Tutorial 4

1) $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

$a = 3 \quad b = 2 \quad f(n) = n^2$

a & b are constant & $f(n)$ is function Master's theorm is applicable

$c = \log_b a$

$= \log_2 3 = 1.58$

$n^c = n^{1.58}$ which is $< n^2$

case 3 is applicable

$T(n) = \Theta(n^2)$

Q2 $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

$a = 4 \quad b = 2 \quad f(n) = n^2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2 \Rightarrow f(n)$

Case 2 $T(n) = \Theta(n^2 \log n)$

Q3 $T(n) = T\left(\frac{n}{2}\right) + 2^n$

$a = 1 \quad b = 2 \quad f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0 \Rightarrow n^c = n^0 = 1$

$f(n) > n^c$

Case 3

$T(n) = \Theta(2^n)$

4) $T(n) = 16T\left(\frac{n}{4}\right) + n$

$a = 16 \quad b = 4 \quad f(n) = 4$

$c = \log_b a = \log_4 16 = 2$

$n^c > f(n)$

Case 1 $T(n) = \Theta(n^2)$

5) $T(n) = 16T(n/4) + n$

$a = 16 \quad b = 4 \quad f(n) = 4$

$c = \log_b a = \log_4 16 = 2$

$n^c > f(n)$

Case 1 $\quad T(n) = \Theta(n^2)$

6) $T(n) = 2T\left(\frac{n}{2}\right) + n \log n$

$a = 2 \quad b = 2 \quad f(n) = n \log n$

$c = \log_b a = \log_2 2 = 1$

$n^c = n$

Case 3 is applied

$T(n) = \Theta(n \log n)$

7) $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

$a = 2 \quad b = 2 \quad f(n) = \frac{n}{\log n}$

$c = \log_2 2 = 1$

$n^c = n$

non polynomial diff between n^c & $f(n)$.

$\therefore$ Master theorm not applicable

8) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a=2 \quad b=4 \quad f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$f(n) > n^c$

Case 3 is applicable

$T(n) = \Theta(n^{0.51})$

9) $T(n) = 0.5T\left(\frac{n}{2}\right) + \left(\frac{1}{n}\right)$

$a < 1$ $\therefore$ Master theorm is not applicable

10) $T(n) = 16T\left(\frac{n}{4}\right) + n!$

$a = 16 \quad b = 4 \quad f(n) = n!$

$c = \log_4 16 = 2$

$n^c = n^2$

$f(n) > n^c$

Case 3 ..

$T(n) = \Theta(n!)$

11) $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

$a = 4 \quad b = 2 \quad f(n) = \log n$

$c = \log_2 4 = 2$

$n^c = n^2$

$n^c > f(n)$

case 1 $\quad T(n) = \Theta(n^2)$

12) $T(n) = \sqrt{n}\, T\left(\frac{n}{2}\right) + \log n$

a is not constant so Master theorm not applicable

13) $T(n) = 3T\left(\frac{n}{2}\right) + n$

$a = 3 \cdot b = 2 \quad f(n) = n$

$c = \log_b a = \log_2 3 = 1.58$

$n^c = n^{1.58} > f(n)$

case 1: $T(n) = \Theta(n^{1.58})$

14) $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

$a = 3 \quad b = 3 \quad f(n) = \sqrt{n}$

$c = 1$

$n^c = n > \sqrt{n}$

Case 1 $\cdot$ $T(n) = \Theta(n)$

15.) $T(n) = 4T\left(\frac{n}{2}\right) + Cn$

$a = 4 \quad b = 2 \quad f(n) = Cn$

$n^c = n^2 > f(n)$

Case 1 $T(n) = O(n^2)$

16) $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$

$a = 3 \quad b = 4 \quad f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.78$

$n^c = n^{0.78} < f(n)$

case 3: $T(n) = \Theta(n\log n)$

17) $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

$a = 3,\quad b = 3 \cdot f(n) = \frac{n}{2}$

$c = 1 \cdot n^c = n > f(n)$

Case 1: $T(n) = \Theta(n)$

18) $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$

$a = 6 \cdot b = 3 \cdot f(n) = n^2 \log n$

$c = \log_3 6 = 1.63$

$n^c = n^{1.63} < f(n)$

Case 3: $T(n) = \Theta(n^2 \log n)$

19) $T(n) = 4T\left(\frac{n}{2}\right) + n \log n$

$c = \log_2 4 = 2$

$n^c = n^2\ f(n) = n \log n$

$n^c > f(n)$

Case 1: $T(n) = \Theta(n^2)$

20) $T(n) = 64T\left(\frac{n}{8}\right) + n^2 \log n$

$a = 64 \cdot \quad c = \log_8 64 = 2$

$b = 8 \quad n^c = n^2 < fn$

Case 3: $T(n) = \Theta(n^2 \log n)$

21) $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a = 7\ b = 3 \cdot f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77} < f(n)$

Case 3: $T(n) = \Theta(n^2)$

22) $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$

$f(n)$ is not regular function

So, Master Theorm not applicable