偏导数与微分

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Outline

1. 偏导数
   - 定义

偏导数

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$z = f(x, y)$

在M_0处关于x的偏导数: $\lim\limits_{\triangle x \to 0} \dfrac{f(x_0 + \triangle x, y_0) - f(x_0, y_0)}{\triangle x}$, 记作 $\dfrac{\partial z}{\partial x}|_{M_0}$, or $f_x(x_0, y_0)$.

在M_0处关于y的偏导数: $\lim\limits_{\triangle y \to 0} \dfrac{f(x_0, y_0 + \triangle y) - f(x_0, y_0)}{\triangle y}$, 记作 $\dfrac{\partial z}{\partial y}|_{M_0}$, or $f_y(x_0, y_0)$.

$z=f(x,y)$ 在M_0 处的梯度：$gradz(M_0)=\nabla z=\{\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}\}|_{M_0}$

四则运算法则，复合运算法则，OK！

例1. 求偏导数$\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$

(1) $z = x^3 + 2x^2y^3 + e^x y$

(2) $z = \arctan \dfrac{y}{x}$

(3) $u = z^{y^x}$

例2. 设$f(x, y) = \begin{cases} \dfrac{xy^2}{x^2 + y^4} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$

(1) 讨论$f(x, y)$在$(0, 0)$处的连续性

(2) 求$f_x(0, 0)$, $f_y(0, 0)$