Q.1

1. Let X be a r.v. with p.d.f.

$$f_X(x) = \begin{cases} c\,(x+1), & \text{if } -1 < x < 1 \\ 0, & \text{otherwise} \end{cases},$$

where c is a real constant.
(a) Find the value of c;
(b) Find the mean and variance of X.

2. . Let X be a r.v. with p.d.f.

$$f_X(x) = \begin{cases} \frac{1}{x^2}, & \text{if } x > 1 \\ 0, & \text{otherwise} \end{cases}.$$

Show that $E(X)$ does not exists.

Q.2

Let X be a r.v. with d.f.

$$F_X(x) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{x}{4}, & \text{if } 0 \leq x < 1 \\ \frac{x}{2}, & \text{if } 1 \leq x < \frac{4}{3} \\ \frac{3x}{5}, & \text{if } \frac{4}{3} \leq x < \frac{5}{3} \\ 1, & \text{if } x \geq 1 \end{cases}.$$

(i) Then $S_X = \ldots\ldots\ldots\ldots$

(ii) Then X is

(a) discrete (b) Continuous
(c) A.C. (d) neither discrete nor A.C.

(iii) $P\left(\{1 \leq X < \frac{4}{3}\}\right) = \ldots\ldots\ldots\ldots;$

(iv) $P\left(\{\frac{1}{2} < X < \frac{3}{4}\}\right) = \ldots\ldots\ldots\ldots$

Q.3

Question 3. Suppose that an RV X has the DF F_X given by any of the following functions. In each case, check if X is discrete or continuous. If so, also find the corresponding p.m.f./p.d.f. (as appropriate) of X.

$$F_X(x) = \begin{cases} 0, \ \text{if } x < 0, \\ \frac{1}{2}, \ \text{if } 0 \le x < 2, \\ \frac{3}{4}, \ \text{if } 2 \le x < 3, \\ 1, \ \text{if } x \ge 3. \end{cases}$$

$$F_X(x) := \begin{cases} 0, \ \text{if } x < 0, \\ \frac{x^2}{2}, \ \text{if } 0 \le x < 1, \\ \frac{x}{2}, \ \text{if } 1 \le x < 2, \\ 1, \ \text{if } x \ge 2. \end{cases}$$

Q.4

Let X be a r.v. with p.m.f.

$$f_X(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, & \text{if } x \in \{0, 1, 2, \ldots\} \\ 0, & \text{otherwise} \end{cases},$$

where $\lambda > 0$ is a fixed constant.

Thus

$$M_X(t) \quad =$$

$$E(X) \quad =$$

$$E(X^2) \quad =$$

$$E(X^3) \quad =$$

$$\mathrm{Var}(X) \quad =$$

Q.5

Example 1.237 (Geometric (p) RV). Fix $p \in (0,1)$. Note that $\sum_{k=0}^{\infty} p(1-p)^k = 1$ and hence the function $f : \mathbb{R} \to [0,1]$ given by

$$f(x) = \begin{cases} p(1-p)^x, & \text{if } x \in \{0,1,2,\cdots\}, \\ 0, & \text{otherwise.} \end{cases}$$

Example 1.240 (Exponential(λ) RV). Let $\lambda > 0$. Note that $\int_0^\infty \exp(-\frac{x}{\lambda})\,dx = \lambda$ and hence the function $f : \mathbb{R} \to [0,\infty)$ given by

$$f(x) = \begin{cases} \frac{1}{\lambda}\exp(-\frac{x}{\lambda}), & \text{if } x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

These 2 are standard Random Variable distributions. There are many others like them. Explore them out.

For these 2, calculate expectation values, mean, variance, moment generating function.

Also try to find out where you can find such distribution in application (Bonus Question)

Submission Link :
https://docs.google.com/forms/d/1Y3wAAEveNKBsqnPM3Av4W578LfINIFCB0EszlKrWOIA/edit