Assignment 3:

Q1:

Assignment-3

Q1) (a)

Payoff Matrix:

		Caleb: Go to party	Caleb: Not go to party
Roger	Clown	$8x, 0$	$3x, 1$
	No clown	4,4	2,3

(b) $x=0$

Payoff Matrix:

		Caleb: Go to party	Caleb: Not go to party
Roger	Clown	0, 0	0, 1
	No clown	4,4	2,3

No clown is strictly dominated by clown for player 1 (Roger)

	Go to Party	Not go to party
No Clown	4,4	2,3

Going to party is strictly dominated by not going for Caleb.

Thus, No clown and going to the party is a pure strategy Nash Equilibrium. (By iterated deletion of dominated strategies)

Equilibrium Payoffs:

- Roger's Payoff : 4
- Caleb's Payoff : 4

So, equilibrium payoffs when $x = 0$ are (4,4)

(C) $x=2$

Payoff Matrix:

		Caleb: Go to Party	Caleb: Not go to Party
Roger	Clown	16, 0	6, 1
	No Clown	4,4	2,3

Q2)

		Player 2 (Pickpocket): At Market	Player 2 (Pickpocket): At Home
Player 1 (Police constable)	Patrol	30, -15	0, 0
	Relax	10, 10	10, 0

~~No, there is no dominated strategy~~

Similar to part(B),

For Roger, hiring clown is a strictly dominated strategy.

Thus eliminate hiring clown case.

Now for Caleb, not going to the party is dominant strategy.

Eliminate this as well.

Mixed Strategy Nash Equilibrium-

Let σ be the probability that Roger hires a clown (C)

$(1-\sigma)$ be the probability that Roger doesn't hire a clown

ρ be the probability that Caleb goes to party and $(1-\rho)$ that Caleb doesn't go the party.

(Here, there is no pure strategy Nash Equilibrium as Roger's and Caleb's strategies do not form mutual best responses in any combination)

Caleb's expected Payoff's,

$$E(\overset{\text{going}}{G}) = E(\overset{\text{not going}}{D})$$

$$\sigma(0) + (1-\sigma)4 = \sigma(1) + (1-\sigma)(3)$$

$$4 - 4\sigma = \sigma + 3 - 3\sigma$$

$$1 = 2\sigma$$

$$\boxed{\sigma = 1/2}$$

Roger's expected Payoff's,

$$E(C) = E(N)$$

$$\rho(16) + (1-\rho)6 = \rho(4) + (1-\rho)(2)$$

$$16\rho + 6 - 6\rho = 4\rho + 2 - 2\rho$$

$$10\rho + 6 = 2\rho + 2$$

$$8\rho = -4$$

$$\rho = -1/2$$

We are getting -ve probability, Thus neither mixed strategy Nash equilibrium exists.

Q2:

Q2)

		Player 2 (Pickpocket)	
		(1/3) At (M) market	At (H) home (2/3)
Player 1 (Police)	(2/5) Patrol (P)	30, −15	0, 0
	(3/5) Relax (R)	10, 10	10, 0

As we can observe, there are no dominant strategy in this game for either of the players. Thus no pure strategy nash equilibrium exist.

Mixed strategy Nash Equilibrium,

Let σ be probability that police patrols the market and $(1-\sigma)$ that police relaxes $(1-\rho)$ be the probability that pickpocket stays at home and ~~Q~~ (ρ) that he prowls the market

Police's Expected Payoff:

$$E(P) = E(R)$$

$$30(\rho) + 0(1-\rho) = 10(\rho) + 10(1-\rho)$$

$$30\rho = 10$$

$$\boxed{\rho = 1/3}$$

Pickpocket's Expected Payoff:

$$E(M) = E(H)$$

$$\sigma(-15) + (1-\sigma)(10) = 0$$

$$10 - 25\sigma = 0$$

$$\boxed{\sigma = 2/5}$$

Thus,

Police patrols the market with probability 2/5 and relaxes with probability 3/5 and the pickpocket stays at home with probability 1/3 and prowls the market with probability 2/3.

Equilibrium Payoffs:

Police's expected Payoff:

$$E(\text{Police}) = \frac{2}{5}\left(30 \times \frac{1}{3} + 0 \times \frac{2}{3}\right) + \frac{3}{5}\left(10 \times \frac{1}{3} + 10 \times \frac{2}{3}\right)$$

$$= \frac{2}{\cancel{5}}(\cancel{10})^{2} + \frac{3}{\cancel{5}}(\cancel{10})^{2} = 4 + 6 = 10 \text{ Ans.}$$

Pickpocket's expected Payoff:

$$E(\text{Pickpocket}) = \frac{1}{3}\left(-15 \times \frac{2}{5} + 10 \times \frac{3}{5}\right) + \frac{2}{3}\left(\frac{2}{5} \times 0 + \frac{3}{5} \times 0\right)$$

$$= \frac{1}{3}(-6 + 6) = 0 \text{ Ans.}$$

Q3:

Q3) (a)

Firm 2

Firm 1

	Doesn't Enter Market	Produce A	Produce B
Doesn't enter market	0,0	0,15	0,30
Enter Market Produce A	15,0	-10,-10	10,10
Enter Market Produce B	30,0	10,10	5,5

(b) Our employee is correct as entering the market and producing B product is better than not entering the market as we can see that in any of the strategy of Firm 2, Firm 1's 1st strategy is dominated one.

[30 > 0 , 10 > 0, 5 > 0]

(c) If Firm 1 enters and produces B,

- Firm 2's payoff if it produces B is 5.
- " " " " " A is 10
- " " " " " doesn't enter is 0.

As 10 > 5, Firm 2 can deviate from producing B to producing A if Firm 1 is producing A.

Similar is the case for Firm 1.

Since both the firms have an incentive to deviate from producing B to producing A, this strategy doesn't form a Nash Equilibrium.

(They are getting higher payoff by producing A rather than B)

(d) If we check by iterative deletion of dominant strategy, we are left with:

Firm 2

Firm 1

	A	B
A	-10,-10	$\boxed{10,10}$
B	$\boxed{10,10}$	5,5

By observing this payoff matrix, we can see 2 nash equilibrium (Pure Strategy)

(Enter & produce B, Enter & product A)

(Enter B " A, " " " B)

Q4.

Q4)

(a) For bidder a, who never makes a mistake, we need to evaluate whether bidding this true value is a dominant strategy.

- If $v_a = 1$

Case 1 Bidder A can bid 1.
If Bidder B bids 0, A wins and pays nothing (0)
If Bidder B bids 1, A wins half the time and pays 1.

Case 2 Bidder A bids less than 1 ($x < 1$), then bidder B wins if he bids 1.
The payoff for A is 0

Case 3 Bidder A bids more than 1 ($y > 1$), he wins if B bids 0, but the payment is still 0, same as bidding 1.

- If $v_a = 0$

Case 1 Bidder A can bid 0.
If Bidder B bids 0, payoff is 0.
If Bidder B bids 1, A loses and payoff is 0.

Case 2 If A bids more than 0, he risks winning the auction and paying B's bid which leads to a -ve payoff because A's value is 0

In Both the cases, Bidding true value is optimal.
Thus, Bidding true value is still a dominant strategy for bidder A.

(b) To determine seller's expected revenue, we have to consider all possible cases

Case 1 Bidder a and b both have $v = 0$

- Both bid 0, Revenue = 0
- Probability $= \frac{1}{2} \times \frac{1}{4} = 1/8$

Case 2 Bidder a has $v = 1$ and bidder b has $v = 0$

- Bidder a bids 1 and b bids 0 ($\frac{1}{2}$ probability)
- Revenue = 0 (Bidder B bids 0)
- Probability $= \frac{1}{2} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{16}$

Case 3 Bidder a has $v = 0$ and b has $v = 1$

- Bidder a bids 0 and b bids 1
- Revenue = 0 (a bids 0)
- Probability $= \frac{1}{2} \times \frac{3}{4} = 3/8$

Case 4 Both bid $v = 1$

- Both will bid 1
- Revenue $= 1$ (winner is selected at random but price is 1)
- Probability $= \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

Case 5 Bidder a has $v = 1$, bidder b mistakenly bid 1. (value $= 0$)

- Revenue $= 1$
- Probability $= \frac{1}{2} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{16}$

Expected Revenue (Revenue * Probability)

$$= \left(0 \times \frac{1}{8}\right) + \left(0 \times \frac{1}{16}\right) + \left(0 \times \frac{3}{8}\right) + \left(1 \times \frac{3}{8}\right) + \left(1 \times \frac{1}{16}\right)$$

$$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$$ Ans.

Q5.

Q5) (a) In this situation, since I am not the member of that group, I am going to bid the value that I think that rare wine glass is worthy for because since I don't know the value of v, ~~I don't know~~ I can't decide my bid according to it. Thus, best option for me would be to bid the actual value that I think of.

(b) In this situation also I would bid the value that I think the glass is worthy of. Here, seller is using trick upon the bidders and benefitting himself. Here, I cannot cross-trick the seller and get benefits. Thus only option is to bid the actual value that I think of.