Dynamical Systems

Penn Engineering

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- $x(t)$ gives the values of these states over time

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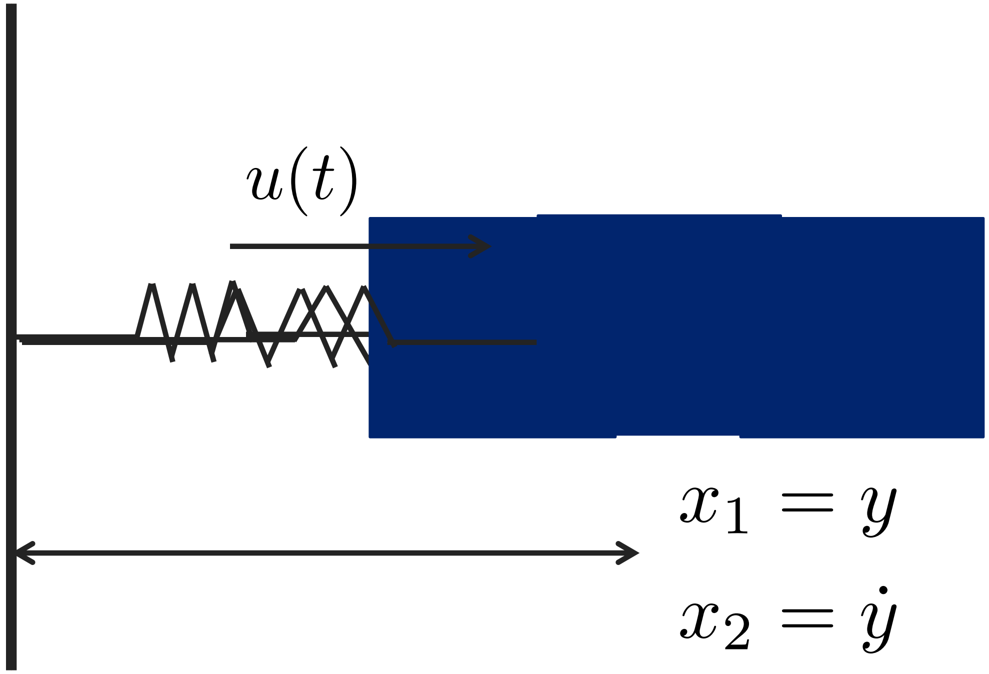
Evolution of these states over time is often given by a set of governing ordinary differential equations

- Order: highest derivative that appears in the equations

$$\boxed{\ddot{x}(t)} = u(t)$$

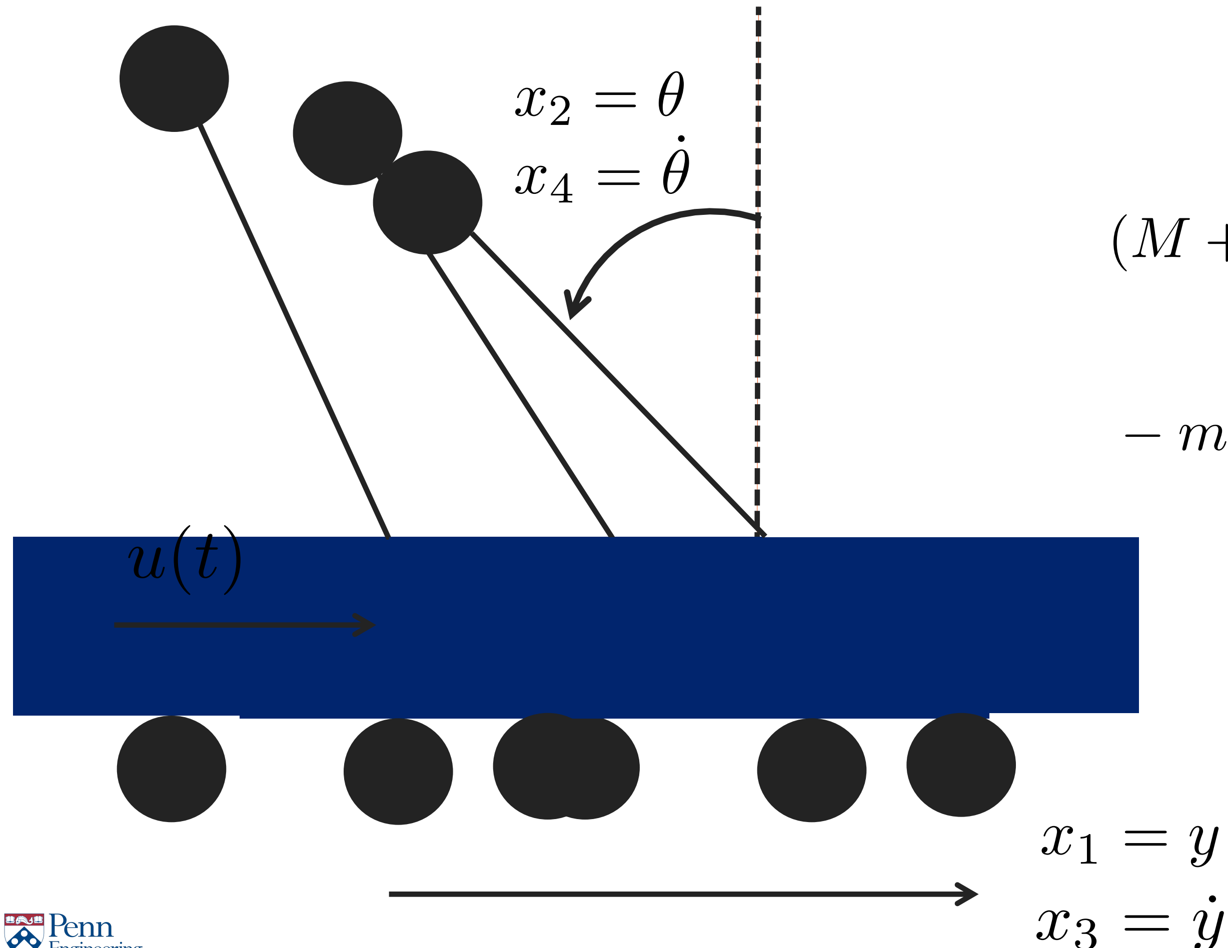
Second-order system

Example 1: Mass-Spring System



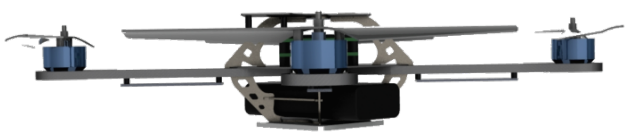
$$m\ddot{y}(t) + ky(t) = u(t)$$

Example 2: Pendulum on a Cart



$$
\begin{aligned}
(M+m)\ddot{y}(t) - ml\cos(\theta(t))\ddot{\theta}(t) \\
+ ml\sin(\theta(t))\dot{\theta}(t)^2 = u(t) \\
-ml\cos(\theta(t))\ddot{y}(t) + (J+ml^2)\ddot{\theta}(t) \\
- mgl\sin(\theta(t)) = 0
\end{aligned}
$$

Example 3: Quadrotor



$$x_1 = x \qquad x_7 = \dot{x}$$

$$x_2 = y \qquad x_8 = \dot{y}$$

$$x_3 = z \qquad x_9 = \dot{z}$$

$$x_4 = \phi \qquad x_{10} = p$$

$$x_5 = \theta \qquad x_{11} = q$$

$$x_6 = \psi \qquad x_{12} = r$$