换元法

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例9. 如下做法对否

(1)

$$\int_0^4 \frac{dx}{1+\sqrt{x}} = \int_0^4 \frac{2t}{1+t}dt = 2\int_0^4 (1-\frac{1}{1+t})dt$$

$$= \quad 2(t-\ln(1+t))|_0^4 = 8-2\ln 5.$$

(2)

$$\int_{-1}^1 \frac{1}{1+x^2}dx = \int_{-1}^1 \frac{t^2}{1+t^2}(-\frac{1}{t^2})dt = -\int_{-1}^1 \frac{1}{1+t^2}dt$$

$$\Rightarrow$$

$$\int_{-1}^1 \frac{1}{1+x^2}dx = 0$$

(3)

$$\int_0^{\pi} \sqrt{\sin x - \sin^3 x}\,dx = \int_0^{\pi} \sqrt{\sin x \cos^2 x}\,dx$$

$$= \int_0^{\pi} \sqrt{\sin x}\cos x\,dx = \int_0^{\pi} \sqrt{\sin x}\,d\sin x = \frac{2}{3}(\sin x)^{\frac{3}{2}}|_0^{\pi} = 0$$

(4)

$$\int_0^{3} x\sqrt{1-x^2}\,dx,$$

令$x = \sin t$

例10. 设$f \in C_{[a,b]}$, 试证

$$\int_{-a}^{a} f(x)dx = \int_{0}^{a} (f(x) + f(-x))dx.$$

If f 是偶函数, $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

If f 是奇函数, $\int_{-a}^{a} f(x)dx = 0$

例11. (1) $\int_{-2}^{2} \frac{x^2 + x \ln(x^4 + 1)}{2 + \sqrt{4 - x^2}} dx$

(2) $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x (\ln \frac{1 + x}{1 - x} + \sin^2 x) dx$

例12. 设$f(x)$是以T为周期的函数，且连续，则

$$\int_a^{a+T} f(x)dx = \int_0^T f(x)dx$$

例13. 求$\int_{\frac{40\pi}{n}}^{\frac{50\pi}{n}} |\sin nx|dx$

例14. $f \in C_{[0,1]}$，证明：

(1) $\int_0^{\frac{\pi}{2}} f(\sin x)dx = \int_0^{\frac{\pi}{2}} f(\cos x)dx$

(2) $\int_0^{\pi} xf(\sin x)dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)dx$

并计算$\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$.

例15. 设$f(x)=\begin{cases} xe^{-x^2} & x\geq 0 \\ \frac{1}{1+\cos x} & -2<x<0, \end{cases}$ 计算$\int_0^3 f(x-2)dx$.