ASSIGNMENT 1 STAMATICS

(Q1) (1) $f_X(X) = \begin{cases} c(x+1) & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

(a) $c = 1/2$

(b) Mean $= 1/3$

Variance $= 2/9$

Q4) ② $f_X(X) = \begin{cases} \frac{1}{x^2} & \text{if } x > 1 \\ 0 & \text{otherwise} \end{cases}$

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)\, dx$$

$$= \int_{1}^{\infty} \frac{1}{x}\, dx \; \cancel{= \left.\frac{-1}{x^2}\right|_1^{\infty} = (0+1)}$$

$$\cancel{= \left.\frac{-1}{x^2}\right|_{-\infty}^{1}} = \ln x \Big|_1^{\infty} = \text{DNE}$$

Thus $E(X)$ does not exist.

Q2) $F_X(x) = \begin{cases} 0 & x < 0 \\ x/4 & 0 \leq x < 1 \\ x/2 & 1 \leq x < 4/3 \\ 3x/5 & 4/3 \leq x < 5/3 \\ 1 & x \geq 5/3 \end{cases}$

(i) $S_X = \{x \in \mathbb{R} : F_X(x+\varepsilon) - F_X(x-\varepsilon) > 0, \ \forall \ \varepsilon > 0\}$

$= \left[0, \frac{5}{3}\right]$

(ii) $D_X = \{1, 4/3\} \neq \phi$. So X is not continuous and hence also not A.C.

$P(\{X \in D_X\}) = P(\{X = 1\}) + P(\{X = 4/3\})$

$= \left[\frac{1}{2} - \frac{1}{4}\right] + \left[\frac{4}{5} - \frac{2}{3}\right]$

$= \frac{1}{4} + \frac{2}{15} = \frac{15+8}{60} = \frac{23}{60} < 1$

Therefore, X is neither a discrete r.v.

(d)

(iii) $P(\{1 \leq X < 4/3\}) = P(\{X < 4/3\}) - P(\{X < 1\})$

~~$= P(\{X = 4/3\}) - P($~~ $= F_X(4/3-) - F_X(1-)$

$= \frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12}$ Ans

(iv) $P(\{\frac{1}{2} < X < \frac{3}{4}\}) = P(\{X < 3/4\}) - P(\{X \leq 1/2\})$

$= F_X(3/4-) - F_X(1/2)$

$= \frac{3}{16} - \frac{1}{8} = \frac{1}{16}$ Ans.

Q3) case-1 $F_X(x) = \begin{cases} 0 & x < 0 \\ 1/2 & 0 \leq x < 2 \\ 3/4 & 2 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$

$D_X = \{0, 2, 3\} \neq \phi$, Thus X is not continuous

$$P(\{X \in D_X\}) = P(\{X = 2\}) + P(\{X = 3\}) + P(\{X = 0\})$$

$$= \left(\frac{3}{4} - \frac{1}{2}\right) + \left(1 - \frac{3}{4}\right) + \left[\frac{1}{2} - 0\right)$$

$$= 1$$

Thus X is a discrete random variable

case-2 $F_X(x) = \begin{cases} 0 & x < 0 \\ x^2/2 & 0 \leq x < 1 \\ x/2 & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$

$D_X = \phi$

X is continuous and not discrete.

p.d.f (for case 2) $= \begin{cases} 0 & x < 0 \\ x & 0 \leq x \leq 1 \\ 1/2 & 1 \leq x < 2 \\ 0 & x \geq 2 \end{cases}$

Q4) $f_X(X) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!} & \text{if } x \in \{0,1,2 \dots\} \\ 0 & \text{otherwise} \end{cases}$

$\lambda > 0 = \text{const}^n$

$$M_X(t) = E(e^{tX})$$
$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^t)^x}{x!} = e^{-\lambda} e^{\lambda e^t} = e^{\lambda(e^t - 1)}$$

$$M_X(t) = 1 + \lambda(e^t - 1) + \frac{\lambda^2}{2!}(e^t - 1)^2 \dots$$

$$= 1 + \lambda s + \frac{\lambda^2 s^2}{2!} \dots = 1 + \lambda t + (\lambda + \lambda^2)\frac{t^2}{2!} + \left(\frac{\lambda}{3!} + \frac{2\lambda^2}{(2!)^2} + \frac{\lambda^3}{3!}\right)\frac{t^3}{3!} \dots$$

where $s = e^t - 1$

$E(X) = $ coeff. of t in $M_X(t) = \lambda$

$E(X^2) = $ " " $\frac{t^2}{2!}$ in $M_X(t) = \lambda + \lambda^2$

$E(X^3) = $ " " $\frac{t^3}{3!}$ in $M_X(t) = \lambda^3 + 3\lambda^2 + \lambda$

$\text{Var}(X) = E(X^2) - (E(X))^2 = \lambda$

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$$f(x) = \begin{cases} \frac{1}{\lambda} e^{(-x/\lambda)} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$M_x(t) = E(e^{tX})$$

$$= \int_{-\infty}^{\infty} e^{tx} f_x(x)\, dx$$

$$= \int_0^{\infty} e^{tx} \frac{1}{\lambda} e^{-x/\lambda}\, dx = \frac{1}{\lambda} \int_0^{\infty} e^{\cancel{-}(t+1/\lambda)x}\, dx$$

$$\cancel{= \frac{1}{\lambda}\left[\frac{e^{(t-1/\lambda)x}}{(t-1/\lambda)}\right]_0^{\infty}} \qquad \frac{1}{\lambda} \int_0^{\infty} e^{-\left(\frac{1}{\lambda}-t\right)x}\, dx$$

$$= \frac{1}{\lambda\left(\frac{-1}{\lambda}+t\right)}\left[0-1\right] = \cancel{\frac{1}{\lambda t - 1}} = \frac{-1}{(-1+\lambda t)}$$

$$= \frac{1}{(1-\lambda t)}$$

$$\cancel{= \frac{1}{\lambda}\left(t-\frac{1}{\lambda}\right)^{-1}}$$

$$= (1-\lambda t)^{-1}$$

$$= 1 + \lambda t + (\lambda t)^2 + (\lambda t)^3 \ldots$$

$E(X) =$ coefficient of t in $M_X(t)$

$= \lambda$

$E(X^2) = 2\lambda^2$

$E(X^3) = 6\lambda^3$

Variance $= E(X^2) - (E(X))^2 = 2\lambda^2 - \lambda^2 = \lambda^2$

I was unable to do the Q5 1st part.