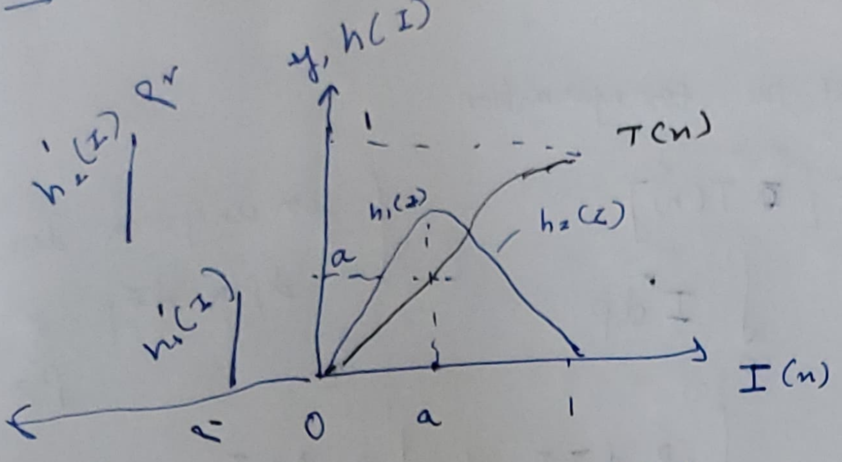
Q3 a



Total mass = 1.

histogram after transformation

In $[0, a)$ $\int_0^a h(I)\,dI = \alpha$

$[a, 1]$ $\int_a^1 h(I)\,dI = 1-\alpha$

Let $h_1'(I) = p_1$, $h_2'(I) = p_2$

Mass conservation (since masses are conserved after transformation independently in both $[0,a]$ and $(a,1]$)

$$\int_0^a p_1\,dI = \int_0^a h(I)\,dI = \alpha$$

$$p_1 a = \alpha \Rightarrow p_1 = \alpha/a$$

Similarly

$$\int_a^1 p_2\,dI = \int_a^1 h(I)\,dI$$

$$p_2(1-a) = (1-\alpha) \Rightarrow p_2 = \frac{1-\alpha}{1-a}$$

Mean intesity past the tranformation

$$= E[T(x)]$$

$$= \int_0^1 I\,dp \qquad \left(\text{for uniform dist. } dp = dI\,p \text{, where } p = h(I) \right)$$

$$= \int_0^a p_1\, dI\, I + \int_a^1 p_2\, dI\, I$$

$$= p_1 \frac{a^2}{2} + \frac{p_2}{2}(1 - a^2)$$

$$= \frac{\alpha}{\cancel{\alpha}} \frac{a^2}{2} + \frac{1}{2} \frac{1-\alpha}{\cancel{1-a}} \cancel{(1-a)}(1+a)$$

$$= \frac{a\alpha}{2} + \frac{(1-\alpha)(1+a)}{2} \qquad - (1)$$

(b) when a is the median intensity,

$\alpha = 1/2$ (by definition)

Mean intensity post the transformation is given by (1)

$$\Rightarrow \bar{I} = \frac{a}{4} + \frac{1}{4}(1+a)$$

$$= \frac{1}{4} + \frac{a}{2}$$

(c) Having independent transformation on pixel values $\leq$ median intensity (a) and pixel values $>$ median intensity (a):

Performing equalization helps increase the contrast of an image by taking the current range of intensities in the image and distributing them uniformly. Consider an grayscale image which has a dark ~~bg~~ background (black) ~ 50% of the pixels with a fore ground object;

This is a common scenario for pictures for eg. picture of the statue (Statue.png) or picture of the moon.

Now if we perform ~~histogram equali~~ median intensity based transformation on this image, median intensity a will be very small and thus

$p_1 = \frac{1}{2a}$ will be large, $p_2 = \frac{1}{2(1-a)}$ will be small.

In this case the intensity values of the foreground will be distributed to a larger range and thus will have better increase in contrast

(p_2 is small, mass is half in both regions, thus greater uniform spread)

while the pixels in the background will not be distributed over a significant range (we don't care about increasing contrast in background region)

If we performed normal Histogram equalization, the dark bg pixels would have covered ~ half of the intensity range and increase in contast of foreground would not be as effective.

3d Transformation function, using conservation of mass

Let $y = T(x)$

$$T(x) = \begin{cases} h(y)dy = h(x)dx & \\ CDF_y(y) = CDF_x(x) & 0 < x \le a \\ p_1 y = CDF_x(x) & \\ y = \dfrac{CDF_x(x)}{p_1} & \\ \hline h(y) = h(x)dx & \\ CDF_y(y) = CDF_x(x) & 1 > x > a \\ p_2(y-a) + \alpha = CDF_x(x) & \\ y = \dfrac{CDF_x(x) - \alpha}{p_2} + a & \end{cases}$$

$\Rightarrow$ Implemented in 3/code