偏导数与微分

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Outline

1 隐函数求导法则

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Theorem (7)

设

1. $F(x,y)$在$P(x_0,y_0)$的某邻域内具有一阶连续偏导数
2. $F(x_0,y_0)=0$
3. $F_y(x_0,y_0)\neq 0$

则方程$F(x,y)=0$在点$P(x_0,y_0)$的某邻域内能唯一确定一个连续且有一阶连续导数的函数$y=f(x)$, $s.t.\ y_0=f(x_0)$, 且$\dfrac{dy}{dx}=-\dfrac{F_x}{F_y}$

例20. $F(x,y)=x^2+y^2-1=0$, 求$\dfrac{dy}{dx}$

Theorem (8)

设

1. $n+1$元函数$F(x_1, x_2, \cdots, x_n, u)$ 在点$P(x_1^0, x_2^0, \cdots, x_n^0, u^0)$的某邻域内具有一阶连续偏导数
2. $F(x_1^0, x_2^0, \cdots, x_n^0, u^0) = 0$
3. $F_u(x_1^0, x_2^0, \cdots, x_n^0, u^0) \neq 0$

则方程$F(x_1, x_2, \cdots, x_n, u) = 0$ 在点$P(x_1^0, x_2^0, \cdots, x_n^0, u^0)$的某邻域内唯一确定一个连续且有一阶连续偏导数$u = f(x_1, x_2, \cdots, x_n)$, 且$\dfrac{\partial u}{\partial x_i} = -\dfrac{F_{x_i}}{F_u}$

例21. 设方程$z^3 - 3xyz = a^2$, 确定z是x, y的函数， 求$\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$

例22. 设$z = z(x, y)$由方程$F(x - az, y - bz) = 0$确定。 a, b为常数。 求证$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = 1$.

Theorem (9)

1. $F(x,y,u,v)$, $G(x,y,u,v)$在点$P(x_0,y_0,u_0,v_0)$的某一邻域内有一阶连续偏导数
2. $F(x_0,y_0,u_0,v_0)=0$, $G(x_0,y_0,u_0,v_0)=0$
3. $Jacobi$ 行列式$J|_P=\frac{\partial(F,G)}{\partial(u,v)}|_P=\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}|_P \neq 0$

$\Rightarrow$ 由方程组$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases}$ 在P 的某一邻域内存在唯一连续且有一阶连续偏导数的函数$u=u(x,y)$, $v=v(x,y)$, 且$u_0=u(x_0,y_0)$, $v_0=v(x_0,y_0)$ 且

$$\frac{\partial u}{\partial x}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(x,v)} \quad \frac{\partial u}{\partial y}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(y,v)}$$
$$\frac{\partial v}{\partial x}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(u,x)} \quad \frac{\partial v}{\partial y}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(u,y)}$$

例23. $u=u(x,y)$， $v=v(x,y)$ 由方程组 $\left\{\begin{array}{l} u^2-v+x=0 \\ u+v^2-y=0 \end{array}\right.$ 确定，求$\dfrac{\partial u}{\partial x}, \dfrac{\partial v}{\partial x}, \dfrac{\partial u}{\partial y}, \dfrac{\partial v}{\partial y}$

例24. 设$u=f(x,y,z)$, $y=g(\sin x)$, $z=z(x)$ 由方程$\varphi(x^2,e^y,z)=0$确定，(f,φ)具有一阶连续偏导数，g可导，$\dfrac{\partial\varphi}{\partial z}\neq 0$, 求$\dfrac{du}{dx}$