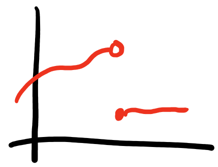
# Review of Calculus

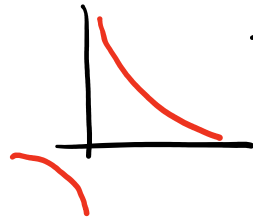
Sunday, August 23, 2020 11:38 AM

Not covering explicitly (see book)

- limits
- definition of continuity
- differentiability
- Rolle's thm
- Mean Value thm (MVT)
- Extreme Value thm (EVT)
- Intermediate Value thm (IVT)

Ex.

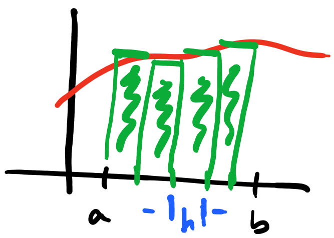




$f(x) = \frac{1}{x}$

not cts on $[-1,1]$

**Integration**




$$\underbrace{\lim_{h \to 0} \text{riemann sum} = \int_a^b f(x)\,dx}_{\text{Definite}} \qquad \underbrace{\int f(x)\,dx}_{\text{Indefinite}}$$

Proper: $f$ is bounded and cts (except at a few pts) then we have a proper integral, and $[a,b]$ is bounded

Improper:

$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{b\to\infty} \underbrace{\int_1^b \frac{1}{x^2}\,dx}_{\text{proper}} = \lim_{b\to\infty} -\frac{1}{b} + 1 = 1 \quad \checkmark$$

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{a\to 0^+} \underbrace{\int_a^1 \frac{1}{\sqrt{x}}\,dx}_{\text{proper}} = \lim_{a\to 0^+} 2 - 2\sqrt{a} = 2 \quad \checkmark$$

$$\int_{-1}^1 \frac{1}{x}\,dx = \lim_{b\to 0^+} \int_{-1}^{-b} \frac{1}{x}\,dx + \lim_{a\to 0^+} \int_a^1 \frac{1}{x}\,dx = -\infty + \infty$$

Does not exist

$a = b$ then cancellation occurs... not ok

**Taylor Series** $f \in C^n([a,b])$ and $f^{(n+1)}$ exists on $[a,b]$

\# of cts. derivatives

Ex: $C([0,1])$ means $f$ is continuous

$C'([0,1])$ means $f'$ is continuous
$f$ is continuous

Any $x_0 \in [a,b]$

$x \in [a,b]$

$$f(x) = \underbrace{f(x_0) + \frac{1}{1!}f'(x_0)(x-x_0) + \frac{1}{2!}f''(x_0)(x-x_0)^2 + \dots + \frac{1}{n!}f^{(n)}(x_0)(x-x_0)^n}_{\text{Taylor Polynomial } P_n(x)} \dots$$

$$+ \underbrace{\frac{1}{(n+1)!}f^{(n+1)}(\xi)(x-x_0)^{n+1}}_{\text{Remainder } R_n(x)}$$

$\xi \in [x_0, x]$ or $[x, x_0]$

UNKNOWN!

**Usefulness:**

$$\sin(\overset{x}{.75}) = \underbrace{\underbrace{\sin(\overset{x_0}{\pi/4})}_{\sqrt{2}/2} + \underbrace{\cos(\pi/4)}_{\sqrt{2}/2}(.75-\pi/4) + \frac{1}{2!}\overbrace{-\sin(\pi/4)}^{\sqrt{2}/2}(.75-\pi/4)^2}_{P_n(x)}$$

$\pi/4 \approx .78$

$\sin(\pi/4) = \sqrt{2}/2$

$\cos(\pi/4) = \sqrt{2}/2$

$$+ R_n(x) = \frac{1}{3!}-\cos(\xi)\,(.75-\pi/4)^3$$

$\xi \in [.75, \pi/4]$

$$|R_n| \leq \frac{1}{6}(.75-\pi/4)^3$$

## Review of logarithms

Q: $\log(a+b) \neq \log(a) + \log(b)$

$\log(0) = -\infty$

$\log(1) = 0$

$\log(a \cdot b) = \log(a) + \log(b)$

"log" = "ln"

$\log_{10}$, $\log_2$

$\log(a^5) = 5 \cdot \log(a)$

$$\underbrace{\log(1+1)}_{\neq 0} \stackrel{?}{=} \underbrace{\log(1)}_{0} + \underbrace{\log(1)}_{0}$$