School of Engineering and Applied Science (SEAS),Ahmedabad University

Probability and Stochastic Processes (MAT277)

Homework Assignment-1

Enrollment No: AU2140161 **Name: Mihir Kotecha**

1. **Alex told us that either face 1 or face 5 had turned up. Calculate the probability that face 5 has turned up.**

Let P(A) be the probability of getting a 1:

$\therefore P(A) = 0.24$

Let P(B) be the probability of getting a 5:

$\therefore P(B) = 0.04$

It is given that either 5 or 1 has turned up, we need to find the probability of getting a 5.

$$P(B|B \cup A) = \frac{P(B \cap (B \cup A)}{P(B \cup A)}$$

$$= \frac{P(B)}{P(A \cup B)}$$

$$= \frac{P(B)}{P(A) + P(B)} \quad \textit{[Since A and B are mutually exclusive]}$$

$$= \frac{0.04}{0.04 + 0.24}$$

$$= \frac{1}{7} = 0.143$$

2. **There are two events E1 and E2 and** $P(E1|E2) = 0.5$ **and** $P(E2|E1) = 0.6$ **and** $P(E1 \cup E2)$ **= 0.5**

   (a) **We need to find** $P(E1 \cap E2)$**:**

$$Given, \quad P(E1|E2) = \frac{P(E1 \cap E2)}{P(E2)} = 0.5$$

$$\therefore \textit{ from above equation:} \quad P(E2) = \frac{P(E1 \cap E2)}{0.5} \qquad ---(1)$$

$$Also, \quad P(E2|E1) = \frac{P(E1 \cap E2)}{P(E1)} = 0.6$$

$$\therefore \textit{ from above equation:} \quad P(E1) = \frac{P(E1 \cap E2)}{0.6} \qquad ---(2)$$

Now,

$$P(E1 \cup E2) = 0.5$$

$$\therefore P(E1) + P(E2) - P(E1 \cap E2) = 0.5$$

$$\therefore \frac{P(E1 \cap E2)}{0.5} + \frac{P(E1 \cap E2)}{0.6} - P(E1 \cap E2) = 0.5 \qquad \textit{[From (1) and (2)]}$$

$$\therefore P(E1 \cap E2) * (\frac{1}{0.5} + \frac{1}{0.6} - 1) = 0.5$$

$$\therefore P(E1 \cap E2) = \frac{0.5}{2.67}$$

$$\boldsymbol{P(E1 \cap E2) = 0.187}$$

(b) **Now let us find if E1 and E2 are dependent or not:**

On substituting the value of $\boldsymbol{P(E1 \cap E2)}$ *in (1):*

$$\boldsymbol{P(E2) = \frac{0.187}{0.5} = 0.374}$$

Similarly, on substituting the value in (2):

$$\boldsymbol{P(E1) = \frac{0.187}{0.6} = 0.312}$$

Now,

$$\boldsymbol{P(E1) * P(E2) = 0.312 * 0.374 = 0.117}$$

We can see that the above value is not equal to $\boldsymbol{P(E1 \cap E2)}$, thus the two events are Dependent.

3. We are tossing an unfair dice repeatedly until we we get two heads in a row. The probability of getting a head $P(H) = p$. We need to find the winning probability. We win when we get two heads in a row.

The winning series can be as follows:

HH, THH, HTHH, THTHH, HTHTHH ...

$$\therefore P(win) = p^2 + (1-p) * p^2 + p * (1-p) * p^2 + p * (1-p)^2 * p^2 + ...$$

$$= p^2 * [1 + (1-p) + p * (1-p) + p * (1-p)^2 + ...]$$

$$= p^2 * [(1 + p(1-p) + p^2(1-p)^2 + ...) + ((1-p) + p * (1-p)^2 + p^2 * (1-p)^3 + ...)]$$

$$= p^2 * [\frac{1}{1-p+p^2} + \frac{1-p}{1-p+p^2}]$$

$$= p^2 * [\frac{2-p}{1-p+p^2}]$$

4. **A ten sided fair dice is rolled until a six is obtained. Let the number of throws be X.**

   (a) **We need to find the probability for X=5**

   For this to happen we need the six to occur exactly on the 5th throw

   $$\therefore P(X=5) = \frac{9}{10} * \frac{9}{10} * \frac{9}{10} * \frac{9}{10} * \frac{1}{10}$$

   $$\therefore P(X=5) = 0.065$$

(b) **Now we the conditional probability i.e. given $X > 5$ what is $X \geq 6$:**

$$P(X \geq 6|X > 5) = \frac{P(X \geq 6 \cap X > 5)}{P(X > 5)}$$

$$= \frac{P(X > 5)}{P(X > 5)}$$

$$= 1$$

5. **We are given the probabilities of arrival and leaving timings of employ. According to the question:**

   Let A be an event where the employ arrives late,

   $\therefore P(A) = 0.15$

   Let B be an event where the employ leaves early,

   $\therefore P(B) = 0.3$

   Also,

   $P(A \cap B) = 0.08$

   We need to find the probability of the employ arriving early given that he leaves late:

   $$P(A'|B') = \frac{P(A' \cap B')}{P(B')}$$

   $$= \frac{P(A \cup B)'}{P(B')}$$

   $$= \frac{(P(A) + P(B) - P(A \cup B))'}{P(B')}$$

   $$= \frac{1 - (P(A) + P(B) - P(A \cup B))}{1 - (P(B))}$$

   $$= \frac{1 - (0.15 + 0.3 - 0.8)}{0.7}$$

   $$= 0.9$$

6. **We are given that A is an event in which the product breaks down in the third year and B is the event in which the product survives the first two years. Also, T is a random variable which represents the lifetime of a product.**

Also,

$P(T \geq t) = e^{\frac{-t}{5}}$, $t \geq 0$

$P(T \geq 2) = 0673$

We need to find the conditional probability A given B i.e. $P(A|B)$**:**

$$P(A|B) = P(X = 3|X \geq 2)$$

$$= \frac{P(X = 3 \cap X \geq 2)}{P(X \geq 2)}$$

$$= \frac{P(X = 3)}{P(X \geq 2)}$$

$$= \frac{P(X \geq 2) - P(X \geq 3)}{P(X \geq 2)}$$

$$= \frac{e^{\frac{-2}{5}} - e^{\frac{-3}{5}}}{0.6703}$$

$$= \frac{0.6704 - 0.5488}{0.6703}$$

$$= \frac{0.1215}{0.6703}$$

$$= 0.1812$$