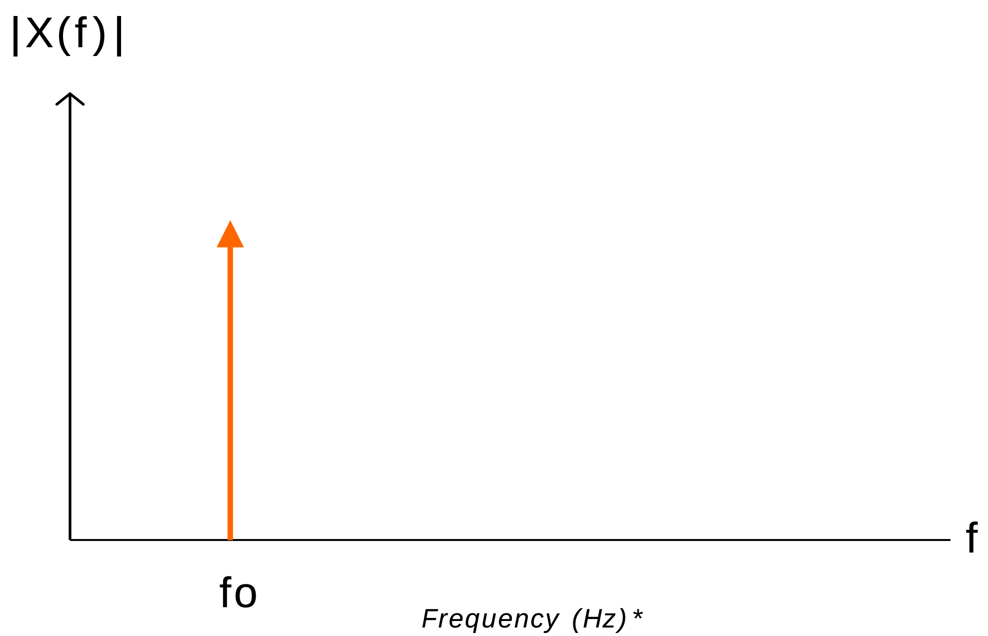
03 Speech Signal Fundamentals II

Deviation from Ideal Spectra

$$x(t) = \cos(2\pi f_0 t)$$

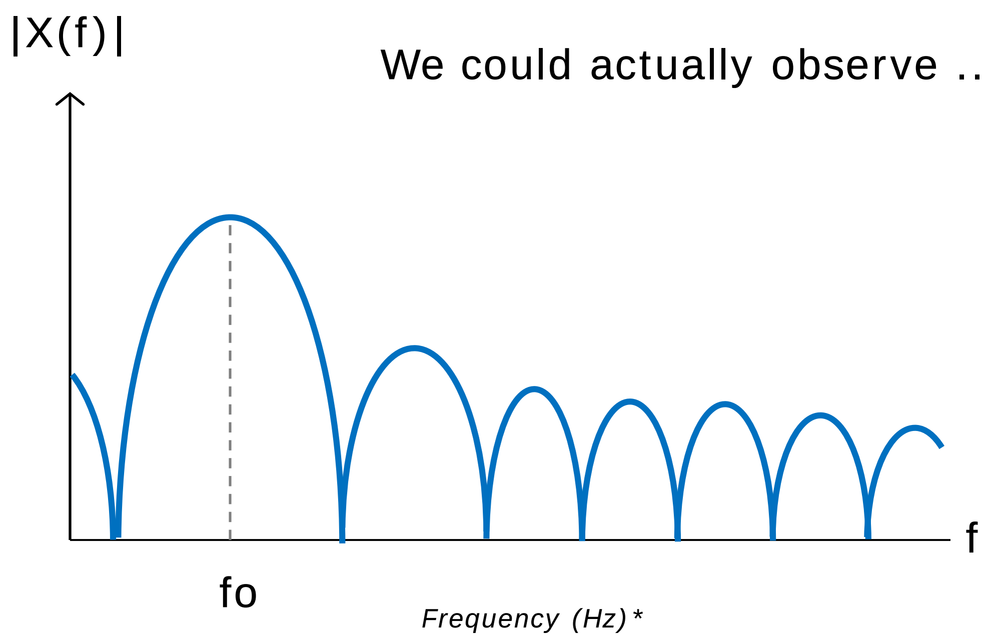
x(t) has infinite length.



* Negative frequencies omitted

Deviation from Ideal Spectra

$$x(t) = \cos(2\pi f_0 t)$$



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Deviation from Ideal Spectra

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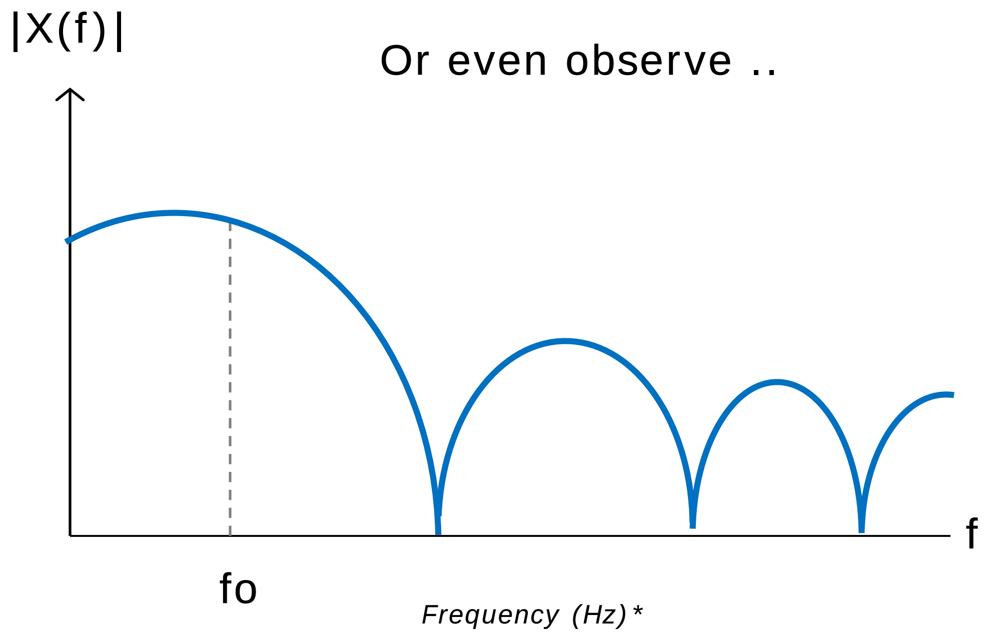


* Negative frequencies omitted


Photo by Paul on Unsplash

Finite-length = Time-windowed

$$x(t) = \cos(2\pi f_0 t)$$ → x(t) has infinite length.

But in real life, we can never have a signal of infinite length.

A portion of sine wave with finite length can be considered as a windowed version of the infinite length sine wave.

E.g.: A sine wave from t_1 to t_2, $x_w(t)$ can be written as:

$$\boxed{x_w(t) = \cos(2\pi f_0 t) w(t)}$$

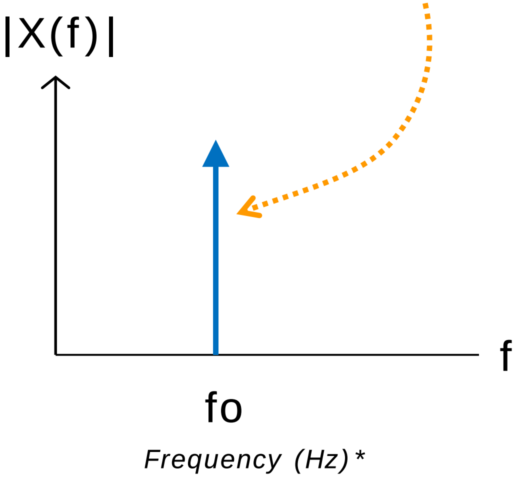
Where:

$$w(t) = \begin{cases} 1 & ; t_1 \le t < t_2 \\ 0 & ; otherwise \end{cases}$$

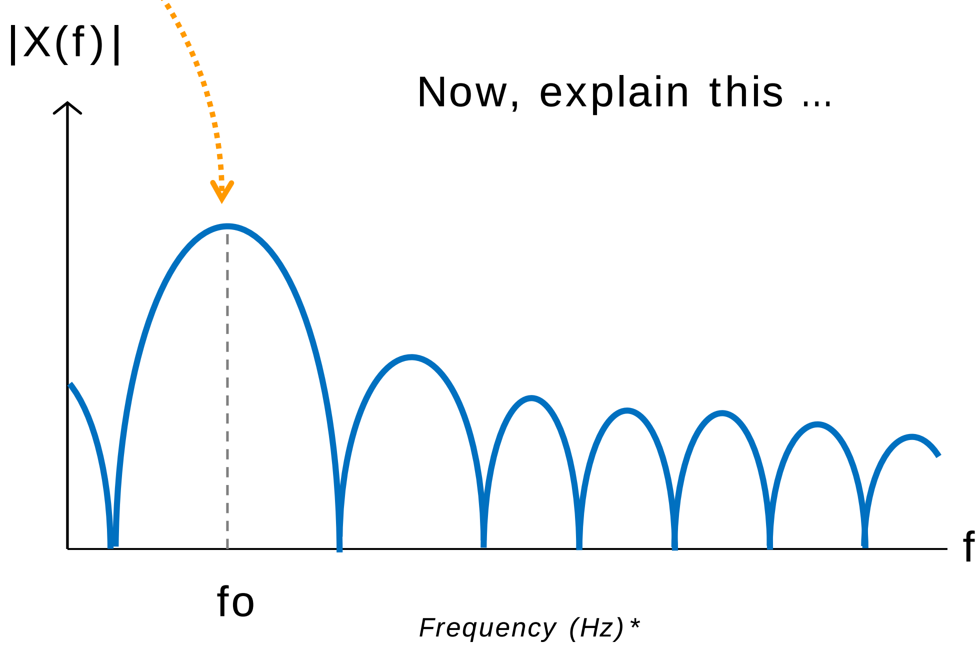
Recall that ...

$$x(t)y(t) \Leftrightarrow X(f) * Y(f)$$

$$x_w(t) = \cos(2\pi f_0 t)w(t)$$

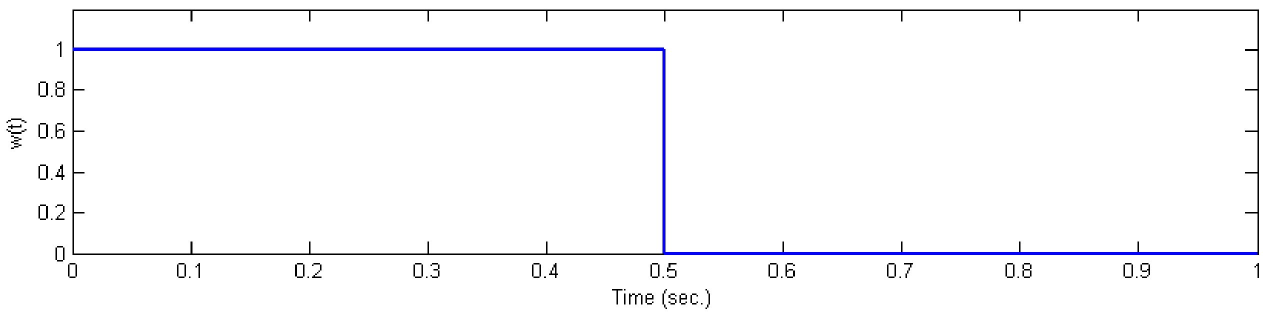


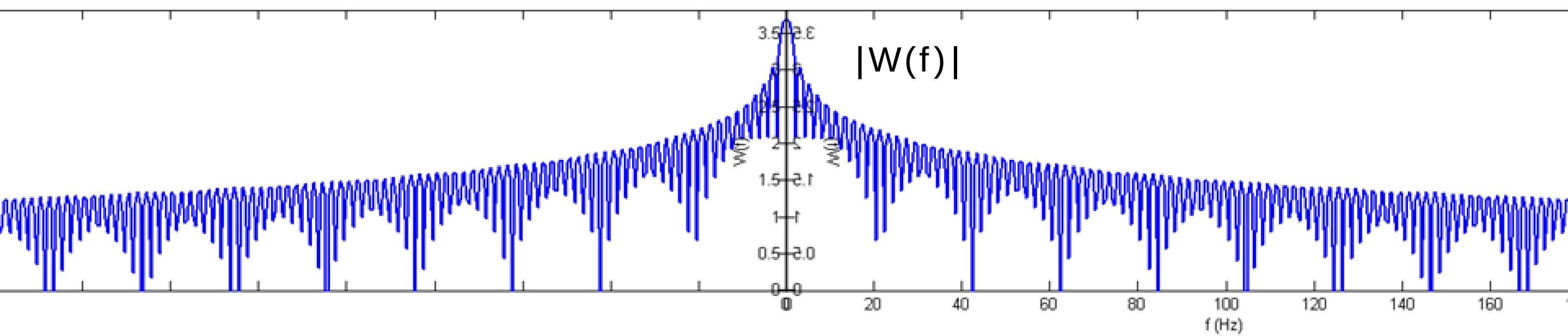
Now, explain this ...



Square Window

$$w(t) = \begin{cases} 1 & ; 0 \le t < 0.5\,\text{sec.} \\ 0 & ; otherwise \end{cases}$$

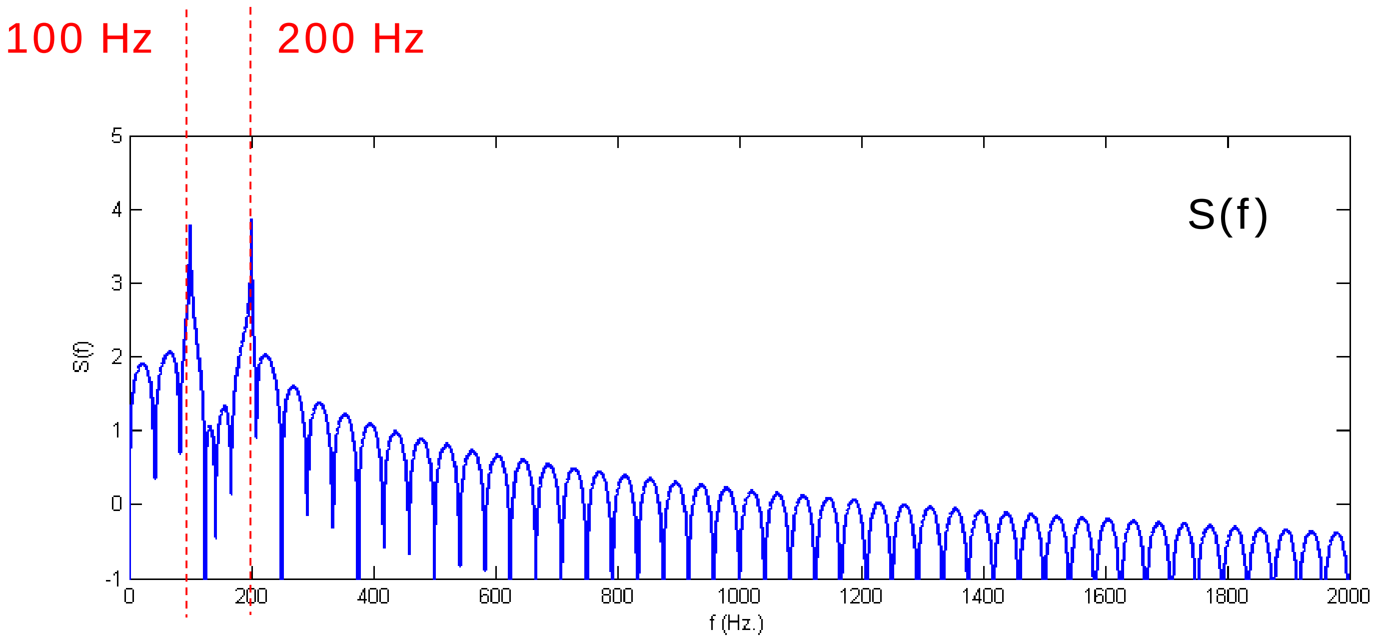




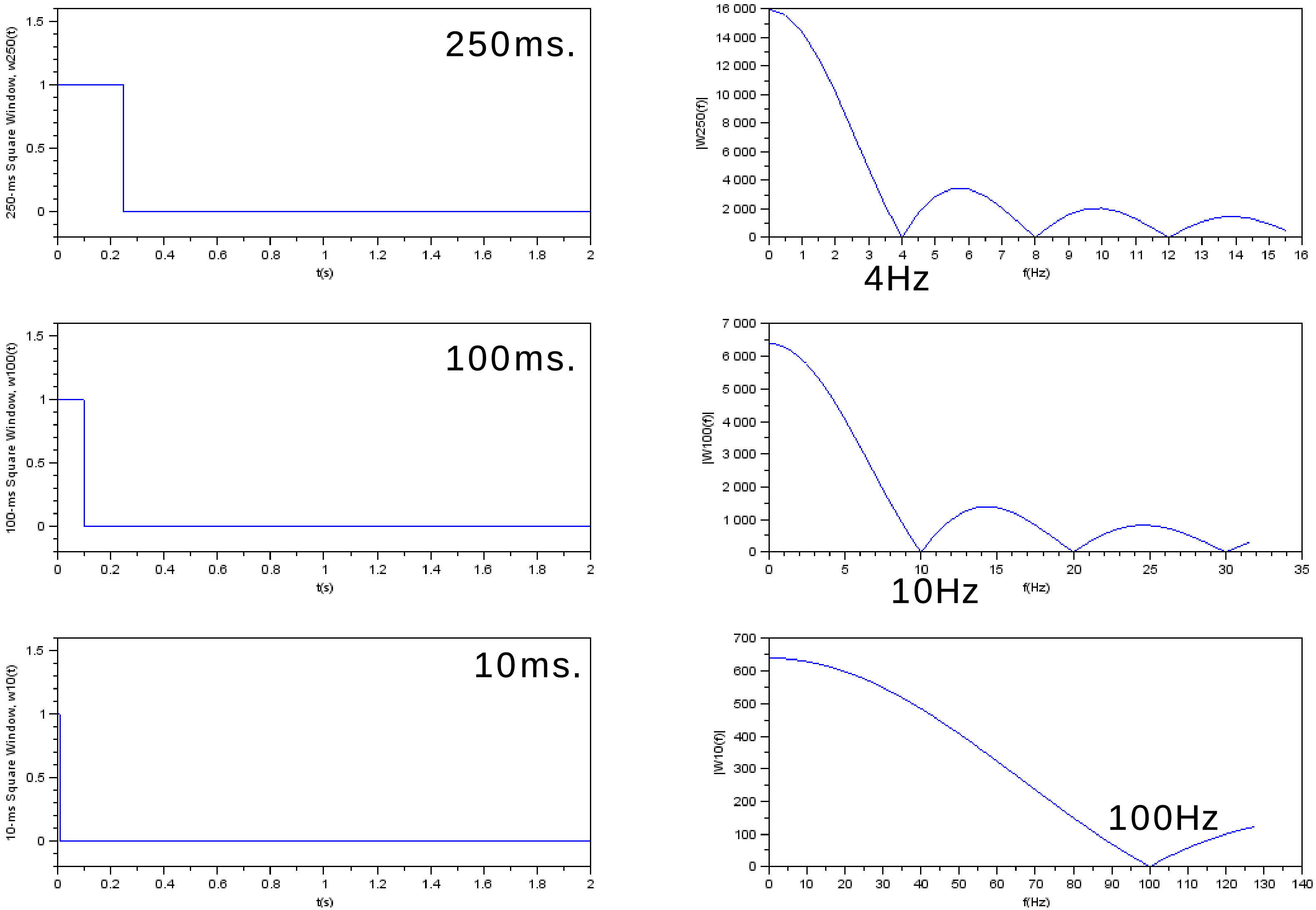
Effect of Time-windowing

$$s(t) = (\sin(2\pi(100)t) + \sin(2\pi(200)t))w(t)$$

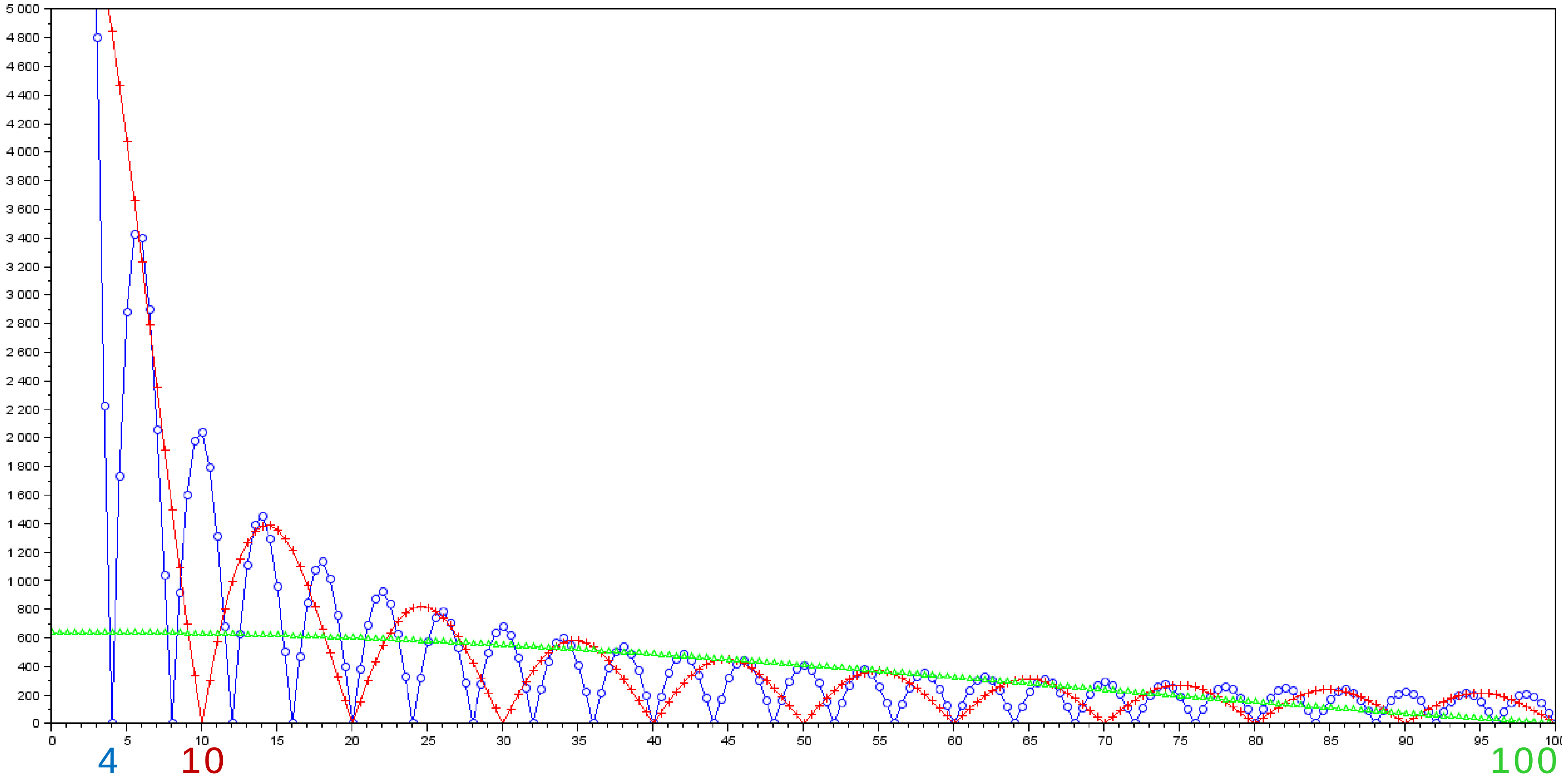
w(t) is a 1-second-long square window



Window Length Vs. Frequency Resolution

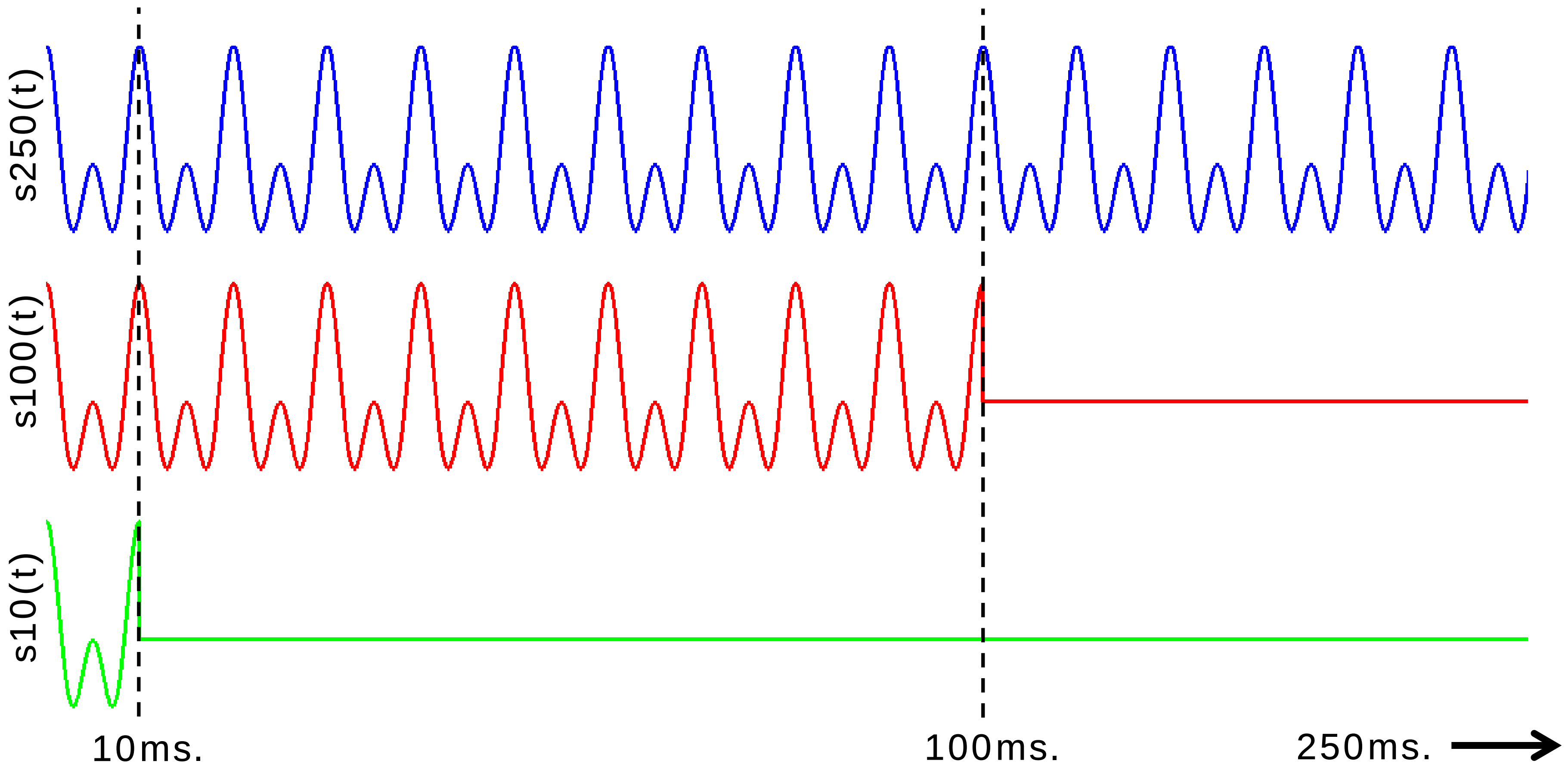


Window Length Vs. Frequency Resolution

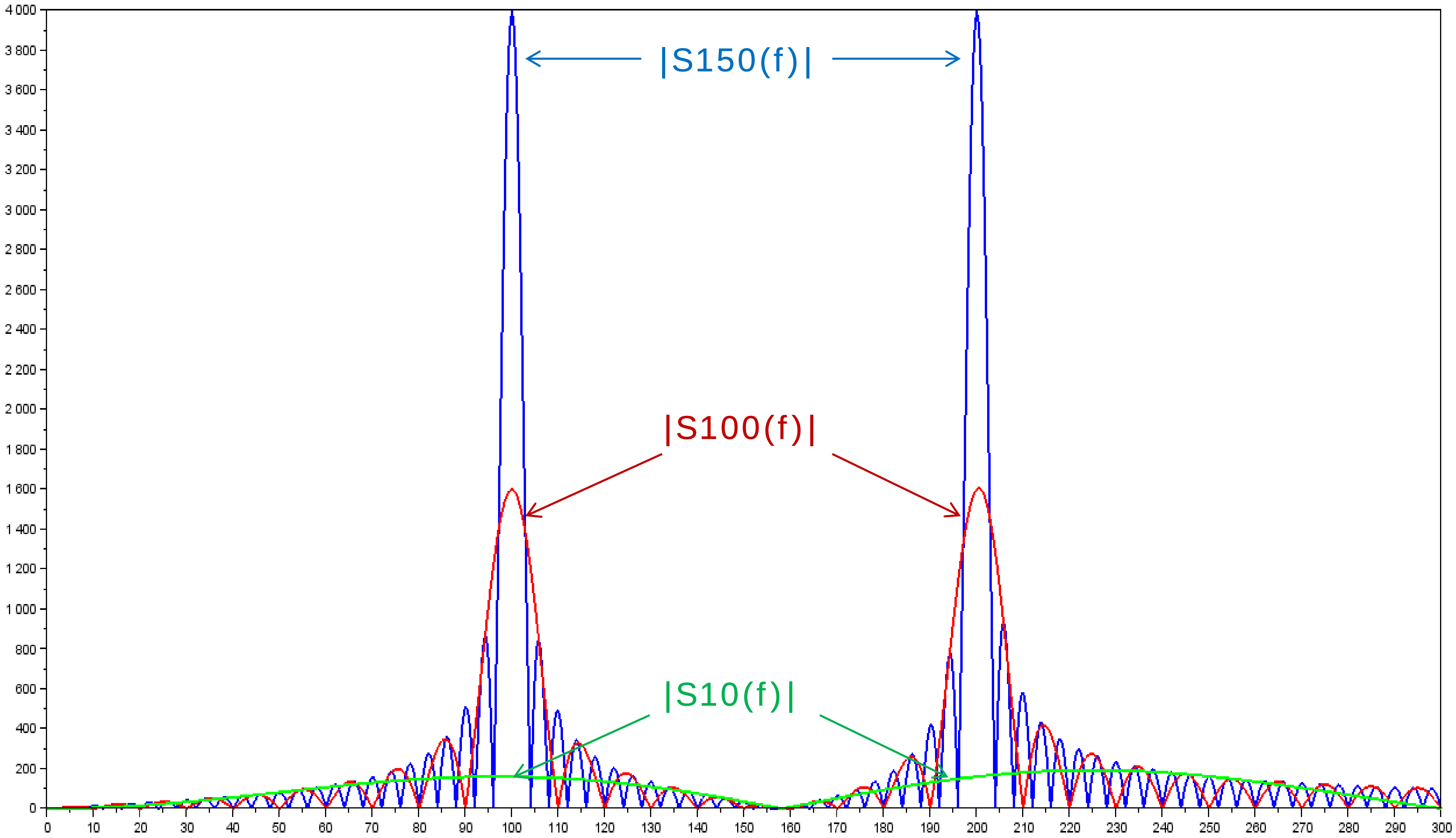


Window Length Vs. Frequency Resolution

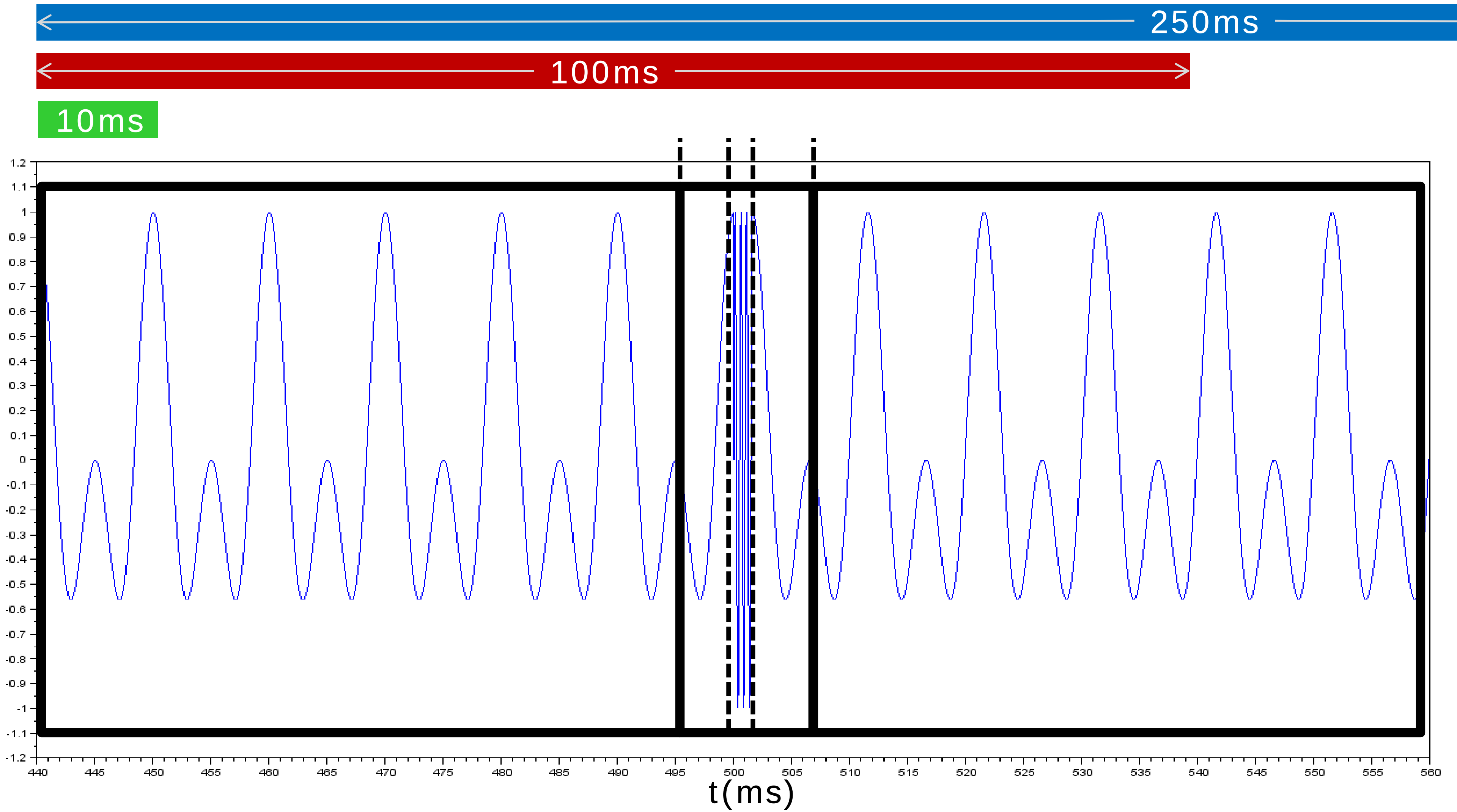
$$s(t) = (\sin(2\pi(100)t) + \sin(2\pi(200)t))w(t)$$



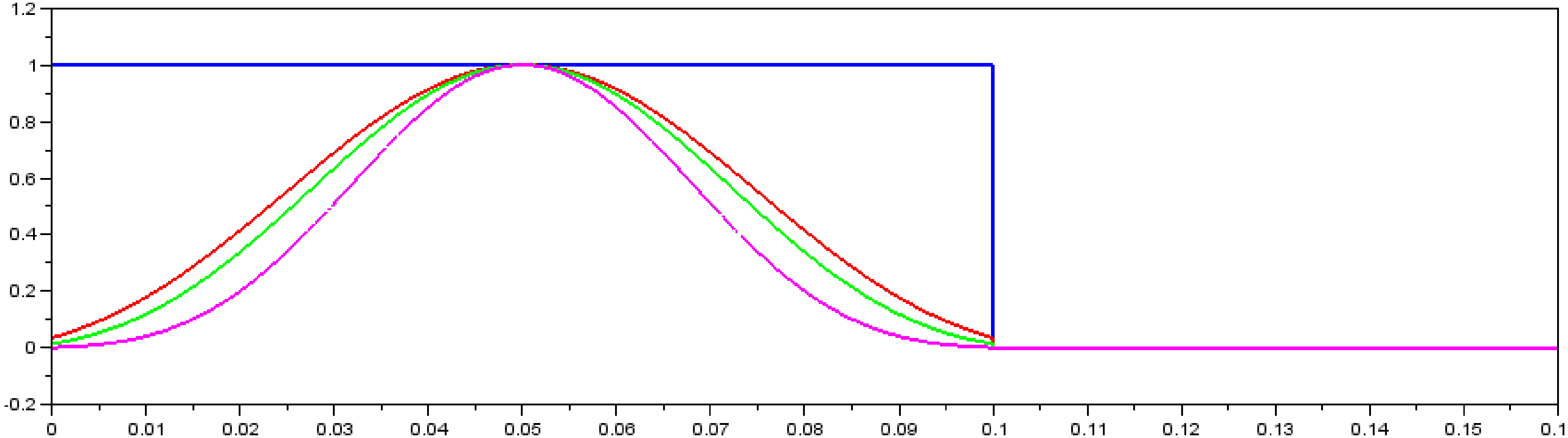
Window Length Vs. Freq.Resolution

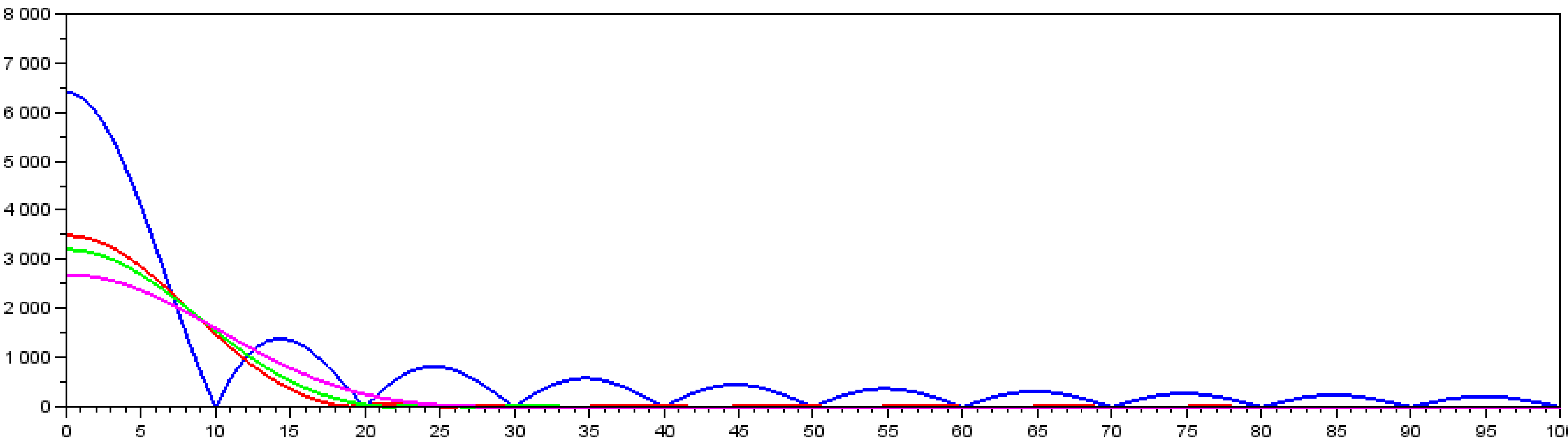


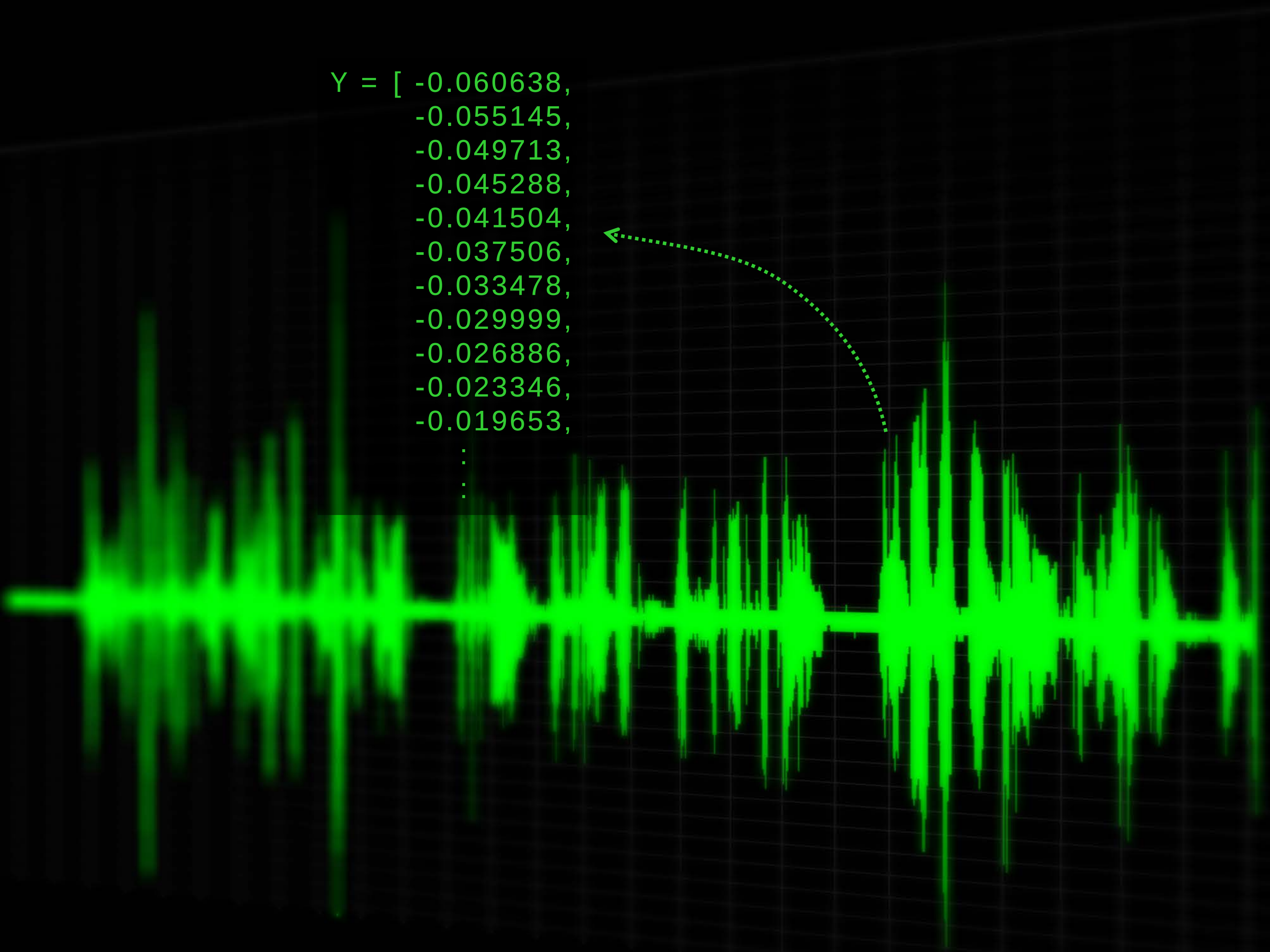
Time Resolution Vs. Frequency Resolution



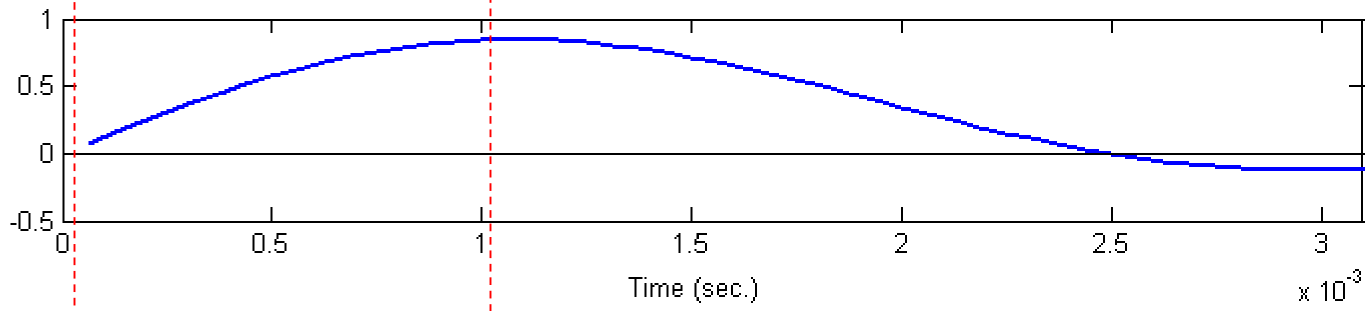
Different Types of Window






Y = [-0.060638,
-0.055145,
-0.049713,
-0.045288,
-0.041504,
-0.037506,
-0.033478,
-0.029999,
-0.026886,
-0.023346,
-0.019653,
⋮

Sampling

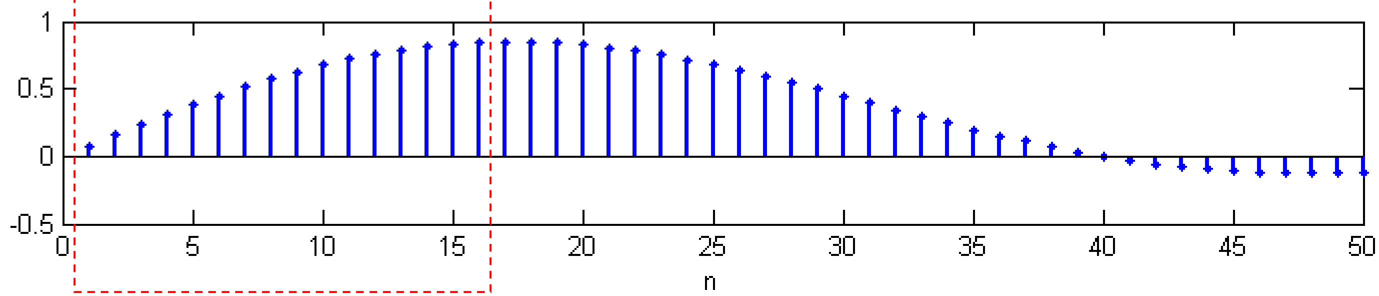


In 1 x 10^{-3} sec.
16 samples are picked

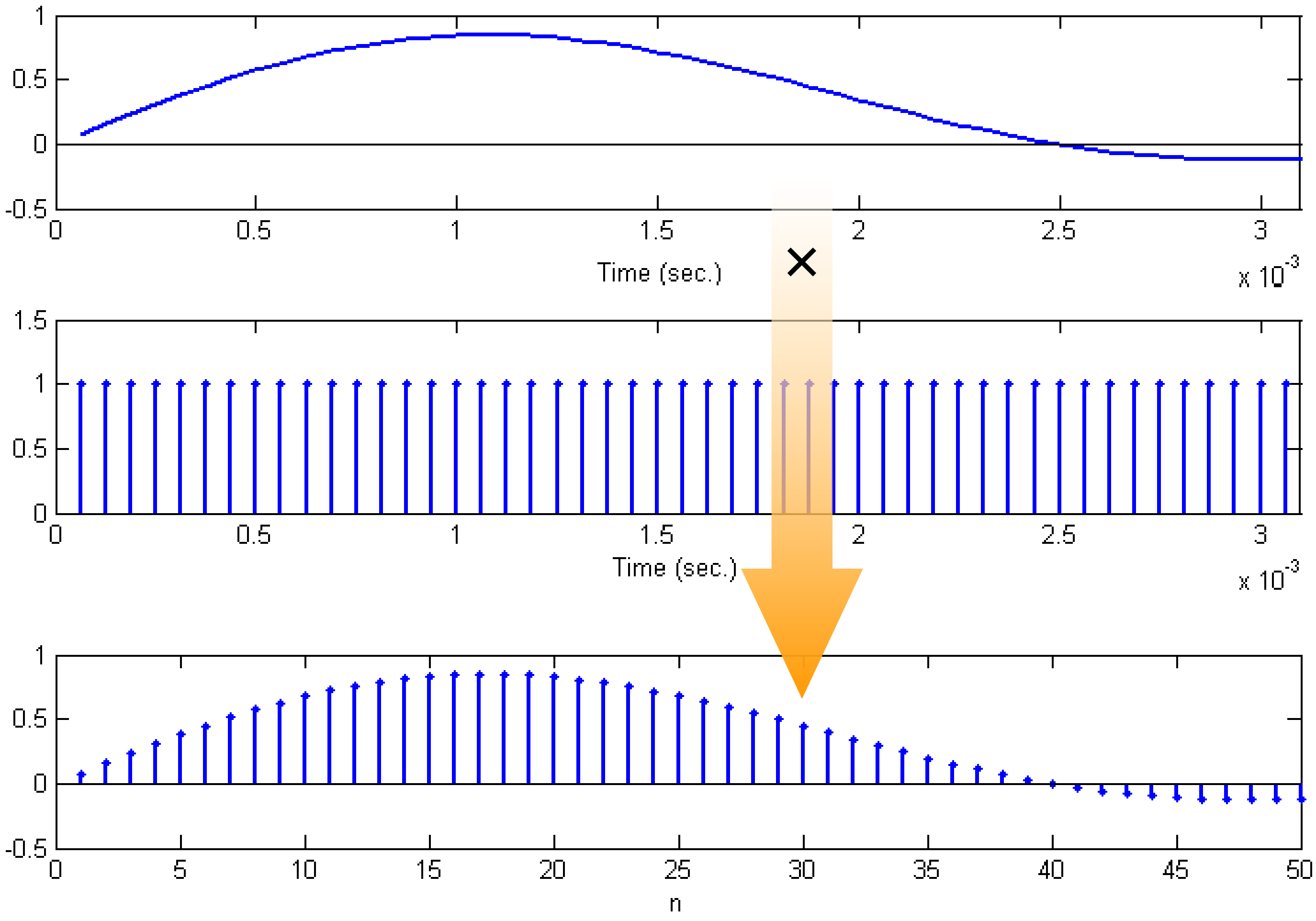
$\therefore$Fs = 16,000 samples/sec.
= 16 kHz.

Sampling Frequency, Fs

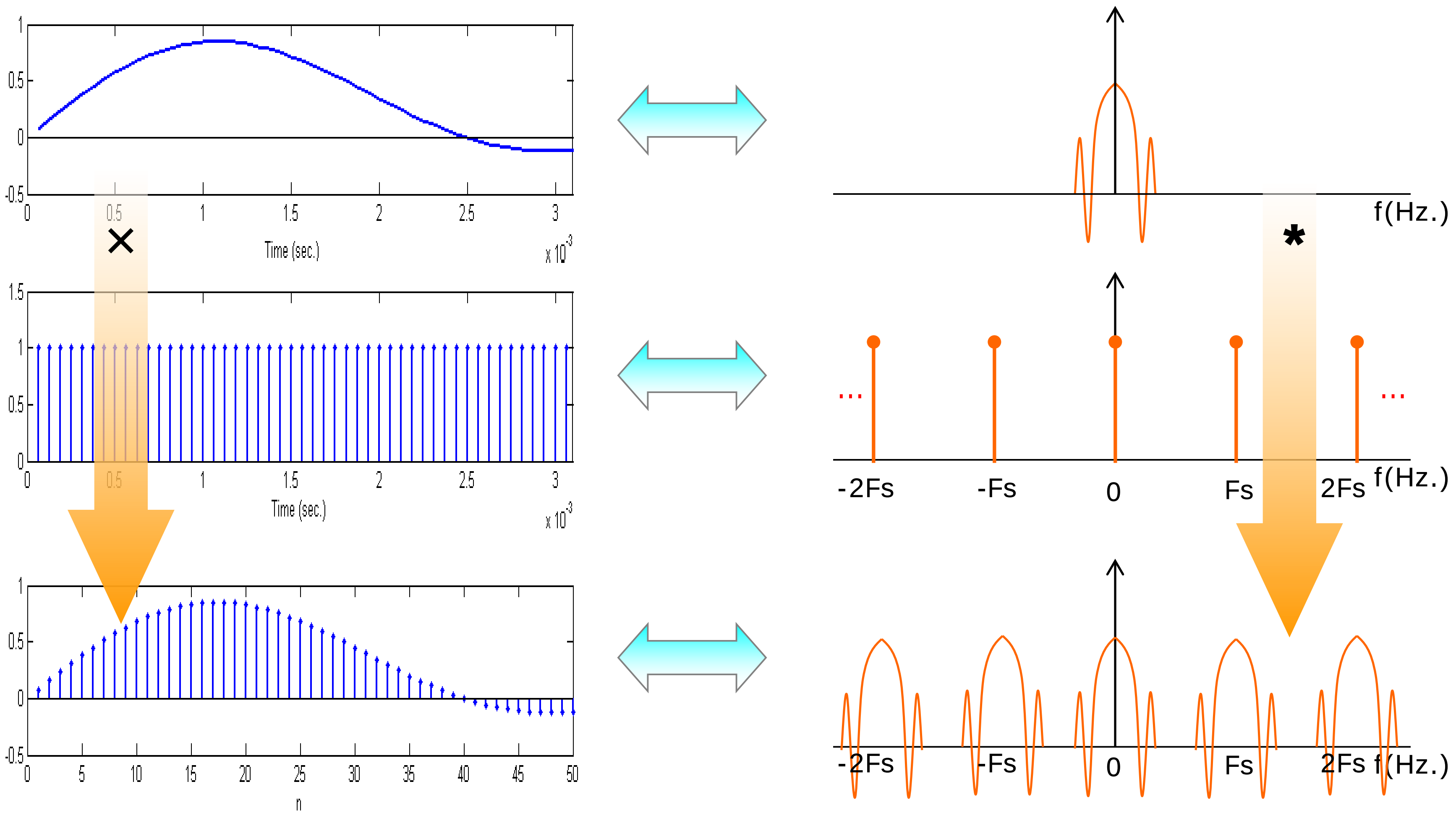
= how many samples are picked in the interval of 1 second.



Sampling

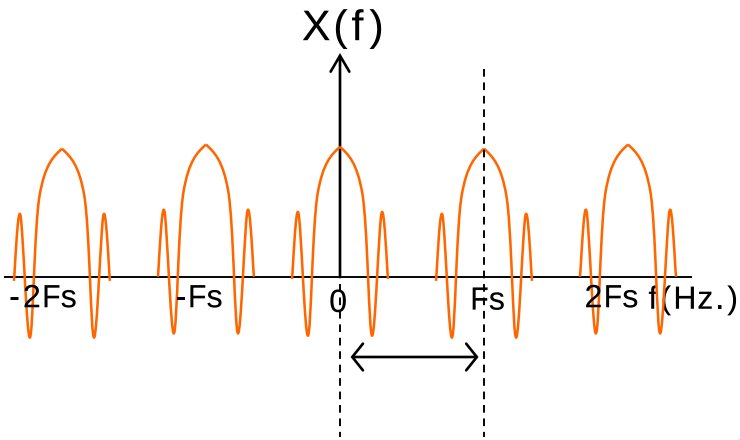

1
0.5
0
-0.5
0 0.5 1 1.5 2 2.5 3
Time (sec.)
x 10^-3
×
1.5
1
0.5
0
0 0.5 1 1.5 2 2.5 3
Time (sec.)
x 10^-3
1
0.5
0
-0.5
0 5 10 15 20 25 30 35 40 45 50
n

Sampling



Normalized Frequency

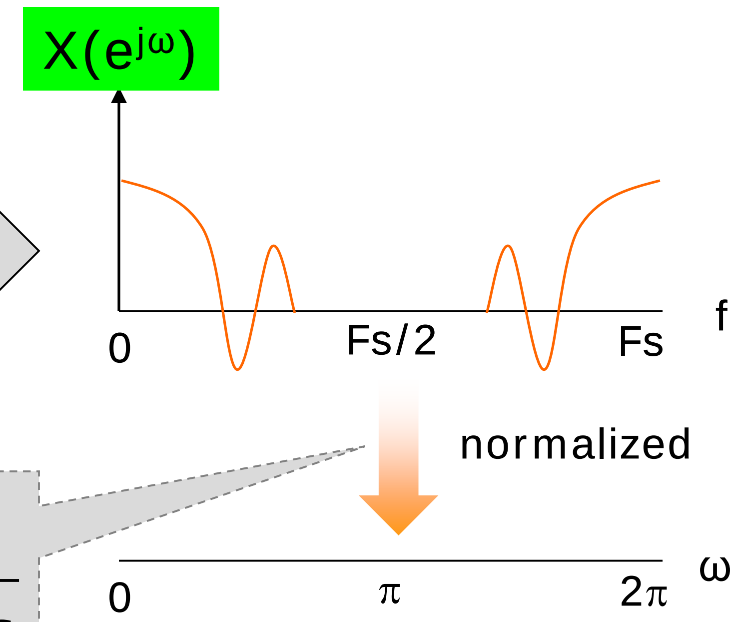
Fourier Transform



Repeats every Fs

f : frequency (Hz.)

Discrete-time Fourier Transform



$$\omega = 2\pi \frac{f}{Fs}$$

ω : Normalized frequency
(Radian per sample)

Discrete-time Fourier Transform

$$\boxed{x[n] \Leftrightarrow X(e^{j\omega})}$$

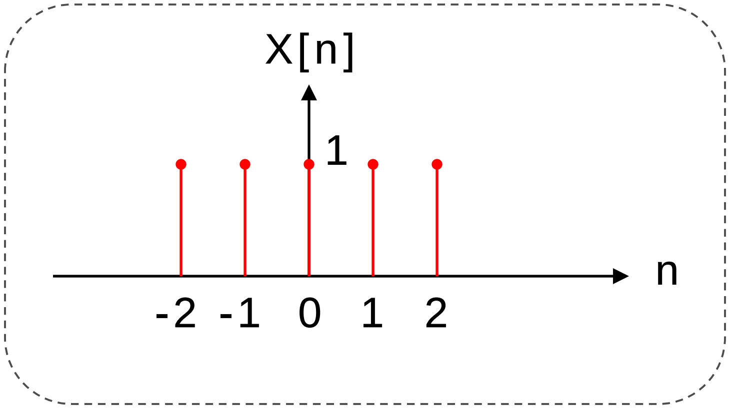
Discrete-time Fourier Transform

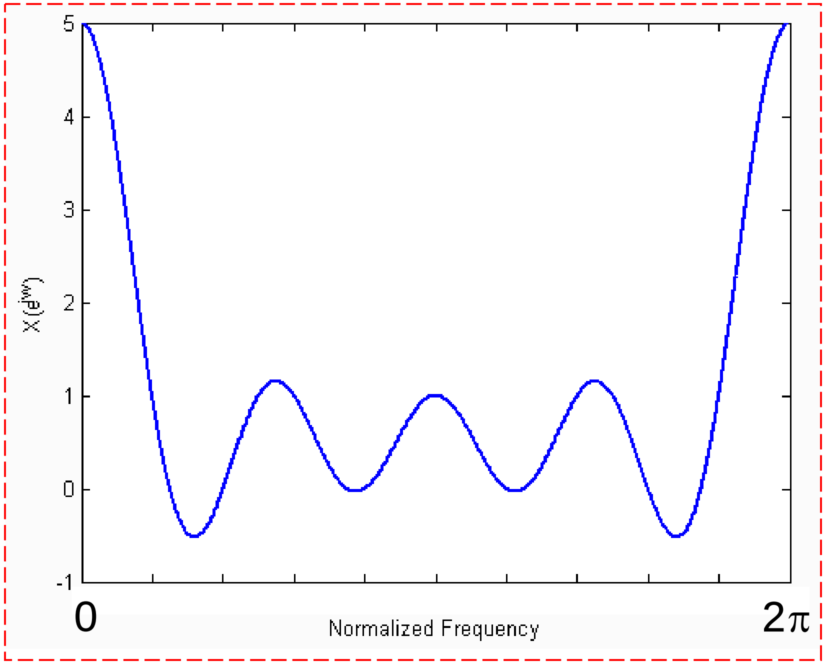
$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

Inverse Discrete-time Fourier Transform

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

Example





$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

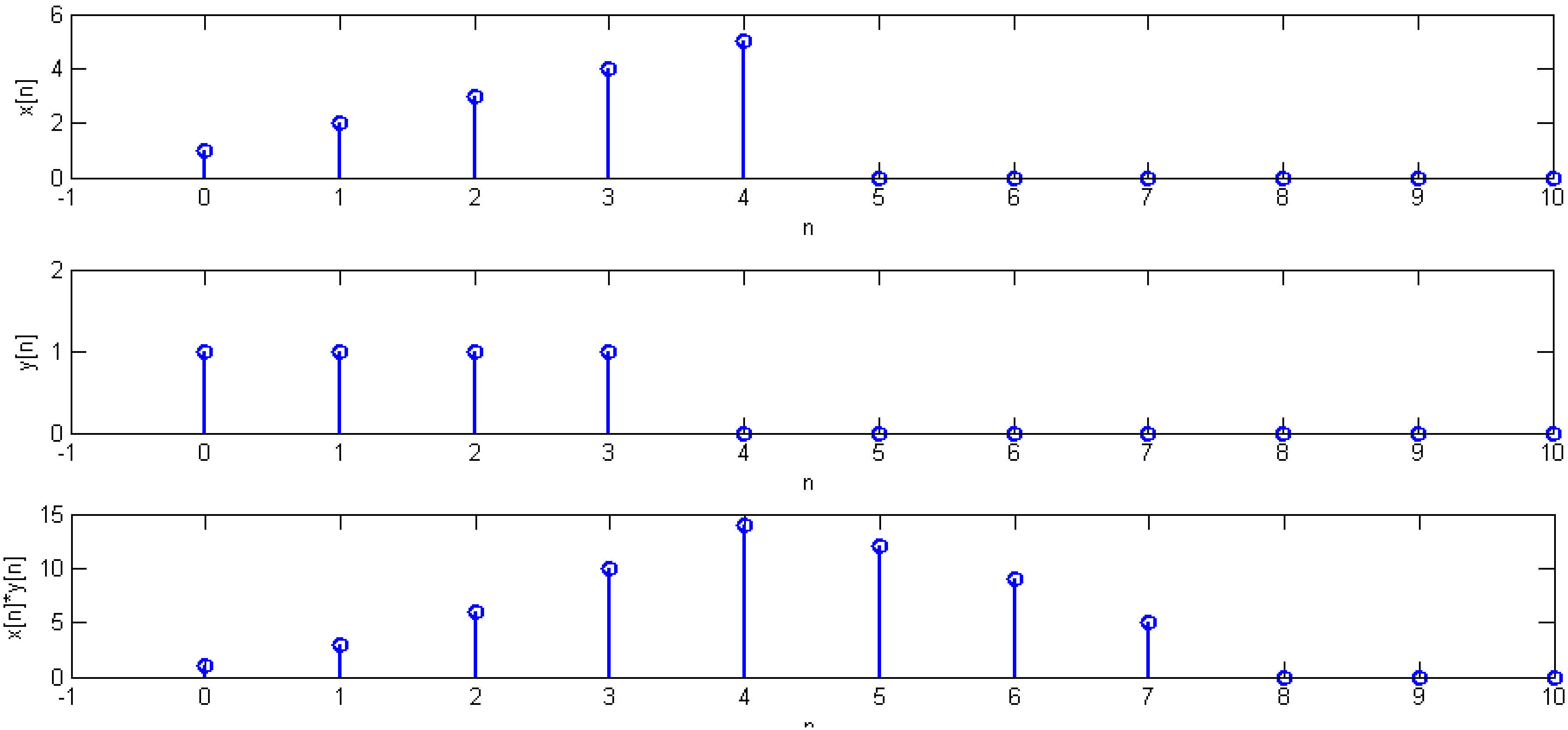
$$= x[-2]e^{-j\omega(-2)} + x[-1]e^{-j\omega(-1)} + x[0]e^{-j\omega(0)} + x[1]e^{-j\omega(1)} + x[2]e^{-j\omega(2)}$$

$$= e^{j\omega(2)} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j\omega(2)}$$

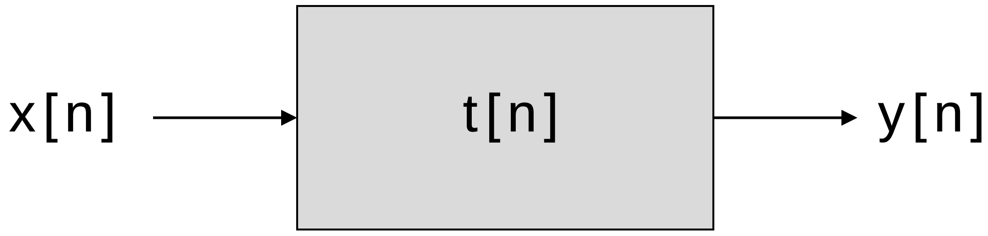
$$= \boxed{1 + 2\cos(\omega) + 2\cos(2\omega)}$$

Discrete-time Convolution

$$x[n] * y[n] = \sum_{m=-\infty}^{\infty} x[m]y[n-m]$$



Discrete-time Filter



$$Y(e^{j\omega}) = X(e^{j\omega})T(e^{j\omega}) \quad \Leftrightarrow \quad y[n] = x[n] * t[n]$$

Family of Fourier Transform

- "Discreteness" in one domain implies "Periodicity" in the other domain.
- "Continuity" in one domain implies "Aperiodicity" in the other domain.

Transform	Time	Freq.
Cont. Fourier Trans.	Cont. & aperiodic	Cont. & aperiodic
Fourier Series	Cont. & periodic	Disc. & aperiodic
DTFT	Disc. & aperiodic	Cont. & periodic
DFT	Disc. & periodic	Disc & periodic

Discrete Fourier Transform

$$x[n] \Leftrightarrow X[k]$$

DFT

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi kn}{N}} \qquad k = 0,1,\ldots, N-1$$

IDFT

$$X[n] = \frac{1}{N} \sum_{k=0}^{N-1} x[k] e^{j\frac{2\pi kn}{N}} \qquad n = 0,1,\ldots, N-1$$

Discrete Fourier Transform

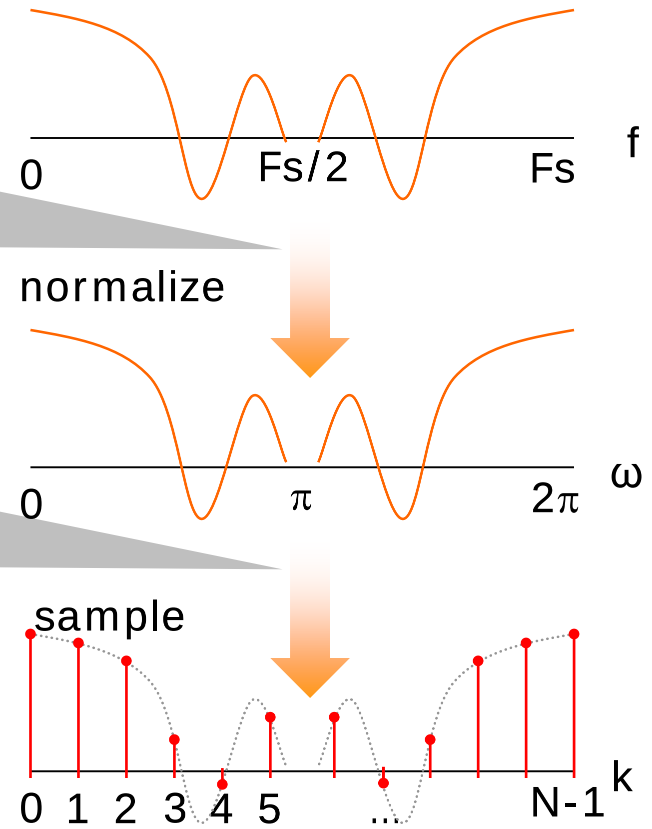
$$X(f)=\int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt$$

$$\omega = 2\pi\frac{f}{Fs}$$

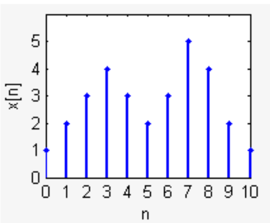
$$X(e^{j\omega})=\sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$k=\frac{\omega}{2\pi}N$$

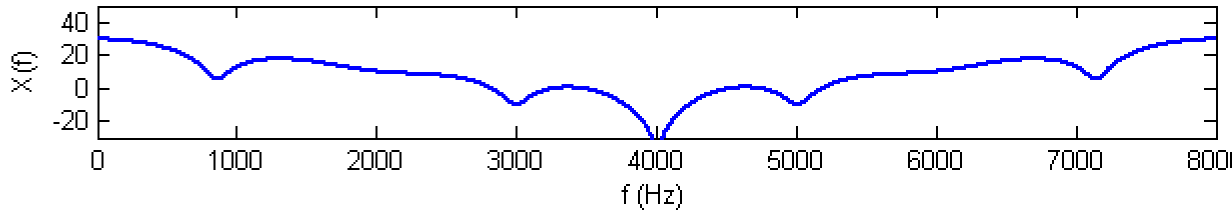
$$X[k]=\sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi kn}{N}}$$



N DFT

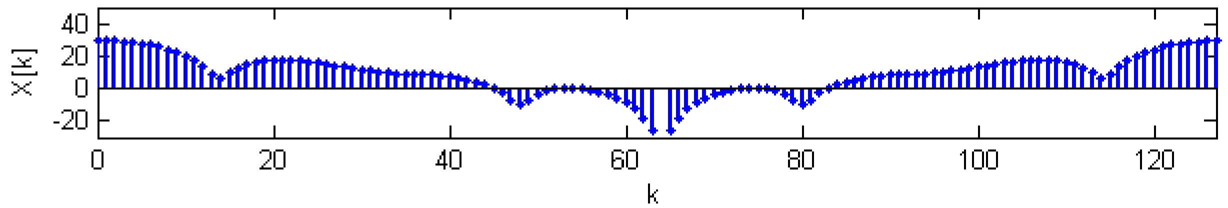


DTFT



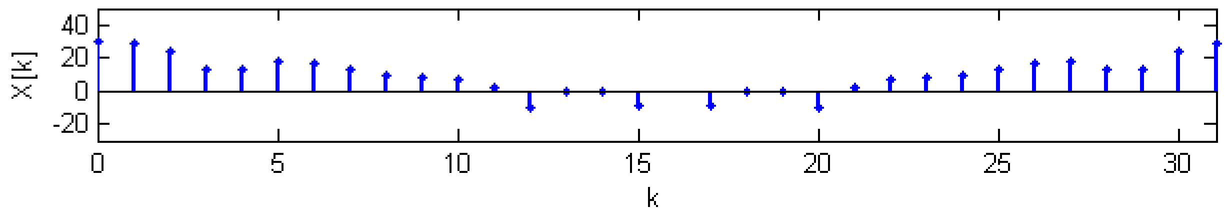
DFT

N=128



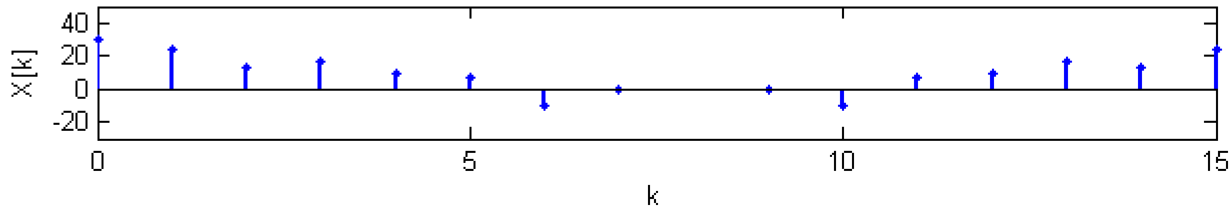
DFT

N=32



DFT

N=16



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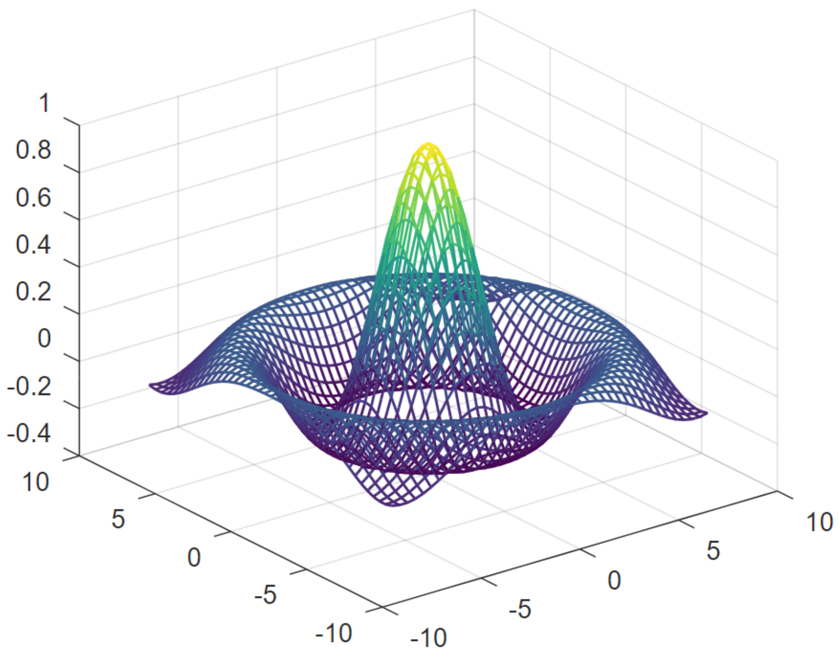
GNU Octave



GNU Octave

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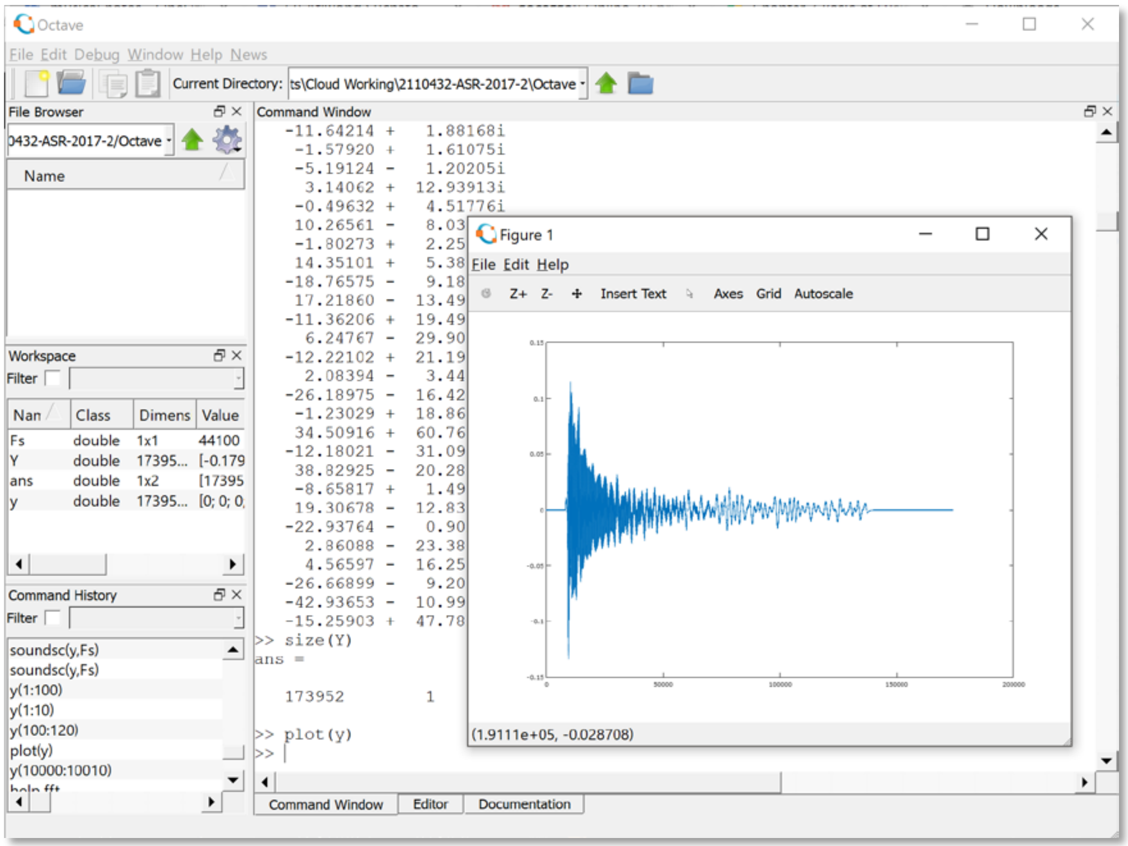
Octave Demo



- Basic operations
  - Variables
  - Colon operator
  - Matrix manipulation
  - Convolution
- Arrays
- Plotting
- Loading/Saving audio
- Help
- Control structures