# Functions For Computer Science

1) **Floor Function** :- let $x$ be a real number. The floor function rounds $x$ down to the closest integer less than or equal to $x$. It is denoted by $\lfloor x \rfloor$.

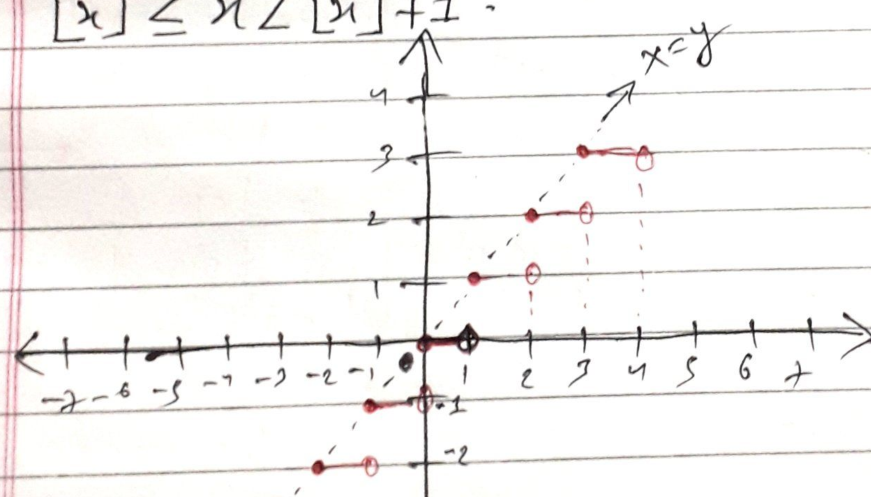
The floor function is often also called the greatest integer function.

Example, $\lfloor 5 \rfloor = 5$, $\lfloor 6.459 \rfloor = 6$, $\lfloor \sqrt{7} \rfloor = 2$, $\lfloor \pi \rfloor = 3$,

$\lfloor -13.24 \rfloor = -14$, $\lfloor -0.1 \rfloor = -1$

In general, $\lfloor x \rfloor$ is the unique integer satisfying

$\lfloor x \rfloor \le x < \lfloor x \rfloor + 1$.




$x \in [0,1) = 0$
$x \in [1,2) = 1$
$x \in [2,3) = 2$
$x \in [3,4) = 3$
⋮

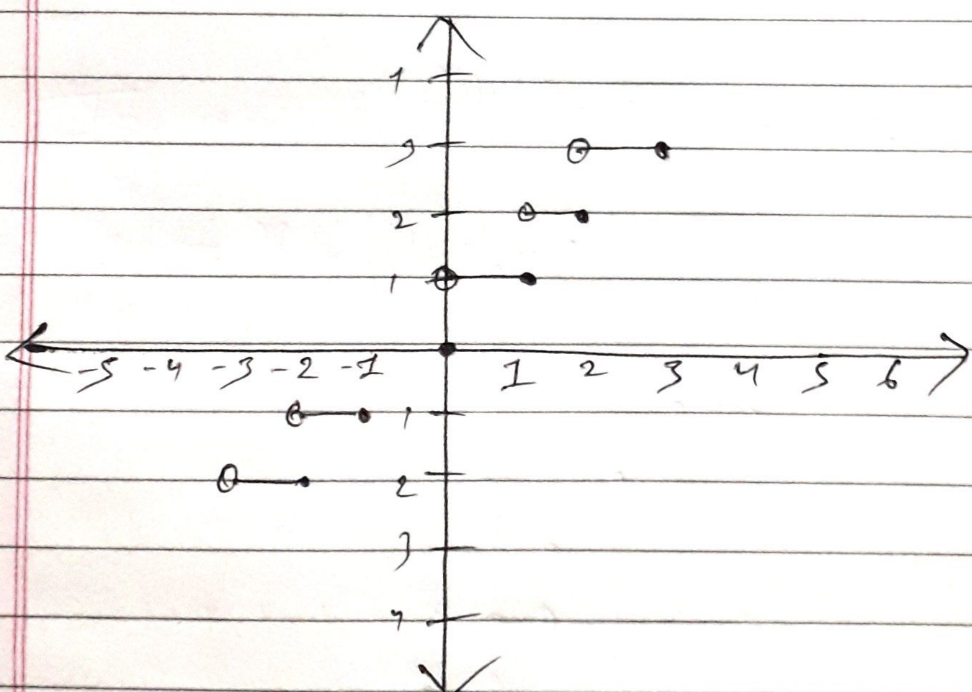
$y = [x]$

fig - floor function

(11) <u>Ceiling function</u> :- Let $x$ be a real number. The ceiling function rounds $x$ up to the closest integer greater than or equal to $x$. It is denoted by $\lceil x \rceil$.

<u>Example</u>, $\lceil \frac{1}{2} \rceil = 1$, $\lceil -\frac{1}{2} \rceil = 0$, $\lceil 3.1 \rceil = 4$, $\lceil 7 \rceil = 7$

$$\left\lfloor \frac{2}{3} + \left\lceil \frac{1}{2} \right\rceil \right\rfloor = \left\lfloor \frac{2}{3} + 1 \right\rfloor = 1$$



(b) $y = \lceil x \rceil$

<u>Fig</u>:- Ceiling function

**EXAMPLE 25** Data stored on a computer disk or transmitted over a data network are usually represented as a string of bytes. Each byte is made up of 8 bits. How many bytes are required to encode 100 bits of data?

*Solution:* To determine the number of bytes needed, we determine the smallest integer that is at least as large as the quotient when 100 is divided by 8, the number of bits in a byte. Consequently, $\lceil 100/8 \rceil = \lceil 12.5 \rceil = 13$ bytes are required. ◀

**EXAMPLE 26** In asynchronous transfer mode (ATM) (a communications protocol used on backbone networks), data are organized into cells of 53 bytes. How many ATM cells can be transmitted in 1 minute over a connection that transmits data at the rate of 500 kilobits per second?

*Solution:* In 1 minute, this connection can transmit $500{,}000 \cdot 60 = 30{,}000{,}000$ bits. Each ATM cell is 53 bytes long, which means that it is $53 \cdot 8 = 424$ bits long. To determine the number of cells that can be transmitted in 1 minute, we determine the largest integer not exceeding the quotient when $30{,}000{,}000$ is divided by 424. Consequently, $\lfloor 30{,}000{,}000/424 \rfloor = 70{,}754$ ATM cells can be transmitted in 1 minute over a 500 kilobit per second connection. ◀

**TABLE 1 Useful Properties of the Floor and Ceiling Functions.**
($n$ is an integer)

| |
|---|
| (1a) $\lfloor x \rfloor = n$ if and only if $n \leq x < n + 1$<br>(1b) $\lceil x \rceil = n$ if and only if $n - 1 < x \leq n$<br>(1c) $\lfloor x \rfloor = n$ if and only if $x - 1 < n \leq x$<br>(1d) $\lceil x \rceil = n$ if and only if $x \leq n < x + 1$ |
| (2) $x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1$ |
| (3a) $\lfloor -x \rfloor = -\lceil x \rceil$<br>(3b) $\lceil -x \rceil = -\lfloor x \rfloor$ |
| (4a) $\lfloor x + n \rfloor = \lfloor x \rfloor + n$<br>(4b) $\lceil x + n \rceil = \lceil x \rceil + n$ |

# Boolean Function

A boolean function is described by an algebraic expression consisting of binary variables, constants '0' and '1' and the logic operation symbols +, ·, ' . It is sometimes referred to as a "switching function", because it assume values from two element set (0,1)

A Boolean function takes the form $f: \{0,1\}^k \rightarrow \{0,1\}$, where {0,1} is called a Boolean domain and k is a non-negative integer.

Example:

[Boolean Function] $F = x'y + z$ [Boolean expression]

The above function is defined in terms of three binary variables x, y and z. The function is equal to 1 if x = 0, y = 1 and z = 1.

In the equation, the left hand side represents the output (Y).

[Truth table Formation]

$F(x,y,z) = x'y + z$

or $Y = x'y + z$

The truth table for this equation is shown below: The number of rows in the truth table is $2^n$ where n is the number of input variables. Hence there are $2^3 = 8$ possible input combination of inputs.

| Inputs | | | Output |
| --- | --- | --- | --- |
| x | y | z | F |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

$F(x, y, z) = x'y + z$

## Exponential Function

A function of the form:

$$f(x) = ab^x$$

is called an exponential function where x is a variable, and a is a constant called the base of the function and b is a positive real numbers not equal to 1.

$f(x) = ab^x$
- where x is a real number
- $a \neq 0$ and $b > 0$ and $b \neq 1$.

An exponential function has a successive output values for each unit increase in the input values have a constant ratio. The constant ratio is the base b.

If a discrete exponential function has inputs that are a set of equally spaced integers, then its outputs are a sequence of numbers called a geometric sequence.

Properties

$c^0 = 1, c \neq 0$ eg; $8^0 = 1$

$c^{-n} = \frac{1}{c^n}, c \neq 0$ eg; $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

Representation of an exponential function:

Question Make a graph for the function $f(x) = 3\left(\frac{2}{4}\right)^x$ for the domain $\{-3, -2, -1, 0, 1, 2, 3\}$.

Soln:

| $x$ | $f(x)$ | $(x, f(x))$ |
|---|---|---|
| $-3$ | 24 | $(-3, 24)$ |
| $-2$ | 12 | $(-2, 12)$ |
| $-1$ | 6 | $(-1, 6)$ |
| 0 | 3 | $(0, 3)$ |
| 1 | $3/2$ | $(1, 3/2)$ |
| 2 | $3/4$ | $(2, 3/4)$ |
| 3 | $3/8$ | $(3, 3/8)$ |

graph: self

Rough

~~$3\left(\frac{2}{4}\right)^{-3} = 3\left(\frac{1}{\frac{4}{8}}\right)$~~

$3\left(\frac{2}{4}\right)^{-3} = 3 \times \frac{1}{\frac{8}{64}}$

$= 3 \times \frac{64}{8}$

$= 24$

# [Fuzzy sets and Membership functions]

Fuzzy sets are those sets whose elements have degree of membership.

- In classical set theory, the membership of elements in a set is assessed in binary terms; i.e an element either belongs or doesnot belong to the set.

- In fuzzy set theory, the set theory permits the gradual assessment of the membership of elements in a set. This is described with the aid of a membership function valued in the real unit interval [0,1].

Fuzzy set are applied in the areas such as linguistics, decision making and clustering and bioinformatics.

# A fuzzy set is a pair $(U, m)$ where $U$ is a set and $m : U \rightarrow [0,1]$ a membership function. The reference set $U$ (sometimes denoted by $X$) is called universe of discourse. For each $x \in U$, the value $m(x)$ is called the grade of membership of $x$ in $(U, m)$.

The function $m = \mu_A$ is called the membership function of the fuzzy set $A = (U, m)$.

Note: For an element $x$ of $X$, the value $\mu_A(x)$ is called the membership degree of $x$ in the fuzzy set $A$.

Let $x \in U$. Then $x$ is called

1. not included in fuzzy set $(U, m)$ if $m(x) = 0$ (not a member)
2. fully included if $m(x) = 1$ (full member)
3. partially included if $0 < m(x) < 1$ (fuzzy member)

## Important points

- A fuzzy set $A = (U, m)$ is empty $(A = \emptyset)$ if and only if: $\forall x \in U : \mu_A(x) = m(x) = 0$
- Two fuzzy sets A and B are equal $(A = B)$ iff: $\forall x \in U : \mu_A(x) = \mu_B(x)$
- A fuzzy set A is included in a fuzzy set B $(A \subseteq B)$ iff: $\forall x \in U : \mu_A(x) \leq \mu_B(x)$

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**Example** $U$ = all students $[x_1, x_2, x_3, \ldots x_n]$
$A$ = good students
$B$ = bad students
$A = \{x_i, \mu_A\}$, $\mu_A$ is degree of goodness.
$B = \{x_i, \mu_B\}$, $\mu_B$ is degree of badness.

$A = \{(x_1, 0.8), (x_2, 0.2), (x_3, 0.9), (x_4, 0.3)\}$
$B = \{(x_1, 0.2), (x_2, 0.9), (x_3, 0.1), (x_4, 0.7)\}$

## Fuzzy set operation

(1) **Intersection:** Let A and B be the two Fuzzy set with their membership functions $\mu_A$ and $\mu_B$ respectively.

$$\therefore \mu_{A \cap B} = \min(\mu_A, \mu_B)$$

Eg:

| X | y | X AND Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

fig: Truth table (logical AND)

| X | y | min (x, y) |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |
| 0.2 | 0.5 | 0.2 |
| 0.7 | 0.2 | 0.2 |
| 0.6 | 0.6 | 0.6 |

fig: Truth table (with membership functions)

② Unions: Let A and B be the two fuzzy sets with their membership function $\mu_A$ and $\mu_B$ respectively.

$\therefore \mu_{A \cup B} = \max(\mu_A, \mu_B)$

Eg:

| X | y | X OR Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

fig: Truth table (logical OR)

| x | y | max (x, y) |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |
| 0.2 | 0.5 | 0.5 |
| 0.7 | 0.2 | 0.2 |
| 0.6 | 0.6 | 0.6 |

fig: Truth table (with membership function)

Complement: let A be a fuzzy set with membership function $\mu_A$.

$\therefore \mu_{\bar{A}} = 1 - \mu_A$

Eg:

| X | NOT X |
|---|---|
| 0 | 1 |
| 1 | 0 |

| X | 1-X |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 0.2 | 0.8 |
| 0.7 | 0.3 |