Intro to 1D optimization

Thursday, August 27, 2020 11:57 PM

Optimization

Many root-finding problems come from optimization

Thm if $f:[a,b]\to\mathbb{R}$ is differentiable, then its minimum (and maximum) is at a critical point (ie., $f'(x)=0$) or at an end-point $\{a,b\}$.

Proof let $f(x_0)=\min_x f(x)$ and for the sake of contradiction,

assume $f'(x_0)\neq 0$ and $x_0\notin\{a,b\}$. Without loss of generality "WLOG"
assume $f'(x_0)<0$ (if $f'(x_0)>0$, pick $h<0$ below)

Then since $f'(x_0)=\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}$, there is some δ s.t.

if $|h|<\delta$, then $\frac{f(x_0+h)-f(x_0)}{h}<0$. Choose such a h with $0<h<\delta$

then $f(x_0+h)-f(x_0)<0$, ie.,
$f(x_0+h)<f(x_0)$, with $h\neq 0$, which is a contradiction. $\square$

In 1D, optimization
and root-finding are easy.
Even the slowest methods in Ch.2 aren't bad.

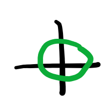
Optimization in higher dimensions

Thm: if $f:\mathbb{R}^n\to\mathbb{R}$ is differentiable, then all minima/maxima occur at critical points ($\nabla f(x)=0$)

For $\min_{x\in C} f(x)$, it's different: now we need $x\in C$, $\nabla f(x)^T(y-x)\geq 0\quad\forall y\in C$

Things are way worse

1) no real equivalent of bisection (sort of: "ellipsoid method")
2) making a grid and just checking is very slow in high-dimensions
3) not only might it be hard to list all critical points
   but in fact there could be an infinite number of them!

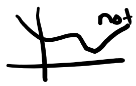
   Ex: $x^2+y^2=1$ has an ∞ # of solutions 

4) checking all boundary points (like $\{a,b\}$) also fails,

Since there can be an ∞ # of boundary points in > 1 dimension

Usually we need more structure

ex: f is convex, C is a convex set



not convex

not convex

Fact: if f is convex, C is a convex set, then all local minimizers of f (over C) are also global.

... and if not convex, finding "the" minimizer (aka "global minimizer") usually impossible

take CSCI 5254 or APPM 5630

ECEN 5008 Special topics