In [2]:
```
# import libraries
import numpy as np
import matplotlib.pyplot as plt
```

In [3]:
```
# Arrival and Service rates

λ = 20              # calls per hour
μ = 5               # calls served per agent per hour
shift_hours = 8     # 8-hour shift

# Simulation settings
np.random.seed(21)
```

In [4]:
```
# single-Run Simulation function
def simulate_queue(s):
    '''    Simulate an M/M/s queue over 'shift_hours'.
    Returns arrays of wait times (hrs) and system sizes at arrivals.
    '''
    # generate arrival times until the end of shift
    inter = np.random.exponential(1/λ, int(λ*shift_hours*1.5))
    arrivals = np.cumsum(inter)
    arrivals = arrivals[arrivals < shift_hours]
    N = len(arrivals)
# generate service times for each caller
    services = np.random.exponential(1/μ, N)

    # track each server's next-free time
    next_free = np.zeros(s)
    wait_times = np.zeros(N)
    system_size = np.zeros(N)
# departure times list
    dep_times = []

    for i, t in enumerate(arrivals):
        # find soonest-available agent
        j = np.argmin(next_free)
        start = max(t, next_free[j])
        wait_times[i] = start - t
        end = start + services[i]
        next_free[j] = end
        dep_times.append(end)
# Count how many callers are still in system at time t
        system_size[i] = np.sum(np.array(dep_times) > t)

    return wait_times, system_size

# quick test for s=1
w1, q1 = simulate_queue(s=1)
print(f"s=1: avg wait {w1.mean()*60:.1f} min, avg queue len {q1.mean():.2f}")
```

```
s=1: avg wait 908.5 min, avg queue len 74.34
```

In [5]:
```
# traffic intensity ρ for s=1
```

```python
ρ1 = λ / μ
L_analytic = ρ1 / (1- ρ1)
W_analytic = L_analytic / λ

print("M/M/1 analytic vs. simulation:")
print(f"  Analytic system size L = {L_analytic:.2f}")
print(f"  Simulated avg system size = {q1.mean():.2f}")
print(f"  Analytic time in system W = {W_analytic*60:.1f} min")
print(f"  Simulated avg wait+service = {w1.mean()*60 + (1/μ)*60:.1f} min")
```

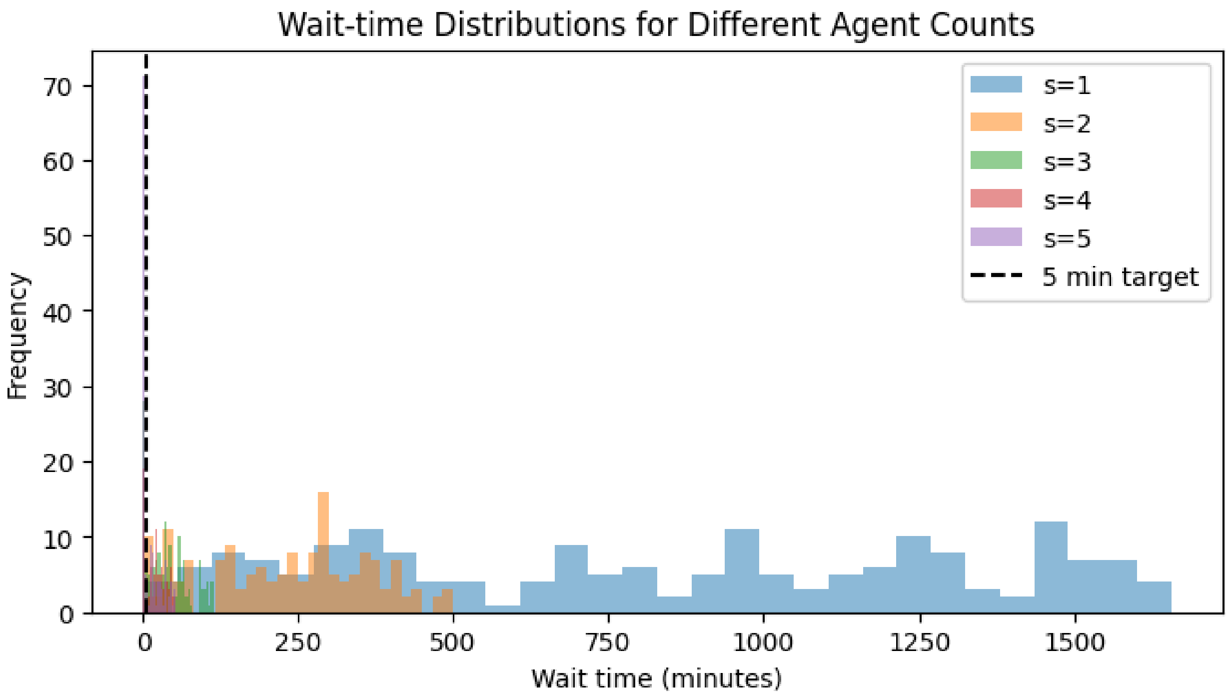
```
M/M/1 analytic vs. simulation:
  Analytic system size L = -1.33
  Simulated avg system size = 74.34
  Analytic time in system W = -4.0 min
  Simulated avg wait+service = 920.5 min
```

In [6]:
```python
# 4. Test Staffing Levels (s=1..5)

threshold = 5   # minutes
results = []

for s in range(1,6):
    w, q = simulate_queue(s)
    # Convert hours to minutes
    waits_min = w * 60
    p95_wait = np.percentile(waits_min, 95)
    results.append((s, waits_min.mean(), p95_wait, q.mean()))

    # tabulate
import pandas as pd
df = pd.DataFrame(results, columns=['Agents', 'Avg wait(min)', '95th-pct wait(
print(df)

# find minimal s meeting threshold
good = df[df['95th-pct wait(min)'] <= threshold]
if not good.empty:
    best_s = int(good.iloc[0]['Agents'])
    print(f"\n→ Schedule at least {best_s} agents to keep 95% of waits ≤ {thre

else:
    print("\nEven 5 agents can’t meet the 5 min 95% wait target.")
```

```
   Agents  Avg wait(min)  95th-pct wait(min)  Avg System Size
0       1     581.970986         1087.740484        54.483221
1       2     237.411975          436.601258        40.892405
2       3     109.263307          210.025871        29.831325
3       4      20.807699           46.457200        11.341040
4       5       1.679563            9.245414         4.647482

Even 5 agents can’t meet the 5 min 95% wait target.
```

In [7]:
```python
# 5. Visualize Wait-Time Distributions

plt.figure(figsize=(8,4))
```

```python
for s in [1,2,3,4,5]:
    w, _ = simulate_queue(s)
    plt.hist(w*60, bins=30, alpha=0.5, label=f's={s}')

plt.axvline(threshold, color='k', linestyle='--', label='5 min target')
plt.xlabel('Wait time (minutes)')
plt.ylabel('Frequency')
plt.title('Wait-time Distributions for Different Agent Counts')
plt.legend()
plt.show()
```



In [8]:

```python
# 6. Time -Varying A
def simulate_queue_timevarying(s, μ):
    # Define piecewise arrival rates per hour: (start, end, arrival rate λ)
    periods = ((0, 2, 30), (2, 6, 20), (6, 8, 40))
    arrivals = []

    for start, end, lam in periods:
        duration = end - start
        # Oversample to ensure enough arrivals
        inter = np.random.exponential(1 / lam, int(lam * duration * 1.5))
        ts = np.cumsum(inter) + start
        arrivals.extend(ts[ts < end])

    arrivals = np.array(arrivals)
    arrivals.sort()

    N = len(arrivals)
    services = np.random.exponential(1 / μ, N)
    next_free = np.zeros(s)
    wait_times = np.zeros(N)
```

```python
    for i, t in enumerate(arrivals):
        j = np.argmin(next_free)  # Choose the next available server
        start = max(t, next_free[j])
        wait_times[i] = start - t
        next_free[j] = start + services[i]

    return wait_times


w_tv = simulate_queue_timevarying(s=3, μ=μ)

print (f"Time-varying λ, s=3 → avg wait = {w_tv.mean()*60:.1f} min")

#**Outcome:** Students will see how peak-hour traffic dramatically
#increases wait times unless staff levels rise.
```

```
Time-varying λ, s=3 → avg wait = 169.1 min
```

In [9]:

```python
# 7. Abandonment
def simulate_with_abandon(s, λ=30, μ=50, shift_hours=8, patience=s/60):
    arrivals = np.cumsum(np.random.exponential(1 / λ, int(λ * shift_hours * 1.
    arrivals = arrivals[arrivals < shift_hours]
    services = np.random.exponential(1/μ, len(arrivals))
    next_free = np.zeros(s)
    waits = []
    abandons = 0

    for i, t in enumerate(arrivals):
        j = np.argmin(next_free)
        start = max(t, next_free[j])
        wait = start - t

        if wait > patience:
            abandons += 1
        else:
            waits.append(wait)
            next_free[j] = start + services[i]

    return np.array(waits), abandons / len(arrivals)


w, ab_rate = simulate_with_abandon(s=4)
print(f"Abandon rate (s=4): {ab_rate:.2%}, avg wait on served calls: {w.mean()
# Shows the trade-off: fewer agents → more abandonments → lost revenue and sat
```

```
Abandon rate (s=4): 0.00%, avg wait on served calls: 0.0 min
```

In [10]:

```python
# 8. Agent Break Scheduling
def simulate_with_breaks(s, break_start=3, break_length=0.25):
    arrivals = np.cumsum(np.random.exponential(1/λ, int(λ * shift_hours * 2)))
    arrivals = arrivals[arrivals <= shift_hours]
    services = np.random.exponential(1/μ, len(arrivals))
    next_free = np.zeros(s)
    waits =[]
```

```python
    for i, t in enumerate(arrivals):
        # if in break window,one fewer agent
        avail = next_free.copy()
        if break_start < t < break_start+break_length:
            avail = np.delete(avail, 0)
            # remove one agent
            j = np.argmin(avail)
            start = max(t, avail[j])
            waits.append(start - t)
            # Update that agent's free time in original array
            idx = j + (1 if break_start < t < break_start + break_length else
            next_free[idx] = start + services[i]

        return np.array(waits)

w_b = simulate_with_breaks(s=3)
print(f"With breaks, s=3 → avg wait = {w_b.mean() * 60:.1f} min")

# See the "break-time  spike" in waiting.
```

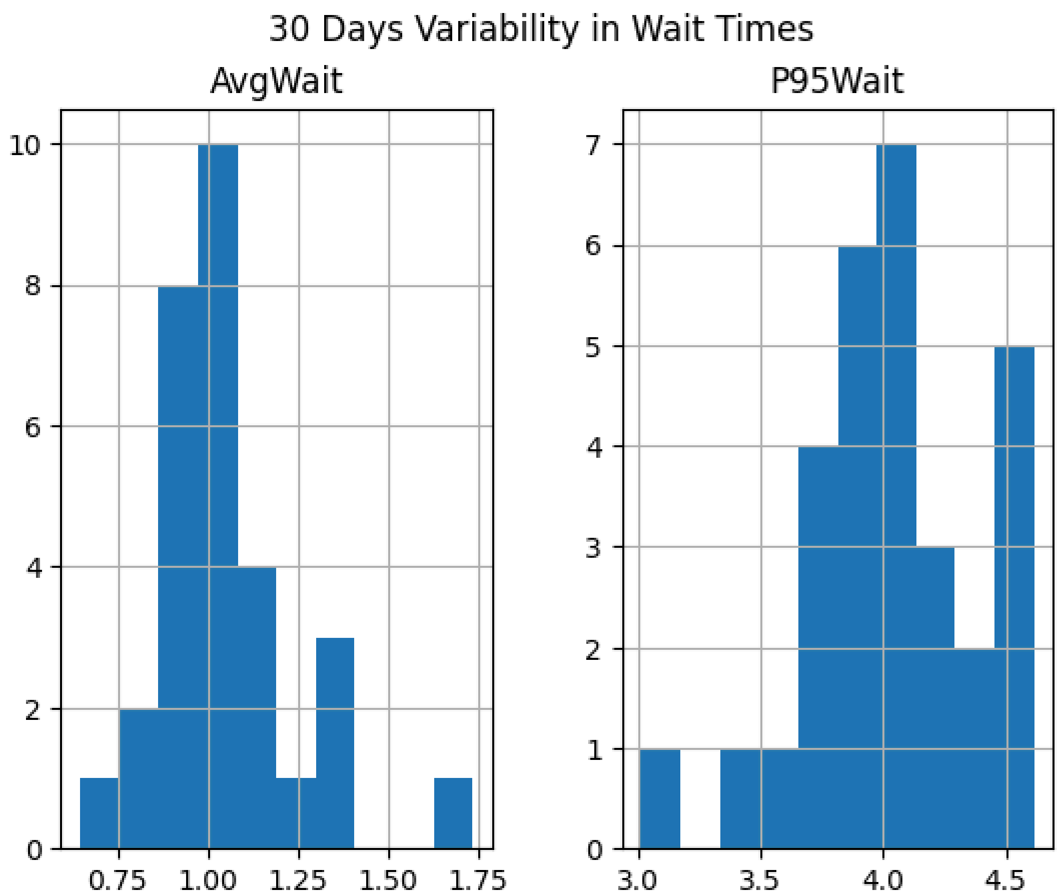
```
With breaks, s=3 → avg wait = nan min
```

```
C:\Users\deves\AppData\Local\Temp\ipykernel_9128\3413400210.py:24: RuntimeWarni
ng: Mean of empty slice.
  print(f"With breaks, s=3 → avg wait = {w_b.mean() * 60:.1f} min")
C:\Users\deves\AppData\Local\Programs\Python\Python313\Lib\site-packages\nump
y\_core\_methods.py:145: RuntimeWarning: invalid value encountered in scalar di
vide
  ret = ret.dtype.type(ret / rcount)
```

In [11]:
```python
# 9. Cost Optimization
c_agent = 20   # $20/hr per agent
c_wait = 0.50  # $0.50 per minute waited

costs = []

for s in range(1,6):
    w, _ = simulate_with_abandon(s) # or choose another sim fn
    total_wait_cost = w.sum()*60*c_wait
    staff_cost = s*c_agent*shift_hours
    costs.append((s, staff_cost+total_wait_cost))

opt = min(costs, key=lambda x:x[1])
print("Agent count, total cost:")

for s, c in costs: print(f" s={s}: ${c:,.0f}")
print(f" → Optimal s by cost = {opt[0]}")
```

```
Agent count, total cost:
 s=1: $271
 s=2: $329
 s=3: $481
 s=4: $640
 s=5: $800
 → Optimal s by cost = 1
```

```
In [12]: # 10. 30-day simulation
days = 30
daily = []

for _ in range(days):
    w, _ = simulate_with_abandon(opt[0])
    daily.append((w.mean()*60, np.percentile(w*60,95)))
df_days = pd.DataFrame(daily, columns=['AvgWait','P95Wait'])
df_days.describe()
df_days.hist(bins=10)
plt.suptitle('30 Days Variability in Wait Times')
plt.show()
```




```
In [13]: df_days
```

Out[13]:

	AvgWait	P95Wait
0	0.994408	3.984950
1	0.992519	4.302430
2	1.019990	3.767121
3	0.927777	3.913086
4	1.320857	4.541893
5	1.084611	3.949125
6	0.915238	3.612630
7	1.079513	3.923195
8	1.011597	3.991571
9	1.379804	4.409735
10	0.989511	3.881583
11	0.773146	3.388260
12	1.733193	4.542913
13	0.952847	4.213788
14	0.825053	4.049794
15	1.054576	3.750638
16	0.646718	3.012475
17	0.969718	3.670075
18	1.148666	4.144479
19	0.974770	4.044488
20	0.905483	3.789281
21	1.032087	4.567976
22	1.034225	4.614714
23	1.083712	3.882002
24	1.097593	4.100393
25	0.916612	3.934497
26	0.959904	4.126553
27	1.402270	4.265232
28	0.950075	4.020376
29	1.240797	4.556372

In [14]:
```
df_days.describe()
```

Out[14]:

	AvgWait	P95Wait
count	30.000000	30.000000
mean	1.047242	4.031721
std	0.206672	0.362615
min	0.646718	3.012475
25%	0.950768	3.881687
50%	1.003002	4.005974
75%	1.084386	4.252371
max	1.733193	4.614714

In [ ]:
```

```