Review Sheet for Midterm 2
APPM/MATH 4650 Fall '20 Numerical Analysis

Instructor: Prof. Becker

Chapter 3: interpolation, mainly Hermite interpolation and cubic spline interpolation

1. Can be bound the error of approximating a function f by its unique degree n-or-less interpolant p (on a specified set of $n+1$ nodes)?
2. What's the idea of Hermite interpolation? Can this be done in a piecewise (composite) fashion?
3. Briefly, what's the difference between Hermite interpolation and cubic splines?

Chapter 4, part 1: numerical differentiation, Richardson extrapolation

1. How are finite difference formulas usually derived?
2. What is the basic forward difference formula? What's the order of error?
3. Why might we prefer a higher-order method?
4. Why don't we use order 100 methods?
5. What's the idea behind Richardson extrapolation?
6. Let $F : \mathbb{R}^m \to \mathbb{R}^n$ be a vector-valued function, with domain $\mathbb{R}^m$ and range $\mathbb{R}^n$. For which values of m and n is automatic differentiation (AD) really useful? For which values are finite difference rules a good idea? What are the downsides of AD?
7. For cubic splines on $n+1$ nodes, how many parameters are there?
8. For cubic splines on $n+1$ nodes, how many conditions to satisfy are there?
9. Running not-a-knot cubic spline interpolation on $f(x) = |x|$ on $[-1, 1]$, in which region of the domain do we expect the error to be the highest?
10. Running natural cubic spline interpolation on $f(x) = x^2$ on $[-1, 1]$, in which region of the domain do we expect the error to be the highest?

Chapter 4, part 2: numerical integration

1. How are quadrature formulas usually derived?
2. For quadrature, do we have a notion of "forward" vs "centered" formulas?
3. What does Newton-Cotes mean? Are all quadrature rules Newton-Cotes?
4. Is it OK to use a higher-order Newton-Cotes formula rather than switching over to a composite rule?
5. What's the idea behind Gauss-Legendre integration?
6. What does it means to say the "order of exactness/precision" of a quadrature rule?
7. What's the order of exactness of Newton-Cotes methods?

8. What's the order of exactness of the Gauss-Legendre method?

9. What's the difference between open and closed Newton-Cotes formulas? Give an example of each

10. Finite difference formulas are unstable; we cannot take $h \to 0$ and get error $\to 0$. What about composite Newton-Cotes? If we take $n \to \infty$, does error go to zero?

11. What are the relevant properties of the Legendre polynomials? Are these the same as Lagrange polynomials?

12. Gauss-Legendre integration only works on $[-1, 1]$

13. For each L-named French mathematician in the following list, list the corresponding topic named after them that we've seen in our course: Legendre, Lagrange, Laguerre, L'Hôpital. Bonus: Laplace (not yet covered in this class), and Lipschitz (actually a German)

14. What are the relevant properties of the Laguerre polynomials?

15. How are Hermite interpolation and Gauss-Hermite quadrature related?

16. The same Romberg integration formula works for both composite midpoint and composite trapezoid rules

17. Is there any limit to Romberg integration?

18. For a 2D integral, we need to rederive quadrature rules

19. If a quadrature rule in d dimensions, then in terms of the spacing h between node points, how expensive to we expect the rule to be? like d^h, h^{-d}, e^d/h, hd, etc?

20. Monte Carlo integration is faster than quadrature rules

21. Basic composite Newton-Cotes formulas cover proper integrals like $\int_0^1 1/x \, dx$ (true/false?)

22. Gauss-Laguerre and Gauss-Hermite formulas can be used to accurately integrand any function over $[0, \infty)$ or $(-\infty, \infty)$, respectively.

23. The Runge phenomenon affects... interpolation? differentiation? integration? When does it arise?

Chapter 5: IVPs and ODEs

1. We talk about IVP for first-order ODE. Why don't we talk about boundary value problems (BVP) for first-order ODE?

2. The wave equation is an example of an ODE

3. All ODEs have analytic solutions, unlike for PDE

4. When running a high-order ODE solver, we get a list of independent variable points $\{t_i\}$ and corresponding points $\{w_i\}$ that approximate $w_i \approx y(t_i)$, where y is the true solution to the IVP. When plotting the solution, if `t` is the vector of $\{t_i\}$ and `w` is the vector of $\{w_i\}$, we get a good idea of how y behaves by plotting `plot(t,w,'-')` (either via Matlab or via Python with `from matplotlib.pyplot import plot`) (true/false?)

5. Just like for multidimensional integration, numerically solving a coupled system of d ODEs is way harder than solving a single system.

6. What is the local truncation error of Euler's method (aka Forward Euler)? What is the global error?