Entropy Balance & Generation

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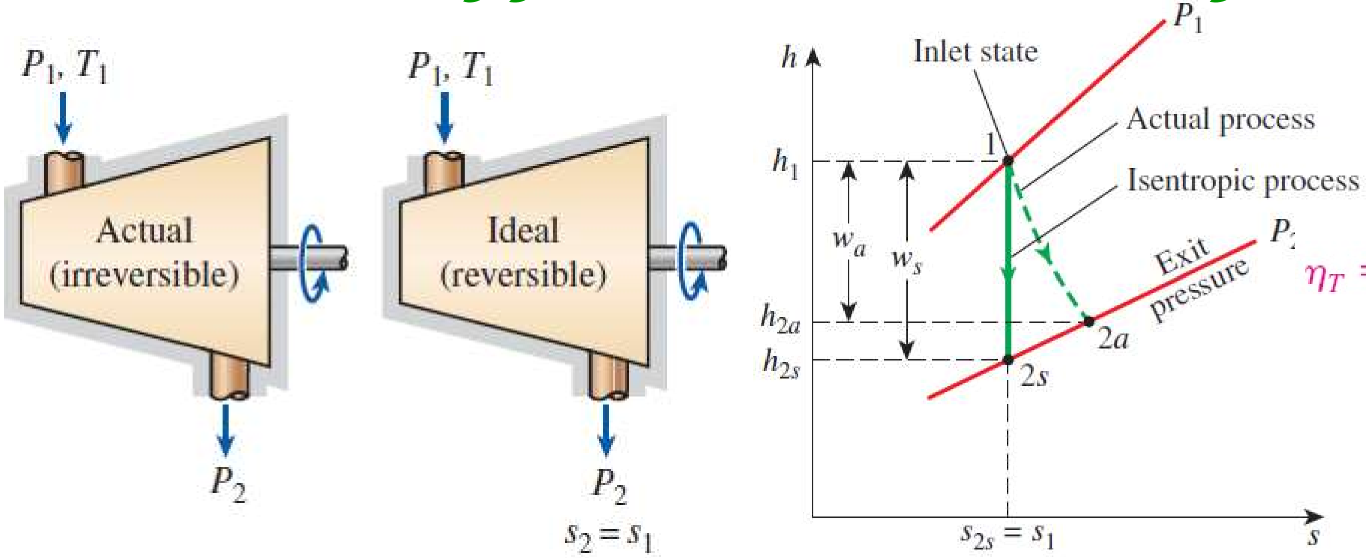
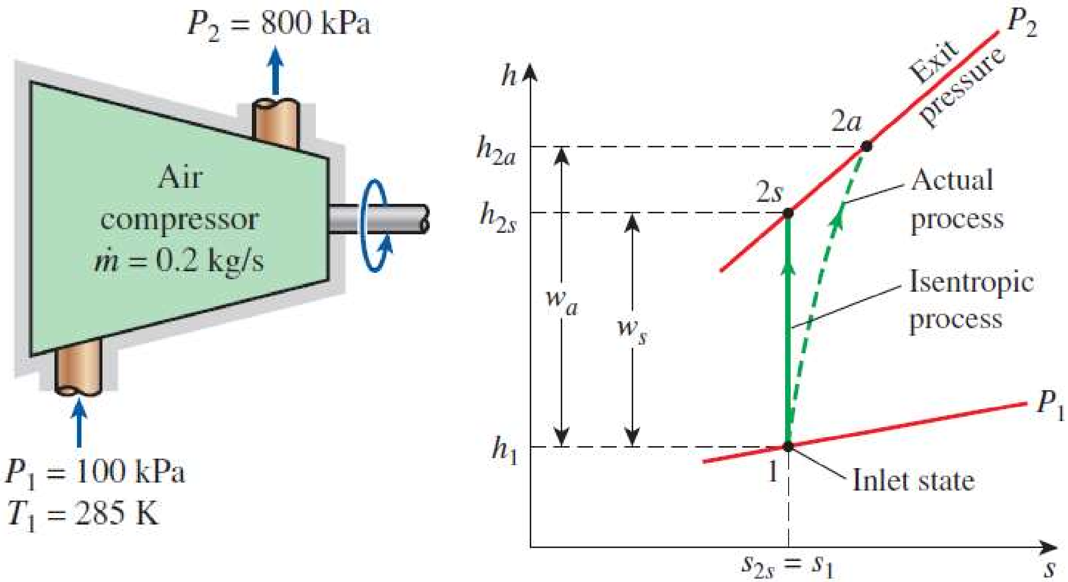
Previously: Efficiencies of steady-flow devices-Isentropic analysis



$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s}$$

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$



$$\eta_C = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a}$$

$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

$$\eta_P = \frac{w_s}{w_a} = \frac{v(P_2 - P_1)}{h_{2a} - h_1}$$ For a pump

Fig-TD: Cengel & Boles

Mass/Energy Balance Vs. Entropy Balance

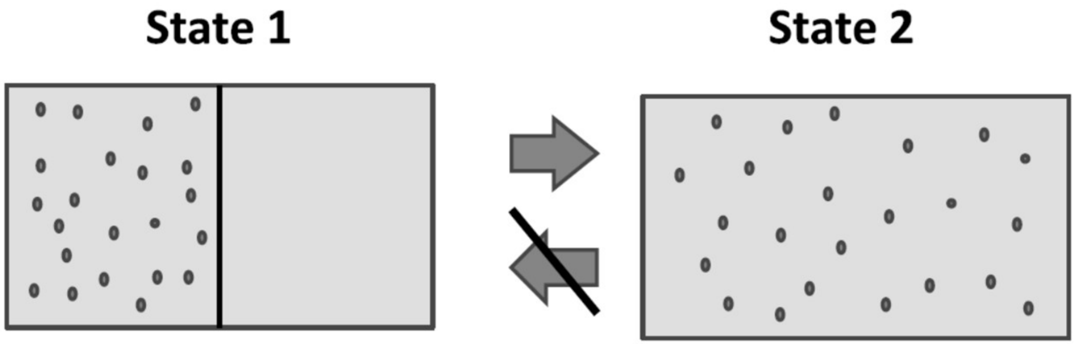
- Mass/Energy can be transferred across boundaries; Mass/Energy are conserved quantities that are neither created nor destroyed; Balance equation is straightforward
- Entropy can be "transferred" across boundaries; Entropy is not a conserved quantities & is generated in an irreversible process

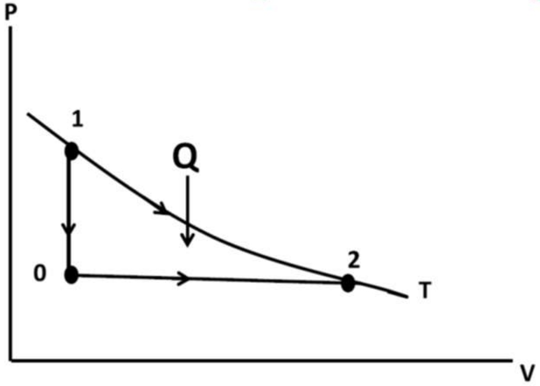
$$\boxed{\text{All cycles:} \quad \oint \frac{\delta Q}{T} \leq 0}$$

= reversible processes; < irreversible processes

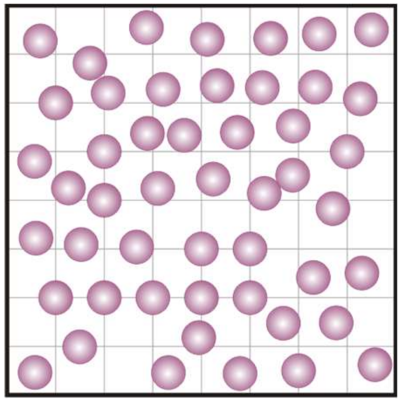
- How do we write a balance equation?

Revisit: Microscopic Interpretation of Entropy





- Equivalent reversible path connects the 2 relevant states
- $\Delta S = NK^* \ln 2$; Result of macroscopic TD
- Each lattice has volume ~ r^3; (r ~ atomic radius)
- # of lattice points: $L_1 = (V_1/r^3)$; $L_1 >>> N$; $L_2 = 2L_1$
- $\Omega_1 = (L_1)^N$; $\Omega_2 = (2L_1)^N = 2^N * (L_1)^N$
- $\Delta S = S_2 - S_1 = K\ln\Omega_2(U,V,N) - K\ln\Omega_1(U,V,N) = K^*\ln 2^N$



- $(\Delta S)_{isolated} \geq 0$

$S(U,V,N) = K^* \ln\Omega(U,V,N)$

$F(T, V, N) = -KT^* \ln Z(T,V,N)$

Figs: Fundamentals of Physics-1, R. Shankar

Entropy balance via "Entropy generation"

$$\begin{pmatrix}\text{Total}\\\text{entropy}\\\text{entering}\end{pmatrix} - \begin{pmatrix}\text{Total}\\\text{entropy}\\\text{leaving}\end{pmatrix} + \begin{pmatrix}\text{Total}\\\text{entropy}\\\text{generated}\end{pmatrix} = \begin{pmatrix}\text{Change in the}\\\text{total entropy}\\\text{of the system}\end{pmatrix}$$

$$S_{\text{in}} - S_{\text{out}} + S_{\text{gen}} = \Delta S_{\text{system}}$$

$$\Delta S_{\text{system}} = S_{\text{final}} - S_{\text{initial}} = S_2 - S_1$$

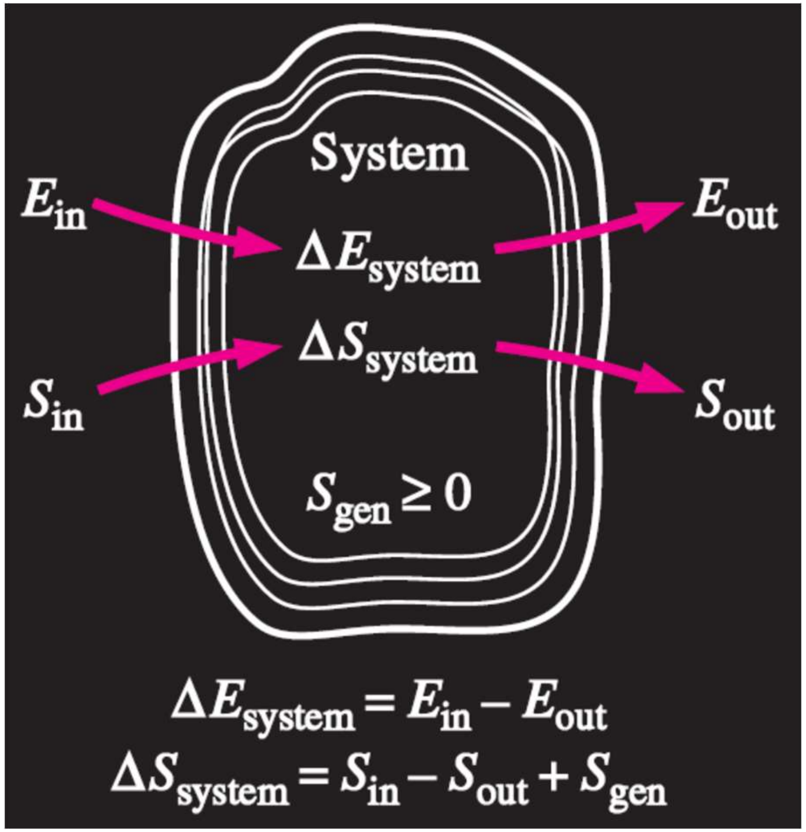
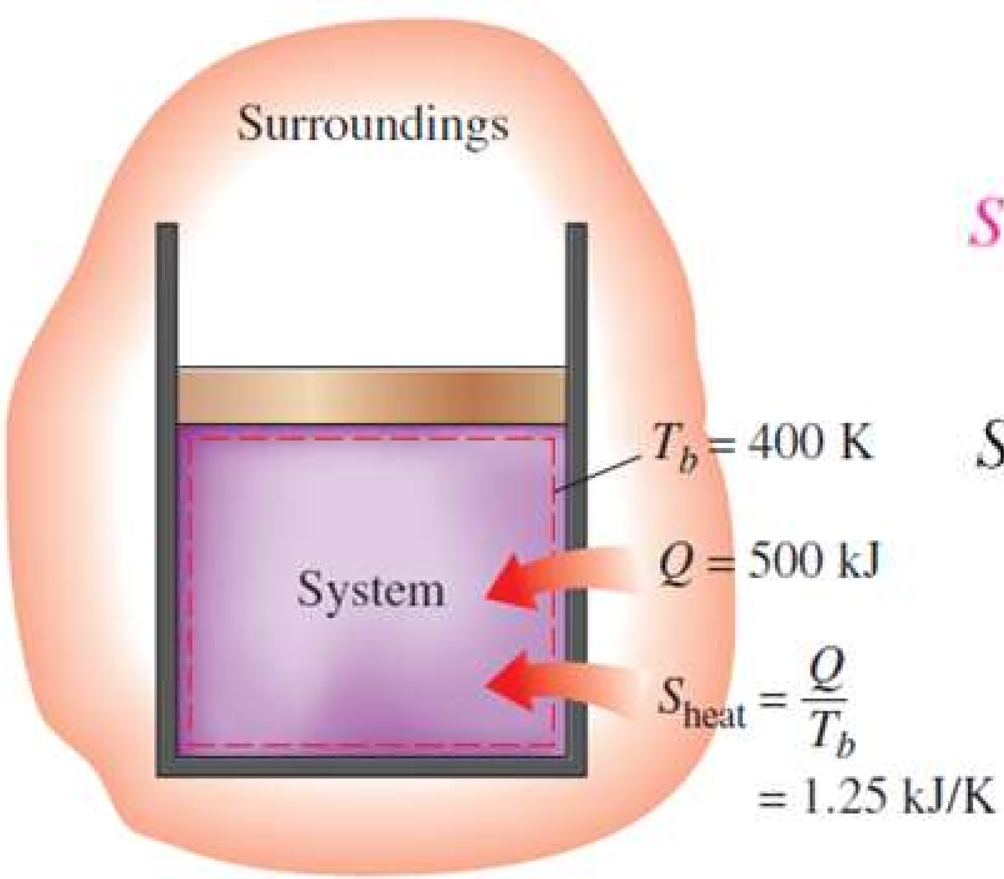


Fig-TD: Cengel & Boles

Entropy transfer via heat



$$S_{heat} = \frac{Q}{T} \qquad (T = \text{constant})$$

$$S_{heat} = \int_1^2 \frac{\delta Q}{T} \cong \sum \frac{Q_k}{T_k}$$

Fig-TD: Cengel & Boles

No Entropy transfer by work but…Irreversibilities

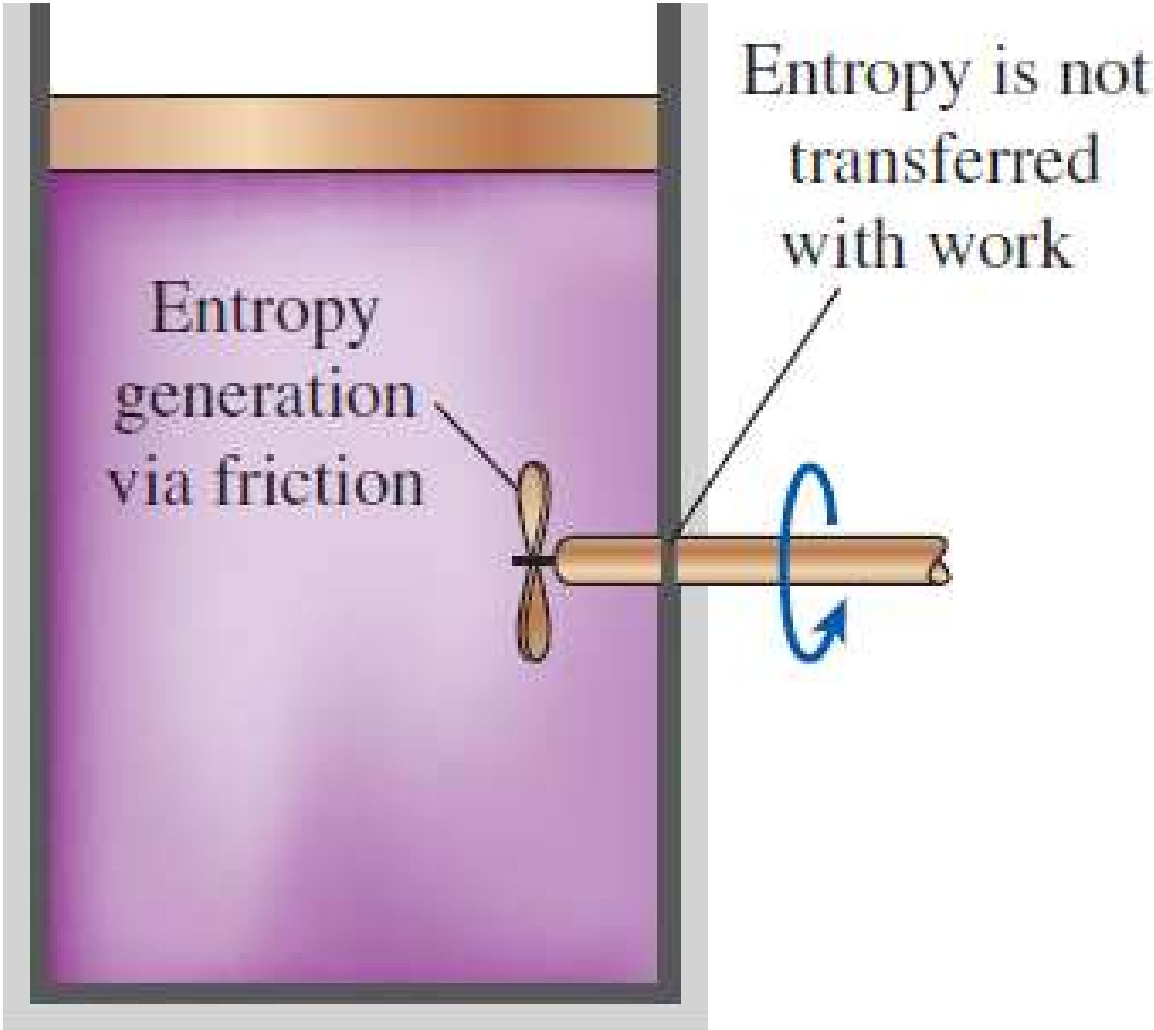
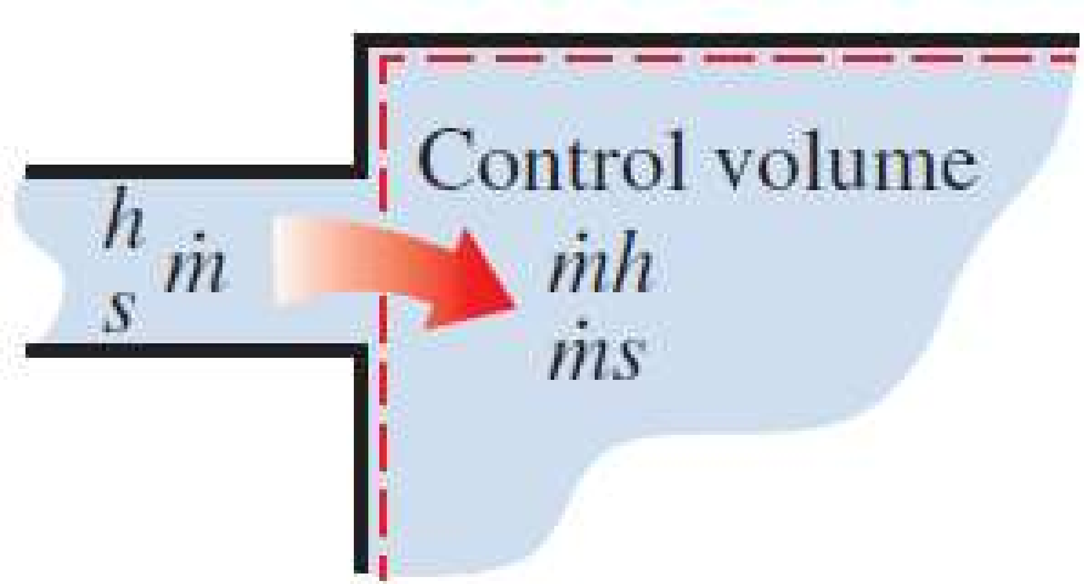


Fig-TD: Cengel & Boles

Entropy transfer by mass flow

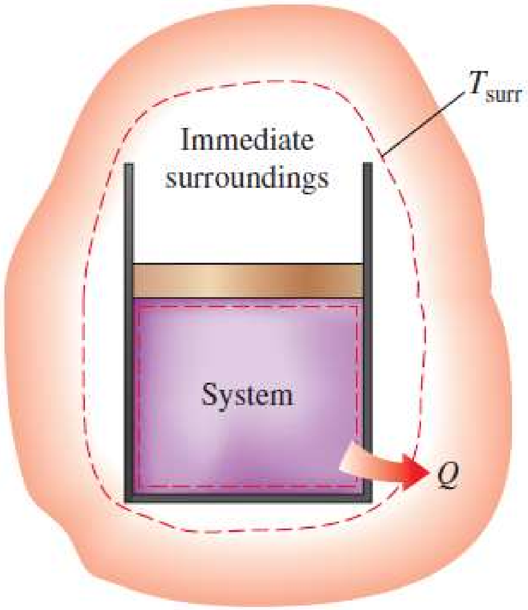


$$\dot{S}_{\text{mass}} = \int_{A_c} s\rho V_n \, dA_c$$

$$S_{\text{mass}} = \int s \, \delta m = \int_{\Delta t} \dot{S}_{\text{mass}} \, dt$$

Fig-TD: Cengel & Boles

Entropy Generation in Closed System



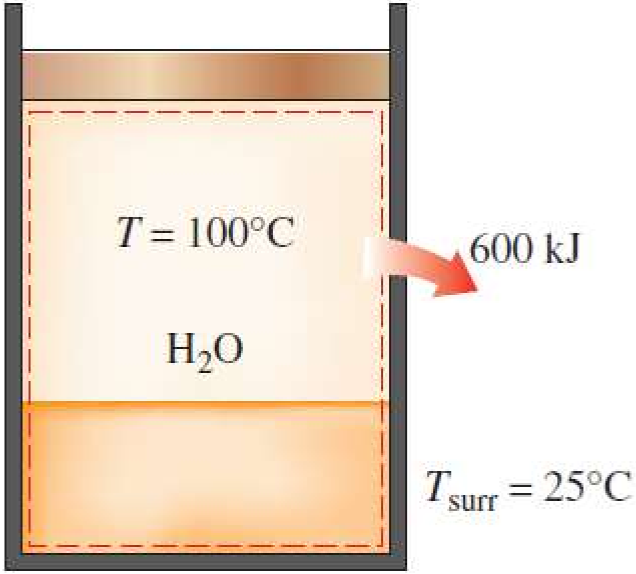
Closed system: $$\sum \frac{Q_k}{T_k} + S_{gen} = \Delta S_{system} = S_2 - S_1 \qquad \text{(kJ/K)}$$

System + Surroundings: $$S_{gen} = \sum \Delta S = \Delta S_{system} + \Delta S_{surroundings}$$

$$\Delta S_{system} = m(s_2 - s_1) \qquad \Delta S_{surr} = Q_{surr}/T_{surr}$$

Fig-TD: Cengel & Boles

Entropy generation to balance entropy transfer

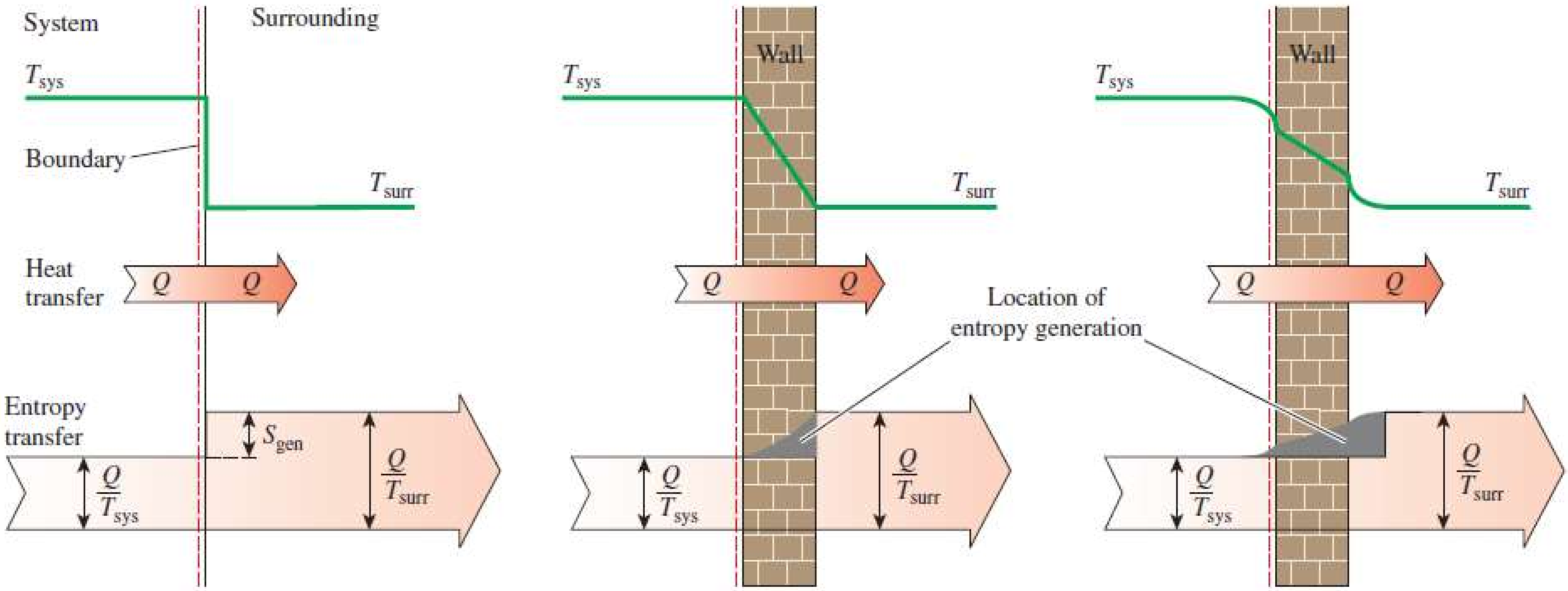


$$\Delta S_{\text{system}} = \frac{Q}{T_{\text{system}}} = \frac{-600 \text{ kJ}}{(100 + 273 \text{ K})} = \mathbf{-1.61 \text{ kJ/K}}$$

$$S_{\text{gen}} = \frac{Q_{\text{out}}}{T_b} + \Delta S_{\text{system}} = \frac{600 \text{ kJ}}{(25 + 273) \text{ K}} + (-1.61 \text{ kJ/K}) = \mathbf{0.40 \text{ kJ/K}}$$

Fig-TD: Cengel & Boles

Entropy balance for heat transfer though a wall



(*a*) The wall is ignored

(*b*) The wall is considered

(*c*) The wall as well as the variations of temperature in the system and the surroundings are considered

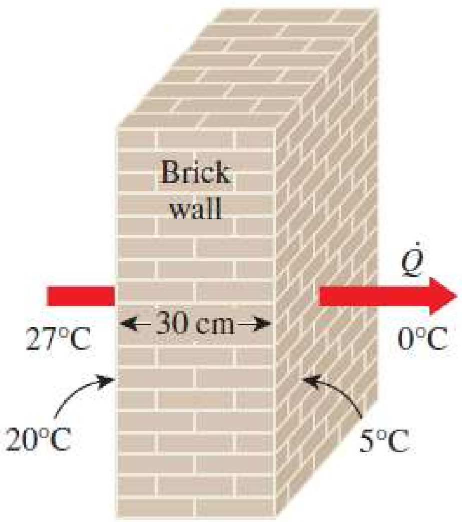
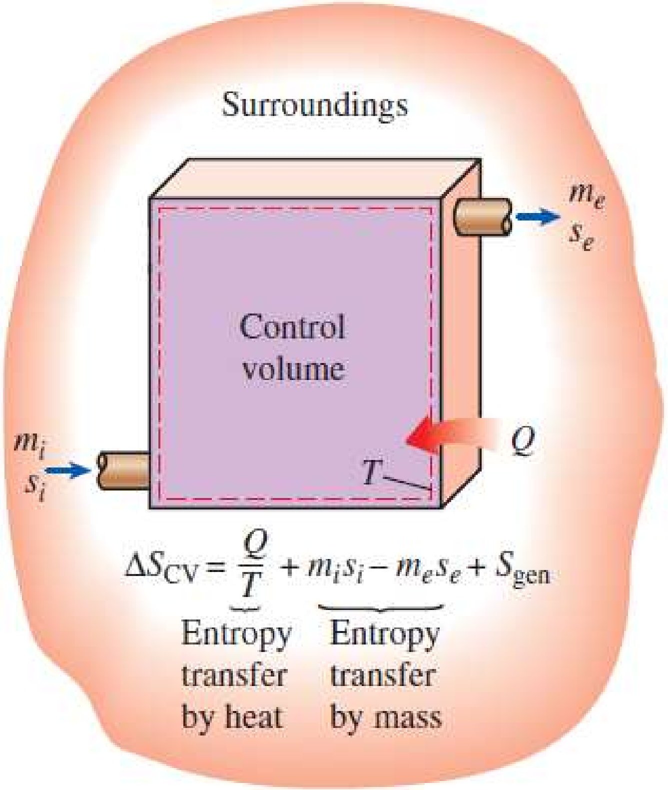
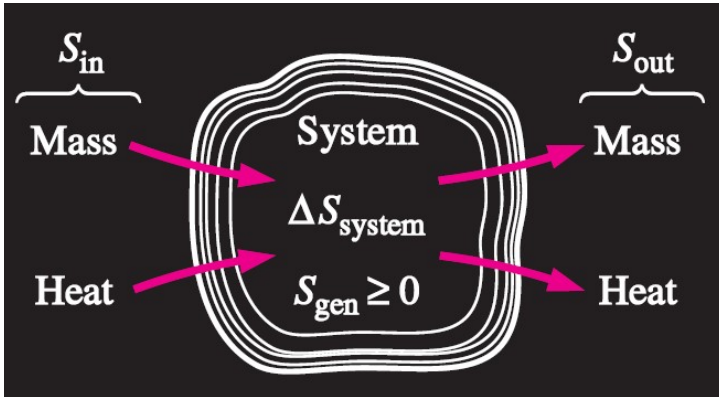


Fig-TD: Cengel & Boles

Entropy generation in a flow device





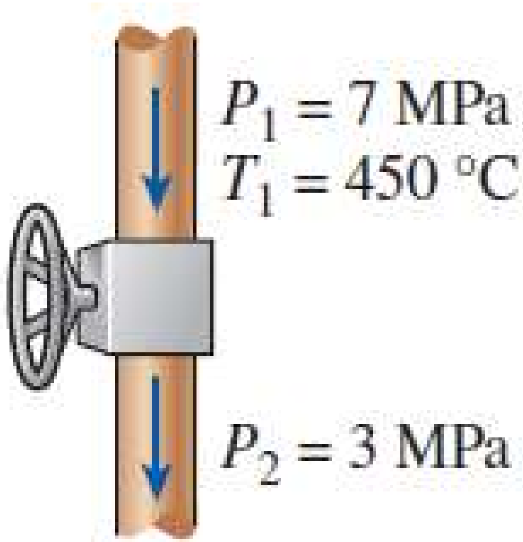
$$\underbrace{S_{in} - S_{out}}_{\substack{\text{Net entropy transfer} \\ \text{by heat and mass}}} + \underbrace{S_{gen}}_{\substack{\text{Entropy} \\ \text{generation}}} = \underbrace{\Delta S_{system}}_{\substack{\text{Change} \\ \text{in entropy}}} \qquad (\text{kJ/K})$$

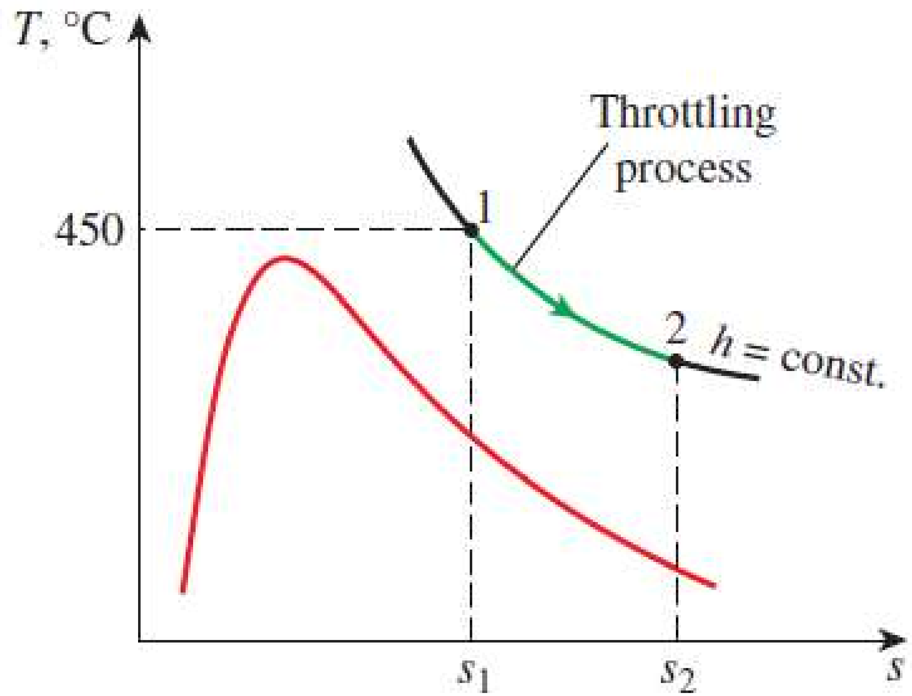
$$(s_{in} - s_{out}) + s_{gen} = \Delta s_{system} \qquad (\text{kJ/kg} \cdot \text{K})$$

$$\underbrace{\dot{S}_{in} - \dot{S}_{out}}_{\substack{\text{Rate of net entropy} \\ \text{transfer by heat} \\ \text{and mass}}} + \underbrace{\dot{S}_{gen}}_{\substack{\text{Rate of entropy} \\ \text{generation}}} = \underbrace{dS_{system}/dt}_{\substack{\text{Rate of change} \\ \text{in entropy}}} \qquad (\text{kW/K})$$

Fig-TD: Cengel & Boles

Entropy balance in throttling





$$\underbrace{\dot{S}_{in} - \dot{S}_{out}}_{\substack{\text{Rate of net entropy}\\ \text{transfer by heat}\\ \text{and mass}}} + \underbrace{\dot{S}_{gen}}_{\substack{\text{Rate of entropy}\\ \text{generation}}} = \underbrace{\cancelto{0\ \text{(steady)}}{dS_{system}/dt}}_{\substack{\text{Rate of change}\\ \text{in entropy}}}$$

$$\dot{m}s_1 - \dot{m}s_2 + \dot{S}_{gen} = 0$$

$$\dot{S}_{gen} = \dot{m}(s_2 - s_1)$$

Fig-TD: Cengel & Boles

What's next?

- Exergy analysis