DFT of the Laplacian

$$F_d(u,v) = \sum_{x=-N/2}^{N/2} \sum_{y=-N/2}^{N/2} f(x,y) \exp\left(-2\pi j \frac{(ux+vy)}{N}\right)$$

$$K_1(x,y) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

N, N point DFT for K_1:

$$F(u,v) = -4 \exp(0) + \exp\left(-2\pi j \left(\frac{u}{N}\right)\right) + \exp\left(-2\pi j \frac{v}{N}\right)$$

$$+ \exp\left(+2\pi j \left(\frac{v}{N}\right) + \exp\left(+2\pi j \frac{v}{N}\right)\right.$$

$$= -4 + \exp\left(-2\pi j \frac{u}{N}\right) + \exp\left(-2\pi j \frac{v}{N}\right) + \exp\left(+2\pi j \frac{v}{N}\right) + \exp\left(\frac{2\pi u}{N}\right)$$

$$K_2(x,y) = \begin{pmatrix} -1 & -1 & -1 \\ -1 & 8 & 1 \\ -1 & -1 & -1 \end{pmatrix}$$

N, N point DFT for K_2:

$$F(u,v) = 8 - \exp\left(\frac{-2\pi j \cdot u}{N}\right) - \exp\left(+\frac{2\pi j u}{N}\right)$$

$$- \exp\left(\frac{-2\pi j v}{N}\right) - \exp\left(\frac{+2\pi v j}{N}\right)$$

$$- \exp\left(\frac{-2\pi j (u+v)}{N}\right) - \exp\left(\frac{+2\pi j (u+v)}{N}\right)$$

$$- \exp\left(\frac{-2\pi j (u-v)}{N}\right) - \exp\left(\frac{+2\pi j (u-v)}{N}\right)$$