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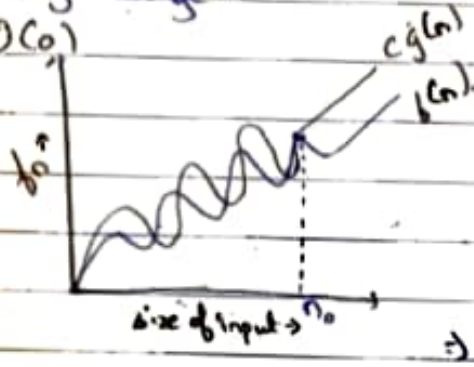
Unit 1 Asymptotic Notations
↳ Tending to infinity

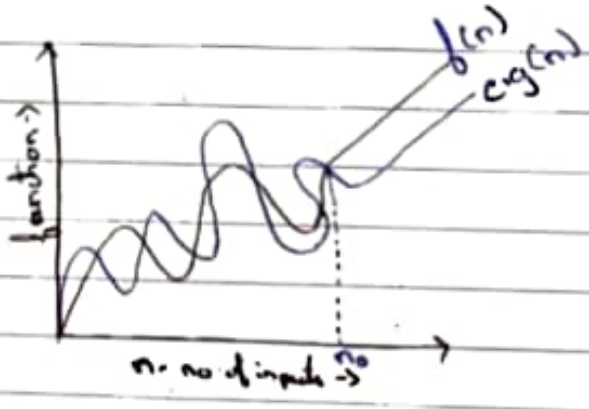
They help you find the complexity an algorithm when input is very large.

i) Big O(o)

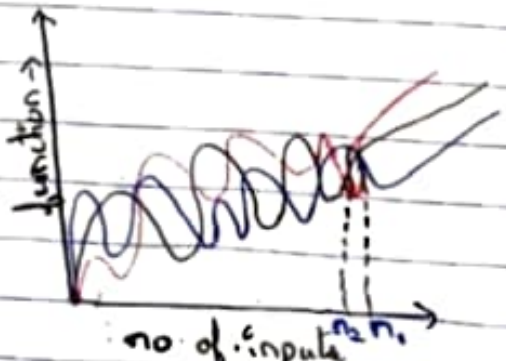


$f(n) = O(g(n))$
iff $f(n) \leq c \cdot g(n)$
$\forall n \geq n_0$
for some constant $c > 0$
$\Rightarrow g(n)$ is 'tight' upper bound of $f(n)$

ii) Big Omega (Ω)
$f(n) = \Omega(g(n))$
$g(n)$ is 'tight' lower bound of $f(n)$
$f(n) = \Omega(g(n))$
iff $f(n) \geq c \cdot g(n)$
$\forall n \geq n_0$ for some constant $c > 0$



iii) Theta (Θ)
$f(n) = \Theta(g(n))$.
$g(n)$ is both 'tight' upper & lower bound of $f(n)$
$f(n) = \Theta(g(n))$
iff
$c_1 g(n) \leq f(n) \leq c_2 \cdot g(n)$
$\forall n \geq \max(n_1, n_2)$
for some constant $c_1 > 0$ & $c_2 > 0$



4) small o(O)

$f(n) = o(g(n))$

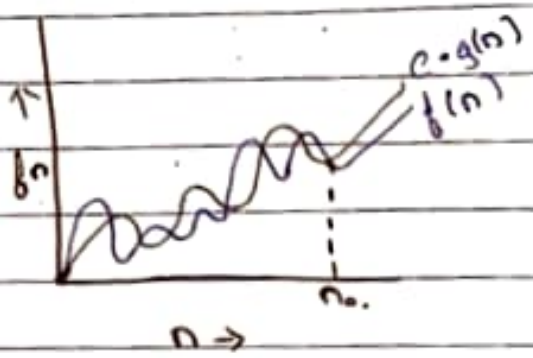
$g(n)$ is upper bound of for $f(n)$

$f(n) = o(g(n))$

when $f(n) < c \cdot g(n)$

$\forall n > n_0$

& $\forall c > 0$



5) small omega (ω)

$f(n) = \omega(g(n))$

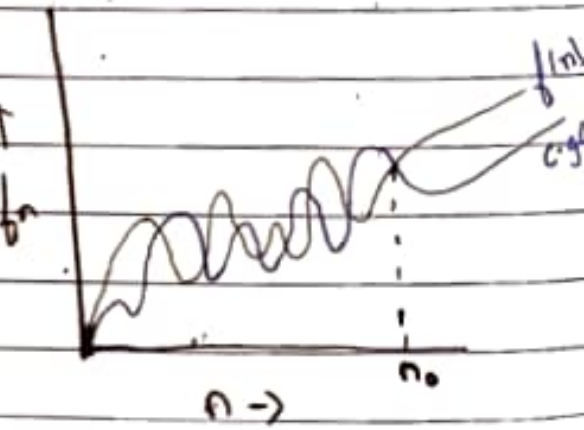
$g(n)$ is lower bound of for $f(n)$

$f(n) = \omega(g(n))$

when $f(n) > c \cdot g(n)$

$\forall n > n_0$

& $\forall c > 0$



Ques-2 What should be time complexity of
for(i=1 to n) {i= i*2}

```
for(i=1 to n)      //   i = 1, 2, 4, 8 ..... n
  { i=i*2 }        //   O(1)
```

$\Rightarrow \sum_{i=1}^{?} 1 + 2 + 4 + 8 + \cdots n.$

GP kth value $\Rightarrow T_k = ar^{k-1}$

$\Rightarrow 1 \times 2^{k-1}$

$\Rightarrow n = 2^{k-1}$

$\Rightarrow 2n = 2^k$

$\Rightarrow \log 2n = k \log 2$

$\Rightarrow \log_2 + \log n = k \log 2$

$\Rightarrow \log n + 1 = k$

$\Rightarrow O(k) = O(1 + \log n)$

$= O(\log n)$

Q-3 $T(n) = \{3T(n-1)$ if $n>0$, otherwise $1\}$

$T(n) = 3T(n-1)$ — (1)

put $n = n-1$

$T(n-1) = 3\{T(n-2)$ — (2)

from 1 & 2

$\Rightarrow T(n) = 3(3T(n-2))$

$= 9T(n-2)$ — (3)

putting $n = n-2$ in (1)

$T(n) = 3(T(n-3)$ - (4)

$\Rightarrow T(n) = 27(T(n-3))$

$\Rightarrow T(n) = 3^k (T(n-k)$

putting $n-k=0$

$\Rightarrow n = k.$

$\Rightarrow T(n) = 3^n [T(n-n)]$

$\Rightarrow T(n) = 3^n T(0)$

$\Rightarrow T(n) = 3^n \times 1$ $[T(0)=1]$

$\Rightarrow T(n) = O(3^n)$

$T(n) = \{2T(n-1)-1$ if $n>0$, otherwise $1\}$

$T(n) = 2T(n-1) - 1$ - (1)

Let $n = n-1$

$\Rightarrow T(n-1) = 2T(n-2) - 1$ - (2)

$\Rightarrow$ from (1) & (2)

$\Rightarrow T(n) = 2[2T(n-2)-1] - 1$

$\Rightarrow T(n) = 4T(n-2) - 2 - 1$ - (3)

Let $n = n-2$

$\Rightarrow T(n-2) = 2T(n-3) - 1$ - (4)

from (3) & (4)

$\Rightarrow T(n) = 4[2T(n-3)-1] - 2 - 1$

$\Rightarrow T(n) = 8T(n-3) - 4 - 2 - 1$

$\Rightarrow T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} - \cdots - 1$

$\Rightarrow$ GP $= 2^{k-1} + 2^{k-2} + 2^{k-3} + \cdots + 1$

$a = 2^{k-1}$

$r = 1/2$

$\Rightarrow S_k = \dfrac{a(1-r^n)}{1-r}$

$= \dfrac{2^{k-1}\left(1-(1/2)^n\right)}{1/2}$

$= 2^k \left(1-(1/2)^k\right)$

$= 2^k - 1$

Let $n-k = 0$

$\Rightarrow n = k.$

$\Rightarrow T(n) = 2^n \cdot T(n-n) - (2^n - 1)$

$\Rightarrow T(n) = 2^n \cdot 1 - (2^n - 1)$

$\Rightarrow T(n) = 2^n - (2^n - 1)$

$\Rightarrow T(n) = O(1)$

what should be time complexity of

```
int i=1, s=1;
while (s<=n)
{ i++; s=s+i;
    printf("#");
}
```

i = 1 2 3 4 5 6 ----

s = 1 + 3 + 6 + 10 + 15 + 21 ---- n ---- ①

s becomes

Sum of s = 1+3+6+10+ ---- + T_n — ①

also s = 1+3+6+10+ ---- + T_{n-1} + T_n — ②

from ① - ②

0 = 1+2+3+4+ ---- n - T_n

⇒ T_k = 1+2+3+4+ ---- k

⇒ T_k = $\frac{1}{2}$ k (k+1)

⇒ for k iterations.

1+2+3+ ---- + k <= n.

⇒ $\frac{k(k+1)}{2}$ <= n.

⇒ $\frac{k^2+k}{2}$ <= n

⇒ $O(k^2)$ <= n

⇒ $k = O(\sqrt{n})$

⇒ $T(n) = O(\sqrt{n})$

Ques6 Time complexity of -

```
void fn(int n)
{ int i, count=0;
  for(i=1; i*i<=n; ++i)
      count++                    // O(1)
}
```

as $i^2 <= n$,

$\Rightarrow i <= \sqrt{n}$.

$i = 1, 2, 3, 4 \ldots, \sqrt{n}$

$\sum_{i=1} 1 + 2 + 3 + 4 + \ldots + \sqrt{n}$.

$\Rightarrow T(n) = \frac{\sqrt{n} \times (\sqrt{n} + 1)}{2}$.

$\Rightarrow T(n) = \frac{n \times \sqrt{n}}{2}$

$\Rightarrow T(n) = O(n)$.

Ques-7 Time complexity of :-

```
void fn(int n)
{ int i, j, k, count=0;
  for(i=n/2; i<=n; ++i)
      for(i=1; j<=n; j=j*2)
          for(k=1; k<=n; k=k*2)
              count++;
}
```

for $k = k * 2$

$k = 1, 2, 4, 8, \cdots n.$

$\Rightarrow$ GP $\Rightarrow$ $a = 1$, $r = 2$.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1(2^k - 1)}{1}$$

$n \Rightarrow 2^k = n$

$\Rightarrow \log n = k$

$\Rightarrow$

i	j	k
1	$\log n$	$\log n * \log n$
2	$\log n$	$\log n * \log n$
$\vdots$	$\vdots$	$\vdots$
n	$\log n$	$\log n * \log n$

$\Rightarrow O(n * \log n * \log n)$

$\therefore O(n \log^2 n)$

8) Time Complexity of

```
function (int n)
{ int (n==1)
    return ;                          // O(1)
    for (i=1 to n)                    // i = 1, 2, 3, 4 --- n => O(n)
      { for (j=1 to n)                // j = 1, 2, 3, 4, --- n => O(n)
          { print (*') ;
          }
      }
    function (n-3) ;                  T(n/3).
}
```

$\Rightarrow$ $T(n) = T(n/3) + n^2$

$\Rightarrow$ $a = 1, \quad b = 3, \quad f(n) = n^2.$

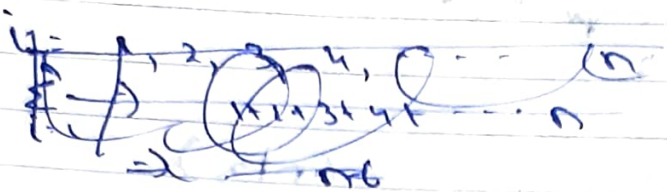
$c = \log_3 1 = 0$

$\Rightarrow$ $n^0 = 1 \quad > (f(n) = n^2).$

$\Rightarrow$ $T(n) = \Theta(n^2)$

Ques 9 Time complexity of -

```
void function (int n)
{ for (i=1 to n)                    // O(n)
  { for (j=1; j<=n; j=j+i)          // O(n)
      print ("*")
  }
}
```



for i=1 ⇒ j = 1, 2, 3, 4 . . . . n. = n.

for i=2 ⇒ j = 1, 3, 5, ... n = n/2

for i=3 ⇒ j = 1, 4, 7, ... n = n/3

⋮

for i=n ⇒ j = 1 . . .

$$\Rightarrow \sum_{j=n}^{1} n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \cdots + 1$$

$$\Rightarrow \sum_{j=n}^{1} n\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \frac{1}{n}\right]$$

$$\sum_{j=n} n[\log n]$$

$$\Rightarrow T(n) = [n \log n]$$

$$T(n) = O(n \log n)$$

Q-10 for functions, n^k & c^n, what is the asymptotic relation between these functions? assume that $k >= 1$, & $c > 1$ are constant. Find out the value of c & n_0 for which relation holds

As given n^k & c^n.

relation blw n^k & c^n is

$n^k = O(c^n)$.

as $n^k \leq ac^n$

$\forall n \geq n_0$ & some constant $a > 0$

for $n_0 = 1$

$c = 2$.

$\Rightarrow$ $1^k \leq a2^1$

$\Rightarrow$ $n_0 = 1$ & $c = 2$