PHYSICS SHEET SOLUTION

Rotational dynamics
Exercise JM

1. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of:-

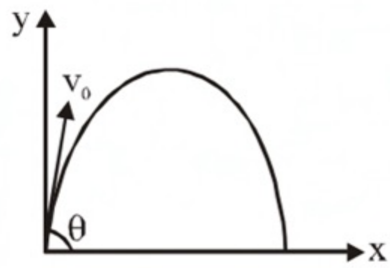
[AIEEE - 2009]

By WET A→B

$$-mgh = 0 - \frac{1}{2} I \omega^2$$

$$h = \frac{1}{6}\frac{l^2\omega^2}{g} \qquad \left(I = \frac{ml^2}{3}\right)$$

2. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is: Where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively. **[AIEEE-2010]**



(1) $\frac{1}{2} mg\, v_0\, t^2 \cos\theta\, \hat{i}$ (2) $-mg\, v_0\, t^2 \cos\theta\, \hat{j}$ (3) $mg\, v_0\, t \cos\theta\, \hat{k}$ (4*) $-\frac{1}{2} mg\, v_0\, t^2 \cos\theta\, \hat{k}$

Sol. $I\omega\, \frac{mgu\cos\theta t^2}{2}\left(-\hat{k}\right)$

$$\vec{V} = v_0 \cos\theta\, \hat{i} + (v_0 \sin\theta - gt)\hat{j}$$

$$\vec{r} = v_0 \cos\theta t\, \hat{i} + (v_0 \sin\theta t + \tfrac{1}{2} - g t^2)\hat{j}$$

$$\vec{L} = \vec{r} \times m\vec{v}$$

$$= -\tfrac{1}{2} m g v_0 t^2 \cos\theta\, \hat{k}$$

M2

Direction can be directly checked by RHRule $-\hat{k}$ is in D option only

3. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion it reversed, is :-

(1) more than 6 but less than 9
(2) more than 9 **[AIEEE-2011]**
(3) less than 3
(4) more than 3 but less than 6

Sol^n 4

$$\tau = Fr = I\alpha$$

$$\alpha = 4t - t^2$$

$$\omega = 2t^2 - t^3/3 = 0$$

$t = 6$ seconds

$$\theta = \int_0^6 \omega\, dt = \left. \frac{2t^3}{3} - \frac{t^4}{12} \right|_0^6$$

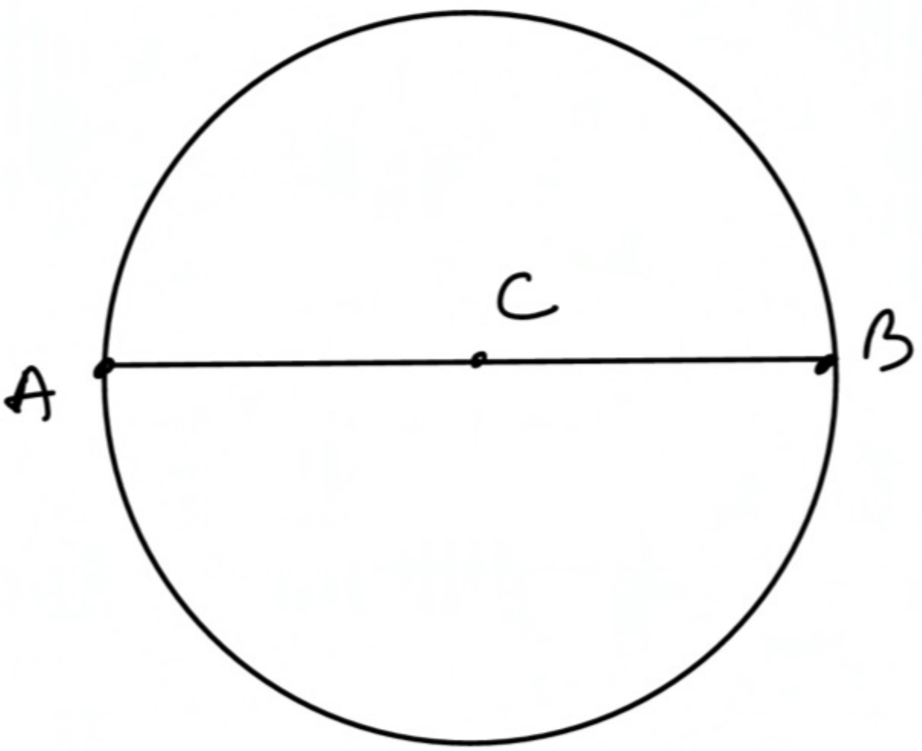
$= 36$ rad

$$\text{Rotations} = \frac{\theta}{2\pi} = \frac{36}{6.14} \approx 5.8$$

3 to 6

4. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. Euring the fjourney of the insect, then angular speed of the disc :-**[AIEEE-2011]**

(1) continuously increases
(2*) first increases and then decreases
(3) remains unchanged
(4) continuously decreses



$A \rightarrow C$ $\quad r$ decrease $\quad I \downarrow \quad \omega \uparrow$

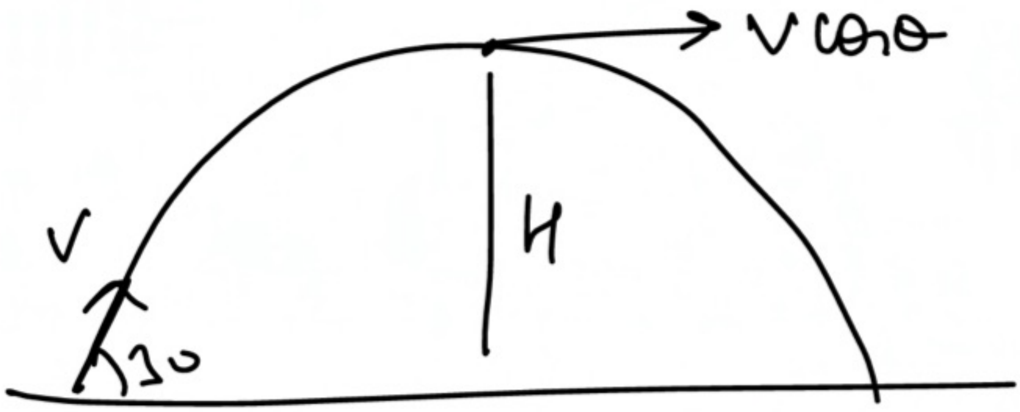
$C \rightarrow B$ $\quad r$ increase $\quad I \uparrow \quad \omega \downarrow$

5. A particle of mass 'm' is projected with a velocity v making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height 'h' is :- **[AIEEE-2011]**

क्षैतिज से 30° के कोण पर वेग v से द्रव्यमान 'm' के एक कण को प्रक्षेपित किया जाता है। जब कण अपनी अधिकतम ऊँचाई 'h' पर है, तब प्रक्षेप बिन्दु के सापेक्ष कण के कोणीय संवेग का परिमाण है :-

(1) $\frac{\sqrt{3}}{2}\frac{mv^2}{g}$ (2) zero (3) $\frac{mv^3}{\sqrt{2}g}$ (4*) $\frac{\sqrt{3}}{16}\frac{mv^3}{g}$

Ans. (4)



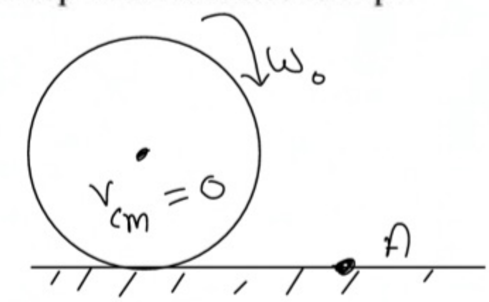
$$L = \vec{r} \times m\vec{v}$$

$$= H\, mv\cos\theta$$

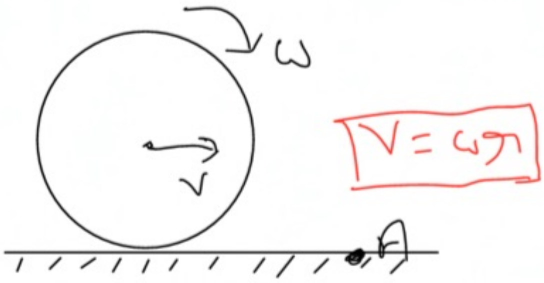
$$= \frac{u^2 \sin^2\theta}{2g} \cdot m v \cos\theta$$

$$= \frac{\sqrt{3}\, mv^3}{16g}$$

6. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip? **[JEE Mains-2013]**



When it is placed on rough surface



When it starts rolling

We can apply conservation of ang. momentum about any point on ground say point A.

$$L_i = L_f$$

$$(mr^2)\omega_0 = mvr + (mr^2)\left(\frac{v}{r}\right)$$

$$(\cancel{m}r^{\cancel{2}})\omega_0 = 2\cancel{m}v\cancel{r} \Rightarrow v = \frac{\omega_0 r}{2}$$

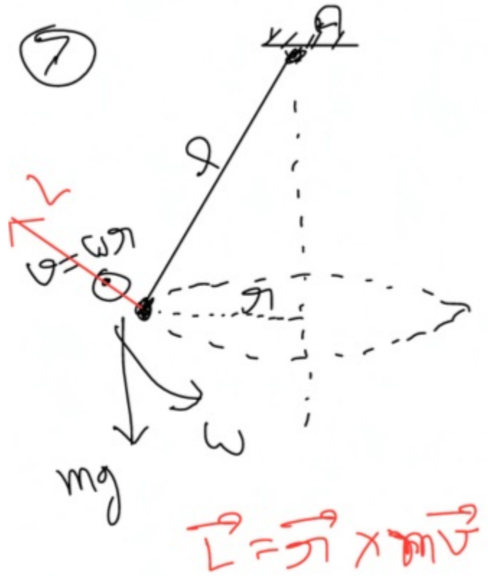
7. A bob of mass m attached to an inextensible string of length ℓ is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension : **[JEE Mains-2014]**

(1) Angular momentum changes in direction but not in magnitude

(2) Angular momentum changes both in direction and magnitude

(3) Angular momentum is conserved

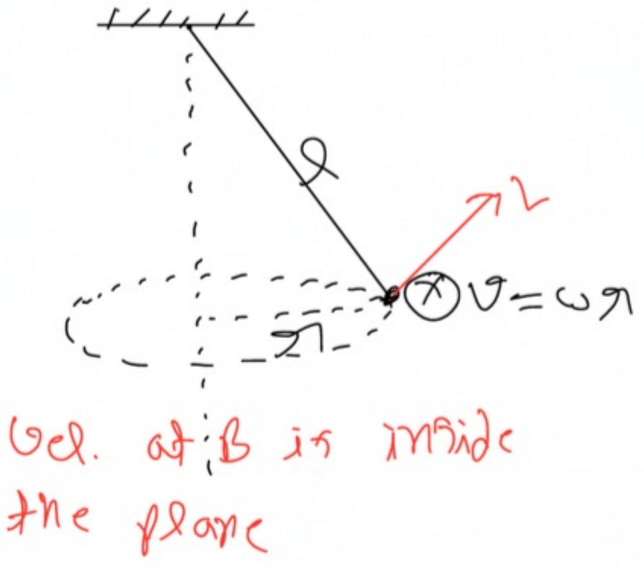
(4) Angular momentum changes in magnitude but not in direction.



As we can see torque of mg about point A is non-zero. so Ang. momentum will not be conserved

⇒ Now if we see the magnitude about A at the shown instant, that will be $L = mvr_{\perp} = mvl$ because vel. is along out of the plane.

Now if bob comes to point B:

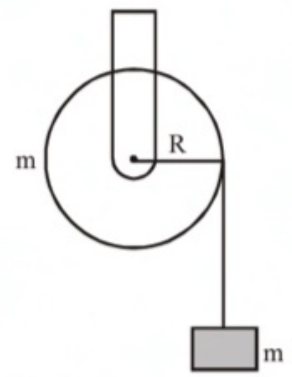


Ang. momentum's magnitude = $mv\ell$ but we can see dirn. has changed, which can be seen by right hand thumb rule.

Infact at every instant magnitude will be same but dirn. will change which can be seen by right hand thumb rule.

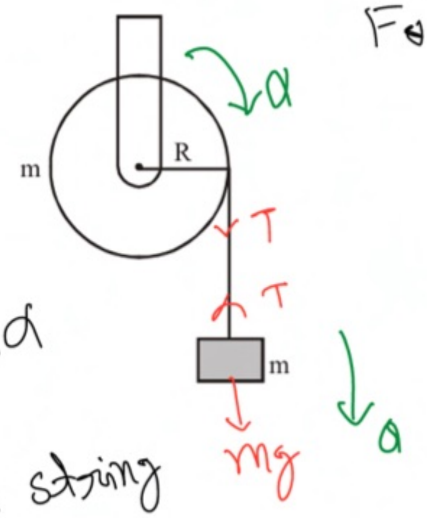
Ans(1)

8. A mass 'm' is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release? **[JEE Mains-2014]**



(1) $\frac{5g}{6}$ (2) g (3) $\frac{2g}{3}$ (4) $\frac{g}{2}$

(8)



For block:

$$mg - T = ma \quad —(1)$$

$$TR = mR^2 \frac{a}{R} \quad —(2)$$

from (1) & (2)

$$a = g/2$$

$a = R\alpha$

↓

because string doesn't slip over pulley.

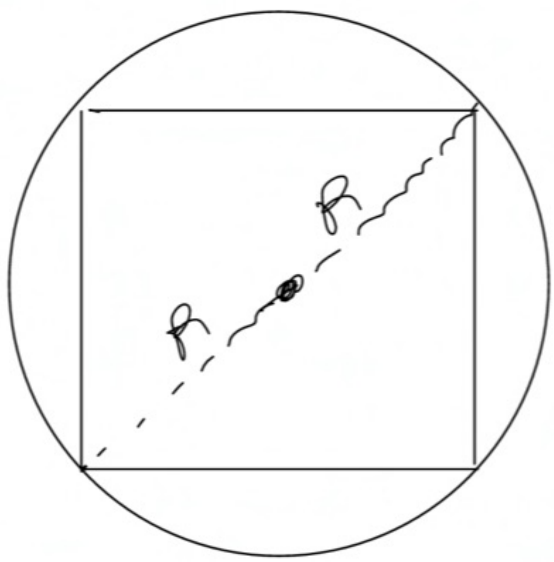
Ans. (4)

9. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is:-

[JEE Mains-2015]

(1) $\frac{4MR^2}{9\sqrt{3}\pi}$ (2) $\frac{4MR^2}{3\sqrt{3}\pi}$ (3) $\frac{MR^2}{32\sqrt{2}\pi}$ (4) $\frac{MR^2}{16\sqrt{2}\pi}$

(9)



Front view

Diagonal of cube will be equal to diameter of sphere.

Suppose side length of cube is a.

So $\sqrt{3}a = 2R$

$$a = \frac{2R}{\sqrt{3}}$$

And we know moment of inertia of cube about axis asked in ques. is given as:

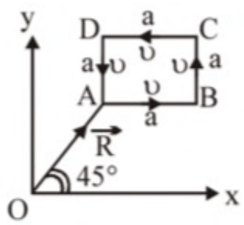
$$I = \frac{ma^2}{6} = \frac{(\rho \times a^3)\, a^2}{6} = \frac{\rho a^5}{6}$$

where $\rho = \frac{M}{\frac{4}{3}\pi R^3}$

So $I = \frac{\cancel{3}M}{4\pi R^3} \times \frac{a^5}{\cancel{6}_2} = \frac{M}{8\pi R^3}\left(\frac{2R}{\sqrt{3}}\right)^5$

$$I = \frac{M}{8\pi R^3} \times \frac{32 R^5}{9\sqrt{3}} = \frac{4}{9\pi\sqrt{3}} MR^2 \qquad \text{Ans (1)}$$

10. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed υ in the x-y plane as shown in the figure : **[JEE Mains-2016]**



Which of the following statement is false for the angular momentum $\vec{L}$ about the origin ?

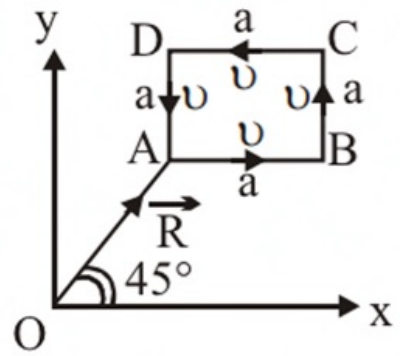
(1) $\vec{L} = \frac{m\upsilon}{\sqrt{2}} R\hat{k}$ when the particle is moving from D to A

(2) $\vec{L} = -\frac{m\upsilon}{\sqrt{2}} R\hat{k}$ when the particle is moving from A to B

(3) $\vec{L} = m\upsilon\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D

(4) $\vec{L} = m\upsilon\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C

(10)

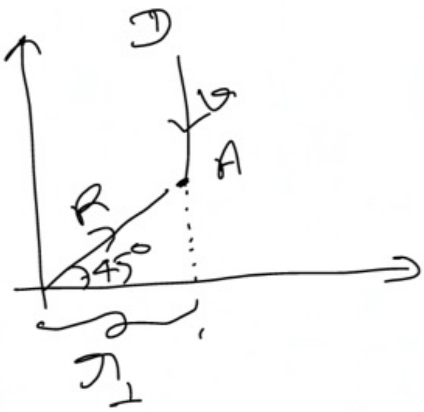


We have to select false statement.

Option:

(1) When particle is moving from D to A:

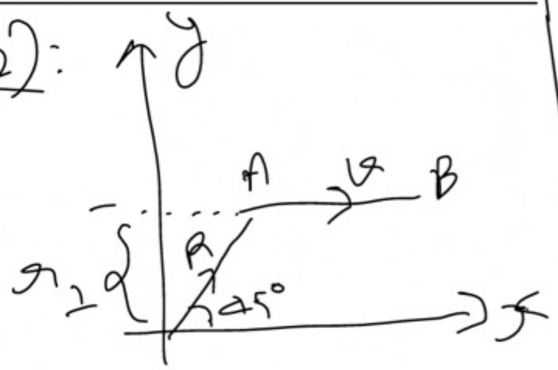
$$L = mvr_{\perp} = mv\frac{R}{\sqrt{2}}$$



and dirⁿ. along $(-\hat{k})$ from RHT rule

So option (1) false

Option (2):



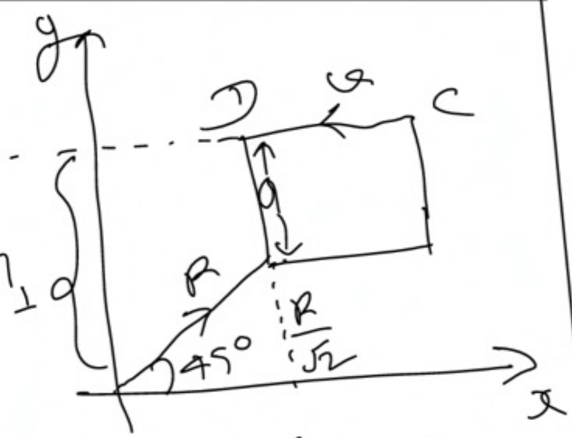
$$L = mvr_{\perp} = mv\frac{R}{\sqrt{2}}$$

dirⁿ. $(-\hat{k})$ using RHT rule

so option (2) correct

Option (3)

$$a + \frac{R}{\sqrt{2}} = r_{\perp}$$

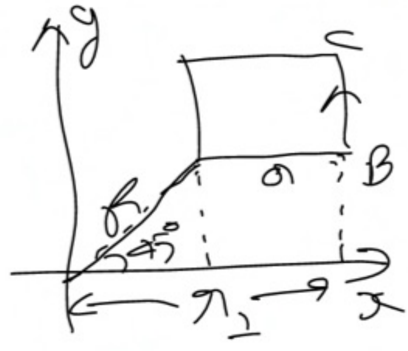


$$L = mvr_{\perp} = mv\left(\frac{R}{\sqrt{2}} + a\right)$$

dirⁿ. $(\hat{k})$

so option (3) false

Option (4)



$$L = mvr_{\perp} = mv\left(\frac{R}{\sqrt{2}} + a\right)$$

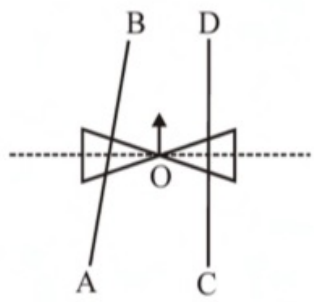
dirⁿ. $\hat{k}$ using RHT rule.

so option (4) correct

So option (1) and (3) both can be answers.

11. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to :-

[JEE Mains-2016]



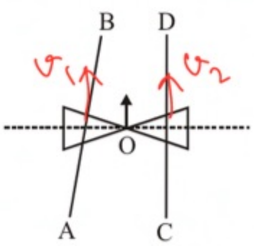
(1) turn left and right alternately.

(2) turen left.

(3) turn right.

(4) go straight.

(11)



Lets consider both cross sections where roller touches the rails AB and CD

Vel. of centre of cross-section which is on rail AB $V_1 = \omega r_1$

vel. of centre of cross-section which lies on rail CD $v_2 = \omega r_2$

As roller proceeds slightly r_1 will decrease as rail AB moves near to point O.

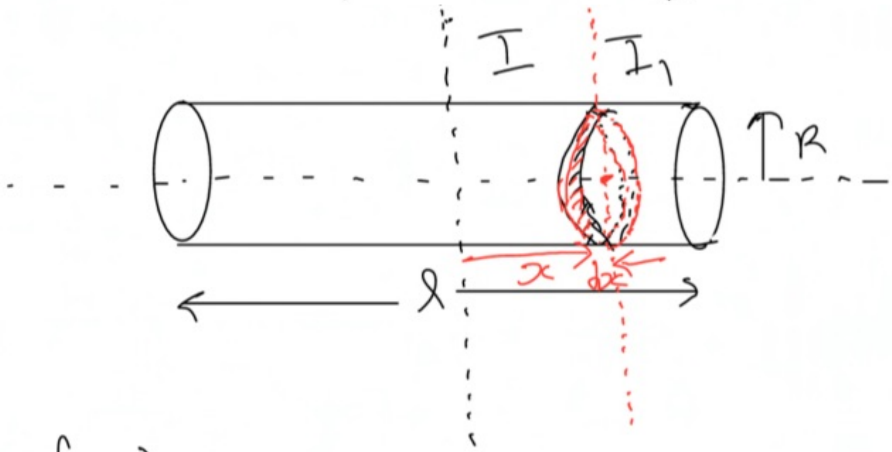
So $v_1 = (\omega r_1)$ will also decrease.

if v_1 becomes less than v_2, roller will turn towards left.

12. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio ℓ/R such that the moment of inertia is minimum ? **[JEE Main-2017]**

(1) 1 (2) $\frac{3}{\sqrt{2}}$ (3) $\sqrt{\frac{3}{2}}$ (4) $\frac{\sqrt{3}}{2}$

12



To find I lets consider an elemental ring as shown above

M.I. of this elemental ring can be given using parallel axis theorem

$$dI = dI_1 + dm\,x^2$$

$$\int dI = \int \frac{dm\,R^2}{4} + \int_{-l/2}^{l/2} \frac{m}{l} dx\, x^2$$

$$I = \frac{MR^2}{4} + \frac{Ml^2}{12} \quad \text{— (1)}$$

Now we have to minimize this I :

Here we have two variables R and l.

Total mass and volume will remain const.

$$V = \pi R^2 l = \text{const.} \quad \text{— (2)}$$

Now to minimize I, lets differentiate

it w.r.t. R (we can also differentiate I with respect to l)

$$\frac{dI}{dR} = \frac{M}{4}2R + \frac{M}{12} \times 2l \frac{dl}{dR} \quad (3)$$

Now differentiating eqn. (2)

$$\frac{dV}{dR} = 0 = \pi\left(R^2 \frac{dl}{dR} + l(2R)\right)$$

because $V =$ const.

$$\frac{dl}{dR} = \left(-\frac{2l}{R}\right)$$

Substituting this value in eqn. (3)

$$\frac{dI}{dR} = \frac{MR}{2} + \frac{Ml}{6}\left(\frac{-2l}{R}\right) = 0$$

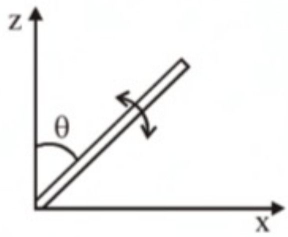
because we have to minimize I.

$$\frac{R}{2} = \frac{l^2}{3R}$$

$$\frac{l^2}{R^2} = \frac{3}{2} \Rightarrow l/R = \sqrt{3/2}$$

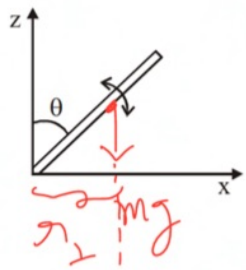
Ans (3)

13. A slender uniform rod of mass M and length ℓ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is : **[JEE Main-2017]**



(1) $\frac{3g}{2\ell}\cos\theta$ (2) $\frac{2g}{3\ell}\cos\theta$ (3) $\frac{3g}{2\ell}\sin\theta$ (4) $\frac{2g}{3\ell}\sin\theta$

13



$$\tau = r_\perp F = \left(\frac{l}{2}\sin\theta\right)mg$$

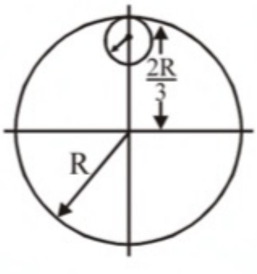
$$\tau = I\alpha$$

$$\tau = \frac{mgl}{2}\sin\theta = \frac{ml^2}{3}\alpha$$

$$\alpha = \left(\frac{3g}{2l}\right)\sin\theta$$

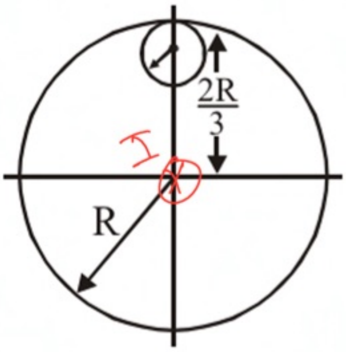
Ans. (3)

14. From a uniform circular disc of radius R and mass 9M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is : **[JEE Main-2018]**



(1) $\frac{40}{9}MR^2$ (2) $10\ MR^2$ (3) $\frac{37}{9}MR^2$ (4) $4\ MR^2$

(14)



$$I = (I \text{ of complete disc}) - (I \text{ of removed disc})$$

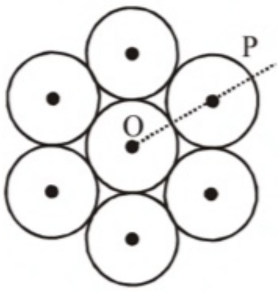
$$= \left(9M\frac{R^2}{2}\right) - \left(\frac{M\left(R/3\right)^2}{2} + M\left(2R/3\right)^2\right)$$

$$= \frac{9}{2}MR^2 - \left(MR^2\left(\frac{1}{18} + \frac{4}{9}\right)\right)$$

$$= 4MR^2$$

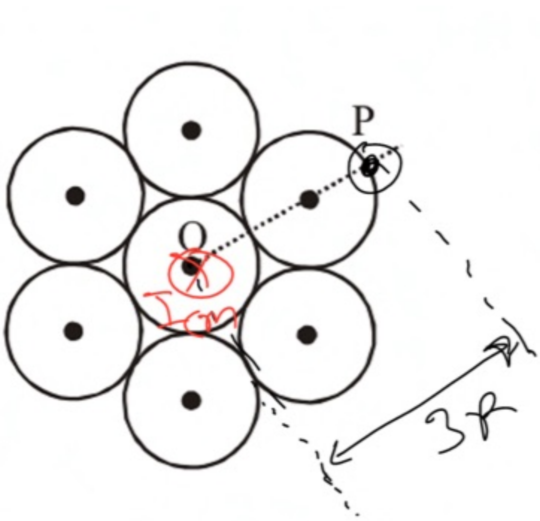
Ans. (4)

15. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is : **[JEE Main-2018]**



(1) $\frac{55}{2}MR^2$ (2) $\frac{73}{2}MR^2$ (3) $\frac{181}{2}MR^2$ (4) $\frac{19}{2}MR^2$

(15)



$$I_P = I_{cm} + \left(7M(3R)^2\right)$$

↑

parallel axis theorem for whole system.

$$I_{cm} = \frac{MR^2}{2} + 6\left(\frac{MR^2}{2} + M(2R)^2\right)$$

$$= \frac{55}{2} MR^2$$

So $I = \frac{55}{2} MR^2 + 63\, MR^2$

$$= \frac{181}{2} MR^2$$

Ans. (3)