Minimum Velocity Trajectories



Minimum Velocity Trajectory

Why is the minimum velocity curve also the shortest distance curve?

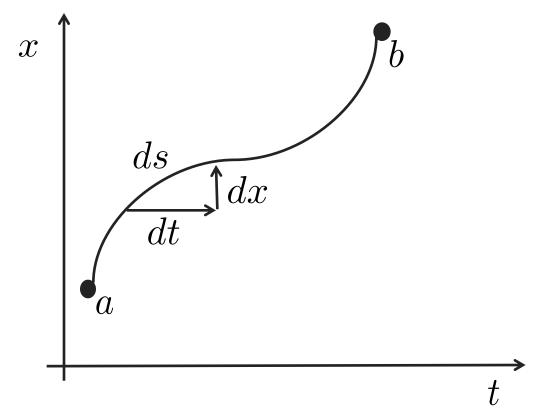
To get the minimum velocity trajectory, we solved:

$$x^{\star}(t) = \underset{x(t)}{\operatorname{argmin}} \int_{0}^{T} \dot{x}^{2} dt$$

From the Euler-Lagrange equations, the solution is:

$$x(t) = c_1 t + c_0$$

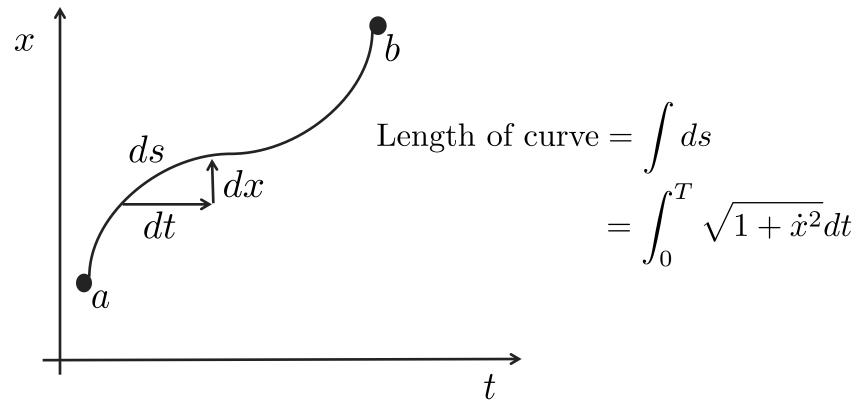




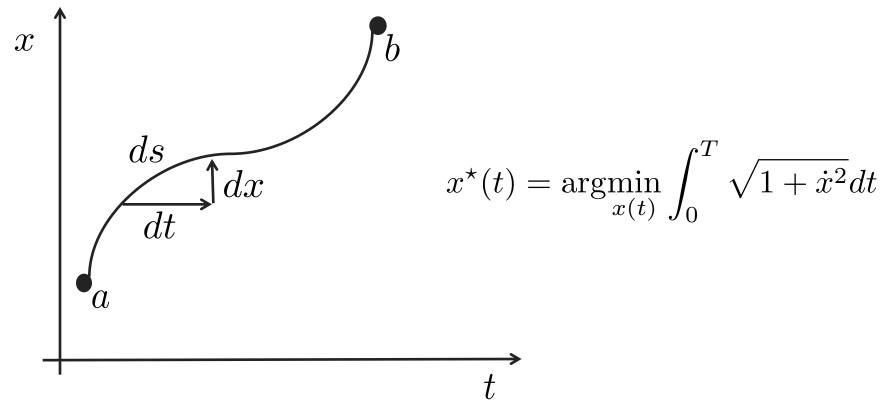
$$ds = \sqrt{dt^2 + dx^2}$$

$$= \sqrt{1 + \left(\frac{dx}{dt}\right)^2} dt$$

$$= \sqrt{1 + \dot{x}^2} dt$$









$$x^{\star}(t) = \operatorname*{argmin}_{x(t)} \int_{0}^{T} \sqrt{1 + \dot{x}^{2}} dt$$

$$\mathcal{L}(\dot{x}, x, t) = \sqrt{1 + \dot{x}^2}$$

Euler-Lagrange equation:

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{x}} \right) - \frac{\partial \mathcal{L}}{\partial x} = 0$$



Euler-Lagrange equation:

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{x}} \right) + \frac{\partial \mathcal{L}}{\partial x} = 0$$

Cost-function:
$$\mathcal{L}(\dot{x},x,t)=\sqrt{1+\dot{x}^2}$$

Euler-Lagrange terms:

$$\left(\frac{\partial \mathcal{L}}{\partial x}\right) = 0$$
 \longleftarrow No x appears in \mathcal{L}

$$\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = \frac{\dot{x}}{\sqrt{1 + \dot{x}^2}}$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{x}} \right) = \frac{d}{dt} \left(\frac{\dot{x}}{\sqrt{1 + \dot{x}^2}} \right)$$

Euler-Lagrange equation:

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{x}} \right) - \frac{\partial \mathcal{L}}{\partial x} = 0$$

Euler-Lagrange terms: $\frac{d}{dt} \left(\frac{\dot{x}}{\sqrt{1 + \dot{x}^2}} \right) = 0$

Integrate to get velocity: $\frac{\dot{x}}{\sqrt{1+\dot{x}^2}}=K \longrightarrow \dot{x}=\sqrt{\frac{K^2}{1-K^2}}=c_1$

Integrate to get position: $x(t) = c_1 t + c_0$ \leftarrow Same as minimum velocity solution