

COL 351 : ANALYSIS & DESIGN OF ALGORITHMS

LECTURE 1

INTRODUCTION TO ALGORITHMS

JULY 23, 2024

|

ROHIT VAISH

COURSE WEBPAGE



WHAT IS AN ALGORITHM ?

WHAT IS AN ALGORITHM ?



WHAT IS AN ALGORITHM ?



Any piece of code ?

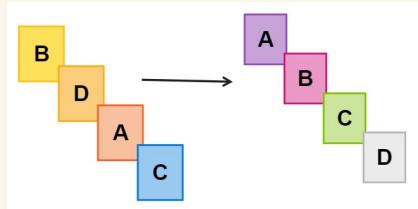
WHAT IS AN ALGORITHM ?



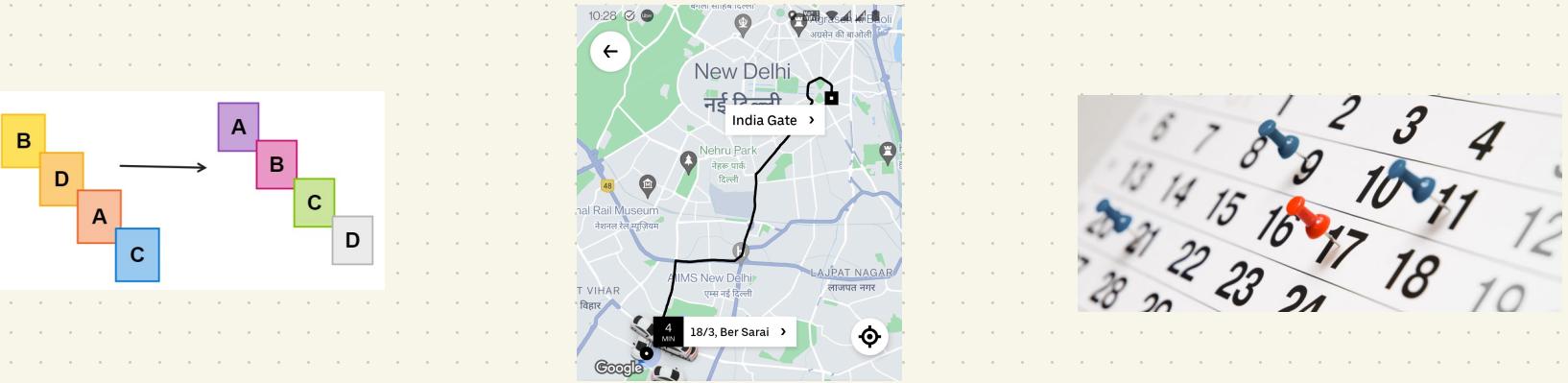
Any piece of code ?

- ♥ A recipe for solving some computational problem.

EXAMPLES



Sorting a list
of objects



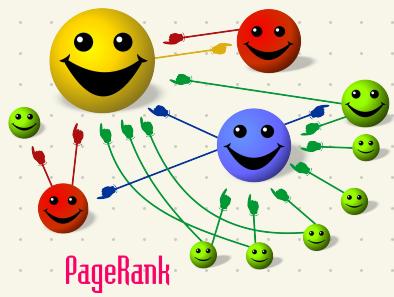
finding shortest path
between two places



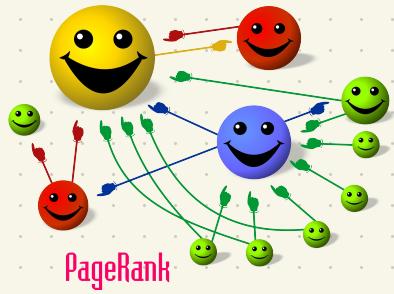
Completing tasks
with deadlines

KILLER APPS

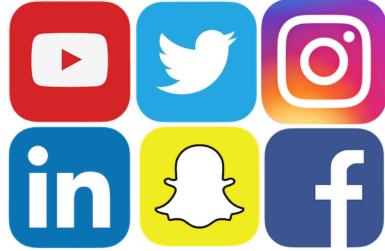
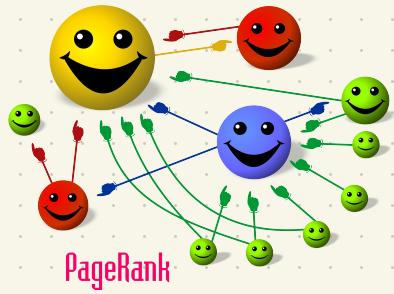
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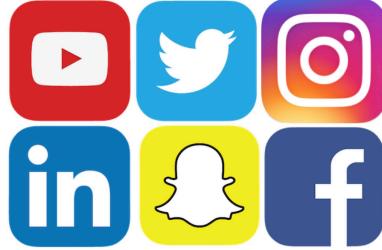
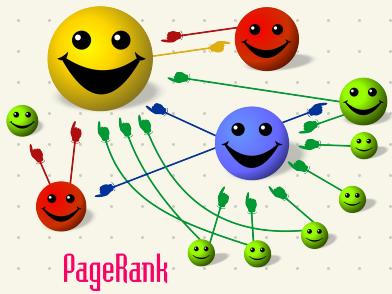


KILLER APPS



68	2,400	35,933	5,970	1,720	9,996	1,120
95	5,970	539,137	1,710	1,720	233,167	1,030
4,542	1,720	48,100	0,314	0,316	778,186	1,000
,900	0,314	833,789	1,180	1,190	68,000	0,781
0,781	1,190	10,000	0,332	0,338	158,294	0,4500
0,4500	0,332	10,000	0,460	0,479	350,000	0,4500

KILLER APPS



68	2,400								
95	5,970	35,933	5,970						
4,542	1,720	539,137	1,710	1,720	1,720	233,167			
1,900	0,314	48,100	0,314	0,316	0,316	778,186			
0,781	1,190	833,789	1,180	1,190	1,190	68,000			
14,500	0,332	10,000	0,332	0,338	0,338	158,294			



WHY SHOULD YOU STUDY ALGORITHMS ?

WHY SHOULD YOU STUDY ALGORITHMS ?

- * Important for all branches of computer science

Networks

shortest-path
algorithms

Cryptography

number-theoretic
algorithms

Graphics

geometric
algorithms

Databases

search tree
data structures

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- * A "lens" for non-CS areas

Physics

quantum computation

Economics

understanding "equilibria"

Biology

evolution as a search algorithm

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- * Challenging puzzles — you will feel smarter!

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- * A "lens" for non-CS areas

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quantum computation

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- * Challenging puzzles — you will feel smarter!

- * Fun !

CAKE - CUTTING

What is a fair way to divide
a cake between two people ?



CAKE - CUTTING

What is a fair way to divide
a cake between two people?



Person 1 cuts the cake into two parts they consider equal in value.

Person 2 chooses their favorite part and Person 1 gets the leftovers.

INTEGER ADDITION

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input: two n digit numbers x and y

output: the sum $x + y$

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input : two n digit numbers x and y

output : the sum $x + y$

e.g.,

5 6 7 8

1 2 3 4

INTEGER ADDITION

input: two n digit numbers x and y

output: the sum $x + y$

e.g.,

$$\begin{array}{r} 5 \ 6 \ 7 \ 8 \\ 1 \ 2 \ 3 \ 4 \\ \hline \end{array}$$

Grade - school addition

INTEGER ADDITION

input: two n digit numbers x and y

output: the sum $x + y$

e.g.,

$$\begin{array}{r} 5 \ 6 \ 7 \ 8 \\ 1 \ 2 \ 3 \ 4 \\ \hline 2 \end{array}$$

Grade - school addition

INTEGER ADDITION

input: two n digit numbers x and y

output: the sum $x + y$

e.g.,

$$\begin{array}{r} 5 \ 6 \ 7 \ 8 \\ + 1 \ 2 \ 3 \ 4 \\ \hline 1 \ 2 \end{array}$$

Grade - school addition

INTEGER ADDITION

input: two n digit numbers x and y

output: the sum $x + y$

e.g.,

$$\begin{array}{r} & 1 & 1 \\ 5 & 6 & 7 & 8 \\ + & 1 & 2 & 3 & 4 \\ \hline & 9 & 1 & 2 \end{array}$$

Grade - school addition

INTEGER ADDITION

input: two n digit numbers x and y

output: the sum $x + y$

e.g.,

$$\begin{array}{r} & 1 & 1 \\ 5 & 6 & 7 & 8 \\ + & 1 & 2 & 3 & 4 \\ \hline & 6 & 9 & 1 & 2 \end{array}$$

Grade - school addition

INTEGER ADDITION

input: two n digit numbers x and y

output: the sum $x + y$

e.g.,

$$\begin{array}{r} & 1 & 1 \\ & 5 & 6 & 7 & 8 \\ & 1 & 2 & 3 & 4 \\ \hline & 6 & 9 & 1 & 2 \end{array}$$

Grade - school addition

This algorithm is **correct**.

INTEGER ADDITION

input: two n digit numbers x and y

output: the sum $x + y$

e.g.,

$$\begin{array}{r} & 1 & 1 \\ & 5 & 6 & 7 & 8 \\ & 1 & 2 & 3 & 4 \\ \hline & 6 & 9 & 1 & 2 \end{array}$$

Grade - school addition

This algorithm is **correct**.

How much **time** does it take?

INTEGER ADDITION

input: two n digit numbers x and y

output: the sum $x + y$

e.g.,

$$\begin{array}{r} \begin{array}{cccc} & 1 & & 1 \\ 5 & 6 & 7 & 8 \\ & 1 & 2 & 3 & 4 \end{array} \\ \hline \begin{array}{cccc} 6 & 9 & 1 & 2 \end{array} \end{array}$$

basic operation
adding two single-digit numbers

INTEGER ADDITION

input: two n digit numbers x and y

output: the sum $x + y$

e.g.,

$$\begin{array}{cccc} & 5 & 6 & 7 \\ & | & | & | \\ 1 & 2 & 3 & 4 \\ \hline & 6 & 9 & 1 & 2 \end{array}$$

basic operation
adding two single-digit numbers

n single-digit addition operations

INTEGER ADDITION

input: two n digit numbers x and y

output: the sum $x + y$

e.g.,

$$\begin{array}{r} 5 \ 6 \ 7 \ 8 \\ 1 \ 2 \ 3 \ 4 \\ \hline 6 \ 9 \ 1 \ 2 \end{array}$$

basic operation
adding two single-digit numbers

n single-digit addition operations

$n-1$ carry operations (at most)

INTEGER ADDITION

input: two n digit numbers x and y

output: the sum $x + y$

e.g.,

$$\begin{array}{r} 5 \ 6 \ 7 \ 8 \\ 1 \ 2 \ 3 \ 4 \\ \hline 6 \ 9 \ 1 \ 2 \end{array}$$

basic operation
adding two single-digit numbers

n single-digit addition operations

$n-1$ carry operations (at most)
adding a 2-digit and a 1-digit number

INTEGER ADDITION

input: two n digit numbers x and y

output: the sum $x + y$

e.g.,

$$\begin{array}{r} 5 \ 6 \ 7 \ 8 \\ 1 \ 2 \ 3 \ 4 \\ \hline 6 \ 9 \ 1 \ 2 \end{array}$$

basic operation

adding two single-digit numbers

n single-digit addition operations

$\left\{ \begin{array}{l} n-1 \text{ carry operations (at most)} \\ \text{adding a 2-digit and a 1-digit number} \\ \leq 2(n-1) \text{ single-digit addition operations} \end{array} \right.$

INTEGER ADDITION

input: two n digit numbers x and y

output: the sum $x + y$

e.g.,

5	6	7	8
1	2	3	4
6	9	1	2

basic operation

adding two single-digit numbers

n single-digit addition operations

$\leq 3n$ operations overall

$\left\{ \begin{array}{l} n-1 \text{ carry operations (at most)} \\ \text{adding a 2-digit and a 1-digit number} \end{array} \right.$

$\leq 2(n-1)$ single-digit addition operations

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

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input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

$$\begin{array}{r} 5 \ 6 \ 7 \ 8 \\ \times \ 1 \ 2 \ 3 \ 4 \\ \hline \end{array}$$

Grade - school algorithm

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

$$\begin{array}{r} 5 \ 6 \ 7 \ 8 \\ 1 \ 2 \ 3 \ 4 \\ \hline 2 \ 2 \ 7 \ 1 \ 2 \end{array}$$

Grade - school algorithm

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

5 6 7 8

1 2 3 4

2 2 7 1 2

1 7 0 3 4

Grade - school algorithm

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

$$\begin{array}{r} 5 \ 6 \ 7 \ 8 \\ 1 \ 2 \ 3 \ 4 \\ \hline \end{array}$$

Grade - school algorithm

$$\begin{array}{r} 2 \ 2 \ 7 \ 1 \ 2 \\ | \ 7 \ 0 \ 3 \ 4 \\ 1 \ 1 \ 3 \ 5 \ 6 \end{array}$$

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

5 6 7 8
1 2 3 4

Grade - school algorithm

$$\begin{array}{r} 22712 \\ 17034 \\ 11356 \\ 5678 \end{array}$$

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

$$\begin{array}{r} 5 6 7 8 \\ 1 2 3 4 \end{array}$$

Grade - school algorithm

$$\begin{array}{r} 2 2 7 1 2 \\ | \quad | \quad | \quad | \quad | \end{array}$$

$$\begin{array}{r} 1 7 0 3 4 \\ | \quad | \quad | \quad | \quad | \end{array}$$

$$\begin{array}{r} 1 1 3 5 6 \\ | \quad | \quad | \quad | \quad | \end{array}$$

$$\begin{array}{r} 5 6 7 8 \\ | \quad | \quad | \quad | \quad | \end{array}$$

$$\begin{array}{r} 7 0 0 6 6 5 2 \\ | \quad | \quad | \quad | \quad | \quad | \quad | \end{array}$$

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

$$\begin{array}{r} 5 6 7 8 \\ 1 2 3 4 \end{array}$$

Grade - school algorithm

$$\begin{array}{r} 2 2 7 1 2 \\ 1 7 0 3 4 \\ 1 1 3 5 6 \\ 5 6 7 8 \end{array}$$

This algorithm is **correct**.

$$7 0 0 6 6 5 2$$

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

$$\begin{array}{r} 5 6 7 8 \\ 1 2 3 4 \end{array}$$

Grade - school algorithm

$$\begin{array}{r} 2 2 7 1 2 \\ 1 7 0 3 4 \\ 1 1 3 5 6 \\ 5 6 7 8 \end{array}$$

This algorithm is **correct**.

How much **time** does it take?

$$7 0 0 6 6 5 2$$

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

$$\begin{array}{r} 5678 \\ \times 1234 \\ \hline \end{array}$$

basic operations: add or multiply
two single-digit numbers

$$\begin{array}{r} 22712 \\ | \\ \end{array}$$

$$\begin{array}{r} 17034 \\ | \\ \end{array}$$

$$\begin{array}{r} 11356 \\ | \\ \end{array}$$

$$\begin{array}{r} 5678 \\ | \\ \end{array}$$

$$\begin{array}{r} 7006652 \\ | \\ \end{array}$$

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

$$\begin{array}{r} 5 \ 6 \ 7 \ 8 \\ \times \ 1 \ 2 \ 3 \ 4 \\ \hline \end{array}$$

basic operations: add or multiply
two single-digit numbers

$$\begin{array}{r} 2 \ 2 \ 7 \ 1 \ 2 \\ \times \ 1 \ 7 \ 0 \ 3 \ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \ 1 \ 3 \ 5 \ 6 \\ \times \ 5 \ 6 \ 7 \ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \ 0 \ 0 \ 6 \ 6 \ 5 \ 2 \\ \hline \end{array}$$

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

5 6 7 8

1 2 3 4

2 2 7 1 2

1 7 0 3 4

1 1 3 5 6

5 6 7 8

basic operations: add or multiply

two single-digit numbers

$\leq 3n$ operations

$\left. \begin{array}{l} n \text{ multiplications} \\ \leq 2n \text{ additions} \\ (\text{including carry}) \end{array} \right\}$

7 0 0 6 6 5 2

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

5 6 7 8

1 2 3 4

basic operations: add or multiply
two single-digit numbers

2 2 7 1 2

$\leq 3n$ operations

1 7 0 3 4

1 1 3 5 6

5 6 7 8

7 0 0 6 6 5 2

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

$$\begin{array}{r} 5 \ 6 \ 7 \ 8 \\ 1 \ 2 \ 3 \ 4 \\ \hline \end{array}$$

basic operations: add or multiply
two single-digit numbers

$$\begin{array}{r} 2 \ 2 \ 7 \ 1 \ 2 \end{array}$$

$\leq 3n$ operations

$$\begin{array}{r} 1 \ 7 \ 0 \ 3 \ 4 \end{array}$$

$$\begin{array}{r} 1 \ 1 \ 3 \ 5 \ 6 \end{array}$$

$$\begin{array}{r} 5 \ 6 \ 7 \ 8 \end{array}$$

$$\begin{array}{r} 7 \ 0 \ 0 \ 6 \ 6 \ 5 \ 2 \end{array}$$

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

5 6 7 8

1 2 3 4

2 2 7 1 2

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1 1 3 5 6

5 6 7 8

basic operations: add or multiply
two single-digit numbers

$\leq 3n$ operations

7 0 0 6 6 5 2

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

$$\begin{array}{r} 5 6 7 8 \\ 1 2 3 4 \end{array}$$

basic operations: add or multiply
two single-digit numbers

$$\begin{array}{r} 2 2 7 1 2 \\ | \quad | \quad | \quad | \quad | \end{array}$$

$\leq 3n$ operations per row

$$\begin{array}{r} 1 7 0 3 4 \\ | \quad | \quad | \quad | \quad | \end{array}$$

$$\begin{array}{r} 1 1 3 5 6 \\ | \quad | \quad | \quad | \quad | \end{array}$$

$$\begin{array}{r} 5 6 7 8 \\ | \quad | \quad | \quad | \quad | \end{array}$$

$$\begin{array}{r} 7 0 0 6 6 5 2 \\ | \quad | \quad | \quad | \quad | \quad | \quad | \end{array}$$

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

$$\begin{array}{r} 5 \ 6 \ 7 \ 8 \\ 1 \ 2 \ 3 \ 4 \end{array}$$

basic operations: add or multiply
two single-digit numbers

$$\begin{array}{r} 2 \ 2 \ 7 \ 1 \ 2 \\ | \ 7 \ 0 \ 3 \ 4 \\ 1 \ 1 \ 3 \ 5 \ 6 \\ 5 \ 6 \ 7 \ 8 \\ \hline 7 \ 0 \ 0 \ 6 \ 6 \ 5 \ 2 \end{array}$$

$\uparrow \leq 3n$ operations per row
 $\downarrow n$ rows

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

$$\begin{array}{r} 5 \ 6 \ 7 \ 8 \\ 1 \ 2 \ 3 \ 4 \\ \hline \end{array}$$

basic operations: add or multiply
two single-digit numbers

$$\begin{array}{r} 2 \ 2 \ 7 \ 1 \ 2 \\ | \ 7 \ 0 \ 3 \ 4 \\ 1 \ 1 \ 3 \ 5 \ 6 \\ 5 \ 6 \ 7 \ 8 \\ \hline \end{array}$$

$\uparrow \leq 3n$ operations per row
 $\downarrow n$ rows

7 0 0 6 6 5 2

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

$$\begin{array}{r} 5 \ 6 \ 7 \ 8 \\ 1 \ 2 \ 3 \ 4 \\ \hline \end{array}$$

basic operations: add or multiply
two single-digit numbers

$$\begin{array}{r} 2 \ 2 \ 7 \ 1 \ 2 \\ | \ 7 \ 0 \ 3 \ 4 \\ 1 \ 1 \ 3 \ 5 \ 6 \\ 5 \ 6 \ 7 \ 8 \\ \hline \end{array}$$

$\uparrow \leq 3n$ operations per row
 $\downarrow n$ rows

$$7 \ 0 \ 0 \ 6 \ 6 \ 5 \ 2$$

$\leq 6n^2$ operations (Exercise)

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

$$\begin{array}{r} 5 \ 6 \ 7 \ 8 \\ 1 \ 2 \ 3 \ 4 \\ \hline \end{array}$$

basic operations: add or multiply
two single-digit numbers

$$\begin{array}{r} 2 \ 2 \ 7 \ 1 \ 2 \\ | \ 7 \ 0 \ 3 \ 4 \\ 1 \ 1 \ 3 \ 5 \ 6 \\ 5 \ 6 \ 7 \ 8 \\ \hline \end{array}$$

$\uparrow \leq 3n$ operations per row
 $\downarrow n$ rows

$\leq 9n^2$ operations overall

$\leq 6n^2$ operations (Exercise)

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

5 6 7 8

1 2 3 4

2 2 7 1 2

1 7 0 3 4

1 1 3 5 6

5 6 7 8

High-level message

operations $\leq \underbrace{\text{constant} \cdot n^2}_{\leq 9}$

7 0 0 6 6 5 2

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

5 6 7 8

1 2 3 4

2 2 7 1 2

1 7 0 3 4

1 1 3 5 6

5 6 7 8

High-level message

operations $\leq \underbrace{\text{constant} \cdot n^2}_{\leq 9}$

double the input size

\Rightarrow no. of operations quadruples
(at most)

7 0 0 6 6 5 2

INTEGER MULTIPLICATION

input: two n digit numbers x and y

output: the product $x \cdot y$

e.g.,

5 6 7 8

1 2 3 4

2 2 7 1 2

1 7 0 3 4

1 1 3 5 6

5 6 7 8

High-level message

operations $\leq \underbrace{\text{constant} \cdot n^2}_{\leq 9}$

Can we do better?

7 0 0 6 6 5 2

The algorithm design space is surprisingly rich !

KARATSUBA MULTIPLICATION

KARATSUBA MULTIPLICATION

$x = 5 \ 6 \ 7 \ 8$

$y = 1 \ 2 \ 3 \ 4$

KARATSUBA MULTIPLICATION

$$x = \begin{matrix} a & b \\ 5 & 6 \\ 7 & 8 \end{matrix}$$
$$y = \begin{matrix} c & d \\ 1 & 2 \\ 3 & 4 \end{matrix}$$

KARATSUBA MULTIPLICATION

①

Compute $a \cdot c = 56 \times 12 = 672$

$$\begin{array}{cc} a & b \\ 5 & 6 \\ 7 & 8 \\ \hline c & d \\ 1 & 2 \\ 3 & 4 \\ \hline \end{array}$$
$$x = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$
$$y = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

KARATSUBA MULTIPLICATION

①

Compute $a \cdot c = 56 \times 12 = 672$

$$a = \begin{matrix} 5 & 6 \\ 7 & 8 \end{matrix}$$
$$b = \begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix}$$
$$x = \begin{matrix} 5 & 6 \\ 1 & 2 \end{matrix}$$
$$y = \begin{matrix} 7 & 8 \\ 3 & 4 \end{matrix}$$
$$c = \begin{matrix} 5 & 6 \\ 7 & 8 \end{matrix}$$
$$d = \begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix}$$

②

compute $b \cdot d = 78 \times 34 = 2652$

KARATSUBA MULTIPLICATION

①

Compute $a \cdot c = 56 \times 12 = 672$

$$x = \begin{matrix} a \\ 5 & 6 \end{matrix} \quad \begin{matrix} b \\ 7 & 8 \end{matrix}$$
$$y = \begin{matrix} c \\ 1 & 2 \end{matrix} \quad \begin{matrix} d \\ 3 & 4 \end{matrix}$$

②

compute $b \cdot d = 78 \times 34 = 2652$

③

compute $(a+b) \cdot (c+d) = 134 \times 46 = 6164$

KARATSUBA MULTIPLICATION

①

Compute $a \cdot c = 56 \times 12 = 672$

$$x = \begin{matrix} a \\ 5 & 6 \end{matrix} \quad \begin{matrix} b \\ 7 & 8 \end{matrix}$$
$$y = \begin{matrix} c \\ 1 & 2 \end{matrix} \quad \begin{matrix} d \\ 3 & 4 \end{matrix}$$

②

compute $b \cdot d = 78 \times 34 = 2652$

③

compute $(a+b) \cdot (c+d) = 134 \times 46 = 6164$

④

compute $③ - ② - ① = 2840$

KARATSUBA MULTIPLICATION

①

Compute $a \cdot c = 56 \times 12 = 672$

$$a = \begin{matrix} 5 & 6 \\ 1 & 2 \end{matrix}$$
$$b = \begin{matrix} 7 & 8 \\ 3 & 4 \end{matrix}$$

②

compute $b \cdot d = 78 \times 34 = 2652$

③

compute $(a+b) \cdot (c+d) = 134 \times 46 = 6164$

④

compute $③ - ② - ① = 2840$

⑤

Compute

$$\begin{array}{r} 6720000 \\ 2652 \\ \hline 284000 \end{array}$$

from ①

from ②

from ④

KARATSUBA MULTIPLICATION

①

Compute $a \cdot c = 56 \times 12 = 672$

$$x = \begin{matrix} a \\ 5 & 6 \end{matrix} \quad y = \begin{matrix} b \\ 7 & 8 \\ 1 & 2 \\ 3 & 4 \\ c & d \end{matrix}$$

②

compute $b \cdot d = 78 \times 34 = 2652$

③

compute $(a+b) \cdot (c+d) = 134 \times 46 = 6164$

④

compute $③ - ② - ① = 2840$

⑤

Compute

$$\begin{array}{r} 6720000 \\ 2652 \\ \hline 284000 \end{array} \quad \begin{array}{l} \text{from } ① \\ \text{from } ② \\ \text{from } ④ \end{array}$$

$$\hline 7006652$$

KARATSUBA MULTIPLICATION

①

Compute $a \cdot c = 56 \times 12 = 672$

$$x = \begin{matrix} a \\ 5 & 6 \end{matrix} \quad y = \begin{matrix} b \\ 7 & 8 \\ 1 & 2 \\ 3 & 4 \\ c & d \end{matrix}$$

②

compute $b \cdot d = 78 \times 34 = 2652$

③

compute $(a+b) \cdot (c+d) = 134 \times 46 = 6164$

④

compute $③ - ② - ① = 2840$

⑤

Compute $672 \ 0000$ from ①

$$2652 \quad \text{from } ②$$

$$2840 \ 00 \quad \text{from } ④$$

$$\begin{array}{r} 672 \ 0000 \\ 2652 \\ \hline 7006652 = x \cdot y \end{array}$$

KARATSUBA MULTIPLICATION

①

$$\text{Compute } a \cdot c = 56 \times 12 = 672$$

$$x = \begin{matrix} a \\ 5 & 6 \end{matrix} \quad y = \begin{matrix} b \\ 7 & 8 \\ 1 & 2 \\ 3 & 4 \end{matrix}$$

$$c = \begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix} \quad d$$

②

$$\text{compute } b \cdot d = 78 \times 34 = 2652$$

③

$$\text{compute } (a+b) \cdot (c+d) = 134 \times 46 = 6164$$

④

$$\text{compute } ③ - ② - ① = 2840$$

⑤

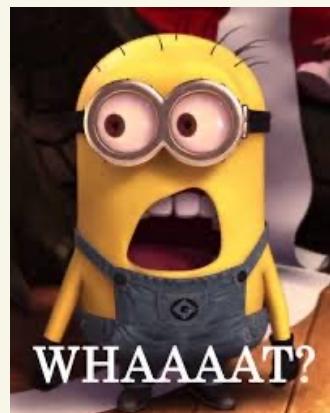
$$\text{Compute } \begin{array}{r} 672 \\ 2652 \\ \hline 284000 \end{array}$$

from ①

from ②

from ④

$$\hline 7006652 = x \cdot y$$



Designing fast algorithms

Divide-and-conquer

Graph algorithms

Greedy algorithms

Dynamic programming

Network flow

Linear programming

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Proving that a fast algorithm
probably doesn't exist

NP-completeness

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- Divide-and-Conquer
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Proving that a fast algorithm
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NP-completeness

Coping with NP-completeness

- Approximation
- Randomization
- Parametrization

COURSE LOGISTICS

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INSTRUCTOR : ROHIT VAISH (Please call me Rohit)
→ Lectures on Tue/Wed/Fri 10-11 AM LH 121

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Check your tutorial group on the website (by July 27)

EVALUATION

~2 hrs MINOR 20%

~2 hrs MAJOR 20%

~45 mins QUIZZES 36% [12% × best 3 out of 4]

~10 mins TUTORIALS 24% [3% × best 8 out of 12]

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AUDITS Not allowed

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~ 45 mins QUIZZES 36% [12% × best 3 out of 4]

~ 10 mins TUTORIALS 24% [3% × best 8 out of 12]

AUDITS Not allowed

ATTENDANCE Not monitored by me (but might be required
by the institute)

↓
based on above-mentioned evaluations

RE-EVALUATION

RE-EVALUATION

* Missed QUIZ :

in-class / tutorial

* Missed MINOR :

and/or MAJOR

* RE-MAJOR :

with E-grade

RE-EVALUATION

* Missed QUIZ : No re-quiz

in-class / tutorial

* Missed MINOR :

and/or MAJOR

* RE-MAJOR :

with E-grade

RE-EVALUATION

- * Missed QUIZ : No re-quiz
in-class / tutorial
- * Missed MINOR : A Common make-up exam (20%)
and/or MAJOR : Prior approval before the date of exam(s)
and $\geq 75\%$ attendance
- * RE-MAJOR :
with E-grade

RE-EVALUATION

- * Missed QUIZ : No re-quiz
in-class / tutorial
- * Missed MINOR : A Common make-up exam (20%)
and/or MAJOR : Prior approval before the date of exam(s)
and $\geq 75\%$ attendance
- * RE-MAJOR : $\geq 75\%$ attendance
with E-grade

TUTORIALS

- * Once (almost) every week

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- * First ~10 mins: Quiz

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- * First ~10 mins: Quiz 1 problem from weekly problem set

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Announced by TA at the
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- * Each tutorial submission is Worth 3% (best eight).

TUTORIALS

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- * First ~10 mins: Quiz 1 problem from weekly problem set
 - Announced by TA at the start of the tutorial
- * Remaining time: Discussion (Ask questions!)
- * Each tutorial submission is Worth 3% (best eight).
- * Problem not asked in quiz \neq Not relevant

REGRADING

- * Gradescope : Sign up **only** with @ dept.iitd, **not** @ iitd
- * "Frivolous" requests : -2% in overall score for first one
Subsequently -4%, -8%, -16%, ...

If in doubt, talk to your group's TA.

PLAGIARISM

Any kind, at any stage : F grade

If in doubt, talk to your TA or me.

NO GARBAGE

(credit : Allan Borodin, Nisarg Shah)

NO GARBAGE

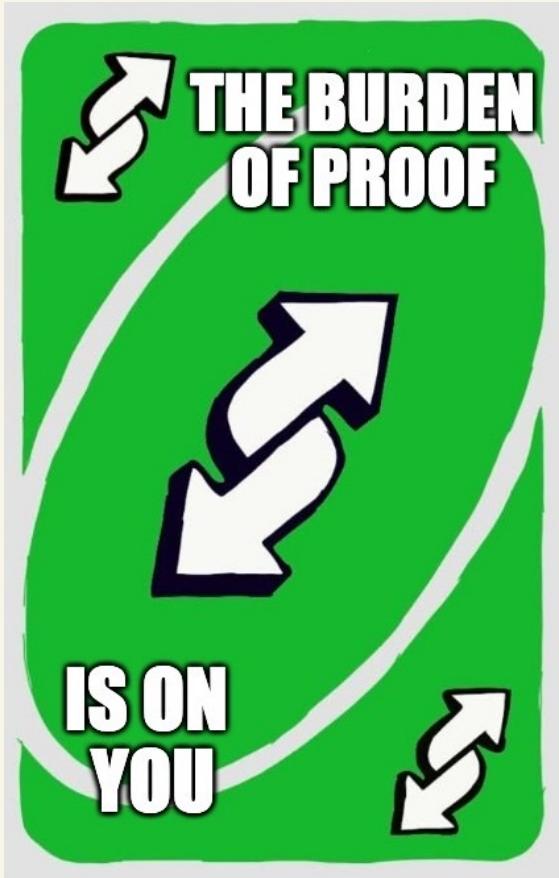
Partial marks for promising approaches [can be extended to give correct answer]

Zero marks if answer makes no sense or is hard to read

20% marks for "I do not know how to approach this problem."

NOTE : $20 > 0$

(credit : Allan Borodin, Nisarg Shah)



COMMUNICATION

- * Most important!
- * Please reach out to me or TAs about ANY issues at any time. Do not wait until the majors.
- * We want you to enjoy learning about algorithms and do well in the course.

All the best!