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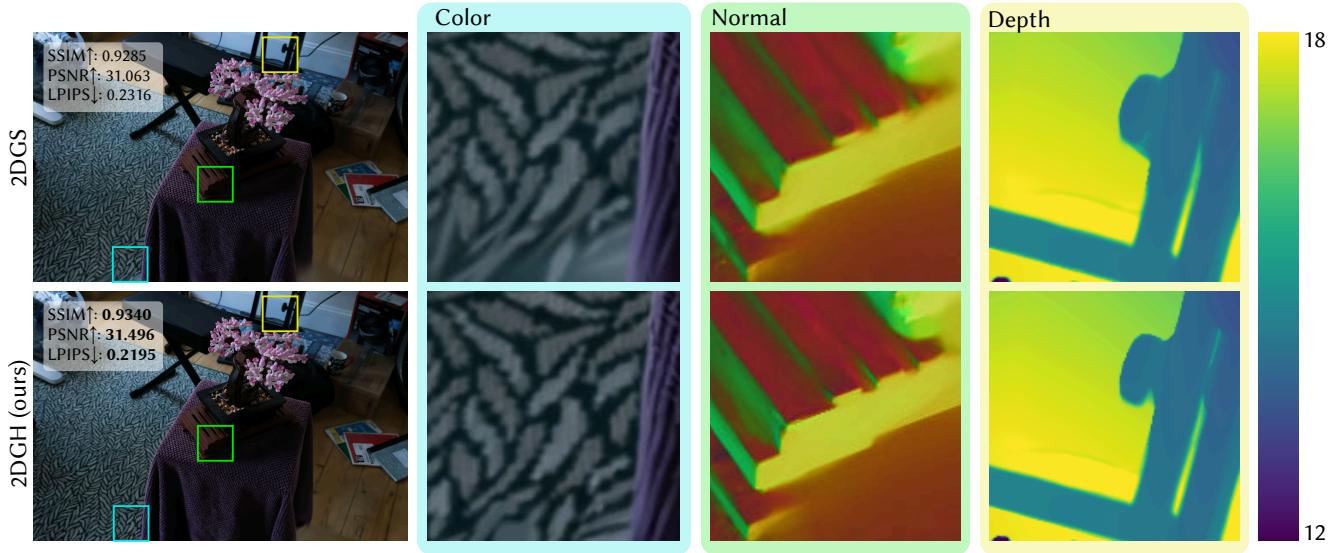


Figure 1. Compared to the current state-of-the-art 2D Gaussian Splatting (2DGS) [19], our proposed method, 2D Gaussian-Hermite Splatting (2DGH), demonstrates superior novel view synthesis performance in highly complex scenes while achieving comparable or even better geometric reconstruction quality under the same number of Gaussians. By modulating the Gaussian functions with Hermite series, the resulting Gaussian-Hermites exhibit a stronger representational capacity than original 2D Gaussians, particularly excelling in the reconstruction of fine complex structures and sharp discontinuous edges.

Abstract

2D Gaussian Splatting has recently emerged as a significant method in 3D reconstruction, enabling novel view synthesis and geometry reconstruction simultaneously. While the well-known Gaussian kernel is broadly used, its lack of anisotropy and deformation ability leads to dim and vague edges at object silhouettes, limiting the reconstruction quality of current Gaussian splatting methods. To enhance the representation power, we draw inspiration from quantum physics and propose to use the Gaussian-Hermite kernel as the new primitive in Gaussian splatting. The new kernel takes a unified mathematical form and extends the Gaussian

function, which serves as the zero-rank term in the updated formulation. Our experiments demonstrate the extraordinary performance of Gaussian-Hermite kernel in both geometry reconstruction and novel-view synthesis tasks. The proposed kernel outperforms traditional Gaussian Splatting kernels, showcasing its potential for high-quality 3D reconstruction and rendering.

1. Introduction

Novel-view synthesis (NVS) and 3D surface reconstruction are long-standing challenges in computer vision and graphics. Recently, 3D Gaussian Splatting (3DGS) [23] has emerged as a promising approach for NVS that strikes the balance between high resolution and real-time performance.

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While its volumetric nature enables various extensions such as global illumination [66], dynamic scenes [20, 49, 52] and anti-aliasing [28, 43, 50, 62], the 3D Gaussian math form essentially lacks of good definition of surfaces and fails to extract high quality mesh.

To achieve better surface reconstruction, subsequent research efforts like SuGAR [15], 2DGS [19], and Gaussian Surfels [9], strive to align the modified kernels with surfaces or make them behave more like surfaces. And from the perspective of mesh-based rendering, these research endeavors can be seen as bridging the gap between splatting primitives and traditional mesh facets.

However, there remains a gap between the shape primitives of polygon meshes and Gaussian splatting. Unlike polygons, which have clear edges, elliptical Gaussians lack the flexibility to be deformed in a way that can express sharp boundaries and complicated shape effectively. Enhancing the representation power of the shape primitives holds potential in improving the reconstruction quality of current Gaussian splatting methods.

On the other hand, the Gaussian function has a well-established history in various fields, such as statistics [4, 30], physics [11, 24, 41], electronics [26, 53], holding an irreplaceable role in signal analysis [25, 51].

The wave function in quantum physics shares conceptual similarities with Gaussian Splatting, in that both describe the spatially-varying distribution of matter or particles using Gaussian functions. Furthermore, quantum physics indicates that beyond the standard Gaussian, there exists a family of higher-rank Gaussian functions, such as Gaussian-Hermite polynomials [39]. These Gaussian-Hermite polynomials serve as the solutions to the quantum harmonic oscillation equation, a fundamental model in quantum physics. And they are used to address the interaction between photons and electrons, which is directly relevant to the underlying physics of volume scattering in physics-based rendering [5].

Consequently, one can expect that the Gaussian-Hermite polynomials, which model electron orbital distributions in quantum physics, could potentially be adopted to model the opacity distribution in Gaussian splatting. The opacity distribution can be seen as a macroscopic perspective of the electron orbital distribution, as both reflect the fundamental forms that compose the matter we observe.

Inspired by research on Gaussian-based functions in quantum physics, we propose using a unified representation, Gaussian-Hermite polynomials as the kernel for Gaussian Splatting. With higher-rank terms, this kernel can deform beyond the elliptical shape and better express sharp boundaries. Modifying the shape primitive is independent of and compatible with recent advancements in other aspects of Gaussian splatting.

Naively adding high-rank functions, however, can result

in invalid opacity values. Therefore, we propose techniques to handle the large and negative coefficients of the high-rank Gaussian-Hermite function, ensuring the resulting opacity falls within the valid range of $[0, 1]$.

We compare our proposed kernel with previous ones on Gaussian Splatting tasks, and our experiments show that the Gaussian-Hermite polynomial kernel achieves state-of-the-art performance in terms of NVS and geometry reconstruction quality compared to previous Gaussian kernels.

In summary, we make the following contributions:

- We first introduce a family of Gaussian-Hermite kernels with higher representation power in Gaussian Splatting, and prove that the original Gaussian kernel is the zero-rank case of the new formulation.
- To enable the use of Gaussian-Hermite polynomials in opacity modeling, we propose a new activation function that can handle high-order coefficients in the alpha blending process.
- We conduct experiments to compare our proposed kernel with previous ones in Gaussian Splatting, showing that our method improves reconstruction around shape boundaries and achieve state-of-the-art performance in surface reconstruction and novel-view synthesis.

2. Related Work

2.1. Novel View Synthesis

Performing novel view synthesis (NVS) from a set of input images has been central to computer graphics and vision research. Neural Radiance Fields (NeRF) [32] utilizes neural networks to model radiance field in space, thereby enabling NVS. Following NeRF, a significant number of NeRF-based works have notably expanded its capabilities, such as alleviating the aliasing issues [1, 2, 18], extending NeRF to unbounded scenes [2, 64], and improving rendering [7, 17, 37, 38, 57, 58] and training [3, 6, 8, 13, 29, 33, 44] efficiency of NeRF.

The emergence of 3D Gaussian Splatting (3DGS) [23] has allowed for real-time NVS, with further extensions [14, 15, 20, 28, 36, 43, 48, 50, 52, 62, 63, 66, 67]. Among the enhancements to 3DGS, GES [16] reformulates the mathematical representation of 3D Gaussians by introducing the Generalized Exponential Function (GEF) to represent 3D scenes, demonstrating a stronger scene representation capability compared to the original 3D Gaussians.

Recently, 2D Gaussian Splatting (2DGS) [19] has gained attention as an innovative approach that simplifies 3D scene representation by reducing volumetric data into 2D oriented Gaussian disks. 2DGS demonstrates superior geometric reconstruction performance compared to 3DGS without sacrificing efficiency; however, it exhibits a decline in both qualitative and quantitative metrics for NVS.

Our work introduces a new mathematical representation

for 2D Gaussians involving Hermite series, which enhances the scene representation capability of 2DGS.

2.2. Multi-view 3D Reconstruction

Multi-view 3D reconstruction has seen substantial advancements with traditional methods like Multi-view Stereo (MVS) [40, 54, 59], but has been further revolutionized by neural approaches. Neural approaches such as Occupancy Networks [31] and Neural Implicit Surfaces [34, 55] use multi-layer perceptrons (MLPs) to represent 3D geometry. Subsequent works [35, 45, 56] employ volume rendering techniques to render implicit surfaces. More recent innovations include scalable approaches [27, 60, 61] and methods targeting high efficiency in complex scenes [6, 46, 58]. Methods based on Neural Implicit Surfaces and volume rendering are not highly efficient. 3DGS [23] employs an explicit scene representation, significantly improving training and rendering speed. However, the lack of well-defined boundaries results in degraded geometric reconstruction quality. To address this issue, various methods, including SuGaR [15], GOF [63], Gaussian Surfels [9], and 2DGS [19], have been proposed. The most recent and well-known 2DGS has demonstrated state-of-the-art geometric reconstruction performance. Building on 2DGS, our approach introduces a new Gaussian mathematical formulation, improving the reconstruction of fine structures and discontinuous surface connections.

3. Methods

3.1. Preliminaries

3D Gaussian Splatting (3DGS) [23] proposes to represent 3D scenes with many translucent 3D Gaussian ellipsoids and render images through rasterization. Specifically, 3DGS parameterizes Gaussian primitives via mean position (μ_k), opacity α and covariance matrix Σ :

$$\mathcal{G}(\mathbf{x}, \alpha, \Sigma) = \alpha \exp \left(-\frac{1}{2} (\mathbf{x} - \boldsymbol{\mu}_k)^\top \Sigma^{-1} (\mathbf{x} - \boldsymbol{\mu}_k) \right) \quad (1)$$

where \mathbf{x} is a point position in 3D world space and the covariance matrix is factorized into a rotation matrix \mathbf{R} and a scaling matrix \mathbf{S} :

$$\Sigma = \mathbf{R} \mathbf{S} \mathbf{S}^\top \mathbf{R}^\top \quad (2)$$

However, due to the lack of view consistency during projection, 3DGS suffers from poor geometry reconstruction performance. 2DGS [19] proposes that diminishing one scaling to zero and making primitive more like surface can contribute to view consistency. In 2DGS, Gaussians can be represented by:

$$\mathcal{G}(\mathbf{x}) = \exp \left(-\frac{u(\mathbf{x})^2 + v(\mathbf{x})^2}{2} \right) \quad (3)$$

where $u(\mathbf{x})$ and $v(\mathbf{x})$ are local UV space coordinates.

In the rasterization process, 3DGS will project the Gaussian primitives onto a 2D manifold with EWA approximation [68]. Previous work [69] has demonstrated that this projection is reliable only at the center of the Gaussian function, with the approximation becoming less precise as the distance from the central point increases. 2DGS [19] shows that it will lead to unstable optimization during differentiable rendering and introduce Ray-Splat Intersection [42] to address this issue.

Given central position \mathbf{p}_k , a scaling vector $\mathbf{S} = (s_u, s_v)$ that controls the covariance of a 2D Gaussian and a 3×3 rotation matrix $\mathbf{R} = [\mathbf{t}_u, \mathbf{t}_v, \mathbf{t}_w]$ that controls 2D Gaussian orientation, the transformation between UV space and world space can be written as the following:

$$\mathbf{H} = \begin{bmatrix} s_u \mathbf{t}_u & s_v \mathbf{t}_v & \mathbf{0} & \mathbf{p}_k \\ \mathbf{0} & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} \mathbf{S} & \mathbf{p}_k \\ \mathbf{0} & 1 \end{bmatrix} \quad (4)$$

Assuming \mathbf{W} is the transformation matrix from world space to screen space, a homogeneous ray emitted from the camera and passing through pixel (x, y) can be obtained by:

$$\mathbf{x} = (xz, yz, z, 1)^\top = \mathbf{W} \mathbf{H} (u, v, 1, 1)^\top \quad (5)$$

where z represents intersection depth. In the rasterization, we input pixel coordinate (x, y) and inquiry intersection in Gaussian's local coordinate. To achieve that, we need to obtain the inverse transformation of the projection (5). The intersection depth z is constrained by the view-consistent 2D Gaussian. Therefore, by solving this constraint equation, we can get the final result, as detailed in [19]:

$$u(\mathbf{x}) = \frac{h_u^2 h_v^4 - h_u^4 h_v^2}{h_u^1 h_v^2 - h_u^2 h_v^1} \quad v(\mathbf{x}) = \frac{h_u^4 h_v^1 - h_u^1 h_v^4}{h_u^1 h_v^2 - h_u^2 h_v^1} \quad (6)$$

$$\mathbf{h}_u = (\mathbf{W} \mathbf{H})^\top (-1, 0, 0, x)^\top \quad \mathbf{h}_u = (\mathbf{W} \mathbf{H})^\top (0, -1, 0, y)^\top \quad (7)$$

where (x, y) is the pixel coordinate and $\mathbf{h}_u^i, \mathbf{h}_v^i$ represent the i -th parameter of the vector.

Distinguished from 3DGS, 2DGS maintains a strict one-to-one mapping between points on the 2D Gaussians and pixel coordinates in screen space. This leads to another unexpected benefit - the ability to freely modify the expression of the Gaussian kernel. This is because the transformation from the local coordinate system to the screen coordinate system is a reversible transformation in 2DGS, whereas 3DGS utilizes an irreversible non-affine transformation.

For our experiments, this reversibility property of 2DGS provides a great deal of convenience.

3.2. Gaussian-Hermite Splatting Kernel

Gaussian-Hermite (GH) polynomials are the product of a Gaussian function and Hermite polynomials. In mathematics, Hermite polynomials frequently appear where a Gaussian distribution is utilized. This family of polynomials

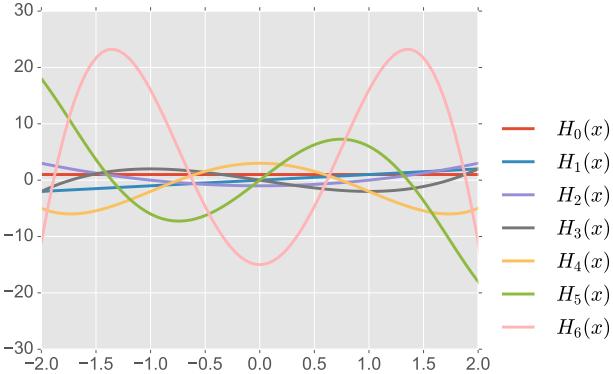


Figure 2. Hermite Polynomials. The figure presents Hermite polynomials $H_n(x)$ for $n = 0, 1, 2, \dots, 6$. These orthogonal polynomials exhibit varying degrees of oscillatory behavior as n increases.

has applications in various fields, such as classical mechanics [12], probability theory [4] and quantum mechanics [39].

The n -th rank Hermite polynomial, denoted as $H_n(x)$, is defined by the Rodrigues formula [10]:

$$H_n(x) = \frac{(-1)^n}{\omega(x)} \frac{\partial^n}{\partial x^n} \omega(x) \quad (8)$$

$$\omega(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \quad (9)$$

$H_0(x)$ to $H_6(x)$ are shown in Figure 2. Appendix 6 provides the complete expressions for the first several Hermite polynomials and proves that they form a complete orthogonal basis in $L^2(\mathbb{R})$, so any function belonging to $L^2(\mathbb{R})$ space (most cases in the real world) can be written as [22]:

$$f(x) = \sum_{n=0}^{\infty} a_n H_n(x) \quad (10)$$

The GH polynomials are also orthogonally complete, as demonstrated in Appendix 6. So any function belonging to $L^2(\mathbb{R})$ space can be written as:

$$f(x) = \sum_{n=0}^{\infty} a_n \mathcal{G}H_n(x) \quad (11)$$

$$\mathcal{G}H_n(x) = e^{-\frac{1}{2}x^2} H_n(x) \quad (12)$$

For 2D Splatting, we propose changing Gaussian kernel into GH kernel:

$$f(\mathbf{x}) = \exp\left(-\frac{u(\mathbf{x})^2 + v(\mathbf{x})^2}{2}\right) \times \left(\sum_{n=0}^N c_n H_n(u(\mathbf{x})) \right) \times \left(\sum_{m=0}^M d_m H_m(v(\mathbf{x})) \right) \quad (13)$$

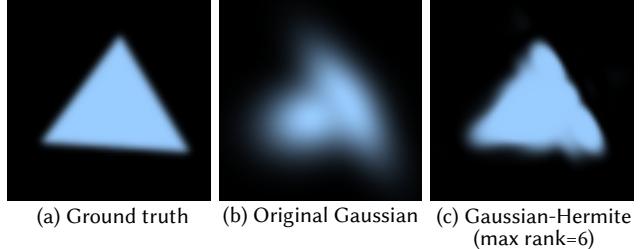


Figure 3. Toy experiment. We use 2 original Gaussians / 2 Gaussian-Hermites to fit the triangle in figure (a). In figure (b), the result produced by fitting with 2 original Gaussians only vaguely represents the general shape of the triangle and fails to clearly capture its edge features. In figure (c), the 2 Gaussian-Hermites more sharply captures the edge features of the triangle.

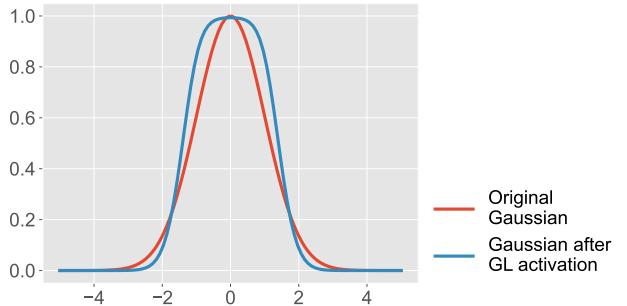


Figure 4. Visualization comparing the original Gaussian with the Gaussian after applying GL activation function. Gaussian after GL activation is smoother near the mean and exhibits a steeper decline as it moves away from the mean.

where c_n and d_m are optimizable coefficients. N and M stand for maximum rank number and we set $N = M = 9$ for our experiment.

A similar approach can also be applied to 3DGS. However, this necessitates modifying the projection method of 3DGS, utilizing a clip over the center of the 3D Gaussian for alpha blending. For simplicity, we choose 2D Gaussian Splatting (2DGS) as our baseline.

In Figure 3 we show how 2D Gaussians can fit a mesh triangle on a 2D plane and we only use 2 Gaussians. We allow optimizing rotation, scaling, position, opacity and GH coefficients. The color is rendered through alpha blending as original Gaussian splatting. Our 2DGH kernel displays great capacity for fitting an anisotropic shape like mesh triangle and also shares a merit of clear edge cut while the original 2DGS kernel causes dim and irregular edges.

3.3. Gaussian-like Activation

To ensure numerical stability and physical significance, we need to restrict formula (13) to the range $[0, 1]$. Thus we apply a new activation, Gaussian-like (GL) activation, to

the formula (13):

$$\mathcal{GL}(x) = 1 - \exp(-\sigma x^2) \quad (14)$$

where σ is a hyperparameter. The comparison between the original Gaussian kernel and the Gaussian kernel after applying our new activation function is shown in Figure 4. We did not choose the Sigmoid function because we wanted to maintain the symmetry and allow negative coefficients to contribute equally with positive coefficients. The hyperparameter σ is theoretically optimizable; however, in our work, we treat it as a constant and set $\sigma = 5$.

4. Experiments and Results

4.1. Datasets and Metrics

We evaluate the performance of our method on both synthetic and real datasets, including Synthetic NeRF dataset [32], DTU dataset [21] and Mip-NeRF 360 dataset [2]. Following the strategy of 2D Gaussian Splashing (2DGS) [19], we evaluate geometric reconstruction and rendering quality on Synthetic NeRF dataset and DTU dataset, and assess NVS performance on Synthetic NeRF dataset and Mip-NeRF 360 dataset. For training and evaluation, we keep the resolution of Synthetic NeRF dataset and we down sample the resolution of DTU dataset to 1/2 and the resolution of Mip-NeRF 360 dataset to 1/4 to accelerate the process.

For novel view synthesis and rendering quality, we compare SSIM [47], PSNR and LPIPS [65] among scenes on Synthetic NeRF dataset, DTU dataset and Mip-NeRF 360 dataset.

For geometry reconstruction, we compute bidirectional Chamfer Distance (CD) between ground truth and TSDF extracted mesh on Synthetic NeRF dataset and DTU dataset.

4.2. Implementation and Baseline

We implement our 2DGH kernel by modifying the original 2DGS CUDA kernel. We simply add Hermite polynomials to the Gaussian function and apply a new Gaussian-like activation function.

During training, we follow the adaptive control strategy from 2DGS and 3DGS to increase the number of 2D Gaussian primitives. However, we only begin the optimization of the Hermite coefficients after the clone-and-split process of Gaussians is complete and the number of Gaussians is fixed. This helps eliminate ambiguity among different parameters.

Moreover, we adopt a coarse-to-fine strategy for the Hermite coefficient optimization, akin to the strategy used for SH degree optimization. Specifically, we increase the Hermite rank every 1000 steps until the maximum rank is reached.

We choose original 2DGS, GES as baselines for comparison. However, for the GES [16] implementation, it is based on 3DGS and it is unfortunate that we found they did not actually implement the optimization of the GES extra parameters in the CUDA kernel. Moreover, the improvements in rendering quality and memory usage reported in their paper were achieved by adding an equivalent optimizable parameter to scaling parameters and applying a new loss in the PyTorch code, outside of the CUDA kernel. Therefore, GES fails to demonstrate its true superiority in the original paper.

Nevertheless, based on the GEF formulation presented in the GES paper, we managed to implement a correct corresponding version on 2DGS, denoted as 2DGES, to serve as a baseline for comparison.

In order to eliminate the randomness in the Gaussian quantity introduced by the control strategy, we use the original 2DGS to initialize all the methods. We train for 15000 iterations using the original 2DGS and then fix Gaussian number. Next, we use different methods to complete the remaining 15000-step training, including the original 2DGS, 2DGES, and our 2DGH approach.

For Synthetic NeRF dataset, we adopt the Mip-NeRF 360 [2] strategy, using random background colors to provide supervision for transparent backgrounds. All experiments are conducted on an RTX 4090 GPU.

4.3. Results and Comparison

Rendering quality & novel view synthesis 2DGS can still represent 3D scenes as radiance fields and enabling high-quality rendering and novel view synthesis. We compare rendering (with train-set views) and novel view synthesis of our 2DGH kernel with original 2DGS kernel on Synthetic NeRF dataset, DTU dataset and Mip-NeRF 360 dataset as shown in Table 1, 2, 3 and Figure 5, 6, 7.

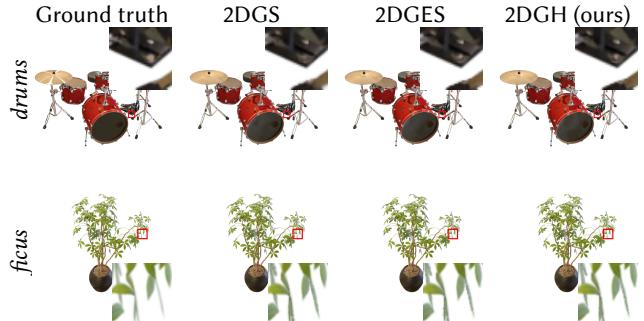


Figure 5. **Visual comparison of rendering quality on Synthetic NeRF dataset.** In the *drum* case, 2DGH provides the best representation of the fine nail structure; in the *ficus* case, 2DGH most effectively captures the slender leaves viewing from the side.

Quantitatively, our method outperforms GES and original 2DGS in almost all metrics in NVS and rendering tasks.

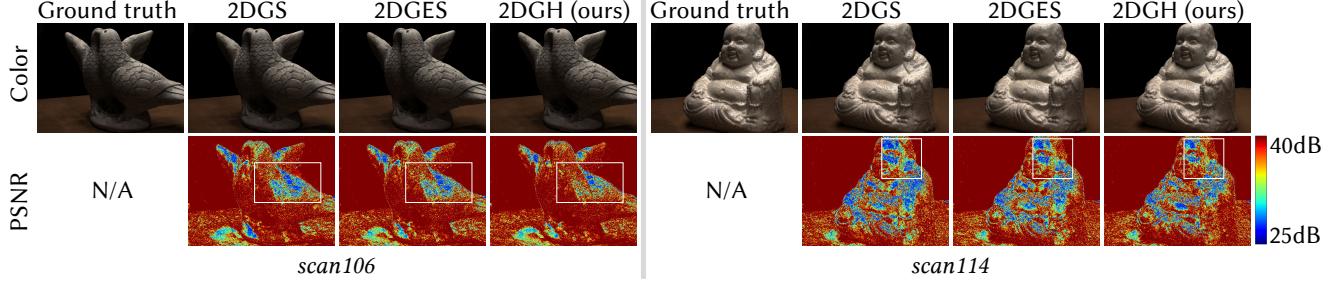


Figure 6. Visual comparison of rendering quality on the DTU dataset. Since the average PSNR values of all methods are relatively high, making it difficult to distinguish them visually, we have visualized the PSNR metrics. It can be observed that 2DGH achieves higher PSNR in a substantial number of regions compared to the original 2DGS and 2DGES.

Method	SSIM↑	PSNR (dB)↑	LPIPS↓
3DGS	0.982	35.89	0.0241
2DGS	0.978	34.95	0.0289
2DGES	0.979	35.38	0.0273
2DGH	0.980	35.75	0.0257

Table 1. Quantitative comparison of rendering quality on Synthetic NeRF dataset. The densify threshold for all experiments is set to 0.0002. We ensure that the number of primitives is kept the same across different methods for the same case except 3DGS.

Method	SSIM↑	PSNR (dB)↑	LPIPS↓
3DGS	0.952	37.69	0.039
2DGS	0.936	36.91	0.062
2DGES	0.937	36.92	0.061
2DGH	0.939	37.12	0.056

Table 2. Quantitative comparison of rendering quality on DTU dataset. We ensure that the number of primitives is kept the same across different methods for the same case except 3DGS.



Figure 7. Visual comparison of NVS results on Mip-NeRF 360 dataset. Compared to other methods, our approach effectively captures fine and sharp discontinuous edges.

We can obtain better rendering details because 2DGH can capture high-frequency information through high-rank Hermite polynomials. Not surprisingly, for rendering quality evaluation on train-set views, our 2DGH kernel also achieves better performances.

However, even using Gaussian-Hermite kernel, 2DGS rendering quality cannot surpass original 3DGS yet. And of course, we expect adding Hermite polynomials on 3DGS

Method	Outdoor scene			Indoor scene		
	SSIM↑	PSNR (dB)↑	LPIPS↓	SSIM↑	PSNR (dB)↑	LPIPS↓
3DGS	0.7286	24.64	0.239	0.9243	31.00	0.187
2DGS	0.7029	24.17	0.287	0.9103	29.88	0.214
2DGES	0.7048	24.14	0.283	0.9118	29.94	0.211
2DGH	0.7054	24.03	0.276	0.9146	30.06	0.203

Table 3. Quantitative comparison of NVS on Mip-NeRF 360 dataset. We ensure that the number of primitives is kept the same across different methods for the same case except 3DGS. We believe the discrepancy in PSNR performance in outdoor scenes, compared to SSIM and LPIPS, arises because the local optima for PSNR do not coincide with those of SSIM/LPIPS. We follow 2DGS setting and use a linear combination of L1 loss and SSIM to generate the loss, resulting in local optima that differ from those of PSNR.

can push pure rendering quality to the limit to the future.

Geometry Reconstruction Regarding Chamfer Distance(CD) on Synthetic NeRF dataset shown by Table 4, we find our results are much better than original 2DGS and slightly better than 2DGES. However, CD on DTU dataset shown by Table 5 reveal that 2DGS is better. We suppose that it's because DTU dataset objects have many dark areas and many of them have a subtle texture thus it can't provide enough supervision on Hermite coefficient optimization. It will finally result in floaters around surface due to 2DGH kernel can have a non-central and non-local alpha distribution compared to kernels used by 2DGES and 2DGS.

Method	chair	drums	ficus	hotdog	lego	materials	mic	ship	mean
2DGS	4.859	2.013	4.559	6.384	1.492	0.655	0.813	4.08	3.107
2DGES	4.951	1.847	4.52	5.640	1.691	0.664	0.734	2.783	2.854
2DGH	4.857	2.057	4.238	4.975	1.609	0.673	0.818	2.782	2.751

Table 4. Quantitative comparison of Chamfer Distance \downarrow ($\times 10^{-3}$) on Synthetic NeRF dataset. Compared to 2DGS and 2DGES, our method significantly outperforms in terms of both the number of best cases and the mean performance. These quantitative results are generally consistent with the findings from qualitative experiments.

Method	24	37	40	55	63	65	69	83	97	105	106	110	114	118	122
2DGS	0.5026	0.9493	0.4321	0.4177	0.9979	0.9678	0.8535	1.4144	1.2782	0.8190	0.7380	1.7761	0.4024	0.7311	0.5551
2DGES	0.5149	0.9626	0.4413	0.4262	1.0070	0.9393	0.8286	1.4494	1.2923	0.8469	0.7261	2.2639	0.4007	0.7353	0.5564
2DGH	0.4901	0.9842	0.4477	0.4062	1.0000	1.1282	0.8083	1.4515	1.2872	0.8573	0.7166	2.0272	0.3936	0.6826	0.5550

Table 5. **Quantitative comparison of geometric reconstruction on DTU dataset.** Our method performs on par with 2DGS in the best cases and outperforms 2DGES. We believe that the quantitative results for geometric reconstruction on the DTU dataset do not effectively compare the three methods. This is primarily due to the significant noise present in the DTU ground truth, where the Chamfer Distance (CD) should typically be on the order of 10^{-3} or lower.

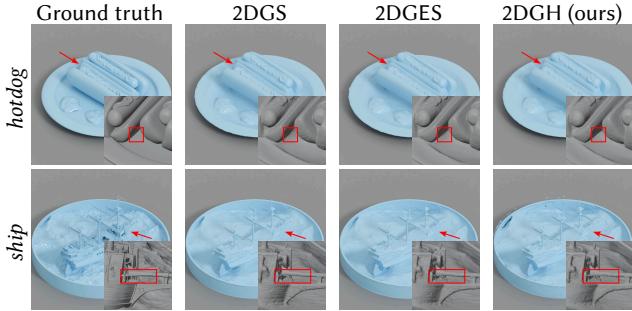


Figure 8. **Visual comparison of geometric reconstruction results on Synthetic NeRF dataset.** In the *hotdog* case, 2DGH best captures the sharp interfaces between surfaces; in the *ship* case, 2DGH most effectively represents the fine structures at the bow of the ship.

Dataset	Synthetic NeRF	DTU	Mip-NeRF 360
2DGS	52.55	141.9	1423
2DGES	53.82	144.3	1447
2DGH	68.58	185.2	1857

Table 6. **Quantitative comparison of the average storage (MB) across different methods in the previous rendering quality, NVS and geometric reconstruction experiments.** To ensure fairness, we set the number of primitives to be the same for each case. Our approach, having the largest number of parameters, understandably results in the highest storage overhead. However, due to its superior expressiveness, 2DGH can actually achieve results that surpass those of 2DGS with equal or even lower storage overhead, as shown in Table 7.

Overhead In the previous experiments, a fair comparison is ensured by using an equal number of primitives across all methods except 3DGS. Given that our method involves more parameters, it naturally incurs a higher memory overhead. Table 6 provides a quantitative illustration of this overhead. However, due to the superior expressiveness power of Gaussian-Hermite, we can reduce the number of primitives in our method while still maintaining stronger performances in rendering quality and geometric reconstruction. Table 7 illustrates that with less memory usage, 2DGH can achieve higher rendering quality and more accurate shape than original 2DGS and is a competitive method compared with GES.

Method	SSIM↑	PSNR (dB)↑	LPIPS↓	CD↓	NUM	Memory (MB)↓
2DGS	0.978	34.95	0.0289	0.003035	78374	52.55
2DGES	0.979	35.38	0.0273	0.002939	78374	53.82
2DGH	0.979	35.47	0.0275	0.002905	59887	52.78

Table 7. **Quantitative comparison on Synthetic NeRF dataset with unequal numbers of primitives.** NUM is referred to the number of primitives. Our method demonstrates superior rendering quality and geometric results with close memory overhead compared to 2DGS.

Method	SSIM↑	PSNR (dB)↑	LPIPS↓	mean CD↓
2DGS	0.978106	34.94331	0.02889	0.003041
2DGS + GL(0 rank 2DGH)	0.978783	35.251918	0.027641	0.002952
2DGES	0.979150	35.377960	0.027281	0.002898
2DGES + GL	0.979017	35.336123	0.027179	0.002865
2DGH(9 rank)	0.980216	35.746217	0.025671	0.002885

Table 8. **Quantitative ablation study on GL activation.** We observed that the new activation function significantly improves the quantitative performance in both rendering quality and geometric metrics.

We prove that there is more space to do trade-off between parameters and number of primitives and we provide a very flexible strategy to achieve it by adjusting the maximum rank of Gaussian-Hermite coefficients.

5. Discussion and Conclusion

5.1. Ablation

Gaussian-like activation Our proposed activation function, referred to as the Gaussian-like (GL) activation, exhibits similarities to the GEF [16] activation when the β parameter of GEF exceeds 2, particularly in terms of having a sharper boundary. We want to ensure fairness between different methods and confirm if there are any merits brought by new activation, so we combine new activation with 2DGES kernel and original 2DGS kernel. We start ablation training from the same 15000-step checkpoint on Synthetic NeRF dataset initialized by original 2DGS. Comparison is shown in Table 8.

We find GL activation does contribute to rendering and geometry metrics. For rendering metrics and geometry metrics, the contribution of GL activation is roughly close to

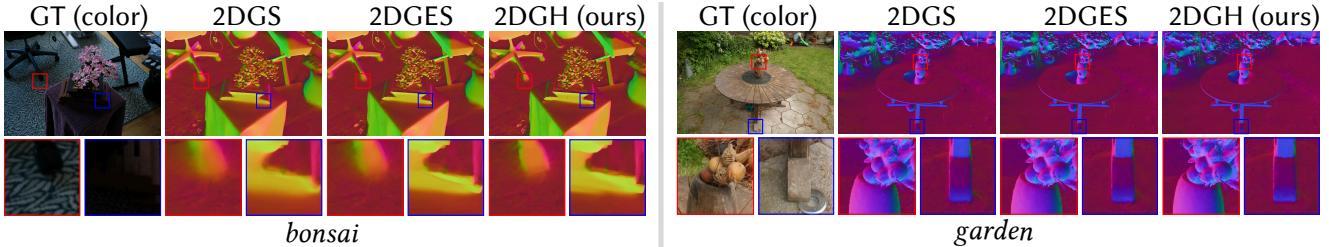


Figure 9. **Visual comparison of normal reconstruction results on Mip-NeRF 360 dataset.** Compared to 2DGS and 2DGES, our method excels at capturing the intersections of surfaces with sharp normal variations and high-frequency fine structures.

Gaussian-Hermite kernel.

2DGH Rank	SSIM↑	PSNR (dB)↑	LPIPS↓	mean CD↓
0 rank	0.978783	35.251918	0.027641	0.002952
3 rank	0.979527	35.576122	0.026676	0.002905
6 rank	0.980007	35.695724	0.026039	0.002884
9 rank	0.980216	35.746217	0.025671	0.002885

Table 9. **Quantitative ablation study on the highest rank of Hermite polynomials.** As the highest order of Hermite polynomials increases, 2DGH exhibits stronger representational capabilities, as evidenced by improvements in both rendering quality and geometric metrics.

Gaussian-Hermite Polynomial Rank The rank of the Gaussian-Hermite coefficients is found to affect rendering quality and geometry reconstruction metrics, as shown in Table 9. While we use Gaussian-Hermite coefficients up to the 9th rank in our experiments, the results suggest that using Gaussian-Hermite coefficients up to the 6th rank may be sufficient for geometry reconstruction.

Moreover, the improvement in the metrics as the rank increases (from 0th rank to 3rd rank, 6th rank, and finally 9th rank) exhibits diminishing returns, which is an expected trend.

5.2. Limitation

While 2DGH demonstrates promising results in 3D reconstruction, it faces challenges when dealing with reflective surfaces and weakly textured objects, shared by many existing reconstruction methods. The former one is generated by the global illumination effects and the latter is closely tied to the inherent lack of distinctive visual cues.

Furthermore, the use of the Truncated Signed Distance Function (TSDF) for mesh extraction is not an optimal solution. TSDF-based methods suffer from boundary artifacts, such as ghosting and blurring, and they solely rely on the first-order boundary condition (depth information) without leveraging additional cues. The quality of the extracted mesh is highly susceptible to the influence of floaters and hyperparameters, which can significantly impact the evaluation accuracy.

Interestingly, our approach has demonstrated the ability to produce sharp normal boundaries, which suggests the potential benefit of incorporating normal information as a second-order boundary condition to improve the mesh extraction process. Exploring methods that effectively combine depth and normal data could be a promising direction for future research to address the limitations of TSDF-based approaches and enhance the overall reconstruction quality of geometry.

5.3. Discussion and Conclusion

Through this research, we have introduced a family of Gaussian-Hermite kernels as a unified mathematical representation in Gaussian Splatting, enabling the creation of sharp boundaries within the Gaussian representation. We have incorporated arbitrary deformation capabilities and significantly enhanced anisotropy in the Gaussian representation, reducing the gap between Gaussian primitives and mesh facets. Devising ways to directly and analytically convert between Gaussian primitives and mesh triangle representations seems like a fascinating research problem.

To handle high-order coefficients, we have developed a novel activation function, while preserving the symmetry and physical meaning of the Hermite coefficients. This also challenges the traditional view that the primitives used for splatting must be Gaussian-based functions.

The 2DGS framework allows arbitrary modification to the kernel function within the local coordinate system. This raises the question of whether there is a complete set of bases that could potentially offer largest improvements over all alternative kernels. One intriguing possibility is the use of Fourier-based functions, which may provide enhanced representational capabilities. We consider this to be a subject that warrants comprehensive investigation in the future.

The proposed Gaussian-Hermite kernel-based approach has demonstrated remarkable performance in both geometry reconstruction and novel-view synthesis tasks, outperforming traditional Gaussian Splatting kernel. It's also very interesting to think over the essence of matter and density, also the similarities between atoms and Gaussians while Gaussian-Hermite polynomials provide an excellent bridge between physics world and graphics world.

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2DGH: 2D Gaussian-Hermite Splatting for High-quality Rendering and Better Geometry Reconstruction

Supplementary Material

6. Details of Gaussian-Hermite Polynomial

Gaussian-Hermite (GH) polynomial is the multiplication of Gaussian function and Hermite polynomial. Hermite polynomial frequently appears where a Gaussian distribution is used, such as the eigenmode of laser beam [41], the solution of Appell's equation of motion in classical mechanics [12], Edgeworth series in probability theory [4] and quantum harmonic oscillator in quantum mechanics [39]. The following are first 9 rank Hermite polynomials:

$$H_0(x) = 1 \quad (15)$$

$$H_1(x) = x \quad (16)$$

$$H_2(x) = x^2 - 1 \quad (17)$$

$$H_3(x) = x^3 - 3x \quad (18)$$

$$H_4(x) = x^4 - 6x^2 + 3 \quad (19)$$

$$H_5(x) = x^5 - 10x^3 + 15x \quad (20)$$

$$H_6(x) = x^6 - 15x^4 + 45x^2 - 15 \quad (21)$$

$$H_7(x) = x^7 - 21x^5 + 105x^3 - 105x \quad (22)$$

$$H_8(x) = x^8 - 28x^6 + 210x^4 - 420x^2 + 105 \quad (23)$$

By multiplying Gaussian and Hermite polynomials, we get Gaussian-Hermite polynomials(GH) and usual Gaussian function is happened to be the zero-order of GH polynomials.

GH polynomial actually has two equivalent definitions given by physicists and statisticians separately. Following the statistical definition, the orthogonality of Hermite polynomial is giving by the following formula:

$$\int_{-\infty}^{\infty} H_m(x) H_n(x) \exp\left(-\frac{x^2}{2}\right) dx = \delta_{mn} \quad (24)$$

where m and n represent the rank of Hermite polynomial. It's straightforward to prove its completeness by demonstrating each monomial x^n with non-negative integer power n can be written as a finite linear combination:

$$x^n = a_0 H_0(x) + a_1 H_1(x) + \dots + a_n H_n(x) \quad (25)$$

where a_0, a_1, \dots, a_n stand for decomposition coefficients. And it is well known that monomials are a complete basis in $L^2(\mathbb{R})$ space which means they are square-integrable functions. Therefore, Hermite polynomials form a set of complete orthogonal basis [22]. Moreover, we can prove that after multiplying Gaussian, the polynomials still

form a set of complete orthogonal basis. But we need a little physics background to prove this conclusion.

As mentioned before, Hermite polynomial often appears with Gaussian function. For example, the 1D quantum harmonic oscillator equation is given by [39]:

$$\mathcal{H}|\psi\rangle = E|\psi\rangle \quad (26)$$

$$\mathcal{H} = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2 \quad (27)$$

where \mathcal{H}, Ψ, E are Hamiltonian, wave function and energy and p, m, ω, x are particle momentum, mass, angular Frequency, position. If using physical definition of Hermite polynomial, we will have solution in the coordinate basis which is called as wavefunction by physicists:

$$\psi_n(x) = \frac{1}{\sqrt{2^n n!}} \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{m\omega x^2}{2\hbar}} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right) \quad (28)$$

$$\psi(x) = \langle x|\psi\rangle = \sum_{n=0}^{\infty} a_n \psi_n(x) \quad (29)$$

where \hbar stands for reduced Planck's constant. It describe the probability density distribution of particles and the probability of a particle being at a specific position in space can be obtained by taking the square of the modulus of the wavefunction. If we ignore some constants and set some constants to 1, the x -related part is so-called Gaussian-Hermite Polynomial:

$$\mathcal{GH}_n(x) = e^{-\frac{1}{2}x^2} H_n(x) \quad (30)$$

Actually, the Hamiltonian of quantum oscillator is a Hermite operator, completeness of which has been well demonstrated in quantum physics [39]. Therefore, the eigenfunctions of a Hermite operator like GH polynomials are completeness in Hilbert Space. Also, it's not difficult to show its orthogonality:

$$\int_{-\infty}^{\infty} \mathcal{GH}_m(x) \mathcal{GH}_n(x) \exp\left(-\frac{x^2}{2}\right) dx = \delta_{mn} \quad (31)$$

So functions in 2D plane can be fitted with GH polynomials:

$$f(\mathbf{x}) = \alpha \exp\left(-\frac{u(\mathbf{x})^2 + v(\mathbf{x})^2}{2}\right) \times \left(\sum_{n=0}^N c_n H_n(u(\mathbf{x})) \right) \times \left(\sum_{m=0}^M d_m H_m(v(\mathbf{x})) \right) \quad (32)$$

where c_n and d_m are optimizable coefficients in our experiments.

7. 3DGES

GES [16] utilizes Generalized Exponential Function as the kernel function of 3D Gaussian Splatting:

$$f(x, \mu, \alpha, \beta, A) = A \exp \left(\left(-\frac{|x - \mu|}{\alpha} \right)^\beta \right) \quad (33)$$

where x is the query location, μ is the center location of Gaussian, α is the scaling parameter, A is the opacity and β is the new shape parameter.

In the original paper, it claims that one can achieve approximated GES by applying the following transformation to the scaling parameter without modifying the CUDA code of 3DGS:

$$\alpha' = \frac{2}{1 + \exp -\rho(\beta - 2)} \alpha \quad (34)$$

where ρ is a hyperparameter that controls shape strength. However, this transformation doesn't change the essence of Gaussian function because the kernel in the actual realization can be written as:

$$f(x, \mu, \alpha, \beta, A) = A \exp \left(\left(-\frac{|x - \mu|}{\alpha'(\alpha)} \right)^2 \right) \quad (35)$$

which is still Gaussian function.

Upon reviewing the official GES repository, we identified this problem and successfully developed a corrected version of GES, leveraging the 3DGS framework. Table 10 is the comparison result.

Method	chair	drums	ficus	hotdog	lego	materials	mic	ship	mean
Official	35.196	26.045	34.767	37.432	35.094	29.646	35.310	30.632	33.015
Our 3DGES	35.090	25.994	35.055	37.362	35.525	29.686	35.027	30.530	33.033
Official	0.9851	0.9537	0.9872	0.9840	0.9802	0.9588	0.9912	0.9035	0.9679
Our 3DGES	0.9854	0.9529	0.9873	0.9839	0.9817	0.9586	0.9906	0.9027	0.9679
Official	0.0154	0.0386	0.0119	0.0237	0.0197	0.0365	0.0064	0.1145	0.0334
Our 3DGES	0.0142	0.0388	0.0117	0.0233	0.0163	0.0364	0.0067	0.1129	0.0325

Table 10. **PSNR↑, SSIM↑ and LPIPS↓ scores of novel view synthesis results on Synthetic NeRF dataset.** Our correct version of 3DGES achieves better results than official release version. And we ensure that the number of Gaussians is kept the same across different versions for the same case.

8. Evaluation Details

Following the strategy of 2D Gaussian Splatting (2DGS) [19], we evaluate geometric reconstruction and rendering quality (train-set views) on Synthetic NeRF dataset and DTU dataset, and assess novel view synthesis performance on Synthetic NeRF dataset and Mip-NeRF 360 dataset. The complete experimental results are presented in Tables 12, 13 and 14. A clearer summary of the experiments can be found in Table 11.

Dataset	Novel View Synthesis						Rendering Quality (train-set view)						Geometry Reconstruction					
	Quantitative			Qualitative			Quantitative			Qualitative			Quantitative			Qualitative		
Synthetic NeRF Mip-NeRF 360 DTU	<i>Sup Table 12</i> <i>MT Table 3, Sup Table 13</i> <i>MT Figure 1, 7, Sup Figure 11</i>	<i>MT Table 1, Sup Table 12</i>	<i>MT Figure 5</i>	<i>MT Table 4</i>	<i>MT Figure 8, Sup Figure 10</i>													
		<i>MT Table 2</i>	<i>MT Figure 6</i>	<i>MT Table 5</i>	<i>Sup Figure 12</i>													

Table 11. **The qualitative and quantitative experimental setups across all datasets.** *MT* stands for Main Text and *Sup* stands for Supplementary. Our experiments generally follow the methodology established by 2DGS [19]. The choices are made because the DTU dataset is specifically designed for geometric reconstruction, the Mip-NeRF 360 dataset is tailored for novel view synthesis, and the Synthetic NeRF dataset, being synthetic, can be customized for various purposes.

Method	chair	drums	ficus	hotdog	lego	materials	mic	ship	mean	chair	drums	ficus	hotdog	lego	materials	mic	ship	mean
3DGS	38.297	28.384	35.586	40.072	38.027	34.297	38.814	33.680	35.894	35.741	26.157	34.848	37.716	35.775	29.999	35.359	30.879	33.309
2DGS	36.753	28.101	36.618	39.104	36.220	36.613	32.750	33.422	34.948	35.003	25.909	34.510	37.193	34.235	29.487	34.407	30.865	32.701
2DGES	37.267	28.355	37.293	39.381	37.096	36.865	32.996	33.810	35.383	35.319	25.984	34.890	37.355	34.729	29.633	34.500	30.976	32.923
2DGH	37.867	28.610	37.702	39.698	37.580	37.195	33.241	34.072	35.746	35.456	25.982	35.023	37.283	34.851	29.650	34.536	30.848	32.954
3DGS	0.9928	0.9720	0.9924	0.9905	0.9898	0.9852	0.9958	0.9351	0.9817	0.9874	0.9546	0.9872	0.9853	0.9829	0.9604	0.9915	0.9064	0.9694
2DGS	0.9892	0.9694	0.9930	0.9884	0.9853	0.9935	0.9246	0.9815	0.9781	0.9843	0.9523	0.9873	0.9838	0.9783	0.9574	0.9902	0.9045	0.9673
2DGES	0.9902	0.9712	0.9936	0.9888	0.9871	0.9938	0.9259	0.9824	0.9791	0.9854	0.9534	0.9879	0.9842	0.9800	0.9581	0.9904	0.9055	0.9681
2DGH	0.9915	0.9726	0.9941	0.9896	0.9881	0.9943	0.9282	0.9832	0.9802	0.9861	0.9532	0.9881	0.9845	0.9806	0.9578	0.9905	0.9055	0.9683
3DGS	0.0081	0.0300	0.0090	0.0152	0.0108	0.0212	0.0037	0.0948	0.0241	0.0117	0.0370	0.0117	0.0201	0.0155	0.0341	0.0061	0.1068	0.0304
2DGS	0.0117	0.0340	0.0089	0.0196	0.0169	0.0054	0.1090	0.0256	0.0289	0.0158	0.0420	0.0132	0.0246	0.0224	0.0400	0.0079	0.1175	0.0354
2DGES	0.0106	0.0316	0.0084	0.0188	0.0134	0.0051	0.1063	0.0241	0.0273	0.0144	0.0394	0.0125	0.0236	0.0187	0.0382	0.0075	0.1143	0.0336
2DGH	0.0093	0.0297	0.0078	0.0168	0.0119	0.0047	0.1024	0.0230	0.0257	0.0132	0.0388	0.0122	0.0221	0.0175	0.0382	0.0074	0.1118	0.0326

Table 12. **PSNR↑, SSIM↑ and LPIPS↓ scores of rendering quality (left) and novel view synthesis result (right) on Synthetic NeRF dataset.** We ensure that the number of Gaussians is kept the same across different methods for the same case. In both settings, the methods show consistent results.

Method	Outdoor scene						Indoor scene						Overall mean
	bicycle	flowers	garden	stump	treehill	mean	room	counter	kitchen	bonsai	mean		
3DGS	25.24	21.53	27.37	26.61	22.45	24.64	31.47	29.01	31.34	32.18	31.00	27.47	
2DGS	24.68	21.04	26.69	26.03	22.39	24.17	30.25	28.05	30.17	31.06	29.88	26.71	
2DGES	24.66	21.06	26.64	26.01	22.33	24.14	30.17	28.10	30.19	31.29	29.94	26.72	
2DGH	24.57	20.99	26.62	25.89	22.06	24.03	30.17	28.25	30.33	31.50	30.06	26.71	
3DGS	0.7662	0.6047	0.8662	0.7730	0.6329	0.7286	0.9192	0.9084	0.9274	0.9423	0.9243	0.8156	
2DGS	0.7308	0.5704	0.8425	0.7535	0.6173	0.7029	0.9053	0.8918	0.9157	0.9285	0.9103	0.7951	
2DGES	0.7337	0.5735	0.8431	0.7551	0.6187	0.7048	0.9066	0.8933	0.9161	0.9311	0.9118	0.7968	
2DGH	0.7348	0.5761	0.8441	0.7546	0.6176	0.7054	0.9085	0.8974	0.9185	0.9340	0.9146	0.7984	
3DGS	0.209	0.337	0.107	0.215	0.326	0.239	0.219	0.200	0.126	0.203	0.187	0.216	
2DGS	0.270	0.379	0.146	0.262	0.377	0.287	0.246	0.232	0.147	0.232	0.214	0.255	
2DGES	0.266	0.373	0.144	0.257	0.372	0.283	0.242	0.228	0.145	0.226	0.211	0.251	
2DGH	0.260	0.365	0.137	0.252	0.364	0.276	0.235	0.218	0.140	0.220	0.203	0.243	

Table 13. **PSNR↑, SSIM↑ and LPIPS↓ scores of novel view synthesis result on Mip-NeRF 360 dataset.** We ensure that the number of Gaussians is kept the same across different methods for the same case.

Method	24	37	40	55	63	65	69	83	97	105	106	110	114	118	122	mean
3DGS	34.41	34.37	34.05	36.24	39.81	37.63	37.12	41.74	37.00	38.33	38.47	39.65	35.97	39.92	40.58	37.69
2DGS	36.34	35.98	35.88	35.83	40.03	36.59	36.90	35.89	35.61	36.27	37.85	37.76	36.01	38.34	38.38	36.91
2DGES	36.46	36.03	36.04	35.70	40.09	36.63	37.04	35.86	35.51	36.23	37.86	37.65	36.02	38.30	38.35	36.92
2DGH	36.82	36.27	36.36	36.31	39.49	36.99	37.34	35.66	35.77	36.16	38.21	37.87	36.32	38.59	38.58	37.12
3DGS	0.931	0.913	0.911	0.964	0.968	0.962	0.925	0.974	0.945	0.954	0.962	0.965	0.951	0.973	0.978	0.952
2DGS	0.960	0.960	0.956	0.931	0.973	0.933	0.929	0.910	0.917	0.921	0.937	0.923	0.930	0.933	0.929	0.936
2DGES	0.961	0.961	0.958	0.928	0.972	0.934	0.932	0.909	0.916	0.921	0.938	0.924	0.931	0.934	0.930	0.937
2DGH	0.963	0.962	0.960	0.938	0.970	0.938	0.937	0.909	0.918	0.923	0.941	0.928	0.935	0.937	0.933	0.939
3DGS	0.034	0.045	0.073	0.027	0.032	0.030	0.075	0.040	0.043	0.054	0.028	0.037	0.032	0.019	0.017	0.039
2DGS	0.018	0.029	0.040	0.062	0.023	0.058	0.068	0.075	0.063	0.063	0.075	0.095	0.081	0.087	0.093	0.062
2DGES	0.018	0.028	0.040	0.065	0.023	0.055	0.063	0.073	0.062	0.063	0.072	0.093	0.079	0.085	0.090	0.061
2DGH	0.016	0.029	0.037	0.052	0.027	0.049	0.055	0.074	0.060	0.057	0.066	0.084	0.072	0.079	0.084	0.056

Table 14. **PSNR↑, SSIM↑ and LPIPS↓ scores of rendering quality on DTU dataset.** We ensure that the number of Gaussians is kept the same across different methods for the same case.

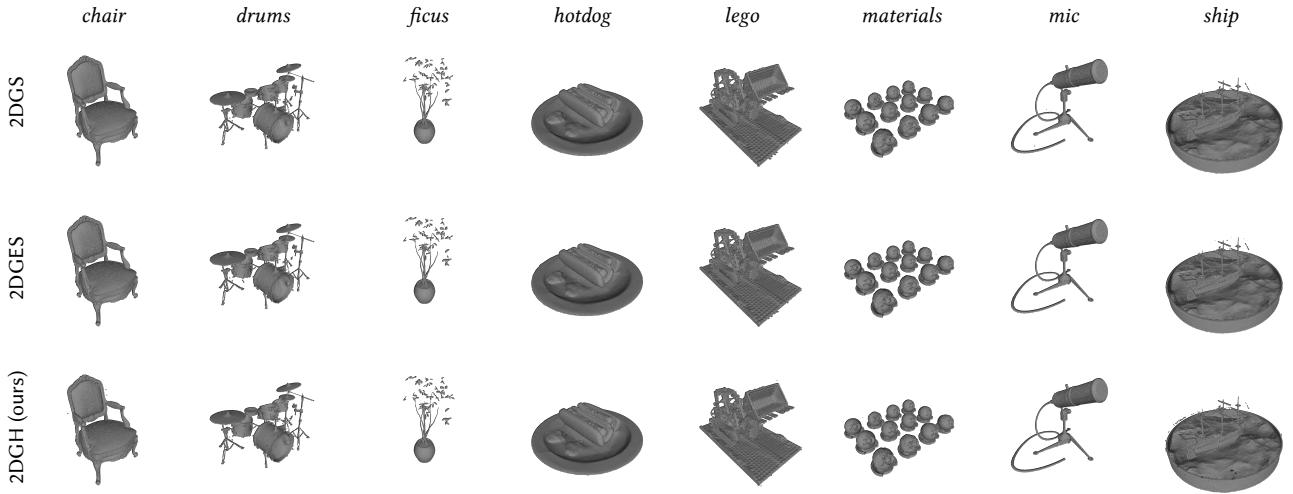


Figure 10. **Full qualitative visualization of geometry reconstruction on Synthetic NeRF dataset.**

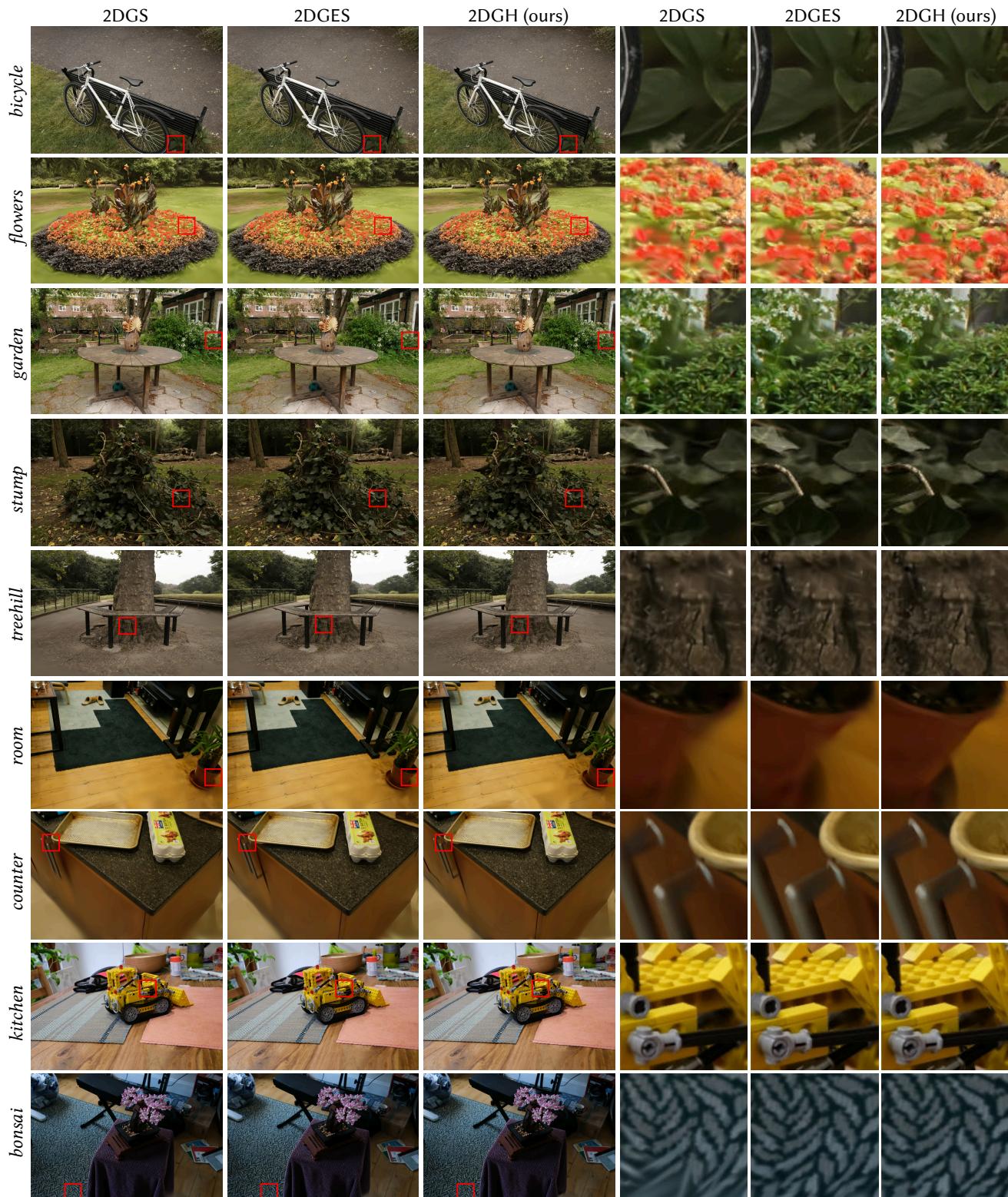


Figure 11. Full qualitative visualization of novel view synthesis on Mip-NeRF 360 dataset.

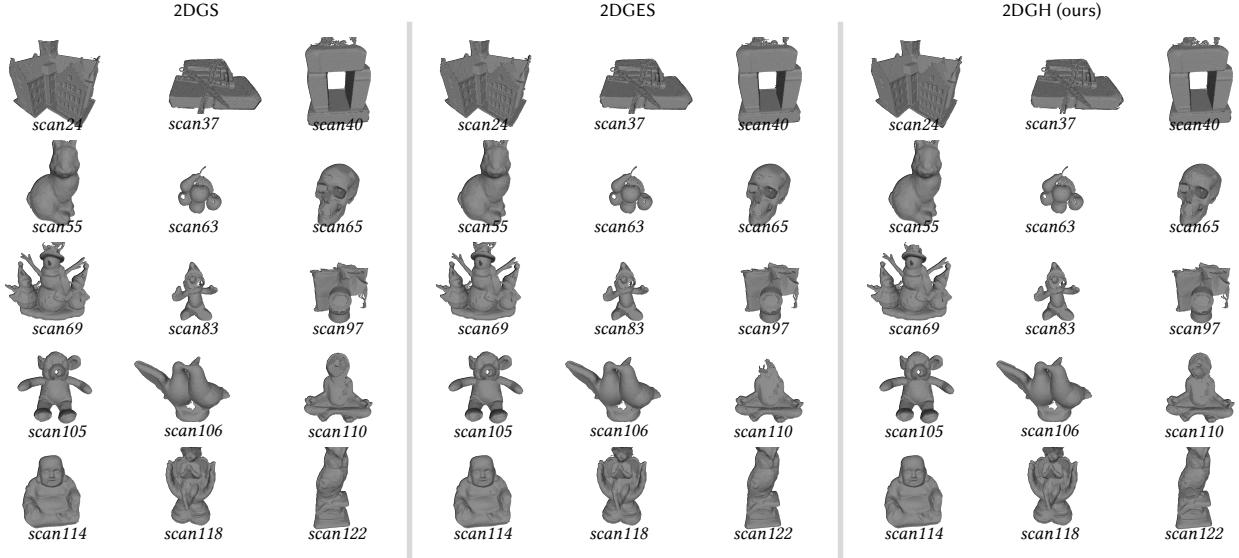


Figure 12. **Full qualitative visualization of geometry reconstruction on DTU dataset.** Our method is generally on par with the original 2DGS and visibly outperforms GES, particularly in cases like *scan110*.

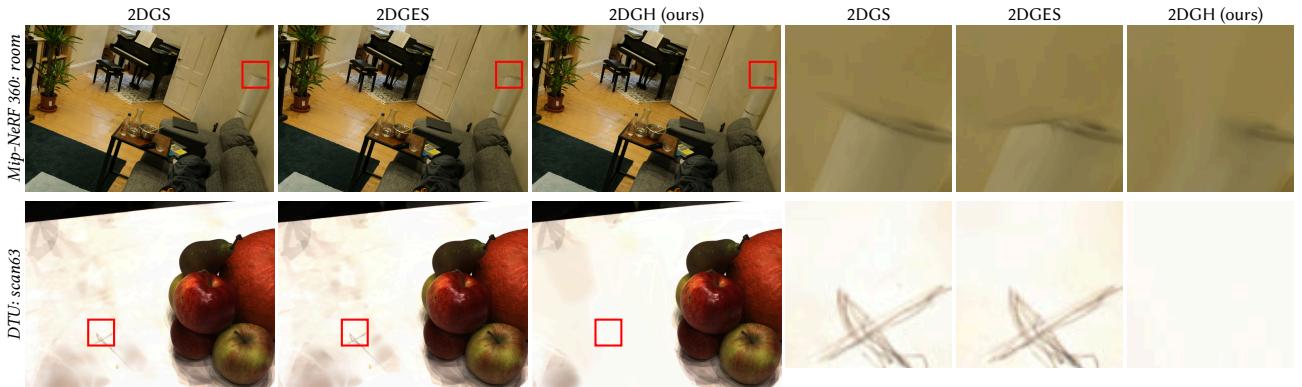


Figure 13. **Failure cases.** Our method, with its larger number of parameters, introduces greater ambiguity, making it challenging to represent certain under-supervised regions in the scene (those with fewer viewpoints).