Ans ⑤ Verification of Predicted Translation

Please note that in the ~~graph~~ images ~~are~~ shown in the report, the X coordinate runs from top to bottom and the Y coordinate runs from left to right. In both noiseless and noisy images, when we took the inverse DFT of the cross-spectrum image, we observed that its maxima occured at $x_0 = 271, y_0 = 71$. However, in the case of noisy image, the fourier ~~a~~ inverse of cross-spectrum is not exactly an impulse, as expected in the noiseless translation case, hence we cannot be ~~sure~~ sure whether this formula / method works correctly for noisy images.

$x_0 = 271, y_0 = 71$ correctly corresponds to $t_x = -30$, $t_y = 70$ due to 1-indexing of matlab and since fourier transform assumes periodic signals. Hence, this method gives correct results.

Time Complexity for $N \times N$ image.

For 2D DFT and 2D-IDFT $\rightarrow O(N^2 \log N)$.

For calculation of cross-spectrum $\rightarrow O(N^2)$.

$\Rightarrow$ Overall complexity $= O(N^2 \log N)$

For naive method, we try different values of translation (N^2 in total) and for each of these value try to match the two images after taking their correlation which requires $O(N^2)$ operations per translation value.

$\Rightarrow$ Complexity of Naive method $= N^2 \times O(N^2)$
$= O(N^4)$.

Approach for Rotation

f_2 is obtained from f_1 by rotating with θ_0 and translating by (x_0, y_0).

$$f_2(x,y) = f_1(x\cos\theta_0 + y\sin\theta_0 - x_0, -x\sin\theta_0 + y\cos\theta_0 - y_0) \quad -(1)$$

Apply fourier transform to (1) :-

$$F_2(u,v) = F_1(u\cos\theta_0 + v\sin\theta_0, -u\sin\theta_0 + v\cos\theta_0) \cdot e^{-2\pi j(ux_0 + vy_0)} \quad -(2)$$

Take absolute value of both sides :-

$$M_2(u,v) = M_1(u\cos\theta_0 + v\sin\theta_0, -u\sin\theta_0 + v\cos\theta_0) \quad -(3)$$

where $M_2(u,v) = |F_2(u,v)|$
and $M_1(u,v) = |F_1(u,v)|$ (in the frequency domain)

Now, convert eq^n (3) to polar coordinates and we get

$$M_1(\rho, \theta) = M_2(\rho, \theta - \theta_0) \quad -(4)$$

Hence, rotation can be seen as a translational displacement in polar coordinates. So, we can use the same method to determine θ_0 as used by us before to determine the translation values x_0, y_0.

Once we have determined θ, we correct one image by rotating it by θ and then use the same method to determine the translational displacements t_x, t_y.