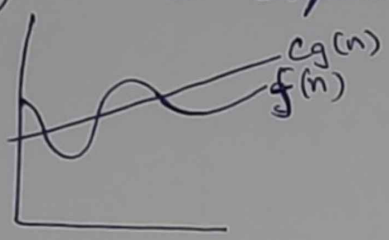
Tutorials - 1

Q1 Asymptotic notations

They are mathematical notations used to describe the running time of an algo when input tends towards a particular value or limiting values.

There are mainly 3 types:

1) Big O - It represents upperbound



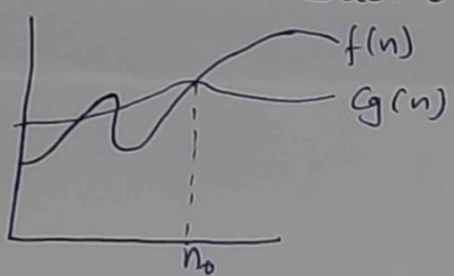
$f(n) = O(g(n))$

there exist +ve const. c & n_0 such that $0 \leq f(n) \leq cg(n)$ for all $n > n_0$

2) Omega - It represents lower bound $\Omega g(n) = f(n)$ then

There exists +ve const. c & no

Such that $0 \leq cg(n) \leq f(n)$ for all $n \geq n_0$



3) Theta notation - It represent lower & upper bound of running time of algo.

$f(n) = \Theta g(n)$ there exist +ve const c_1, c_2 & n_0 such that $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n \geq n_0$

Q2 for (i=1 to n) · { i = i*2 }

i	1	2	4	8	- -	2^k
val	2^0	2^1	2^2	2^3	- - -	n

$$2^k = n$$
$$k \log_2 2 = i \log_2 n$$
$$k = \log n$$
$$T.C = O(\log n)$$

Q3 $$T(n) = \begin{cases} 3T(n-1), & n>0 \\ 1 \end{cases}$$

by forward

$$T(n) = 3T(n-1), \quad T(0) = 1$$
$$T(1) = 3T(1-1) = 3T(0) = 3$$

$$T(2) = 3T(2-1)$$
$$= 3T(1)$$
$$= 3 \times 3 = 3^2$$

$$T(3) = 3T(3-1)$$
$$3T(2)$$
$$3 \times 3^2 = 3^3$$

$$T(n) = 3^n$$

Time Complexity = $O(3^n)$

Q4 $T(n) = \begin{cases} 2T(n-1)-1, & n>0 \\ 1, & n=0 \end{cases}$

$T(0) = 1$

$T(1) = 2T(1-1)-1$

$= 2T(0)-1$

$= 2-1$

$= 1$

$T(2) = 2T(2-1)-1$

$= 2T(1)-1$

$= 2-1$

$= 1$

$T(3) = 2T(3-1)-1$

$= 2T(2)-1$

$= 2-1$

$= 1$

$T(n) = 1$

Time complexity = $O(1)$

Q5)

```
int i=1, s=1
while (s<=n)
{
    i++;
    s = s+i;
    print ("#");
}
```

for k iteration

$S(k) = 1+2+3+\cdots+k$

$= (1+k)\frac{k}{2}$

$\frac{(k+1)k}{2} > n$

$k = O(\sqrt{n})$

Time complexity = $O(\sqrt{n})$

Q6
```
fun (int n)
{
  int i, count = 0
  for (i = 1; i*i <= n; i++)
  {
      c++;
  }
}
```

for $S(k) = 1^2 + 2^2 + 3^2 + \cdots + k^2 <= n$

$= \frac{k(k+1)(2k+1)}{6} \leq n$

$= 2k^3 + 3k^2 + k \leq n$

Time complexity $= 3\sqrt{n}$

Q7
```
fun (int n)
{ int i = n/2; i<=n; i++)
  for (j = 1; j<=n; j = j*2)
   for (k = 1; k<=n; k = k*2)
       count++;
```

Outer loop runs $\frac{n}{2}$ times

Second loop runs $\log n$ times

Third loop runs $\log n$ times

Time complexity $= \frac{n}{2} * \log n * \log n$

$= O(n \log(n)^2)$

Q8 · fun (int n)

```
{ if (n==1) return;
  for (i=1 to n)
    for (j=1 to n)
      print("*")
  fun (n-3)
}
```

for 1st loop n times

for 2nd loop n times

Time complexity = $n * n = O(n^2)$

Q9. Fun (int n)

```
for (i=1 to n)
  for (j=1; j<=n; j=j+i)
    printf("*")
}
```

∴ First loop runs n times

Second loop runs log n times

Time complexity = $n * \log n$

$= O(n \log n)$

Q10 · For functions n^k and c^n what is the asymptotic relationship between these functions?

n^k & c^n

$n^k = O(c^n)$