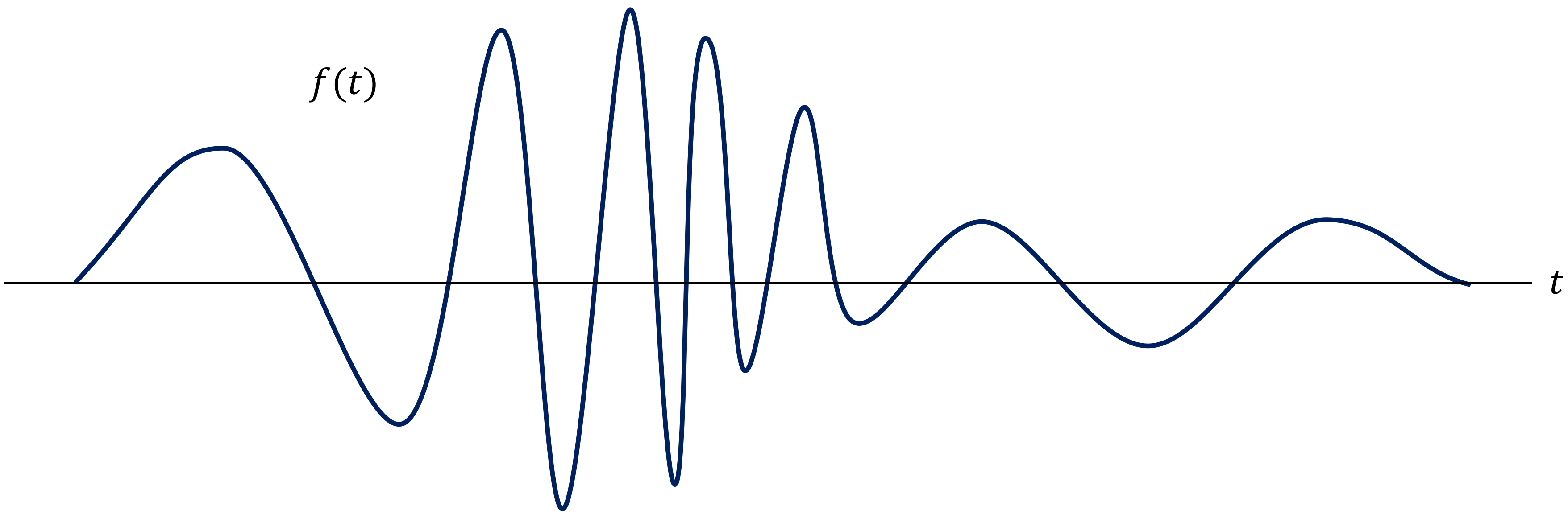
02 Speech Signal Fundamentals I

“Signals”

- Functions of “independent variables”



waveform => time-domain signal

Signal Dimension

- = No. of variables needed to represent the signals

Speech

Image

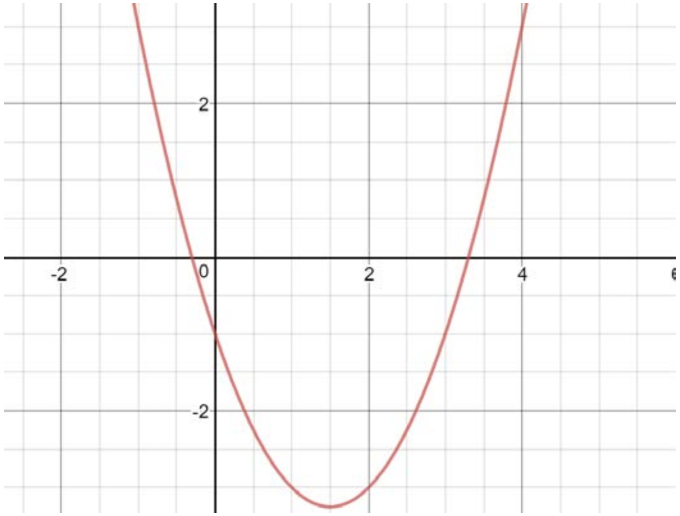
Video

Stereo Sounds

RGB values of images

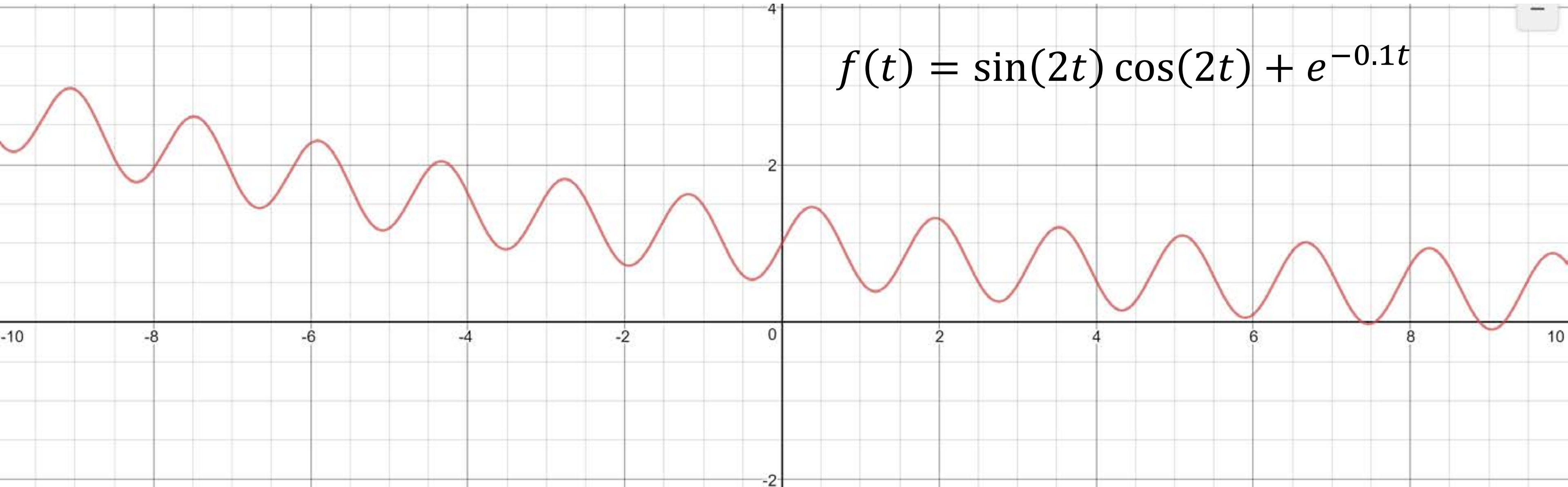
Deterministic Signals

E.g.:

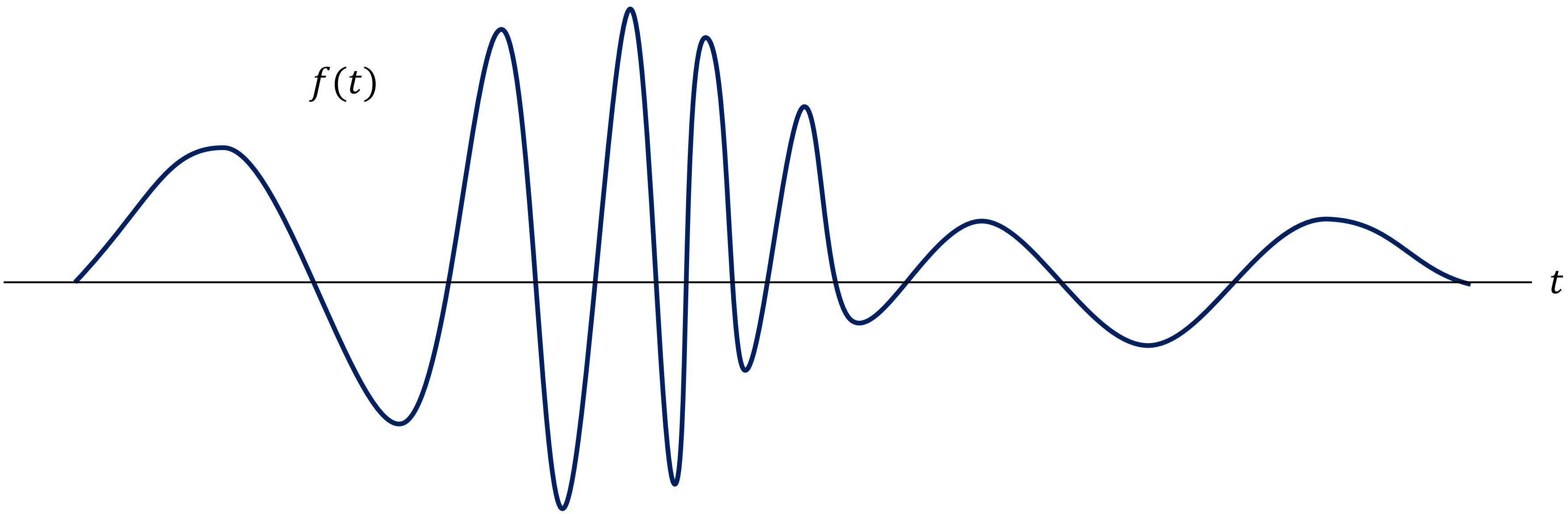


$$f(t) = t^2 - 3t - 8$$

$$f(t) = \sin(2t)\cos(2t) + e^{-0.1t}$$



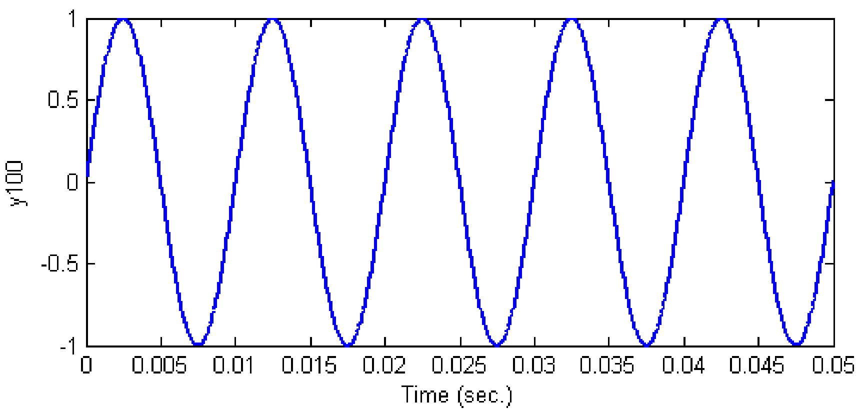
Deterministic Signals

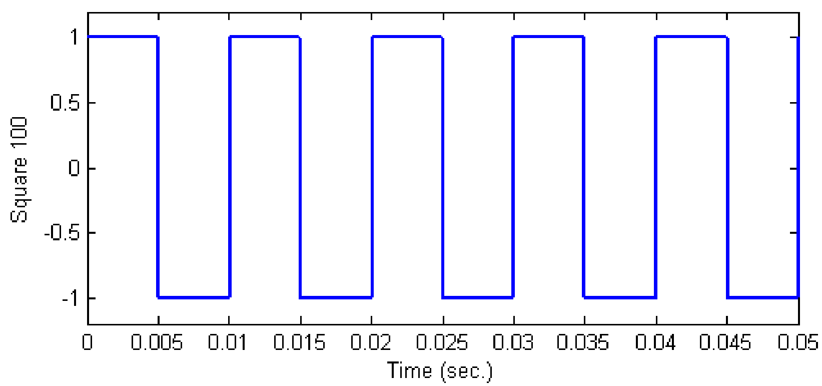


Random Signals

Photo by Alex Chambers on Unsplash

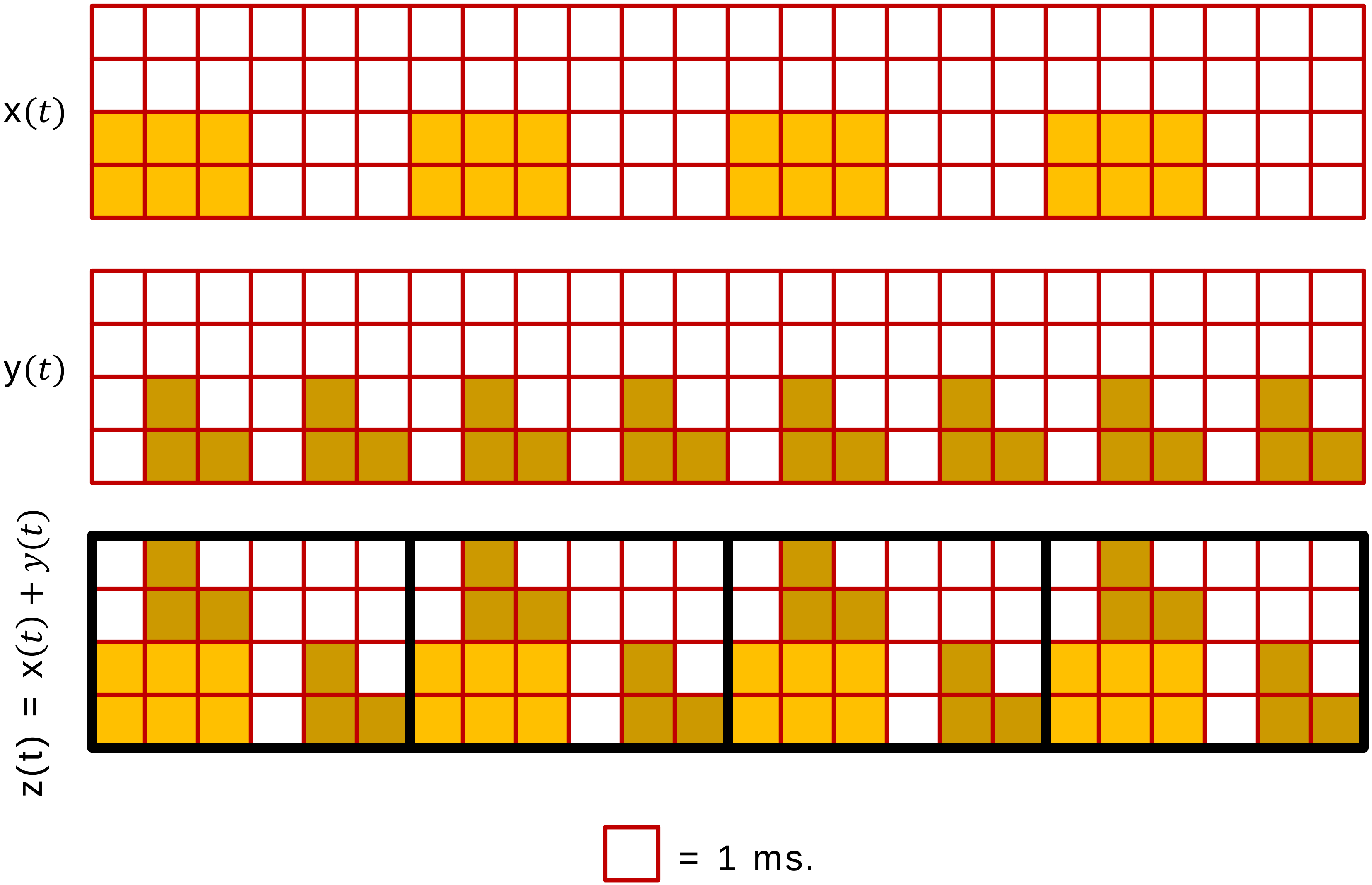
Periodicity



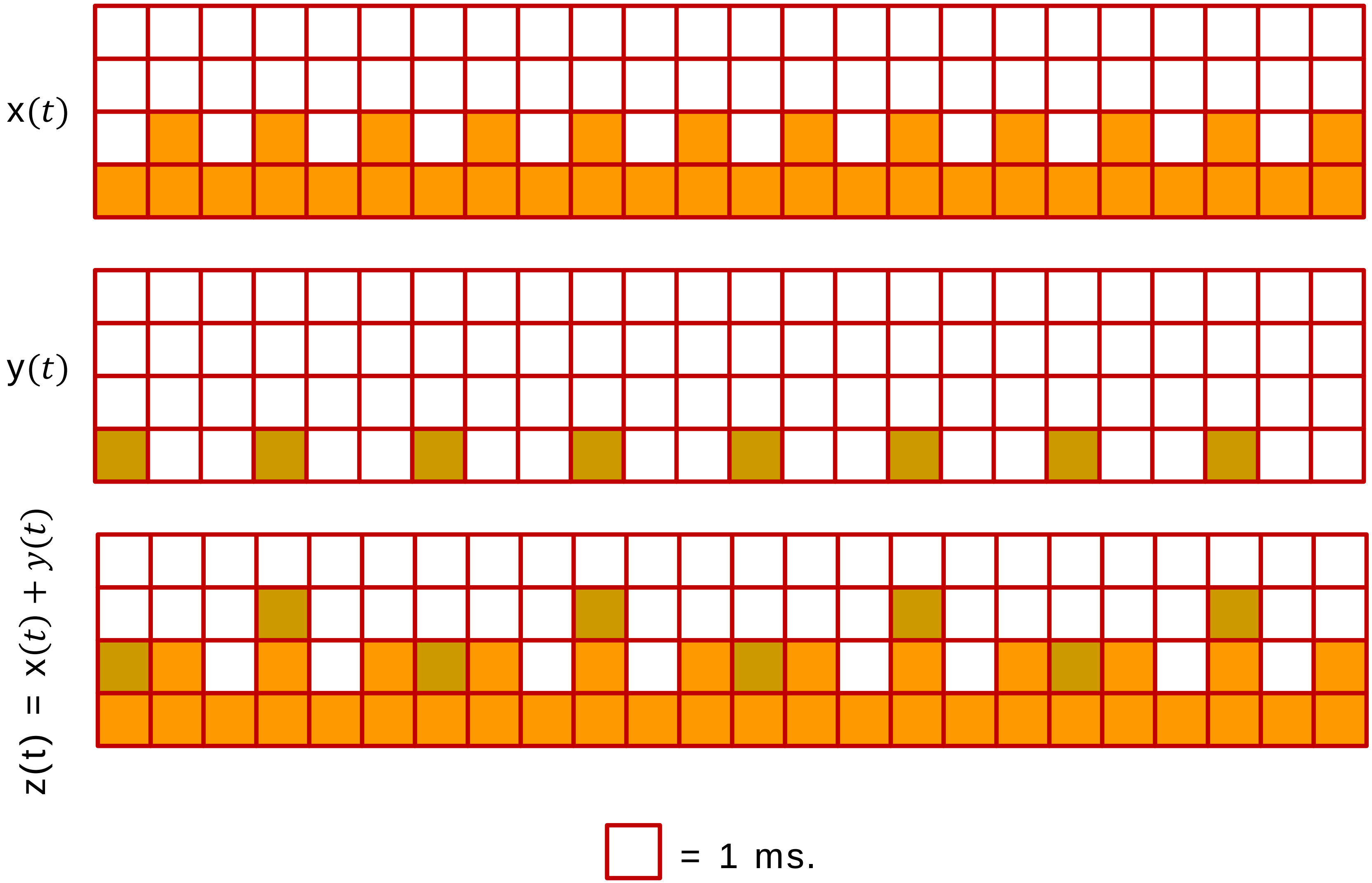


- A “Periodic Signal” is a signal that …

Linear Combination of Periodic Signals



Linear Combination of Periodic Signals



Linear Combination of Periodic Signals

$$s(t) = x_1(t) + x_2(t) + \ \ldots + x_k(t)$$

$$T_s = ?$$

Our Friendly Signals

Impulses

Sinusoids

Complex
Sinusoids

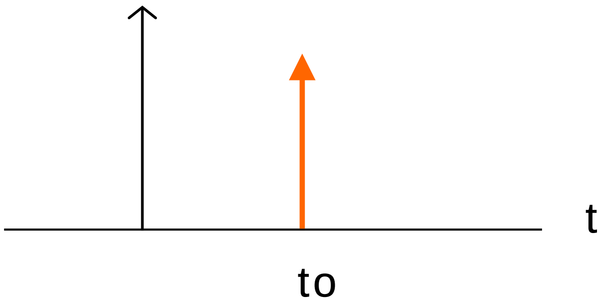
Impulse

$$\delta(t-t_0) = \begin{cases} \infty & ; \quad t = t_0 \\ 0 & ; \quad t \neq t_0 \end{cases}$$

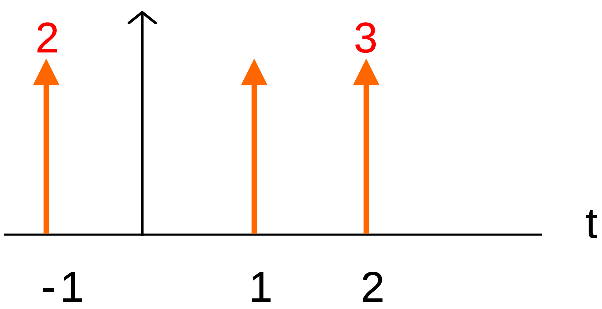
and

$$\int_{-\infty}^{\infty} \delta(t-t_0)\,dt = 1$$

$\delta(t-t_0)$



$\delta(t-1)+3\delta(t-2)+2\delta(t+1)$



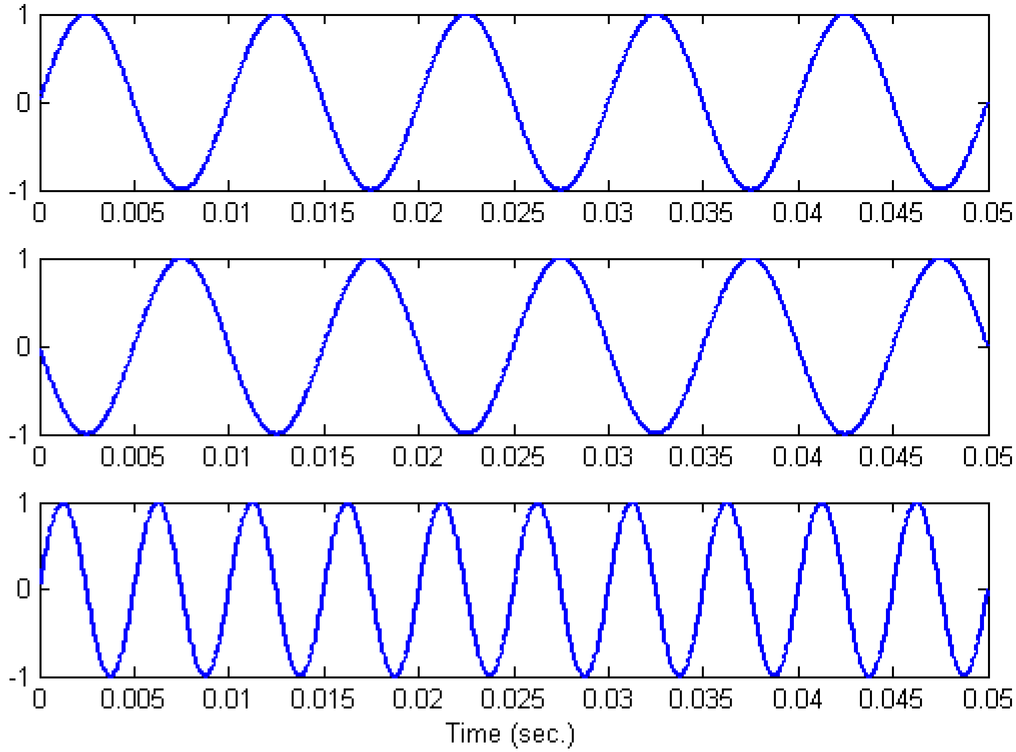
Sinusoidal Signal

$$y = A\sin(2\pi ft + \phi)$$

$f = 100Hz.$
$\phi = 0$

$f = 100Hz.$
$\phi = \pi$

$f = 200Hz.$
$\phi = 0$



$A = 1$

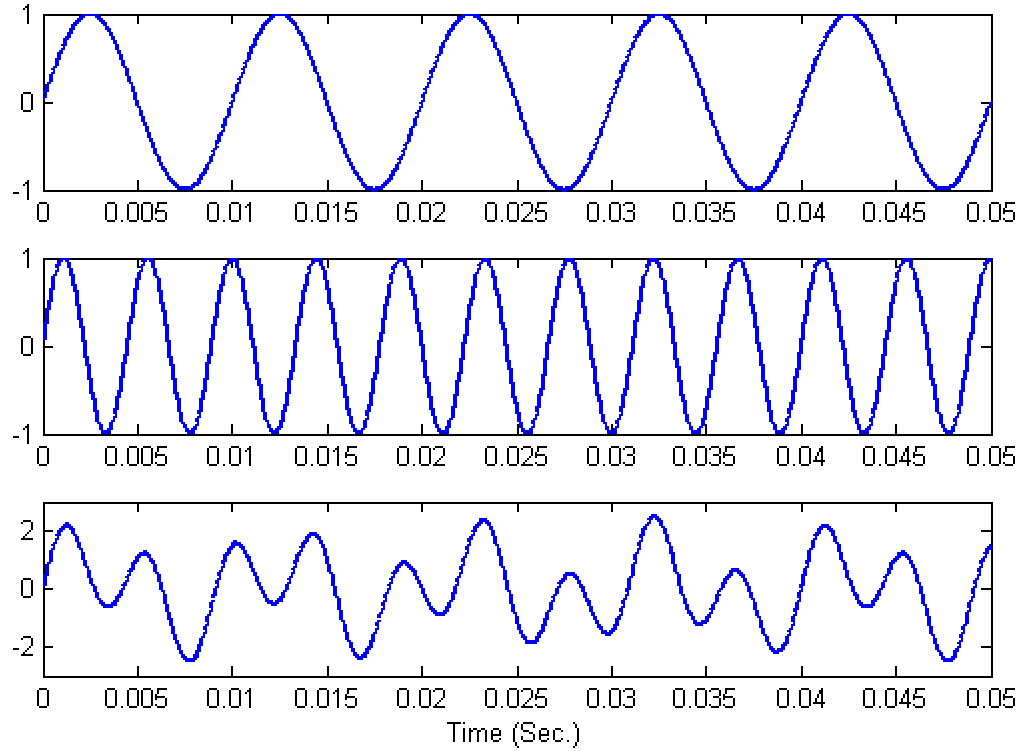
Linear Combination of Signals

$$s = a_1 y_1 + a_2 y_2 + \cdots + a_N y_N$$

$$y_1 = \sin(2\pi(100)t)$$

$$y_2 = \sin(2\pi(225)t)$$

$$s = y_1 + 1.5 y_2$$



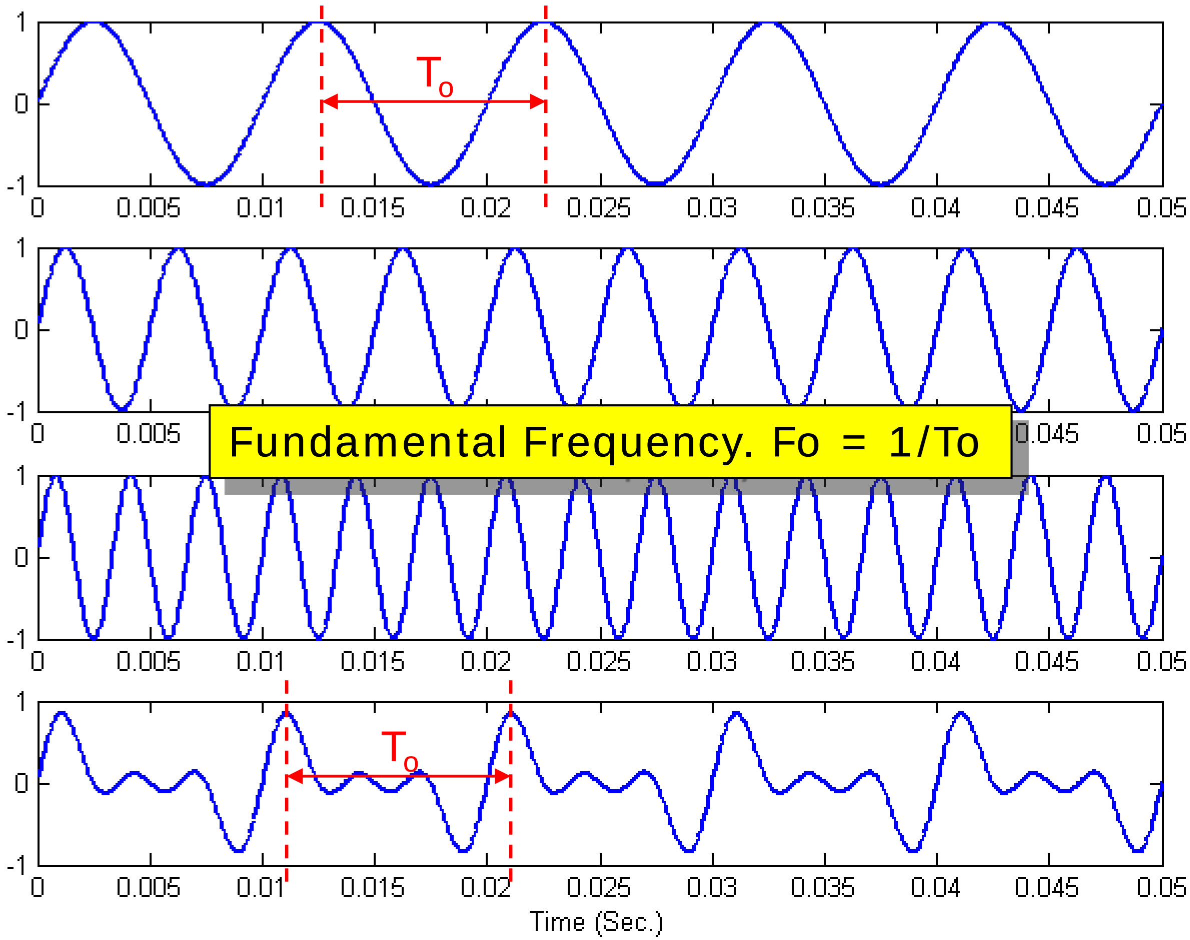
Fundamental Frequency and Harmonics

$$y_1 = \sin(2\pi(100)t)$$

$$y_2 = \sin(2\pi(200)t)$$

$$y_3 = \sin(2\pi(300)t)$$

$$s = 0.3y_1 + 0.4y_2 + 0.3y_3$$



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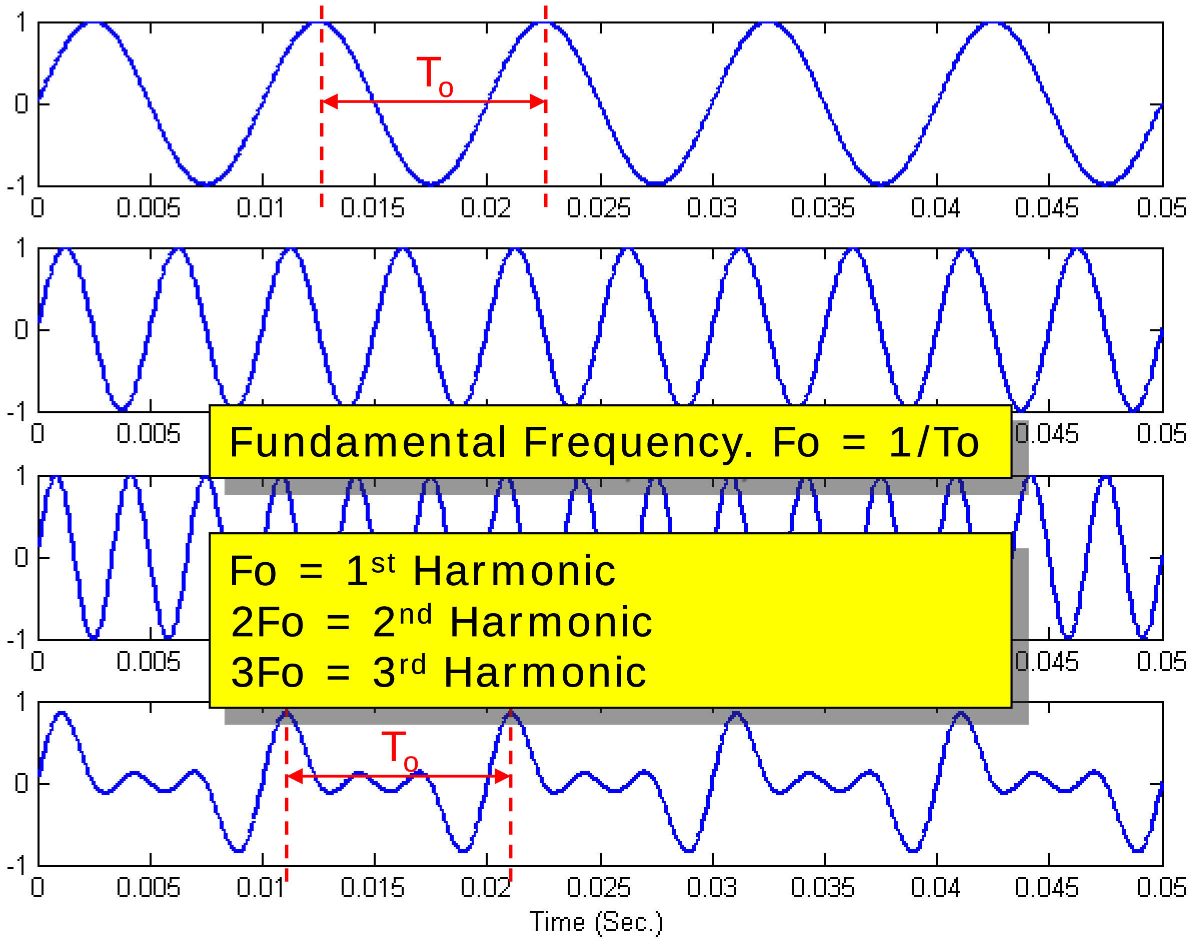
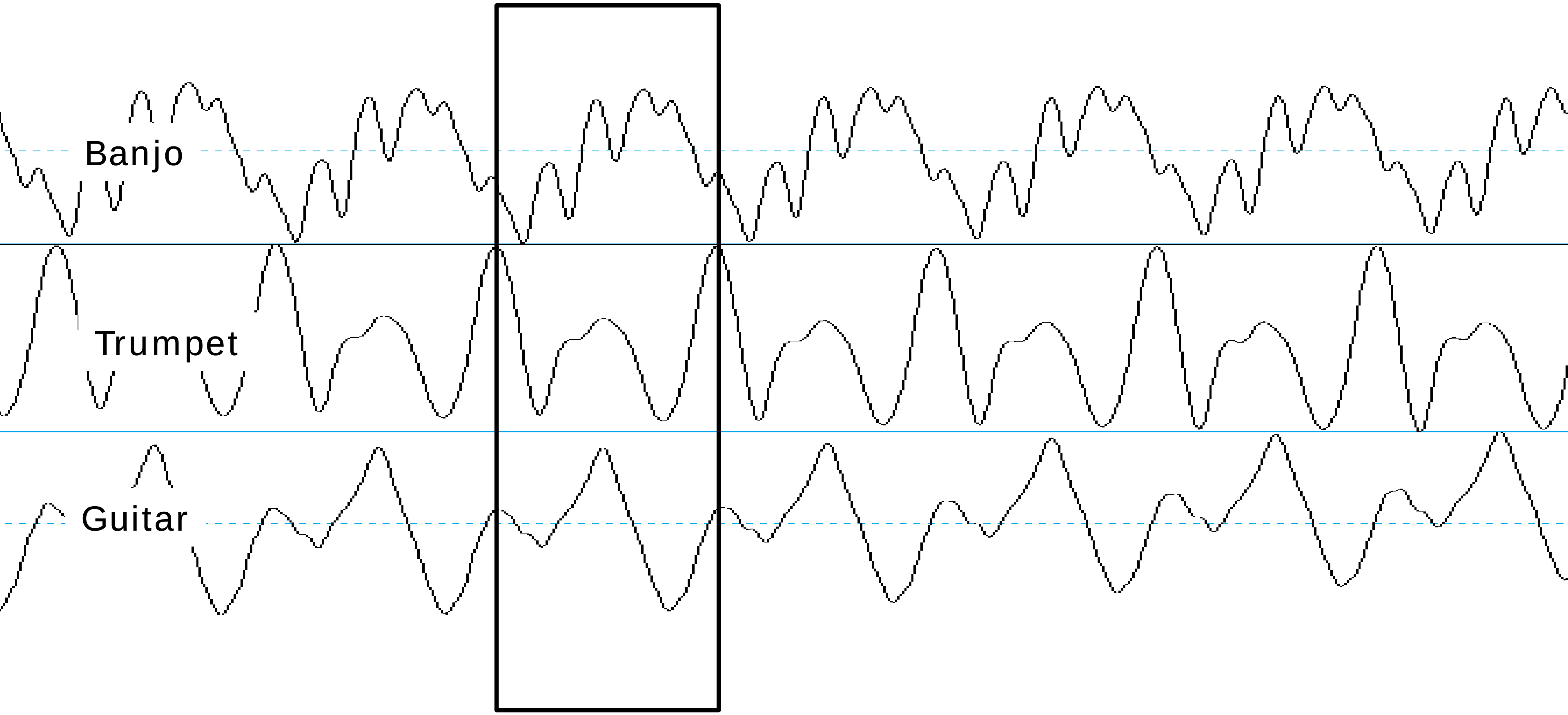


A Single Note of Musical Instruments



Complex Sinusoids

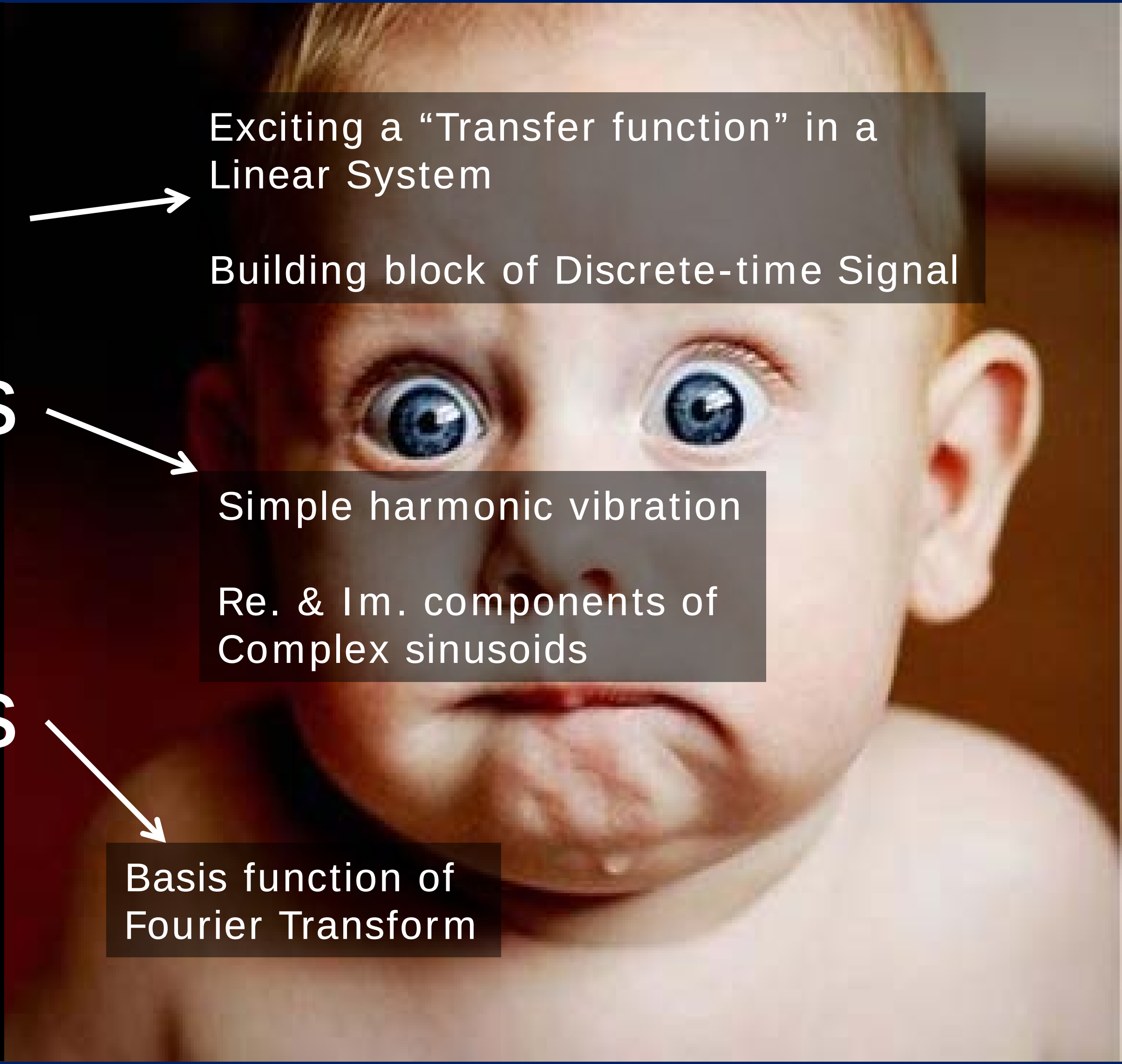
$$s(t) = Ae^{j(2\pi f t + \emptyset)}$$
$$= A\cos(2\pi f t + \emptyset) + j\,A\sin(2\pi f t + \emptyset)$$

Basis function of
Fourier Transform

See animation:

https://betterexplained.com/articles/an-interactive-guide-to-the-fourier-transform/

Our Friendly Signals



Impulses → Exciting a "Transfer function" in a Linear System

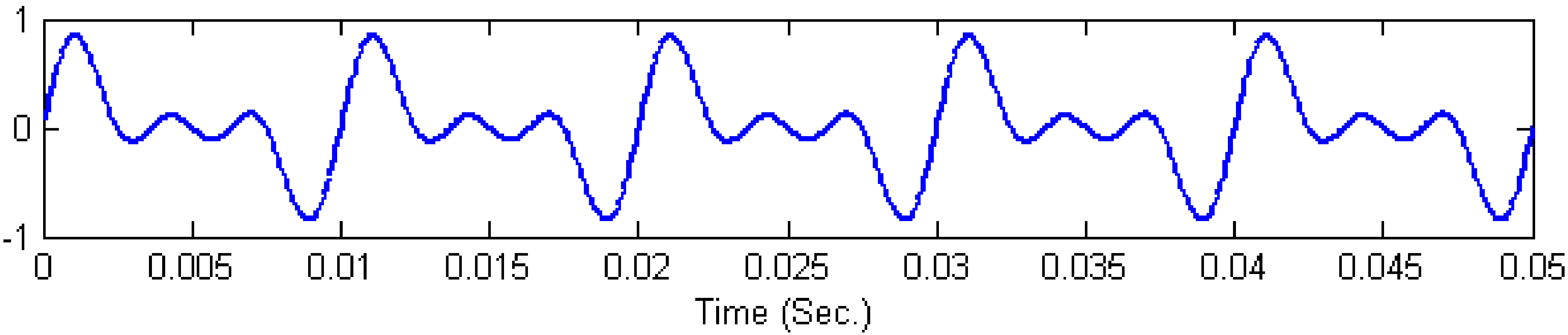
Building block of Discrete-time Signal

Sinusoids → Simple harmonic vibration

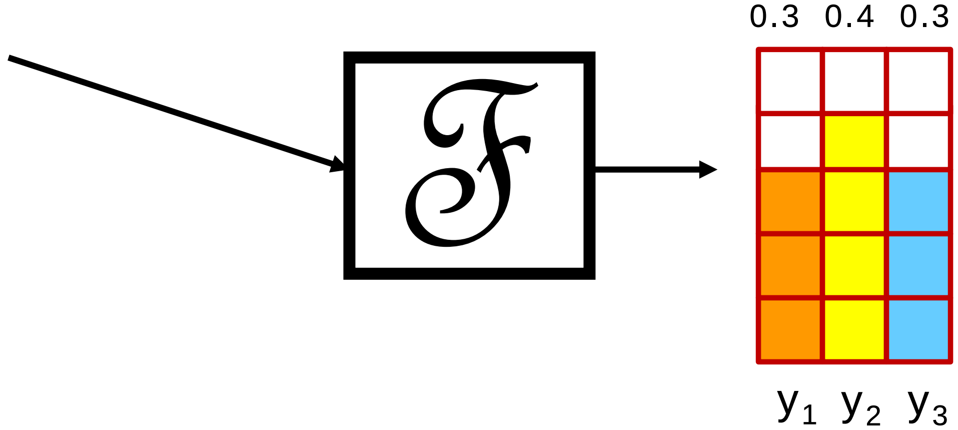
Re. & Im. components of Complex sinusoids

Complex Sinusoids → Basis function of Fourier Transform

Fourier Transform



$s =$????????????



$$y_1 = \sin(2\pi(100)t)$$
$$y_2 = \sin(2\pi(200)t)$$
$$y_3 = \sin(2\pi(300)t)$$

How does Fourier Transform work? : Basis Functions

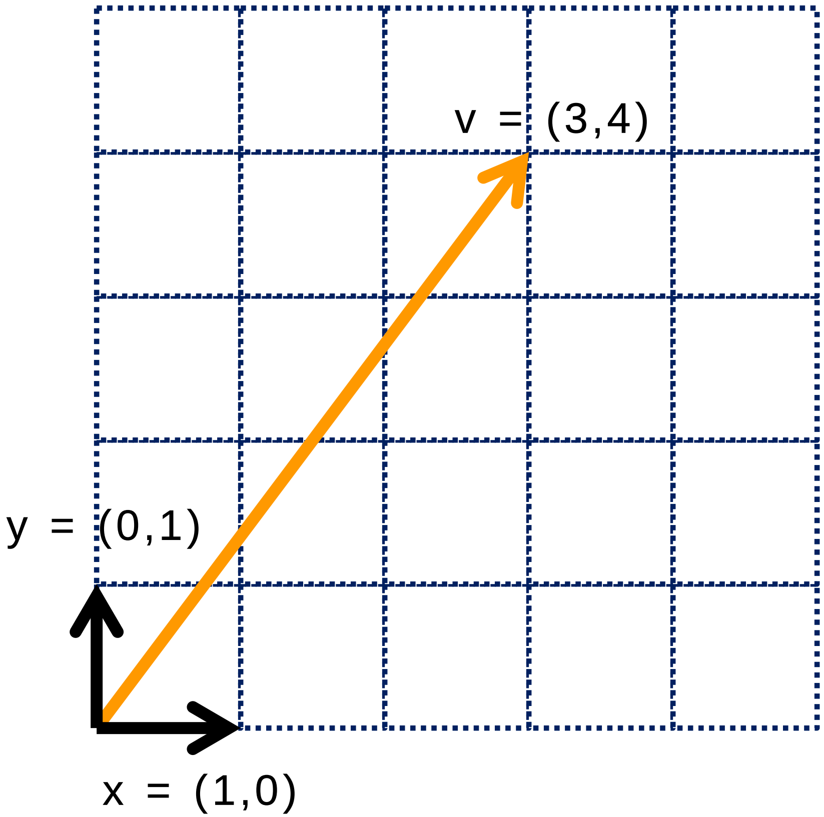
Every continuous function in
the function space can be represented as:

A linear combination of basis functions

(just as every vector in a vector space can be represented as a linear combination of basis vectors)

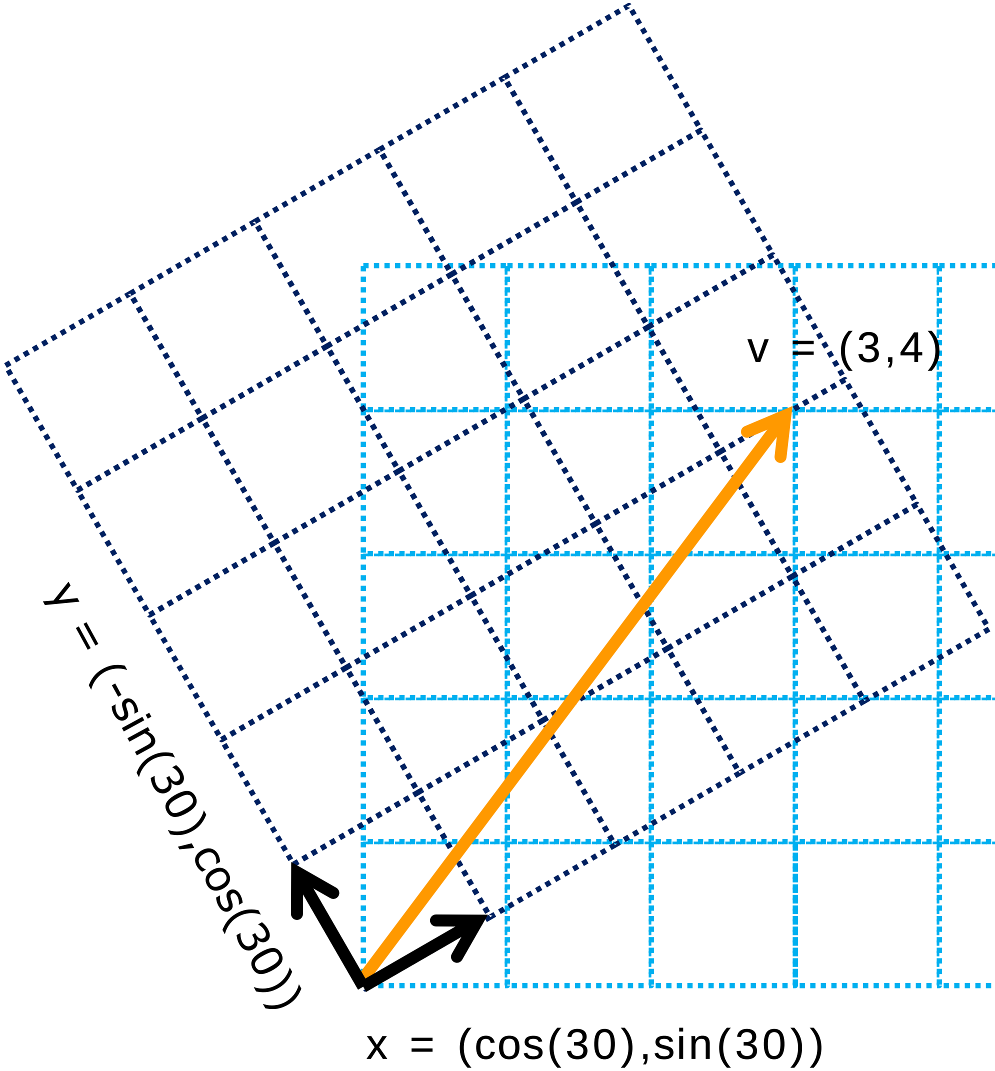
How does Fourier Transform work?: Vector Decomposition

Draw an analogy from a 2D Vector space



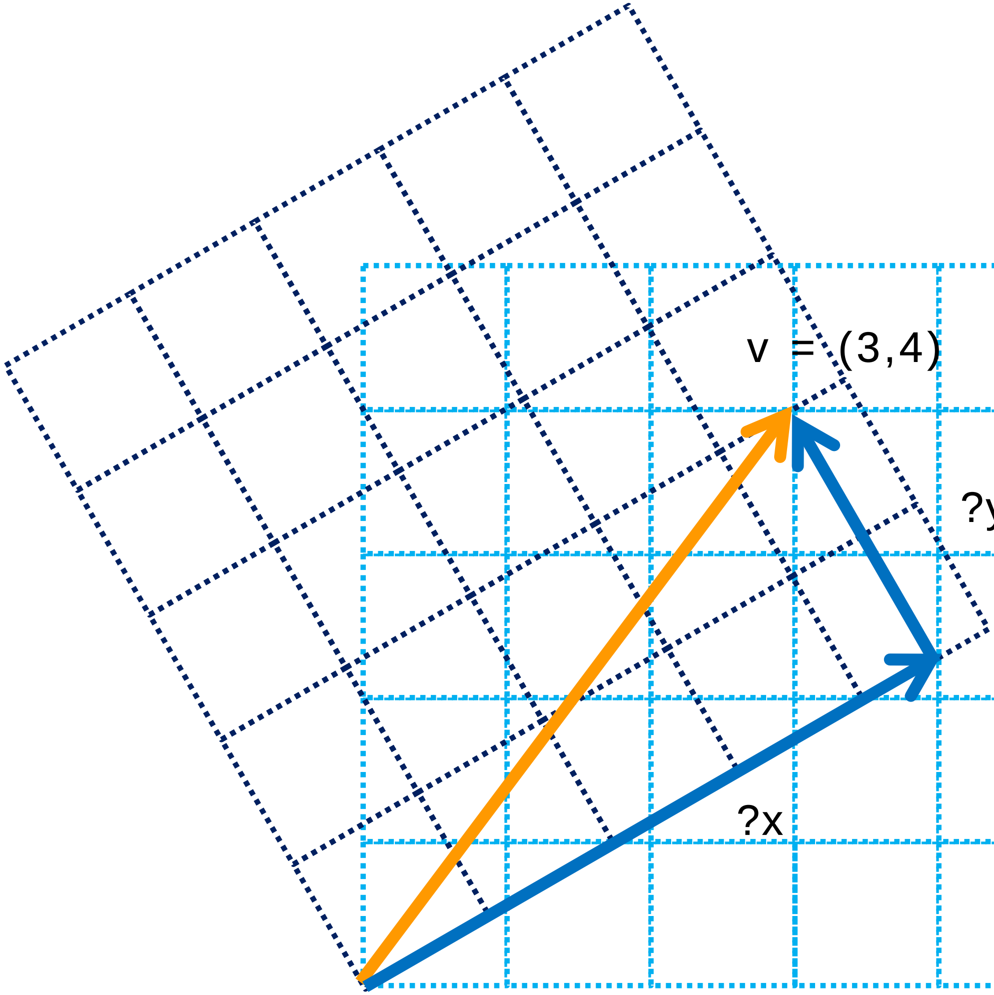
$$v = 3x + 4y$$

How does Fourier Transform work?: Vector Decomposition



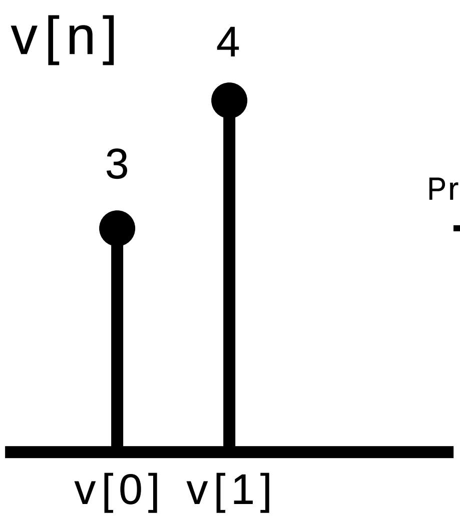
$$v = ?x + ?y$$

How does Fourier Transform work?: Vector Decomposition

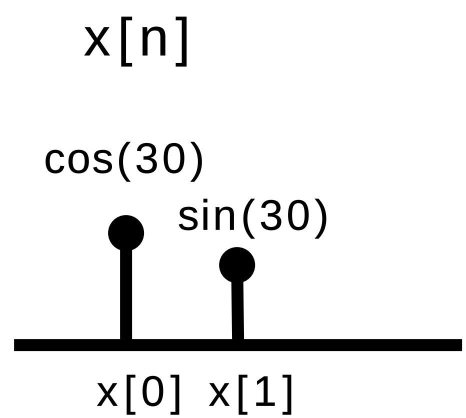


$$v = ?x + ?y$$

How does Fourier Transform work?: Vectors & Functions



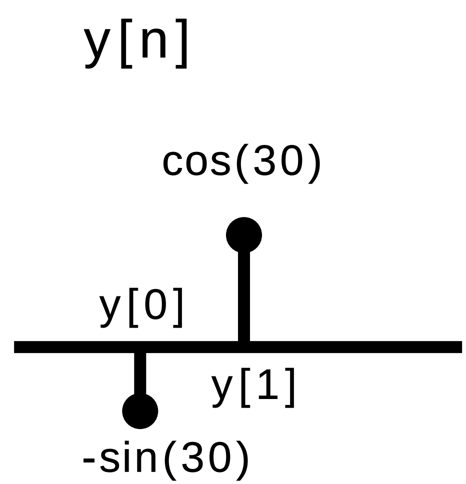
Project onto →



Length of projection of v onto x

$$v[0]x[0]+v[1]x[1]$$

Project onto →



Length of projection of v onto y

$$v[0]x[0]+v[1]x[1]$$

How does Fourier Transform work?: Inner Product of Complex Vectors

$$\langle x, y \rangle = x_1\bar{y}_1 + x_2\bar{y}_2 + \ \ldots + x_k\bar{y}_k$$

Complex Conjugate

Fourier Basis Functions

Every continuous function can be represented as a linear combination of *basis functions*.

$$e^{j(2\pi f t)}$$

any real number

Continuous-Time Fourier Transform (CTFT)

$$x(t) \Leftrightarrow \Im(x(t)) = X(f)$$

Fourier Transform	$\mathfrak{F}(x(t))$	$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$
Inverse Fourier Transform	$\mathfrak{F}^{-1}(X(f))$	$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df$

Continuous-Time Fourier Transform (CTFT)

$$x(t) \Leftrightarrow \Im(x(t)) = X(f)$$

Complex Conjugate of the Fourier basis with Fo = *f*

Fourier Transform	$\mathfrak{F}(x(t))$	$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$
[redacted]		

Projection of x(t) onto this basis

Continuous-Time Fourier Transform (CTFT)

$$x(t) \Leftrightarrow \Im(x(t)) = X(f)$$

Linear Combination of all Fourier basis

Integration by *df*

Inverse Fourier Transform	$\mathscr{F}^{-1}(X(f))$	$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$

Magnitude and Phase

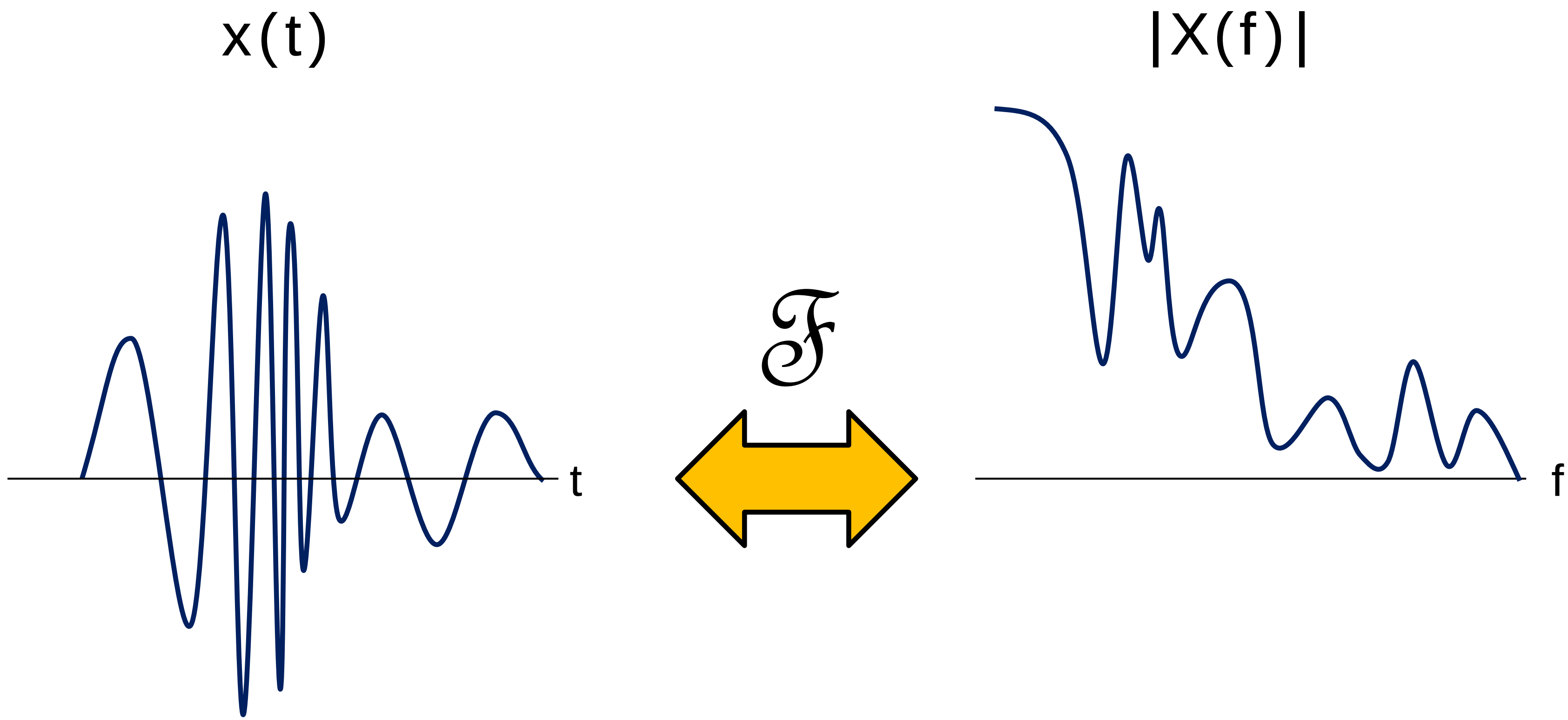
Fourier transform can be a complex number.

$$x(t) \Leftrightarrow X(f) = a + jb$$

$$|X(f)| = \sqrt{a^2 + b^2}$$

$$\angle X(f) = \arctan\left(\frac{b}{a}\right)$$

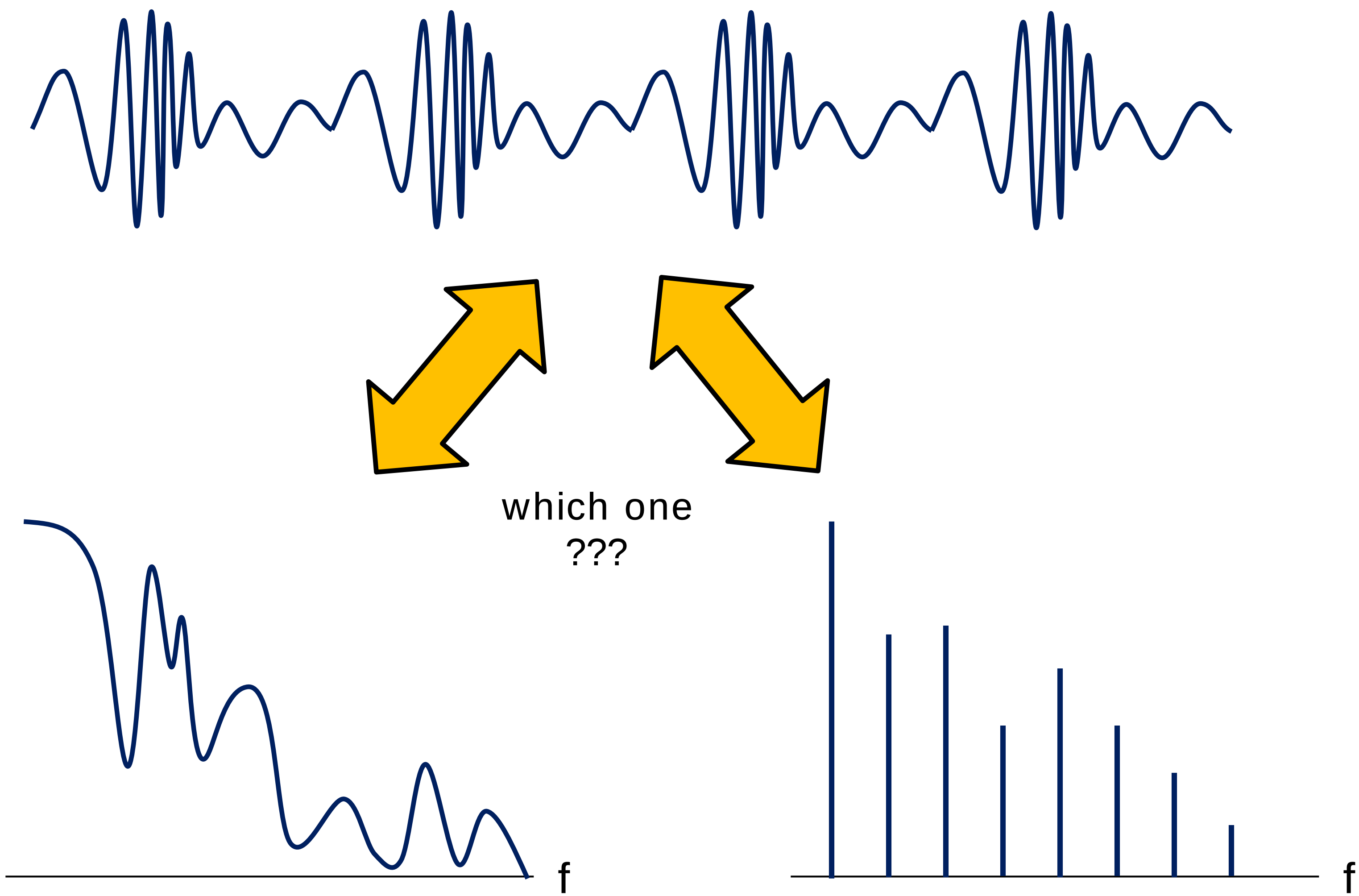
Waveform ⇔ Spectrum



Waveform — *Time-Domain*

Spectrum — *Frequency-Domain*

Spectra of Periodic Signals



Fourier Transform of an Impulse

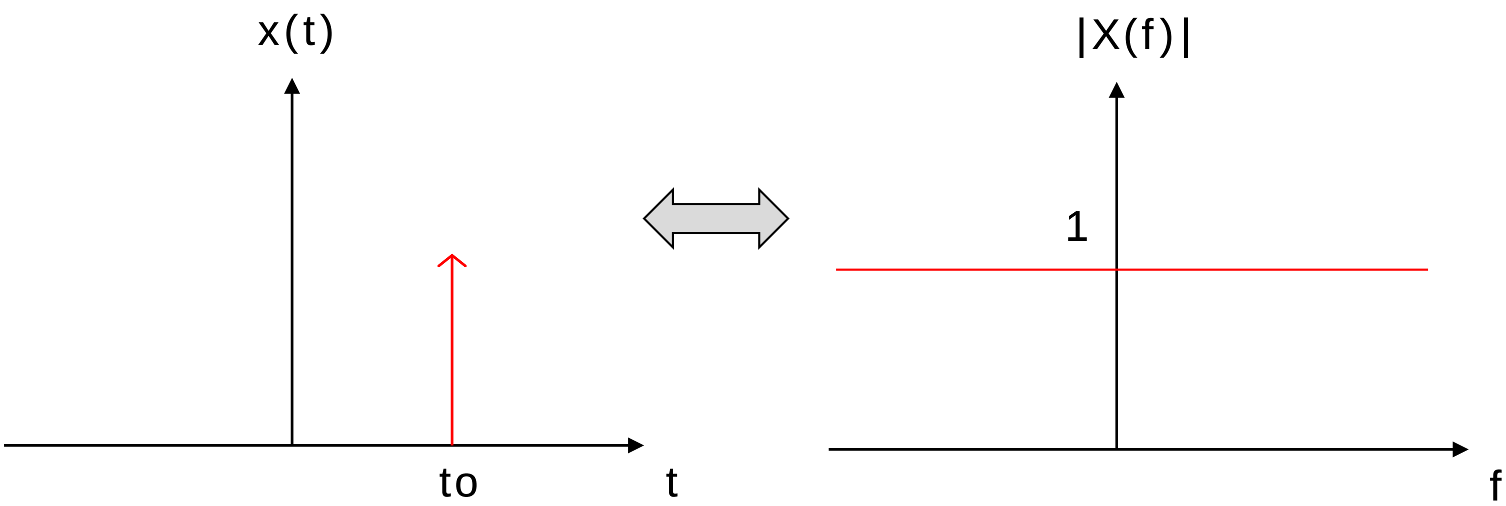
$$x(t) = \delta(t - t_0)$$

$$X(f) = \int_{-\infty}^{\infty} \delta(t - t_0) e^{-j2\pi ft} dt = e^{-j2\pi ft_o}$$

$$\boxed{\therefore \delta(t - t_0) \Leftrightarrow e^{-j2\pi t_0 f}}$$

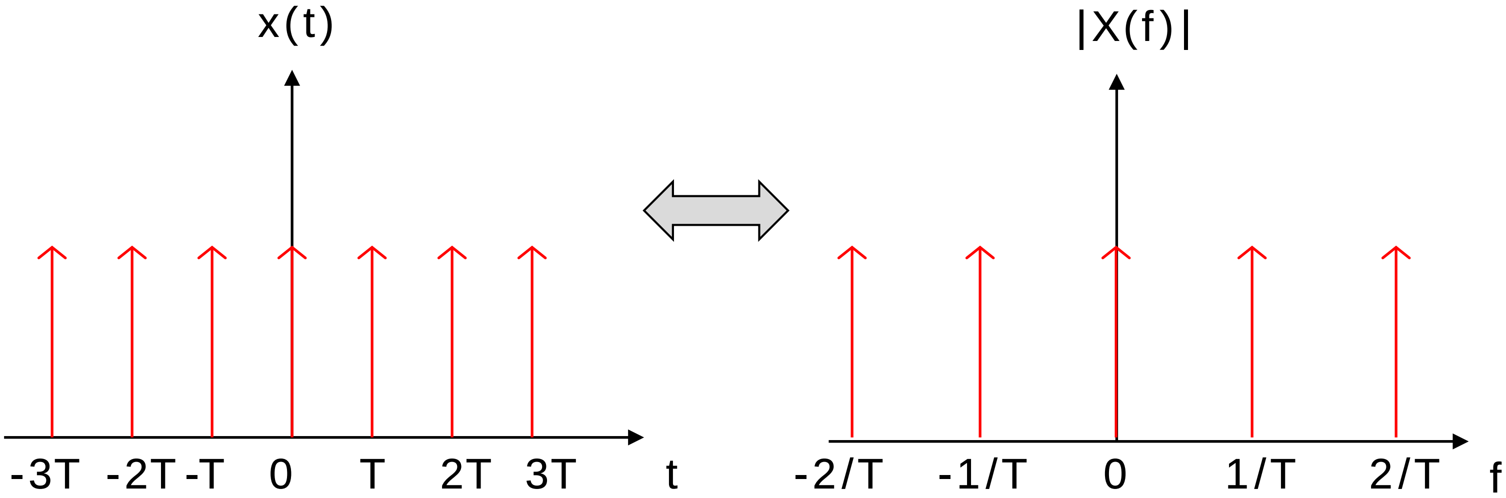
Fourier Transform of an Impulse

$$\delta(t-t_0) \Leftrightarrow e^{-j2\pi t_0 f}$$



Fourier Transform of Impulse Train

$$\sum_{n=-\infty}^{\infty} \delta(t-nT) \Leftrightarrow \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta\left(f-\frac{k}{T}\right)$$



Fourier Transform of Pure Sine

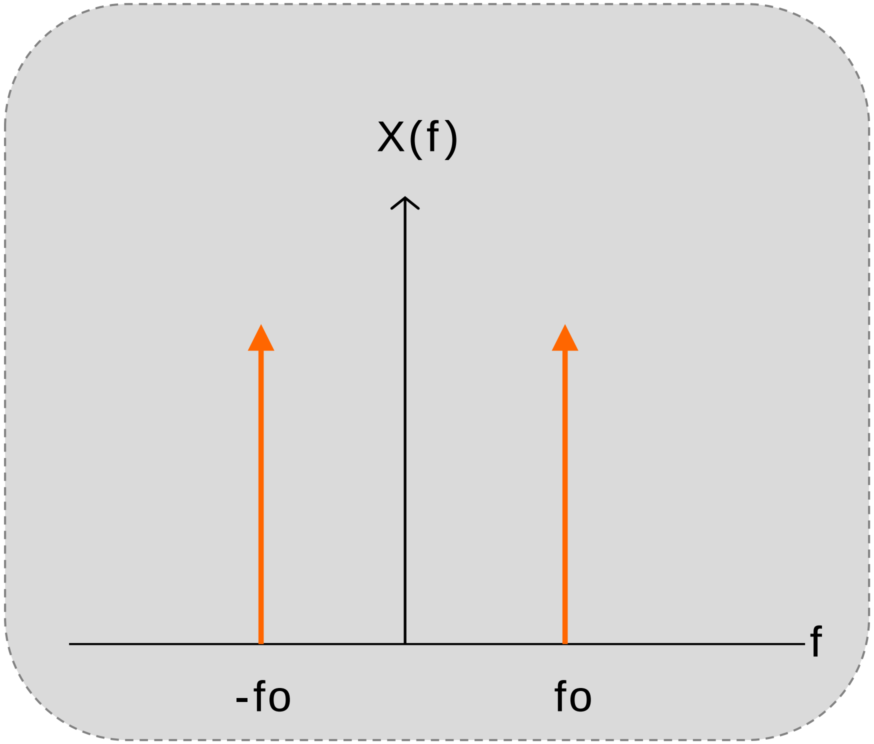
$$x(t) = \cos(2\pi f_0 t)$$

$$X(f) = \int_{-\infty}^{\infty} \cos(2\pi f_0 t) e^{-j2\pi f t} dt$$

$$= \int_{-\infty}^{\infty} \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} e^{-j2\pi f t} dt$$

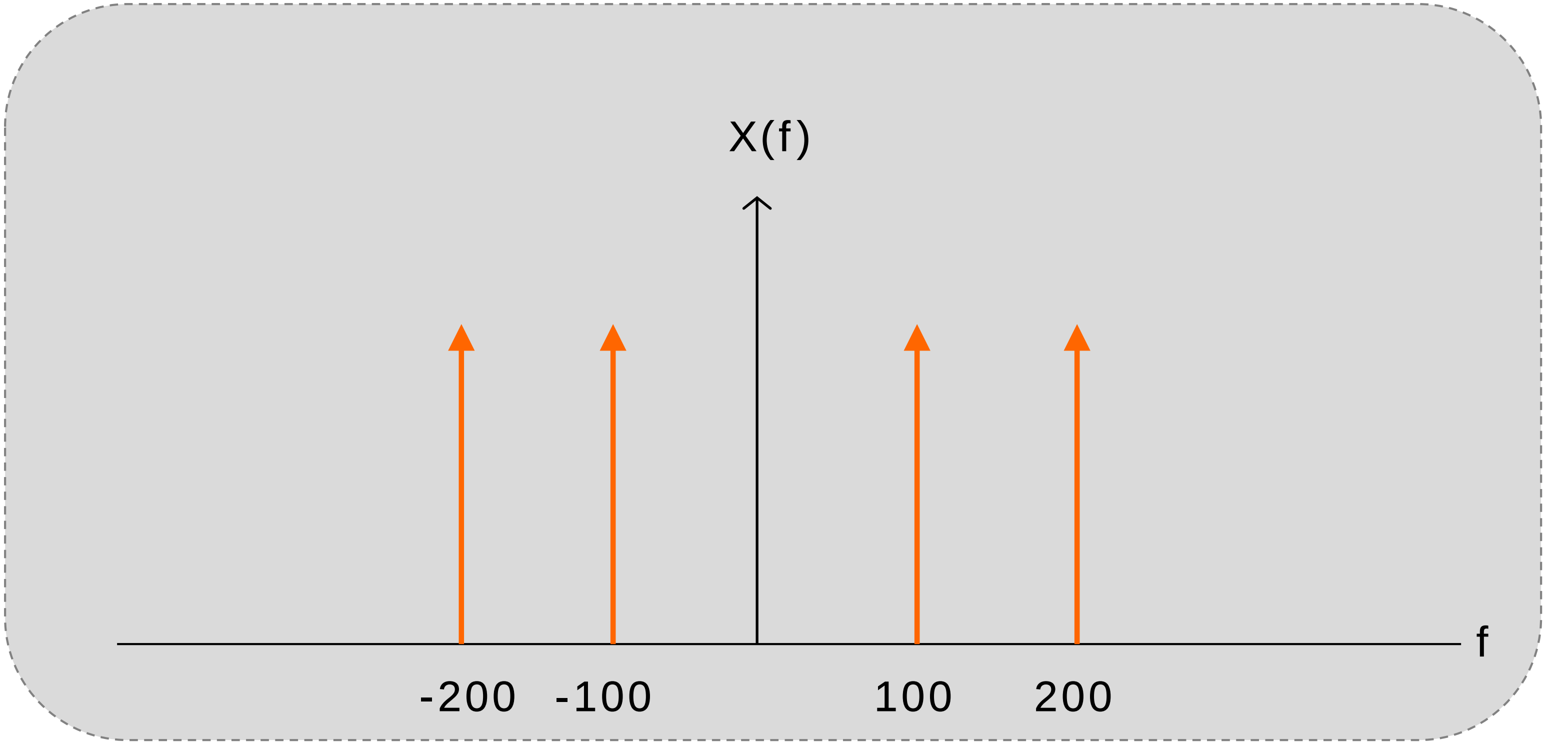
$$= \int_{-\infty}^{\infty} \frac{e^{-j2\pi (f-f_0)t} + e^{-j2\pi (f+f_0)t}}{2} dt$$

$$\boxed{= \frac{1}{2}\left(\delta(f - f_0) + \delta(f + f_0)\right)}$$



Frequency Components

$$x(t) = \cos(2\pi(100)t) + \cos(2\pi(200)t)$$



Fourier Series

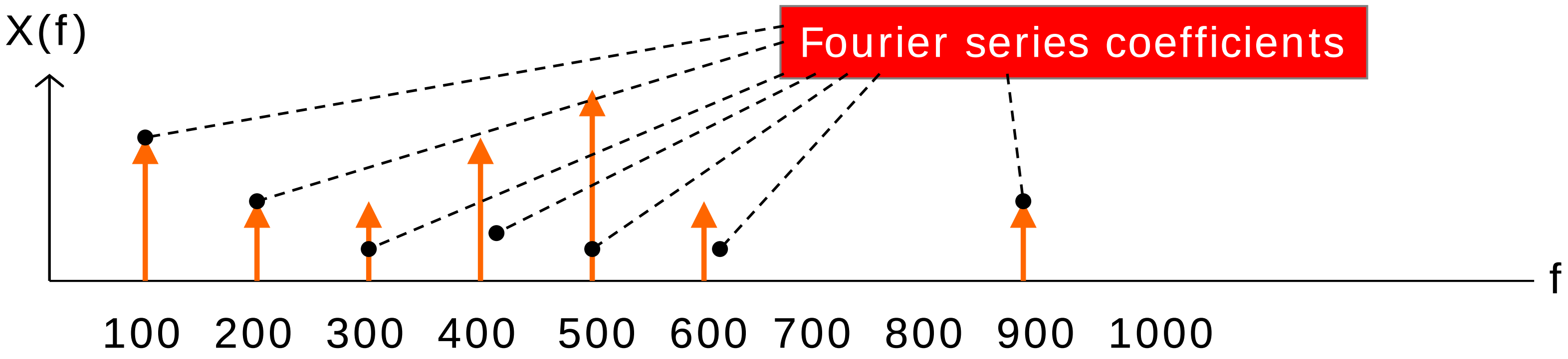
- Linear combination of sinusoidal signals with frequency:

  Fo

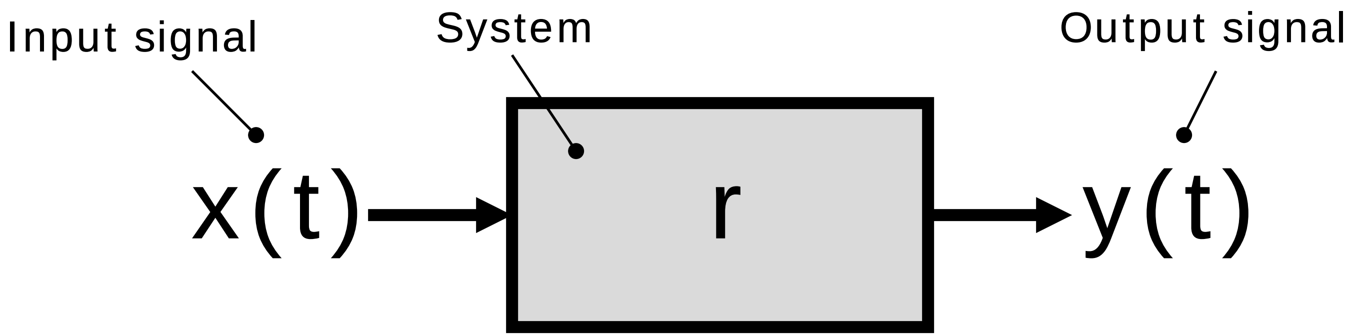
  nFo; n = 2,3,4,...

  results in periodic signals.

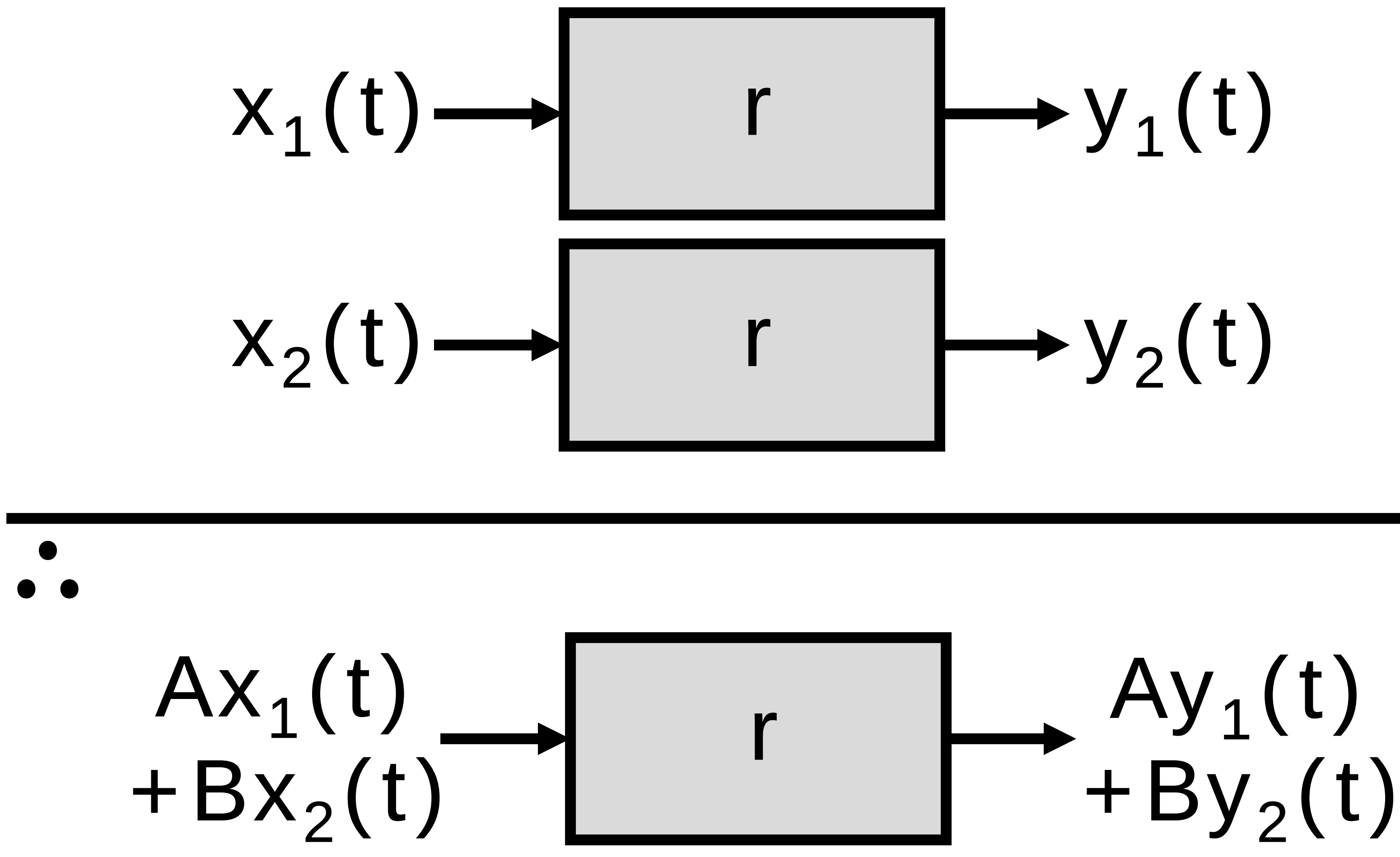
- Their Fourier transforms are discrete.



Linear Time-Invariant (LTI) Systems



Linear Time-Invariant (LTI) Systems



Linear Time-Invariant (LTI) Systems

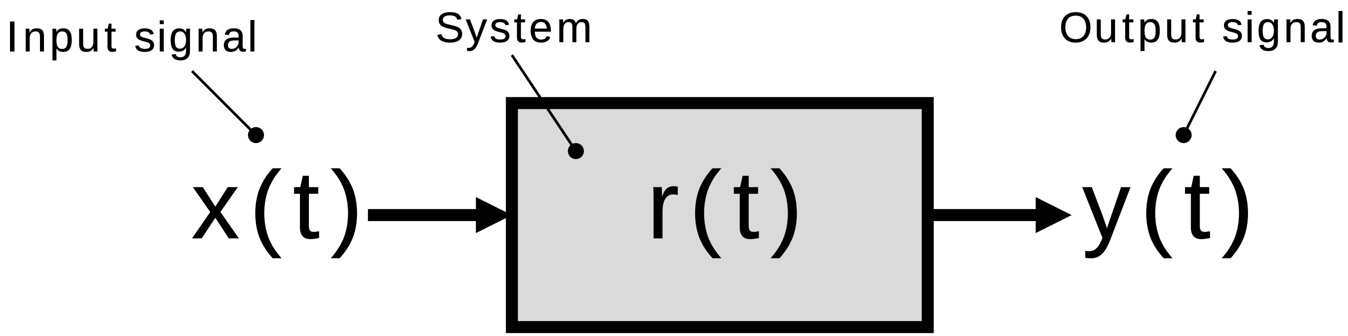
$$x_1(t) \longrightarrow \boxed{\quad r \quad} \longrightarrow y_1(t)$$

$\therefore$

$$x_1(t+\tau) \longrightarrow \boxed{\quad r \quad} \longrightarrow y_1(t+\tau)$$

Linear Time-Invariant (LTI) Systems

Any LTI system can be characterized entirely by *a single function, r(t)*

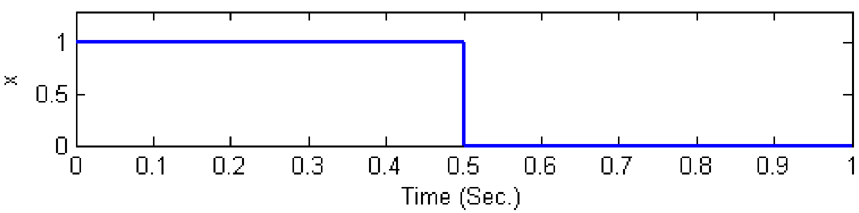


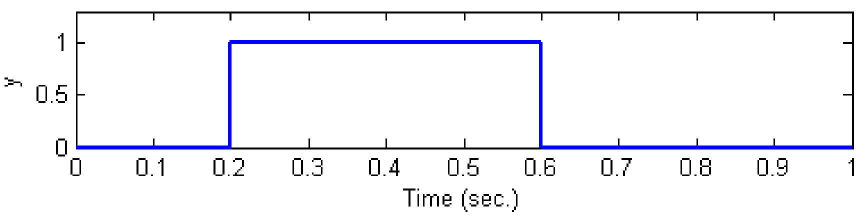
such that:

$$\boxed{y(t) = x(t) * r(t)}$$

Convolution

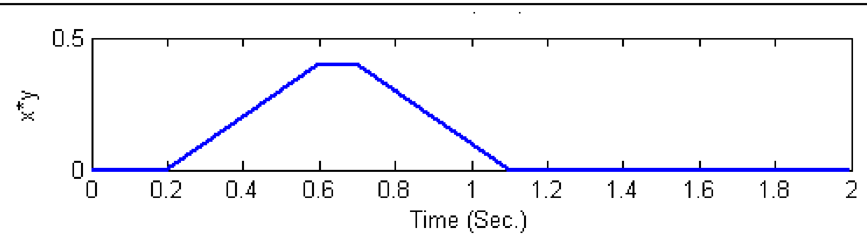
$$x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau) y(t-\tau) d\tau$$





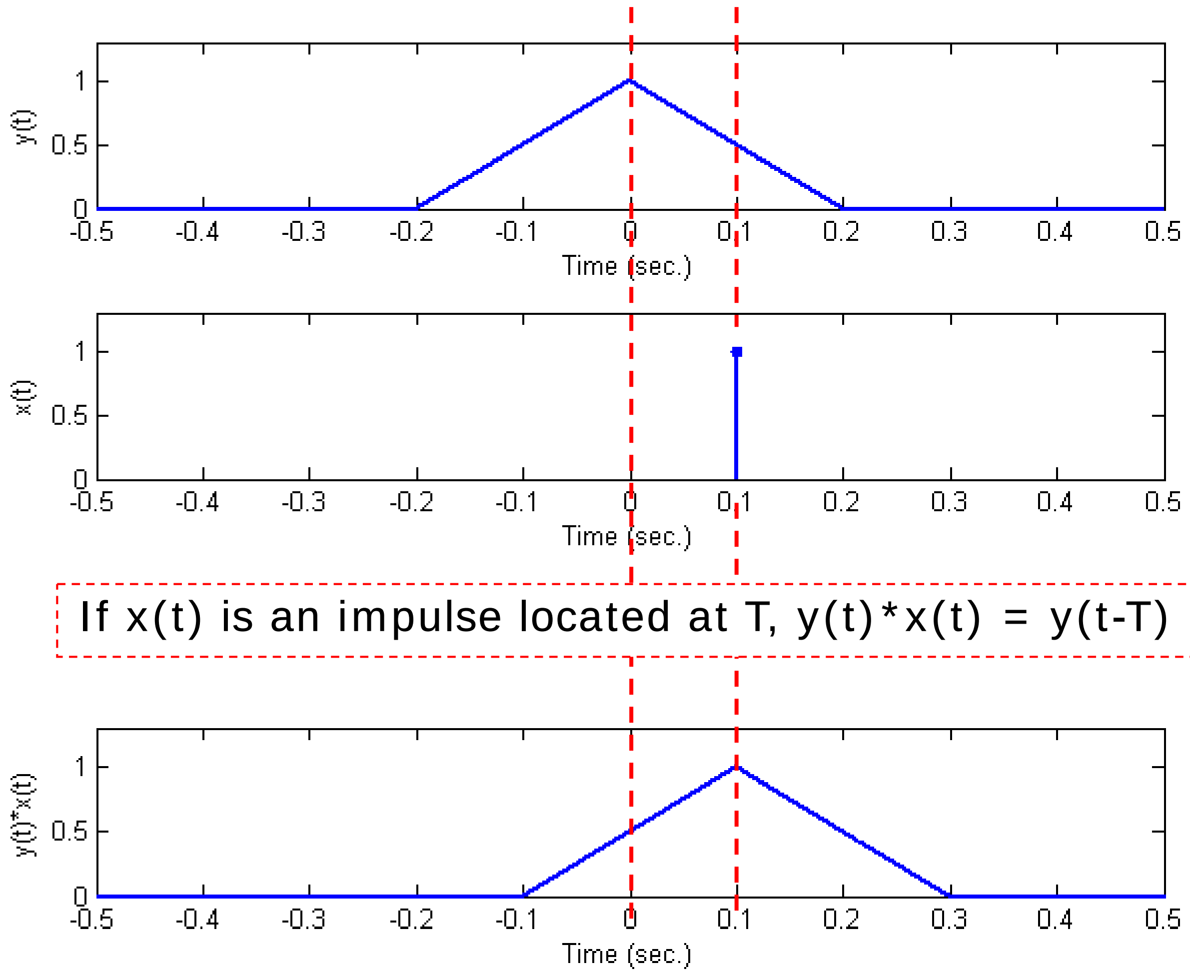
$$x(t) = \begin{cases} 0 & ; \quad -\infty < t < 0, \quad 0.5 \le t < \infty \\ 1 & ; \quad 0 \le t < 0.5 \end{cases}$$

$$y(t) = \begin{cases} 0 & ; \quad -\infty < t < 0.2, \quad 0.6 \le t < \infty \\ 1 & ; \quad 0.2 \le t < 0.6 \end{cases}$$



$$x(t) * y(t) = \begin{cases} 0 & ; -\infty < t < 0.2 \\ t - 0.2 & ; 0.2 \le t < 0.6 \\ 0.4 & ; 0.6 \le t < 0.7 \\ -t - 1.1 & ; 0.7 \le t < 1.1 \\ 0 & ; 1.1 \le t < \infty \end{cases}$$

Convolution with Impulse



If x(t) is an impulse located at T, y(t)*x(t) = y(t-T)

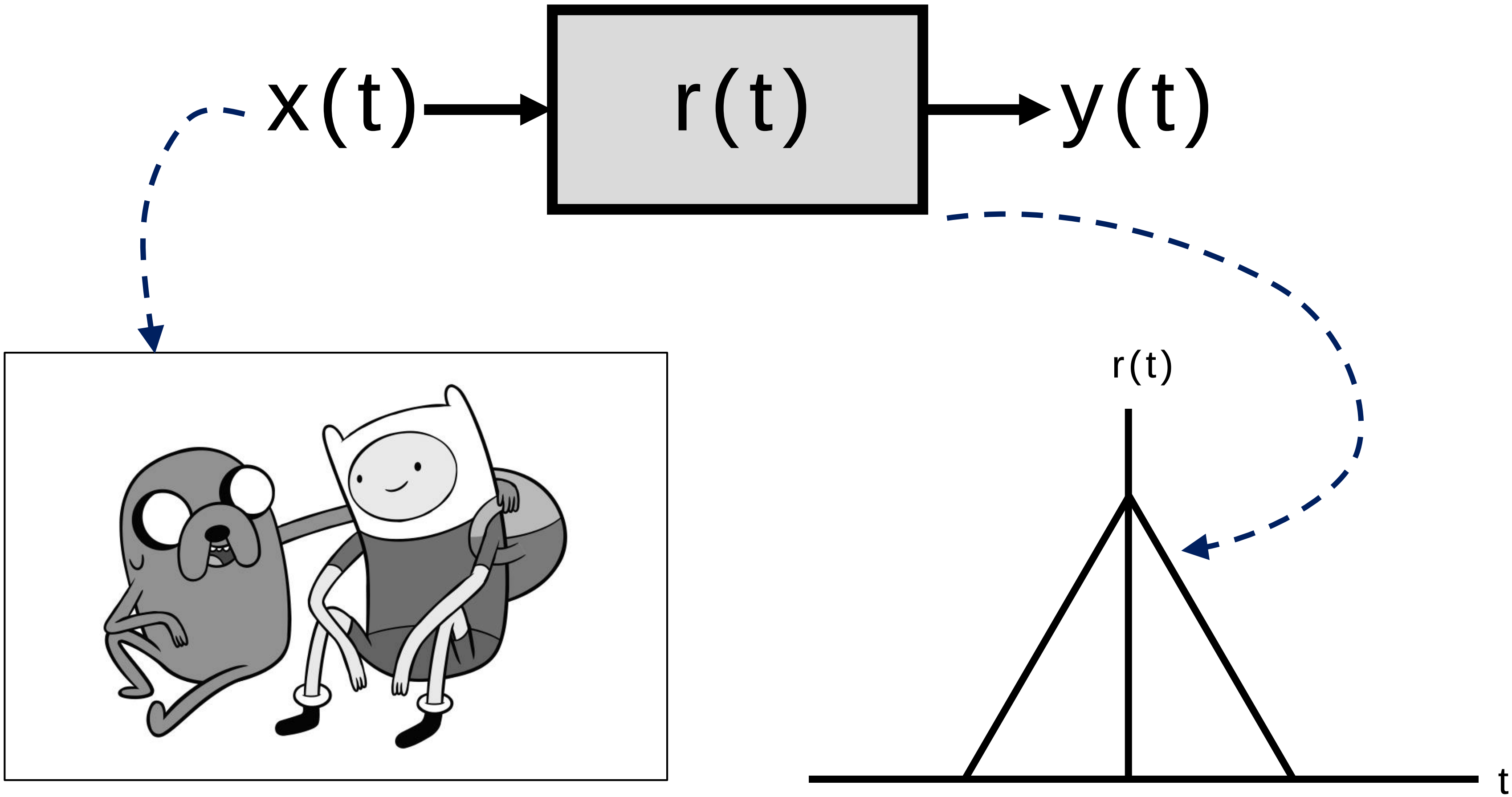
Linear Time-Invariant (LTI) Systems

$$x(t) \longrightarrow \boxed{r(t)} \longrightarrow y(t)$$

$$\boxed{y(t) = x(t) * r(t)}$$

Impulse Response

Linear Time-Invariant (LTI) Systems : Example

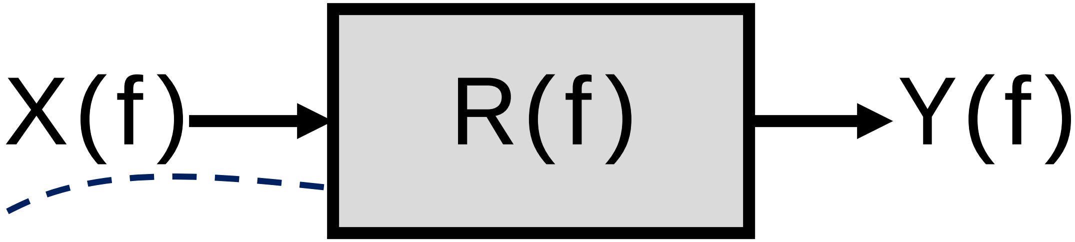


Properties of Fourier Transform

$$x(t) * y(t) \Longleftrightarrow X(f)Y(f)$$

$$x(t)y(t) \Longleftrightarrow X(f) * Y(f)$$

Linear Time-Invariant (LTI) Systems

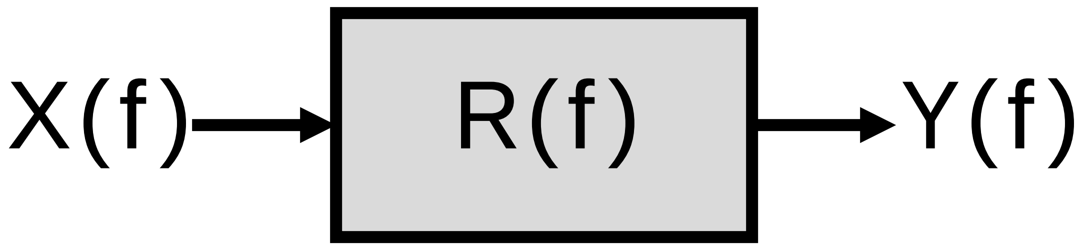


$$\boxed{Y(f) = X(f)R(f)}$$

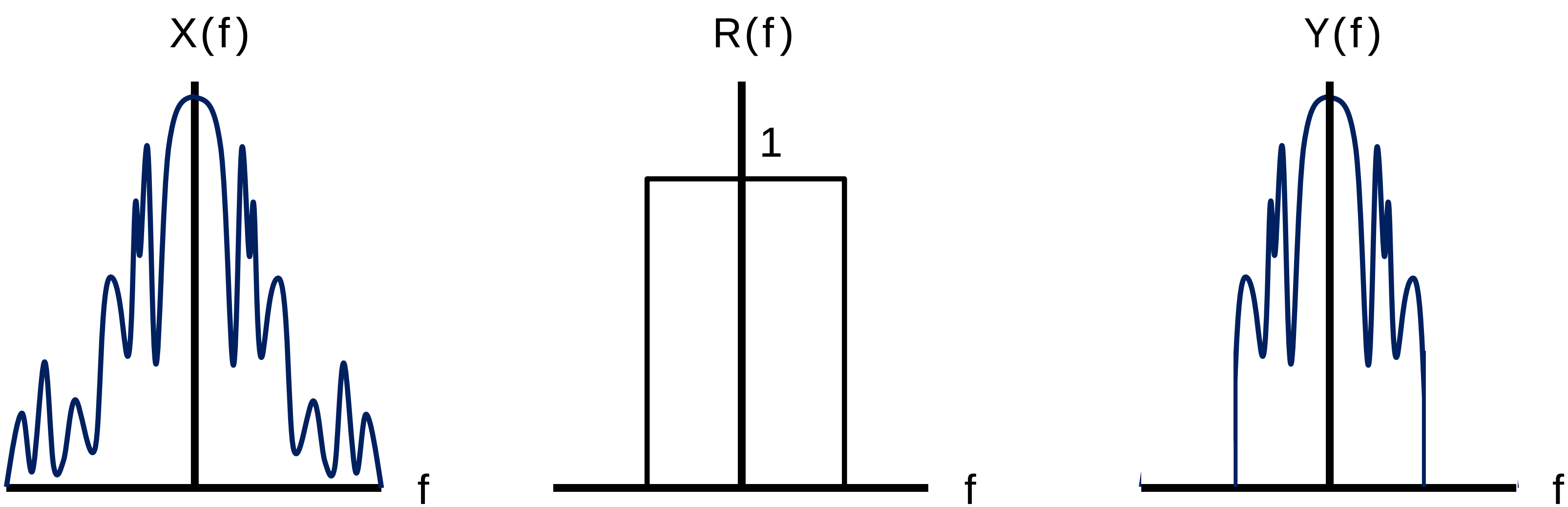
Filter

Transfer Function

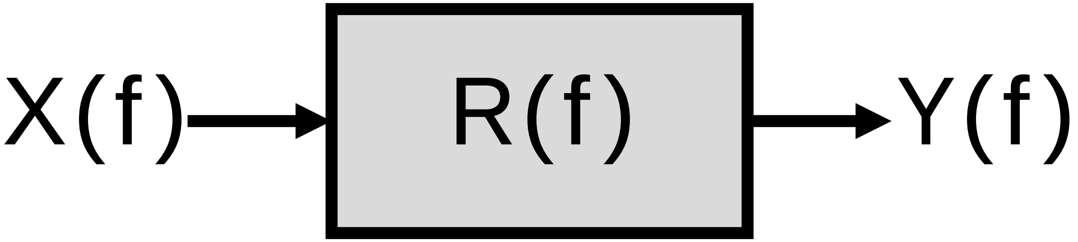
Low-Pass Filter



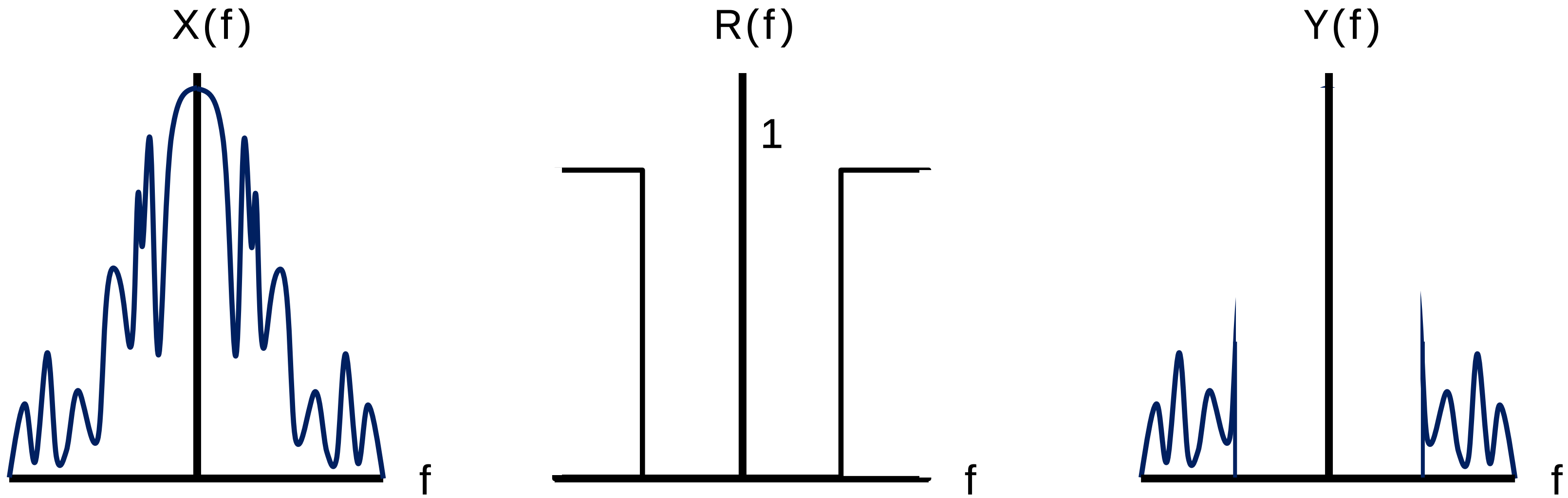
$$Y(f) = X(f)R(f)$$



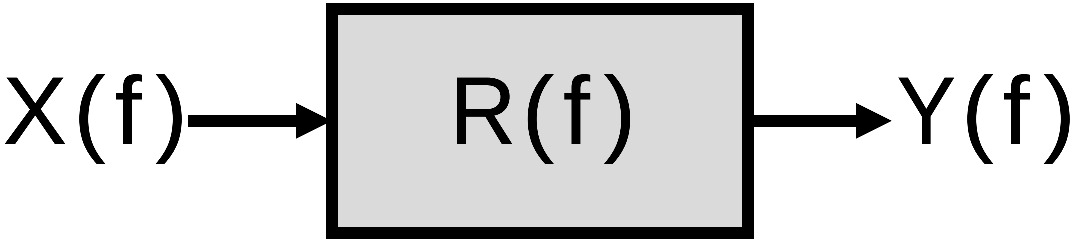
High-Pass Filter



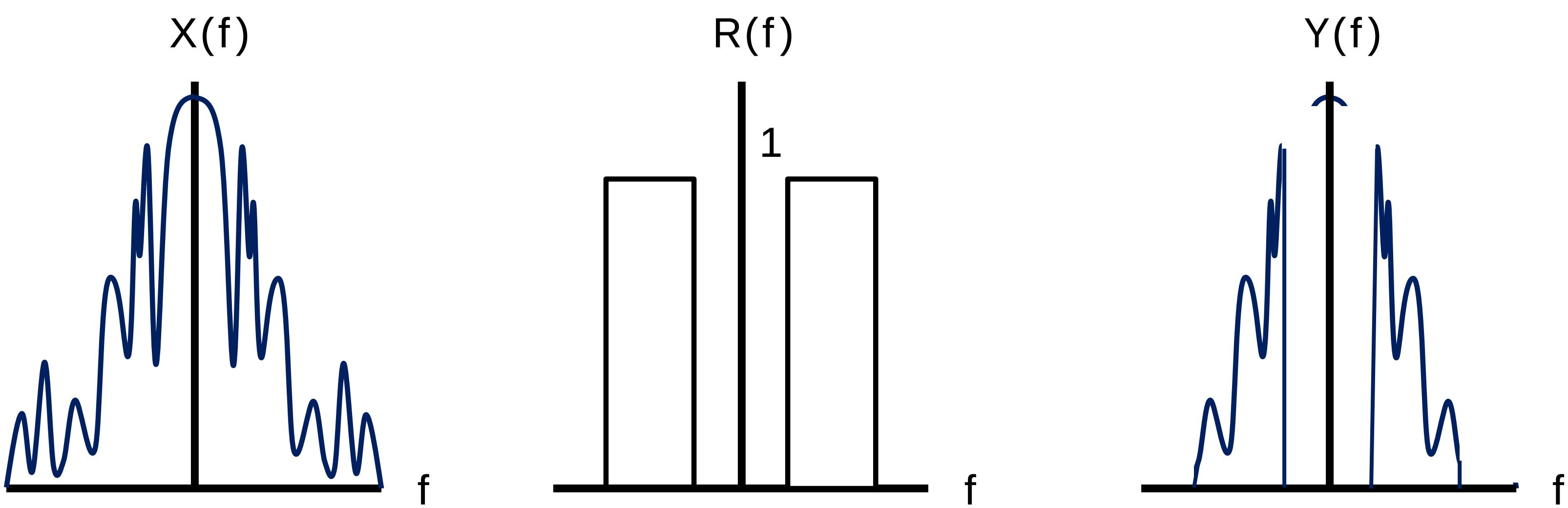
$$Y(f) = X(f)R(f)$$



Band-Pass Filter



$$Y(f) = X(f)R(f)$$

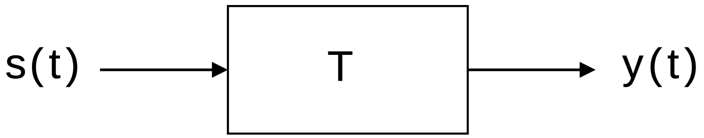


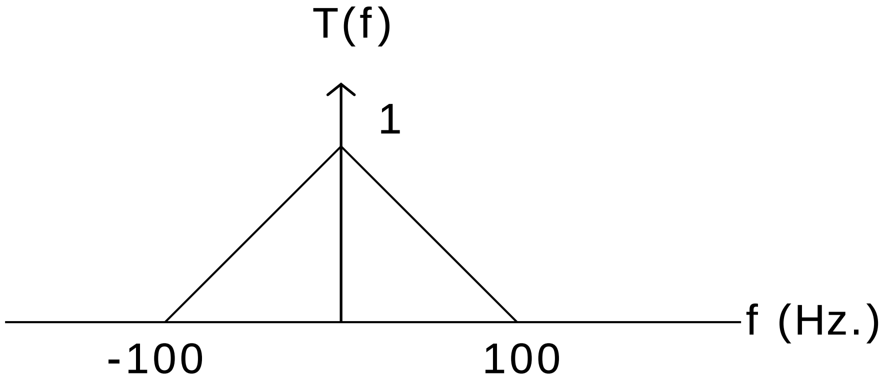
Exercise

- Find the Fourier transform of:

$$s(t) = 3\cos(2\pi(50)t) - \sin(2\pi(100)t)$$

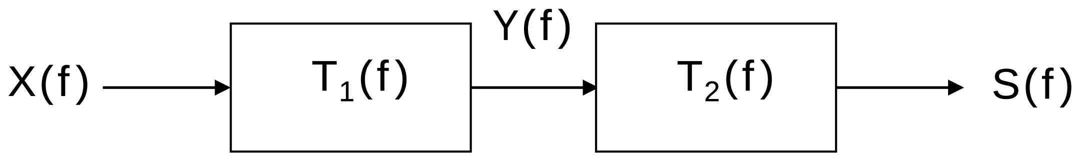
- For a linear system below:

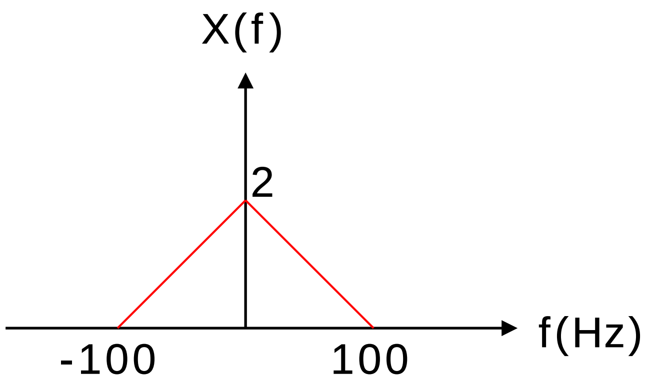


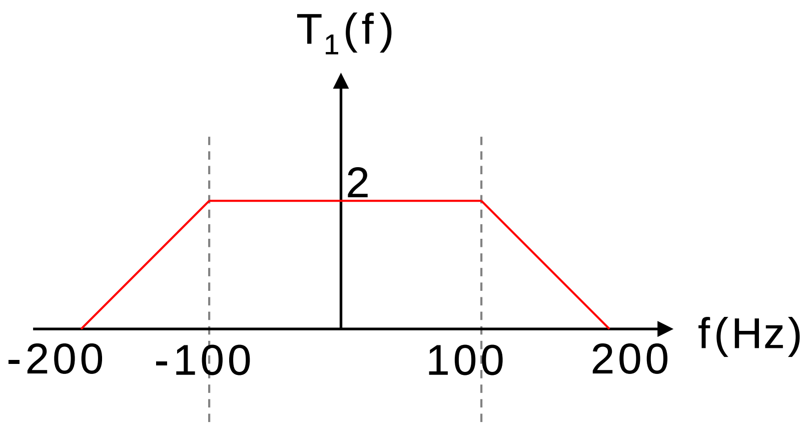


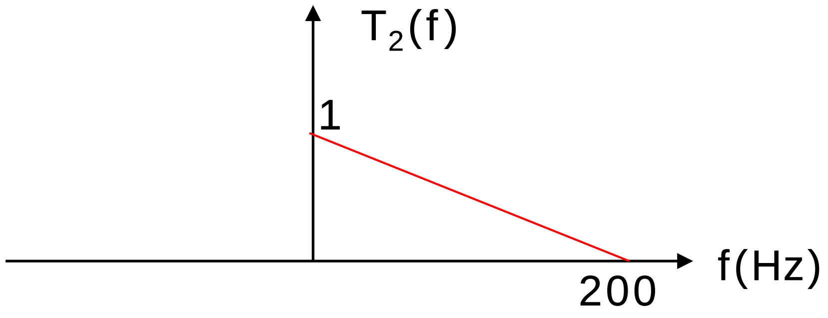
Find y(t).

Exercise









Sketch S(f)

What're Next?

- Real-world limitation
  - Finite signal length
- Sampling & Quantization
- Discrete-Time Fourier Transform (DTFT)
- Discrete Fourier Transform (DFT)
- Signal Analysis / Processing Tools