[Primes]

Every positive integer greater than 1 and divisible by only 1 or itself is called prime. Or positive integers that have exactly two different positive integer factors are called primes.

A positive integer that is greater than 1 and is not prime is called Composite.

Eg: The integer 7 is prime because its only positive factors are 1 and 7, whereas the integer 9 is composite because it is divisible by 3.

THEOREM: THE FUNDAMENTAL THEOREM OF ARITHMETIC.

Every positive integer greater than 1 can be written uniquely as a prime or as the product of two or more primes where the prime factors are written in order of nondecreasing size.

Example: Find the prime factorization of 99, 110, 645 and 875.

Sol: $99 = 3\cdot 3\cdot 11 = 3^2\cdot 11$

$110 = 2\cdot 5\cdot 11$

$645 = 3\cdot 5\cdot 43$

$875 = 5\cdot 5\cdot 5\cdot 7 = 5^3\cdot 7$

Questn: Find the prime factorization of 7007.

Ans: Self.

GCD and LCM

Greatest Common Divisors & least Common Multiples

GCD ⇒ The largest integer that divides both of two integers is called the greatest common divisor of these integers.

OR/ **Definition**: let a and b be integers, not both zero. The largest integer d such that ~~d|b~~ d|a and d|b is called the greatest common divisor of ~~a~~ a and b. The greatest common divisor of a and b is denoted by gcd(a,b).

Question

What is the greatest common divisor of 24 and 36?

Sol - The positive common divisors of 24 and 36 are 1, 2, 3, 4, 6 and 12. Hence, gcd (24, 36) = 12

Relatively Prime:- The integers a and b are relatively prime if their greatest common divisor is 1.

Example: The integers 17 and 22 are relatively prime, because gcd (17, 22) = 1.

LCM ⇒ **Definition**:- The least common multiple of the positive integers a and b is the smallest positive integer that is divisible by both a and b. The least common multiple of a and b is denoted by lcm (a, b).

Prime factorization method to calculate GCD and LCM

⇒ suppose let 'a' and 'b' are two integers which are not equal to zero i.e $a,b \neq 0$.

⇒ The prime factor of $a = p_1^{a_1}, p_2^{a_2}, ---, p_n^{a_n}$ and

The prime factor of $b = p_1^{b_1}, p_2^{b_2}, ---, p_n^{b_n}$

Then,

$$gcd(a,b) = p_1^{min(a_1,b_1)} * p_2^{min(a_2,b_2)} * ---- * p_n^{min(a_n,b_n)}$$

and

$$LCM(a,b) = p_1^{max(a_1,b_1)} * p_2^{max(a_2,b_2)} * ---- * p_n^{max(a_n,b_n)}$$

Example:

1) Use prime factorization to find the gcd of 12 and 30.

Solⁿ

$12 = 2 \cdot 2 \cdot 3$ $\qquad$ $3 = 2 \cdot 3 \cdot 5$

$= 2^2 \cdot 3^1 \cdot 5^0$ $\qquad$ $= 2^1 \cdot 3^1 \cdot 5^1$

$\therefore gcd(12,30) = 2^{min(2,1)} \cdot 3^{min(1,1)} \cdot 5^{min(0,1)}$

$= 2^1 \cdot 3^1 \cdot 5^0 = 6$

2) Use prime factorization to find the LCM of 12 and 18.

Solⁿ

$12 = 2 \cdot 2 \cdot 3$ $\qquad$ $18 = 2 \cdot 3 \cdot 3 \cdot 3$

$= 2^2 \cdot 3^1$ $\qquad$ $= 2^1 \cdot 3^3$

$\therefore LCM(12,18) = 2^{max(2,1)} \cdot 3^{max(1,3)}$

$= 2^2 \cdot 3^3$

$= 4 \cdot 27$

$LCM(12,18) = 108$

Pairwise relative prime :-

The integers $a_1, a_2, \ldots a_n$ are pairwise relatively prime if $\gcd(a_i, a_j) = 1$ where, $1 \leq i \leq j \leq n$.

Example,

Determine whether the integers 10, 17 & 21 are pairwise relatively prime.

Sol

Since $\gcd(10, 17) = 1$

$\gcd(10, 21) = 1$

and $\gcd(17, 21) = 1$

Therefore, the given number sequence 10, 17 and 21 are pairwise relatively prime.