School of Engineering and Applied Science (SEAS), Ahmedabad University

Probability and Stochastic Processes (MAT277)

Homework Assignment-1

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1. **While tossing a biased die, calculate the probability that face 3 has turned up, Given Alex tells either face 3 or face 6 has turned up.**

   (a) *We are given that,*

Face	1	2	3	4	5	6
Probability	0.2	0.22	0.11	0.25	0.15	0.07

Let A be the event that face 3 has turned up and B be the event that face 6 has turned up.

$\therefore P(A) = 0.11 \quad \& \quad P(B) = 0.07$

We know that these two are mutually exclusive events, hence:

$$\begin{aligned}\therefore P(A \cup B) &= P(A) + P(B)\\ &= 0.11 + 0.32\\ &= 0.43\end{aligned}$$

Clearly, here we have to find the conditional probability, $P(A \mid A \cup B)$

$$P(A \mid A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)}$$

$$= \frac{P(A)}{P(A \cup B)}$$

$$= \frac{0.11}{0.43}$$

$$= 0.2558139535.$$

$$\boxed{\therefore\ P(A \mid A \cup B) \quad \approx \quad 0.2559}$$

Hence, the probability that face 3 has turned up, given either face 3 or face 6 has turned up is approx 25.59%

2. **There exists two events E1 & E2 such that** $P(E1 \mid E2) = 0.45$**,** $P(E2 \mid E1) = 0.5$ **and** $P(E1 \cup E2) = 0.4$

   (a) *Calculate* $P(E1 \cap E2)$*:*

   According to Conditional Probability theorem we can say,

   $$\begin{aligned} P(E1 \mid E2)P(E2) &= P(E2 \mid E1)P(E1) \\ 0.45P(E2) &= 0.5P(E1) \end{aligned}$$

   Applying Basic Probaility theorem:

   $$\begin{aligned} P(E1 \cup E2) &= P(E1) + P(E2) + P(E1 \cap E2) \\ 0.4 &= 0.9P(E2) + P(E2) + P(E1 \cap E2) \end{aligned}$$

   $$\therefore P(E1 \cap E2) = 1.9P(E2) - 0.4 \qquad ......(1)$$

   Now, Substituting these equation in conditional probability equation:

   $$P(E1 \mid E2) = \frac{P(E1 \cap E2)}{P(E2)}$$

   $$\begin{aligned} 0.45 &= \frac{1.9P(E2) - 0.4}{P(E2)} \\ 0.4 &= 1.45P(E2) \end{aligned}$$

   $$\therefore P(E2) = 0.275$$

   Substituting value of P(E2) in equation 1, we get

   $$\therefore P(E1) = 0.2475$$

   (b) *Comment on the dependency relation between event E1 and E2:*

   When event E2 occurs, there's a 45% chance that event E1 will occur. On the other hand if event E1 occurs, there's a 50% chance that event E2 will occur. This shows that the events are unsymmetrically related.

   Also upon calculating it was found that, there's some shared occurance happening beteen two of the given events, as probability of them happening together was founf to be 12.25%

3. **Given Probabilies are:**

   (a) **Let's denote the probability of a Red ball as:** $P(R) = 0.45$

   (b) **Let's denote the probability of a Striped ball as:** $P(S) = 0.3$

   (c) **Let's denote the probability of a Red ball with stripes as:** $P(RS) = P(R \cap S) = 0.2$

To find the probability that ball is striped given the ball picked is a Red one.

$$P(S \mid R) = \frac{P(R \cap S)}{P(R)}$$

$$= \frac{0.2}{0.45}$$

$$= 0.4444444...$$

$$\boxed{\therefore\ P(A \mid A \cup B) \quad \approx \quad 0.4445}$$

Hence, the probability that the ball is striped one given the ball in red ball is approx 44.45%

4. **Let A denote the event where number 8 is obtained while tossing a 8-sided unbiased dice**

$\therefore P(A) = \dfrac{1}{8} =$ p

$\therefore P(A') = 1 - p$

X denotes the number of tosses required to get number 8 as an outcome.

(a) The probability that X = 6:
we use the equation:

$$P(X) = P(A')^{(X-1)}.P(A)$$

$$\begin{aligned} P(X_6) &= (1-p)^5.p \\ &= (\frac{7}{8})^5.\frac{1}{8} \\ &= (0.875)^5 * (0.125) \\ &= 0.0641136169 \end{aligned}$$

$$\boxed{\therefore\ P(X_6) \quad \approx \quad 0.06412}$$

(b) Conditional Probaility that $X \leq 6$ given $X < 9$:

$$P(X \leq 6 \mid X < 9) = \frac{P(X \leq 6 \cap X < 9)}{P(X < 9)}$$

$$= \frac{P(X < 9)}{P(X < 9)}$$

$$\boxed{\therefore\ P(X \leq 6 \mid X < 9) \quad = \quad 1}$$

Hence, as the equation suggests both the cases are same, resulting 100% probability.

5. **Given Probabilies are:**

   (a) **probability that an employee arrives late:** $P(A_l) = 0.15$

   (b) **probability that an employee leaves early:** $P(L_e) = 0.25$

   (c) **probability that an employee arrives late and leaves early:** $P(A_l \cap L_e) = 0.8$

   We need to find the probability of the employee arriving early given that he leaves late:

$$P(A_l'|L_e') = \frac{P(A_l' \cap L_e')}{P(L_e')}$$

$$= \frac{P(A_l \cup L_e)'}{P(L_e')}$$

$$= \frac{(P(A_l) + P(L_e) - P(A_l \cup L_e))'}{P(L_e')}$$

$$= \frac{1 - (P(A_l) + P(L_e) - P(A_l \cup L_e))}{1 - P(L_e)}$$

$$= \frac{1 - (0.15 + 0.25 - 0.8)}{0.75}$$

$$= 0.8$$

$$\boxed{\therefore\ P(A_l'|L_e') \quad = \quad 0.8}$$

   Hence, the probability of the employee arriving early given that he leaves late is 80%.