



# Application of Probabilistic Theory

END TERM REPORT

# GAME THEORY

Game theory is designed to address situations in which the outcome of a person's decision depends not just on how they choose among several options, but also on the choices made by the people they are interacting with.

# SOME TERMS RELATED TO GAME THEORY

- GAME: A situation in which your payoff depends not only on what you do, but what the others do.
- ACTION: a move you can make at a stage in a game.
- STRATEGY: a plan conditional on any possible contingency.
- EQUILLIBRIUM: a set of strategies such that neither player wishes to deviate.

## TYPES OF GAMES

1. STATIC GAMES: One that is played just once at the same time
2. DYNAMIC GAMES: One in which players move sequentially or repeatedly.

# TYPES OF STRATEGIES IN STATIC GAMES

- DOMINANT STRATEGY: A strategy that produces a higher payoff than any other possible strategy.
- DOMINATED STRATEGY: A strategy the players will never play, so it can be deleted.
- PURE STRATEGY: If the player chooses a single action
- MIXED STRATEGY: When a player randomizes between two or more actions.

# SOME OTHER IMPORTANT CONCEPTS

NORMAL REPRESENTATION: The players, their strategies and the payoff are shown as a combination of strategies in a payoff matrix. It is just the summary of the game using a table. The row players payoffs are written first, and the column players payoffs are written second.

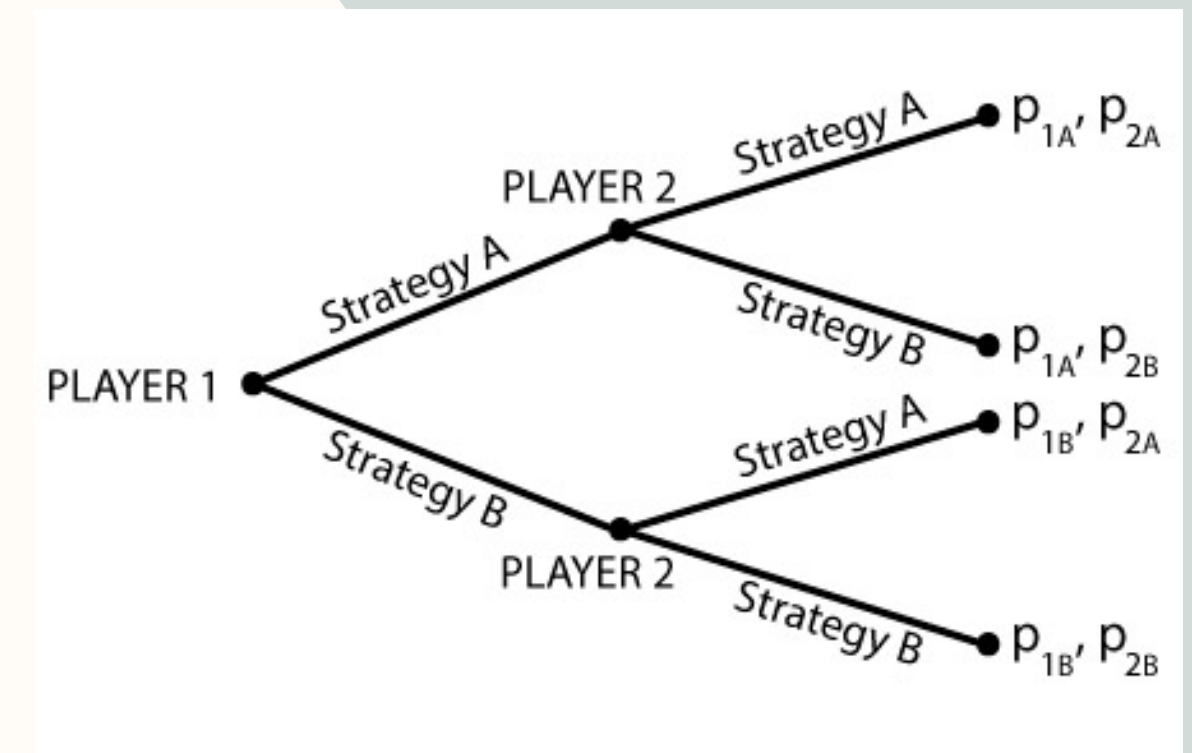
| NICK\TOM | SILENT | BETRAY |
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| SILENT   | -1,-1  | -12,0  |
| BETRAY   | 0,-12  | -3,-3  |

PAYOFF MATRIX FOR PRISONERS'S DILEMMA GAME

NASH EQUILLIBIRUM: A set of strategies is a Nash equilibrium if nobody wishes to deviate from their strategies. There does not exist a profitable deviation. It is self-enforcing. All dominant strategy equilibria are Nash equilibria, the reverse is not true.

# IMPORTANT CONCEPTS OF DYNAMIC GAMES

- EXTENSIVE FORM: Show the sequence of moves and the actions each player can make each move.
- SEQUENTIAL GAME: A game in which one player moves before another. Sequential games contain subgames.
- SUBGAMES: Consists of all subsequent decisions the player can make given the action taken.
- SUBGAME PERFECT NASH EQUILLIBRIUM (SPNE): Strategies are a SPNE if the players strategies a Nash equilibrium in every subgame.



EXAMPLE OF EXTENSIVE FORM

# Auction Theory

- An auction is a sale in which a good or service is sold to the highest bidder
- In an auction, I know how much I value the good, but I don't know how much everybody else values it (I might know the distribution).
- Each player must devise a bidding strategy without complete information

A few type of auctions are as follows:

### **1. ENGLISH AUCTION**

- It is also called ascending-bid auction. The auctioneer starts from a low bid and keeps raising the price until nobody wants to bid more.

### **2. DUTCH AUCTION**

- It is also called descending-bid auction. The seller starts at a really high price. If nobody wants to buy, the price is dropped until somebody accepts.

### **3. SEALED BID AUCTION**

- Everybody submits a bid without seeing the other bids. In a First-price auction, the winner pays her own highest bid. In second-price auctions (or Vickrey), the winner with the highest bid pays the second-highest bid.



Auctioned goods either have a **private value** or **common value**.

### **Private Value Goods**

- Each bidder assigns a personal, subjective value to the good. This value is based on individual preferences, needs, or circumstances.
- Example: A piece of art or a collectible item. One bidder might value it highly because it completes their collection, while another might not value it as much.

### **Common Value Goods**

- The good has the same intrinsic value to everyone, but not everyone knows what that exact value is.
- Example: An oil reserve. The actual amount of oil (and therefore its value) is the same for all bidders, but they may have different estimates of this value based on their information.

Strategies people adopt during auctions

## **Second Price Auction**

- Bidding your valuation is a weakly dominant strategy. This means you are at least as well off by bidding your valuation as any other bid. Bidding something other than your valuation might cause you to lose the auction, or have to pay more than you value the good.

## **English Auction Strategies**

- Once the going bid reaches our maximum bid, one should stop because he/she will not be made better off by purchasing the good at price greater than it.

## **First price sealed and Dutch**

- For Dutch or sealed first-price auctions bidders face a trade-off because they must pay their bid. The lower they make their bid, the more surplus they get if they win. The higher they make their bid, the greater is the probability that they win. Bidders will bid less than their valuations in these auctions.

# Interlinking Probability and Game theory:

We were introduced to interlinking of Probability and Game theory through models and examples.

## **1. Bidding Strategies in Ad Auctions**

- Interlinking Probability Theory and Game Theory: These fields provide powerful frameworks to analyze and predict outcomes in competitive environments like ad auctions.
- Ad auctions involve strategic decision-making under uncertainty, making them an ideal application for both probability and game theory.

We were then introduced to a model regarding an online ad auction.

The outline of the model mainly focused on these parts:

1. Understand Auction Mechanisms.
2. Analyze Bidding Strategies Using Game Theory.
3. Model Bidders' Valuations with Probability Theory.
4. Calculate Expected Revenue.
5. Simulate and Optimize Auction Process.

## **Auction Mechanisms:**

Second-Price Auction (Vickrey Auction):

- Highest bidder wins and pays the price of the second-highest bid.

Generalized Second-Price Auction:

- Extends the Vickrey auction to multiple ad slots.

## **Bidder's Valuation:**

- Each advertiser's valuation is considered as a random variable.
- Assume valuations follow a known distribution, such as uniform distribution  $U[0, 100]$ .

## **Bidding Strategies:**

Optimal Strategy in Second-Price Auction:

- Dominant strategy for each bidder is to bid their true valuation.

Bayesian Nash Equilibrium in Generalized Second-Price Auction:

- Bidders use probabilistic beliefs about others' valuations to determine optimal bids.

Then, the expected revenue of the auctioneer was calculated.

We were then introduced to an example regarding a sample auction with 3 bidders where we theoretically stimulated the whole auction process and calculated the expected revenue for the auctioneer.

## 2. Risk Management in Supply Chain Management

- Supply Chain Management (SCM): Coordinating suppliers, manufacturers, and distributors for efficient production and distribution.
- Game Theory: Strategizes interactions among supply chain entities.
- Probability: Manages uncertainties related to demand, supply, and disruptions.

We were then introduced to an example for understanding the concept behind it.

### **Problem Setup:**

Entities Involved:

- Suppliers: Two suppliers (Supplier 1 and Supplier 2).
- Manufacturer: One manufacturer.

Objectives:

- Suppliers: Maximize profits by determining the quantity and pricing of raw materials.
- Manufacturer: Minimize costs while ensuring a steady supply of raw materials.



## **Game - Theory Application:**

Players: Supplier 1 and Supplier 2.

Strategies:

- Supply a high quantity (HQ) or a low quantity (LQ) of raw materials.

Payoffs: Depend on the strategies chosen by both suppliers and the resulting costs and profits.

Interpretation

## **Probability Application:**

Modeling Demand and Supply Uncertainties:

- Demand Uncertainty: Modeled using probability distributions (e.g., normal distribution).
- Supply Uncertainty: Includes delays and quality issues, also modeled probabilistically.

Risk Management:

- Estimating Likelihoods: Manufacturer estimates the likelihood of different scenarios (e.g., high demand, supply disruptions).
- Mitigation Strategies: Maintaining safety stock, diversifying suppliers.

Another detailed example was also used to show the co-relation between both the theories.

Game Theory helps in understanding the competitive dynamics between suppliers, while Probability aids in managing uncertainties and risks.



Thank You!