Lab 03 Report

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1 Parameter Estimation:

1.1

- Given $X = \{3, 3, 3, 3, 3, 7, 7\}$, $P(3) = \theta,\ P(7) = 1 - \theta$
- **Using Method of Moments**
  - Equating first population moment and first sample moment $\mu_1 = m_1$.
  - $m_1 = \bar{X} = \mu_1 = E(X)$

$$E(X) = \sum_i x_i \ P(X = x_i) = \bar{X}$$

$$E(X) = 3\theta + 7(1 - \theta) = \frac{\sum_{i=1}^{N} x_i}{N}$$

$$\therefore \quad \theta = \frac{7 - \bar{X}}{4} = 0.625$$

- **Using Maximum Likelihood**

$$L(\theta) = \Pi_{i=1}^{N} P(x_i|\theta)$$

$$L(\theta) = \theta^5(1 - \theta)^3$$

$$\log(L(\theta)) = 5\log(\theta) + 7\log(1 - \theta)$$

$$\frac{\partial(\log(L(\theta)))}{\partial(\theta)} = \frac{5}{\theta} - \frac{7}{1 - \theta} = 0$$

$$\therefore \quad \theta = \frac{5}{8} = 0.625$$

- **Standard Error**
  - **For MOM**

$$\theta = \frac{7 - \bar{X}}{4}$$

$$SE(\hat{\theta}) = sd(\hat{\theta}) = \sigma(\hat{\theta})$$

$$\sigma^2(\theta) = var(\theta) = var(\frac{7 - \bar{X}}{4})$$

$$= \frac{var(\bar{X})}{16} = \frac{var(\frac{\sum_{i=1}^{N} x_i)}{N}}{16}$$

$$= \frac{\sum_{i=1}^{N} var(x_i)}{16N^2} = \frac{\sigma^2}{16N}$$

$$\therefore \quad \sigma(\theta) = \frac{\sigma}{4\sqrt{N}}$$

$$s(\theta) = \frac{s}{4\sqrt{N}}$$

$$\therefore \quad SE(\theta) = s(\theta) = 0.1829813$$

$>$ **For MLE** $= SE(\theta) = s(\theta) = \frac{s}{4\sqrt{N}} = 0.1829813$

1.2

- Given number of runs which follows geometric distributions: $3, 7, 5, 3, 2$.
- **Using MoM**
  - $\mu_1 = E(X) = m_1 = \bar{X}$
  - $E(X) = \frac{1}{p} = \bar{X} = 4$
  - $\therefore \;\; p = 0.25$

- **Using MLE**

$$L(\theta) = \Pi_{i=1}^{N} P(x_i|\theta)$$

$$L(\theta) = \Pi_{i=1}^{N} p(1-p)^{x_i - 1}$$

$$= p^5(1-p)^{15}$$

$$\log(L(\theta)) = 5\log(p) + 15\log(1-p)$$

$$\frac{\partial(\log(L(\theta)))}{\partial(\theta)} = \frac{5}{p} - \frac{15}{1-p} = 0$$

$$p = \frac{1}{4} = 0.25$$

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2 Confidence Interval

2.1

- Given $N = 100$, $\bar{X} = 37.7$ and $\sigma = 9.2$.
- $(1-\alpha)$% Confidence level of $\hat{\mu} = [\bar{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \quad \bar{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}]$
- 90% C.I. $= [36.1866, 39.2134]$

2.2

- Given $X = \{30000, 50000, 70000\}$
- $(1-\alpha)$% Confidence level of $\hat{\mu} = [\bar{X} - t_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \quad \bar{X} + t_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}]$
- For $(1-\alpha)\% = 90$ C.I., $\bar{X} = 50000$, $\sigma = 20000$
- 90% C.I. $= [16281.7552, 83718.2448]$