Review Sheet for Midterm 1
APPM/MATH 4650 Fall '20 Numerical Analysis

Instructor: Prof. Becker

Chapter 1: basics

1. Why do we use floating point numbers? What are the alternatives? What are the advantages/disadvantages?
2. When is a good time to use absolute accuracy/error (and what are the definitions)? When is a good time to use relative accuracy/error?
3. How does "number of correct digits" relate to relative accuracy? To absolute accuracy?
4. On a computer, is $a + (b + c) = (a + b) + c$ always true? Can you think of examples?
5. What's the difference between *conditioning* and *stability*?
6. Is it more accurate to evaluate a function f at x if $|f'(x)|$ is large or small?
7. Is it more accurate to find a root p of the function f if $|f'(p)|$ is large or small?
8. What is the relative condition number? (formula and meaning)
9. How does the relative condition number relate to relative error and the number of correct digits?
10. What is the relative condition number of $f(x) = x^2 + 10^{10}$ at $x = 1$? (an approximate value is OK). Can you interpret this? Can we take advantage of this in double precision floating point?
11. Consider the approximation $\cos(x) \approx 1 - x^2/2$. What is the *absolute* error of this approximation on the interval $[-.1, .1]$? [we are asking about mathematical error, now stability issues of an *algorithm*].
12. True/false? If f is continuous then $\forall x > 0$, $\exists \xi \in [0, x]$ such that $\int_0^x f(y)\,dy = xf(\xi)$?
13. What does it mean to say $f(n) = O(n^2)$ as $n \to \infty$? Is $n = O(n^2)$? Is $n^3 = O(n^2)$?
14. What does it mean to say $f(n) = o(n^2)$ as $n \to \infty$?
15. What does it mean to say $f(n) = \Theta(n^2)$ as $n \to \infty$?
16. What does it mean to say $f(h) = O(h^2)$ as $h \to 0$? Is $h = O(h^2)$? Is $h^3 = O(h^2)$?
17. What is quadratic convergence?
18. Does $e_n = 1/n$ converge linearly to 0?
19. What kind of plot makes $e_n = 1/n$ look like a straight line?
20. Describe how you would evaluate the polynomial $p(x) = 5x^3 + 2x^2 + 3x + 4$ using Horner's method.

Chapter 2: root finding

1. What are pros/cons of the bisection method?
2. What are pros/cons of Newton's method?
3. True/False: all root-finding problems can be recast as fixed point problems?
4. True/False: for a root-finding problem, there is only one equivalent fixed point problem?
5. When might the bisection method not work even when there is a root?
6. How might the bisection method give misleading results?
7. What is the rate of convergence of the bisection method? What error is this? (e.g., is it an error about finding the fixed point? Is it an error in the objective f?)
8. Does Newton's method always converge?
9. If Newton's method converges, is it always at a linear rate?
10. Are there any possible issues with modified Newton's method?
11. What controls the eventual convergence rate of fixed-point problem?
12. True/False: a *contraction* is a function f such that $\forall x, y$, $|f(x) - f(y)| < |x - y|$.
13. If f is a linear function, what do we know about Newton's method?

Chapter 3: interpolation

1. Polynomial interpolation is a good idea if we have many noisy data points and want to approximate them with something smooth
2. For nodes $\{x_0 = 1, x_1 = 2, x_2 = 3\}$, draw a rough sketch of the Lagrange polynomials
3. Describe the idea behind Lagrange interpolation
4. Find the minimum degree interpolating polynomial for nodes $\{-1, 0, 1\}$ and values $\{2, 3, 4\}$.
5. Describe the idea behind Newton's divided difference formula
6. How is $f[x_4, x_5, x_6]$ defined?
7. What are pros/cons of Lagrange vs Newton interpolation?
8. What is the Runge phenomenon. What are two ways to avoid it?