Review Sheet for Midterm 2
APPM/MATH 4650 Fall '20 Numerical Analysis

Instructor: Prof. Becker

Chapter 3: interpolation, mainly Hermite interpolation and cubic spline interpolation

1. Can be bound the error of approximating a function f by its unique degree n-or-less interpolant p (on a specified set of $n+1$ nodes)?
2. What's the idea of Hermite interpolation? Can this be done in a piecewise (composite) fashion?
3. Briefly, what's the difference between Hermite interpolation and cubic splines?

Chapter 4, part 1: numerical differentiation, Richardson extrapolation

1. How are finite difference formulas usually derived?
2. What is the basic forward difference formula? What's the order of error? What is the basic centered difference formula? What's the order of error?
3. Why might we prefer a higher-order method?
4. Why don't we use order 100 methods?
5. What's the idea behind Richardson extrapolation?
6. Let $F : \mathbb{R}^m \to \mathbb{R}^n$ be a vector-valued function, with domain $\mathbb{R}^m$ and range $\mathbb{R}^n$. For which values of m and n is automatic differentiation (AD) really useful? For which values are finite difference rules a good idea? What are the downsides of AD?
7. For cubic splines on $n+1$ nodes, how many parameters are there?
8. For cubic splines on $n+1$ nodes, how many conditions to satisfy are there?
9. Running not-a-knot cubic spline interpolation on $f(x) = |x|$ on $[-1, 1]$, in which region of the domain do we expect the error to be the highest?
10. Running natural cubic spline interpolation on $f(x) = x^2$ on $[-1, 1]$, in which region of the domain do we expect the error to be the highest?

Chapter 4, part 2: numerical integration

1. How are quadrature formulas usually derived?
2. For quadrature, do we have a notion of "forward" vs "centered" formulas?
3. What does Newton-Cotes mean? Are all quadrature rules Newton-Cotes?
4. Is it OK to use a higher-order Newton-Cotes formula rather than switching over to a composite rule?
5. What's the idea behind Gauss-Legendre integration?
6. What does it means to say the "order of exactness/precision" of a quadrature rule?

7. What's the order of exactness of Newton-Cotes methods?

8. What's the order of exactness of the Gauss-Legendre method?

9. What's the difference between open and closed Newton-Cotes formulas? Give an example of each

10. Finite difference formulas are unstable; we cannot take $h \to 0$ and get error $\to 0$. What about composite Newton-Cotes? If we take $n \to \infty$, does error go to zero?

11. What are the relevant properties of the Legendre polynomials? Are these the same as Lagrange polynomials?

12. Gauss-Legendre integration only works on $[-1, 1]$

13. For each L-named French mathematician in the following list, list the corresponding topic named after them that we've seen in our course: Legendre, Lagrange, Laguerre, L'Hôpital. Bonus: Laplace (not yet covered in this class), and Lipschitz (actually a German)

14. What are the relevant properties of the Laguerre polynomials?

15. How are Hermite interpolation and Gauss-Hermite quadrature related?

16. The same Romberg integration formula works for both composite midpoint and composite trapezoid rules

17. Is there any limit to how many times we can apply Romberg integration?

18. For a 2D integral, we need to rederive quadrature rules

19. If a quadrature rule in d dimensions, then in terms of the spacing h between node points, how expensive to we expect the rule to be? like d^h, h^{-d}, e^d/h, hd, etc?

20. Monte Carlo integration is faster than quadrature rules

21. Basic composite Newton-Cotes formulas cover proper integrals like $\int_0^1 1/x \, dx$ (true/false?)

22. Gauss-Laguerre and Gauss-Hermite formulas can be used to accurately integrand any function over $[0, \infty)$ or $(-\infty, \infty)$, respectively.

23. The Runge phenomenon affects... interpolation? differentiation? integration? When does it arise?

Chapter 5: IVPs and ODEs

1. We talk about IVP for first-order ODE. Why don't we talk about boundary value problems (BVP) for first-order ODE?

2. The wave equation is an example of an ODE

3. All ODEs have analytic solutions, unlike for PDE

4. When running a high-order ODE solver, we get a list of independent variable points $\{t_i\}$ and corresponding points $\{w_i\}$ that approximate $w_i \approx y(t_i)$, where y is the true solution to the IVP. When plotting the solution, if `t` is the vector of $\{t_i\}$ and `w` is the vector of $\{w_i\}$, we get a good idea of how y behaves by plotting `plot(t,w,'-')` (either via Matlab or via Python with `from matplotlib.pyplot import plot`) (true/false?)

5. Just like for multidimensional integration, numerically solving a coupled system of d ODEs is way harder than solving a single system.

6. What is the local truncation error of Euler's method (aka Forward Euler)? What is the global error?