Tutorial 2

Q1)

```
fun (int n)
{  int j=1, i=0
   while (i<n)
    { i = i+j
    } j++
}
```

$$\frac{k(k+1)}{2} = n$$

$k^2 = n$

$k = \sqrt{n}$

Time complexity = $O(\sqrt{n})$

Q2 $T(0) = 0$

$T(1) = 0$

$T(n) = T(n-1) + T(n-2) + 1$

let $T(n-1) = T(n-2)$

$T(n) = T(n-2) + T(n-2) + 1$

$2T(n-2) + 1$

using backward substitution,

$T(n) = 2 \cdot 2(T(n-2) + 1) + 1 \Rightarrow 4(T(n-2)) + 3$

$T(n-2) = 2T(n-3) + 1$

$= 2(2(2(T(n-3) + 1) + 1) + 1$

$= 8T(n-3) + 7$

$T(n) = 2^k T(n-k) + 2^k - 1$

$T(0) = 0 \quad n - k = 0 \Rightarrow n = k$

$T(n) = 2^n + (n-n) + 2^n - 1$

$= 2^n + 2^n$

Time Complexity: $O(2^n)$

Q3. log (log n)

```
fun (int n)
{ for (int i=n; i>=2; pow(i,i/2)
   {  Some i(1)
   }
}
```

n log n

```
for (int i=1; i<=n; i++)
  for (int j=1; j<=n; j=j*2)
    {  Some O(1)
    }
}
```

n^3

```
for (int i=1; i<n; i++)
  for (int j=1; j<n; j++)
    for (int k=1; k<n; k++)
      Some O(n)
```

Q4.) $T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + cn^2$

assum $T\left(\frac{n}{2}\right) >= T\left(\frac{n}{4}\right)$

$$T(n) = 2T\left(\frac{n}{2}\right) + cn^2$$

$$c = \log_b a$$

$$c = \log_2 2 = 1$$

$$n^2 < f(n)$$

Time Complexity $= O(n^2)$

Q5. i j

1	n
2	$n/2$
3	$n/3$
⋮	⋮
n	n/n

Time Complexity = $O(n \log n)$

Q6 $i = 2, 2^k, (2^k)^k, ((2^k)^k)^k \ldots 2^{k \log_k (\log n)}$

$$2^{k \log_k (\log n)} = n$$

Time Complexity = $O(\log(\log n))$

Q7 $T(n) = T\left(\frac{99}{100} n\right) + \frac{n}{100}$

$T(1) = 0$

Putting $n = \frac{99}{100} n$

$$T\left(\frac{99}{100} n\right) = T\left(\left(\frac{99}{100}\right)^2 n\right) + \frac{99 n}{(100)^2}$$

$$T(n) = T\left(\frac{99^k}{100} n\right) + \cdots \frac{(99)^{k-1}}{(100)^k} n$$

$$\left(\frac{99}{100}\right)^k n = 1$$

$$n = \left(\frac{100}{99}\right)^k$$

$$K = \log_{\frac{100}{99}} n$$

Time Complexity = $n \log n$

Q8 Arrange the following in increasing order

a) n, n!, log n, log log n, root(n), log (n!), n log n, log^2(n)
-2^n, 2^(2^n), 4^n, n^2, 100

$\Rightarrow$ $100 < \log(\log n) < \log n < \sqrt{n} < \log(n) < n \log n < n^2$
$< 2^n < 2^{2n} < 4^n < n!$

b) 2(2^n), 4n, 2n, 1, log(n), log(log(n)), $\sqrt{\log(n)}$; log 2n
2 log (n), n, log (n!), n!, n2, n log (n)

$\Rightarrow$ $1 < \log(\log n) < \sqrt{\log n} < \log(n) < 2n < 4n < 2(2^n) < \log(2n)$
$< 2 \log n < n < \log(n!) < n \log n < n!$

c) 8^(2n); $\log_2(n)$, $n \log_6(n)$, $n \log_2(n)$, log (n!), n!
$\log_8(n)$, 96, 8n2, 7n3, 5n

$\Rightarrow$ $96 < \log_2 n = \log_8(n) < n \log_6 n = n \log_2(n) < 5n < 8n^2$
$< 7n^3 < 8^{2n}$