Exercise 3

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1 Exercise 2: Derivation of Essential Matrix

The epipolar constraint states that the vectors $\overline{O_LX}$, $\overline{O_RX}$, and $\overline{O_LO_R}$ lie in a plane. Hence, if we take the cross-product of any two of these vectors the resulting vector must be perpendicular to all three. If we take $\overline{O_LX}_L$ to be in the coordinate system of camera L and $\overline{O_RX}_R$ to be in the coordinate system of camera R, and we know the translation and rotation between the coordinate systems to be t and R, respectively, we can write the above observation as follows: where $[t]_x$ denotes the skew-symmetric matrix corresponding to t.

$$\begin{split} 0 &= \overline{O_L X} \cdot \left(\overline{O_L O_R}_L \times \overline{O_R X}_L \right) \\ &= \overline{O_L X} \cdot \left(t \times R_L^R \overline{O_R X}_R \right) \\ &= \overline{O_L X}^T [t]_x R_L^R \overline{O_R X}_R \qquad = \overline{O_L X}^T E \overline{O_R X}_R \end{split}$$

We define $E = [t]_x R_L^R$ and call this the "Essential Matrix", denoting the relationship between the two cameras.