# Momentum Library

Minified

Competitive Programming Library

of

https://github.com/OmarBazaraa/Competitive-Programming

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### Data Structures

#### Sparse Table

```
// The array to compute its sparse table and its size.
int n, a[N];
// Sparse table related variables. Don't access them directly.
int ST[LOG_N][N], LOG[N];
// Builds the sparse table for computing min/max/gcd/lcm/...etc
// for any contiquous sub-segment of the array in O(n.log(n)).
//
// This is an example of computing the index of the minimum value.
void buildST() {
   LOG[0] = -1;
   for (int i = 0; i < n; ++i) {</pre>
      ST[0][i] = i;
      LOG[i + 1] = LOG[i] + !(i & (i + 1));
   }
   for (int j = 1; (1 << j) <= n; ++j) {
      for (int i = 0; (i + (1 << j)) <= n; ++i) {
         int x = ST[j - 1][i];
         int y = ST[j - 1][i + (1 << (j - 1))];
         ST[j][i] = (a[x] <= a[y] ? x : y);
      }
   }
}
// Queries the sparse table for the computed value of the interval [l, r] in O(1).
int query(int 1, int r) {
   int g = LOG[r - l + 1];
   int x = ST[g][1];
   int y = ST[g][r - (1 << g) + 1];
   return (a[x] <= a[y] ? x : y);</pre>
}
Monotonic Queue
 * Monotonic queue to keep track of the minimum and the maximum
 * elements so far in the queue in amortized time of O(1).
template<class T>
class monotonic queue {
   queue<T> qu;
   deque<T> mx, mn;
public:
```

```
void push(T v) {
      qu.push(v);
      while (mx.size() && mx.back() < v) mx.pop_back();</pre>
      mx.push_back(v);
      while (mn.size() && mn.back() > v) mn.pop back();
      mn.push back(v);
   }
   void pop() {
      if (mx.front() == qu.front()) mx.pop_front();
      if (mn.front() == qu.front()) mn.pop_front();
      qu.pop();
   }
   T front() const {
      return qu.front();
   T max() const {
      return mx.front();
   }
   T min() const {
      return mn.front();
   }
   size_t size() const {
      return qu.size();
};
Disjoint-Set Union (DSU)
 * Disjoint-set data structure to tracks a set of elements partitioned
 * into a number of disjoint subsets.
class DSU {
   int setsCount;
   vector<int> siz;
   mutable vector<int> par;
public:
   DSU(int n) {
      setsCount = n;
      siz.resize(n, 1);
      par.resize(n);
      iota(par.begin(), par.end(), 0);
   }
   int findSetId(int u) const {
      return u == par[u] ? u : par[u] = findSetId(par[u]);
   }
   bool areInSameSet(int u, int v) const {
      return findSetId(u) == findSetId(v);
```

```
bool unionSets(int u, int v) {
      u = findSetId(u);
      v = findSetId(v);
      if (u == v) {
         return false;
      setsCount--;
      siz[v] += siz[u];
      par[u] = v;
      return true;
   }
   int getSetSize(int u) const {
      return siz[findSetId(u)];
   }
   int getSetsCount() const {
      return setsCount;
   }
};
Fenwick Tree (Binary Indexed Tree)
* Regular Fenwick tree class to compute and update prefix sum in O(log(N)).
 * Note that the tree is is 1-indexed.
template<class T>
class fenwick_tree {
   T BIT[N];
public:
   fenwick_tree() {
      memset(BIT, 0, sizeof(BIT));
   void update(int idx, T val) {
      while (idx < N) {</pre>
         BIT[idx] += val;
         idx += idx \& -idx;
   }
   T operator[](int idx) {
      T res = 0;
      while (idx > 0) {
         res += BIT[idx];
         idx -= idx & -idx;
      return res;
   }
};
```

```
* Fenwick tree class to compute and update range sum in O(log(N)).
 * Note that the tree is is 1-indexed.
template<class T>
class range_fenwick_tree {
   fenwick_tree<T> M, C;
  void update(int l, int r, T val) {
     M.update(1, val);
     M.update(r + 1, -val);
      C.update(1, -val * (1 - 1));
      C.update(r + 1, val * r);
  }
   T operator[](int idx) {
      return idx * M[idx] + C[idx];
};
Segment Tree as Multiset
* Segment tree node struct.
struct node {
  int size;
   node* childL, * childR;
   node() {
      size = 0;
      childL = childR = this;
   node(int s, node* 1, node* r) {
      size = s;
      childL = 1, childR = r;
   }
};
* Multiset that store integers in the range of [-N, N].
* The multiset is implemented using segment tree.
* Note that the multiset is 0-indexed.
 * The most complex function in this class is done in time complexity of O(\log(N)).
class segment_multiset {
  const int N;
  node* nil, * root;
public:
   segment_multiset(int N) : N(N) {
      root = nil = new node();
```

```
~segment_multiset() {
   destroy(root);
   delete nil;
}
void clear() {
   destroy(root);
   root = nil;
}
int size() {
   return root->size;
void insert(int val, int cnt = 1) {
   insert(root, val, cnt, -N, N);
}
int erase(int val, int cnt = 1) {
   return erase(root, val, cnt, -N, N);
int count(int val) {
   node* cur = root;
   int l = -N, r = N;
   while (1 < r) {
      int mid = 1 + (r - 1) / 2;
      if (val <= mid) {
         cur = cur->childL;
         r = mid;
      } else {
         cur = cur->childR;
         l = mid + 1;
   }
   return cur->size;
}
int operator[](int idx) {
   node* cur = root;
   int l = -N, r = N;
   while (1 < r) {
      int mid = 1 + (r - 1) / 2;
      if (idx < cur->childL->size) {
         cur = cur->childL;
         r = mid;
      } else {
         idx -= cur->childL->size;
         cur = cur->childR;
         1 = mid + 1;
      }
   }
```

```
return r;
   }
   int lower_bound(int val) {
      node* cur = root;
      int 1 = -N, r = N, ret = 0;
      while (1 < val) {</pre>
         int mid = 1 + (r - 1) / 2;
         if (val <= mid) {</pre>
            cur = cur->childL;
            r = mid;
         } else {
            ret += cur->childL->size;
            cur = cur->childR;
            1 = mid + 1;
         }
      }
      return ret;
   }
   int upper_bound(int val) {
      return lower_bound(val + 1);
   }
private:
  void insert(node*& root, int val, int cnt, int l, int r) {
      if (val < 1 || val > r) {
         return;
      }
      if (root == nil) {
         root = new node(0, nil, nil);
      root->size += cnt;
      if (1 == r) {
         return;
      }
      int mid = 1 + (r - 1) / 2;
      insert(root->childL, val, cnt, 1, mid);
      insert(root->childR, val, cnt, mid + 1, r);
   int erase(node*& root, int val, int cnt, int l, int r) {
      if (val < 1 || val > r) {
         return 0;
      }
      if (root == nil) {
         return 0;
      }
```

```
if (1 == r) {
      return remove(root, cnt);
   int mid = 1 + (r - 1) / 2;
   int ret = 0;
   ret += erase(root->childL, val, cnt, 1, mid);
   ret += erase(root->childR, val, cnt, mid + 1, r);
   return remove(root, ret);
}
int remove(node*& root, int cnt) {
   int ret = min(cnt, root->size);
   root->size -= cnt;
   if (root->size <= 0) {
      destroy(root);
      root = nil;
   }
   return ret;
}
void destroy(node* root) {
   if (root == nil) return;
   destroy(root->childL);
   destroy(root->childR);
   delete root;
}
```

**}**;

## Strings

#### **KMP**

```
// KMP longest match array. Don't access it directly.
int F[N];
// KMP failure function.
int failure(const char* pat, char cur, int len) {
   while (len > 0 && cur != pat[len]) {
      len = F[len - 1];
   }
   return len + (cur == pat[len]);
}
// Computes the length of the longest suffix ending at the i-th character
// that match a prefix of the string, and fills the results in the global "F" array.
void KMP(const char* pat) {
   for (int i = 1; pat[i]; ++i) {
      F[i] = failure(pat, pat[i], F[i - 1]);
}
Z-Algorithm
// Z-Algorithm longest match array. Don't access it directly.
int Z[N];
// Computes the length of the longest prefix starting at the i-th character
// that match a prefix of the string, and fills the results in the global "Z" array.
void z function(const char* str) {
   for (int i = 1, l = 0, r = 0; str[i]; ++i) {
      if (i <= r)
         Z[i] = min(r - i + 1, Z[i - 1]);
      while (str[i + Z[i]] && str[Z[i]] == str[i + Z[i]])
         Z[i]++;
      if (i + Z[i] - 1 > r)
         l = i, r = i + Z[i] - 1;
   }
}
```

#### Trie

```
// The total length of all the string, and the size of the alphabet.
const int N = 100100, ALPA SIZE = 255;
                         // The trie.
int trie[N][ALPA_SIZE];
                          // The number of nodes in the trie.
int nodesCount;
int distinctWordsCount;
                          // The number of distinct word in the trie.
                          // Number of words sharing node "i".
int wordsCount[N];
                          // Number of words ending at node "i".
int wordsEndCount[N];
// Initializes the trie. This must be called before each test case.
void init() {
   nodesCount = 0;
   memset(trie, -1, sizeof(trie));
}
// Outs a new edge with character "c" from the given node if not exists .
int addEdge(int id, char c) {
   int& nxt = trie[id][c];
   if (nxt == -1) {
      nxt = ++nodesCount;
   }
  return nxt;
}
// Inserts a new word in the trie.
void insert(const char* str) {
   int cur = 0;
   for (int i = 0; str[i]; ++i) {
      wordsCount[cur]++;
      cur = addEdge(cur, str[i]);
   }
   wordsCount[cur]++;
   distinctWordsCount += (++wordsEndCount[cur] == 1);
}
// Removes a word from the trie assuming that it was added before.
void erase(const char* str) {
   int cur = 0;
   for (int i = 0; str[i]; ++i) {
      wordsCount[cur]--;
      int nxt = trie[cur][str[i]];
     if (wordsCount[nxt] == 1) {
         trie[cur][str[i]] = -1;
      cur = nxt;
   }
   wordsCount[cur]--;
   distinctWordsCount -= (--wordsEndCount[cur] == 0);
}
```

```
// Searches for a word in the trie and returns its number of occurrences.
int search(const char* str) {
   int cur = 0;
   for (int i = 0; str[i]; ++i) {
      int nxt = trie[cur][str[i]];
      if (nxt == -1) {
         return 0;
      }
      cur = nxt;
   return wordsEndCount[cur];
}
Suffix Array
              : the length of the string not the number of suffixes.
// str
              : the string itself.
               Note that "str[n+1]" must be smaller than any value of "str"
// SA
              : the suffix array, holding all the suffixes in lexicographical order.
// suffixRank : array holding the order of the i-th suffix after sorting.
// LCP
              : array holding the length of the longest common prefix between "SA[i]"
                and "SA[i - 1]".
//
int n, SA[N], suffixRank[N], LCP[N];
char str[N];
// Temporary arrays needed while computing the suffix array.
int sortedSA[N], sortedRanks[N], rankStart[N];
// Comparator struct to be used internally from "buildSuffixArray" function.
struct comparator {
   int h;
   comparator(int h) : h(h) {}
   bool operator()(int i, int j) const {
      if (suffixRank[i] != suffixRank[j]) {
         return suffixRank[i] < suffixRank[j];</pre>
      return suffixRank[i + h] < suffixRank[j + h];</pre>
   }
};
// To be called internally from "buildSuffixArray" function.
void computeSuffixRanks(int h) {
   comparator comp(h);
   for (int i = 1; i <= n; ++i) {</pre>
      int& r = sortedRanks[i] = sortedRanks[i - 1];
      if (comp(sortedSA[i - 1], sortedSA[i])) {
         rankStart[++r] = i;
      }
   }
```

```
for (int i = 0; i <= n; ++i) {</pre>
      SA[i] = sortedSA[i];
      suffixRank[SA[i]] = sortedRanks[i];
   }
}
// Builds the suffix array of the given string in time complexity of O(n.log(n)).
void buildSuffixArray() {
   for (int i = 0; i <= n; ++i) {</pre>
      sortedSA[i] = i;
      suffixRank[i] = str[i];
   }
   sort(sortedSA, sortedSA + n + 1, comparator(0));
   computeSuffixRanks(0);
   for (int h = 1; sortedRanks[n] != n; h <<= 1) {</pre>
      for (int i = 0; i <= n; ++i) {</pre>
         int k = SA[i] - h;
         if (k >= 0) {
            sortedSA[rankStart[suffixRank[k]]++] = k;
         }
      }
      computeSuffixRanks(h);
   }
}
// Computes the longest common prefix (LCP) for every two consecutive suffixes in the
// suffix array in time complexity of O(n).
void buildLCP() {
   int cnt = 0;
   for (int i = 0; i < n; ++i) {</pre>
      int j = SA[suffixRank[i] - 1];
      while (str[i + cnt] == str[j + cnt]) ++cnt;
      LCP[suffixRank[i]] = cnt;
      if (cnt > 0) --cnt;
   }
}
```

## Graphs

#### Shortest Path (Floyd Warshal's Algorithm)

```
int n;
                    // The number of nodes.
int adj[N][N];
                    // The graph adjacency matrix.
                    // par[u][v] : holds the parent node of "v" in the shortest
int par[N][N];
                                    path from "u" to "v".
// Initializes the graph. Must be called before each test case.
void init() {
    for (int i = 0; i < n; ++i)</pre>
        for (int j = 0; j < n; ++j)</pre>
            adj[i][j] = (i == j ? 0 : oo), par[i][j] = i;
}
// Computes all-pair shortest paths using Floyd Warshall's algorithm in O(n^3).
void floyd() {
    for (int k = 0; k < n; ++k)
        for (int i = 0; i < n; ++i)</pre>
            for (int j = 0; j < n; ++j)</pre>
                if (adj[i][j] > adj[i][k] + adj[k][j])
                    adj[i][j] = adj[i][k] + adj[k][j], par[i][j] = par[k][j];
}
// Checks whether the graph has negative cycles or not.
bool checkNegativeCycle() {
    bool ret = false;
    for (int i = 0; i < n; ++i) {</pre>
        ret = ret || (adj[i][i] < 0);
    return ret;
}
// Prints the shortest path from node "u" to node "v".
void printPath(int u, int v) {
    if (u != v) {
        printPath(u, par[u][v]);
    }
    printf("%d", v + 1);
}
Shortest Path (Bellman Ford's Algorithm)
int n;
                                   // The number of nodes.
int dis[N];
                                   // dis[v] : holds the shortest distance between
                                   //
                                               the source and node "v".
vector<pair<int, int>> edges[N];
                                   // The graph adjacency list.
// Computes signle-source shortest paths using Bellman Ford's algorithm in O(n^2).
// And returns whether the graph contains negative cycles or not.
```

```
bool bellmanFord(int src) {
    memset(dis, 0x3F, sizeof(dis));
    dis[src] = 0;
    bool updated = 1;
    for (int k = 0; k < n && updated; ++k) {</pre>
        updated = ∅;
        for (int u = 1; u <= n; ++u) {</pre>
             for (auto& e : edges[u]) {
                 int v = e.first;
                 int w = e.second;
                 if (dis[v] > dis[u] + w) {
                     dis[v] = dis[u] + w;
                     updated = 1;
                 }
            }
        }
    }
    return updated;
}
```

#### Shortest Path (Dijkstra's Algorithm)

```
* Edge structs to holds the needed information about an edge.
struct edge {
    int to, weight;
    edge() {}
    edge(int t, int w) : to(t), weight(w) {}
    bool operator<(const edge& rhs) const {</pre>
        return weight > rhs.weight;
    }
};
                        // The number of nodes.
int n;
int dis[N];
                        // dis[v] : holds the shortest distance between the source
                                   and node "v".
vector⟨edge⟩ edges[N]; // The graph adjacency list.
// Computes signle-source shortest paths using Dijkstra's algorithm in O(n.log(n)).
void dijkstra(int src) {
    priority_queue<edge> q;
    q.push(edge(src, 0));
    memset(dis, 0x3F, sizeof(dis));
    while (!q.empty()) {
        int u = q.top().to;
        int w = q.top().weight;
```

```
q.pop();

if (dis[u] <= w) {
          continue;
}

dis[u] = w;

for (edge& e : edges[u]) {
          if (w + e.weight < dis[e.to]) {
                q.push(edge(e.to, w + e.weight));
          }
}</pre>
```

#### Minimum Spanning Tree (Kruskal's Algorithm)

```
* Edge structs to holds the needed information about an edge.
struct edge {
    int from, to, weight;
    edge() {}
    edge(int f, int t, int w) : from(f), to(t), weight(w) {}
    bool operator<(const edge& rhs) const {</pre>
        return (weight < rhs.weight);</pre>
    }
};
int n;
                        // The number of nodes.
int par[N];
                        // The DSU parent array.
                       // The edges of the graph.
vector<edge> edges;
// Finds and returns the set id of an element using the DSU data structure.
int findSetId(int u) {
    return u == par[u] ? u : par[u] = findSetId(par[u]);
}
// Computes and returns the minimum spanning tree of a weighted graph.
int kruskalMST() {
    int MST = 0;
    sort(edges.begin(), edges.end());
    for (int i = 1; i <= n; ++i) {
        par[i] = i;
    for (auto& e : edges) {
        int x = findSetId(e.from);
        int y = findSetId(e.to);
        if (x != y) {
            par[x] = y;
```

```
MST += e.weight;
        }
    }
    return MST;
}
SCC (Kosaraju's Algorithm)
                            // Number of nodes.
int n;
bool vis[N];
                            // Nodes visited array.
vector<int> edges[N];
                            // Graph adjacency list.
vector<int> edgesT[N];
                            // Transposed graph adjacency list (i.e. reversed edges).
                            // Strongly connected components.
vector<vector<int>> scc;
// Sorts the nodes in a topological order.
void topoSortDFS(int u, vector<int>* edges, vector<int>& nodes) {
    vis[u] = 1;
    for (int v : edges[u]) {
        if (!vis[v]) {
            topoSortDFS(v, edges, nodes);
        }
    }
    nodes.push back(u);
}
// Extracts the strongly connected components (SCC) of the given directed graph
// using Kosaraju's algorithm, and fills them in the global "scc" vector".
void kosaraju() {
    vector<int> sortedNodes;
    memset(vis, 0, sizeof(vis));
    for (int i = 1; i <= n; ++i) {</pre>
        if (!vis[i]) {
            topoSortDFS(i, edges, sortedNodes);
        }
    }
    memset(vis, 0, sizeof(vis));
    for (int i = sortedNodes.size() - 1; i >= 0; --i) {
        int u = sortedNodes[i];
        if (!vis[u]) {
            vector<int> tmp;
            topoSortDFS(u, edgesT, tmp);
            scc.push_back(tmp);
        }
    }
}
```

#### Topological Sort (Khan's Algorithm)

```
int n;
                            // Number of nodes.
bool vis[N];
                            // Nodes visited array.
vector<int> edges[N];
                            // Graph adjacency list.
                            // List of topologically sorted nodes.
vector<int> sortedNodes;
// Sorts the graph in a topological order using Khan BFS algorithm,
// and fills the result in the global "sortedNodes" vector
void topoSortBFS() {
    queue<int> q;
    vector<int> inDeg(n + 1, 0);
    for (int i = 1; i <= n; ++i) {</pre>
        for (int v : edges[i]) {
            ++inDeg[v];
        }
    }
    for (int i = 1; i <= n; ++i) {</pre>
        if (inDeg[i] == 0) {
            q.push(i);
        }
    }
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        sortedNodes.push_back(u);
        for (int v : edges[u]) {
            if (--inDeg[v] == 0) {
                q.push(v);
        }
    }
}
```

#### Tree Diameter

```
for (auto v : edges[u]) {
            if (dis[v] == -1) {
                dis[v] = dis[u] + 1;
                q.push(v);
            }
        }
    }
    return u;
}
// Computes and returns the length of the diameter of the tree.
int calcTreeDiameter(int root) {
    int u = bfs(root);
    int v = bfs(u);
    return dis[v];
}
Bipartite Graph Check
int color[N];
                      // The set each node belongs to.
vector<int> edges[N]; // The graph adjacency list.
// Do not call this directly.
bool dfs(int u = 1) {
    for (int v : edges[u]) {
        if (color[v] == color[u]) {
            return false;
        }
        if (color[v] == -1) {
            color[v] = color[u] ^ 1;
            if (!dfs(v)) {
                return false;
            }
        }
    }
    return true;
}
// Checks whether the given graph is bipartite or not.
bool isBipartiteGraph() {
    memset(color, -1, sizeof(color));
    color[1] = 0;
    return dfs();
}
Bridge Tree
                                   // The number of nodes.
int n;
                                   // The graph adjacency list.
vector<int> edges[N];
```

```
// Bridge tree related variables
//
int T
int root
int par[N];
int tin[N;
int low[N];
vector<int> tree[N]; vector<pair<int, int>> bridges;
// Do not call this directly.
int findSetId(int u) {
    return (par[u] == u ? u : par[u] = findSetId(par[u]));
}
// Do not call this directly.
void findBridges(int u = 1, int p = -1) {
    tin[u] = low[u] = ++T;
    for (int v : edges[u]) {
        if (v == p) {
            continue;
        }
        if (tin[v] == 0) {
            findBridges(v, u);
            if (low[v] > tin[u]) {
                bridges.push_back({u, v});
            } else {
                par[findSetId(u)] = findSetId(v);
            }
        }
        low[u] = min(low[u], low[v]);
    }
}
// Builds the bridge tree of a graph in O(n+m).
void buildBridgeTree() {
    for (int i = 1; i <= n; ++i) {</pre>
        par[i] = i;
    }
    findBridges();
    for (auto& b : bridges) {
        int u = findSetId(b.first);
        int v = findSetId(b.second);
        tree[u].push_back(v);
        tree[v].push_back(u);
        root = u;
    }
}
```

#### LCA (Euler Walk + RMQ)

```
int n;
                         // The number of nodes.
vector<int> edges[N];
                       // The graph adjacency list.
// LCA related variables.
int dep[N];
int ST[LOG_N][N << 1];</pre>
int LOG[N << 1];</pre>
int F[N];
vector<int> E;
// Do not call this directly.
void dfs(int u = 1, int p = -1, int d = 0) {
    dep[u] = d;
    F[u] = E.size();
    E.push_back(u);
    for (int v : edges[u]) {
        if (v != p) {
            dfs(v, u, d + 1);
            E.push_back(u);
        }
    }
}
// Do not call this directly.
void buildRMQ() {
    LOG[0] = -1;
    for (int i = 0; i < E.size(); ++i) {</pre>
        ST[0][i] = i;
        LOG[i + 1] = LOG[i] + !(i & (i + 1));
    }
    for (int j = 1; (1 << j) <= E.size(); ++j) {</pre>
        for (int i = 0; (i + (1 << j)) <= E.size(); ++i) {</pre>
            int x = ST[j - 1][i];
            int y = ST[j - 1][i + (1 << (j - 1))];
            ST[j][i] = (dep[E[x]] < dep[E[y]]) ? x : y;
        }
    }
}
// Builds the LCA data structure once per test case in O(n.log(n)).
void buildLCA() {
    dfs();
    buildRMQ();
}
// Do not call this directly.
int query(int 1, int r) {
    if (1 > r) swap(1, r);
    int g = LOG[r - l + 1];
    int x = ST[g][1];
    int y = ST[g][r - (1 << g) + 1];
```

```
return (dep[E[x]] < dep[E[y]]) ? x : y;
}
// Returns the LCA of node "u" and node "v" in O(1).
int getLCA(int u, int v) {
    return E[query(F[u], F[v])];
}
// Returns the distance between node "u" and node "v" in O(1).
int getDistance(int u, int v) {
    return dep[u] + dep[v] - 2 * dep[getLCA(u, v)];
}
LCA (Parent Sparse Table)
                        // The number of nodes.
vector<int> edges[N];
                      // The graph adjacency list.
// LCA related variables.
int dep[N];
int par[LOG N][N];
int LOG[N];
// Do not call this directly.
void dfs(int u = 1, int p = -1, int d = 0) {
    dep[u] = d;
    par[0][u] = p;
    for (int i = 1; (1 << i) <= d; ++i) {
        par[i][u] = par[i - 1][par[i - 1][u]];
    for (int v : edges[u]) {
        if (v != p) {
            dfs(v, u, d + 1);
    }
}
// Do not call this directly.
void computeLog() {
    LOG[0] = -1;
    for (int i = 1; i <= n; ++i) {</pre>
        LOG[i] = LOG[i - 1] + !(i & (i - 1));
    }
}
// Builds the LCA data structure once per test case in O(n.log(n)).
void buildLCA() {
    dfs();
    computeLog();
}
// Returns the k-th ancestor of a node "u".
int getAncestor(int u, int k) {
```

```
while (k > 0) {
        int x = k \& -k;
        k -= x;
        u = par[LOG[x]][u];
    return u;
}
// Returns the LCA of node "u" and node "v" in O(log(n)).
int getLCA(int u, int v) {
    if (dep[u] > dep[v]) {
        swap(u, v);
    v = getAncestor(v, dep[v] - dep[u]);
    if (u == v) {
        return u;
    for (int i = LOG[dep[u]]; i >= 0; --i) {
        if (par[i][u] != par[i][v]) {
            u = par[i][u];
            v = par[i][v];
        }
    }
    return par[0][u];
}
// Returns the distance between node "u" and node "v" in O(log(n)).
int getDistance(int u, int v) {
    return dep[u] + dep[v] - 2 * dep[getLCA(u, v)];
}
```

#### Max Flow (Edmonds Karp's Algorithm)

```
int n, m;
                   // The number of nodes and number of edges
int edgeId;
                   // The next edge id to be inserted.
                   // head[u] : the id of the last edge added from node "u".
int head[N];
                   // nxt[e]
                                  : the next edge id pointed from the same node
int nxt[M];
                   //
int to[M];
                   // to[e]
                                 : the id of the node pointed by edge "e".
int capacity[M];
                   // capacity[e] : the maximum capacity of edge "e".
int flow[M];
                   // flow[u]
                                : the current flow of edge "e".
int src, snk;
                   // The id of source and sink nodes.
                   // dist[u]
                                 : the shortest distance between the source and
int dist[N];
                   //
                                    node "u".
int from[N];
                   // from[u]
                                  : the id of the edge that leads to node "u" in the
                                    path from source to sink.
// Initializes the graph. Must be called before each test case.
void init() {
    edgeId = 0;
    memset(head, -1, sizeof(head));
```

```
}
// Adds a new directed edge in the graph from node "f" to node "t"
// with maximum capacity "c".
void addEdge(int f, int t, int c) {
    int e = edgeId++;
    to[e] = t;
    capacity[e] = c;
    flow[e] = 0;
    nxt[e] = head[f];
    head[f] = e;
}
// Adds a new augmented edge in the graph between node "f" and node "t"
// with maximum capacity "w".
void addAugEdge(int f, int t, int c) {
    addEdge(f, t, c);
    addEdge(t, f, ∅);
}
// Do not call this directly.
bool findPath() {
    queue<int> q;
    q.push(src);
    memset(dist, -1, sizeof(dist));
    dist[src] = 0;
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        for (int e = head[u]; ~e; e = nxt[e]) {
            int v = to[e];
            int c = capacity[e];
            int f = flow[e];
            if (f >= c) {
                continue;
            }
            if (dist[v] == -1) {
                dist[v] = dist[u] + 1;
                from[v] = e;
                q.push(v);
            }
            if (v == snk) {
                return true;
            }
        }
    }
    return false;
}
```

```
// Do not call this directly.
int augmentPath() {
    int f = INT_MAX;
    for (int u = snk, e, r; u != src; u = to[r]) {
        e = from[u]; // x ---e--> u
        r = e^{1};
                        // x <--r-- u
        f = min(f, capacity[e] - flow[e]);
    }
    for (int u = snk, e, r; u != src; u = to[r]) {
        e = from[u];  // x ---e--> u
r = e ^ 1;  // x <--r-- u
        flow[e] += f;
        flow[r] -= f; // Reversed edge for flow cancelation
    }
    return f;
}
// Returns the the maximum flow/minimum cut of the graph.
int maxFlow() {
    int f = 0;
    while (findPath()) {
        f += augmentPath();
    return f;
}
```

### Math

#### **GCD**

```
// Computes the greatest common divisors GCD(a, b).
template<class T>
T gcd(T a, T b) {
   while (b) {
      int tmp = a % b;
      a = b;
      b = tmp;
   }
   return a;
}
LCM
// Computes the least common multiple LCM(a, b).
template<class T>
T lcm(T a, T b) {
   return a / gcd(a, b) * b;
Extended Euclid
// Computes the coeffs. of the smallest positive linear combination of "a" and "b".
// (i.e. GCD(a, b) = s.a + t.b).
template<class T>
pair<T, T> extendedEuclid(T a, T b) {
   if (b == 0) {
      return {1, 0};
   }
   pair<T, T> p = extendedEuclid(b, a % b);
   T s = p.first;
   T t = p.second;
   return {t, s - t * (a / b)};
}
Fast Power
// Computes ((base^exp) % mod).
template<class T>
T power(T base, T exp, T mod) {
   T ans = 1;
   base %= mod;
   while (exp > 0) {
      if (exp & 1) ans = (ans * base) % mod;
```

```
exp >>= 1;
      base = (base * base) % mod;
   }
   return ans;
Modular Inverse
// Computes the modular inverse of "a" modulo "m".
template<class T>
T modInverse(T a, T m) {
   return power(a, m - 2, m);
Combinations (nCr)
// Computes "n" choose "r".
int nCr(int n, int r) {
   if (n < r)
      return 0;
   if (r == 0)
      return 1;
   return n * nCr(n - 1, r - 1) / r;
}
Pascal's Triangle
// comb[n][r] : holds the value of "n" choose "r" modulo "mod".
int comb[N][N];
// Builds Pascal's triangle for computing combinations (i.e. "nCr").
void buildPT(int n, int mod) {
   for (int i = comb[0][0] = 1; i \le n; ++i)
      for (int j = comb[i][0] = 1; j <= i; ++j)</pre>
         comb[i][j] = (comb[i - 1][j] + comb[i - 1][j - 1]) \% mod;
}
Prime Check
// Checks whether an integer is prime or not.
template<class T>
bool isPrime(T n) {
   if (n < 2)
      return 0;
   if (n % 2 == 0)
      return (n == 2);
   for (int i = 3; i * i <= n; i += 2)</pre>
      if (n % i == 0)
         return 0;
   return 1;
}
```

#### Prime Check (Miller Rabin's Algorithm)

```
// Do not call it directly.
template<class T>
bool millerRabin(T k, T q) {
   T n = (1LL << k) * q + 1;
   T a = 2 + rand() \% (n - 2);
  T x = power(a, q, n);
   if (x == 1) {
      return true;
   while (k--) {
      if (x == n - 1) {
         return true;
      x = (x * x) % n;
   return false;
}
// Checks whether an integer is prime or not using the probabilistic method of
// Miller Rabin algorithm in O(t.log(n)).
template<class T>
bool isPrimeMillerRabin(T n, int t = 10) {
   if (n == 2) {
      return 1;
   }
   if (n < 2 || n % 2 == 0) {
      return 0;
   }
   T k = 0;
   Tq = n - 1;
   while ((q & 1) == 0) {
      k++;
      q >>= 1;
   }
   while (t--) {
      if (!millerRabin(k, q)) {
         return false;
      }
   return true;
Generate Primes
// prime[i] : true if integer "i" is prime; false otherwise.
bool prime[N];
```

```
// Generates all the prime numbers of the integers from 1 to "n"
void generatePrimes(int n) {
   memset(prime, true, sizeof(prime));
   prime[0] = prime[1] = false;
   for (int i = 2; i * i <= n; ++i) {</pre>
      if (!prime[i]) continue;
      for (int j = i * i; j <= n; j += i) {</pre>
         prime[j] = false;
      }
   }
}
Generate Prime Divisors
// divs[i] : holds a list of all the prime divisors of integer "i".
vector<int> primeDivs[N];
// Generates all the prime divisors of the integers from 1 to "n".
void generatePrimeDivisors(int n) {
   for (int i = 2; i <= n; ++i) {</pre>
      if (primeDivs[i].size()) continue;
      for (int j = i; j <= n; j += i) {
         primeDivs[j].push_back(i);
      }
   }
}
Generate Divisors
// Computes all the divisors of a positive integer.
template<class T>
vector<T> getDivisors(T n) {
   vector<T> divs;
   for (T i = 1; i * i <= n; ++i) {
      if (n % i != 0) continue;
      divs.push back(i);
      if (i * i == n) continue;
      divs.push back(n / i);
   sort(divs.begin(), divs.end());
   return divs;
}
// divs[i] : holds a list of all the divisors of integer "i".
vector<int> divs[N];
// Generates all the divisors of the integers from 1 to "n".
void generateDivisors(int n) {
   for (int i = 1; i <= n; ++i)</pre>
      for (int j = i; j <= n; j += i)
         divs[j].push_back(i);
}
```

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### Others

#### Longest Increasing Sub-sequence

```
// The array to compute its LIS and its length.
int n, a[N];
// Computes and returns the length of the longest increasing subsequence (LIS) of
// the global array "a" in time complexity of O(n.log(n)).
int getLIS() {
   int len = 0;
   vector<int> LIS(n, INT_MAX);
   for (int i = 0; i < n; ++i) {</pre>
      // To get the length of the longest non decreasing subsequence
      // replace function "lower_bound" with "upper_bound"
      int idx = lower_bound(LIS.begin(), LIS.end(), a[i]) - LIS.begin();
      LIS[idx] = a[i];
      len = max(len, idx);
   }
   return len + 1;
}
```