For the 1-D case we have given

$$g = h * f$$

Taking fourier transform on both sides and applying convolution theorem, we get -

$$G(u) = H(u)F(u) \text{ where}$$
$$G = \mathcal{F}(g), H = \mathcal{F}(h), F = \mathcal{F}(f)$$

Now since g and h are given, we know G and H as well. Thus F can be computed using G and H, and f can be obtained by IDFT operation over F. i.e. -

$$f = IDFT(G/H)$$

However, since h is a gradient matrix, H would be a high pass filter. Hence, for very small u, $H(u)$ would take very small values. Now, any error/noise in g would propagate as noise/error in G. For small u and hence small $H(u)$, $G(u)/H(u)$ blows up the noise/error in $F(u)$ and consequently in $f(x)$. Although, for large u this is not an issue, for smaller u this is a serious concern.

For the 2-D case we have -

$$g_x = h_x * f$$
$$g_y = h_y * f$$
$$\text{or}$$
$$G_x(u, v) = H_x(u, v)F(u, v) \tag{1}$$
$$G_y(u, v) = H_y(u, v)F(u, v) \tag{2}$$

Now, here, similar to the issue faced above consider the following 4 cases -

(1) **u and v both large** In this case either of 1 or 2 can be used to compute F even if the corresponding $G's$ are noisy because in both the cases the corresponding $H's$ would be large (H being a high pass filter as before)
(2) **u is large but v is small** If v is small then $H_y(u, v)$ will be small and we won't be able to use 2 as earlier. However, $H_x(u, v)$ still being large, we can still use 1 to compute F without making the error blow up
(3) **u is small but v is large** If u is small then $H_x(u, v)$ will be small and we won't be able to use 1 as earlier. However, $H_y(u, v)$ still being large, we can still use 2 to compute F without making the error blow up
(4) **u and v both are small** This is the case where both $H_y(u, v)$ and $H_x(u, v)$ would be small and neither of 1 or 2 can be used to compute $F(u, v)$ without making the error in G_x or G_y blow up.

Thus, for both the tasks, it is difficult to obtain the exact same image f back if the corresponding f and g are given, assuming non-ideal noisy conditions.