ESC102T : Introduction to Electronics

L10: Transient Analysis of RLC Circuits

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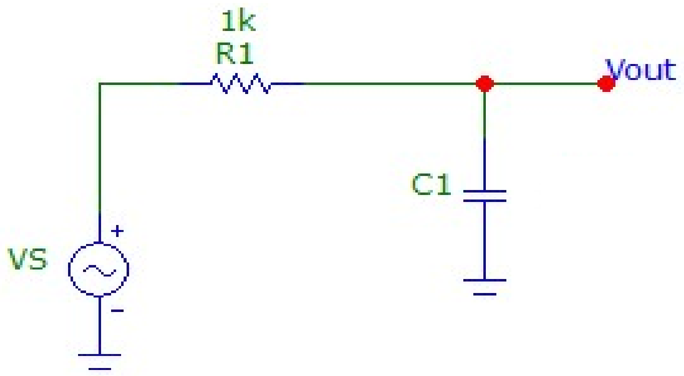
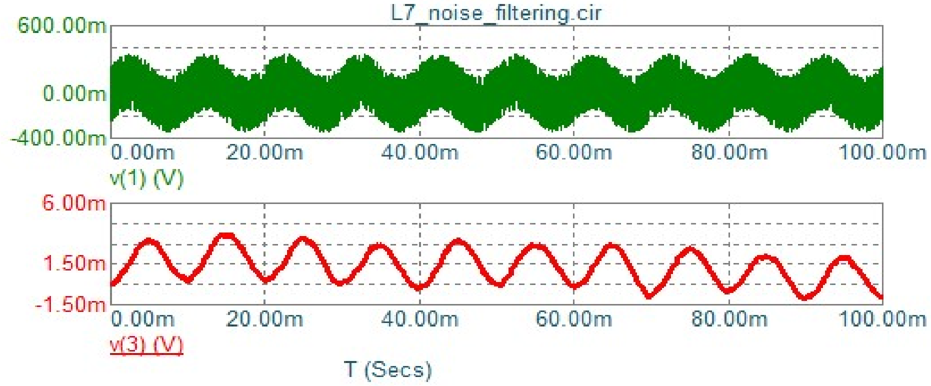

1k
R1
Vout
C1
VS
+
-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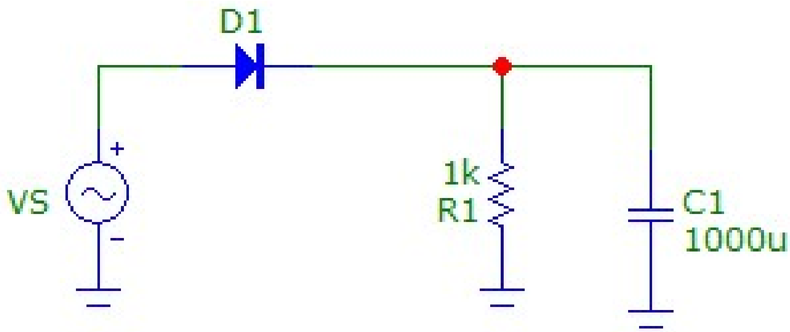
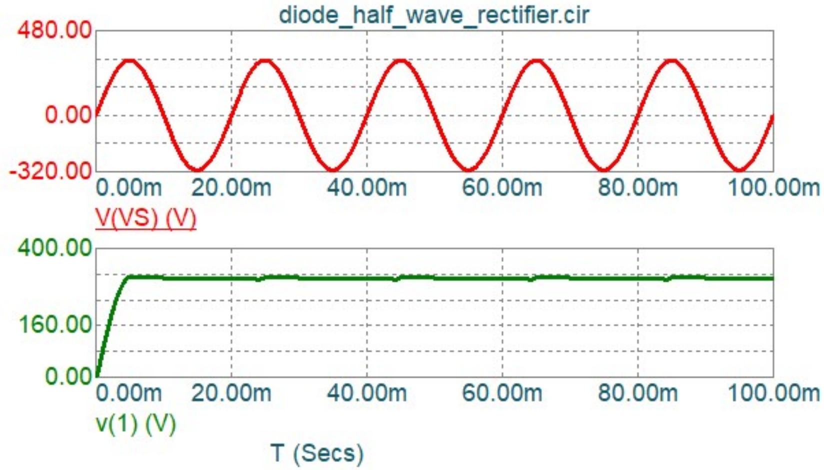
0.00m
-400.00m
0.00m
20.00m
40.00m
60.00m
80.00m
100.00m
v(1) (V)
6.00m
1.50m
-1.50m
0.00m
20.00m
40.00m
60.00m
80.00m
100.00m
v(3) (V)
T (Secs)


D1
VS
+
-
1k
R1
C1
1000u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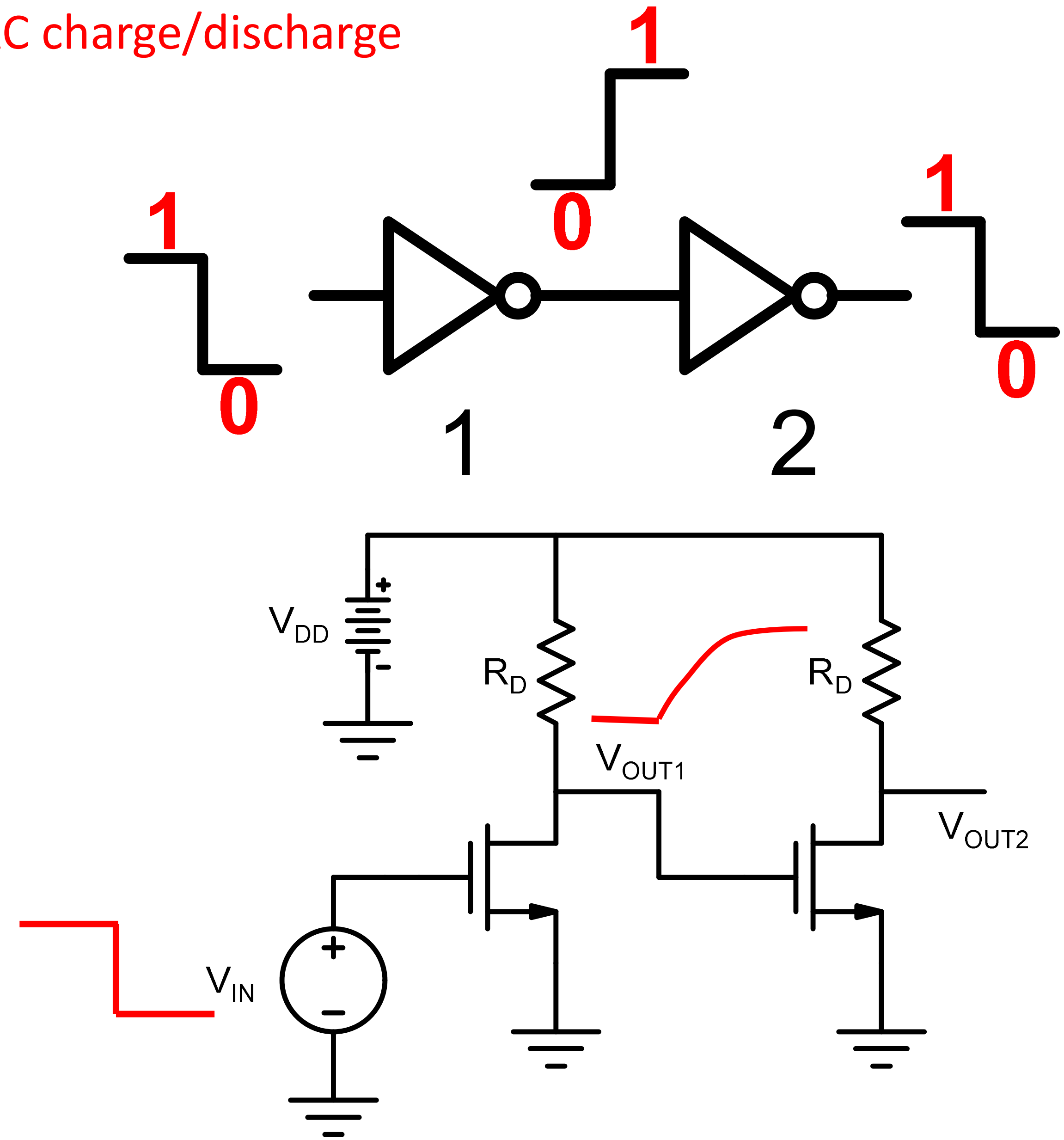
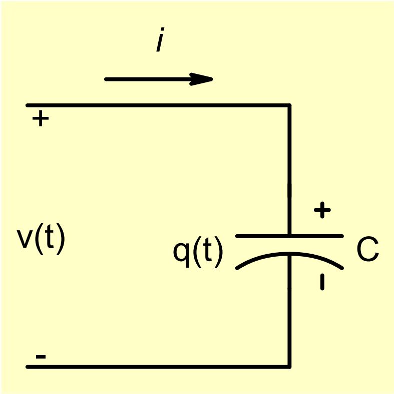
0.00
-320.00
0.00m
20.00m
40.00m
60.00m
80.00m
100.00m
V(VS) (V)
400.00
160.00
0.00
0.00m
20.00m
40.00m
60.00m
80.00m
100.00m
v(1) (V)
T (Secs)

Circuit Delay due to RC charge/discharge





$$q = C \times v$$

Coulombs

Farad

Volt

$$i = \frac{dq}{dt}$$

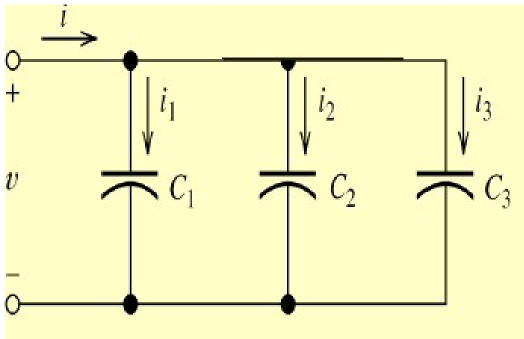
$$i = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int_{t_o}^{t} i\,dt + v(t_o)$$

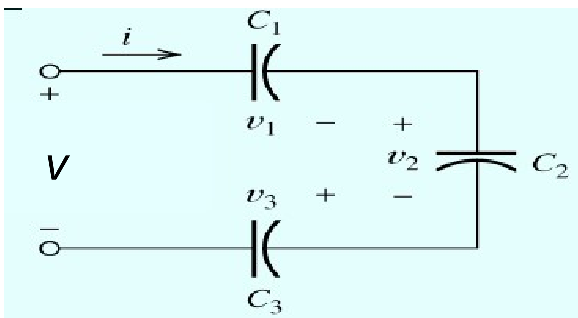
$$w_c(t) = \frac{1}{2}C \times v_c^2(t)$$

For dc or steady state when the voltage does not vary with time $i = 0$

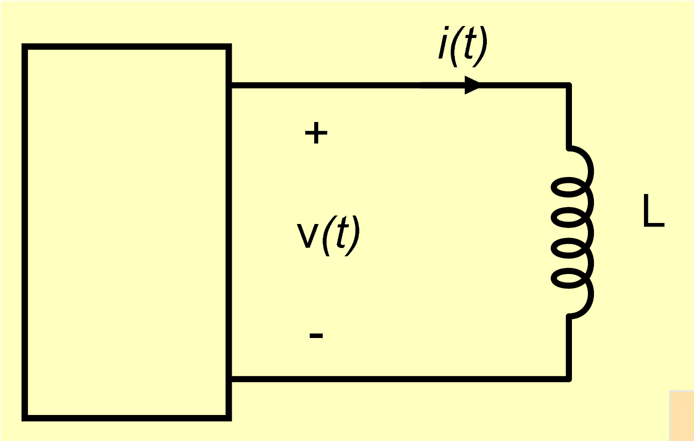
A capacitor under dc or steady state acts like an open circuit



$$C_{eq} = C_1 + C_2 + C_3$$



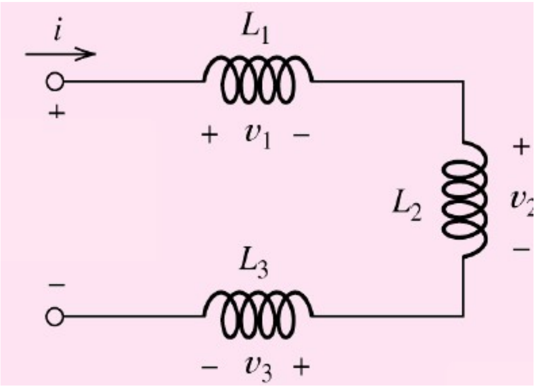
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

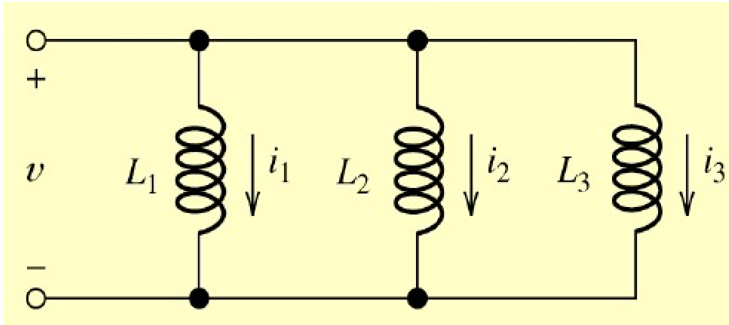


$$v = L \times \frac{di}{dt}$$

For dc or steady state when the current does not vary with time, an inductor under dc or steady state acts like a short circuit

$$w_L(t) = \frac{1}{2} L \times i^2(t)$$



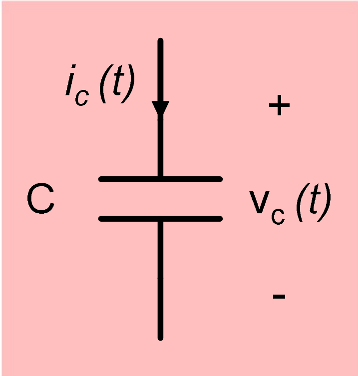


$$L_{eq} = L_1 + L_2 + L_3$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

Two important concepts

Voltage across a capacitor cannot change instantaneously

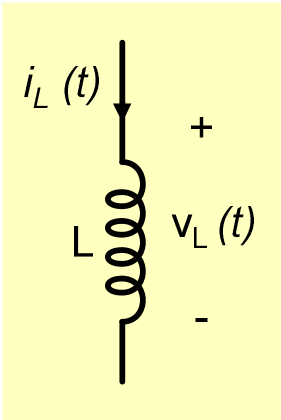


$$i_c = C\frac{dv_c}{dt}$$

$$v_C = \frac{1}{C}\int i_C(t)\,dt$$

Instantaneous change implies infinite current!

Current through an inductor cannot change instantaneously



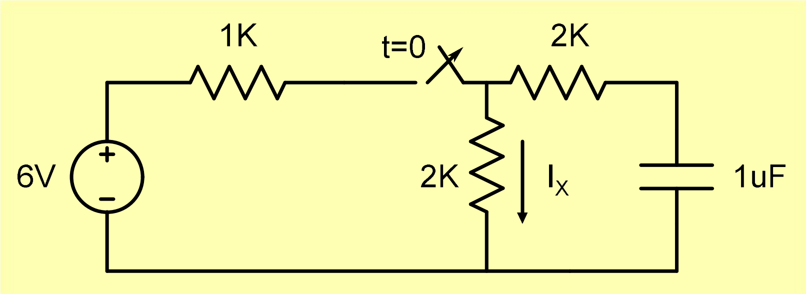
$$v_L = L\frac{di_L}{dt}$$

$$i_L = \frac{1}{L}\int v_L(t)\,dt$$

Instant change in voltage implies infinite voltage!

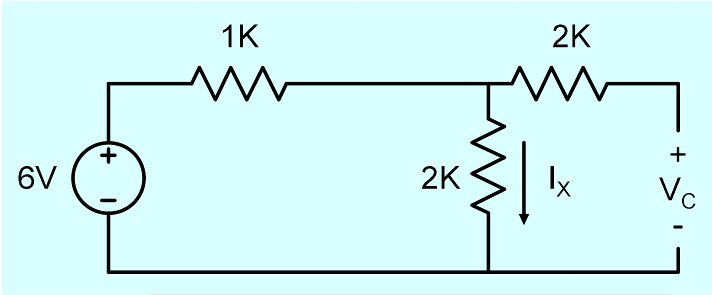
Example

Determine the current I_X immediately after switch is opened.



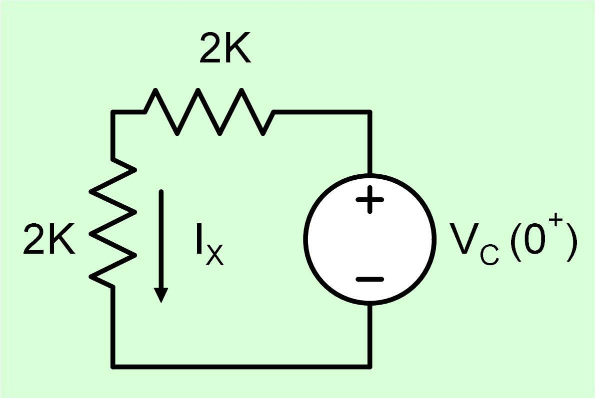
First find voltage $V_C(0^-)$

Circuit for t<0



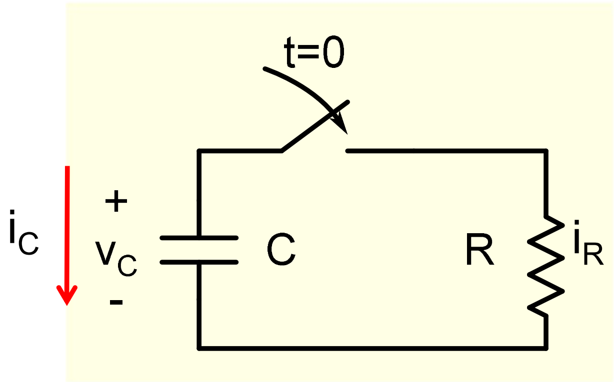
$$v_C(0^+) = \frac{2}{3} \times 6 = 4V$$

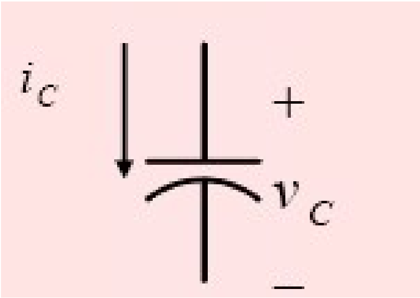
Circuit for t>0



$$i_X(0^+) = \frac{v_C(0^+)}{4K} = 1mA$$

Discharge of a capacitor through a resistor





$$i_c = C\,\frac{dv_c}{dt}$$

$$C\,\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R} = 0$$

$$\frac{dv_C(t)}{dt} = -\frac{1}{RC}\,v_C(t)$$

First Order Differential Equation

$$\frac{dy}{dt} = -a\, y$$

Solution: $$y(t) = K\, e^{st}$$

$$K\, s\, e^{st} = -a\, K\, e^{st}$$

$$s = -a$$

$$y(t) = K\, e^{-at}$$

Constant K is often found from the initial condition

$$K = y(0)$$

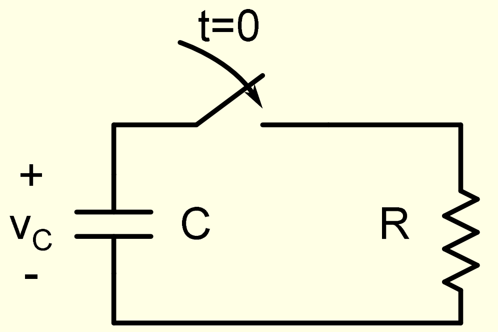
$$y(t) = y(0)\, e^{-at}$$

$$\frac{dx}{dt} = -a_1 x + a_2$$

Solution: $$x(t) = K_1 + K_2 e^{st}$$

$$s = -a_1$$

$$x(t) = K_1 + K_2 e^{-a_1 t}$$

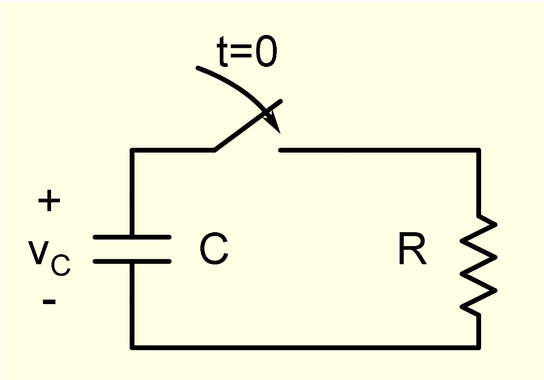


$$x(\infty) = K_1$$

$$x(t) = x(\infty) + K_2 e^{-a_1 t}$$

Use initial condition: $$x(0) = x(\infty) + K_2$$

$$x(t) = x(\infty) + \{x(0) - x(\infty)\} e^{-a_1 t}$$



$$\frac{dv_C(t)}{dt} = -\frac{1}{RC} v_C(t)$$

$$v_C(t) = v_C(0)\, e^{-\frac{t}{RC}}$$

$$v_C(t) = v_C(0^+)\, e^{-\frac{t}{RC}}$$

$$\frac{dy}{dt} = -a\, y$$

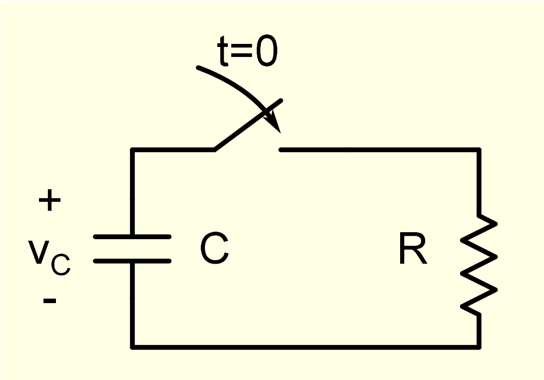
We know: $$v_C(0^-) = V_i$$

$$y(t) = y(0)\, e^{-at}$$

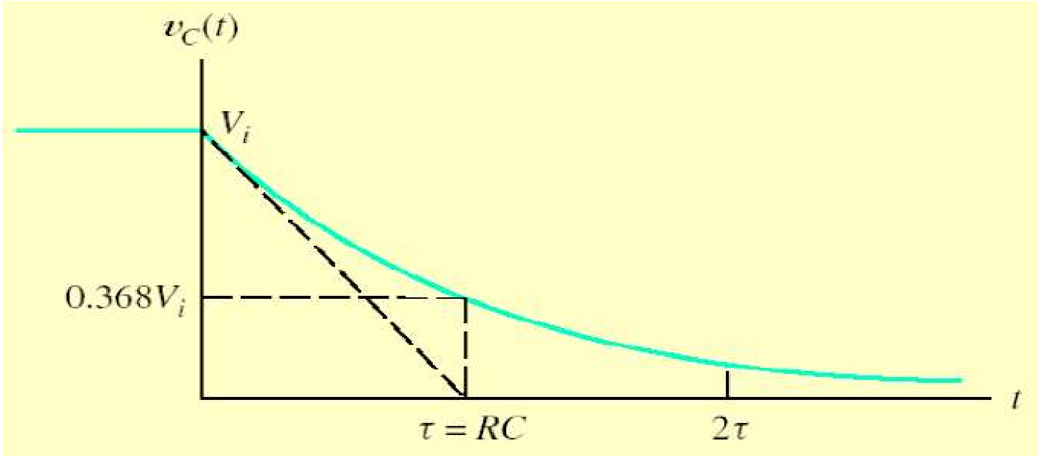
Voltage across a capacitor cannot change instantaneously

$$v_C(0^+) = v_C(0^-) = V_i$$

$$v_C(t) = V_i\, e^{-\frac{t}{RC}}$$

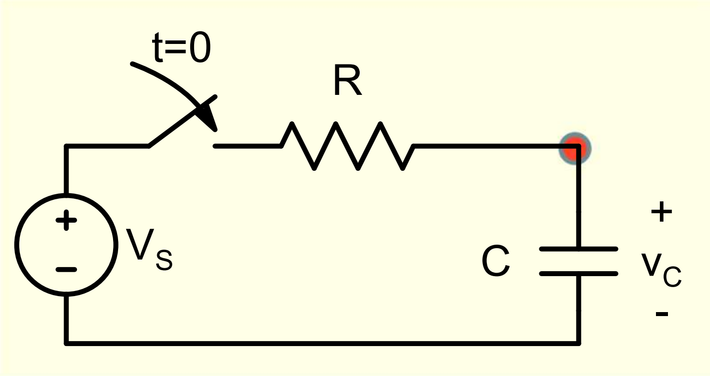


$$v_C(t) = V_i e^{-t/RC}$$



Time	τ	2τ	3τ	4τ	5τ
$V(t)/V_i$	0.368	0.135	.05	0.018	0.0067

Charging a capacitor



$$i_c = C\frac{dv_c}{dt}$$

Application of KCL at the indicated node gives

$$C\frac{dv_c(t)}{dt} + \frac{v_c(t) - V_s}{R} = 0$$

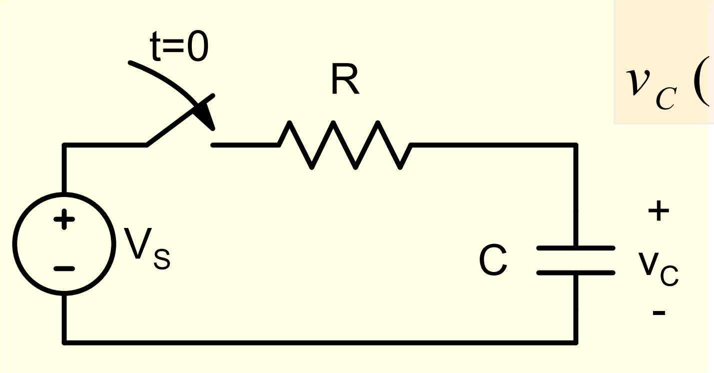
$$RC\frac{dv_c(t)}{dt} + v_c(t) = V_s$$

$$\frac{dx}{dt} = -a_1 x + a_2$$

$$x(t) = x(\infty) + \{x(0) - x(\infty)\}\, e^{-a_1 t}$$

$$a_1 = \frac{1}{RC}$$

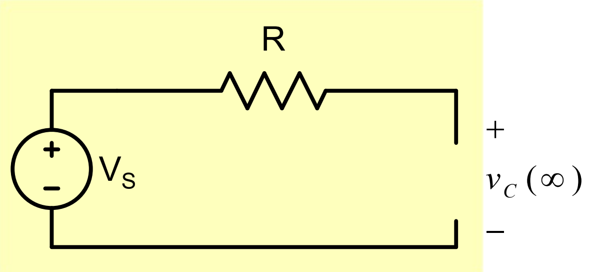
$$v_C(t) = v_C(\infty) + \{v_C(0^+) - v_C(\infty)\}\, e^{-\frac{t}{RC}}$$



$$v_C(t) = v_C(\infty) + \{v_C(0^+) - v_C(\infty)\}\, e^{-\frac{t}{RC}}$$

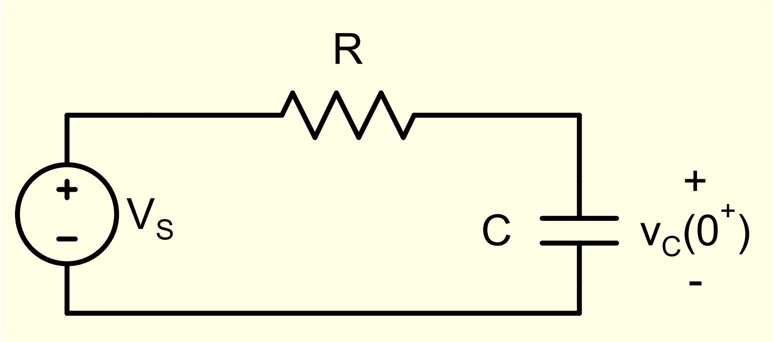
What is $v_C(\infty)$?

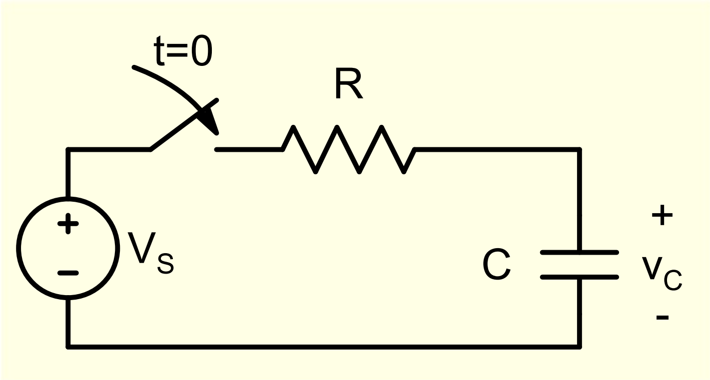
A capacitor under dc or steady state acts like an open circuit



$$v_C(\infty) = V_S$$

What is $v_C(0^+)$?





$$v_C(0^+) = v_C(0^-)$$

We use the fact that voltage across a capacitor cannot change instantly

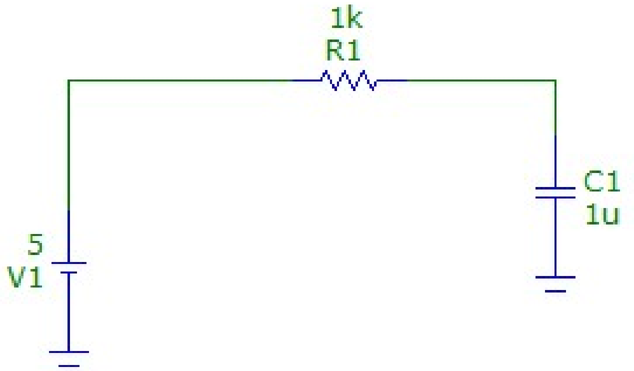
If the capacitor does not have any initial charge, then

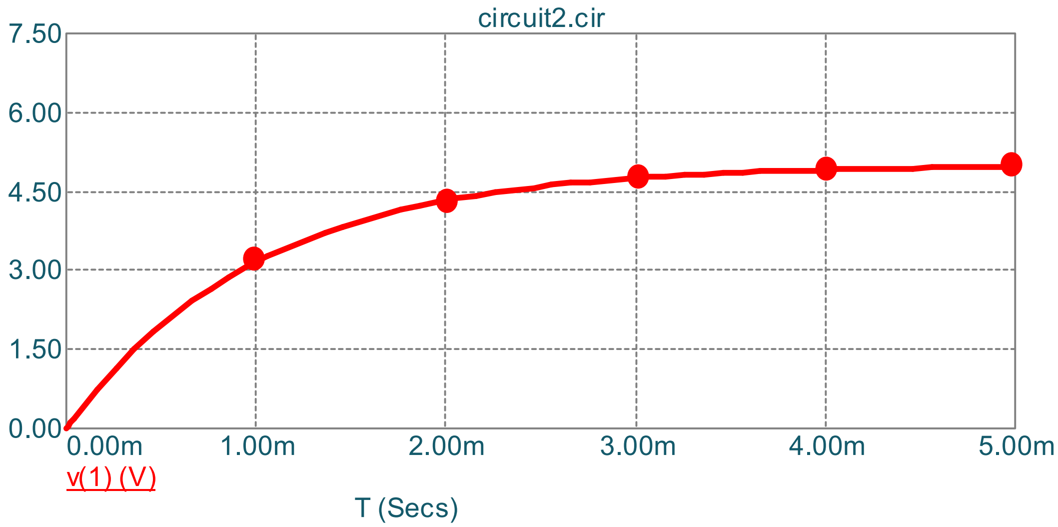
$$v_C(0^+) = v_C(0^-) = 0$$

$$v_C(t) = V_S(1 - e^{-\frac{t}{RC}})$$

$$\tau = RC$$

Charging of a Capacitor

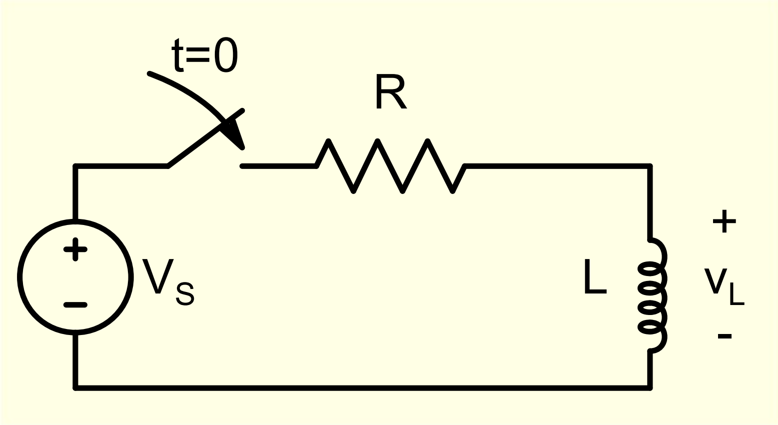




$$v_C(t) = V_S\left(1 - e^{-\frac{t}{RC}}\right)$$

Time	τ	2τ	3τ	4τ	5τ
$V_C(t)/V_i$	0.632	0.865	.95	0.982	0.993

R-L Transients



$$v = L\frac{di}{dt}$$

$$Ri(t) + L\frac{di}{dt} = V_s$$

$$\frac{dx}{dt} = -a_1 x + a_2$$

$$x(t) = x(\infty) + \{x(0) - x(\infty)\}\, e^{-a_1 t}$$

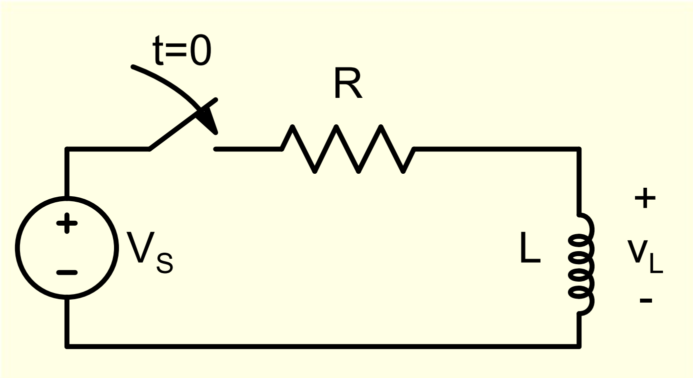
$$i(t) = i(\infty) + \{i(0) - i(\infty)\}\, e^{-\frac{R}{L}t}$$

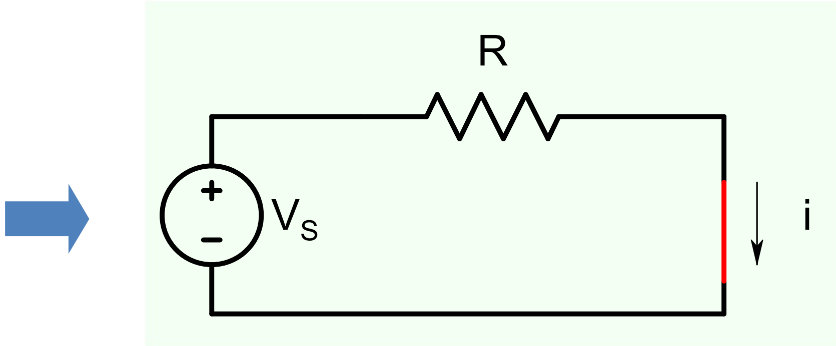
$$e^{-\frac{t}{\tau}}$$

$$\text{Time Constant}: \tau = \frac{L}{R}$$

What is $i(\infty)$?

Remember that inductor in steady state is like a short circuit





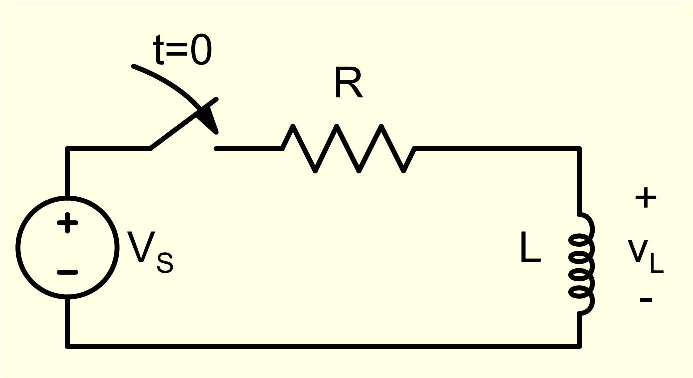
$$i(\infty) = \frac{V_S}{R}$$

$$i(t) = \frac{V_S}{R} + \{i(0) - \frac{V_S}{R}\}\, e^{-\frac{R}{L}t}$$

We also note that inductor current cannot change instantly

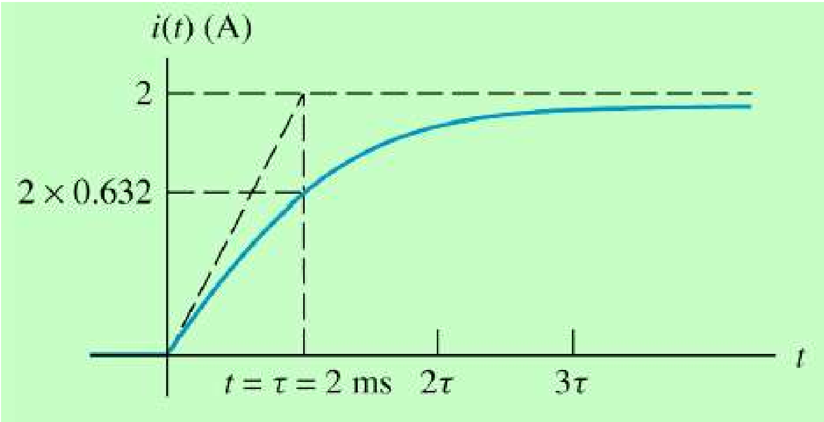
Current through an inductor cannot change instantaneously

$$i(0^+) = i(0^-)$$

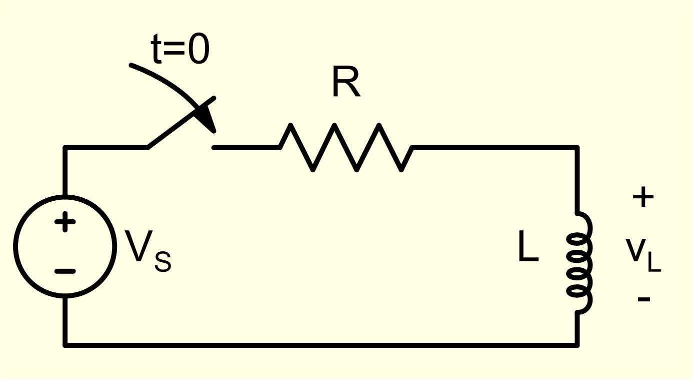


$$i(0^+) = i(0^-) = 0$$

$$i(t) = \frac{V_S}{R} \times (1 - e^{-\frac{t}{\tau}})$$

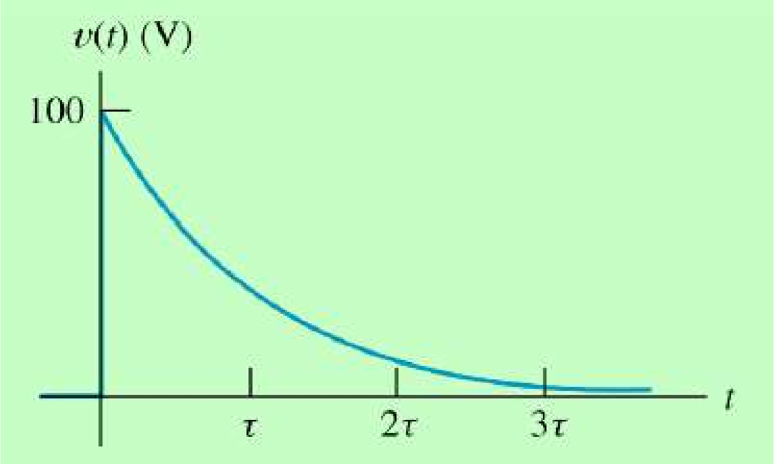


What about voltage across the Inductor?

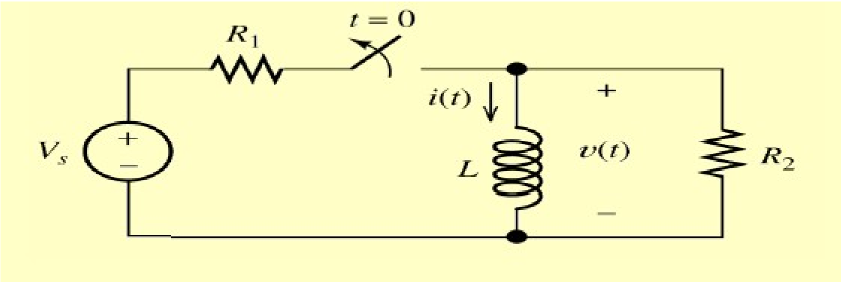


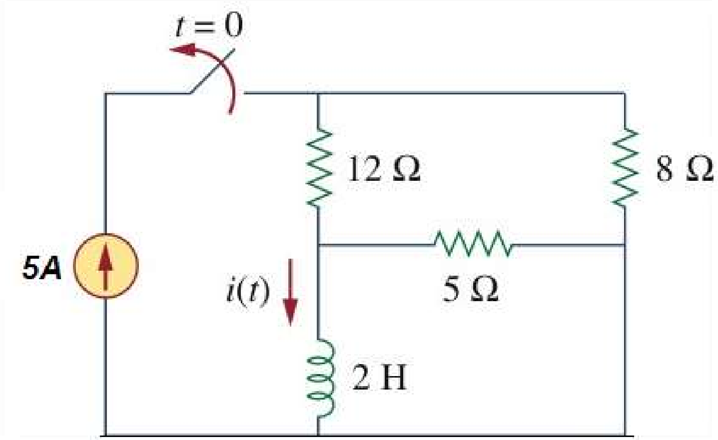
$$i(t) = \frac{V_S}{R} \times (1 - e^{-\frac{t}{\tau}})$$

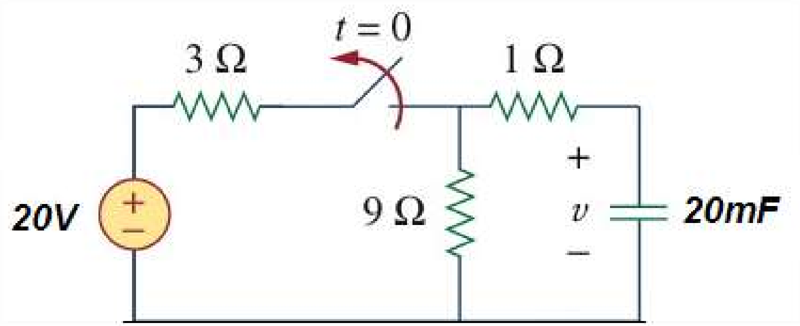
$$v = L\frac{di}{dt} = \frac{L}{R} V_S \times e^{-\frac{t}{\tau}}$$

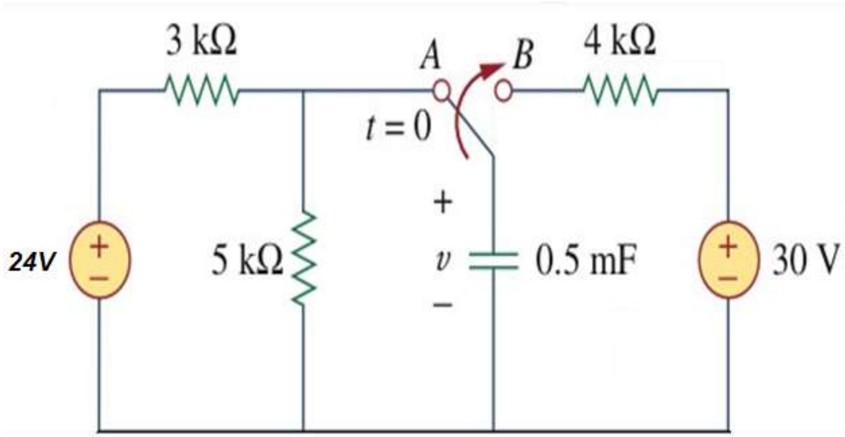


How do we solve more complex circuits containing a single inductor or a capacitor?



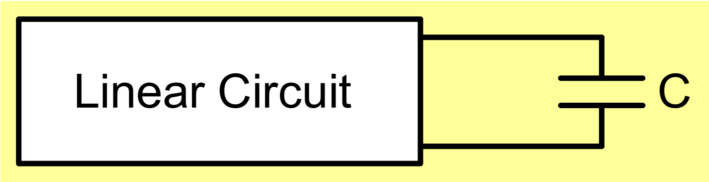


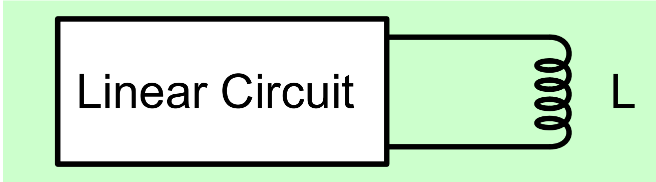


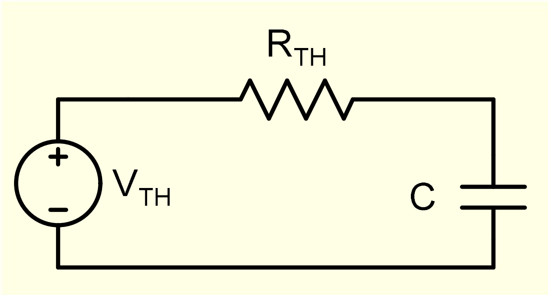


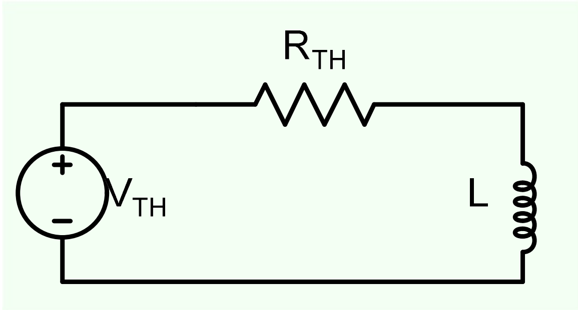
Method for circuits containing a single capacitor or inductor

Circuit for t > 0





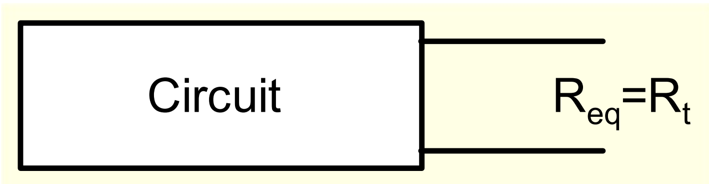




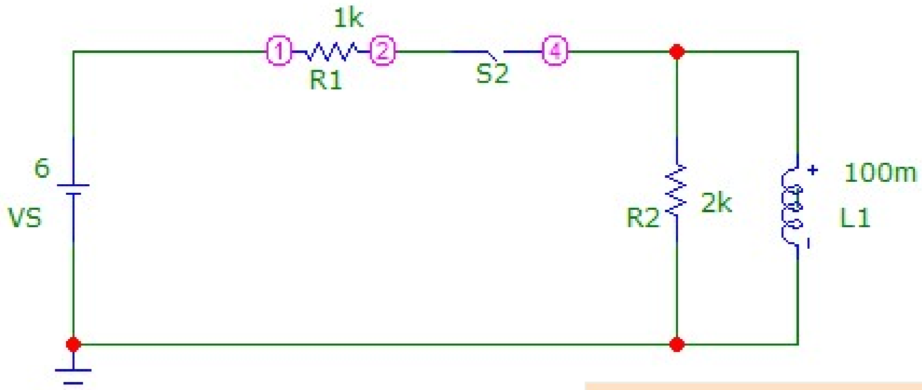
$$x(t) = x(\infty) + \{x(0^+) - x(\infty)\}e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R_{eq}} \; or \; R_{eq}C$$

Where x is capacitor voltage or inductor current



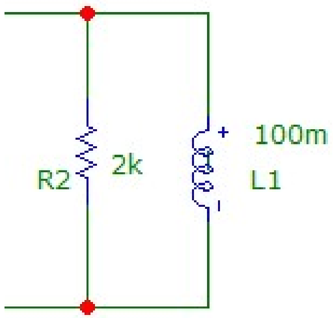
Example-1



Switch is opened at t = 0

$$i(t) = i(\infty) + \{i(0^+) - i(\infty)\} \times e^{-\frac{t}{\tau}}$$

Circuit for t > 0



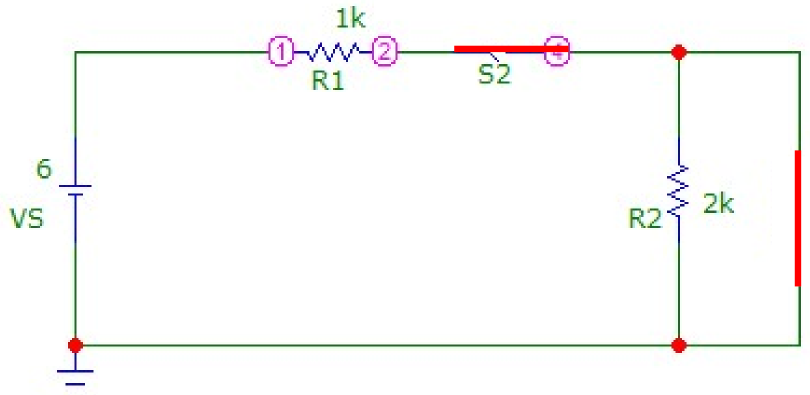
$$\tau = \frac{L}{R_2}$$

Steady state Solution: $i(t \to \infty) = 0$

$$i(t) = i(0^+) \times e^{-\frac{t}{\tau}}$$

Initial condition

Circuit for t < 0

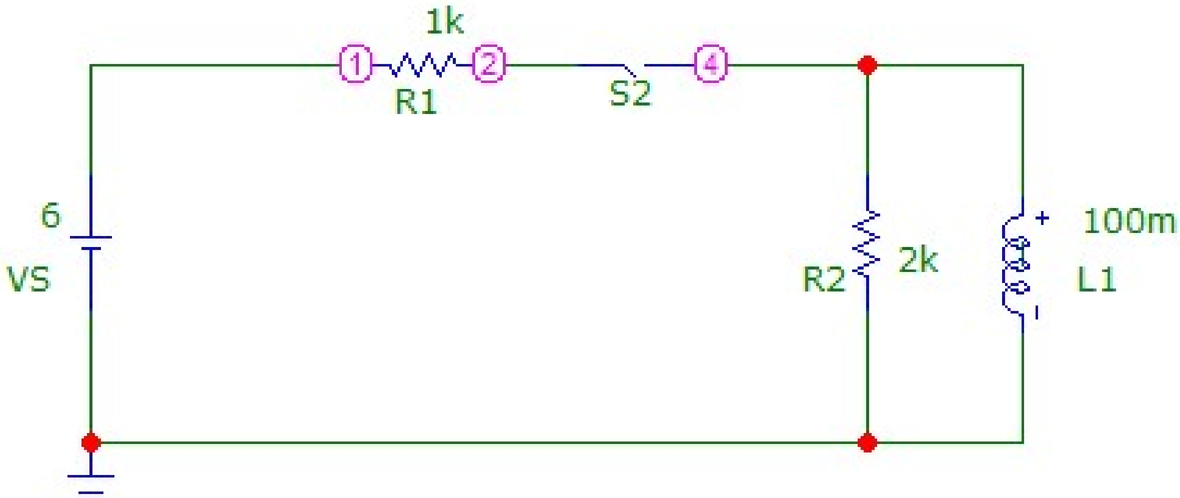


$$i(0^+) = i(0^-) = \frac{V_S}{R_1}$$

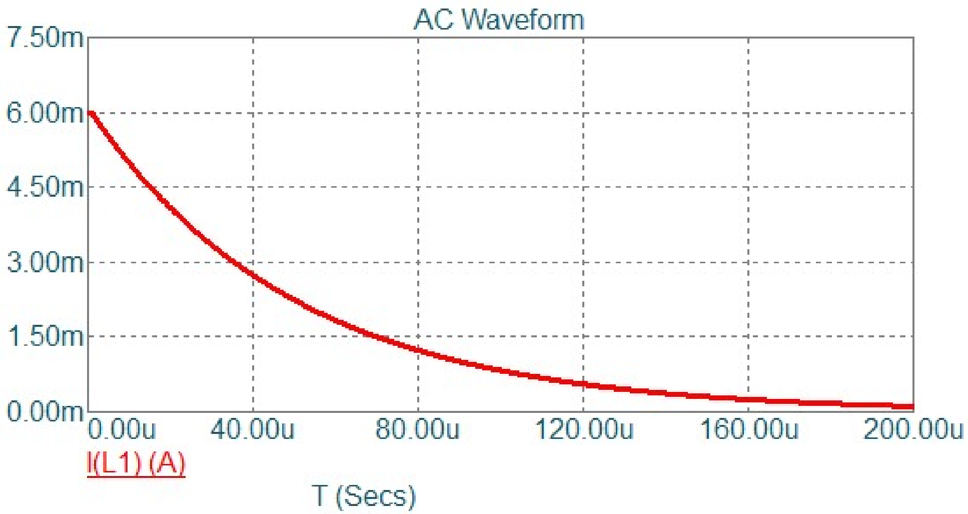
$$i(t \to \infty) = 0$$

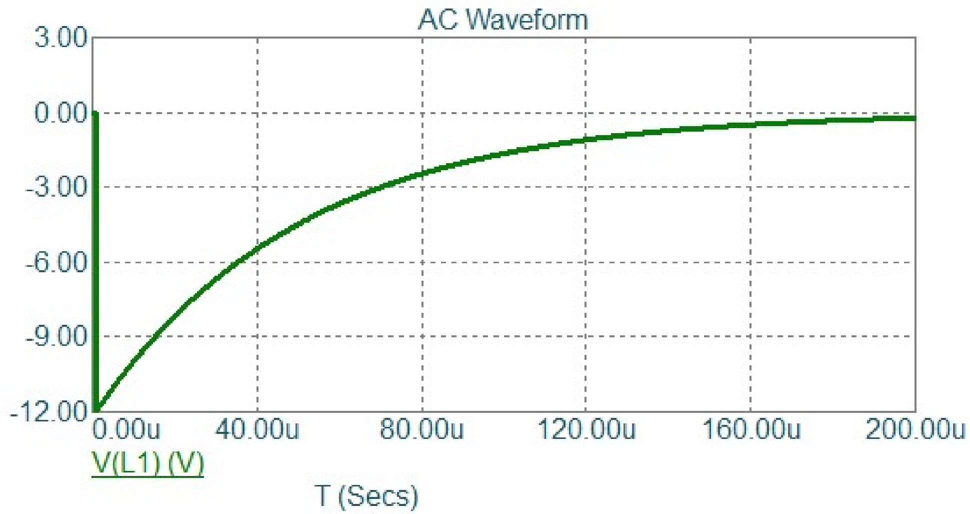
$$i(t) = i(\infty) + \{i(0^+) - i(\infty)\} \times e^{-\frac{t}{\tau}}$$

$$i(t) = \frac{V_S}{R_1} e^{-\frac{R_2}{L}t}$$



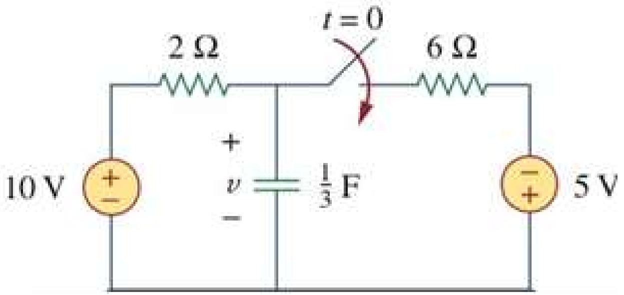
$$x(t) = x(\infty) + \{x(0^+) - x(\infty)\}e^{-\frac{t}{\tau}}$$





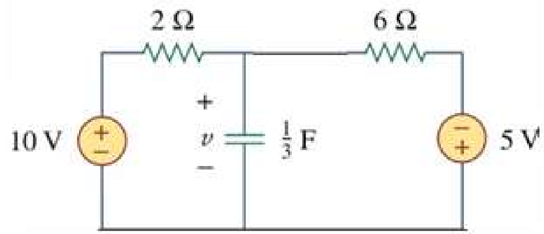
Example-2

Determine the current through the capacitor as a function of time.

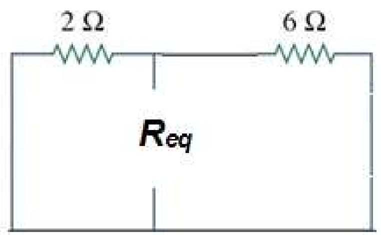


$$v(t) = v(\infty) + \{v(0^+) - v(\infty)\}e^{-\frac{t}{\tau}}$$

Circuit for t > 0



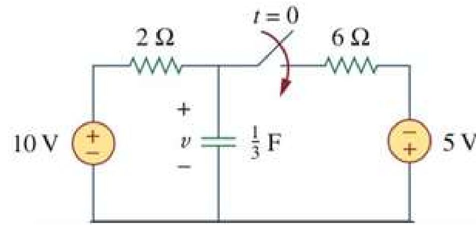
Determine the equivalent resistance seen by the capacitor:



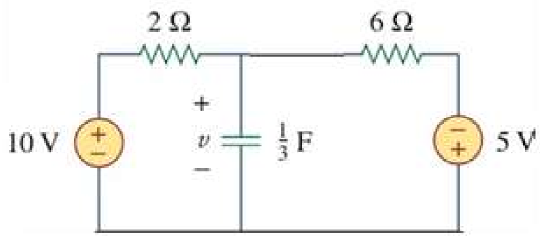
$$R_{eq} = 2\|6 = 1.5\Omega$$

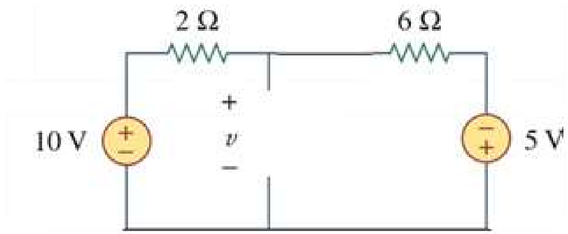
$$\tau = C \times R_{eq} = 0.5s$$

We next find voltage long after closing the switch $v(t \rightarrow \infty) = 0$



Circuit after closing the switch

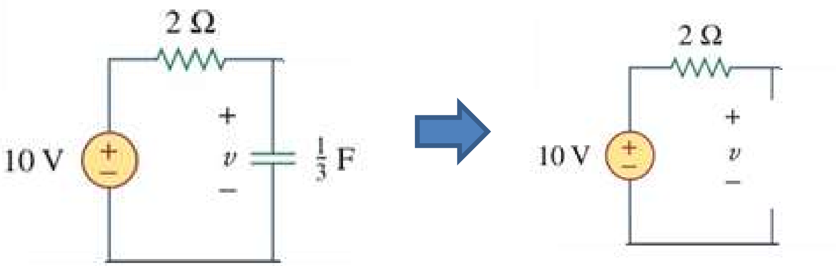




In steady state the capacitor acts like an open circuit $v(\infty) = \frac{25}{4}$

To find v(0⁺) we note that capacitor voltage does not change instantly so it will remain the same as before the switch was closed

Circuit for t < 0

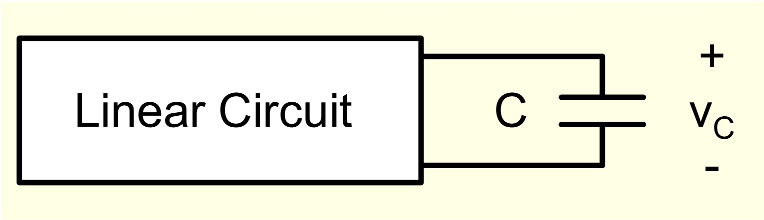


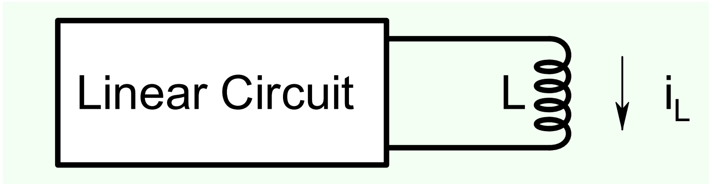
$$v(0^+) = 10$$

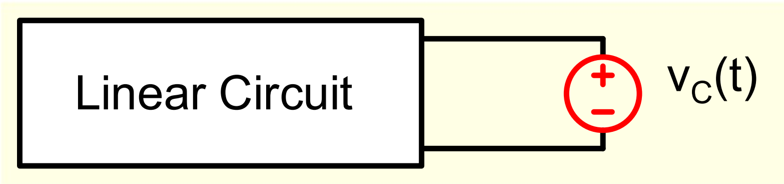
$$v(t) = \frac{25}{4} + \frac{15}{4} e^{-2t}$$

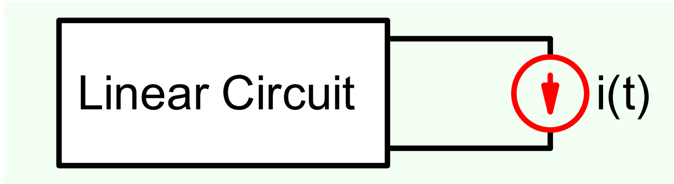
$$I = C \times \frac{dV}{dt} = -\frac{15}{2} \times e^{-2t}$$

How do we find voltages and currents elsewhere in the circuit?



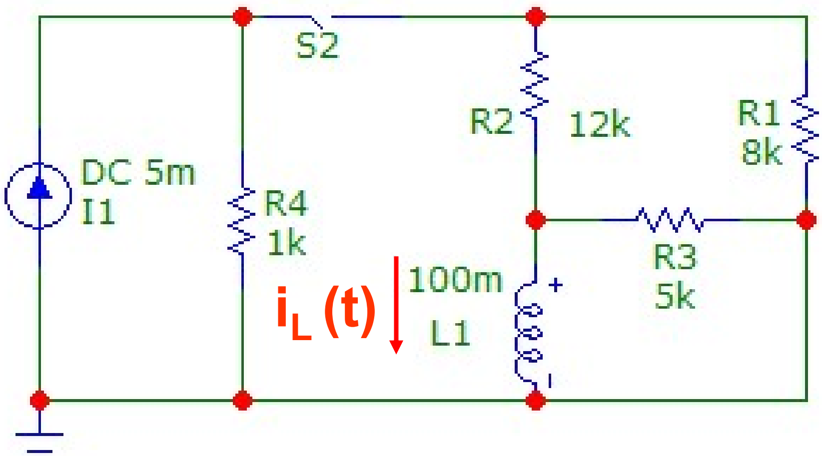




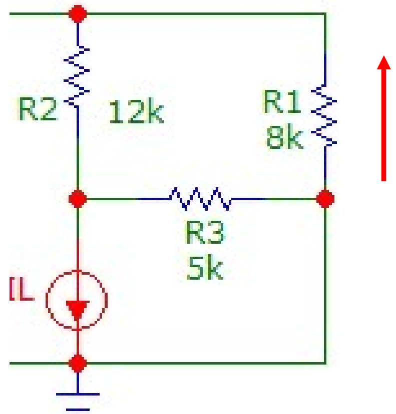


Example-3

Find current in 8 kΩ resistor as a function of time after the switch is opened

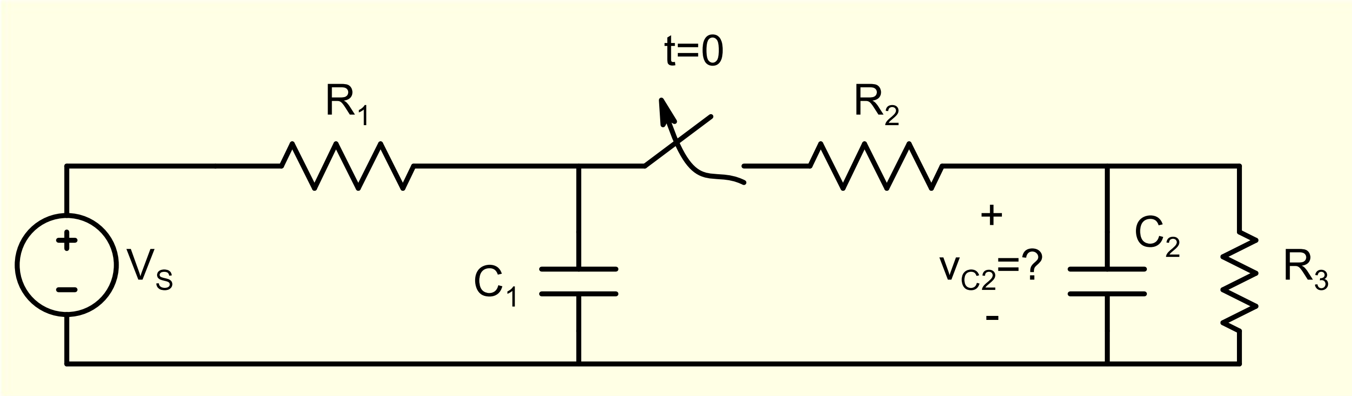


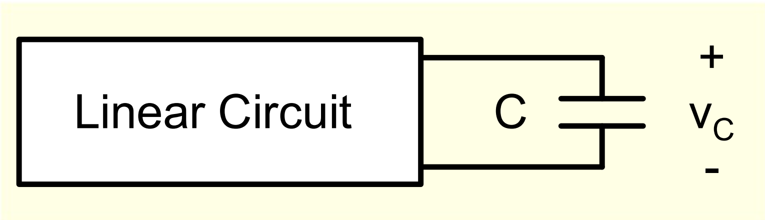
$$i_L(t) = 0.34mA \times e^{-\frac{t}{25\mu s}}$$



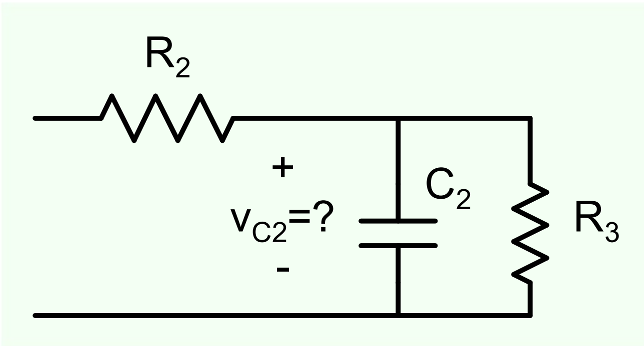
$$i_8 = i_L(t) \times \frac{5}{5+20} = 0.069mA \times e^{-\frac{t}{25\mu s}}$$

Can we solve this 2 capacitor problem using our present approach?





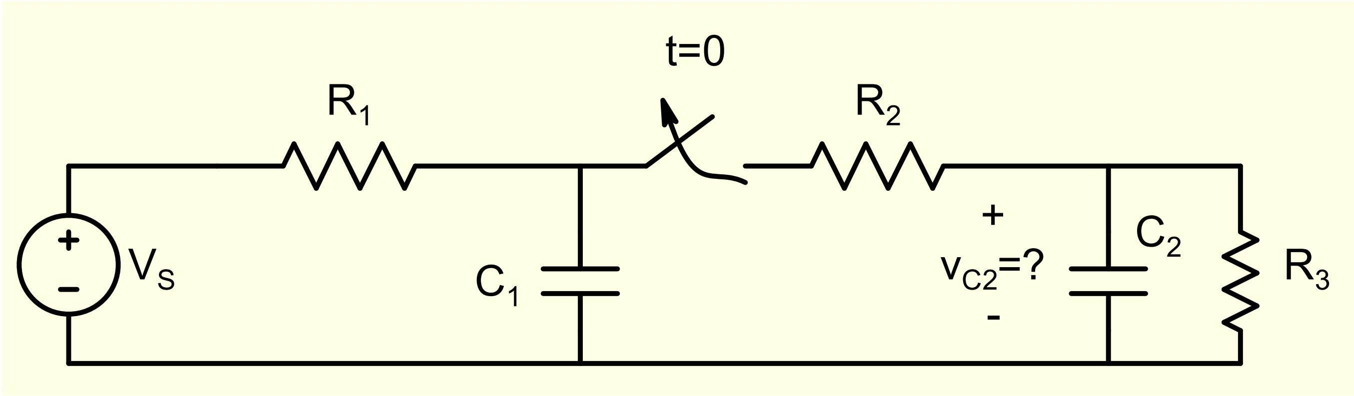
Circuit for t > 0

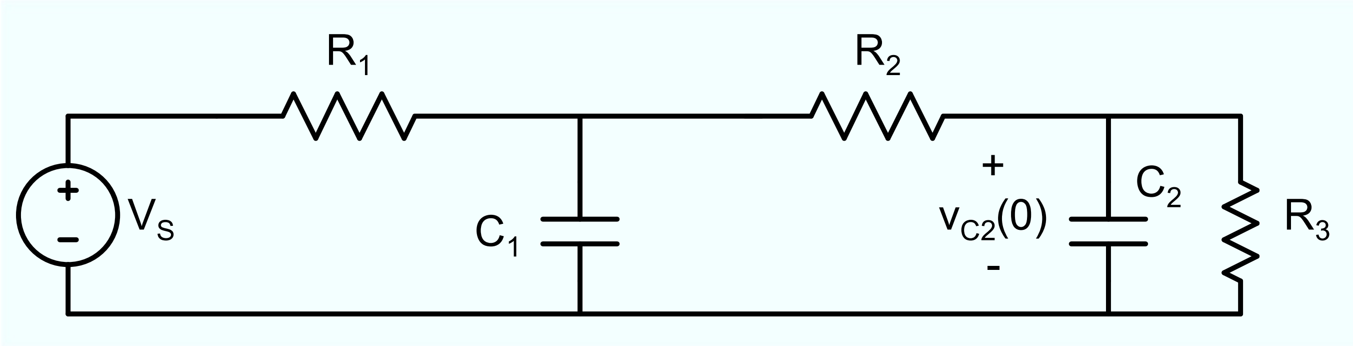


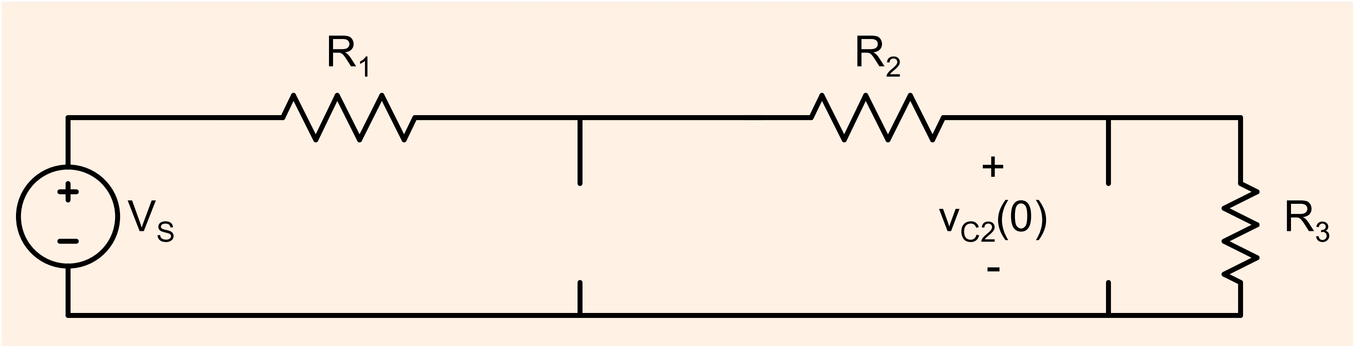
$$v_{c2}(t) = v_{c2}(\infty) + \{v_{c2}(0^+) - v_{c2}(\infty)\}e^{-\frac{t}{\tau}}$$

$$v_{c2}(0^+) = v_{c2}(0^-)$$

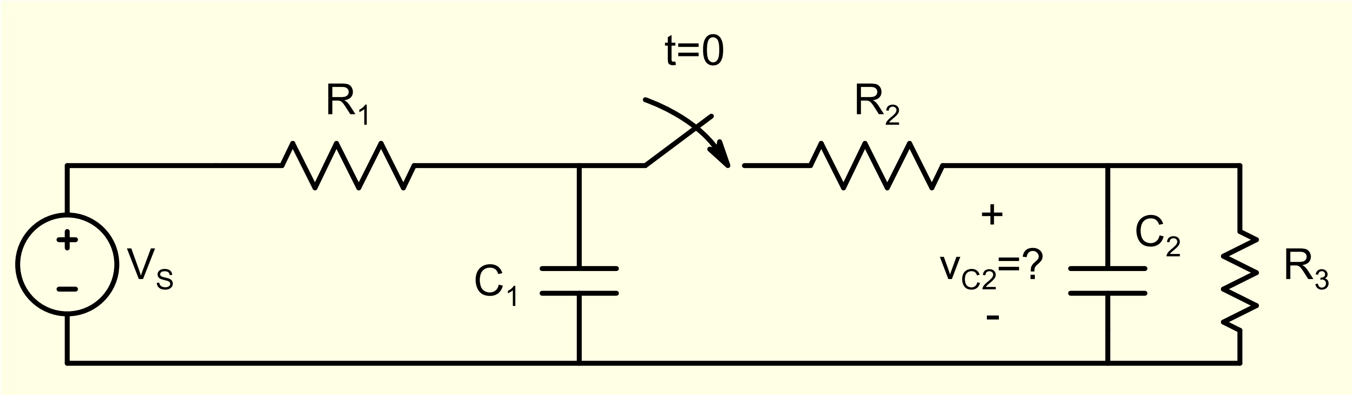
$$v_{c2}(0^+) = v_{c2}(0^-)$$







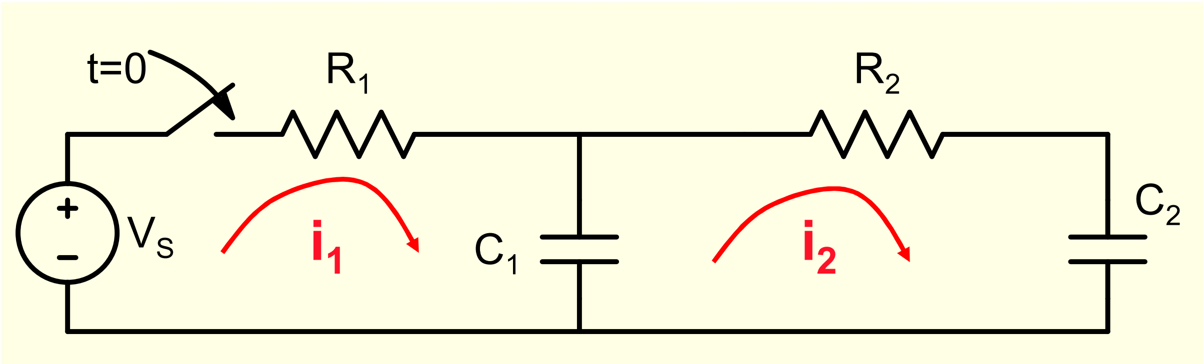
Will our approach work here?



No, because circuit for $t > 0$ has two capacitances

So as long as circuit has single capacitance or inductance for the time interval for which analysis is being carried out, the stated approach will work fine.

What happens when there is more than one storage element?



$$V_S = i_1 R_1 + v_{C1} \quad (1)$$

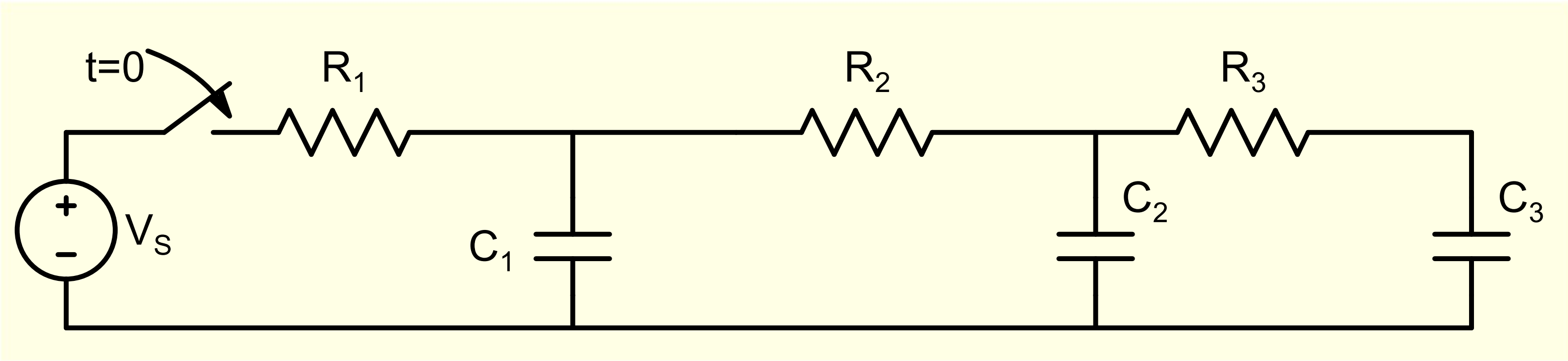
$$v_{C1} = i_2 R_2 + v_{c2} \quad (2)$$

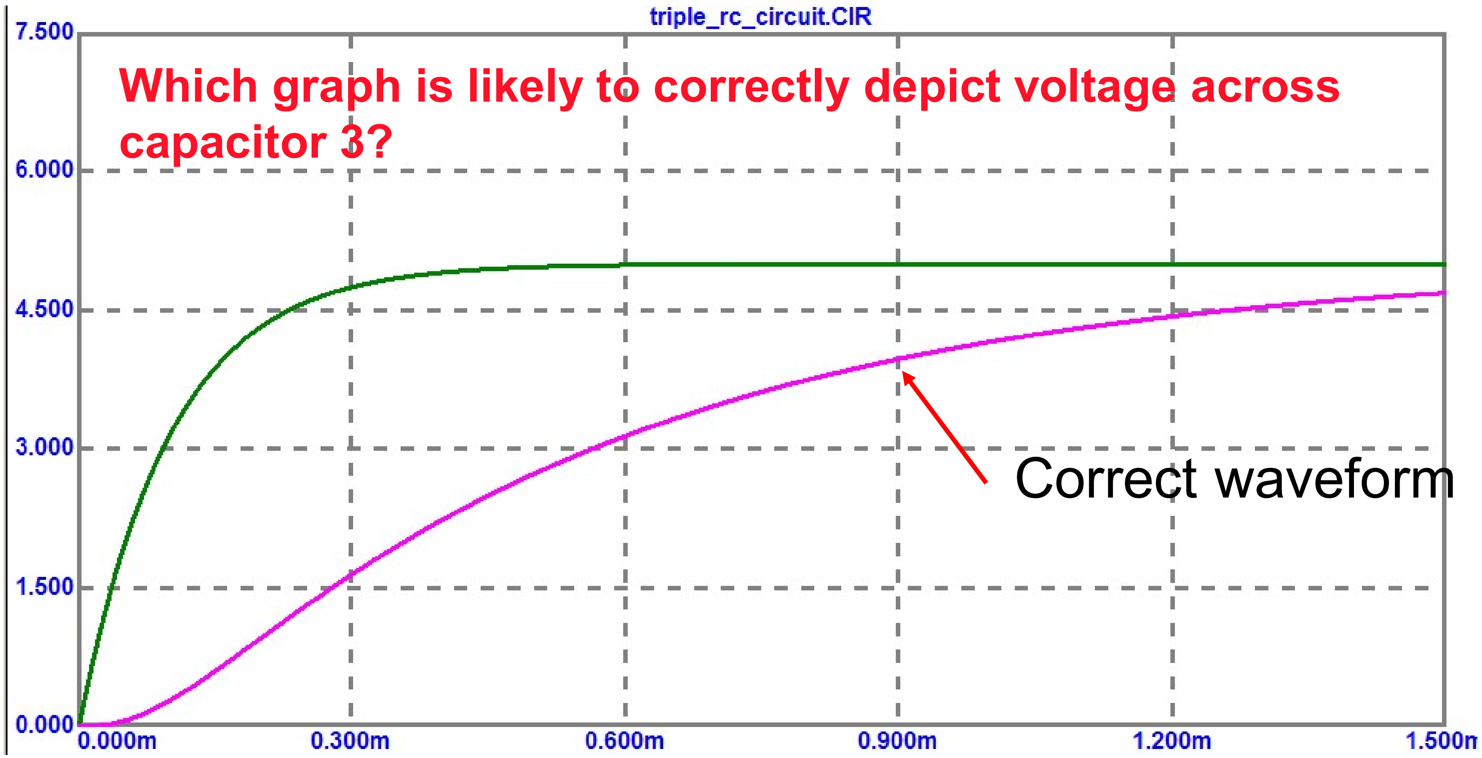
$$i_1 - i_2 = C_1 \frac{dv_1}{dt} (3)$$

$$i_2 = C_2 \frac{dv_2}{dt} (4)$$

$$R_1 R_2 C_1 C_2 \frac{d^2 v_{c2}}{dt^2} + (R_1 C_1 + R_1 C_2 + R_2 C_2) \frac{dv_{c2}}{dt} + v_{c2} = V_S$$

$$v_{c2}(t) = K_0 + K_1 e^{s_1 t} + K_2 e^{s_2 t}$$


t=0
R1
R2
R3
Vs
C1
C2
C3


triple_rc_circuit.CIR
Which graph is likely to correctly depict voltage across capacitor 3?
Correct waveform
7.500
6.000
4.500
3.000
1.500
0.000
0.000m
0.300m
0.600m
0.900m
1.200m
1.500m