatrix}, \begin{bmatrix}5\\2\\2\end{bmatrix}$

Soln: Here,

let $v_1 = \begin{bmatrix}1\\0\\2\end{bmatrix}$, $v_2 = \begin{bmatrix}3\\1\\1\end{bmatrix}$, $v_3 \begin{bmatrix}-2\\-1\\1\end{bmatrix}$ $v_4 = \begin{bmatrix}5\\2\\2\end{bmatrix}$

$\therefore H = \text{span}\{v_1, v_2, v_3, \}$.

which shows that H is a linear combination of v_1, v_2 and v_3. where v_4 doesnot has

let $A = \begin{bmatrix}1 & -2 & 5\\0 & -1 & 2\\2 & 1 & 2\end{bmatrix}$

$R_3 \to R_3 - 2R_1$

$\begin{bmatrix}1 & 3 & -2 & 5\\0 & 1 & -1 & 2\\0 & -5 & 5 & -8\end{bmatrix}$

$R_3 \to R_3 + 5R_2$

$\begin{bmatrix}1 & 3 & -2 & 5\\0 & 1 & -1 & 2\\0 & 0 & 0 & 2\end{bmatrix}$

Since Here x_1, x_2 and x_4 are basic variable and x_3 are free variable so A is spanned by $\{v_1, v_2, v_4\}$ and Hence Now it is linearly independent and it is basis for A. So

dim. of A = 3.

(2) $\begin{bmatrix}1\\-2\\0\end{bmatrix} \begin{bmatrix}-3\\-6\\0\end{bmatrix} \begin{bmatrix}-2\\3\\5\end{bmatrix} \begin{bmatrix}-3\\5\\5\end{bmatrix}$

Soln: Here,

let $H = \begin{bmatrix}1 & -3 & -2 & -3\\-2 & -6 & 3 & 5\\0 & 0 & 5 & 5\end{bmatrix}$

$R_2 \to R_2 + 2R_1$

$\begin{bmatrix}1 & -3 & -2 & -3\\0 & -12 & -1 & -1\\0 & 0 & 5 & 5\end{bmatrix}$

~~$R_2 \to R_2$~~ $-$ $R_3 \to \frac{1}{5}R_3$

$\begin{bmatrix}1 & -3 & -2 & -3\\0 & -12 & -1 & -1\\0 & 0 & 1 & 1\end{bmatrix}$

$R_2 \to R_2 + R_3$

$\begin{bmatrix}1 & -3 & -2 & -3\\0 & -12 & 0 & 0\\0 & 0 & 1 & 1\end{bmatrix}$

Here x_1, x_2 and x_3 are basic variable and x_4 are free variable and is spanned by $\{v_1, v_2, v_3\}$ so it is basis for H

so dim of H = 3

Exercise:- 6.2

Row Space:-
Let A be an $m \times n$ matrix. Each row of A has n entries and thus can be identified with a vector in R^n. The set of all linear combination of the row vectors is called the row space of A. and is denoted by Row A. Each row has n entries, so row A is a subspace of R^n.

Theorem:- If two matrix A and B are equivalent then their row space are the same. If B is in echelon form, the non-zero row of B form a basis for row space of A as well as for that of B.

Rank of Matrix. :-
The rank of matrix A is the dimension of the column space of A.

Rank Theorem:-
The dimension of the column space and row space of $m \times n$ matrix A are equal. This common dimension, the rank of A also equal to the number of pivot column in A and ~~stat~~ satisfies the eqn.

$$\text{rank } A + \dim(\text{Nul } A) = n$$

Exercise:-
Find the bases for row space, the column space and the null space of the matrix

$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$$

Soln; Here

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 3 & -5 & 1 & 5 \\ -2 & -5 & 8 & 0 & -17 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & 13 & 5 & -3 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 3 & -5 & 1 & 5 \\ 0 & 1 & -2 & 2 & -7 \\ 0 & 2 & -4 & 4 & -14 \\ 0 & 4 & -8 & 4 & -8 \end{bmatrix}$$

$R_1 \to R_1 - 3R_2$, $R_3 \to R_3 - 2R_2$, $R_4 \to R_4 - 4R_2$.

$$\begin{bmatrix} 1 & 0 & 1 & -5 & 26 \\ 0 & 1 & -2 & 2 & -7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 & 20 \end{bmatrix}$$

$R_4 \to -\frac{1}{4}R_4$ and $R_4 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 0 & 1 & -5 & 26 \\ 0 & 1 & -2 & 2 & -7 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 + 5R_3,\ R_2 \to R_2 - 2R_3$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -2 & 0 & 3 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = B.$$

Here the non-zero rows are 1^{st}, 2^{nd} and 3^{rd} row.

$\therefore$ bases for row space are.
$(1,0,1,0,1)$, $(0,1,-2,0,3)$ and $(0,0,0,1,-5)$
The number of pivot position 1^{st}, 2^{nd}, and 4^{th} so that the col A is

$$\left\{ \begin{bmatrix} -2 \\ 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -5 \\ 3 \\ 11 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 7 \\ 5 \end{bmatrix} \right\}$$

For bases for NulA.
Here x_3, and x_5 are free variable.
The corresponding equation are

$x_1 + x_3 + x_5 = 0 \Rightarrow x_1 = -x_3 - x_5$
$x_2 - 2x_3 + 3x_5 = 0 \Rightarrow x_2 = 2x_3 - 3x_5$
$x_4 - 5x_5 = 0 \Rightarrow x_4 = 5x_5$
$x_5 = \text{free}$

$$\therefore x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -x_3 - x_5 \\ 2x_3 - 3x_5 \\ x_3 + 0 \cdot x_5 \\ 0 \cdot x_3 + 5x_5 \\ 0 \cdot x_3 + x_5 \end{bmatrix}$$

$$= x_3 \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ -3 \\ 0 \\ 5 \\ 1 \end{bmatrix}$$

$\therefore$ bases for NULA are

$$\left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 0 \\ 5 \\ 1 \end{bmatrix} \right\}$$

Exercise :- 6.2

1. Assume that the matrix A is row equivalent to B without calculations. list rank A and dim NUL A. Also find bases for col A, Row A and NUL A.

(i) $A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & -1 & 5 \\ 0 & -2 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Sol^n: Here,

Given matrix $A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$

The reduced echelon form matrix of A is B i.e.

$B = \begin{bmatrix} 1 & 0 & -1 & 5 \\ 0 & -2 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Therefore, basis for row space of the matrix A is,

$(1,0,-1,5)$ and $(0,-2,5,-6)$.

For bases for col$A$ is

$\left\{ \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -6 \end{bmatrix} \right\}$.

Basis for NULA is

Here x_3 and x_4 are free variable then the corresponding eq^n are,

$x_1 - x_3 + 5x_4 = 0$

$x_1 = x_3 - 5x_4$

$-2x_2 + 5x_3 - 6x_4 = 0$

$-2x_2 = -5x_3 + 6x_4$

$x_2 = \frac{5}{2} x_3 - 3x_4$

x_3 = free

x_4 = free

$\therefore x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_3 - 5x_4 \\ \frac{5}{2} x_3 - 3x_4 \\ x_3 + 0 \cdot x_4 \\ 0 \cdot x_3 + x_4 \end{bmatrix}$

$= x_3 \begin{bmatrix} 1 \\ 5/2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix}$

$\therefore$ Basis of NUL A is

$\left\{ \begin{bmatrix} 1 \\ 5/2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}$

$\therefore$ Rank A = dimension of col A

$= 2$.

dim of NULA = 2

Ans

(11) $A = \begin{bmatrix} 2 & 6 & -6 & 6 & 3 & 6 \\ -2 & -3 & 6 & -3 & 0 & -6 \\ 4 & 9 & -12 & 9 & 3 & 12 \\ -2 & 3 & 6 & 3 & 3 & -6 \end{bmatrix}$.

$B = \begin{bmatrix} 2 & 6 & -6 & 6 & 9 & 6 \\ 0 & 3 & 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Solⁿ: Here,
The given matrix is,

$$A = \begin{bmatrix} 2 & 6 & -6 & 6 & 3 & 6 \\ -2 & -3 & 6 & -3 & 0 & -6 \\ 4 & 9 & -12 & 9 & 3 & 12 \\ -2 & 3 & 6 & 3 & 3 & -6 \end{bmatrix}.$$

The reduced-echelon form of the matrix A is, B i.e.

$$B = \begin{bmatrix} 2 & 6 & -6 & 6 & 3 & 6 \\ 0 & 3 & 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Bases for col A, row space of the matrix A is

$(2, 6, -6, 6, 3, 6)$, $(0, 3, 0, 3, 3, 0)$ and $(0, 0, 0, 0, 3, 0)$.

$\therefore$ Bases for col A is.

$$\left\{ \begin{bmatrix} 2 \\ -2 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 3 \\ 3 \end{bmatrix} \right\}$$

Bases for NUL A.

Here x_3, x_4 and x_6 are free variable then the corresponding eqⁿ are.

$$2x_1 + 6x_2 - 6x_3 + 6x_4 + 3x_5 + 6x_6 = 0$$
$$2x_1 + 6x_2 + 3x_5 = 6x_3 - 6x_4 - 6x_6 \quad \text{--- (1)}$$

$$3x_2 + 3x_4 + 3x_5 = 0$$
$$3x_2 + 3x_5 = -3x_4 \quad (\because x_5 = 0),$$
$$x_2 = -x_4$$
$$: 3x_5 = 0$$
$$x_5 = 0$$
$$x_6 = \text{free}$$

from (1)

$$2x_1 + 6x_2 + 3x_5 = 6x_3 - 6x_4 - 6x_6$$
$$2x_1 + 6(-x_4) + 3 \cdot 0 = 6x_3 - 6x_4 - 6x_6$$
$$2x_1 - \cancel{6x_4} = 6x_3 - \cancel{6x_4} - 6x_6$$
$$2x_1 = 6x_3 - 6x_6$$
$$x_1 = 3x_3 - 3x_6$$

$$\therefore x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 3x_3 - 3x_6 \\ -x_4 \\ x_3 \\ x_4 \\ 0 \\ x_6 \end{bmatrix} = \begin{bmatrix} 3x_3 + 0 \cdot x_4 - 3x_6 \\ 0 \cdot x_3 - x_4 + 0 \cdot x_6 \\ x_3 + 0 \cdot x_4 + 0 \cdot x_6 \\ 0 \cdot x_3 + x_4 + 0 \cdot x_6 \\ 0 \cdot x_3 + 0 \cdot x_4 + 0 \cdot x_6 \\ 0 \cdot x_3 + 0 \cdot x_4 + x_6 \end{bmatrix}$$

$$= x_3 \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Bases for NUL A is

$$\left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$\therefore$ rank of A = dimension of col A
$= 3$

dimension of NUL A = 3.

(iii) $A = \begin{bmatrix} 1 & 1 & -2 & 0 & 1 & -2 \\ 1 & 2 & -3 & 0 & -2 & -3 \\ 1 & -1 & 0 & 0 & 1 & 6 \\ 1 & -2 & 2 & 1 & -3 & 0 \\ 1 & -2 & 1 & 0 & 2 & -1 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 1 & -2 & 0 & 1 & -2 \\ 0 & 1 & -1 & 0 & -3 & -1 \\ 0 & 0 & 1 & 1 & -13 & -1 \\ 0 & 0 & 0. & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Sol^n: Here,
The given matrix is,

$A = \begin{bmatrix} 1 & 1 & -2 & 0 & 1 & -2 \\ 1 & 2 & -3 & 0 & -2 & -3 \\ 1 & -1 & 0 & 0 & 1 & 6 \\ 1 & -2 & 2 & 1 & -3 & 0 \\ 1 & -2 & 1 & 0 & 2 & -1 \end{bmatrix}$

The reduced-echelon form of matrix A is B i.e.

$B = \begin{bmatrix} 1 & 1 & -2 & 0 & 1 & -2 \\ 0 & 1 & -1 & 0 & -3 & -1 \\ 0 & 0 & 1 & 1 & -13 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Therefore Bases for Row spaces of the matrix A is

$(1,1,-2,0,1,-2)$, $(0,1,-1,0,-3,-1)$, $(0,0,1,1,-13,-1)$ and $(0,0,0,0,1,-1)$ and $(0,0,0,0,0,1)$

Bases for Col A is =

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 6 \\ 0 \\ -1 \end{bmatrix} \right\}.$$

Bases for NUL A

Here x_4 is a free variable then the corresponding equations are

$x_1 + x_2 - 2x_3 + x_5 - 2x_6 = 0$ | $x_1 + x_3 - 2x_3 + 0 + 0 = 0$

$x_2 - x_3 - 3x_5 - x_6 = 0$ | $x_1 + 3x_3 = 0$

$x_3 + x_4 - 13x_5 - x_6 = 0$ | $x_1 + x_4 = 0$

$x_6 = 0$, $x_5 - x_6 = 0 \Rightarrow x_5 = 0$ | $x_1 = -x_4$

$x_1 + x_2 - 2x_3 + x_5 = 0$

$x_2 - x_3 - 3x_5 = 0$

$x_3 + x_4 - 13x_5 = 0$

$x_1 = -x_2 - 2x_3 + x_5 \rightarrow$ | $x_1 = -x_3 - 3x_5 - 2x_3 + x_5$

$x_2 = x_3 + 3x_5$ | $x_1 = -3x_3 - 2x_5$

$x_3 = -x_4 + 13x_5$ | $x_1 = -3(-x_4 + 13x_5) - 2x_5$

$x_1 = 3x_4 - 39x_5 - 2x_5$

$x_1 = 3x_4$

$x_2 = x_3 + 3x_5$

$x_2 = -x_4 + 13x_5 + 3x_5$

$x_2 = -x_4 + 16x_5$

$x_2 = -x_4$

$x_3 = -x_4$

$x_4 = \text{free}$

$x_5 = 0$

$x_6 = 0.$

$$\therefore x \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} -2x_4 \\ -x_4 \\ -x_4 \\ x_4 \\ 0 \\ 0 \end{bmatrix} = x_4 \begin{bmatrix} -2 \\ -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Bases for NULA is

$$\left\{ \begin{bmatrix} -2 \\ -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$\therefore$ Rank of A = col. of A

= 5.

dim for NUL A = 1 Ans

Find Rank A, dim NULA and bases for colA, RowA and NUL A from the given matrix.

$$A = \begin{bmatrix} 2 & -1 & 1 & -6 & 8 \\ 1 & -2 & -4 & 3 & -2 \\ -7 & 8 & 10 & 3 & -10 \\ 4 & -5 & -7 & 0 & 4 \end{bmatrix}$$

Sol^n: Here

$$A = \begin{bmatrix} 2 & -1 & 1 & -6 & 8 \\ 1 & -2 & -4 & 3 & -2 \\ -7 & 8 & 10 & 3 & -10 \\ 4 & -5 & -7 & 0 & 4 \end{bmatrix}$$

$R_2 \leftrightarrow R_1$

$$\begin{bmatrix} 1 & -2 & -4 & 3 & -2 \\ 2 & -1 & 1 & -6 & 8 \\ -7 & 8 & 10 & 3 & -10 \\ 4 & -5 & -7 & 0 & 4 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1,\ R_3 \to R_3 + 7R_1,\ R_4 \to R_4 - 4R_1$

$$\begin{bmatrix} 1 & -2 & -4 & 3 & -2 \\ 0 & 3 & 9 & -12 & 12 \\ 0 & -6 & -18 & 24 & -24 \\ 0 & 3 & 9 & -12 & 12 \end{bmatrix}$$

$R_2 \to \frac{1}{3}R_2,\ R_3 \to -\frac{1}{6}R_2,\ R_4 \to \frac{1}{3}R_4$

$$\begin{bmatrix} 1 & -2 & -4 & 3 & -2 \\ 0 & 1 & 3 & -4 & 4 \\ 0 & 1 & 3 & -4 & 4 \\ 0 & 1 & 3 & -4 & 4 \end{bmatrix}$$

$R_3 \to R_3 - R_2$ and $R_4 \to R_4 - R_2$

$$\begin{bmatrix} 1 & -2 & -4 & 3 & -2 \\ 0 & 1 & 3 & -4 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -4 & 3 & -2 \\ 0 & 1 & 3 & -4 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Now

Therefore basis for Row space of matrix A is

$(1,-2,-4,3,-2)$ and $(0,1,3,-4,4)$.

Basis for col A is

$$\left\{ \begin{bmatrix} 2 \\ 1 \\ -7 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 8 \\ -5 \end{bmatrix} \right\}.$$

Basis for NUL A is.

Here x_3, x_4 and x_5 are free variable then the corresponding equation is

$x_1 - 2x_2 - 4x_3 + 3x_4 - 2x_5 = 0$

$x_1 - 2x_2 = 4x_3 - 3x_4 + 2x_5$

$x_2 + 3x_3 - 4x_4 + 4x_5 = 0$

$x_2 = -3x_3 + 4x_4 - 4x_5$

$x_3 = 0$

$x_4 = 0$

$\therefore x_2 = -3\cdot0 + 4\cdot0 - 4x_5$

$x_2 = -4x_5$

$x_1 + (4x_5)\cdot2 = 0 - 0 + 2x_5$

$x_1 + 8x_5 = 2x_5$

$x_1 = -6x_5$

$x_1 - 2x_2 - 4x_3 + 3x_4 - 2x_5 = 0$

$x_1 - 2x_2 = 4x_3 - 3x_4 + 2x_5$

$x_2 = -3x_3 + 4x_4 - 4x_5$

$x_3 =$ free and $x_3 = 0$

$x_4 =$ free $x_4 = 0$

$x_5 =$ free

$x_1 - 2(-3x_3 + 4x_4 - 4x_5) = 4x_3 - 3x_4 + 2x_5$

$x_1 + 6x_3 - 8x_4 + 8x_5 = 4x_3 - 3x_4 + 2x_5$

$x_1 = -2x_3 + 5x_4 - 6x_5$

$$\therefore X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2x_3 + 5x_4 - 6x_5 \\ -3x_3 + 4x_4 - 4x_5 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$= \begin{bmatrix} -2x_3 + 5x_4 - 6x_5 \\ -3x_3 + 4x_4 - 4x_5 \\ x_3 + 0\cdot x_4 + 0\cdot x_5 \\ 0\cdot x_3 + 0\cdot x_4 + x_5 \end{bmatrix}$$

$$= x_3 \begin{bmatrix} -2 \\ -3 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 5 \\ 4 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -6 \\ -4 \\ 0 \\ 1 \end{bmatrix}$$

Basis for NUL A is

$$\left\{ \begin{bmatrix} -2 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ -4 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Rank of A = dimension of col A = 2

dim of NUL A = 3

Ans

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3. If 4×7 matrix A has 3 rank. find dim NUL A, dim Row A, and rank A^T.

Sol^n: Here
we have
$\text{rank}A + \dim(\text{NUL}A) = n = 7$
$3 + \dim(\text{NUL}A) = 7$
$\dim(\text{NUL}A) = 4$
Since $\dim.\text{Row}A = \text{rank}A$
$= 3$
$\text{rank } A^T = \dim(\text{col}.A^T)$
$= \dim.\text{row}(A)$
$= 3$ Ans

(4) Suppose a 4×7 matrix, A has four pivot column. Is col $A = R^4$? Is NUL $A = R^3$? Explain your answer.

Sol^n: Here,
the matrix A has order 4×7, Such that,

$$A = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 \\ c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 \\ d_1 & d_2 & d_3 & d_4 & d_5 & d_6 & d_7 \end{bmatrix}$$

According to Question,
A has four pivot column that mean's the bases for col A is 4.
therefore rank of A = bases of col A = bases of row A
rank of $A = 4$
$\therefore$ rank of $A + \cancel{\dim(\text{col}A)}^{\dim of NUL(A)} = n$
$4 + \dim of(\text{NUL}A) = 7$
$\dim of(\text{NUL}A) = 3$

from above matrix it is clear that a_1, b_2, c_3 and d_4 are basic so that it represented basic for col A. Similarly d_5, d_6 and d_7 are the entities of free Variable so it form a basis for NUL. therefore NUL A is basis of R^3.

EIGENVALUES AND EIGENVECTORS

UNIT-7

Eigen Value and Eigen Vector.

If A is $n \times n$ matrix, then a scalar λ is called eigen value of matrix A if the equation $Ax = \lambda x$ has a non-trivial soln. Such x is called eigen vector corresponding to eigen value λ.

If A is $n \times n$ matrix, then a non-zero vector $x \in R^n$ is called an eigen vector of matrix A if $Ax = \lambda x$ where λ is scalar.

Example.

Is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ an eigen vector $\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$?

Soln. here,

$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$Ax = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3+0 \\ 8-2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$Ax = 3 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$\therefore \boxed{Ax = \lambda x}$ where $\lambda = 3$

$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is an eigen vector corresponding to eigen value 3.

Ans

Exercise:- 7.1

1. let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$, $u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$. Is u is eigen vector of A?

Soln. here,

$$A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}, \quad u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$$

$$Au = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ -5 \end{bmatrix}$$

$$= \begin{bmatrix} 6-30 \\ 30-10 \end{bmatrix}$$

$$= \begin{bmatrix} -24 \\ 20 \end{bmatrix}$$

$$= -4 \begin{bmatrix} 6 \\ -5 \end{bmatrix}$$

$\therefore Au = \lambda u$ where $\lambda = -4$

$Au = -4u$

$\therefore$ ~~x~~ $u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$ is an eigen vector of A

2. Is $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ an eigen vector of $\begin{bmatrix} -3 & 1 \\ -3 & 8 \end{bmatrix}$?

Soln. Here,

$$A = \begin{bmatrix} -3 & 1 \\ -3 & 8 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$A \cdot x = \begin{bmatrix} -3 & 1 \\ -3 & 8 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -3+4 \\ -3+32 \end{bmatrix} = \begin{bmatrix} 1 \\ 29 \end{bmatrix}$$

$$Ax \neq \lambda x$$

$\therefore \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ is not an eigen vector of $\begin{bmatrix} -3 & 1 \\ -3 & 8 \end{bmatrix}$

Ans

3. Is $\begin{bmatrix}2\\4\end{bmatrix}$ an eigen vector of $\begin{bmatrix}3 & 1\\2 & -1\end{bmatrix}$?

Sol^n: Here,

let $A = \begin{bmatrix}3 & 1\\2 & -1\end{bmatrix}$ and $x = \begin{bmatrix}2\\4\end{bmatrix}$

$A.x = \begin{bmatrix}3 & 1\\2 & -1\end{bmatrix}.\begin{bmatrix}2\\4\end{bmatrix}$

$= \begin{bmatrix}6+4\\4-4\end{bmatrix}$

$= \begin{bmatrix}10\\0\end{bmatrix}$

$\because Ax \neq \lambda x$

$\therefore \begin{bmatrix}2\\4\end{bmatrix}$ is not an eigen vector of A.

4. Is $\begin{bmatrix}4\\-3\\1\end{bmatrix}$ an eigenvector of $\begin{bmatrix}3 & 7 & 9\\-4 & -5 & 1\\2 & 4 & 4\end{bmatrix}$? If so, find the eigen value,

Sol^n: Here,

let $A = \begin{bmatrix}3 & 7 & 9\\-4 & -5 & 1\\2 & 4 & 4\end{bmatrix}$, $x = \begin{bmatrix}4\\-3\\1\end{bmatrix}$

$Ax = \begin{bmatrix}3 & 7 & 9\\-4 & -5 & 1\\2 & 4 & 4\end{bmatrix}.\begin{bmatrix}4\\-3\\1\end{bmatrix} = \begin{bmatrix}12-21+9\\-16+15+1\\8-12+4\end{bmatrix}$

$Ax = \begin{bmatrix}21-21\\16-16\\12-12\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$Ax = 0\begin{bmatrix}4\\-3\\1\end{bmatrix}$

$Ax = \lambda x$ where $\lambda = 0$

$Ax = 0.x.$

$\therefore \begin{bmatrix}4\\-3\\1\end{bmatrix}$ is an eigen vector of A. and corresponding to its eigen value (λ) is 0.

Ans

5. Is $\lambda = 1$ an eigenvalue of matrix $\begin{bmatrix}2 & 2\\3 & 5\end{bmatrix}$?

Sol^n: Here,

let $A = \begin{bmatrix}2 & 2\\3 & 5\end{bmatrix}$ and $\lambda = 1$.

If 1 is an eigen value, then

$Ax = \lambda x$

$Ax - \lambda x = 0$

$(A - \lambda . I)x = 0$

$A - \lambda . I = \begin{bmatrix}2 & 2\\3 & 5\end{bmatrix} - 1\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$= \begin{bmatrix}2-1 & 2-0\\3-0 & 5-1\end{bmatrix}$

$= \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$

The augmented matrix of $(A-\lambda I)x = 0$ is

$[A-\lambda I, 0] = \left[\begin{array}{cc:c}1 & 2 & 0\\3 & 4 & 0\end{array}\right]$

$R_2 \to R_2 - 3R_1$

$= \left[\begin{array}{cc:c}1 & 2 & 0\\0 & -2 & 0\end{array}\right] \xrightarrow{R_1 \to R_1 + R_2} \left[\begin{array}{cc:c}1 & 0 & 0\\0 & -2 & 0\end{array}\right]$

Here the solution of Augmented matrix is trivial so it doesnot taken any eigen value to define the eigen vector of A. so 1 is not an eigen value of A.

GOOD MORNING PAGE NO.: DATE: / /

6. Is $\lambda = 5$ an eigenvalue of matrix $\begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}$? If so find one corresponding eigen vector.

Sol^n: Here,

let $A = \begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}$ and $\lambda = 5$

If 5 is an eigen value of matrix A then,

$Ax = \lambda x$

$Ax - \lambda x = 0$

$(A - \lambda)x = 0$

$(A - \lambda I)x = 0$

$A - \lambda I = \begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 5-5 & 0-0 \\ 2-0 & 1-5 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 2 & -4 \end{bmatrix}$

The augmented matrix of $(A - \lambda I).x = 0$ is

$[A - \lambda I \; 0] = \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 2 & -4 & 0 \end{array}\right]$.

$R_2 \to \frac{1}{2} R_2$

$= \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & -2 & 0 \end{array}\right] \xleftrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right]$

Here x_2 is a free variable so the system has trivial solution, then correspond. $\lambda = 5$ is an eigen value of matrix A such that,

the corresponding equation is

$x_1 - 2x_2 = 0$

$2x_2 = x_1$

$2x_2 = x_1$

$\therefore x_1 = 2x_2$

$\therefore X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\therefore \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is an eigen vector of given matrix A.

GOOD MORNING PAGE NO.: DATE: / /

7. Is $\lambda = 3$ eigenvalue of $\begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$? If so find one corresponding eigen vector.

Sol^n: Here,

let $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and $\lambda = 3$

If $\lambda = 3$ is an eigen value of A then

$Ax = \lambda x$

$Ax - \lambda x = 0$

$x(A - \lambda I) = 0$

$A - \lambda I = \begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} -2 & 2 & 2 \\ 3 & -5 & 1 \\ 0 & 1 & -2 \end{bmatrix}$

The augmented matrix of $(A-I\lambda)x=0$

$$[A-\lambda I \quad 0] = \begin{bmatrix} -2 & 2 & 2 & : 0 \\ 3 & -5 & 1 & : 0 \\ 0 & 1 & -2 & : 0 \end{bmatrix}$$

$R_1 \to -\frac{1}{2} R_1$

$$\begin{bmatrix} 1 & -1 & -1 & : 0 \\ 3 & -5 & 1 & : 0 \\ 0 & 1 & -2 & : 0 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$$\begin{bmatrix} 1 & -1 & -1 & : 0 \\ 0 & -2 & 4 & : 0 \\ 0 & 1 & -2 & : 0 \end{bmatrix}$$

$R_2 \to \frac{-1}{2} R_2$

$$\begin{bmatrix} 1 & 1 & -1 & : 0 \\ 0 & 1 & -2 & : 0 \\ 0 & 1 & -2 & : 0 \end{bmatrix}$$

$R_1 \to R_1 - R_2$ and $R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 & : 0 \\ 0 & 1 & -2 & : 0 \\ 0 & 0 & 0 & : 0 \end{bmatrix}$$

Here x_3 is a free variable so the system has non-trivial solution then $\lambda = 3$ is an eigen value of A. then the corresponding equation is

$x_1 + x_3 = 0$

$x_1 = -x_3$

$x_2 - 2x_3 = 0$

$x_2 = 2x_3$

$x_3 =$ free

$$\therefore X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_3 \\ 2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

(i) Show that 2 is eigen value of matrix $\begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$

Sol^n: Here,

let $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$ and $\lambda = 2$

If $\lambda = 2$ is an eigen value then of matrix A then

$Ax = \lambda x$

$Ax - \lambda x = 0$

$x(A - \lambda I) = 0$

$$\therefore A - \lambda I = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-2 & -1-0 & 6-0 \\ 2-0 & 1-2 & 6-0 \\ 2-0 & -1-0 & 8-2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 6 \\ 2 & -1 & 6 \\ 2 & -1 & 6 \end{bmatrix}$$

The augmented matrix of $(A-\lambda I)x = 0$ is

$$[A-\lambda I \quad 0] = \begin{bmatrix} 2 & -1 & 6 & : 0 \\ 2 & -1 & 6 & : 0 \\ 2 & -1 & 6 & : 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 2 & -1 & 6 & : 0 \\ 0 & 0 & 0 & : 0 \\ 0 & 0 & 0 & : 0 \end{bmatrix}$$

GOOD MORNING PAGE NO. DATE / /

$$\left[\begin{array}{ccc|c} 2 & -1 & 6 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Here x_2 and x_3 ~~are~~ free variable then the system has ~~the~~ non-trivial solution so $\lambda = 2$ is an eigen value of matrix A then,

the corresponding equation is.

$2x_1 - x_2 + 6x_3 = 0$

$2x_1 = x_2 - 6x_3$

$x_1 = \frac{1}{2}x_2 - 3x_3$

$x_2 =$ free

$x_3 =$ free .

$$\therefore x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}x_2 - 3x_3 \\ x_2 + 0 \cdot x_3 \\ 0 \cdot x_2 + 1 \cdot x_3 \end{bmatrix} = x_2 \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore x = \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}.$$

Here, we show that x is an eigen vector of A

$\therefore Ax = \lambda x$

$$\begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix} = \lambda x .$$

where ~~λ=2~~

$$\begin{bmatrix} 4 \times \frac{1}{2} - 1 + 0 \\ 2 \times \frac{1}{2} + 1 + 0 \\ 2 \times \frac{1}{2} - 1 + 0 \end{bmatrix} = \lambda x .$$

GOOD MORNING PAGE NO. DATE / /

$$\begin{bmatrix} 2-1 \\ 1+1 \\ 0 \end{bmatrix} = 2 \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad \text{Verified,}$$

Similarly for other, taking $x = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

$Ax = \lambda x$

$$\begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} = \lambda x$$

$$\begin{bmatrix} -12+0+6 \\ -6+0+6 \\ -6+0+8 \end{bmatrix} = 2 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -6 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 2 \end{bmatrix} \quad \text{verified,}$$

$\therefore$ Hence Here $\lambda = 2$ is an eigen value of matrix A.

(10) Find a basis for the eigenspace corresponding to listed eigenvalue.

(a) $A = \begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}$ and $\lambda = 1$

Soln: Here,

$A = \begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}$ and $\lambda = 1$

If $\lambda = 1$ is an eigen value of of matrix A is then,

$Ax = \lambda x$

$Ax - \lambda x = 0$

$x(A - \lambda I) = 0$

$\therefore A - \lambda I = \begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix} - 1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5-1 & 0-0 \\ 2-0 & 1-1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 \\ 2 & 0 \end{bmatrix}$

The augmented matrix of $(A - \lambda I)x = 0$ is

$\begin{bmatrix} (A-\lambda I) & 0 \end{bmatrix} = \begin{bmatrix} 4 & 0 & : & 0 \\ 2 & 0 & : & 0 \end{bmatrix}$

$R_1 \to \frac{R_1}{4}$ and $R_2 \to \frac{1}{2} R_2$

$\begin{bmatrix} 1 & 0 & : & 0 \\ 1 & 0 & : & 0 \end{bmatrix}$

$R_2 \to R_2 - R_1$

$\begin{bmatrix} 1 & 0 & : & 0 \\ 0 & 0 & : & 0 \end{bmatrix}$

Here x_2 is an free variable then the system has non-trivial solution, then $\lambda = 1$ is an eigen value of A then.

The corresponding equation is,

$x_1 = 0$

$x_2 = \text{free},$

$\therefore x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\therefore$ eigen vector of A is $x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\therefore$ eigen space of A is $= \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

(b) $A = \begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix}$ and $\lambda = 4$

Soln: Here,

$A = \begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix}$ and $\lambda = 4$

If $\lambda = 4$ is an eigen value of matrix A then

$Ax = \lambda x$

$Ax - \lambda x = 0$

$(A - \lambda I)x = 0$

$\therefore A - \lambda I = \begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

$= \begin{bmatrix} 10-4 & -9-0 \\ 4-0 & -2-4 \end{bmatrix} = \begin{bmatrix} 6 & -9 \\ 4 & -6 \end{bmatrix}$

GOOD MORNING PAGE NO: DATE: / /

The augmented matrix of $(A-\lambda I)X=0$ is

$$[(A-\lambda I)\ 0] = \begin{bmatrix} 6 & -9 & : & 0 \\ 4 & -6 & : & 0 \end{bmatrix}$$

$R_1 \to \frac{1}{3}R_1$ and $R_2 \to \frac{1}{2}R_2$.

$$\begin{bmatrix} 2 & -3 & : & 0 \\ 2 & -3 & : & 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 2 & -3 & : & 0 \\ 0 & 0 & : & 0 \end{bmatrix}$$

Here x_2 is a free variable then the system has non-trivial Sol^n. So $\lambda=4$ is an eigen value of matrix A. then the corresponding equation is,

$$2x_1 - 3x_2 = 0$$
$$2x_1 = 3x_2$$
$$x_1 = 3/2\, x_2$$
$$x_2 = \text{free}$$

$$\therefore x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3/2\, x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 3/2 \\ 1 \end{bmatrix}$$

$\therefore$ The eigen vector of A is, $x = \begin{bmatrix} 3/2 \\ 1 \end{bmatrix}$

$\therefore$ The eigen space of A is, $\left\{ \begin{bmatrix} 3/2 \\ 1 \end{bmatrix} \right\}$

Ans

GOOD MORNING PAGE NO: DATE: / /

(c) $A = \begin{bmatrix} 4 & -2 \\ -3 & 9 \end{bmatrix}$ and $\lambda = 10$

Sol^n: Here,

$A = \begin{bmatrix} 4 & -2 \\ -3 & 9 \end{bmatrix}$ and $\lambda = 10$,

If $\lambda = 10$ is an eigen value of matrix A then

$$Ax = \lambda x$$
$$Ax - \lambda x = 0$$
$$(A-\lambda I)x = 0$$

$$\therefore A-\lambda I = \begin{bmatrix} 4 & -2 \\ -3 & 9 \end{bmatrix} - 10 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -2 \\ -3 & 9 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 4-10 & -2-0 \\ -3-0 & 9-10 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -2 \\ -3 & -1 \end{bmatrix}$$

The augmented matrix of $(A-\lambda I)X=0$ then,

$$[A-\lambda I,\ 0] = \begin{bmatrix} -6 & -2 & : & 0 \\ -3 & -1 & : & 0 \end{bmatrix}$$

$R_1 \to \frac{-R_1}{3}$

$$\begin{bmatrix} 3 & 1 & : & 0 \\ -3 & -1 & : & 0 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$$\begin{bmatrix} 3 & 1 & : & 0 \\ 0 & 0 & : & 0 \end{bmatrix}$$ → Here x_2 is a free variable then the system has non-trivial Sol^n so that $\lambda=10$ is an eigen value of A so that the corresponding eq^n is,

GOOD MORNING PAGE NO. DATE: / /

$5x_1 + x_2 = 0$

$3x_1 = -x_2$

$x_1 = -\frac{1}{3} x_2$

$x_2 = $ free.

$\therefore$ The $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3}x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} -\frac{1}{3} \\ 1 \end{bmatrix}$

$\therefore$ The eigen vector of A is $x = \begin{bmatrix} -\frac{1}{3} \\ 1 \end{bmatrix}$

$\therefore$ the eigen space of A is $\left\{ \begin{bmatrix} -\frac{1}{3} \\ 1 \end{bmatrix} \right\}$

Ans.

(d) $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$ and $\lambda = 2$

Sol: Here,

$A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$ and $\lambda = 2$,

If $\lambda = 2$ is an eigen value of A then,

$Ax = \lambda x$

$Ax - \lambda x = 0$

$(A - \lambda I)x = 0$

$\therefore (A - \lambda I) = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

GOOD MORNING PAGE NO. DATE: / /

$= \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4-2 & -1-0 & 6-0 \\ 2-0 & 1-2 & 6-0 \\ 2-0 & -1-0 & 8-2 \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 & 6 \\ 2 & -1 & 6 \\ 2 & -1 & 6 \end{bmatrix}$

The augmented matrix of $(A - \lambda I)x = 0$ is

$[(A - \lambda I), 0] = \left[\begin{array}{ccc:c} 2 & -1 & 6 & 0 \\ 2 & -1 & 6 & 0 \\ 2 & -1 & 6 & 0 \end{array}\right]$

$R_3 \to R_3 - R_1$ & $R_2 \to R_2 - R_1$ $\left[\begin{array}{ccc:c} 2 & -1 & 6 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Here x_2 and x_3 are free variable then the System has non-trivial Soln. So that $\lambda = 2$ is an eigen value of A then the corresponding eqn is,

$2x_1 - x_2 + 6x_3 = 0$

$2x_1 = x_2 - 6x_3$

$x_1 = \frac{1}{2}x_2 - 3x_3$

$x_2 = $ free

$x_3 = $ free,

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}x_2 - 3x_3 \\ x_2 + 0 \cdot x_3 \\ 0 \cdot x_2 + 1 \cdot x_3 \end{bmatrix} = x_2 \begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

GOOD MORNING PAGE NO DATE

$\therefore$ The eigen vector of A is $x = \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

The eigen space of A is $\left\{ \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$.

Ans

ⓒ $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ and $\lambda = 1$ and $\lambda = 2$

Sol^n: Here,

$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ and $\lambda = 1$ and $\lambda = 2$

$\therefore$ If $\lambda = 1$ is an eigen value of A then

$Ax = \lambda x$

$Ax - \lambda x = 0$

$\therefore (A - \lambda I)x = 0$

$A - \lambda I = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix} - 1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 & -2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

The augmented matrix of $(A - \lambda I)x = 0$ is

$[A - \lambda I \; 0] = \left[\begin{array}{ccc:c} -1 & 0 & -2 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 2 & 0 \end{array}\right]$.

GOOD MORNING PAGE NO DATE

$R_1 \leftrightarrow R_3$

$\left[\begin{array}{ccc:c} 1 & 0 & 2 & 0 \\ 1 & 1 & 1 & 0 \\ -1 & 0 & -2 & 0 \end{array}\right]$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 + R_1$

$\left[\begin{array}{ccc:c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Here x_3 is a free variable then the system has non-trivial sol^n. then $\lambda = 1$ is an eigen value of A.

then, the corresponding eq^n is

$x_1 + 2x_2 = 0$

$x_2 - x_3 = 0$

$x_2 = x_3$

$x_3 =$ free

$x_1 + 2(x_3) = 0$

$x_1 = -2x_3$

$\therefore x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

The eigen vector of A is $x = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

$\therefore$ The eigen space of A is $\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\}$

If $\lambda = 2$ then then the eigen value of matrix A is,

$Ax = \lambda x$

$Ax = \lambda x$

$Ax - \lambda x = 0$

$(A - \lambda I)x = 0$

$$\therefore (A-\lambda I) = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0-2 & 0-0 & -2-0 \\ 1-0 & 2-2 & 1-0 \\ 1-0 & 0-0 & 3-2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & -2 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

The augmented matrix of $(A-\lambda I)x = 0$ is

$$[A - \lambda I \quad 0] = \left[\begin{array}{ccc|c} -2 & 0 & -2 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right]$$

$R_1 \to \frac{-R_1}{2}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right]$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Here x_2 and x_3 are free variable then the system has non-trivial soln. If $\lambda = 2$ is an eigen value of A then the corresponding equation is

$x_1 + x_3 = 0$

$x_1 = -x_3$

$x_2 = \text{free}$

$x_3 = \text{free}$

$$\therefore x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 - x_3 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \cdot x_2 - x_3 \\ x_2 + 0 \cdot x_3 \\ 0 \cdot x_2 + x_3 \end{bmatrix}$$

$$= x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ The eigen vector of A is $x = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$\therefore$ The eigen space of A is $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

GOOD MORNING
PAGE NO.:
DATE: / /

(11) Find the eigenvalue of following matrices.

(a) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 5 \\ 0 & 0 & -1 \end{bmatrix}$

Sol^n: Here.

Since, given matrix is upper triangular matrix, so, eigen value of given matrix are diagonal element of that matrix i.e ~~(0,2,-1)~~ 0, 2 and -1.

$\therefore \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 5 \\ 0 & 0 & -1 \end{bmatrix}$

$\therefore \lambda = 0, 2, -1$

(b) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 3 \end{bmatrix}$

Sol^n: Here,

$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 3 \end{bmatrix}$

Since, given matrix is ~~upper~~ Lower triangular matrix, So eighen value of given matrix are diagonal element of that given matrix i.e 4, 0 and 3

$\therefore \lambda = 4, 0, 3$

GOOD MORNING
PAGE NO.:
DATE: / /

(c) $\begin{bmatrix} 3 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & -4 & 1 \end{bmatrix}$

Sol^n: Here,

$\begin{bmatrix} 3 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & -4 & 1 \end{bmatrix}$

Since, given matrix is Lower triangular matrix So eigen value of given matrix are diagonal element of that given matrix i.e. 3, 2 and 1.

$\therefore \lambda = 3, 2, 1$