School of Engineering and Applied Science (SEAS), Ahmedabad University

Probability and Stochastic Processes (MAT277)

Homework Assignment-4

Enrollment No: AU2140096 **Name: Ansh Virani**

1.

2. A random variable X is uniformly distributed over the interval (0, 1) and related to Y by,

$$\tan\left(\frac{\pi Y}{2}\right) = e^X \implies Y = \frac{2}{\pi}\arctan(e^X)$$

$$\therefore \quad \frac{dY}{dX} = \frac{2}{\pi}\cdot\frac{1}{1+e^{2X}}$$

Applying the the transformation rule, we get:

$$f_Y(y) = f_X(x)\left|\frac{dY}{dX}\right| = 1 \times \frac{2}{\pi}\cdot\frac{1}{1+e^{2X}}$$

Since X is expressed in terms of Y through the initial transformation, $e^X = \tan\left(\frac{\pi Y}{2}\right)$, the PDF can be expressed in terms of Y as follows:

$$f_Y(y) = \left(\frac{2}{\pi}\right)\left(\frac{1}{1+\tan^2\left(\frac{\pi y}{2}\right)}\right)$$

Using the identity $1+\tan^2(z) = \sec^2(z)$, we get:

$$f_Y(y) = \left(\frac{2}{\pi}\right)\left(\frac{1}{sec^2\left(\frac{\pi y}{2}\right)}\right) = \frac{2}{\pi}\cos^2\left(\frac{\pi y}{2}\right)$$

By solving for Y, computing the derivative with respect to X, and applying the transformation rule, the resulting PDF for Y is $f_Y(y) = \frac{2}{\pi}\cos^2\left(\frac{\pi y}{2}\right)$, valid for y in the interval $(0, 1)$.