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A Production Characterization of the Eagle Ford Shale, Texas - A Bayesian Analysis Approach

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Abstract

We began this research by asking "Can we use Bayes' theorem to supplement available decline models and improve the accuracy of our estimates of ultimate recovery?" This study focuses on the Eagle Ford Shale, and in particular, on oil wells in the Greater Core Eagle Ford Area. Our goal was to develop a method based on a probabilistic approach to identify, characterize, and better model well production based on standard decline models

To attempt to answer this question, we first obtained data for 68 wells in the Greater Core of the Eagle Ford Shale, Texas. As process, we eliminated the wells that did not have enough production data, wells that did not show a production decline and wells that had too much data noise, leaving eight wells. We then performed decline curve analysis (DCA) using the Modified Hyperbolic (MH) and Power-Law Exponential (PLE) models (the two most common DCA models), consisting in user-guided analysis software. Then, the Bayesian paradigm was implemented to calibrate the same two models on the same set of wells.

The primary focus of the research was the implementation of the Bayesian paradigm on the eight-well data set. We first performed a "best fit" parameter estimation using least squares optimization, which provided an optimized set of parameters for the two decline models. This was followed by using the Markov Chain Monte Carlo (MCMC) integration of the Bayesian posterior function for each model, which provided a full probabilistic description of its parameters. This allowed for the simulation of a number of likely realizations of the decline curves, from which first order statistics were computed to provide a confidence metric on the calibration of each model as applied to the production data of each well.

Results showed variation on the calibration of the MH and PLE models. The forward models (MH and PLE) overestimated the ultimate recovery in the majority of the wells compared with the Bayesian calibrations, proving that the Bayesian paradigm was able to capture a more accurate trend of the data and thus able to determine more accurate estimates of reserves.

In industry, the same decline models are used for unconventional wells as for conventional wells, even though we know that the same models may not apply. Based on the proposed results, we believe that Bayesian inference yields more accurate estimates of ultimate recovery for unconventional reservoirs than

deterministic DCA methods. Moreover, it provides a measure of confidence on the prediction of production as a function of varying data and varying decline models.

Introduction

The petroleum industry in the U.S. has shifted its focus to unconventional plays due to the enormous estimated reserves and the unconventionals' ability to revolutionize the oil and gas industry, notably with new technology. The majority of the world's proved oil reserves were located in the Middle East as of January 2011, as were the majority of the world's natural gas reserves. The United States has 322.7 trillion cubic feet of gas reserves and 33.4 billion barrels of oil reserves (EIA, 2012-13.) This being said, there is a growing shift to gas both in exploration and production, as seen in Fig. 1.

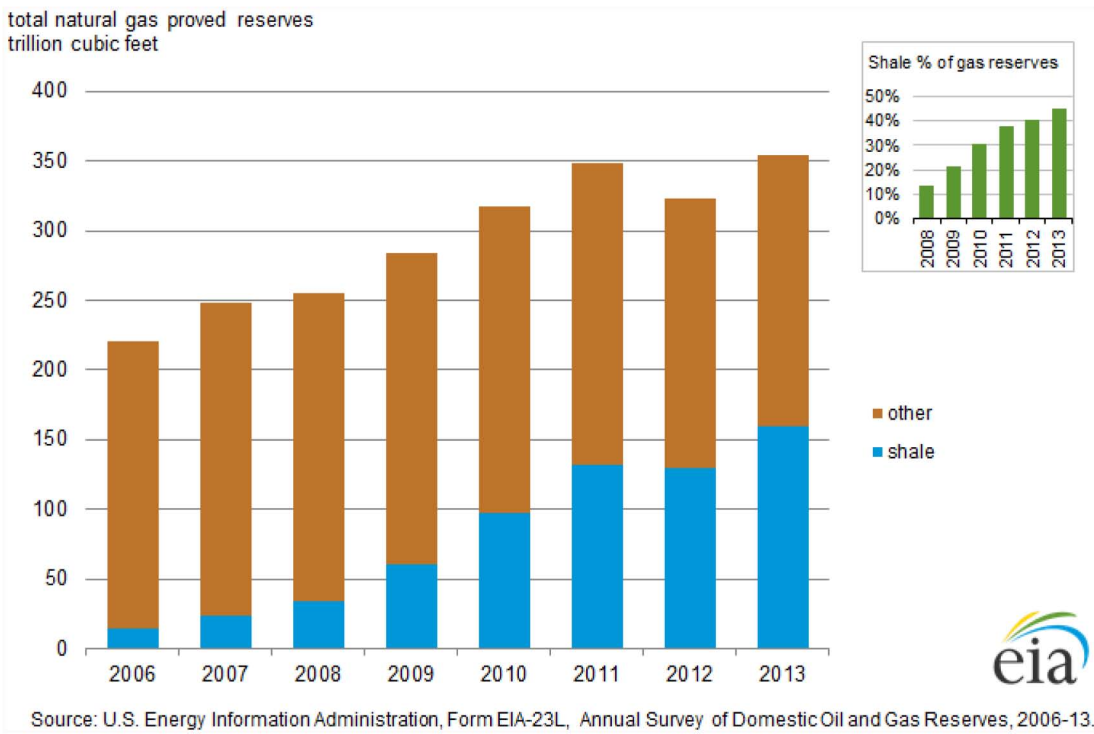

Figure 1—U.S. total natural gas proved reserves shows an increase in shale gas reserves, which in 2008 was approximately 12% of total gas reserves, and in 2013 nearly 50% of total gas reserves (U.S. EIA, 2013)

As seen in Fig. 2, the largest source of U.S. energy today is natural gas, which is forecasted to continue to be the main source of energy through 2040.

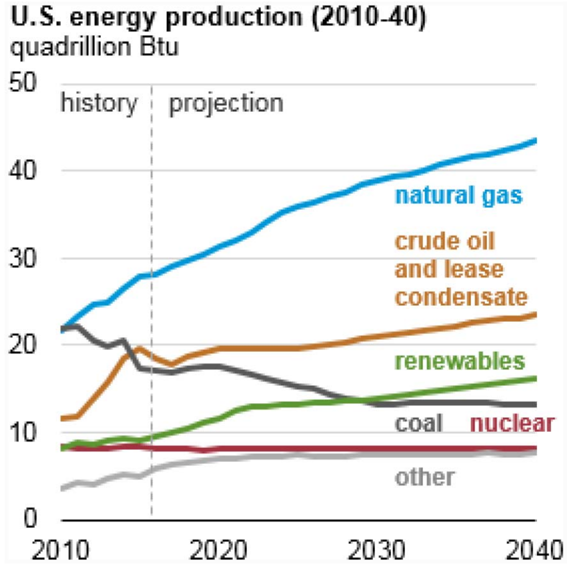

Figure 2—U.S. energy production by fuel, 2010-2040, quadrillion BTU (U.S. DOE EIA 2016)

Natural gas is produced from shale gas, tight gas and coalbed methane reservoirs. Due to the extremely low permeability of such reservoirs, horizontal drilling and hydraulic fracturing are usually used to produce them economically. According to the "resource triangle" (Martin *et al.*, 2010) shown in Fig. 3 below, conventional reservoirs are of high to medium quality, are difficult to find but easy to produce, and thus less expensive. However, as we shift to unconventional reservoirs, the reservoir quality is low, and although the locations of unconventional reservoirs are relatively well-known (as source rocks), these reservoirs are difficult to exploit and complex technologies are required to extract the hydrocarbons, leading to increased prices for drilling and completions, which ultimately requires high oil and gas prices to achieve profitability.

Figure 3—The resource triangle (Martin *et al.*, 2010).

In the resource triangle, conventional resources are located in the apex of the triangle, and "are difficult to find but easy to extract," and as we progress lower into the resource triangle, the resources are easier to find — but harder to extract due to the necessity of improved technologies, and are thus more expensive to extract (Martin *et al.*, 2010).

This study focuses on the Eagle Ford Shale, and in particular, on oil wells in the Greater Core Eagle Ford Area. Our goal was to develop a method based on a probabilistic approach to identify, characterize, and better model well production based on standard decline models.

The Eagle Ford Shale is 50 miles wide and 400 miles long, covering 23 counties in South Central Texas (Gong *et al.,* 2013). The Eagle Ford Shale is the source rock for the Austin Chalk but is now being developed as its own self-sourcing reservoir (Tian *et al*., 2013). This play is composed of Cretaceous mudstone and carbonates which are especially brittle due to the high carbonate and low clay content, meaning that hydraulic fracturing is especially effective. The productive portion of the Eagle Ford shale ranges from 2,500 to 14,000 ft, and the thickness ranges from 120 to 350 ft (Gong *et al*., 2013). The geology of the Eagle Ford shale is quite complex and the calcareous makeup of the rock leads to the "condensate rich environment of this play" which presents "unique fracture design challenges" (Bazan *et al.*, 2012).

The Eagle Ford Shale has been in development since 2008 and is being exploited using horizontal wells with multi-stage hydraulic fracture treatments. The estimated resources in the Eagle Ford Shale are 21 trillion cubic feet (TCF) of gas and 3.35 billion barrels of oil (BBO), however this resource estimate is highly uncertain (Gong *et al.,* 2013).

According to Ayers *et al.,* (2011) the Greater Core of the Eagle Ford is the region of highest oil production, and this is where the focus of this work lies. In the first month of production, a well will generally produce more than 5,000 bbl. The regions of highest gas production are between the Stuart City and Sligo Shelf Margins, where the first month of the wells' production exceeds 60 MMcf (Tian *et al.,* 2013).

Although the production differs throughout the play, the most productive gas wells "are located south of the Stuart City Shelf Margin, where production commonly exceeds 80 MMcf/month/well" while oil production is highest in Karnes and Gonzalez counties, typically exceeding 16 Mbbl/month/well (Tian *et al.*, 2013).

The Eagle Ford Shale is the source rock for the Austin Chalk; however, it is now being developed as its own self-sourcing reservoir (Tian *et al.,* 2013). The lower part of the reservoir is shale-rich, the upper part is carbonate-rich, and the whole Eagle Ford lies over the Buda Limestone, which is overlain by the Austin Chalk (Tian *et al*., 2013). This being said, the Eagle Ford Shale is an unconventional reservoir, which "consists of a wide variety of liquid sources including oil sands, extra heavy oil, gas to liquids and other liquids." (IEA, 2013). Furthermore, unconventional reservoirs are more difficult to produce because they require advanced technology. Shale plays specifically are self-sourcing reservoirs; therefore, the hydrocarbon does not migrate, but stays in place. These shale reservoirs have also been the source rocks for many conventional reservoirs, and only recently have we found that we could economically produce these unconventional reservoirs. Conventional reservoirs are defined as "a petroleum system is a dynamic hydrocarbon generating system, functioning in a geologic space and time scale" that requires "the timely convergence of geologic elements and events essential to the formation of petroleum deposits that include mature source rock, expulsion, secondary migration to reservoir rock, accumulation in a trap and retention." (Ayers, 2011).

Since 2009, the production from the Eagle Ford Shale has grown significantly, due to the use of hydraulic fractures which promote strong well performance. This paper focuses primarily on the gas, condensate and oil windows of the Eagle Ford Shale, where over 200 rigs were active as of February 2012 (Bazan *et al.*, 2012).

Shale-gas formations (such as the Eagle Ford Shale) are naturally fractured, which increases the complexity of the "growth patterns of hydraulic fractures." Furthermore, when multistage hydraulic-fracturing treatments are implemented, they create "conductive networks that could be considered as stimulated reservoir volumes which have been effectively contacted and contribute to economically viable production profiles." (Fazelipour, 2011(b)).

In unconventional reservoirs, one of the primary goals when designing the completion, is to contact as much of the reservoir as possible. This makes these reservoirs economical for development, and leads to increased connectivity between the hydraulic fractures and the wellbore (Bazan *et al.,* 2012).

Reservoir simulation, type-curve and decline-curve analysis have been considered to be the most useful methods for estimating ultimate recovery (Gong et al.). Several methods have been used to to estimate and forecast the production of the Eagle Ford. Dong et al. used Monte Carlo simulation and reservoir simulation of dry gas wells to forecast production and determine ultimate recovery. Arps' decline curve analysis has often been used in unconventional reservoirs to forecast production and estimate ultimate recovery; however, this approach is inaccurate due to the low permeability of thereservoir (Gong *et al*., 2013).

Methodology

In this study, we apply the Modified Hyperbolic and the Power Law Exponential Models to our data set. We start with the basic equations used in these decline models.

Eqs. 1 and 2 are the basis for Arps' decline models. Eq. 2 models exponential decline and Eq. 1 models harmonic decline when $b = 1$ and hyperbolic decline when $b \neq 1$. These two equations are standard for decline curve analysis, and are ideal for application to conventional reservoirs.

$$q(t) = \frac{q_i}{(1 + bD_i t)^{1/b}}; b \neq 0 \tag{1}$$

$$q(t) = q_i \exp(-Dt); b = 0 \tag{2}$$

In these equations, b is Arps' dimensionless hyperbolic decline constant. In conventional reservoirs, the exponent b is between 0 and 1 and the producing well is in boundary-dominated flow. However, in unconventional low-permeability reservoirs, the flow is not boundary-dominated initially (perhaps for several years), and b is usually greater than 1. Though this is an incorrect use of Arps' hyperbolic decline model, this approach is often used to estimate ultimate recovery (Gong *et al*., 2013).

Eq. 3 is the Power Law Exponential (PLE) equation.

$$q(t) = q_i \exp(-D_\infty t - D_i t^n) \tag{3}$$

where $-D_\infty$ is the power law decline rate constant at infinite time, n is the dimensionless time exponent, typically between 0 and 1, and D_i is the power law decline constant. D_i is actually calculated by determining D_1, which is the instantaneous decline rate at t=1:

$$D_i = \frac{D_1}{n} \tag{4}$$

D_i, in this model, is not the initial instantaneous decline rate but the instantaneous decline rate at t=1 divided by n when D_∞ is equal to one. Also, many values of D_∞ term can fit production data equally well at early-times, and it is only at late times rates that "forecasts become sensitive to D_∞" (Mattar et al., 2008)

Eq. 5 is the Modified Hyperbolic (MH) equation.

$$q(t) = \begin{cases} \dfrac{q_i}{[1 + bD_i t]^{1/b}}; & t < t^* \\ q_{i\exp} \exp[-D_{\lim} t]; & t > t^* \end{cases} \tag{5}$$

$$D = \frac{1}{\frac{1}{D_i} + bt} \tag{6}$$

The decline rate, D, is not a constant but decreases continuously, as seen in Eq. 6. "When D becomes too small, the gas rate no longer declines significantly, and the reserves can be over-predicted. To circumvent the problem of D becoming too small, Long and Davis (1988) introduced the Modified Hyperbolic Decline method, that imposes a limit below which D is not allowed to decline (D_{lim})." (Mattar *et al.,* 2008)

Neither of these decline models (nor other published models) quantify the uncertainty in production forecasts and reserves estimates by themselves; they must be combined with other models to properly quantify these uncertainties (Gong et al., 2012).

The Markov Chain Monte Carlo (MCMC) method was presented by Dong *et al.* (2011). It does not modify the actual production data, and has been well calibrated for a limited number of test cases. MCMC was combined with Bayes' theorem by Liu and McVay in 2009 and Xie et al. in 2011 to quantify uncertainty in reservoir simulation (Gong *et al*., 2012).

These methods were all developed based on Arp's method, and there is only limited work published on the use of these methods (particularly in unconventional reservoirs) (Gonzalez, 2013). The methods presented above were all done on Barnett Shale wells.

We decided to perform this work in barrels of oil equivalent (BOE) to incorporate both oil and gas production from the set of wells. To do this, we converted the gas production to oil production using the equation below.

$$bbl = \frac{Mscf}{6} \tag{7}$$

After converting the production data to BOE, we proceeded to perform the decline curve analysis using the Power Law Exponential and the Modified Hyperbolic decline curve models. The equations for the two models are presented below, first being the MH and the second being the PLE.

$$q(t) = \begin{cases} \dfrac{q_i}{[1 + bD_i t]^{1/b}}; & t < t^* \\ q_{i\exp} \exp[-D_{\lim} t]; & t > t^* \end{cases} \tag{8}$$

Where:

$$D_{\lim} = -\frac{\ln[1-p]}{365} \tag{9}$$

$$t^* = \frac{\left(\dfrac{D_i}{D_{\lim}} - 1\right)}{bD_i} \tag{10}$$

We set the decline limit (p) to 10%, which is a conservative decline limit in the industry. This number can be changed to what a company prefers; however, for this study we believe it is best to be conservative. This value determines both the D_{lim} and the t^* values, indicating the point at which decline switches from the hyperbolic to the exponential model.

$$q(t) = q_i \exp(-D_\infty t - D_i t^n) \tag{11}$$

In the PLE model presented in equation 11, we set D_∞ to 0 and n to 1. This allows the power law to change to an exponential decline. The reason that D_∞ is set in the PLE model is to "place a limit on how low the decline can become to avoid reserve over-prediction." (Mattar et al, 2008)

The next step was to run the two decline curve models using proprietary software. This yielded a set of parameters for both equations, along with values of the Estimated Ultimate Recovery (EUR). From here, we applied least squares optimization to obtain an optimized set of parameters. We then used these parameters to implement the MCMC, and set the number of iterations until the values converged - between two and 40 million. After the results converged from the MCMC, we applied them in the Bayesian paradigm on the MH and PLE equations.

To perform the inverse problem, we used the equations below:

$$P(\boldsymbol{\theta}|\mathbf{D}) \propto P(\boldsymbol{\theta})P(\mathbf{D}|\boldsymbol{\theta}) \tag{12}$$

This equation demonstrates that the posterior, $P(\theta|D)$, is proportional to the prior, $P(\theta)$, times the likelihood, $P(D|\theta)$. The θ is the set of known parameters, and the likelihood function indicates the likelihood of the event to occur. In this study, the priors are the initial rate (q_i), the initial decline (D_i), the b-factor and the time exponent (n), and we will assume that they are non-informative priors, meaning that they will all follow a uniform distribution.

We can obtain the posterior distribution using Bayes' Theorem with Eq. 13 and Eq. 14 below.

$$P(\boldsymbol{\theta}|\mathbf{d}) = \frac{P(\boldsymbol{\theta})P(\mathbf{d}|\boldsymbol{\theta})}{\int P(\boldsymbol{\theta})P(\mathbf{d}|\boldsymbol{\theta})d\boldsymbol{\theta}} \tag{13}$$

$$E[f(\boldsymbol{\theta})|\mathbf{d}] = \frac{\int f(\boldsymbol{\theta})P(\boldsymbol{\theta})P(\mathbf{d}|\boldsymbol{\theta})d\boldsymbol{\theta}}{\int P(\boldsymbol{\theta})P(\mathbf{d}|\boldsymbol{\theta})d\boldsymbol{\theta}} \tag{14}$$

We can re-write the above equations to find the following equation:

$$E[f(X)] = \frac{\int f(x)\pi(x)dx}{\int \pi(x)dx} \tag{15}$$

where $\pi(x)$ is the likelihood.

We then move to apply the MCMC, which is "a class of algorithms for sampling from a probability distribution based on constructing a Markov chain that has the desired distribution as its equilibrium distribution. The state of the chain after a number of steps is then used as a sample of the desired distribution." (Wikipedia). We used the Metropolis-Hasting algorithm, which is a "MCMC method for obtaining a sequence of random samples from a probability distribution for which direct sampling is difficult" (Wikipedia). The Metropolis-Hastings criteria follows Eq. 16 below.

$$\alpha(x_t, y) = \min\left\{1, \frac{\pi(y)q(x_t|y)}{\pi(x_t)q(y|x_t)}\right\} \tag{16}$$

where $q(x_t|y)$ is the proposed distribution. In this research, we will assume that the priors will follow a uniform distribution. The constant values in in Eq. 16 will cancel from the numerator and the denominator.

We will assume that the likelihood is normally distributed, and will be determined using Eq. 17 below.

$$P(D|\boldsymbol{\theta}) = (2\pi)^{\frac{-n}{2}} |\boldsymbol{\sigma}|^{-0.5} e^{-0.5(\mathbf{d}-\boldsymbol{\theta})'\boldsymbol{\sigma}^{-1}(\mathbf{d}-\boldsymbol{\theta})} \tag{17}$$

where $\boldsymbol{\sigma}$ is the standard deviation, $\boldsymbol{d}$ is the observed data and that $\boldsymbol{\theta}$ is the forward model, so in this research, either the MH or the PLE models.

The random and the constant variables of the two decline curve analysis models. For the MH model, we will set, D_i and q_i the b-factor as the set of random variables, and they are all greater than 0, and will keep t and $D_{\lim}$ as a constant variable. For the PLE model, we will set q_i, D_i, and n as the random variables, and keep t and D_∞ as the constant variables.

This yielded a set of results that show the discrepancy between the two forward equations and the Bayesian results. The final step of this work was to apply an integration to the results to obtain numerical EUR results to be able to compare them with the EUR results from the forward equations.

Discussion

The initial results of Well 41 using the proprietary software are presented in the Table 1 and Table 2 below.

Table 1—Parameter Estimation and EUR for the MH Model

Modified Hyperbolic					
	D-parameter			t = 30 vrs	qboe = 5 BOE/d
Well	Intercept (Di)	qboie	b	EUR (BOE)	EUR (BOE)
41	0.0035	526	0.73	380,685	372.341

Table 2—Parameter Estimation and EUR for the PLE Model

Power-Law Exponential						
	D-parameter				t = 30 vi s	qboe = 5 BOE/d
Well	Slope (11)	Intercept (Di)	D_infinity7	qboie	EUR (BOE)	EUR (BOE)
41	0.65	0.021	0	577	289.908	282.623

As previously stated, we used these initial results to run the least squares optimization to obtain a set of optimized results, and from there we implemented the MCMC and then the Bayesian paradigm on the models. We then graphed the standard deviation of the two Bayesian models that shows which model is more accurate for the given well.

It is evident from Fig. 4 that the MH model overestimates the reserves and from Fig. 5, the PLE model underestimates the reserves. We see two distinct behaviors from these two models, and one shows an underestimation and the other an overestimation. To determine which of these two models is more correct, we have plotted the standard deviations of the two Bayesian models, shown below in Fig. 6.

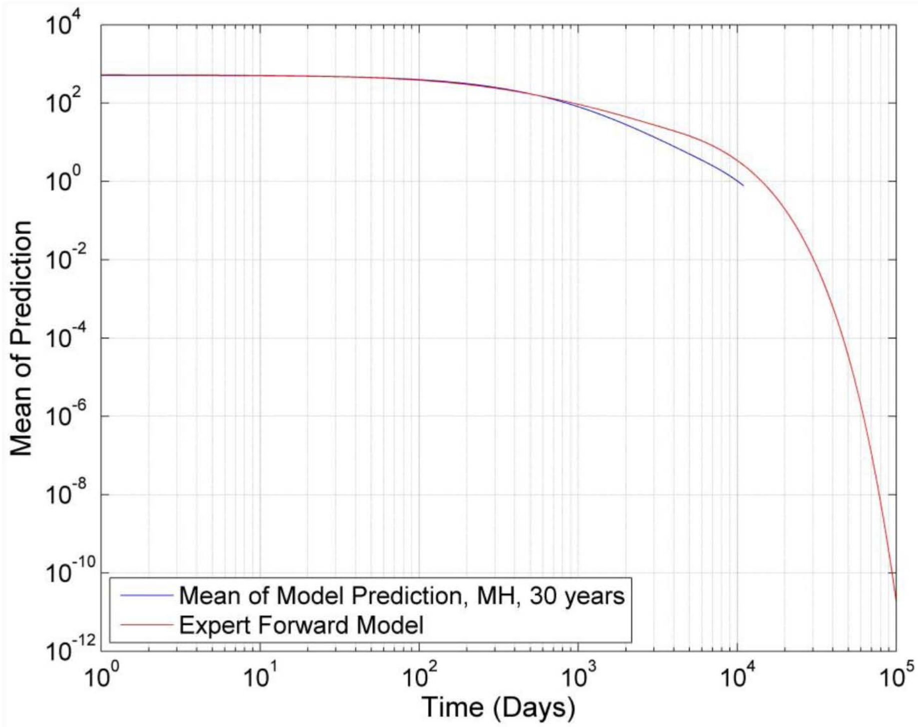

Figure 4—The mean of the Bayesian realizations (blue curve) and the MH model (red curve), plotted for 30 years

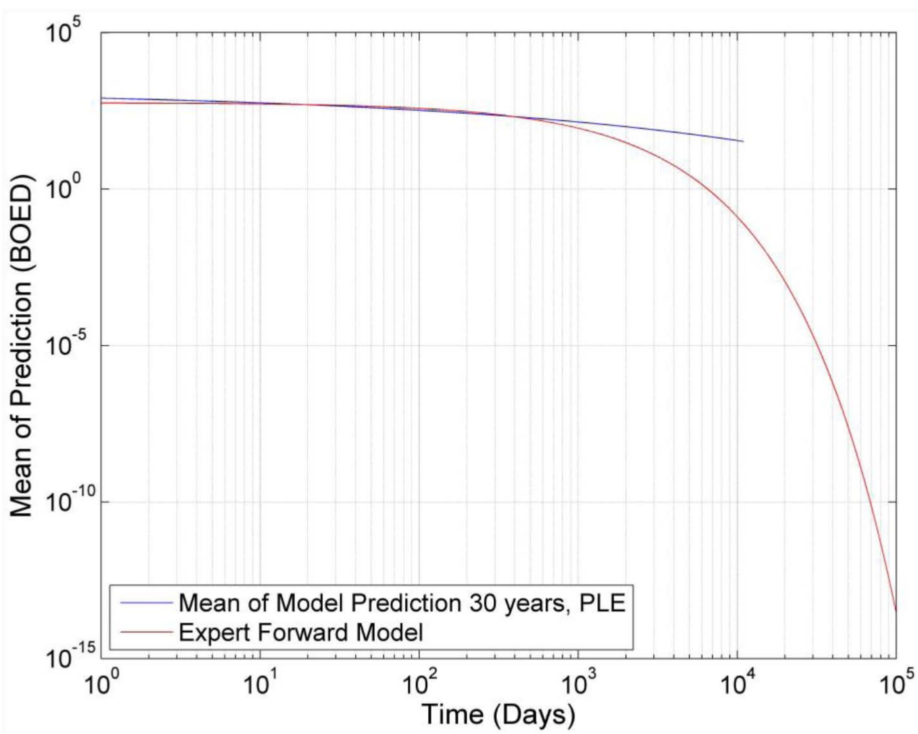

Figure 5—The mean of the Bayesian realizations (blue curve) and PLE model (red curve), plotted for 30 years

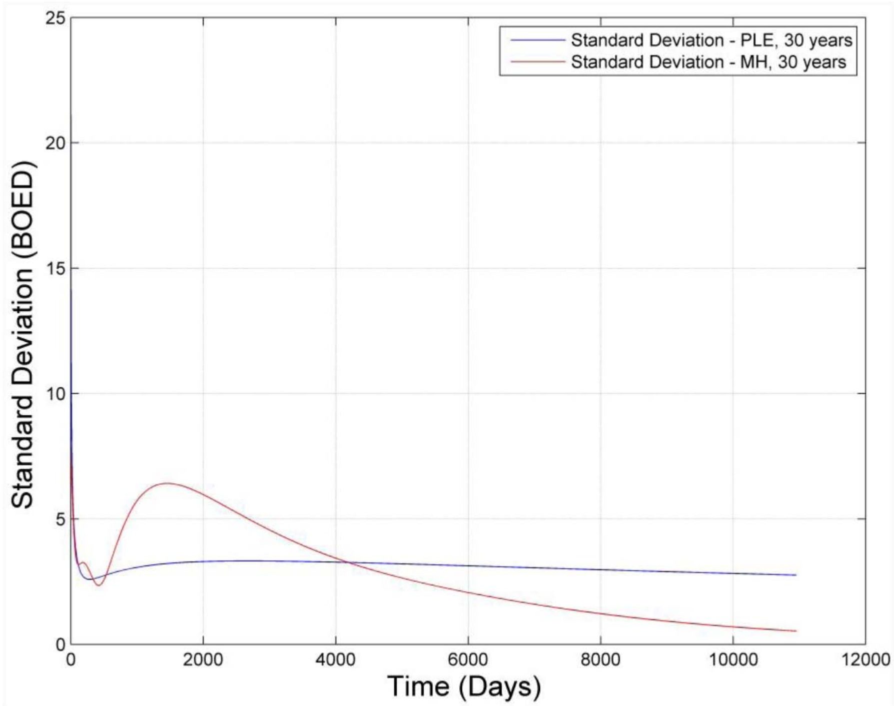

Figure 6—The comparison of the standard deviations of the two Bayesian models

We see from the figure above that the uncertainty of the MH Bayesian model (red curve) decreases with time. This is indicative that the statistical model is accurately capturing the trend of the data and reducing its uncertainty. However, we see that with the PLE Bayesian model (blue curve) that the uncertainty remains constant with time, indicating that the model is not properly capturing the trend of the data. These results indicate that the the MH Bayesian model is more accurate for this well.

We performed the same study on the other seven wells used in this work. Six of the eight wells showed that the MH Bayesian model was more accurate than the PLE Bayesian model. We showed this by comparing the standard deviations of each of the two models for each well (as done in Fig. 6).

The final step of this study was to obtain numerical EUR results. The results are presented in Table 3 below.

Table 3—Comparison between the Bayesian EUR and Forward Model EUR for Well 41

Well 41 (BOE)	
EUR Bayes MH	3.07E+05
EUR MH	3.81E+05
EUR Bayes PLE	8.12E+05
EUR PLE	2.90E+05

The MH model overestimates the EUR, while the PLE model underestimates the EUR. However, the MH model and the Bayesian MH model have relatively close values, whereas the results of the PLE are very different. As previously discussed, the Bayesian PLE model is not accurate because the standard deviation remains constant with time therefore the results are inconclusive. An indicator of a successful model in the Bayesian paradigm is that it's uncertainty decreases with time (McClelland, 2000) — as such, this work suggests that the MH model should yield the most reliable results when the Bayesian paradigm is applied. We note for completeness that these results and conclusions are valid for this well, and may not necessarily apply for the other wells.

Summary

We used proprietary software to apply the MH and the PLE models to the data that resulted in the initial set of parameters along with the EUR results for the eight wells.

We then implemented the least squares optimization to obtain an optimized set of the forward models parameters. We used these optimized results and implemented the MCMC, which resulted in more accurate results of the parameters of the two models. We then implemented the Bayesian paradigm on the two models and compared the Bayesian results with the forward model results. Finally, we plotted the standard deviation of the two Bayesian models for each well (as seen in Fig. 6) which identifies the more accurate model rate for a given well.

We noticed that the MH model overestimates the EUR results compared with the Bayesian MH model. However, the PLE underestimated the EUR in three wells and overestimated reserves in the remaining five wells. We also noticed that the majority of the wells follow the Bayesian MH model, and we reached this conclusion from comparative graphs of the standard deviations of the two models. This indicates that the Bayesian MH model results are more accurate than the Bayesian PLE results. This is an interesting discovery and it would be interesting to see if this trend is true in a different region of the Eagle Ford, and even in different shale plays.

The next step in this study is to change the prior estimate. Instead of assuming that the prior estimate is unknown, we can begin to update this value using the different stages of information. This should show that the more information we have, the more accurate the Bayesian forecast will be. Using these results we might be able to show directly that uncertainty decreases over time, indicating that the more information available, the less the uncertainty. Such a process will also indicate whether there is a point beyond which no more information is needed in the prior estimations, and that having information on a certain (minimum) number of data points will lead to unchanged forecasts. We can assume that the model will be more accurate as more information is provided — however, we would like to prove this conjecture.

Conclusions

Specific to Well 41:

- The MH forward model underestimates the EUR
- The PLE forward model overestimates the EUR
- The uncertainty of the model decreases when the MH model is applied to the Bayesian paradigm
- The uncertainty of the model remains constant when the PLE model is applied to the Bayesian paradigm
- The EUR results for the Bayesian PLE and forward model PLE are inconclusive

Nomenclature

b = Arp's dimensionless hyperbolic constant
BOE = Barrels of Oil Equivalent (BOE)
BOE/D = Barrels of Oil Equivalent per Day (BOE/d)
D_i = Initial decline (1/day)
D_∞ = Decline parameter for the Power-Law Exponential DCA model (1/day)
D_{lim} = Limit below which D cannot decline (1/day)
EUR = Estimated Ultimate Recovery (BOE)
LSQ = Least Squares
MCMC = Markov Chain Monte Carlo

MH	= Modified Hyperbolic DCA model
n	= Time exponent parameter for the Power-Law Exponential DCA model
PLE	= Power Law Exponential DCA model
$P(\theta\|D)$	= Posterior
$P(\theta)$	= Prior
$P(D\|\theta)$	= Likelihood
q_i	= Initial flowrate (BOE/D)
$q(t)$	= Flowrate (BOE/D)
t	= Time (days)
x	= Observed data
θ	= Set of known parameters
μ	= Mean of parameters
σ	= Standard deviation

References

Al-Attar, Hazim Hassan, and Sulaiman Al-Zuhair: "A General Approach for Deliverability Calculations of Gas Wells", paper SPE 111380 presented at the 2008 SPE North Africa Technical Conference and Exhibition, Marrakech, Morrocco, March 12-14

Arps, J. J.: "Analysis of Decline Curves", *Trans. AIME* (1944), paper SPE 945228, vol **160**, issue 01, pp 228–247

Aster, Richard C., Borchers, Brian, and Thurber, Clifford H.: *Parameter Estimation and Inverse Problems* in Elsevier, Amsterdam, The Netherlands (2013)

Ayers, Walter B.: "PETE 612 - Unconventional Oil and Gas Reservoirs", *Course Notes*, Texas A&M University (2011)

Bahadori, Alireza, and Hari Vuthaluru: "A simple decline-curve analysis approach for evaluating gas reserves and predicting future production in gas reservoirs", paper SPE 139057 presented at the 2010 SPE Eastern Regional Meeting, Morgantown, West Virginia, USA, October 12-14

Bazan, Lucas W., Lattibeaudiere, Michael G., and Palisch, Terry: "Hydraulic Fracture Design and Well Production Results in the Eagle Ford Shale: One Operator's Perspective", paper SPE 155779 presented at the 2012 Americas Unconventional Resources Conference, Pittsburgh, Pennsylvania, USA, June 5-7

Benedict, Jon: "The Mathematics Of Decline Curves", *paper SPE 10537* (1981)

Benjamin, Jack R., and C. Allin Cornell: *Probability, Statistics, and Decision for Civil Engineers*, McGraw-Hill Book Company (1970)

Bhattacharya, Srimoyee, and Nikolaou, Michael: "Using Data From Existing Wells To Plan New Wells in Unconventional Gas Field Development", paper CSUG/SPE 147658 presented at the 2011 Canadian Unconventional Resources Conferences, Alberta, Canada, November 15-17

Breyer, John A., et al.: "Stratigraphy and Sedimentary Facies of the Eagle Ford Shale (Cretaceous) between the Maverick Basin and the San Marcos Arch, Texas, USA," Search and Discovery Article #50899 presented at the 2013 AAPG Annual Convention and Exhibition, Pittsburgh, Pennsylvania, USA, May 19-22

Chen, G., D. H. Tehrani, and J. M. Peden: "Calculation of Well Productivity in a Reservoir Simulator (II)", paper SPE 29932 presented at the 1995 International Meeting on Petroleum Engineering, Beijing, China, November 14-17

Corbett, Kevin: "Eagleford Shale Exploration Models: Depositional Controls on Reservoir Properties," Search and Discovery Article #10242 presented at the 2010 AAPG Annual Convention and Exhibition, Denver, Colorado, USA, June 7-10

Cuesta, J. F.: "Reservoir Parameter Calculations From D.S.T. Pressure Analysis," Paper No. 80-31-40 presented at the 1980 Petroleum Society of CIM, Calgary, May 25-28

Fazelipour, W.: "Development of Technicques to Integrate Hydraulic Fracturing Design and Reservoir Simulation Technologies - Application to Forecast Production of Stimulated Wells in Unconventional Gas Reservoirs", paper SPE 142337 presented at the 2011(a) SPE Middle East Unconventional Gas Conference and Exhibition, Muscat, Oman, January 31- February 2

Fazelipour, W.: "Innovative Reservoir Modeling and Simulation of Unconventional Shale Gas Reservoirs Powered by Microseismic Data", paper SPE 141877 presented at the 2011(b) SPE Middle East Unconventional Gas Conference and Exhibition, Muscat, Oman, January 31- February 2

Fetkovich, M. J.: "Decline Curve Analysis Using Type Curves", paper SPE 4629 presented at the 1973 SPE 48th Fall Meeting, Las Vegas, Nevada, USA, September 30-October 3

Gentry, Robert W.: "Decline-Curve Analysis", *paper SPE 3356, Journal of Petroleum Technology* (1972), vol **24**, issue 01, pp 38–41

Gong, Xinglai, et al.: "Assessment of Eagle Ford Shale Oil and Gas Resources," paper SPE 167241 presented in the 2013 SPE Unconventional Resources Conference Canada, Calgary, Alberta, Canada, November 5-7

Gonzalez, R., Gong, X. and McVay, D.A..: "Prior Information Enhances Uncertainty Quantification in Shale Gas Decline Curve Forecasts," paper SPE 167150 presented in the 2013 SPE Unconventional Resources Conference Canada, Calgary, Alberta, Canada, November 5-7

Gross, Henry Emmett: "Decline-curve Analysis", *Trans. AIME, SPE 939101* (1939), vol **132**, issue 01, pp 101–102

Guo, Boyun, and Xiance Yu: "A Simple and Accurate Mathematical Model for Predicting Productivity of Multifractured Horizontal Wells", paper SPE 114452 presented at the 2008 CIPC/SPE Gas Techonology Symposium Joint Conference, Calgeary, Alberta, Canada, June 16-19

Hao, Mingqiang, et al.: "Establishment and Application of the Multi-Peak Forecasting Model", paper IPTC 16897 presented at the 2013 International Petroleum Technology Conference, Beijing, China, March 26-28

Ilk, D., and T. A. Blasingame: "Decline Curve Analysis for Unconventional Reservoir Systems — Variable Pressure Drop Case", paper SPE 167253 presented at the 2013 SPE Unconventional Resources Conference-Canada, Calgary, Alberta, Canada, November 5-7

Ilk, Dilhan, Neal J. Broussard, and Thomas Alwin Blasingame: "Production Analysis in the Eagle Ford Shale — Best Practices for Diagnostic Interpretations, Analysis, and Modeling", paper SPE 160076 presented at the 2012 SPE Annula Technical Conference and Exhibition, San Antonio, Texas, USA, October 8-10

Ilk, Dilhan, et al.: "Hybrid Rate-Decline Models for the Analysis of Production Performance in Unconventional Reservoirs", paper SPE 135616 presented at the 2010 SPE Annual Technical Conference and Exhibition, Florence, Italy, September 19-22

Ilk, Dilhan, Creties David Jenkins, and Thomas Alwin Blasingame: "Production Analysis in Unconventional Reservoirs - Diagnostics, Challenges, and Methodologies", paper SPE 144376 presented at the 2011 North American Unconventional Gas Conference and Exhibition, the Woodlands, Texas, USA, June 14-16

Ilk, Dilhan, Jay Alan Rushing, and Thomas Alwin Blasingame: "Decline-Curve Analysis for HP/HT Gas Wells: Theory and Applications", paper SPE 125031 presented at the 2009 SPE Annual Technical Conference and Exhibition, New Orleans, Louisiana, USA, October 4-7

Ilk, Dilhan, Jay Alan Rushing, and Thomas Alwin Blasingame: "Integration of Production Analysis and Rate-Time Analysis via Parametric Correlations — Theoretical Considerations and Practical Applications", paper SPE 140556 presented at the 2011 SPE Hydraulic Fracturing Technology Conference, The Woodlands, Texas, USA, January 24-26

International Energy Agency. http://www.iea.org/ (2014)

Jensen, Jerry L., et al.: *Statistics for Petroleum Engineers and Geoscientists, Handbook of Petroleum Exploration and Production* in Elsevier, Amsterdam, The Netherlands (2000)

Johnson, Nathalie Louise, et al.: "A Simple Methodology for Direct Estimation of Gas-in-place and Reserves Using Rate-Time Data", paper SPE 123298 presented at the 2009 SPE Rocky Mountain Petroleum Technology Conference, Denver, Colorado, April 14-16

Jordan, C. L., M. J. Fetkovich, and L. R. Sibbald: "Simple, Accurate Gas Production Analysis for Forecasting and Reserves," paper PETSOC-2005-030 presented at the 2005 Canadian International Petroleum Conference, Calgary, Alberta, Canada, June 7-9

Klins, M. A., and M. B. Biterge: "A Semianalytical Approach to Pseudopressure Calculations", *paper SPE 15914, Journal of Petroleum Technology* (1987), vol **39**, issue 4, pp 468–472

Larsen, Leif: "Productivity Computations for Multilateral, Branched and Other Generalized and Extended Well Concepts", paper SPE 36754 presented at the 1996 SPE Annual Technical Conference & Exhibition, Denver, Colorado, USA, October 6-9

Ling, Kegang, and Jun He: "Theoretical Bases of Arps Empirical Decline Curves," paper SPE 161767 presented at the 2012 Abu Dhabi International Petroleum Conference and Exhibition, Abu Dhabi, UAE, November 11-14

Lolon, Elyezer, et al.: "New Semianalytical Solutions for Multilayer Reservoirs", paper SPE 114946 presented at the 2008 CIPC/SPE Gas Technology Symposium Joint Conference, Calgary, Alberta, Canada, June 16-19

Long, D.R., Davis, M.J.: "A New Approah to the Hyperbolic Curve", paper SPE 16237 presented at the 1987 SPE Proudction Operations Symposium, Oklahoma City, Oklahoma, USA, March 8-10

Mannon, Robert W.: "Oil Production Forecasting By Decline Curve Analysis," paper SPE 1254 presented in the 1965 Fall Meeting of the Society of Petroleum Engineers of AIME, Denver, Colorado, USA, October 3-6

Martin, S.O., Holditch, S.A., Ayers, W.B., McVay, D.A.: "PRISE Validates Resource Triangle Concept", *paper SPE 117703, SPE Economics & Management* (2010), vol **2**, issue 1, pp 51–60

Martinez, Wendy L., and Angel R. Martinez: *Computational Statistics Handbook with Matlab* in Chapman & Hall/CRC, Boca Raton, Florida, USA, (2008)

Mattar, Louis: "Production Analysis and Forecasting of Shale Gas Reservoirs: Case History-Based Approach", paper SPE 119897 presented in the 2008 SPE Shale Gas Production Conference, Fort Worth, Texas, USA, November 16-18
McClelland, Gary H.: "Increasing statistical power without increasing sample size", *American Psychologist* (2000), vol **55**, issue 8, pp 963–964
McNulty, Robert R., and Roy M. Knapp: "Statistical Decline Curve Analysis", paper SPE 10279 presented in the 1981 SPE Annual Technical Conference and Exhibition, San Antonio, Texas, USA, October 4-7
Mead, Homer N.: "Modifications to Decline Curve Analysis", *paper SPE 464, original copyright American Institute of Mining, Mettalurgical and Petroleum Engineering* (1956)
Medina-Cetina, Zenon: "Probabilistic Calibration of a Soil Model", Ph.D. Dissertation, Johns Hopkins University (2006)
Miller, Bradley M., and David L. Ranum: *Python Programming in Context* in Jones and Bartlett Publishers, Sudbury, MA, USA, (2009)
Mullen, J.: "Petrophysical Characterization of the Eagle Ford Shale in South Texas", paper CSUG/SPE 138145 presented at the 2010 Canadian Unconventional Resrources & International Petroleum Conference, Calgary, Alberta Canada, October 19-21
Okouma Mangha, Viannet, et al.: "Practical Considerations for Decline Curve Analysis in Unconventional Reservoirs - Application of Recently Developed Rate-Time Relations", paper SPE 162910 presented at the 2012 SPE Hydrocarbon Economics and Evaluation Symposium, Calgary, Alberta, Canada, September 24-25
Redden, Jim: "Eagle Ford Rig count down; operators cautiously bullish as permits, new wells soar", *World Oil Online* (2010), vol **233**, issue 7, pp 1–13
Richardson, J. G., and R. J. Blackwell: "Use of Simple Mathematical Models for Predicting Reservoir Behavior", *paper SPE 2928, Journal of Petroleum Technology* (1971), vol **23**, issue 9, pp 1145–1154
Schlumberger, *Oilfied Glossary. In* http://glossary.oilfield.slb.com/ (2014)
Seshadri, Jagan Nathan, and Louis Mattar: "Comparison of Power Law and Modified Hyperbolic Decline Methods", paper SPE 137320 presented at the 2010 Canadian Unconventional Resources and International Petroleum Conference, Calgary, Alberta, Canada, October 19-21
Solhjell, Ida K.: "Bayesian Forecasting and Dynamic Models Applied to Strain Data from the Gota River Bridge" *in* M.Sc. Thesis, Department of Mathematics, University of Oslo (2009)
Tarantola, Albert: *Inverse Problem Theory and Methods for Model Parameter Estimation*, in The Society for Industrial and Applied Mathematics, Philadelphia, Pennsylvania, USA (2005)
Tian, Yao, Walter Barton Ayers, and William D. McCain: "The Eagle Ford Shale Play, South Texas: Regional Variations in Fluid Types, Hydrocarbon Production and Reservoir Properties", paper IPTC 16808 presented at the 2013 International Petroleum Technology Conference, Beijing, China, March 26-28
Tucker, Delos R.: "Lower Cretaceous Geology, Northwestern Karnes County, Texas", *The American Association of Petroleum Geologists* (1968), vol **52**, issue 5, pp 820–851
U.S. Energy Information Administration, *Independent Statistics & Analysis* http://www.eia.gov/ (2013)
Wang, Lei, et al.: "A Semianalytical Solution for Multifractured Horizontal Wells in Box-Shaped Reservoirs," *Mathematical Problems in Engineering* (2014), vol **2014**, Article ID 716390, pp 12
West, Mike, and Jeff Harrison: *Bayesian Forecasting and Dynamic Models*, in Springer, New York, New York (1997)
Wikipedia: Least squares. http://en.wikipedia.org/wiki/Least_squares (2015)
Wikipedia: Markov chain Monte Carlo, http://en.wikipedia.org/wiki/Markov_chain_Monte_Carlo. (2015)
Xu, Bingxiang, et al.: "Production Data Analysis in Eagle Ford Shale Gas Reservoir", paper SPE 153072 presented at the 2012 SPE/EAGE European Unconventional Resources Conference and Exhibition, Vienna, Austria, March 20-22
Yao Tian, Walter B. Ayers, William D. McCain Jr.: "Regional Analysis of Stratigraphy, Reservoir Characteristics, and Fluid Phases in the Eagle Ford Shale, South Texas", *The Gulf Coast Association of Geological Societies* (2013), vol **62**, pp 471–483