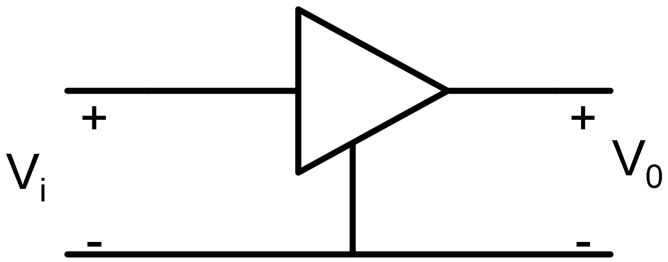
ESC201T : Introduction to Electronics

Lecture 25: Amplifiers-1

B. Mazhari
Dept. of EE, IIT Kanpur

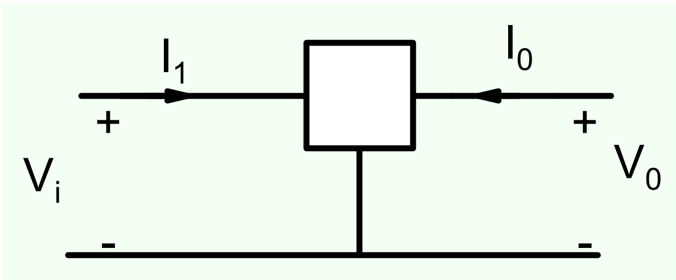
“Amplification is the heartbeat of Electronics”

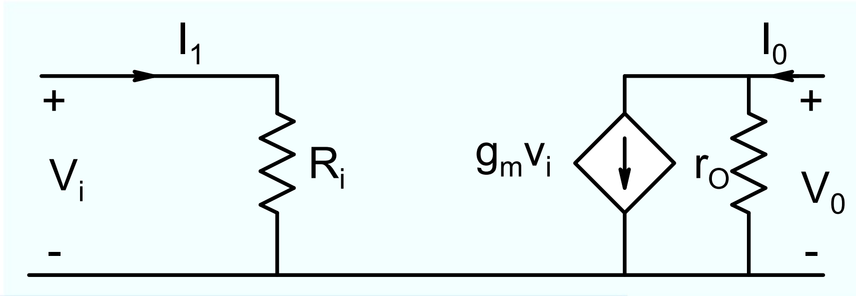
Voltage Amplification



$$V_o = G \times V_i$$
$$G > 1 \text{ and constant}$$

Consider a 3-terminal unilateral linear device

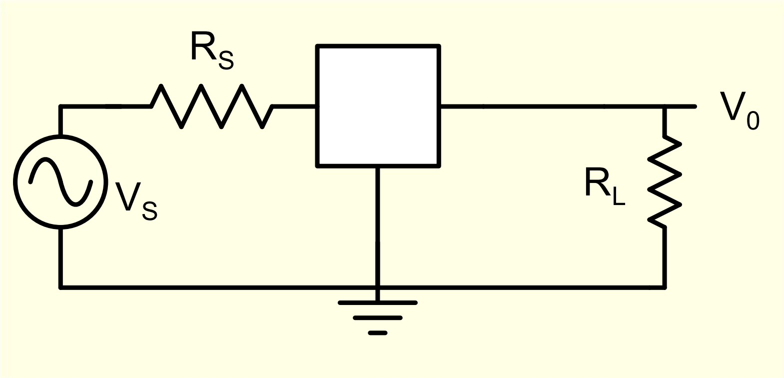


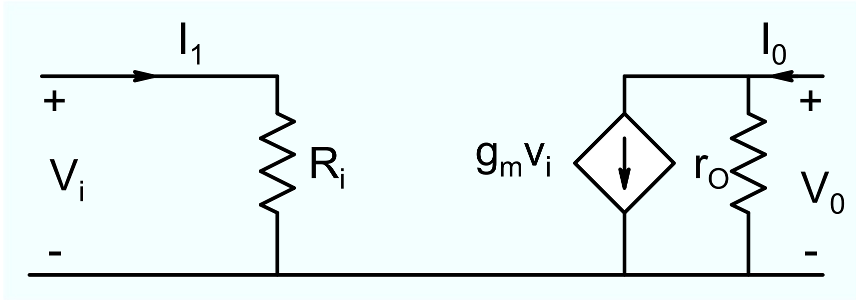


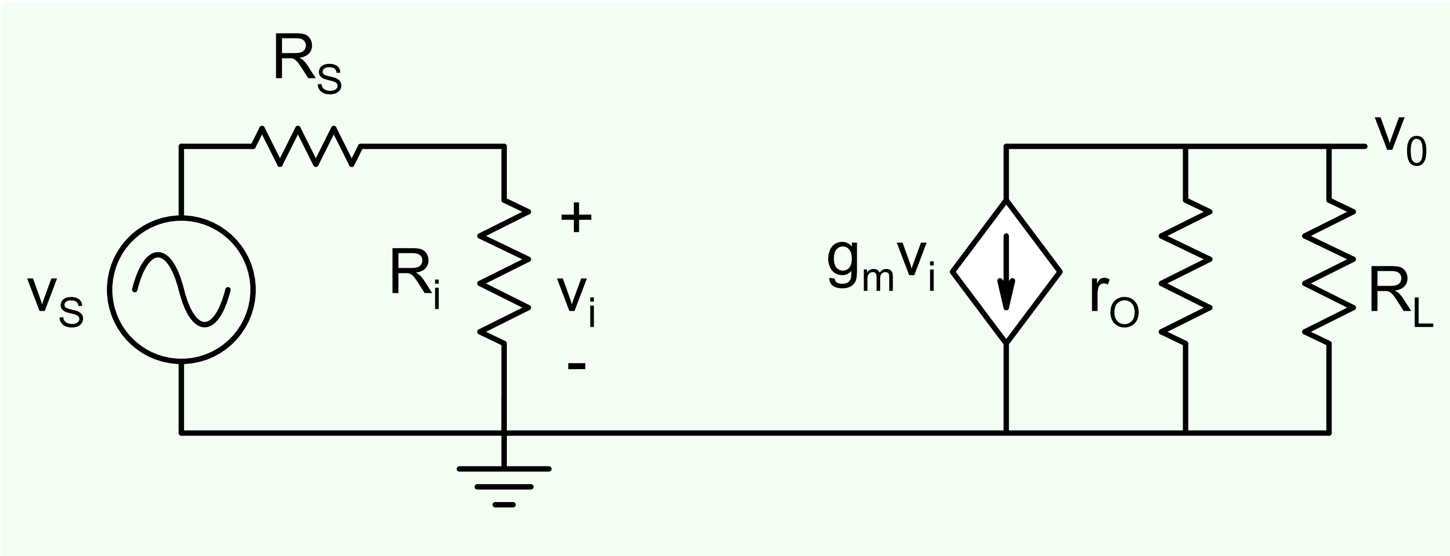
R_i=input resistance=V_i/I_i ; Transconductance:$g_m = \left.\dfrac{I_o}{V_i}\right|_{V_O=0}$

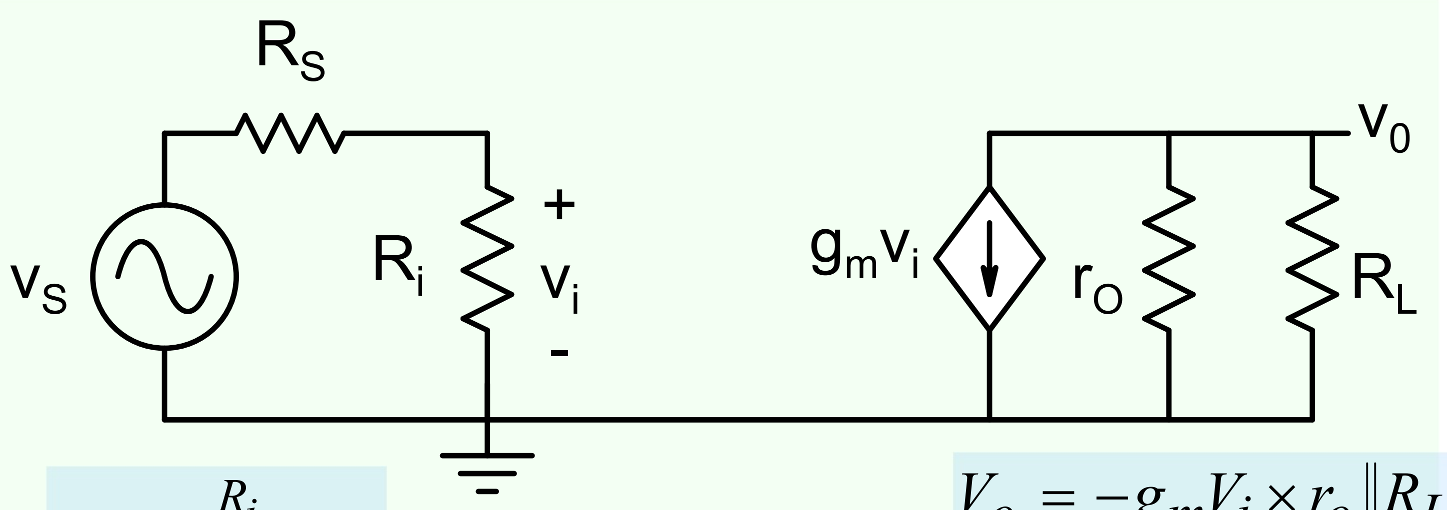
Output conductance:$g_o = 1/r_o = \left.\dfrac{Io}{V_o}\right|_{V_i=0}$

Voltage Amplifier









$$V_i = \frac{R_i}{R_i + R_S} V_S$$

$$V_o = -g_m V_i \times r_o \| R_L$$

$$A_V = \frac{V_o}{V_s} = -g_m r_o \times \frac{R_L}{r_o + R_L} \times \frac{R_i}{R_i + R_S}$$

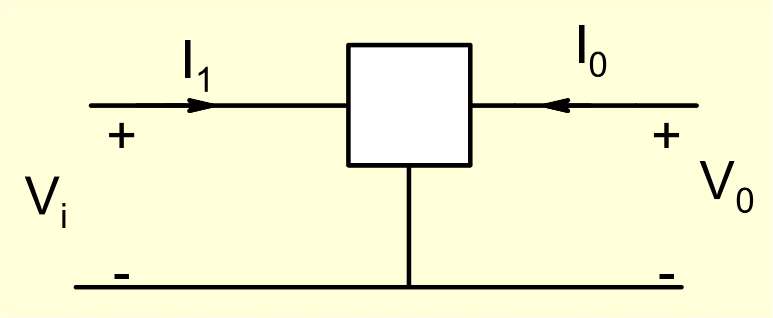
$$\left| A_V \right| \leq g_m \times r_o$$

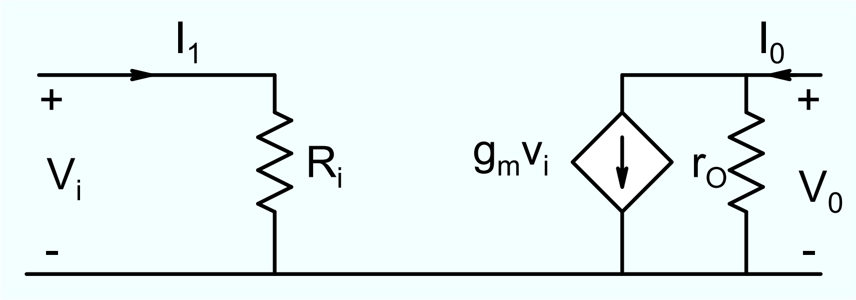
Necessary Condition for Voltage Amplification : $g_m \times r_o > 1$

Voltage Amplification

$$g_m r_o >> 1 \qquad g_m >> g_o$$

Transconductance >> Output Conductance



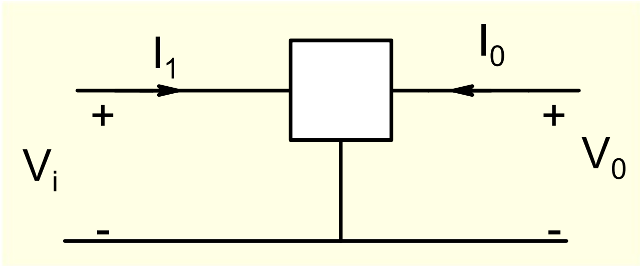


Transistor

Transistor

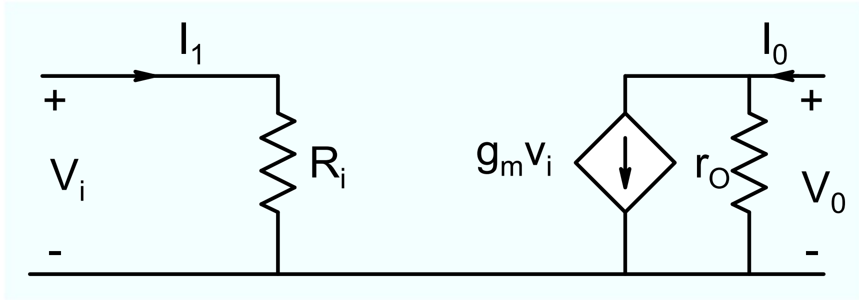
Trans-resistor

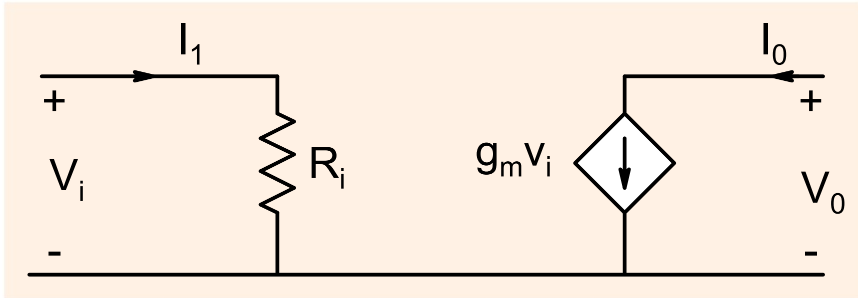
Current I_O is much more sensitive to V_{IN} than V_O



$$g_m r_o >> 1$$

In the ideal case r_o is infinite





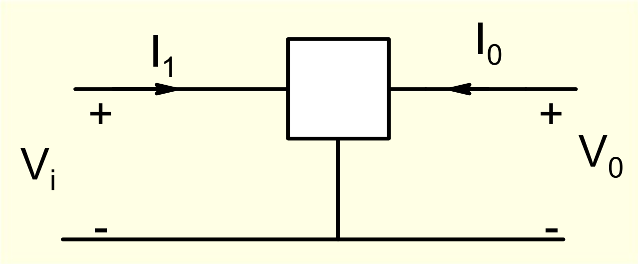
$$A_V = \frac{V_o}{V_s} = -g_m r_o \times \frac{R_L}{r_o + R_L} \times \frac{R_i}{R_i + R_S} = -g_m R_L \times \frac{R_i}{R_i + R_S}$$

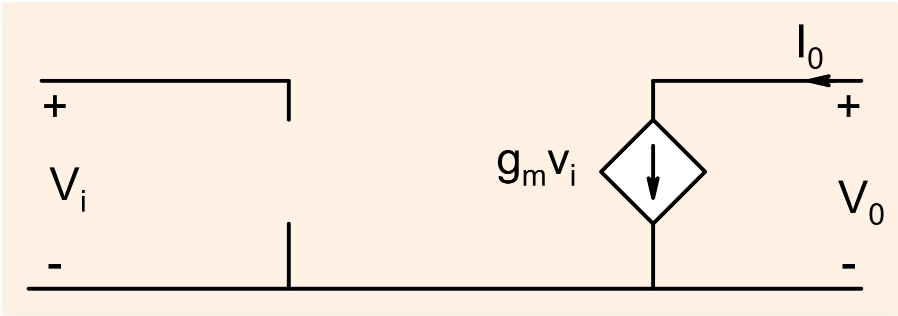
We would ideally like input resistance R_i to be infinite as well !

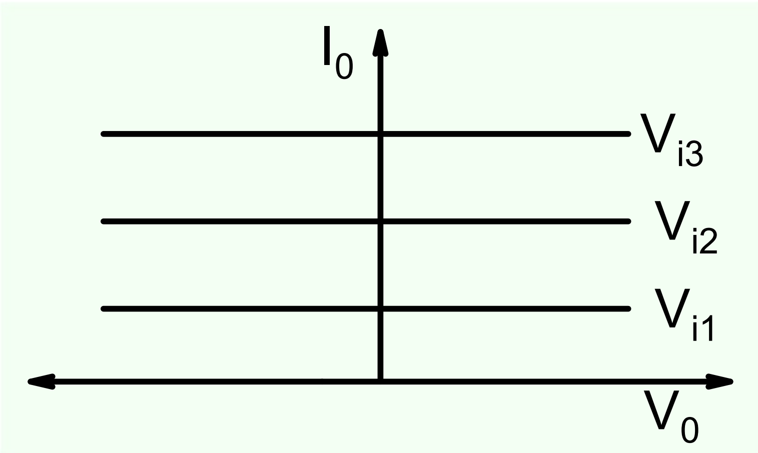
$$A_V = -g_m R_L$$

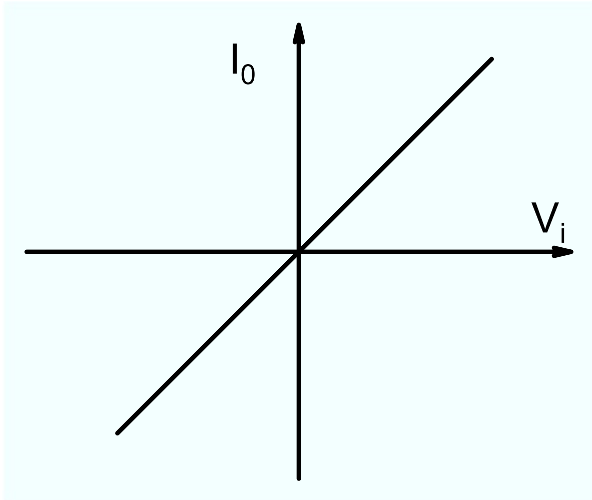
Note that we have power gain as well which is essential for calling a device as an Amplifier

An ideal 3-terminal device for Voltage Amplification





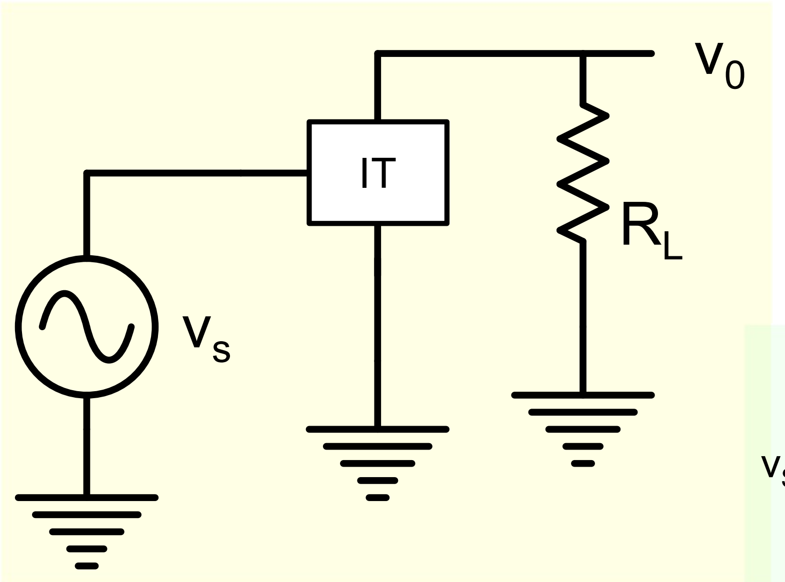


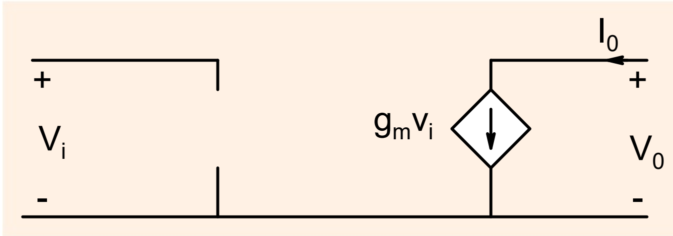


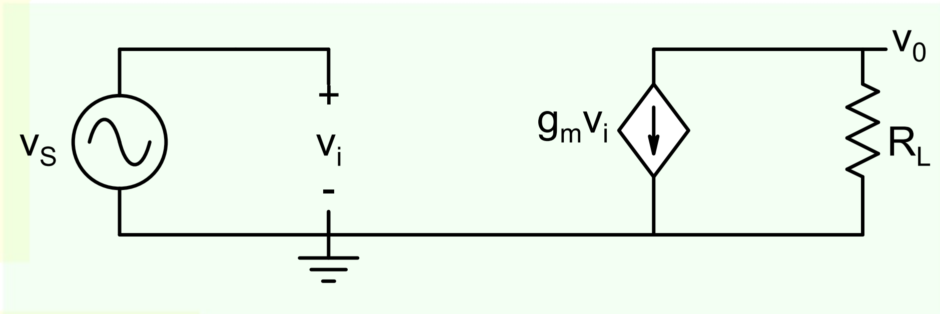
Ideal Transistor Characteristics

Ideal Transistor (IT)

Making a voltage amplifier with an ideal transistor is straightforward



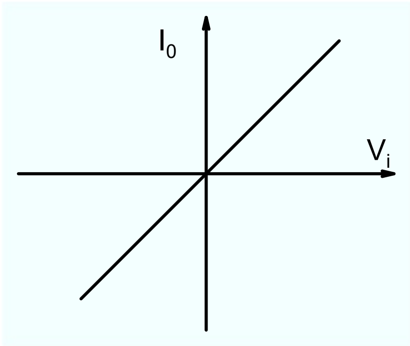


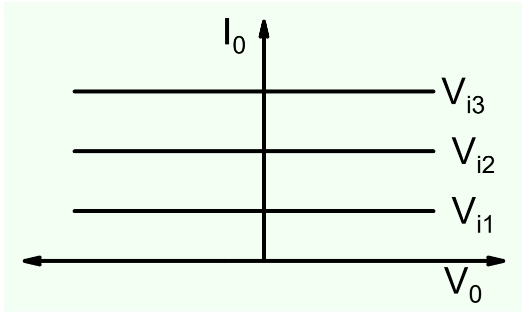


$$A_V = \frac{v_o}{v_s} = -g_m R_L$$

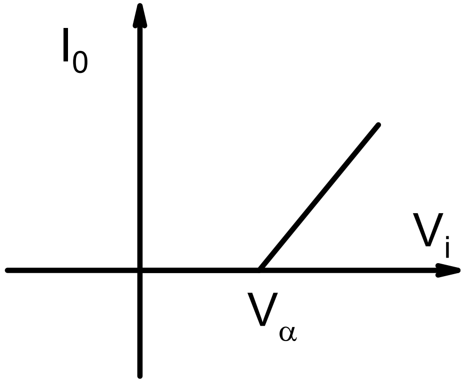
In practice there is no element which has the characteristics of ideal transistor !

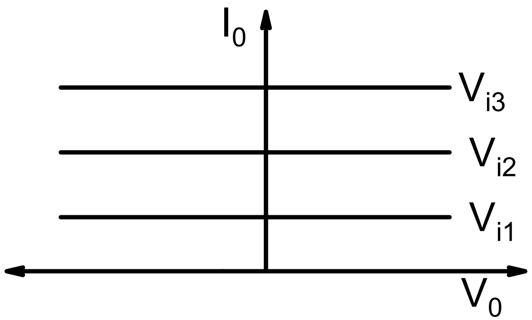
Ideal transistor



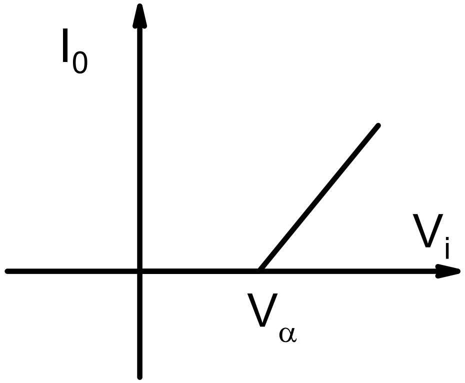


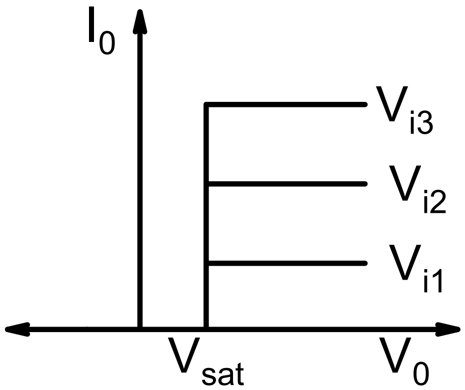
Device X





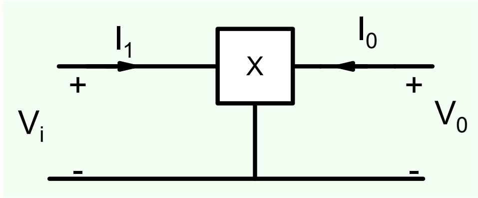
Device Y

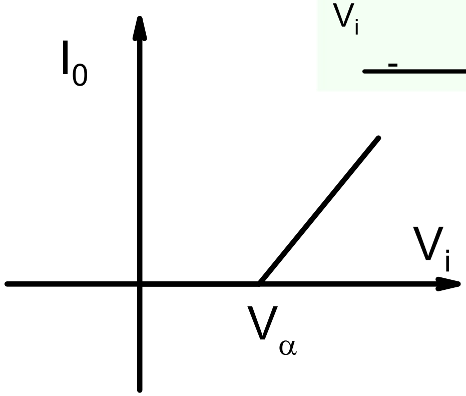


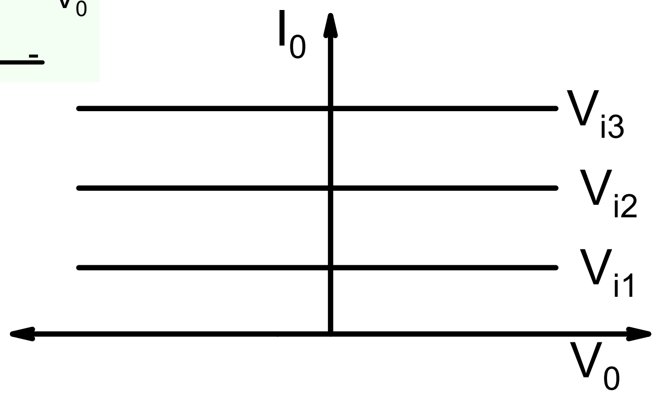


How do we use elements such as X, Y etc to make amplifiers?

Device X

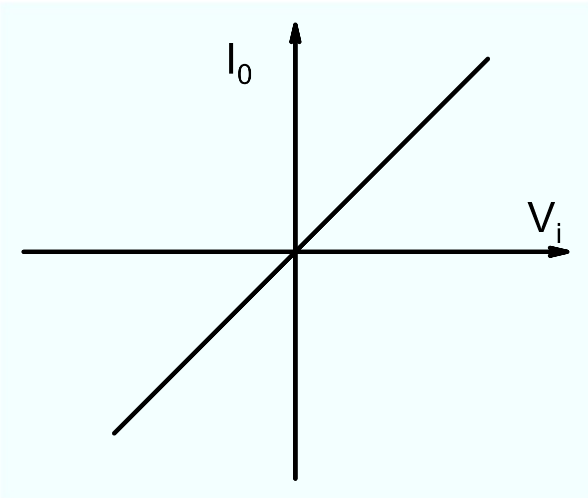


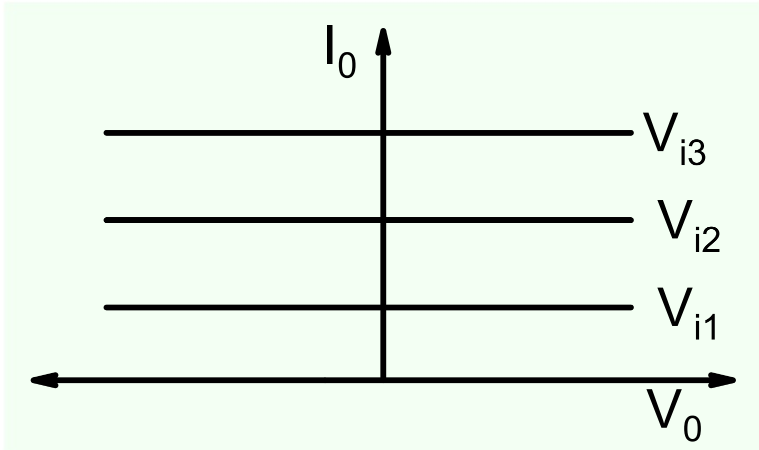




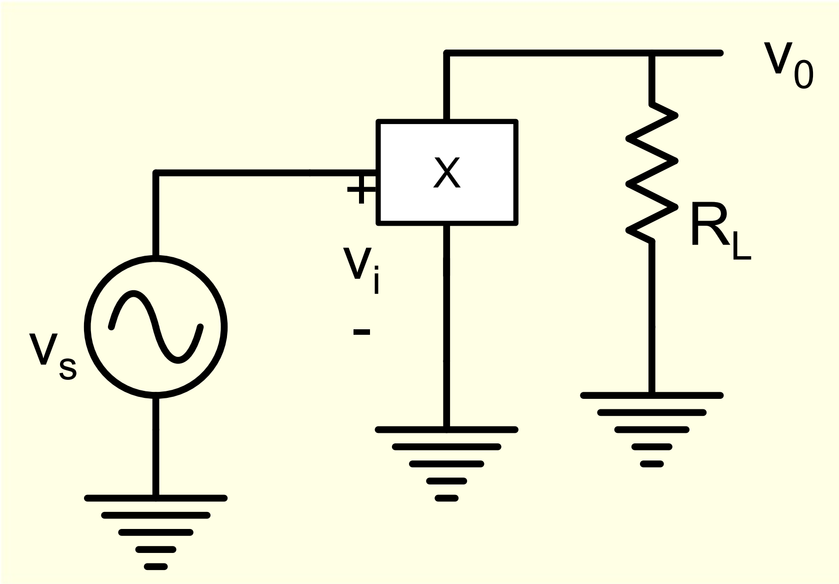
$$
\begin{aligned}
I_o &= 0 && \text{for } V_i \le V_\alpha \\
&= g_m \times (V_i - V_\alpha) && \text{for } V_i > V_\alpha
\end{aligned}
$$

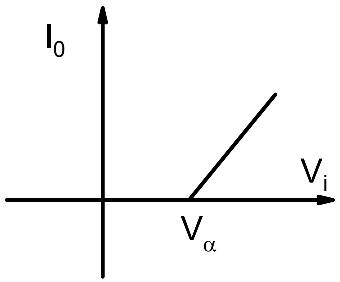
Ideal Characteristics

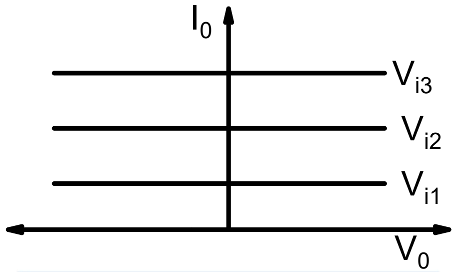




How do we use device X to make an amplifier?

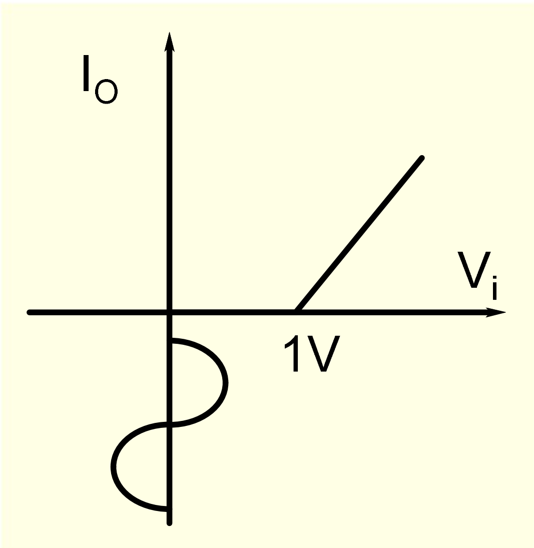






$V_\alpha = 1V;\ g_m = 0.01\Omega^{-1}$

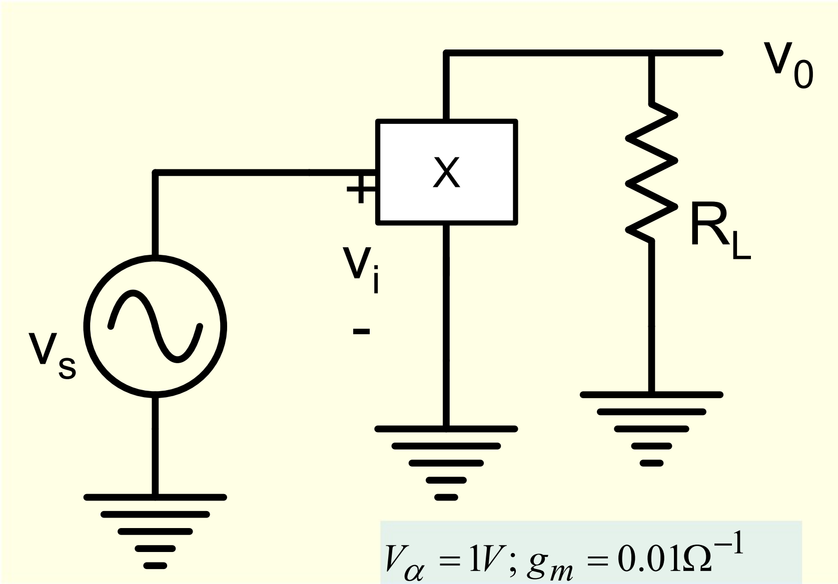
$R_L = 1K\ ;\ v_s = 0.5V\ Sin\ \omega t$



$$I_O = 0 \Rightarrow V_O = 0$$

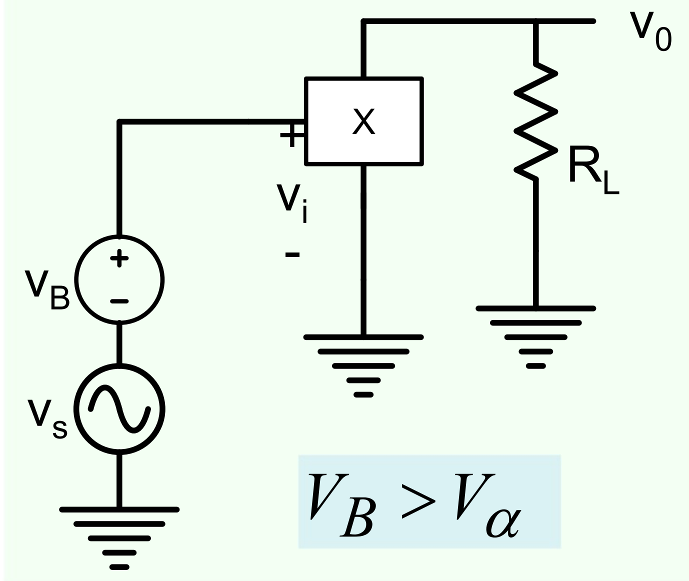
No Amplification

How do we use device X to make an Amplifier?

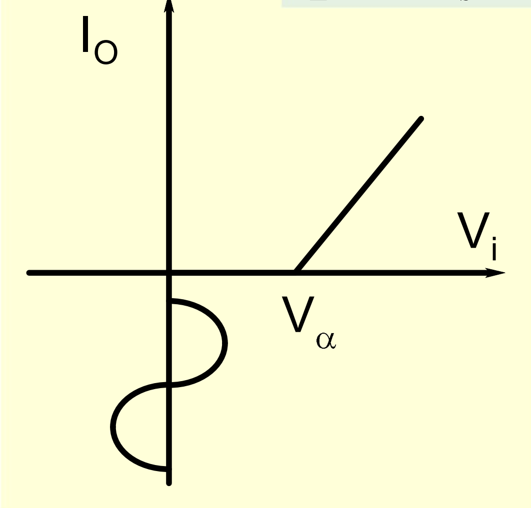


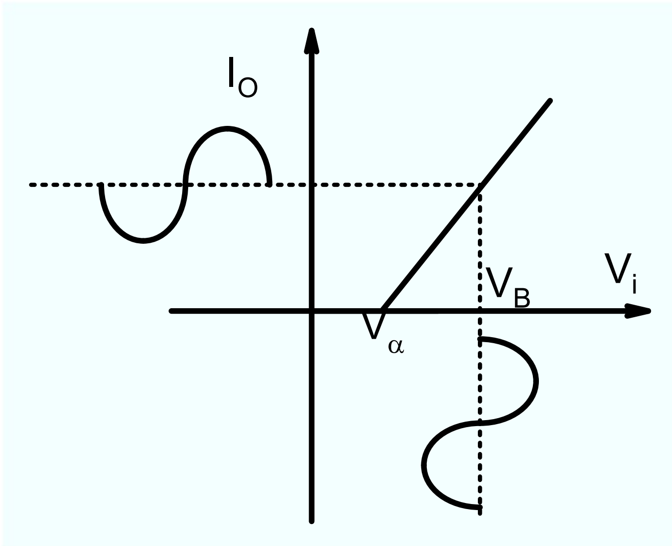
$$V_\alpha = 1V;\ g_m = 0.01\Omega^{-1}$$
$$R_L = 1K\ ;\ v_s = 0.5V\ Sin\ \omega t$$

$$V_O = -I_O\ R_L$$

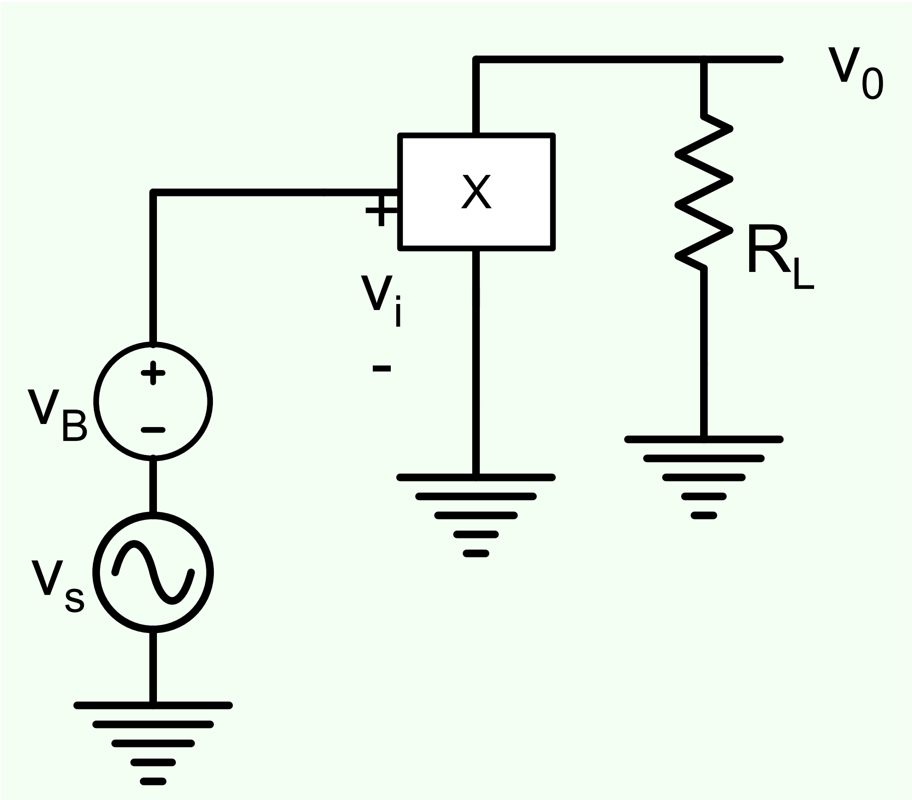


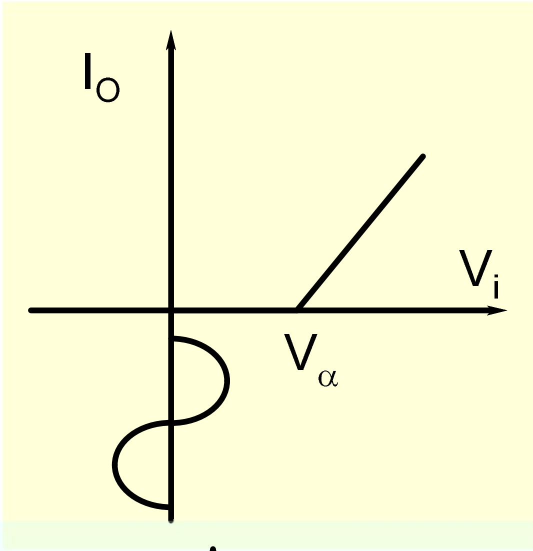
$$V_B > V_\alpha$$

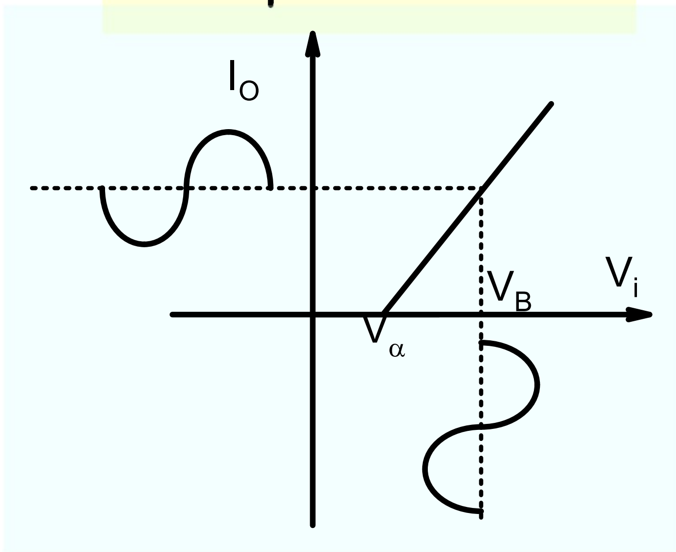




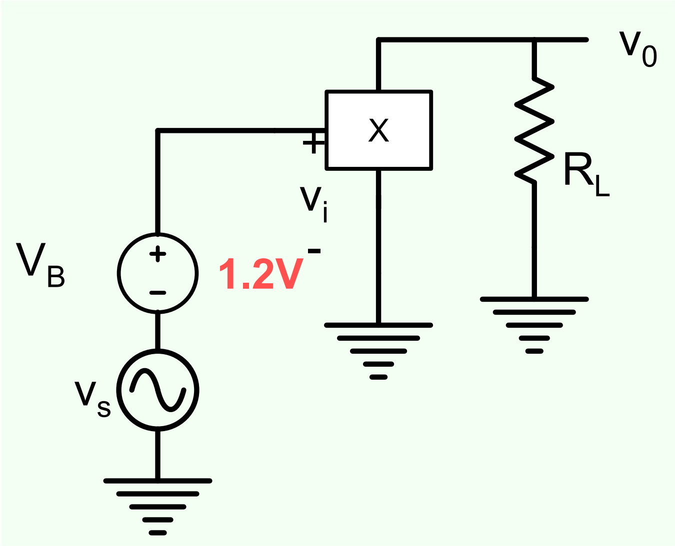
When only a part of device characteristics is suitable for amplification, then we need to push the device into that region by applying suitable bias voltages. This process is called BIASING



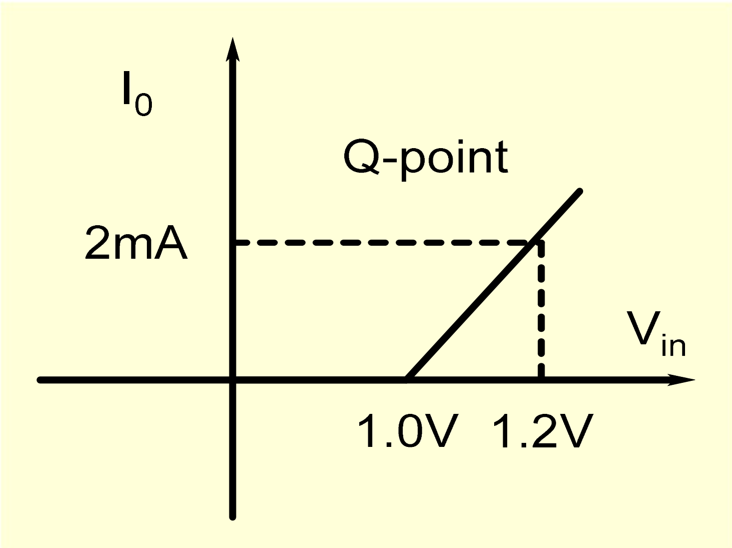




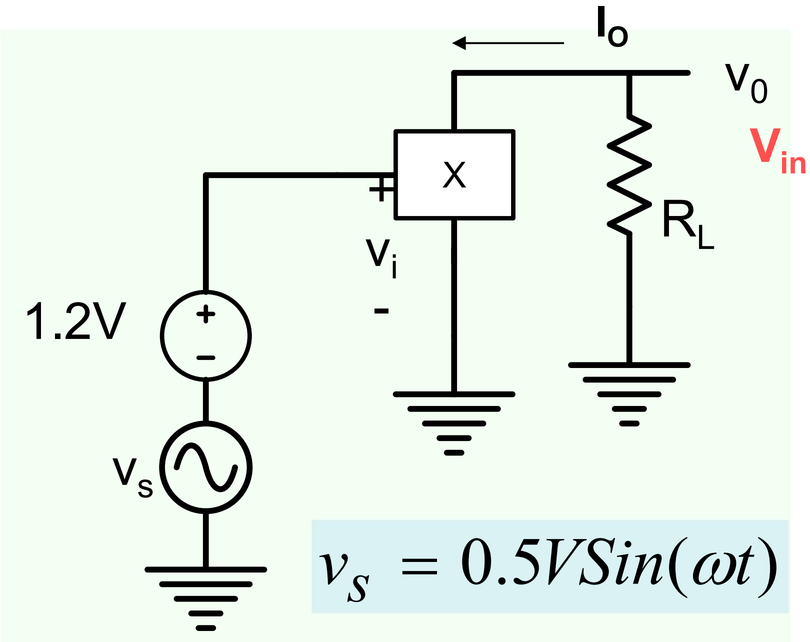
How should one choose the bias voltage V_B ?



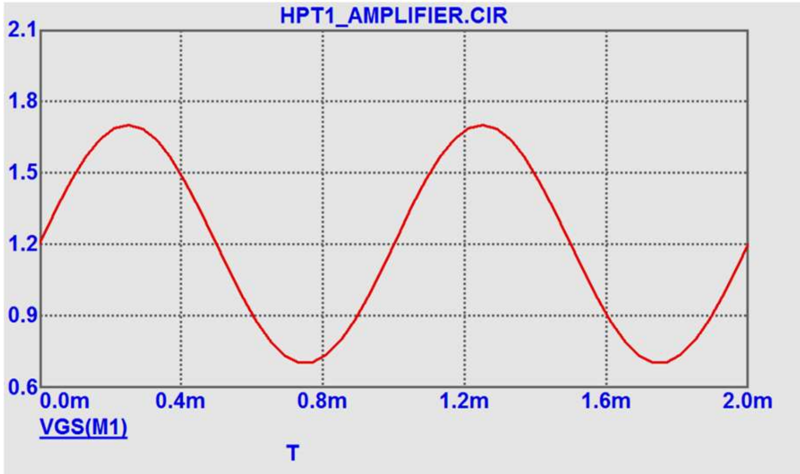
$$v_s = 0.5V\ Sin\ \omega t$$

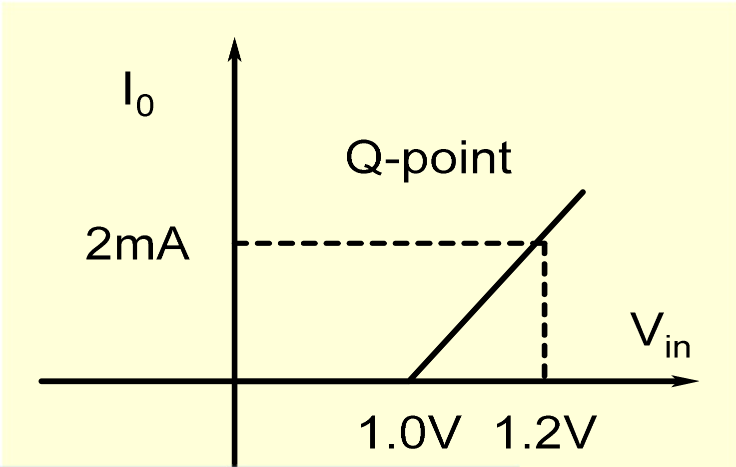


Quiescent point or Bias point

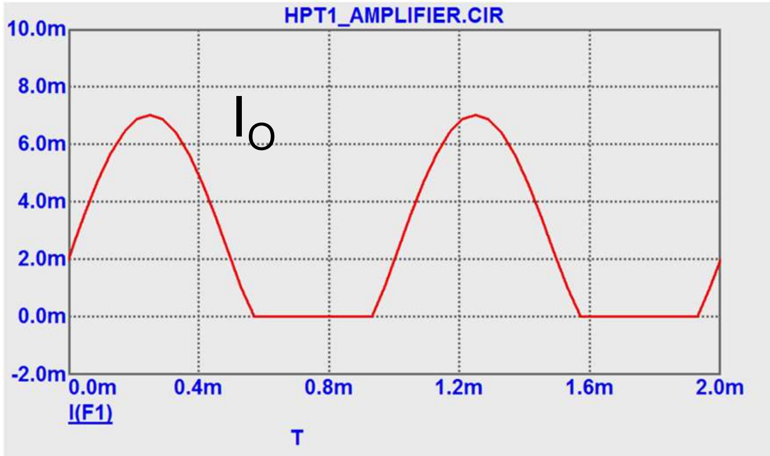


$$v_s = 0.5V Sin(\omega t)$$



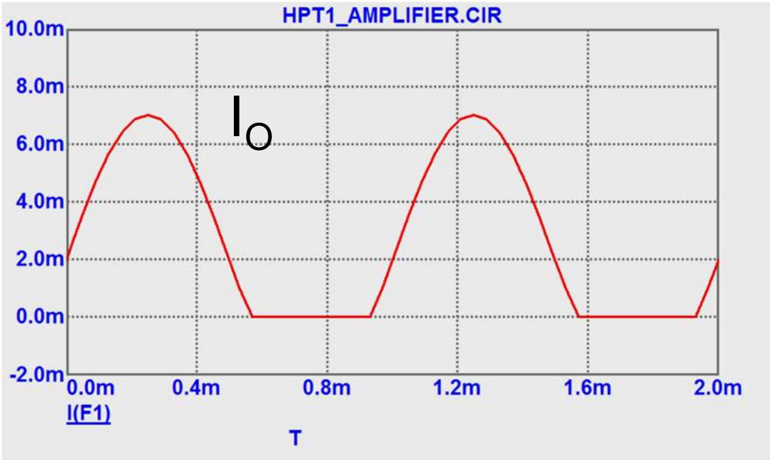


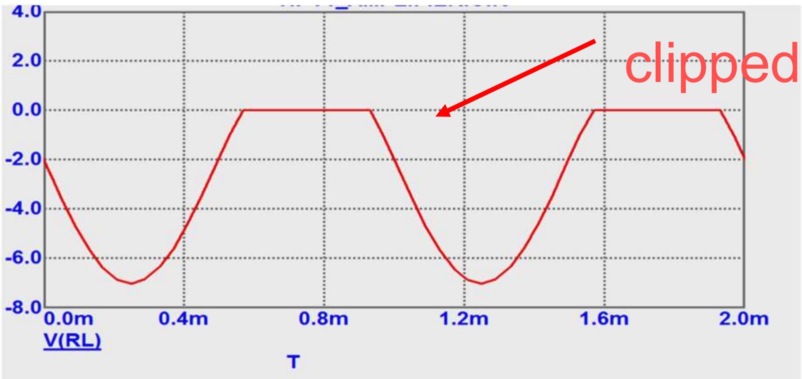
$$V_\alpha = 1V; \; g_m = 0.01\Omega^{-1}$$

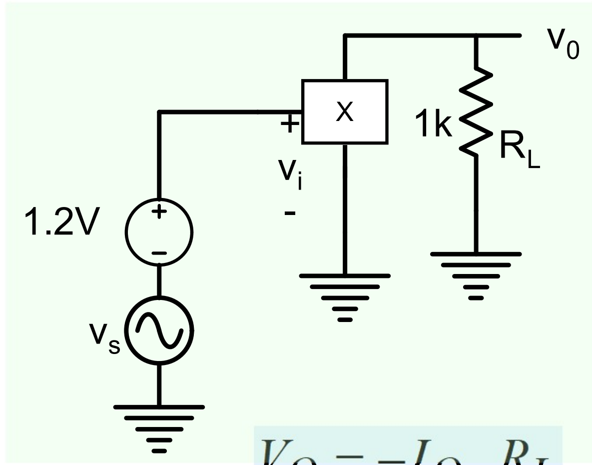


$$V_O = -I_O \; R_L$$

Output voltage is distorted !



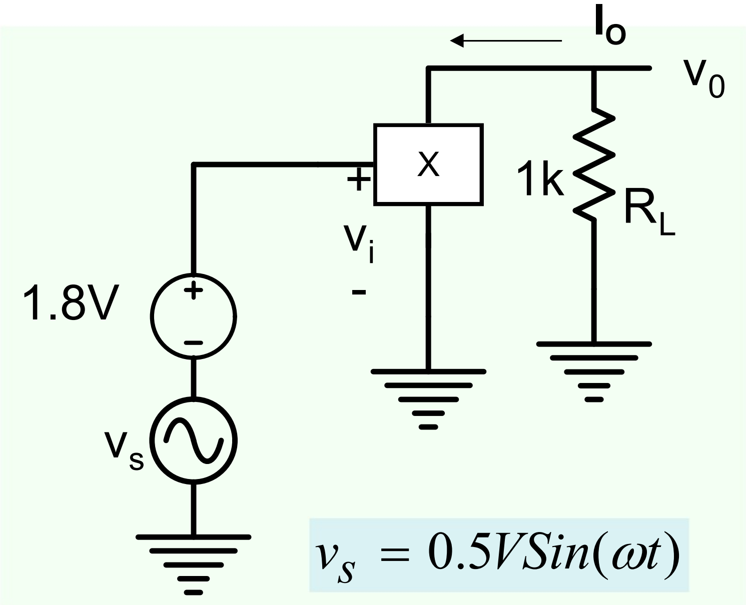




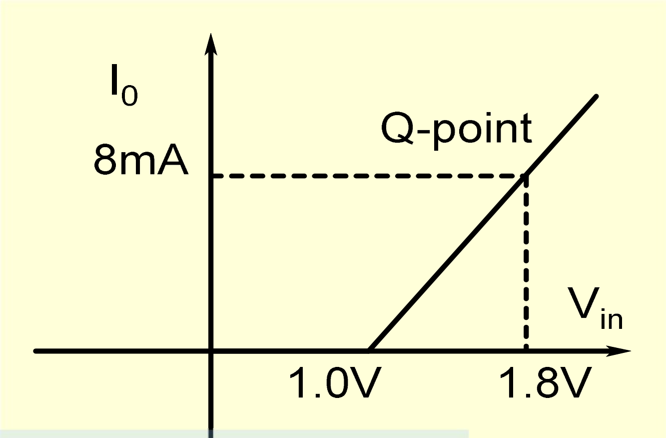
$$V_O = -I_O \ R_L$$

Need to choose a proper value of biasing Voltage

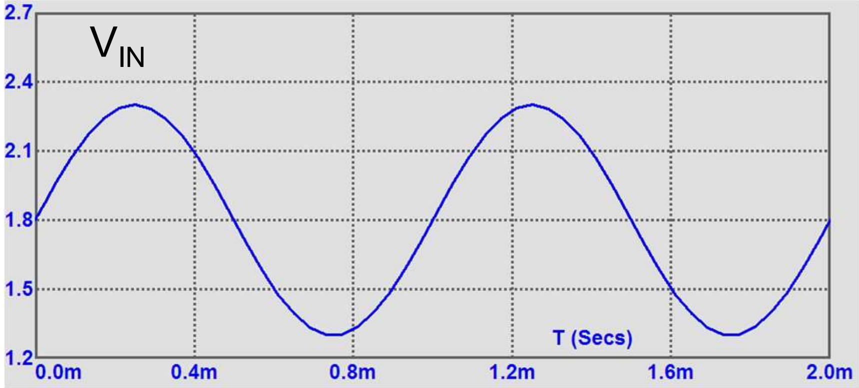
Unnecessary Power Dissipation

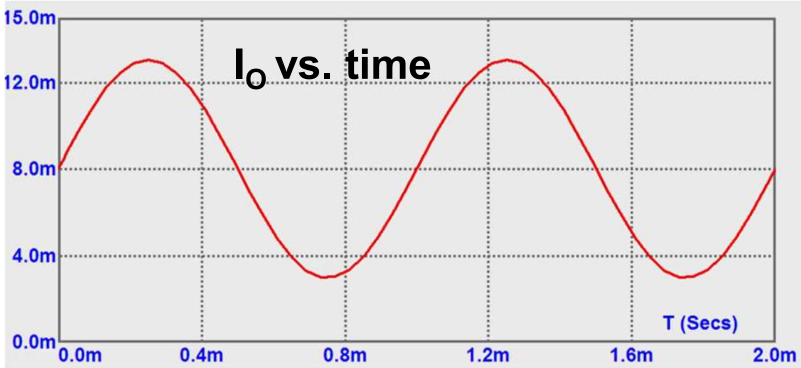


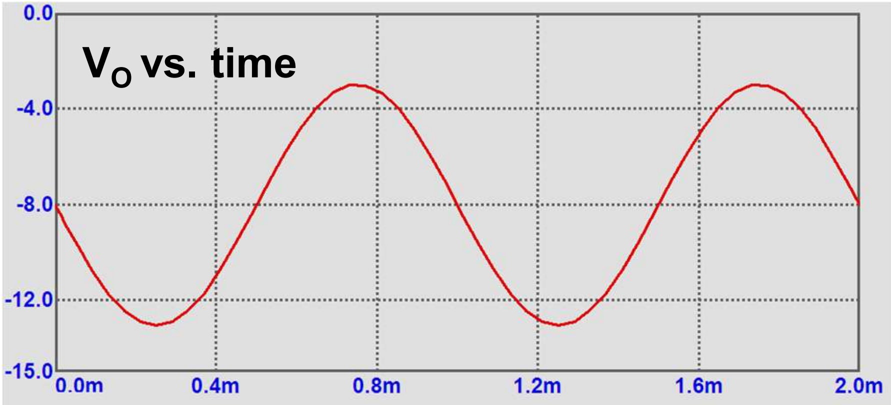
$$v_s = 0.5VSin(\omega t)$$



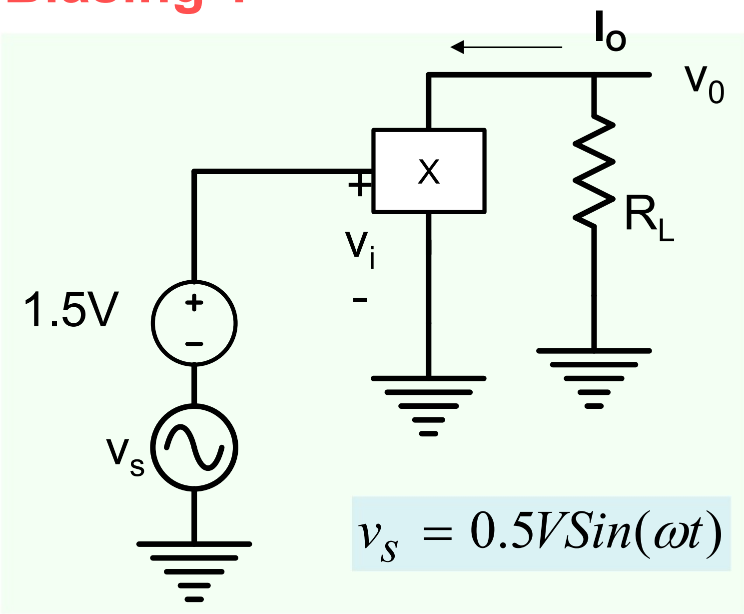
$$V_\alpha = 1V;\ g_m = 0.01\Omega^{-1}$$



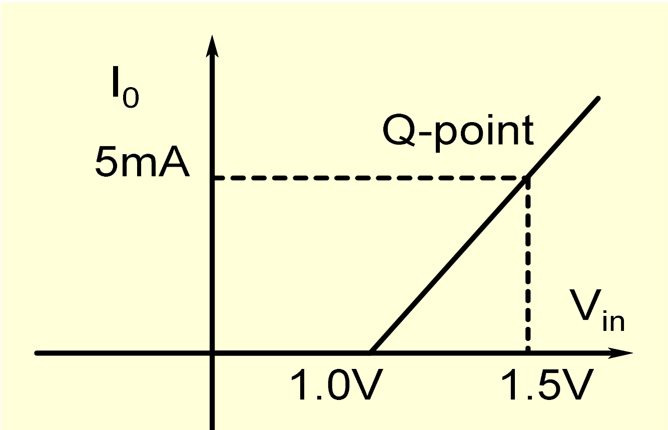




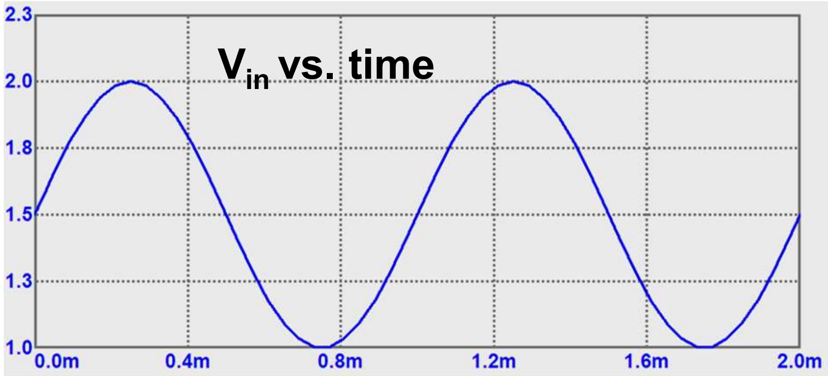
Optimum Biasing ?

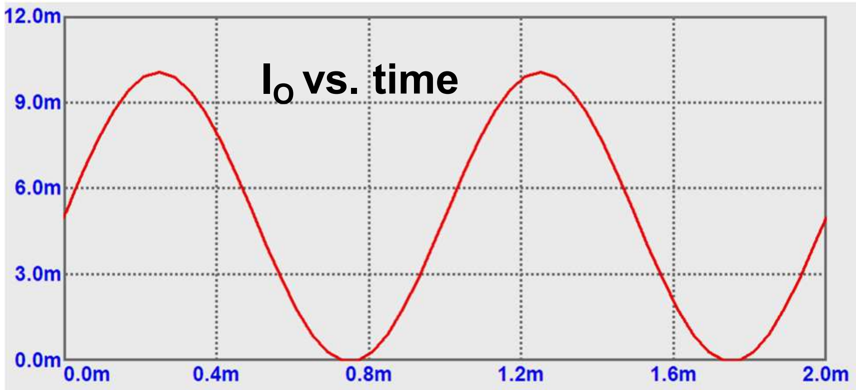


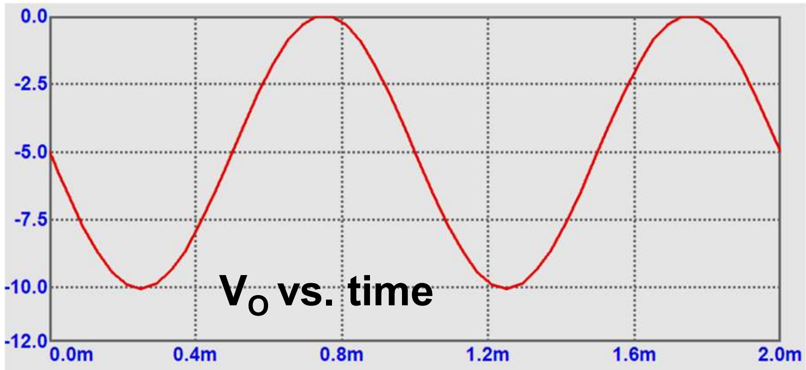
$$v_s = 0.5VSin(\omega t)$$



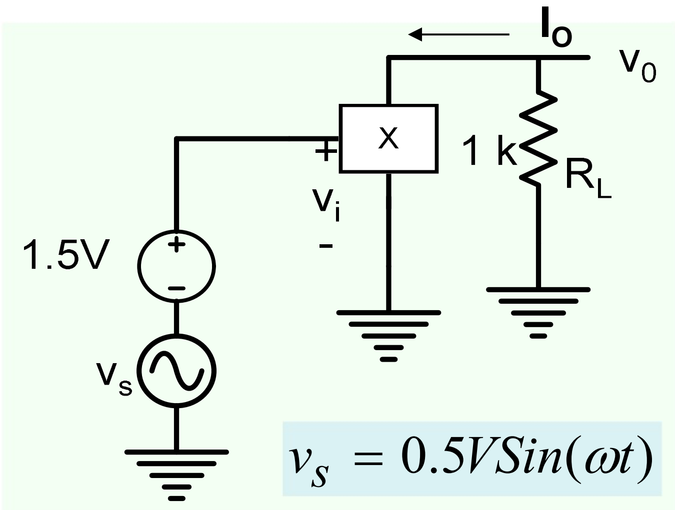
$$V_\alpha = 1V;\ g_m = 0.01\Omega^{-1}$$

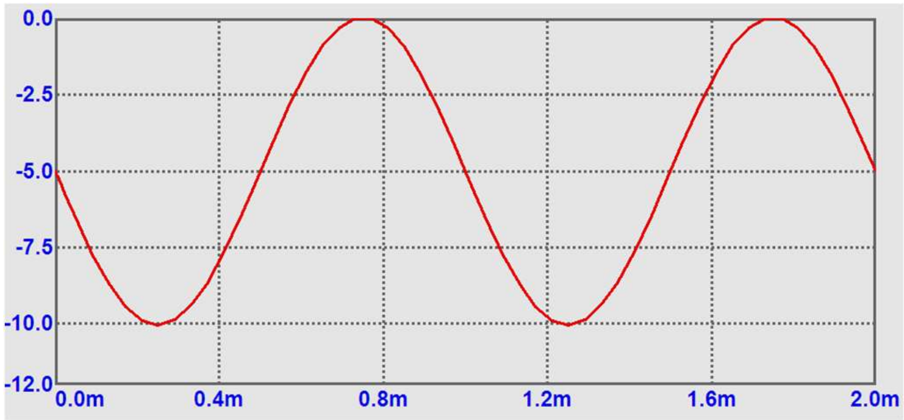


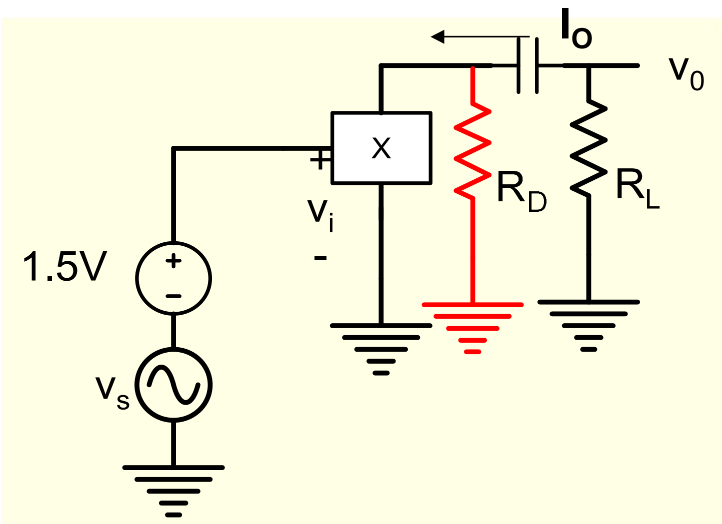


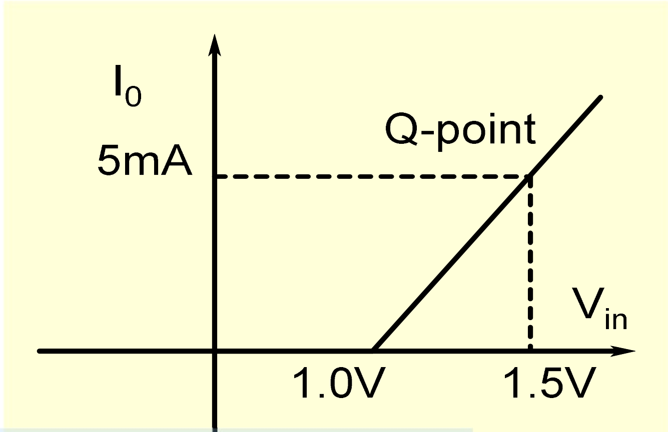


How do we get rid of unwanted dc voltage at the output ?

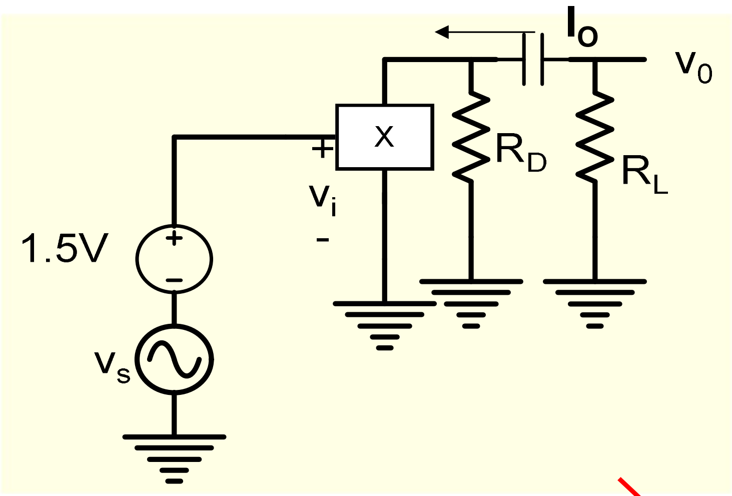


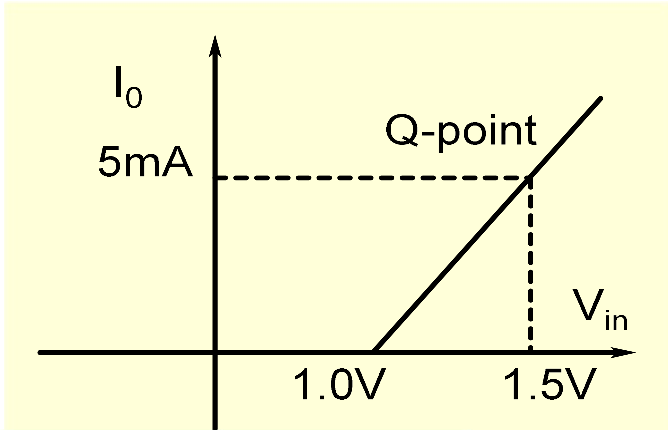






$$V_\alpha = 1V;\; g_m = 0.01\Omega^{-1}$$

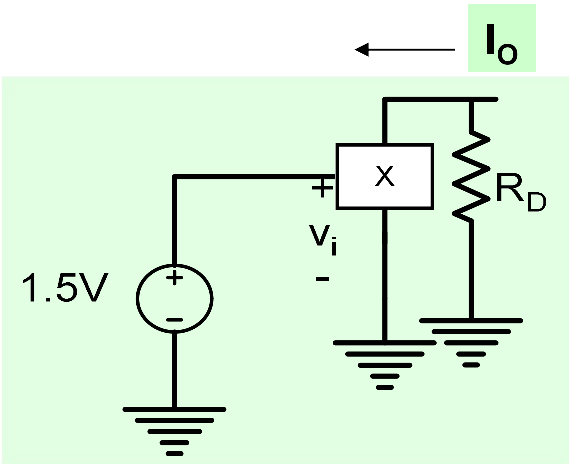


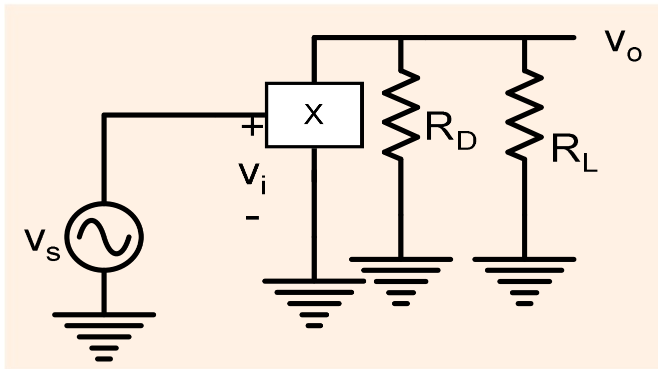


$$V_\alpha = 1V;\ g_m = 0.01\Omega^{-1}$$

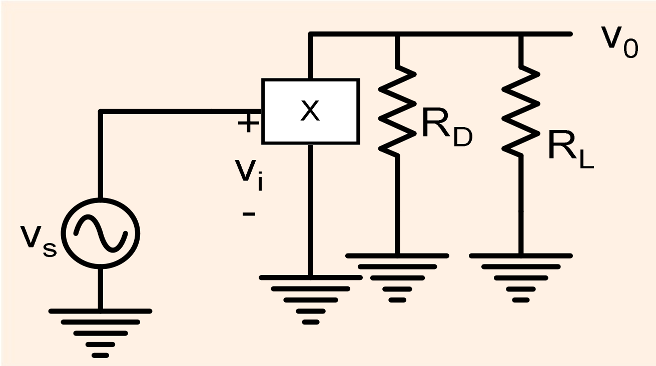
dc

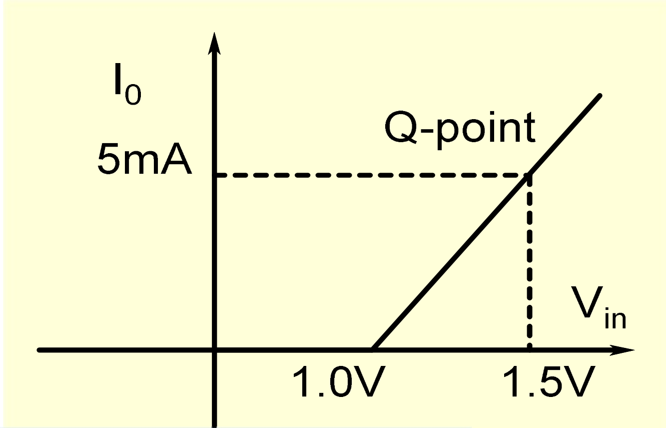
ac (signal)





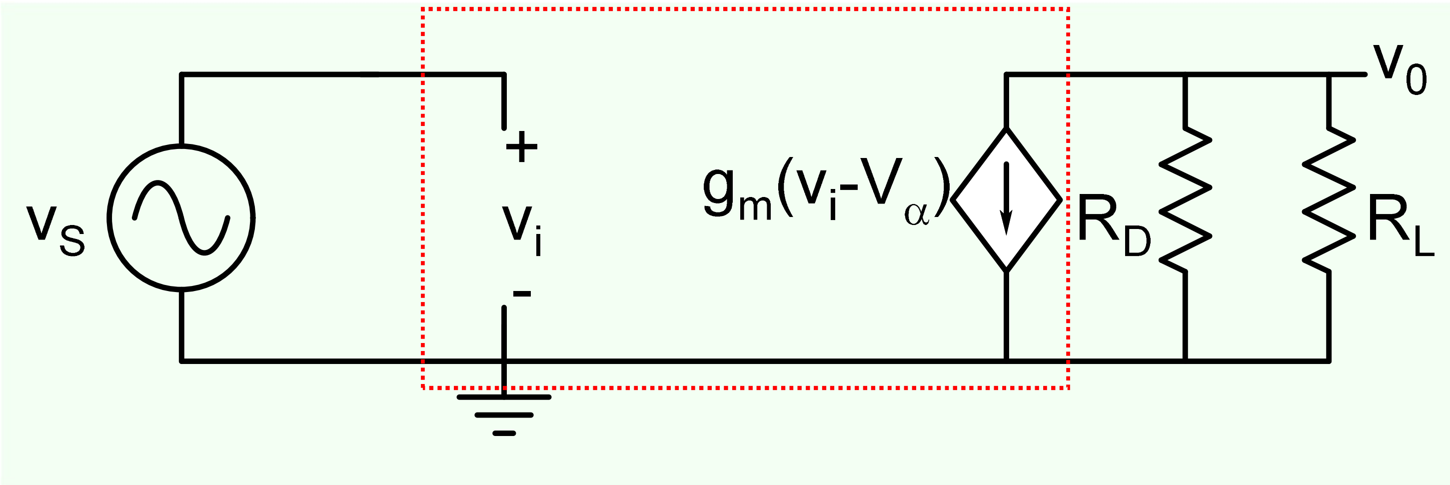
Capacitor is chosen large enough so that at the signal frequency $1/j\omega C \sim 0$.



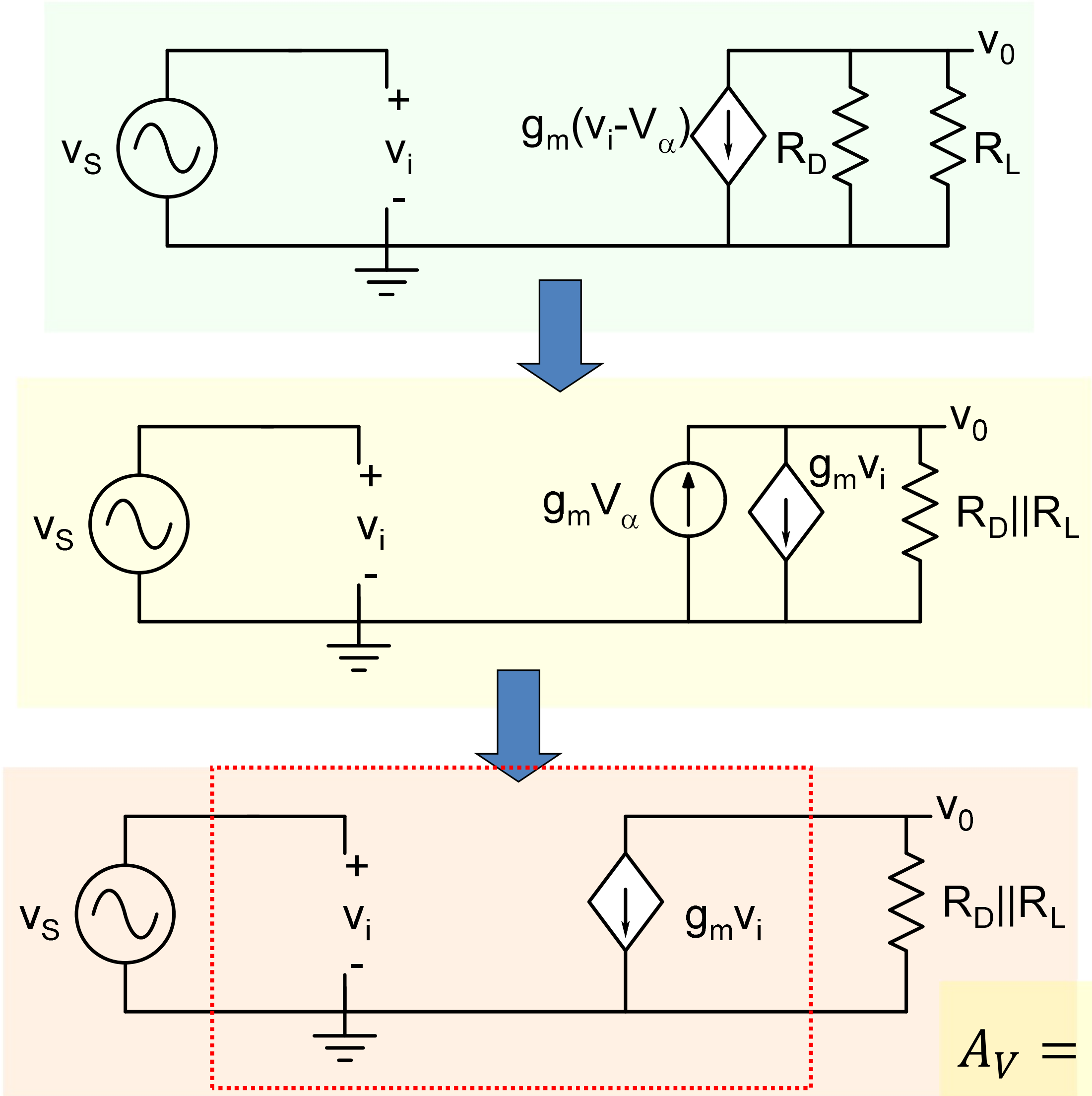


$V_\alpha = 1V;\ g_m = 0.01\Omega^{-1}$

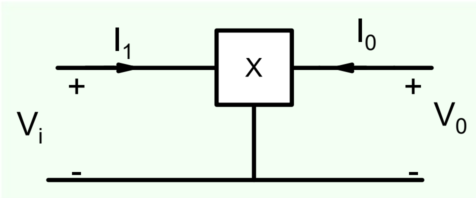
$I_o = g_m \times (V_i - V_\alpha)$ for $V_i > V_\alpha$

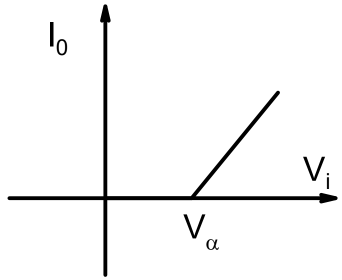


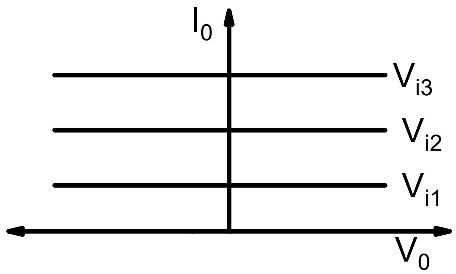
Ac Analysis

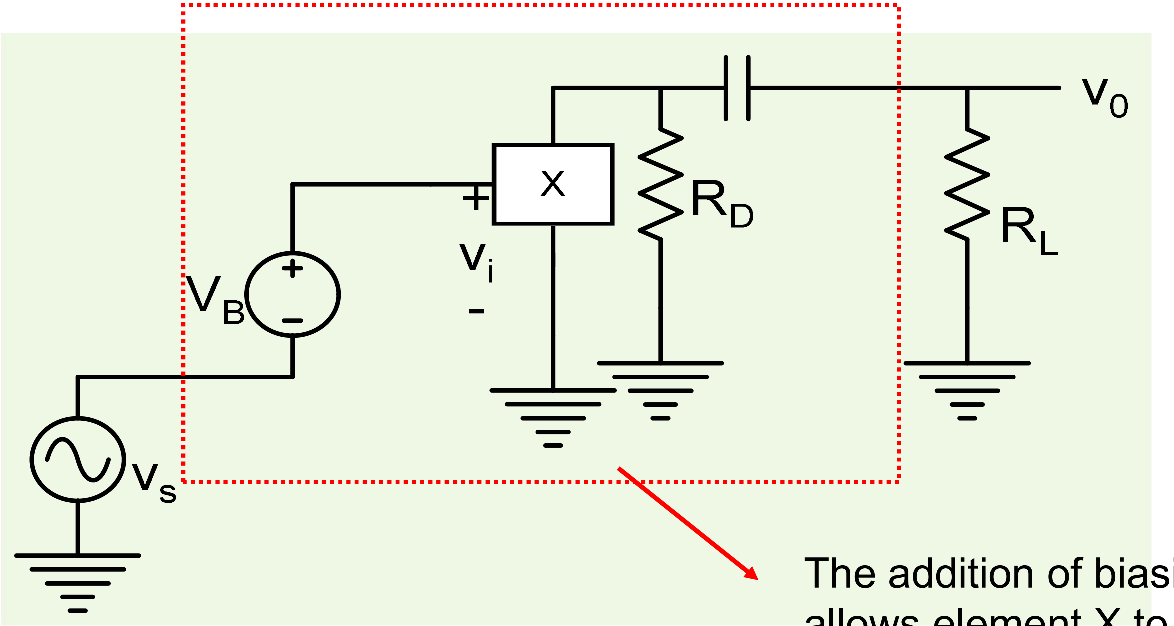


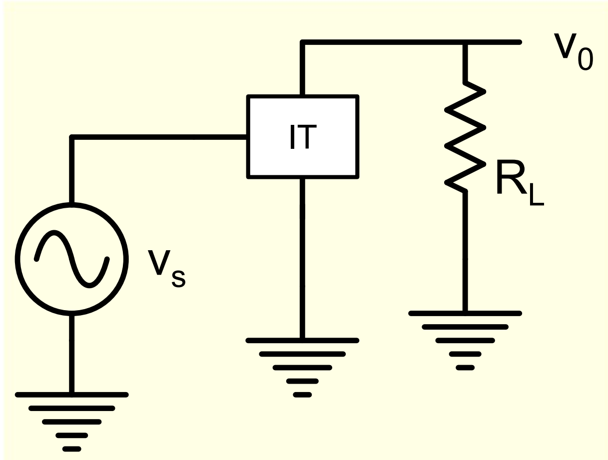
$$A_V = \frac{v_o}{v_s} = -g_m R_D \| R_L$$





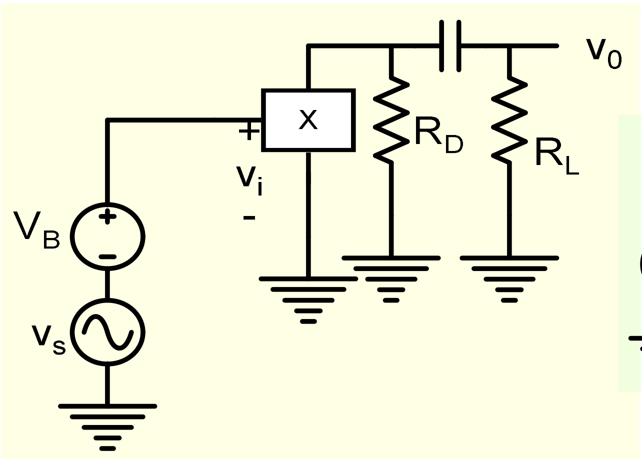


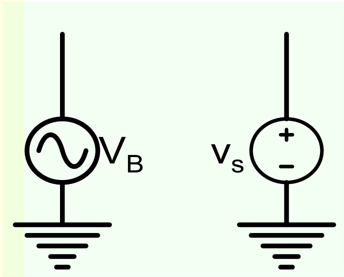


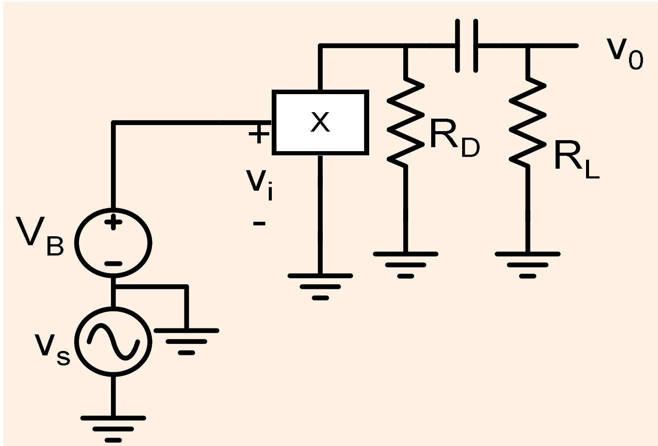


The addition of biasing network allows element X to appear as an ideal transistor to the signal source

What happens if both dc voltage source and signal source have one terminal as ground?

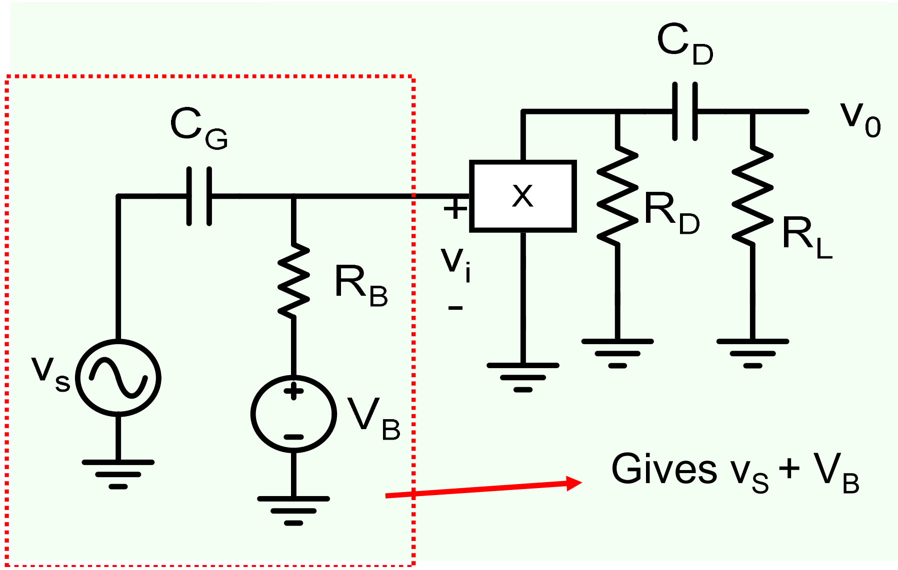


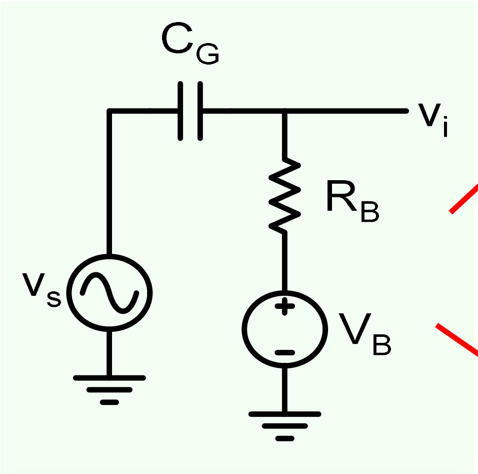


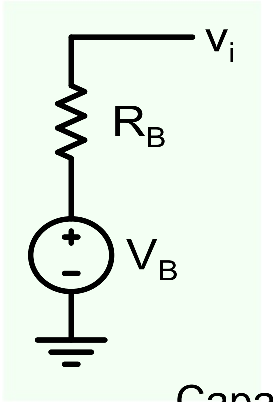


Shorts the signal source

Solution

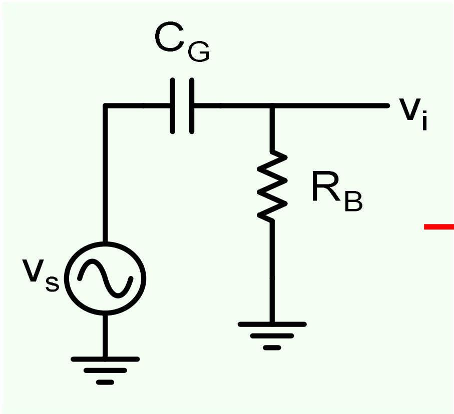


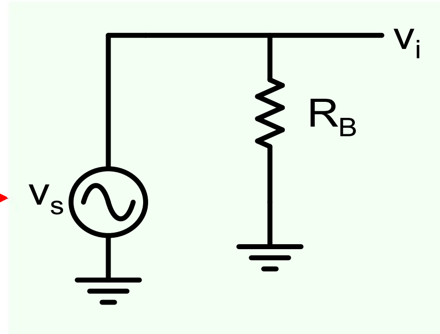




$$v_i = V_B$$

Capacitor is chosen large enough so that at the signal frequency $1/j\omega C \sim 0$.



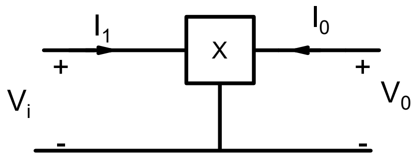


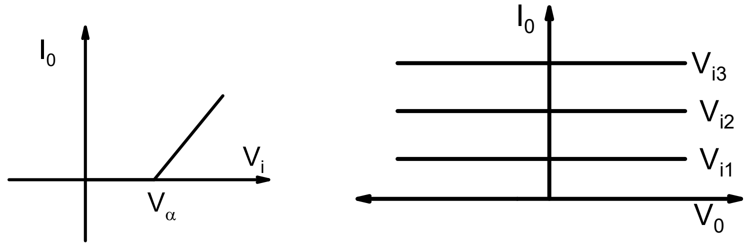
$$v_i = v_s$$

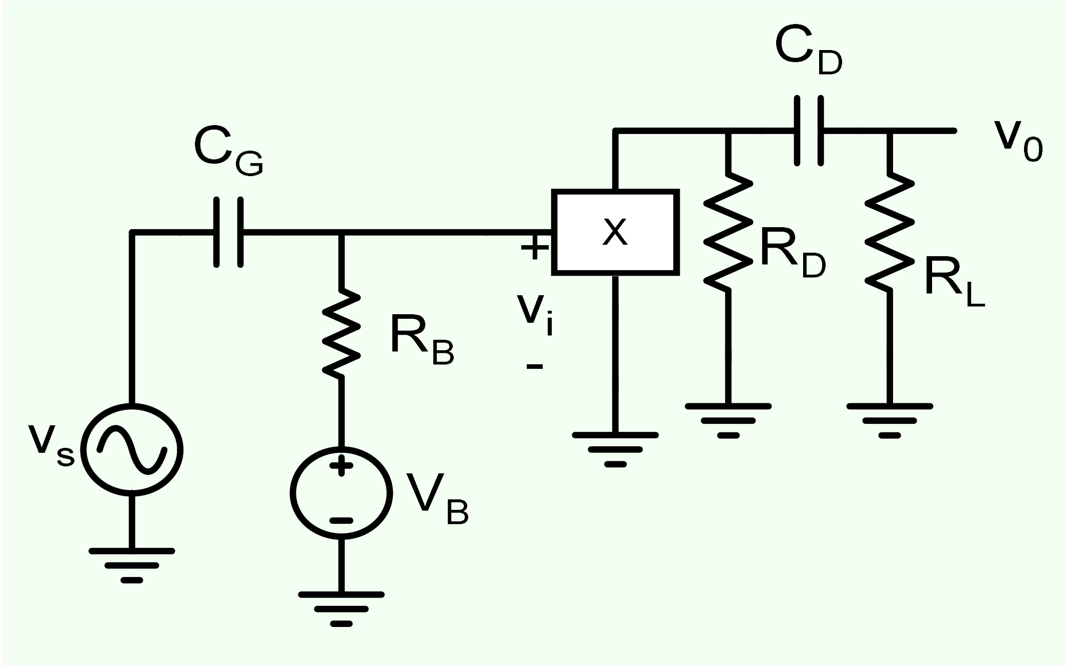
$$v_i(total) = v_s + V_B$$

Note the role of R_B

Amplifier Schematic





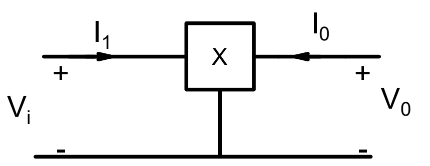


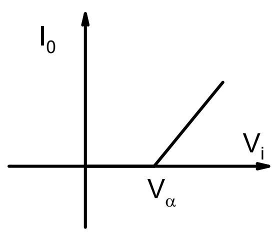
ESC201T : Introduction to Electronics

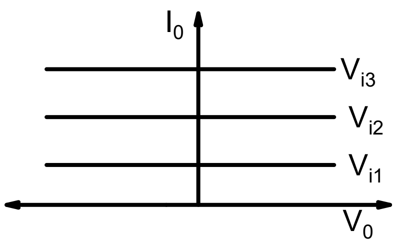
Lecture 26: Amplifiers-2

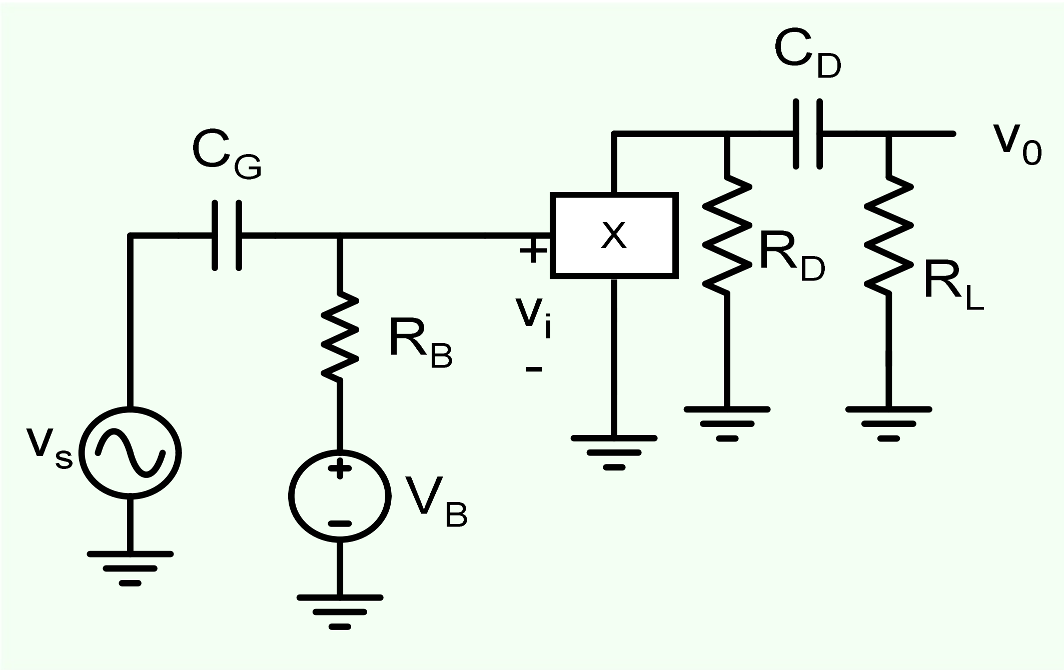
B. Mazhari
Dept. of EE, IIT Kanpur

Amplifier Schematic

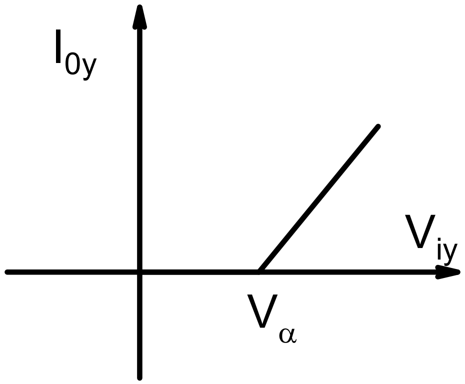


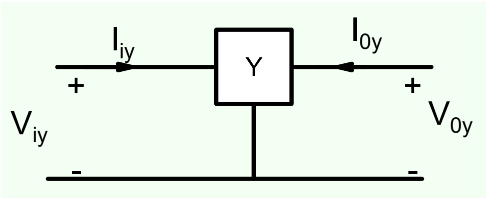






Device Y



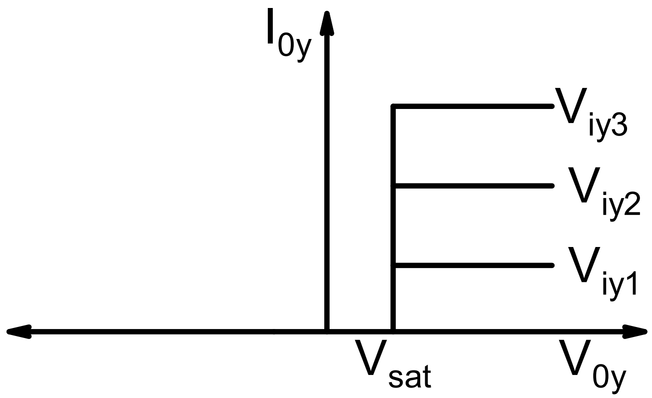


$I_{oy} = 0$ for $V_{OY} < V_{sat}$

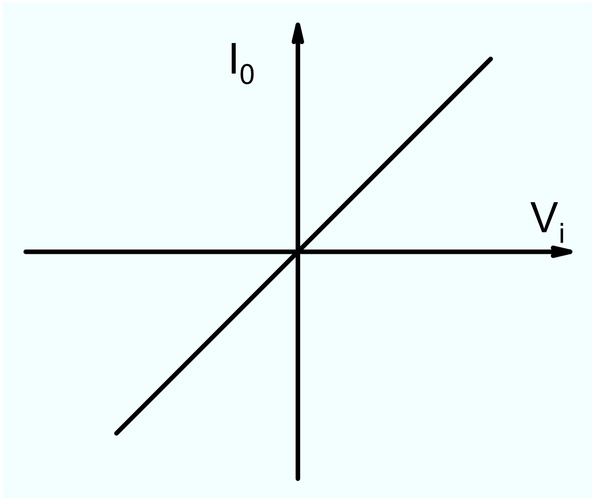
for $V_{OY} \geq V_{sat}$

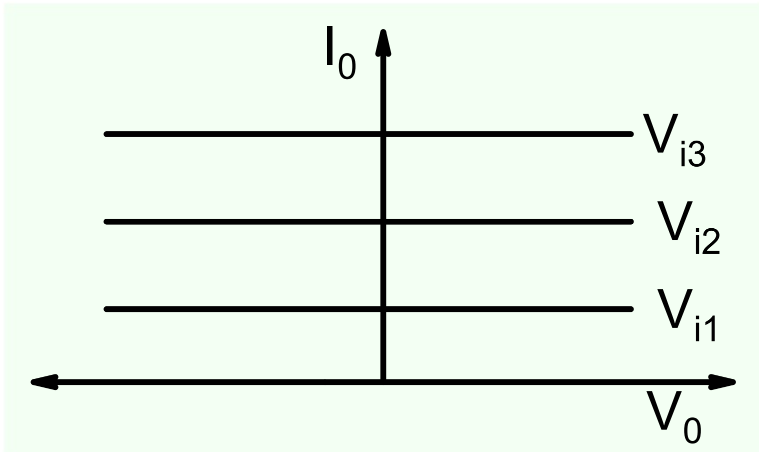
$I_{oy} = 0 \qquad$ for $V_{iy} \leq V_\alpha$

$= g_m \times (V_{iy} - V_\alpha)$ for $V_{iy} > V_\alpha$

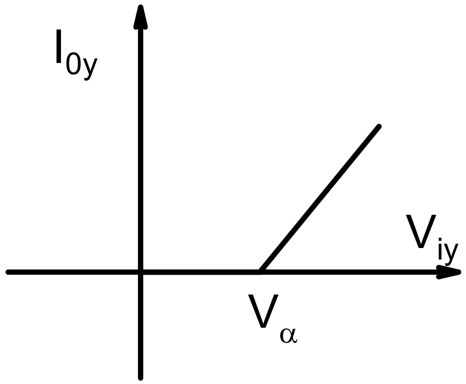


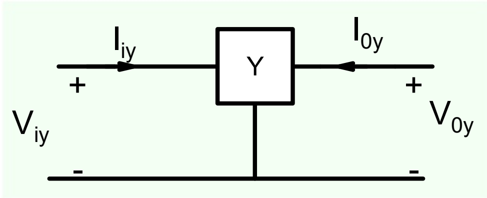
Ideal Characteristics





How do we use device Y to make an amplifier?



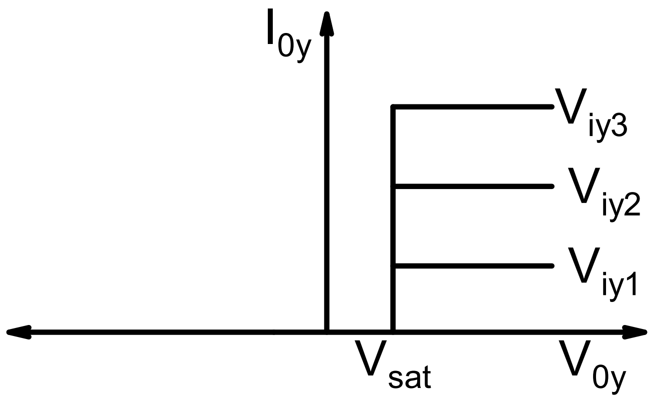


$I_{oy} = 0$ for $V_{OY} < V_{sat}$

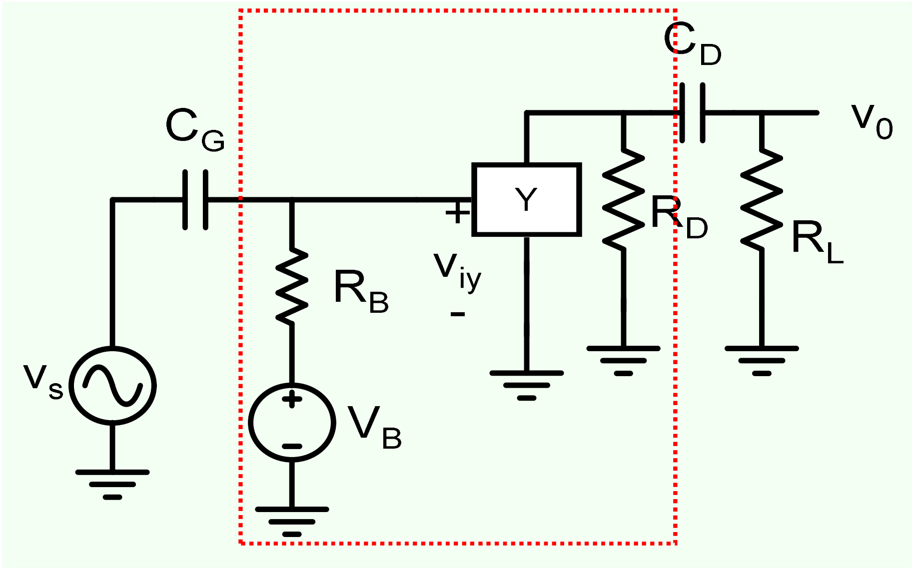
for $V_{OY} \geq V_{sat}$

$I_{oy} = 0$ for $V_{iy} \leq V_\alpha$

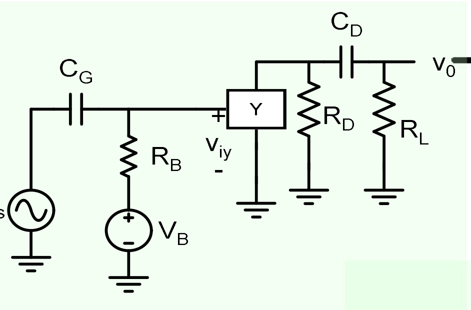
$= g_m \times (V_{iy} - V_\alpha)$ for $V_{iy} > V_\alpha$

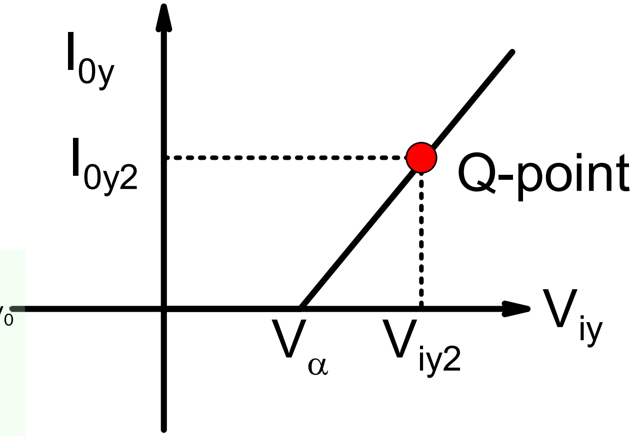


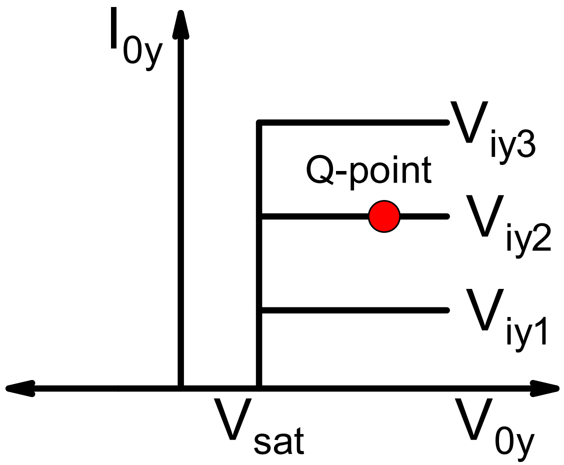
Will the earlier solution work?



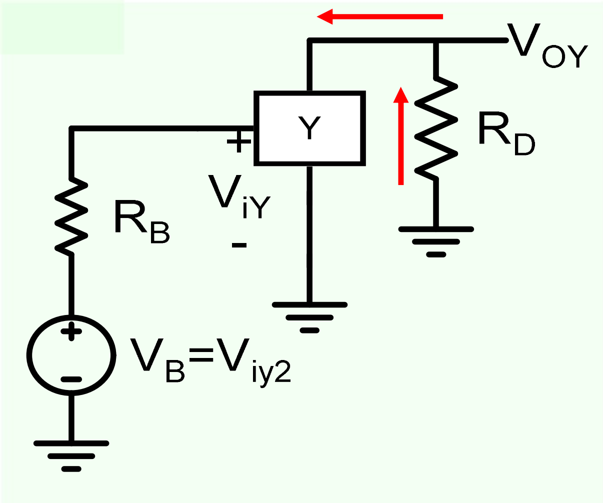
The purpose of biasing network is to operate the device in a region which resembles ideal transistor

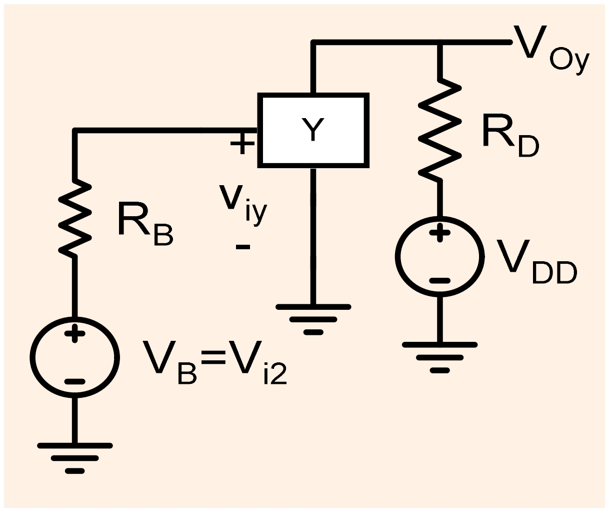




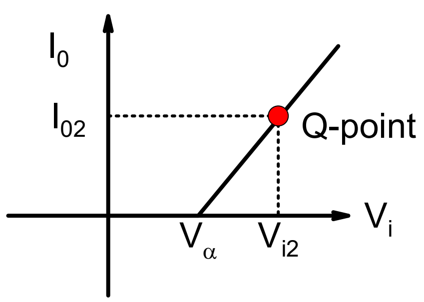


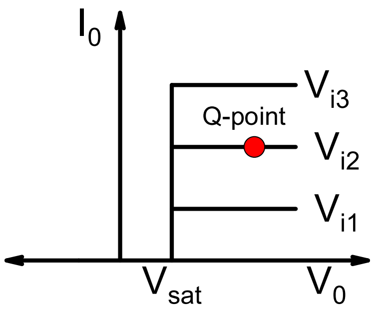
V_{Oy} = -ve which is not possible

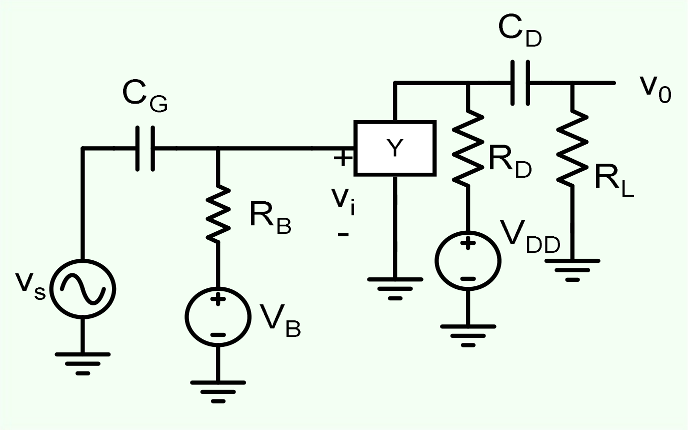


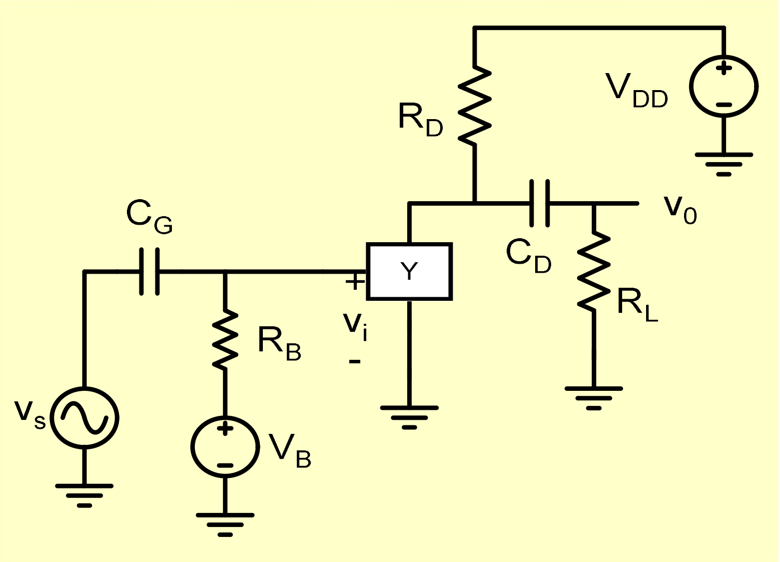


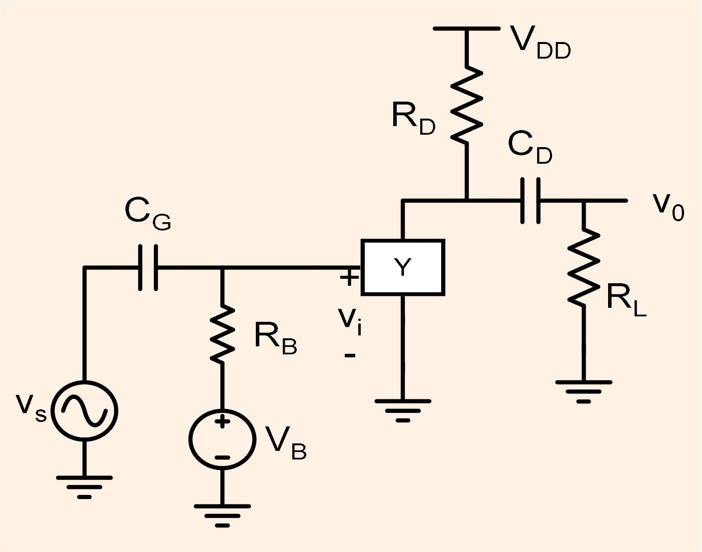
Revised Amplifier Schematic



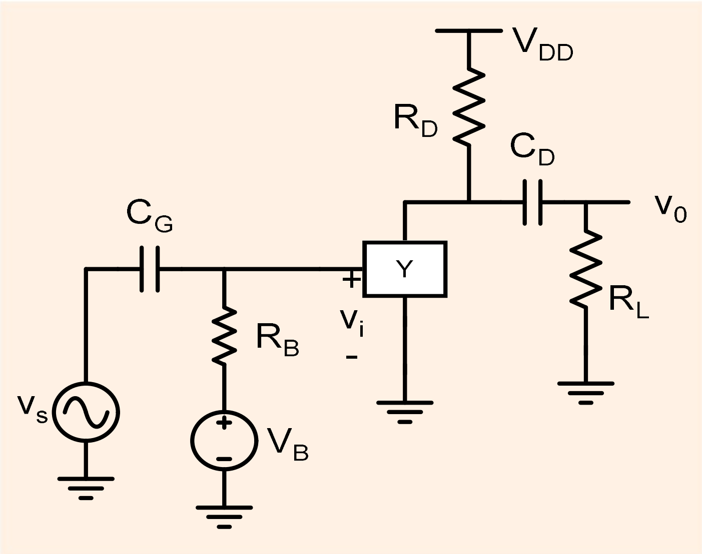


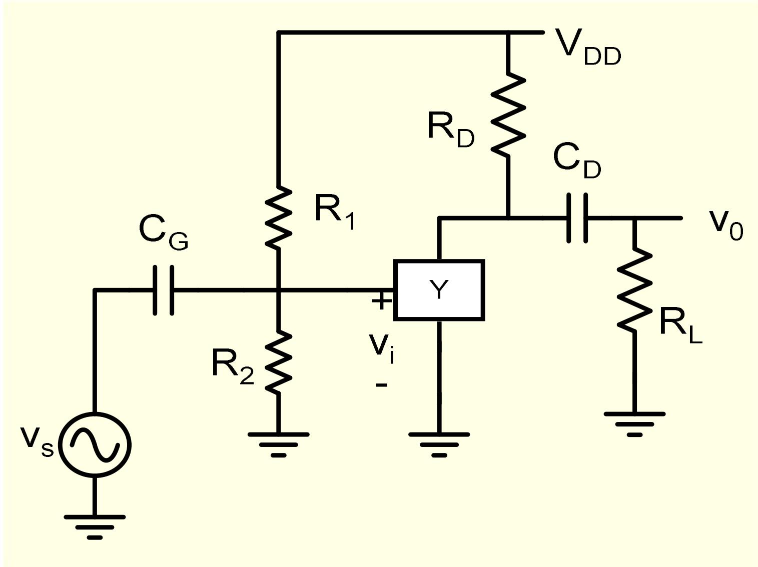




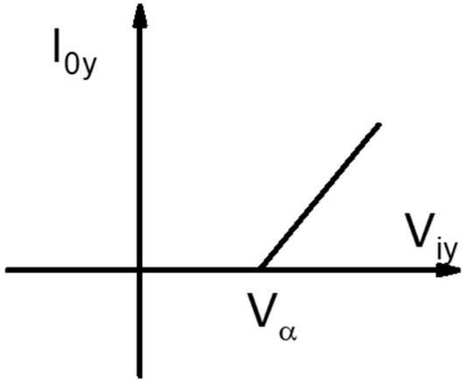


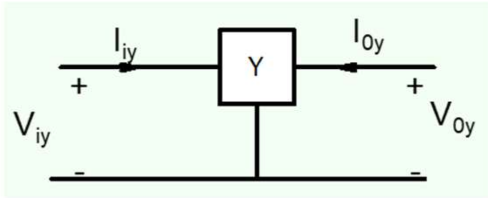
Can we Bias using one dc voltage source only?





$$V_B = V_{DD} \times \frac{R_2}{R_1 + R_2}$$



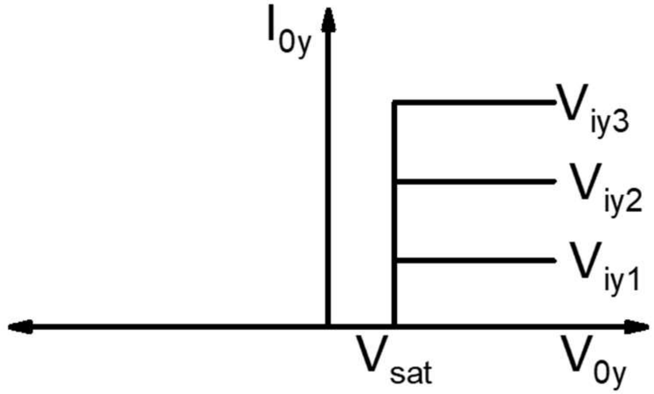


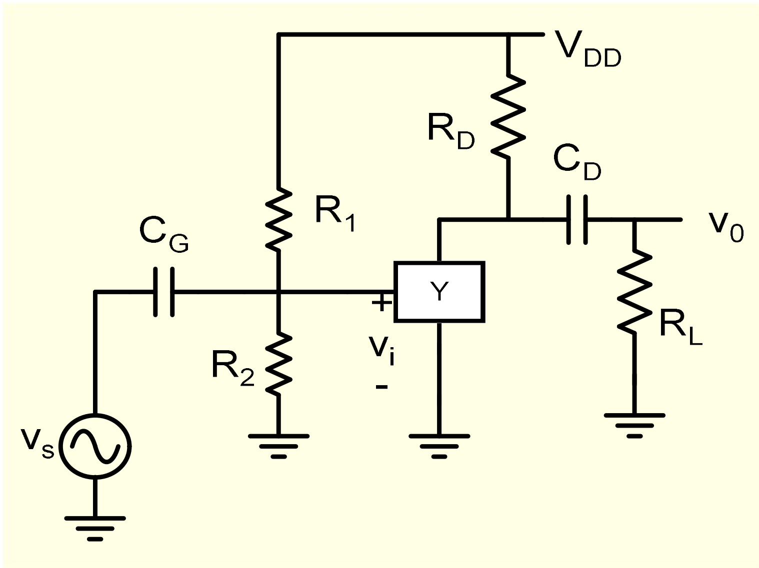
$$I_{oy} = 0 \text{ for } V_{OY} < V_{sat}$$

$$\text{for } V_{OY} \geq V_{sat}$$

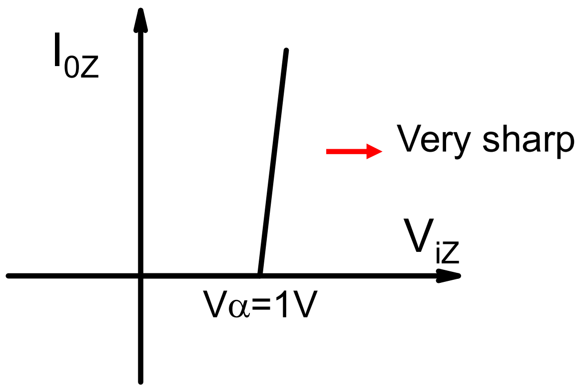
$$I_{oy} = 0 \qquad \text{for } V_{iy} \leq V_{\alpha}$$

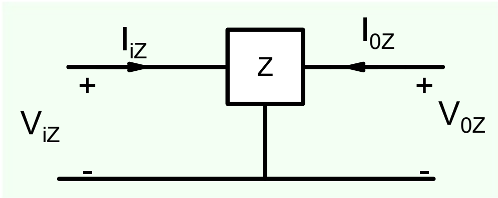
$$= g_m \times (V_{iy} - V_{\alpha}) \;\; \text{for } V_{iy} > V_{\alpha}$$



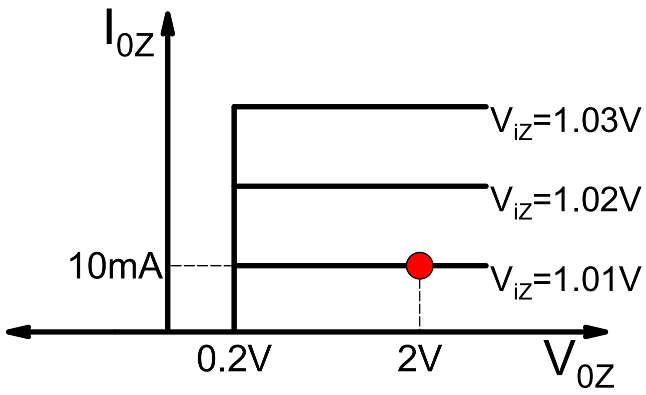


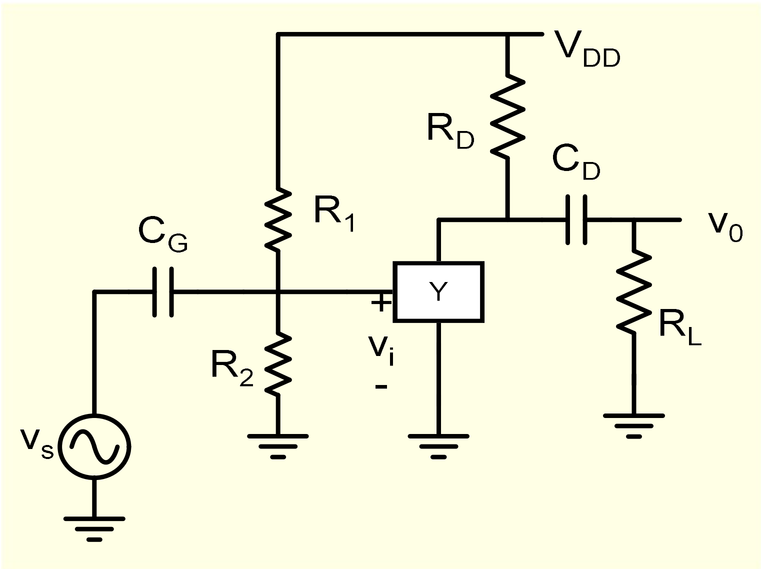
Device Z





$$I_{oz} = 0 \text{ for } V_{OZ} < 0.2V$$
$$\text{for } V_{OZ} \geq 0.2$$
$$I_{oz} = 0 \quad \text{for } V_{iz} \leq 1V$$
$$= 1 \times (V_{iz} - 1V) \quad \text{for } V_{iz} > 1V$$





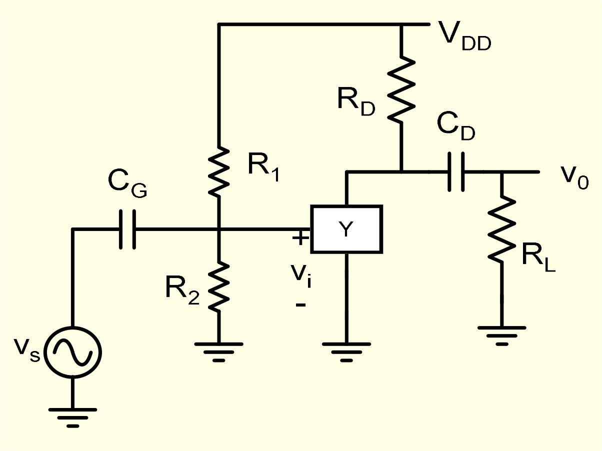
Circuit is very sensitive to variations in resistor values, power supply, device parameters such as $V\alpha$

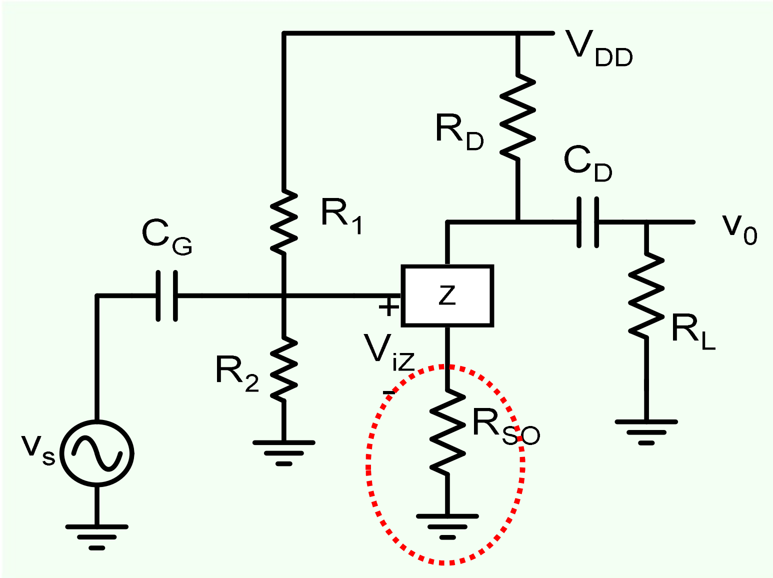
$$V_{DD} = 5V; R_2 = 1K; R_1 = 3.95K$$
$$\Rightarrow V_{iz} = 1.01 \Rightarrow I_{OZ} = 10mA$$

$$V_{DD} = 5V; R_2 = 0.99K; R_1 = 3.95K$$
$$\Rightarrow V_{iz} = 1.002 \Rightarrow I_{OZ} = 1.9mA$$

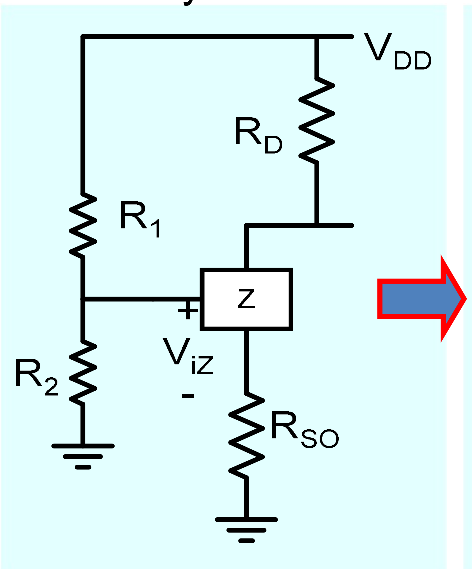
$$V_{DD} = 5V; R_2 = 0.98K; R_1 = 3.95K$$
$$\Rightarrow V_{iz} = 0.994V \Rightarrow I_{OZ} = 0$$

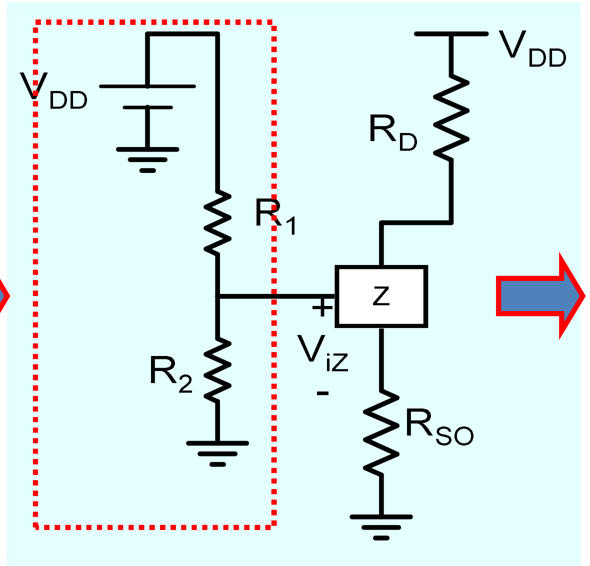
Solution

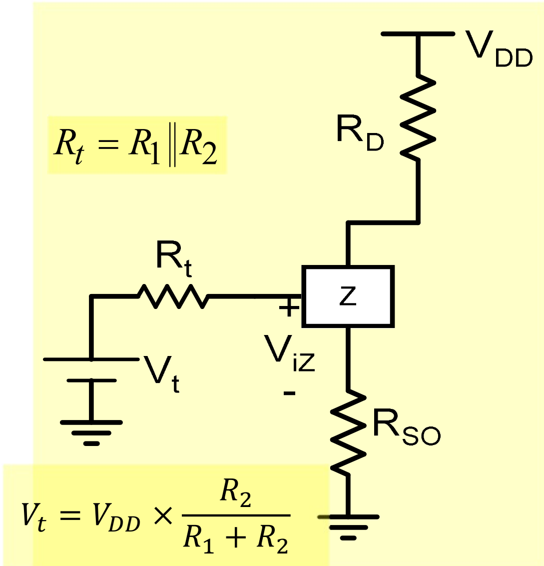


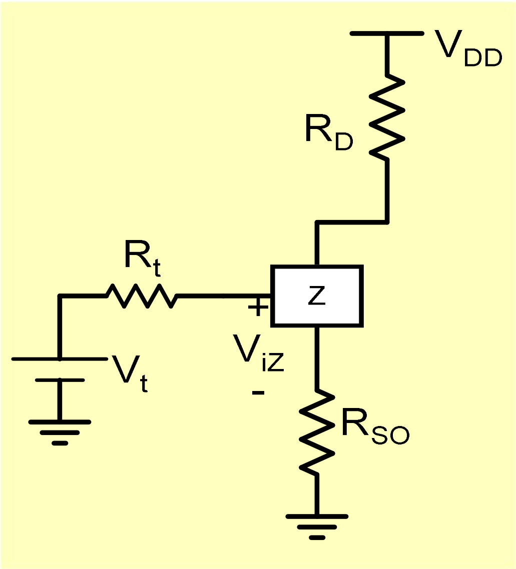


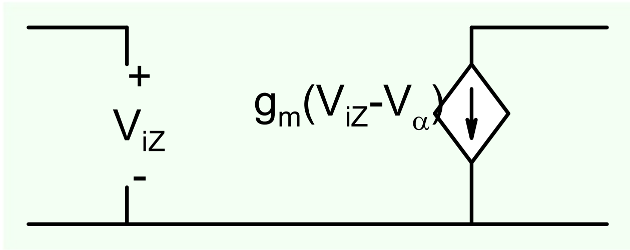
dc Analysis

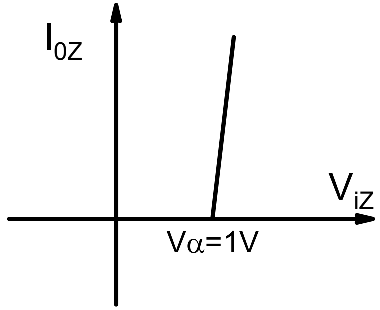


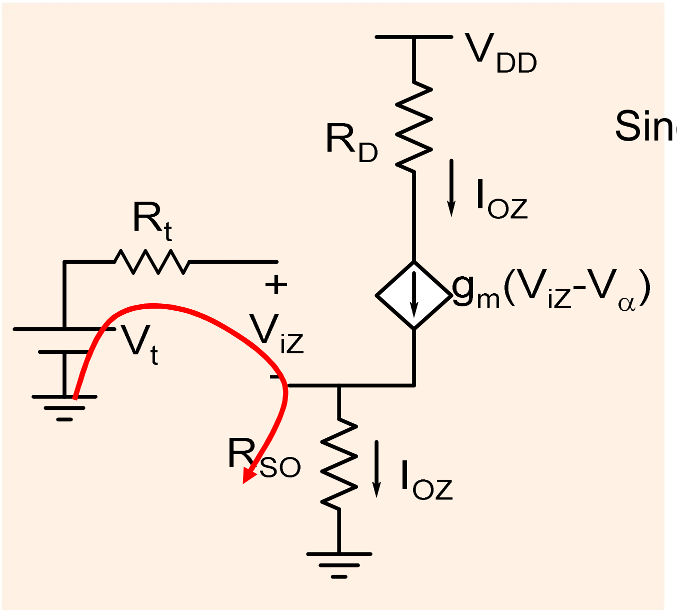












$$-V_t + 0 \times R_t + V_{iz} + I_{OZ}R_{SO} = 0$$

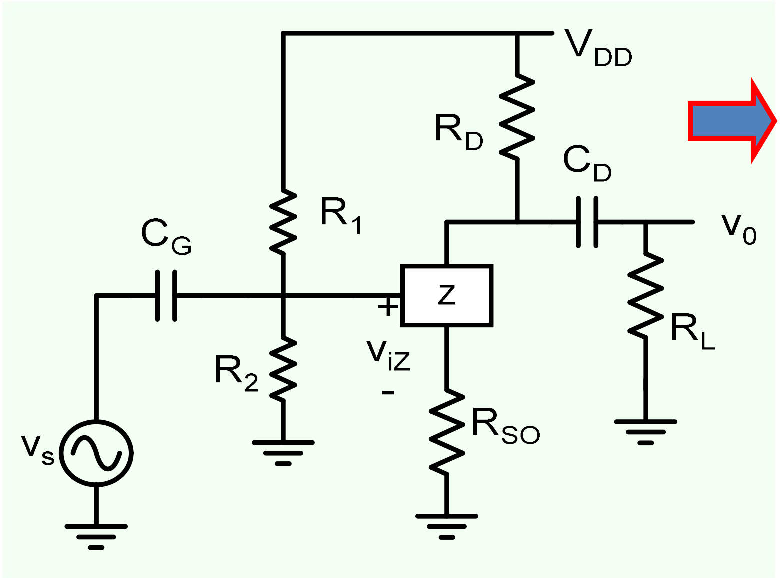
Since I_{oZ} vs. V_{iZ} characteristics is very sharp, $V_{iZ} \sim V_{\alpha} = 1V$

$$I_{OZ} \cong \frac{V_t - V_\alpha}{R_{SO}}$$

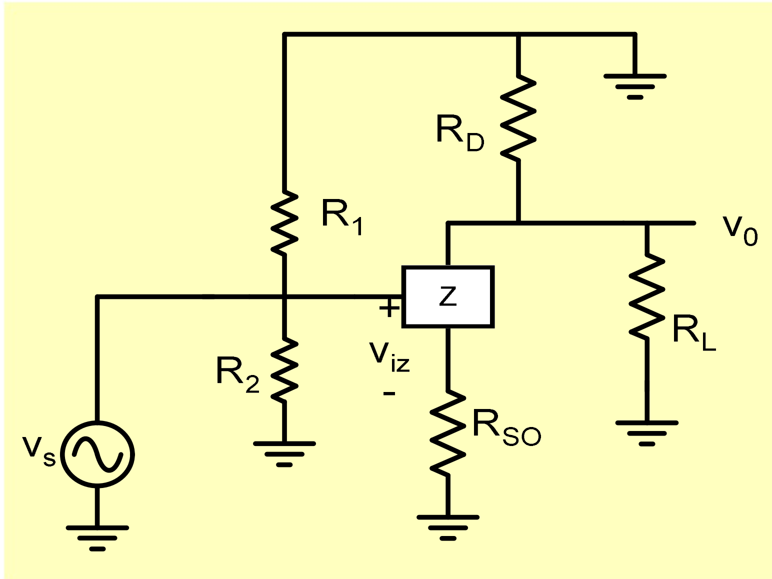
If V_t changes by 1% due to variation in resistor values then the change in output current is proportional.

Circuit is much less sensitive to variations in circuit parameters

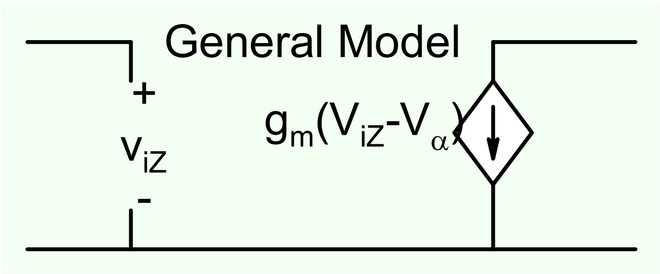
Ac analysis



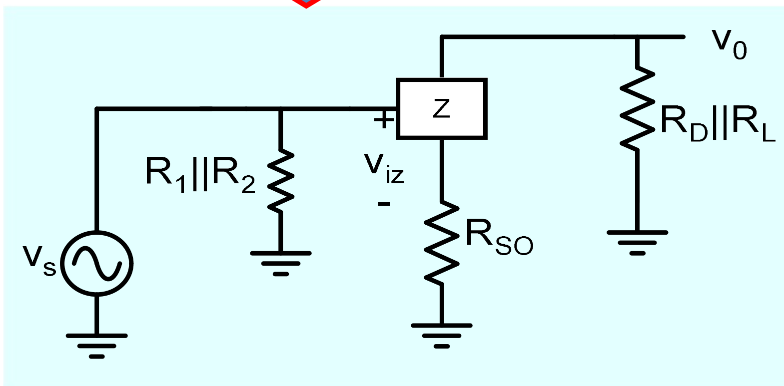
V_{DD}
R_D
C_D
R_1
C_G
v_0
Z
+
v_{iZ}
-
R_L
R_2
R_{SO}
v_s



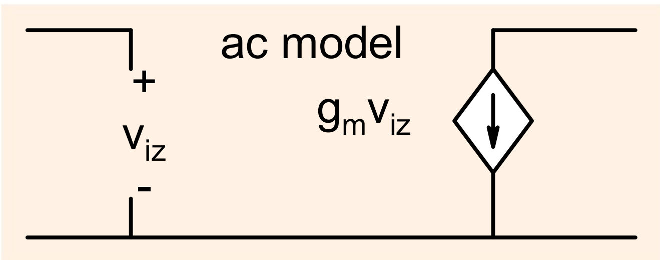
R_D
R_1
v_0
Z
+
v_{iz}
-
R_L
R_2
R_{SO}
v_s



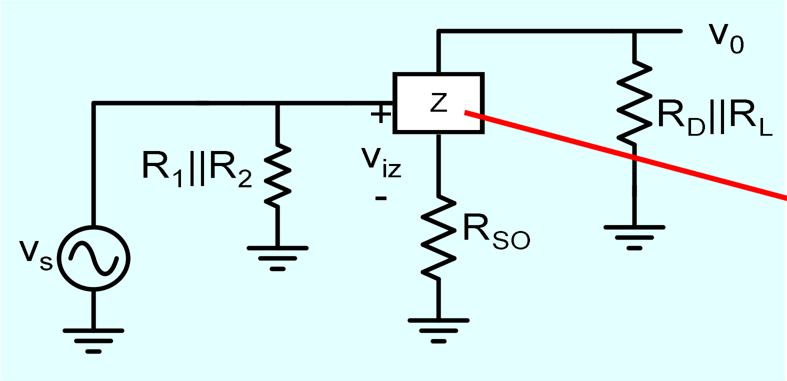
General Model
+
v_{iZ}
-
$g_m(V_{iZ}-V_\alpha)$

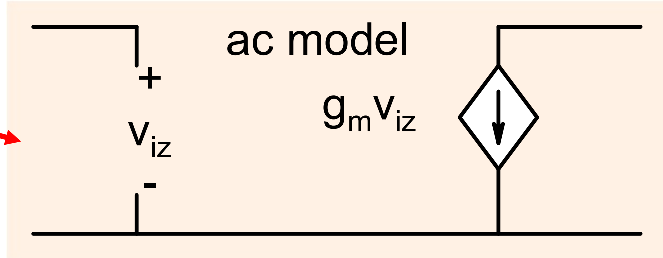


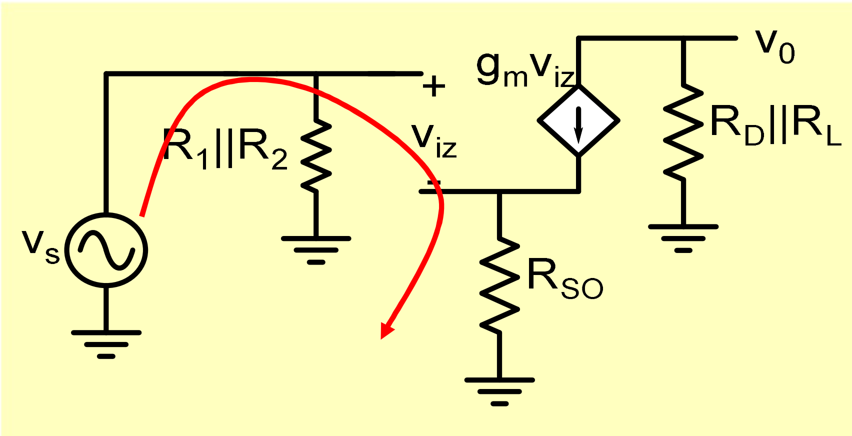
v_0
Z
+
v_{iz}
-
$R_D||R_L$
$R_1||R_2$
R_{SO}
v_s



ac model
+
v_{iz}
-
$g_m v_{iz}$



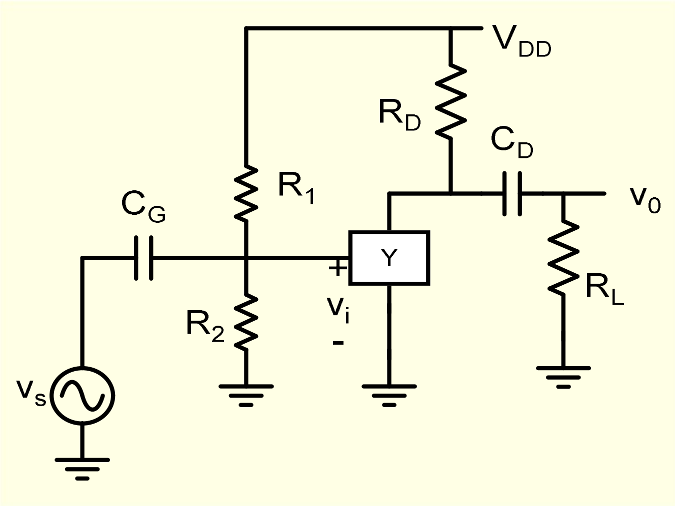




$$v_S = v_{iz} + g_m v_{iz} R_{SO}$$

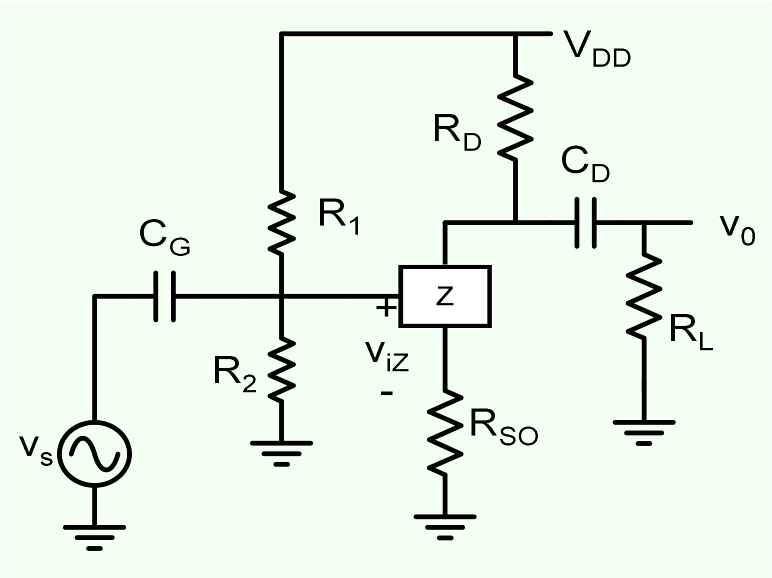
$$v_O = -g_m \times R_D \| R_L$$

$$A_V = \frac{v_o}{v_s} = -\frac{g_m R_D \| R_L}{1 + g_m R_{SO}}$$



Circuit is very sensitive to variations in resistor values, power supply, device parameters such as Vα

$$A_V = \frac{v_o}{v_s} = -g_m R_D \| R_L$$

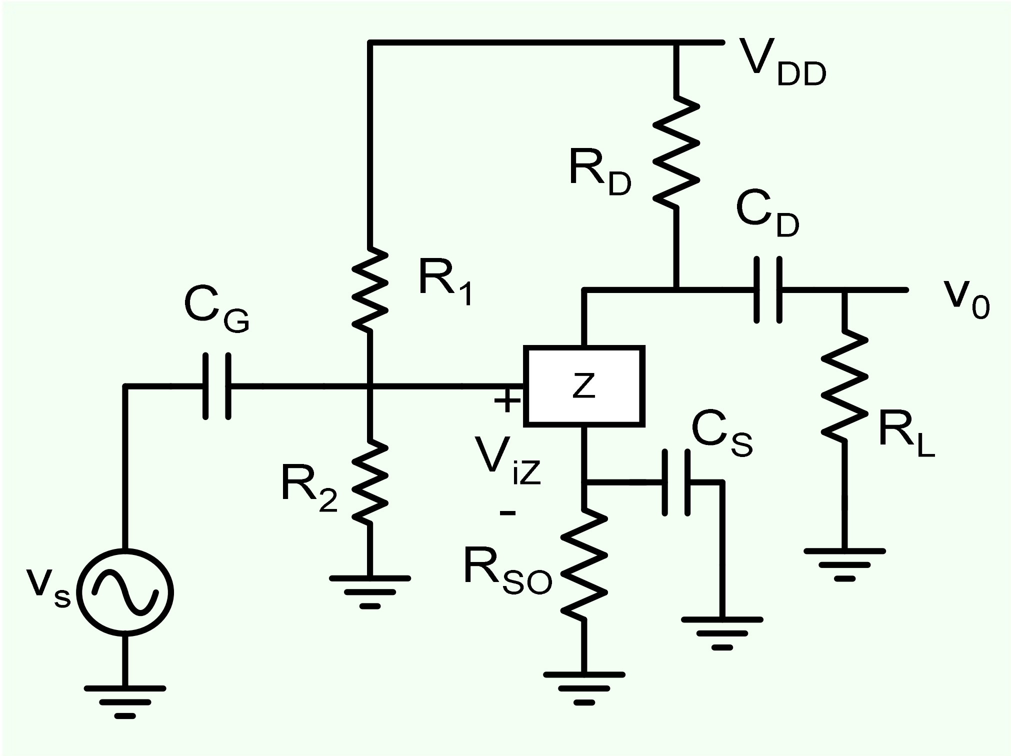


Circuit is much less sensitive to variations in circuit parameters

$$A_V = \frac{v_o}{v_s} = -\frac{g_m R_D \| R_L}{1 + g_m R_{SO}}$$

But gain is smaller

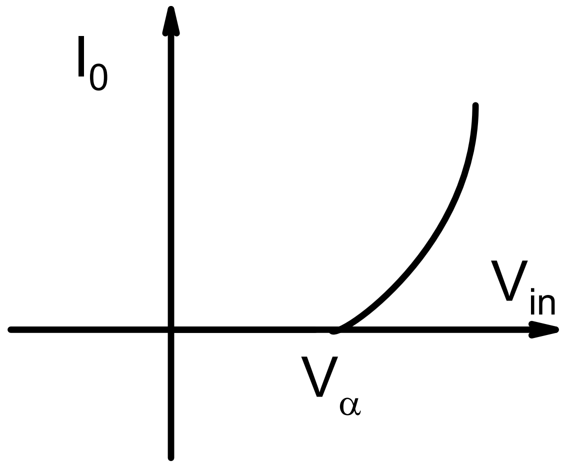
Simple Solution

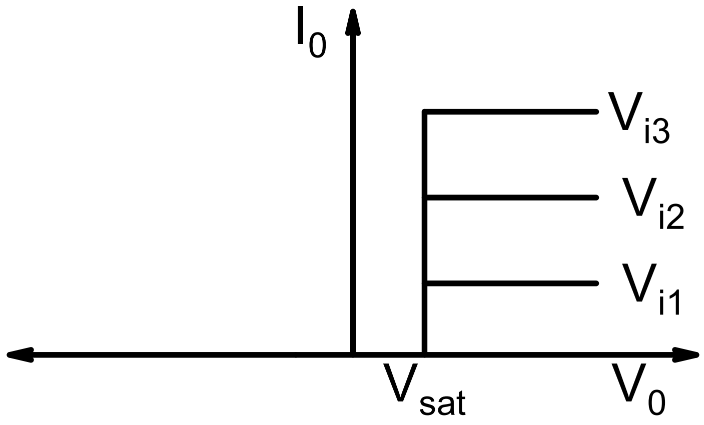


For dc, Capacitor C_S acts as open allowing R_{SO} to reduce variations in current

For ac, Capacitor C_S acts as a short circuit ($1/j\omega C \sim 0$) allowing high voltage gain to be obtained

Device NL:



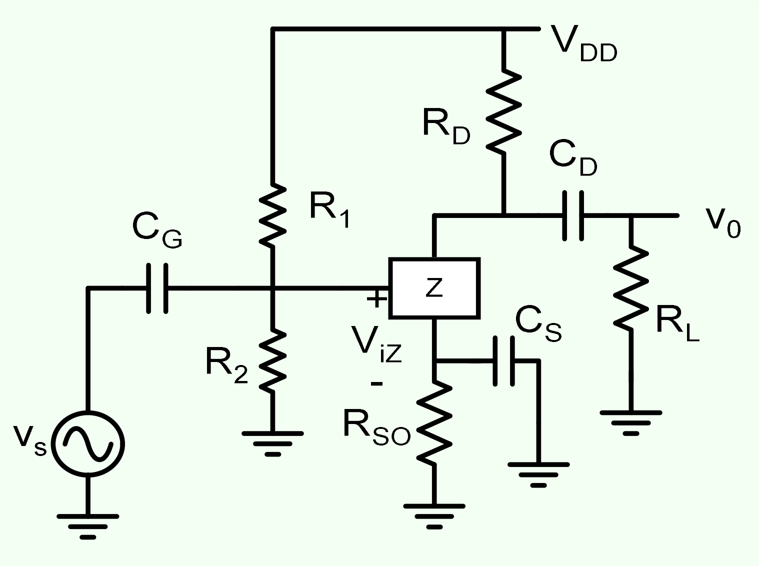


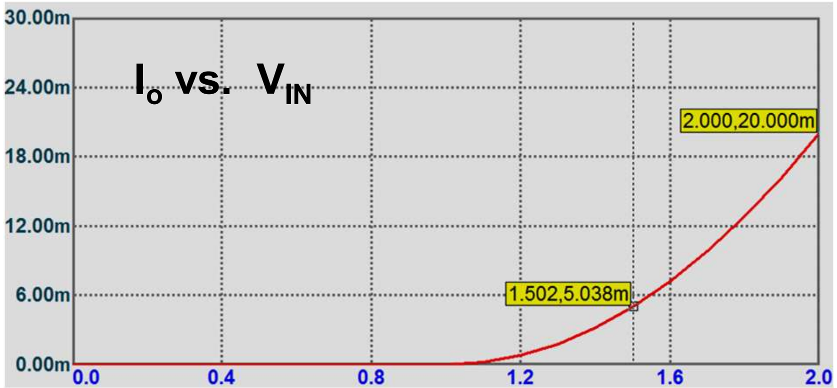
$$I_o = K \times (V_{in} - V_\alpha)^2 \text{ for } V_{in} \geq V_\alpha$$

$$V_\alpha = 1.0V \; ; \; K = 0.02$$

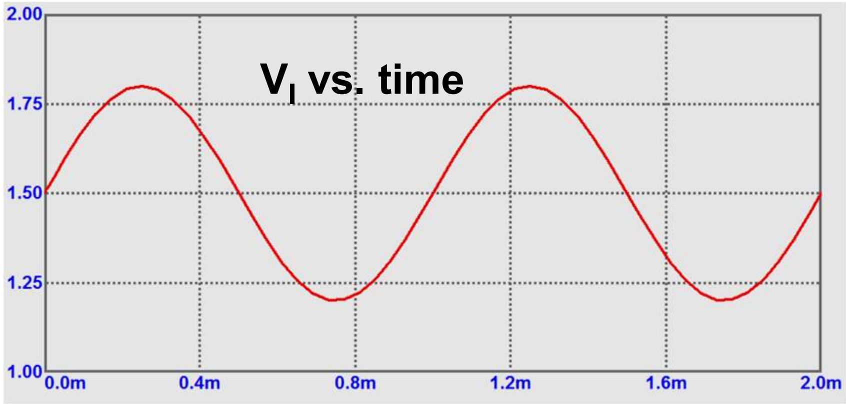
$$V_B = 1.5V$$

$$v_s = 0.3V Sin(\omega t)$$

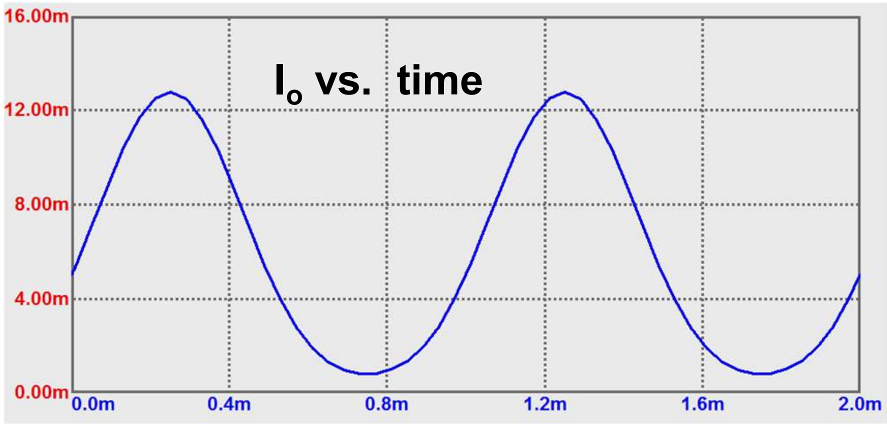




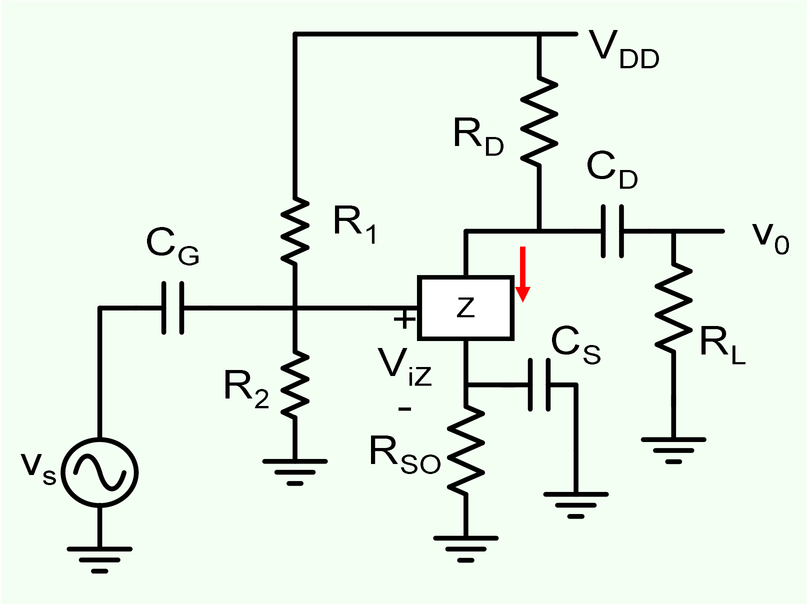
I_o vs. V_{IN}
30.00m
24.00m
18.00m
12.00m
6.00m
0.00m
0.0
0.4
0.8
1.2
1.6
2.0
2.000,20.000m
1.502,5.038m



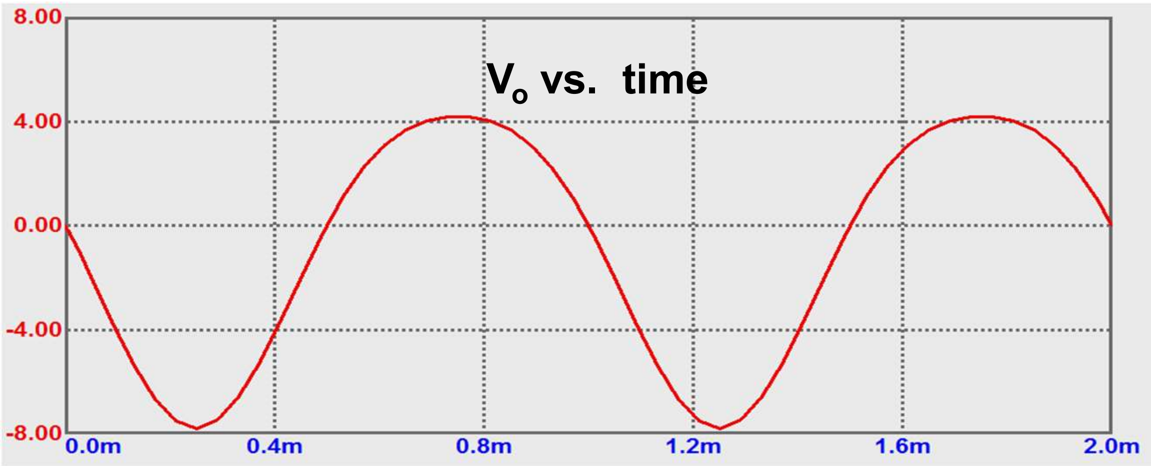
V_I vs. time
2.00
1.75
1.50
1.25
1.00
0.0m
0.4m
0.8m
1.2m
1.6m
2.0m

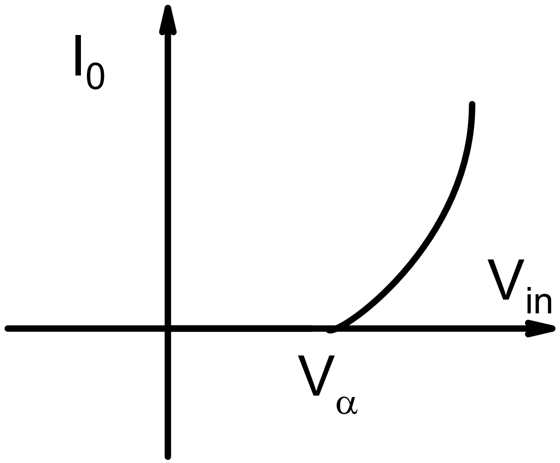


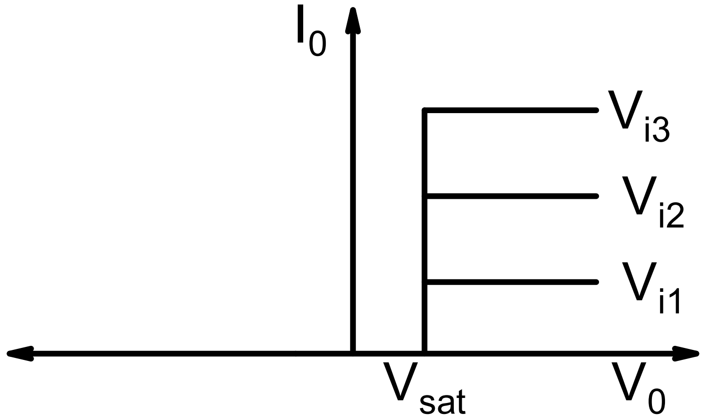
I_o vs. time
16.00m
12.00m
8.00m
4.00m
0.00m
0.0m
0.4m
0.8m
1.2m
1.6m
2.0m



V_{DD}
R_D
C_D
R_1
C_G
v_0
Z
+
V_{iZ}
-
C_S
R_L
R_2
R_{SO}
v_s

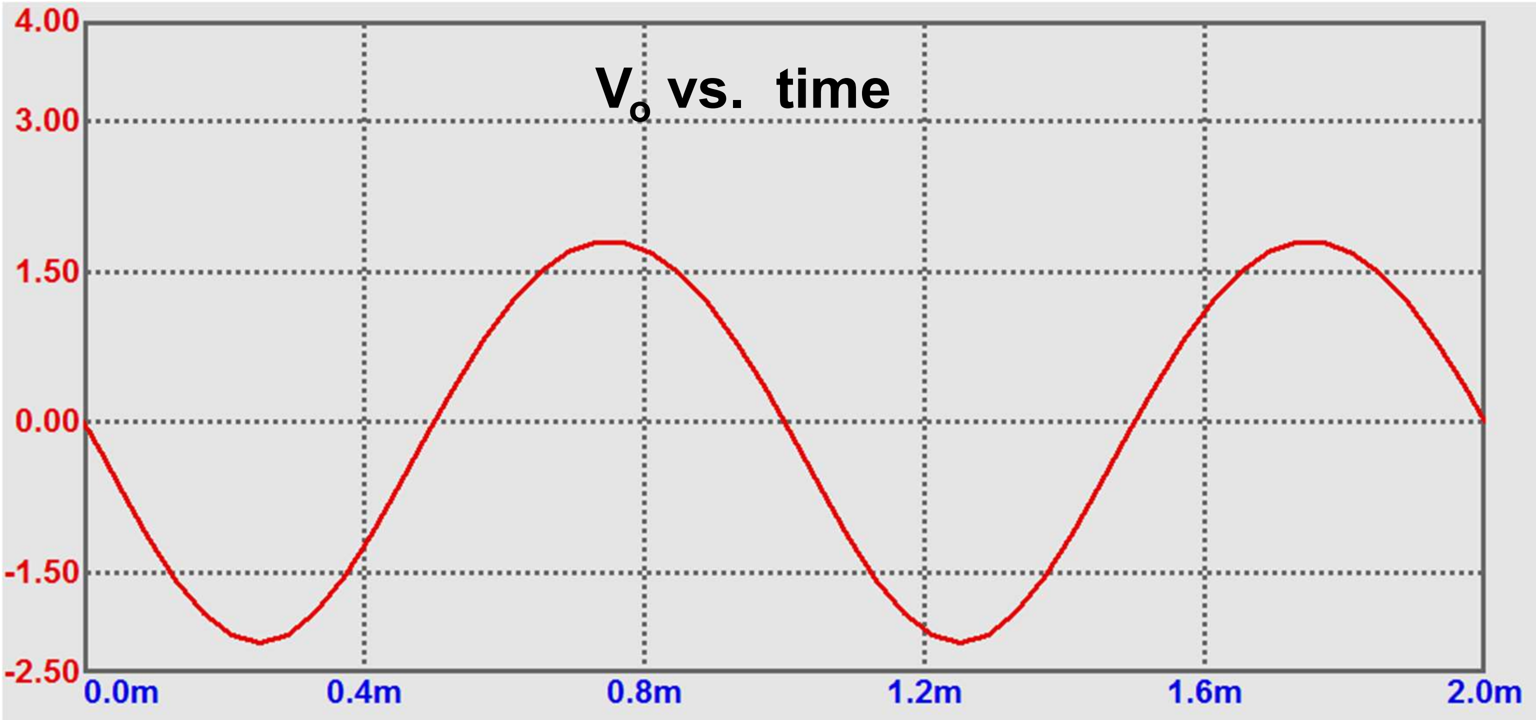






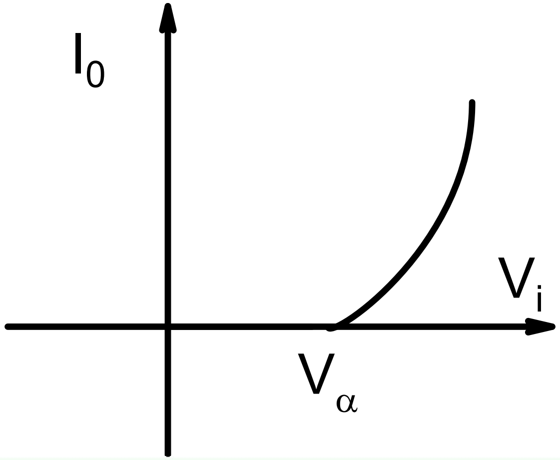
Because of Non-linearity the output waveform is distorted !

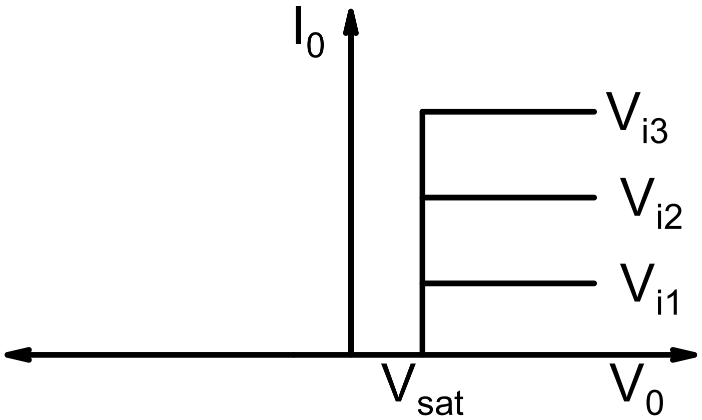
Suppose input is reduced to $v_s = 0.1V\ Sin(\omega t)$

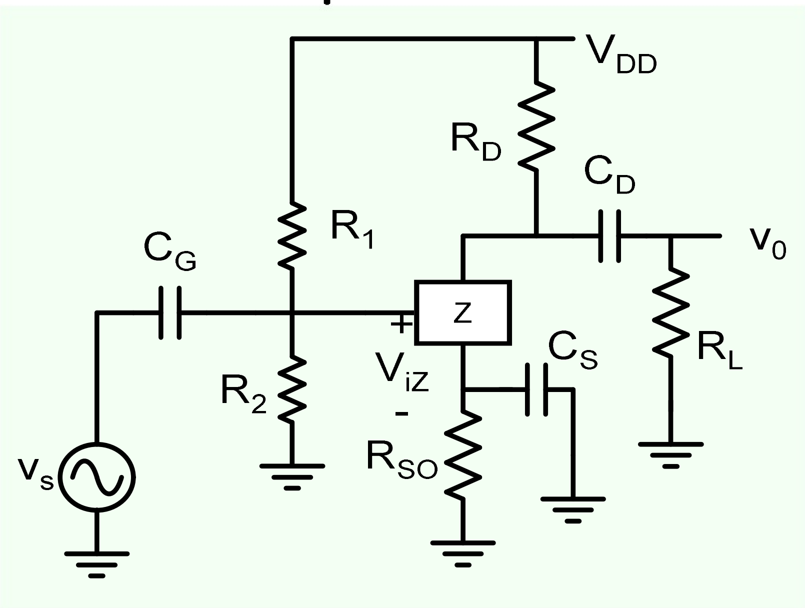


Distortion is much smaller if we restrict input voltage to a small value !

Building Amplifiers with non-linear devices







Amplifier will work properly (with small distortion only if we restrict the amplitude of input signal to small values.

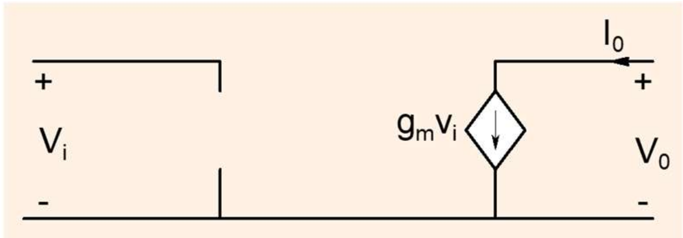
How small depends on the nature of non-linearity. The stronger the non-linearity the lesser the signal amplitude.

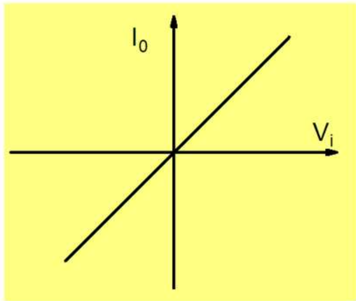
ESC201T: Introduction to Electronics

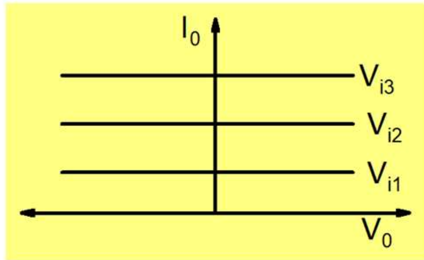
Lecture 27: Transistors

B. Mazhari
Dept. of EE, IIT Kanpur

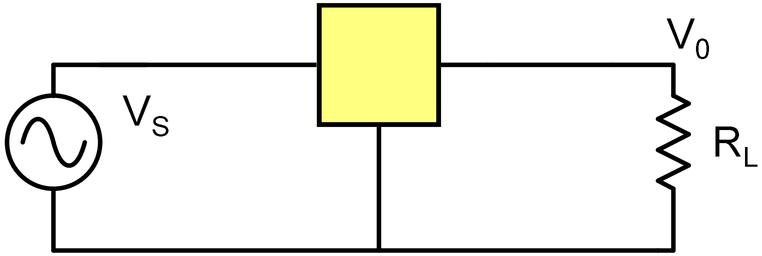
Transistor

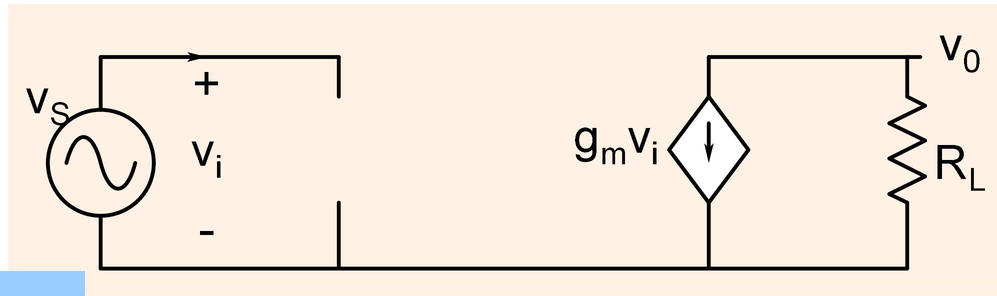






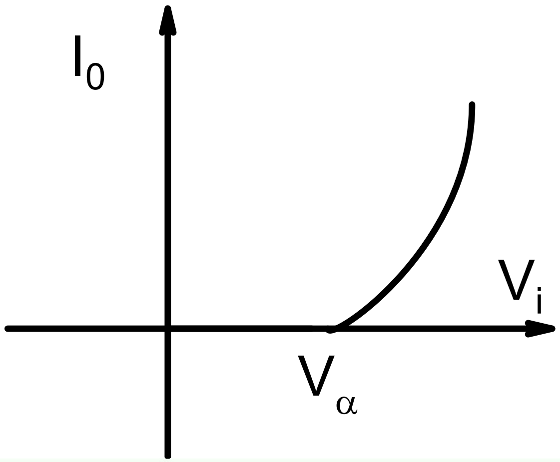
Current I_O is much more sensitive to V_{IN} than V_O

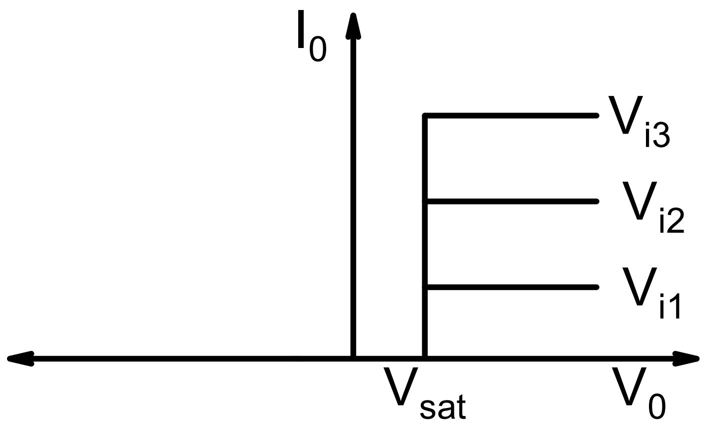


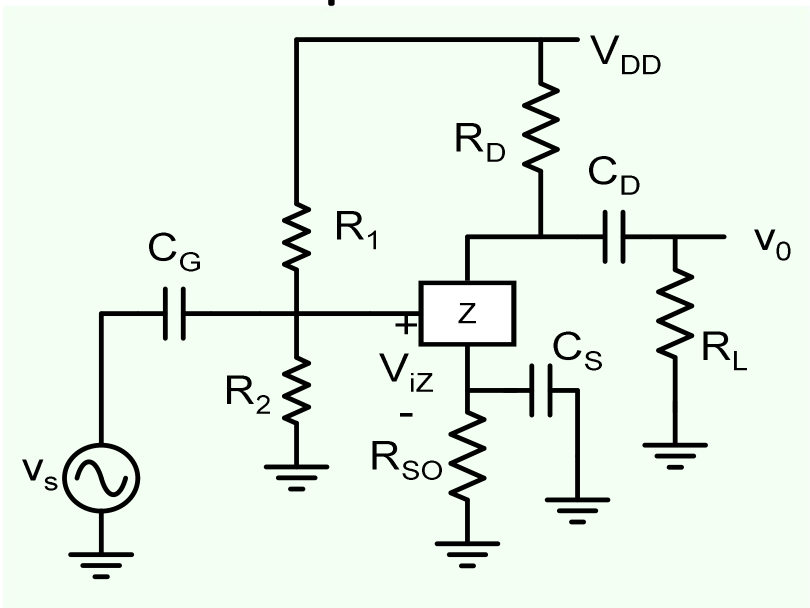


$$A_V = \frac{V_o}{V_s} = -g_m \times R_L$$

Building Amplifiers with non-linear devices



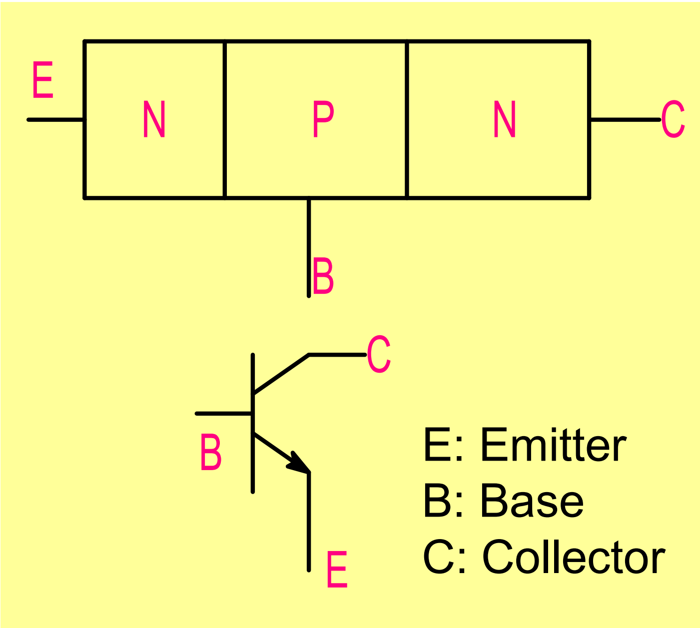




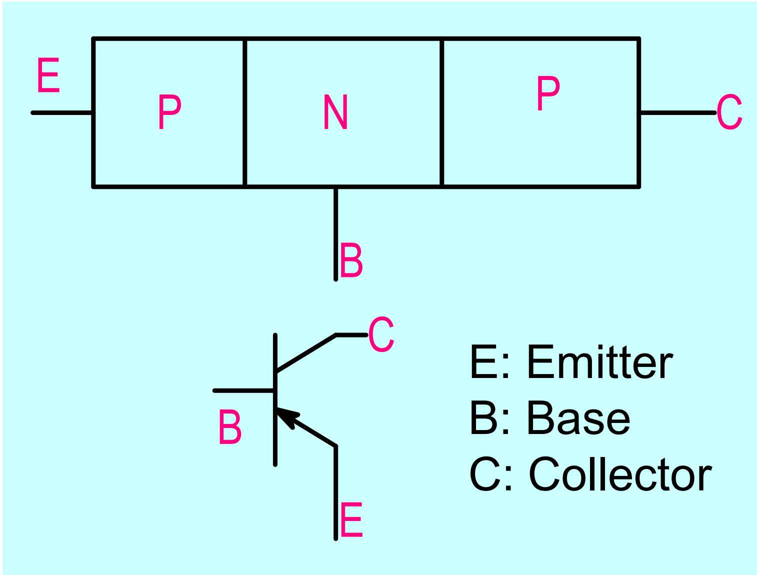
Amplifier will work properly (with small distortion only if we restrict the amplitude of input signal to small values.

How small depends on the nature of non-linearity. The stronger the non-linearity the lesser the signal amplitude.

Bipolar Junction Transistor (BJT)

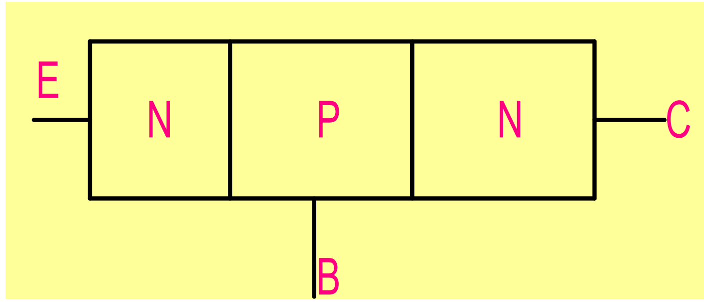


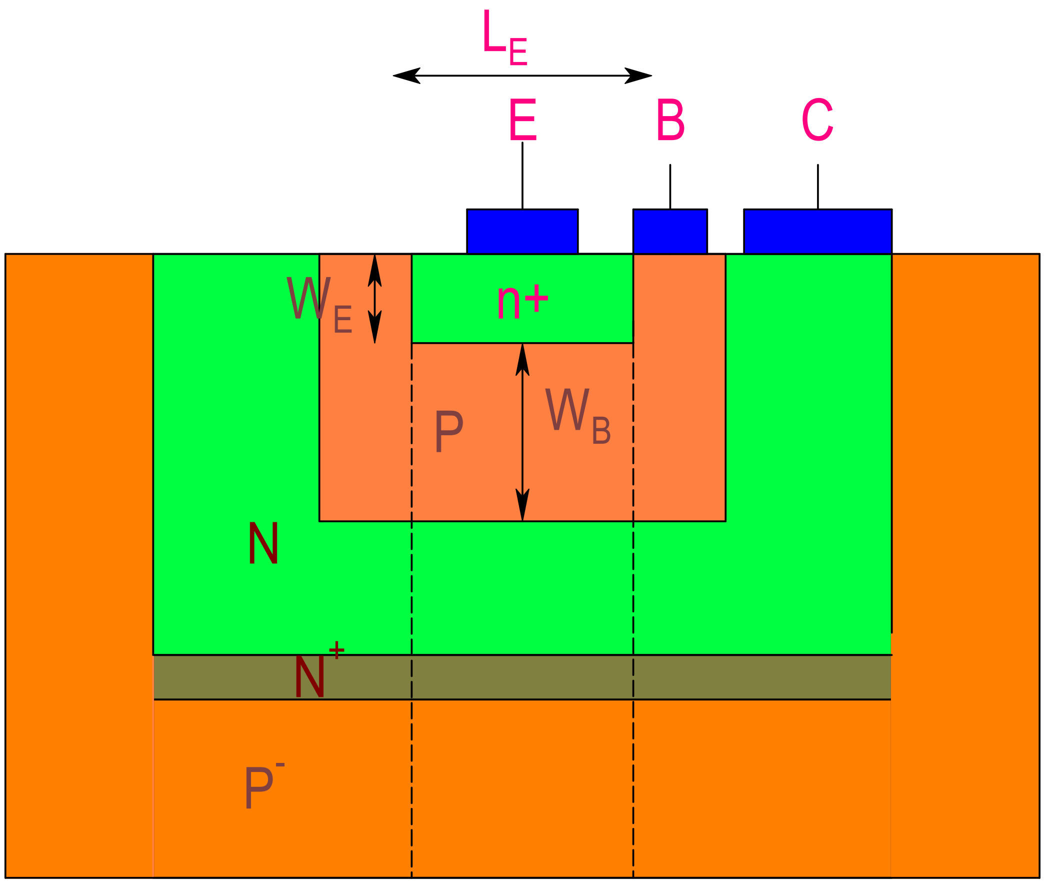
NPN



PNP

More Realistic View





$N_{DE} \sim 10^{19}\, cm^{-3}$

$N_{AB} \sim 10^{17}\, cm^{-3}$

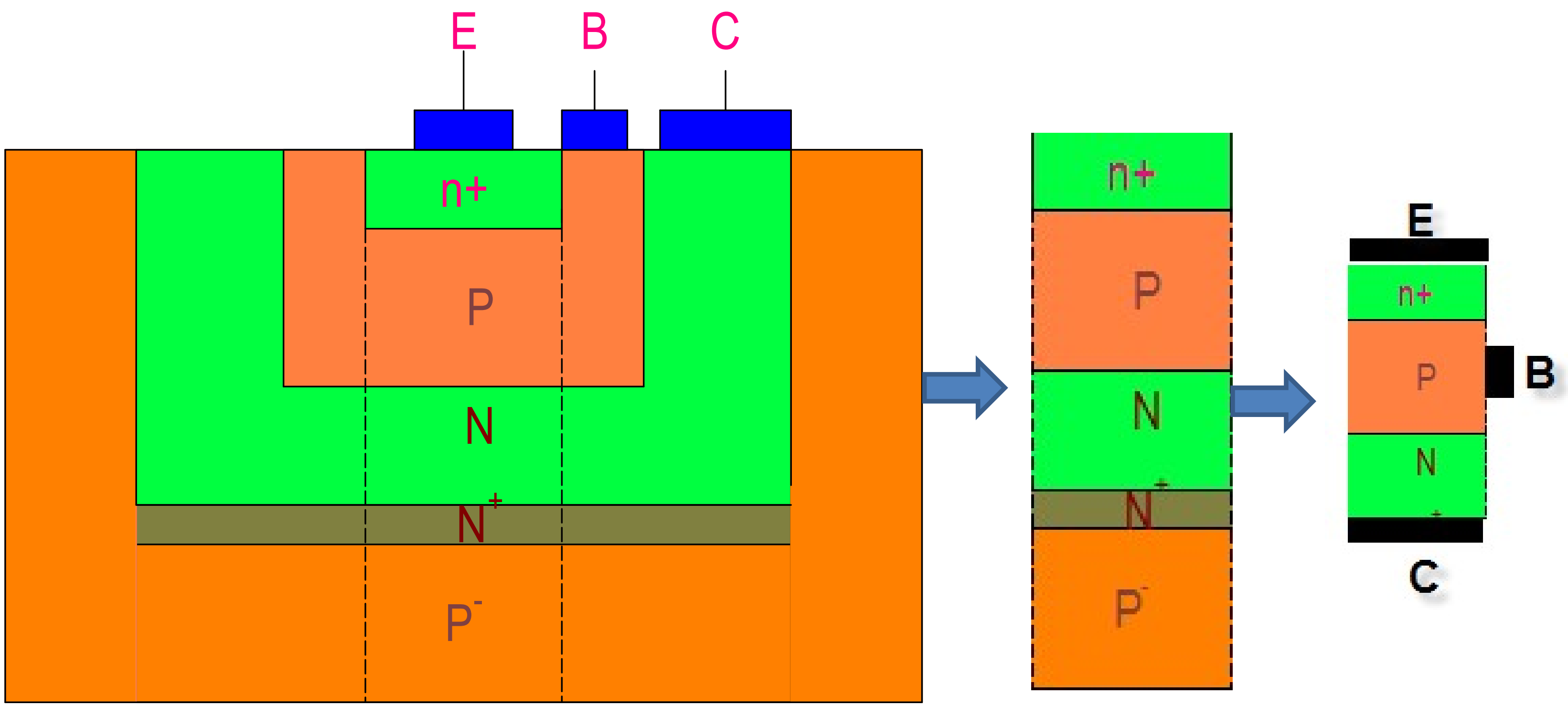
$N_{DC} \sim 10^{16}\, cm^{-3}$

$W_B \sim 2000\, A^{\circ}$

$W_E \sim 1000\, A^{\circ}$

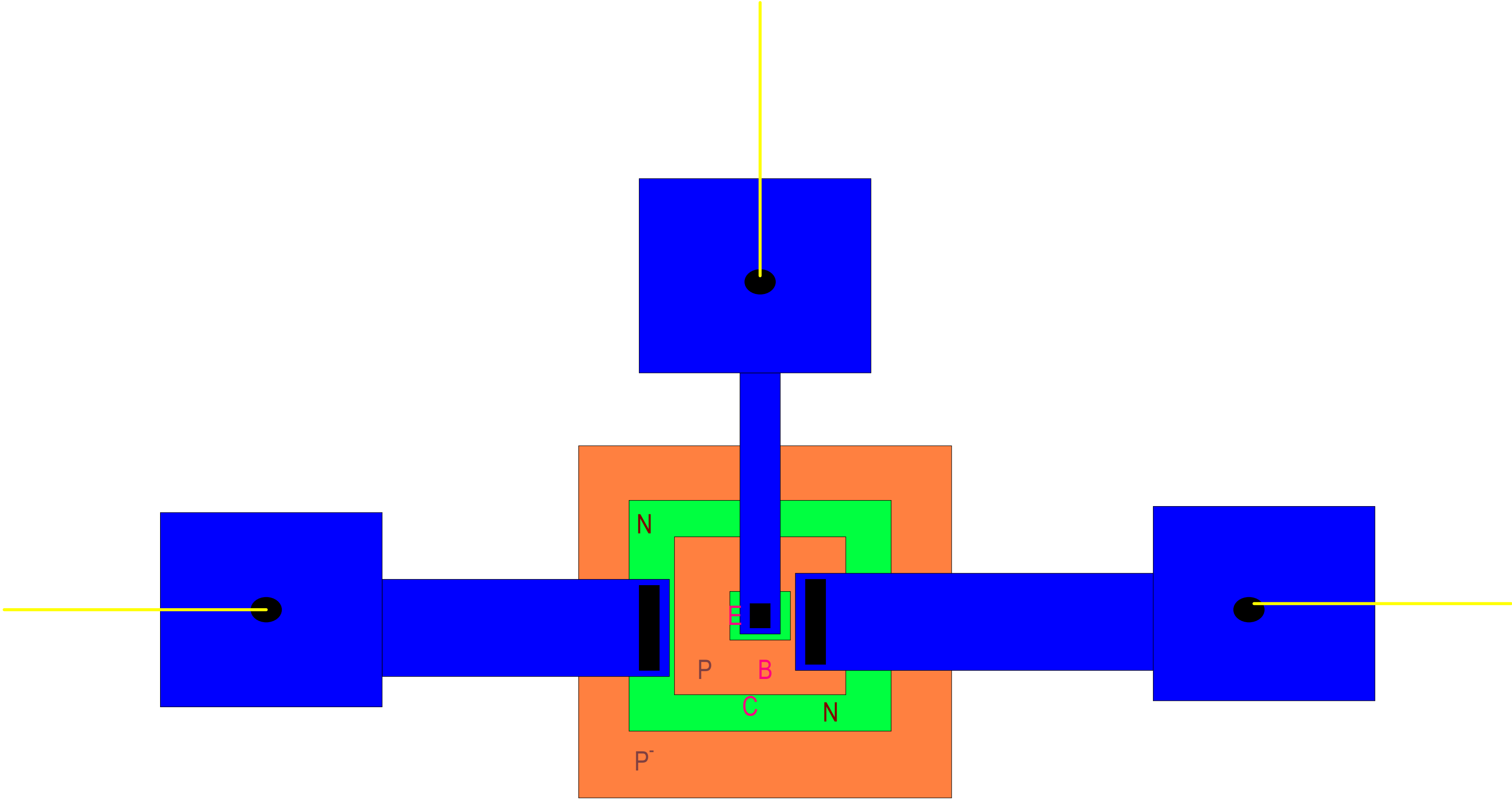
$L_E \sim 1 \mu m$

BJT is not symmetric: emitter and collector cannot be simply interchanged

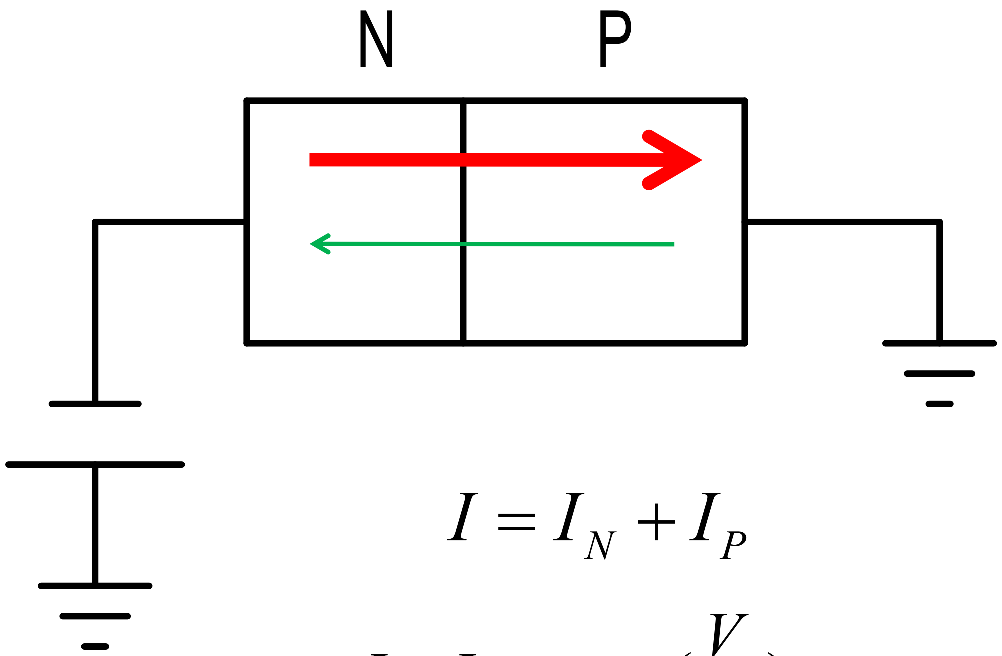


E
B
C
n+
P
N
N^+
P^-
n+
P
N
N^+
P^-
E
n+
P
B
N
C

Top View



Background

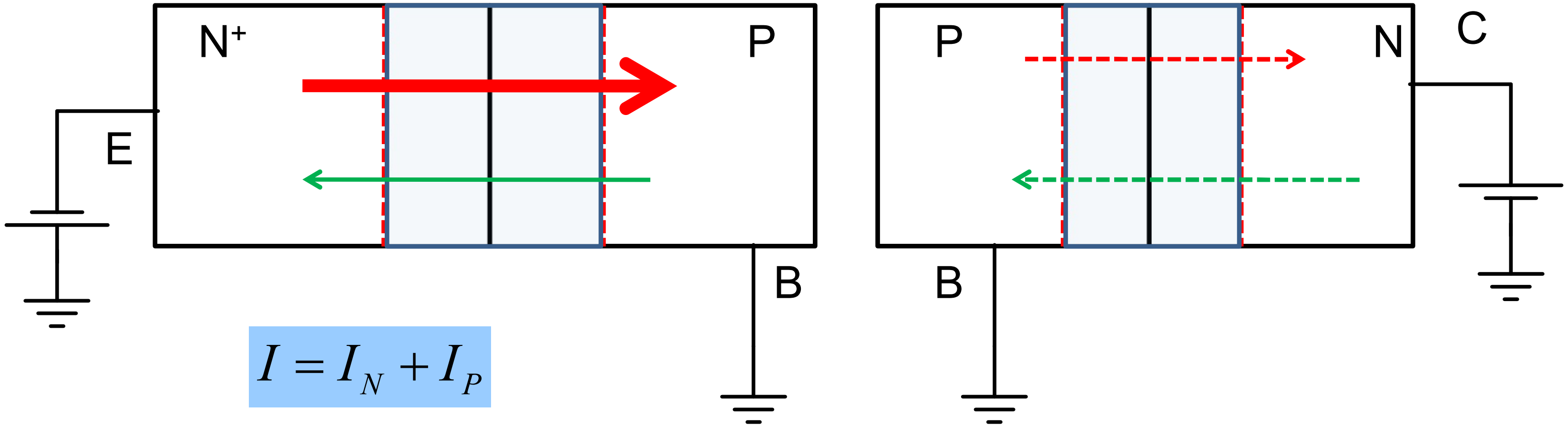


$$I = I_N + I_P$$

$$I_N ; I_P \propto \exp(\frac{V}{V_T})$$

If doping in N region is much larger than doping in p region then $I_N >> I_P$

Basic Transistor Operation



$$I = I_N + I_P$$

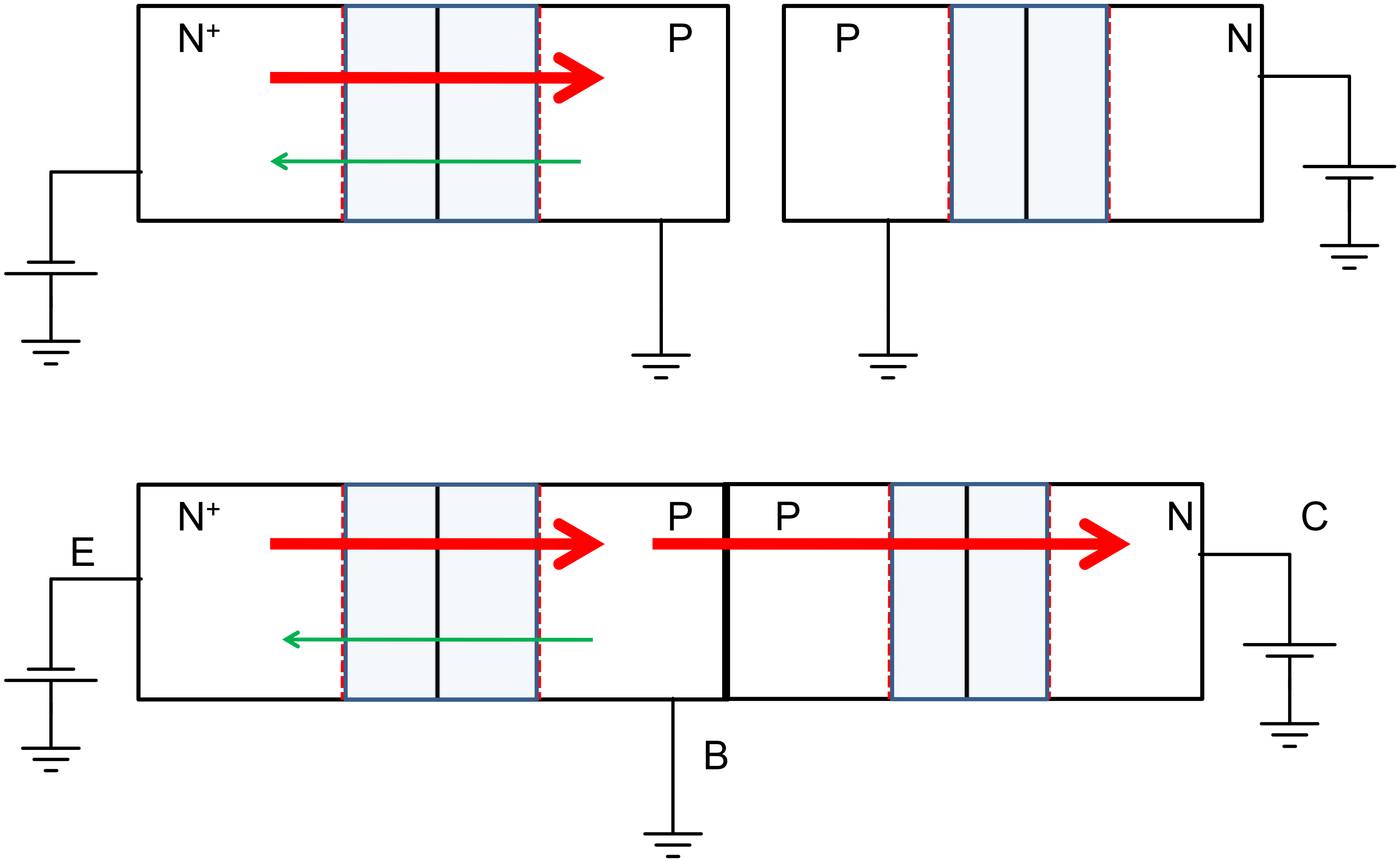
We will assume that doping in emitter is much more than base so that electron current is much larger than hole current

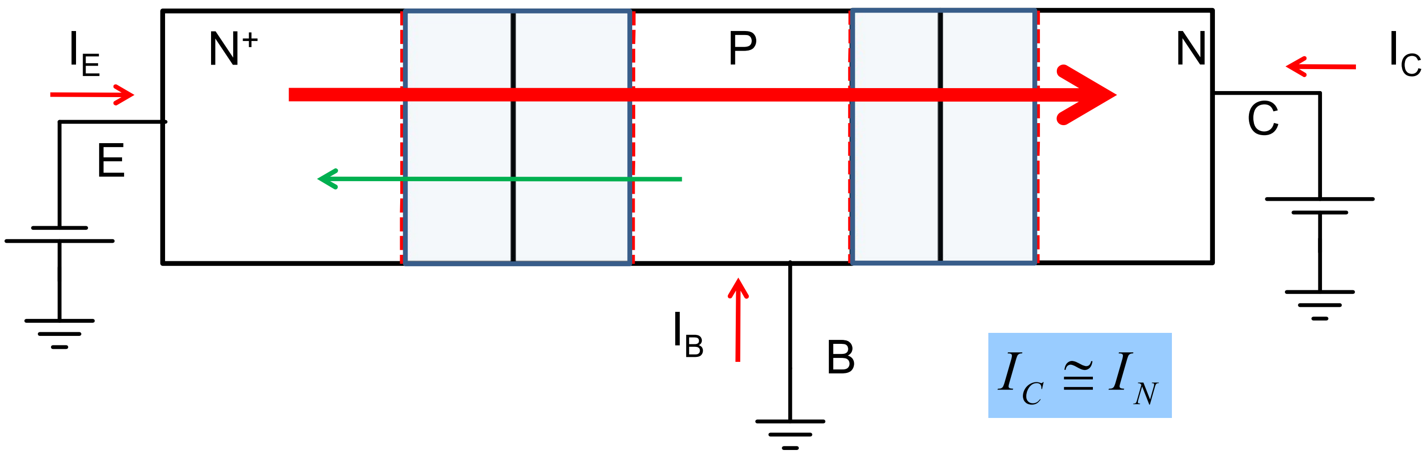
$$I_N >> I_P$$

In the reverse biased junction current is small because there are very few electrons in P and holes in N-region

Basic Transistor Operation

$$I = I_N + I_P$$



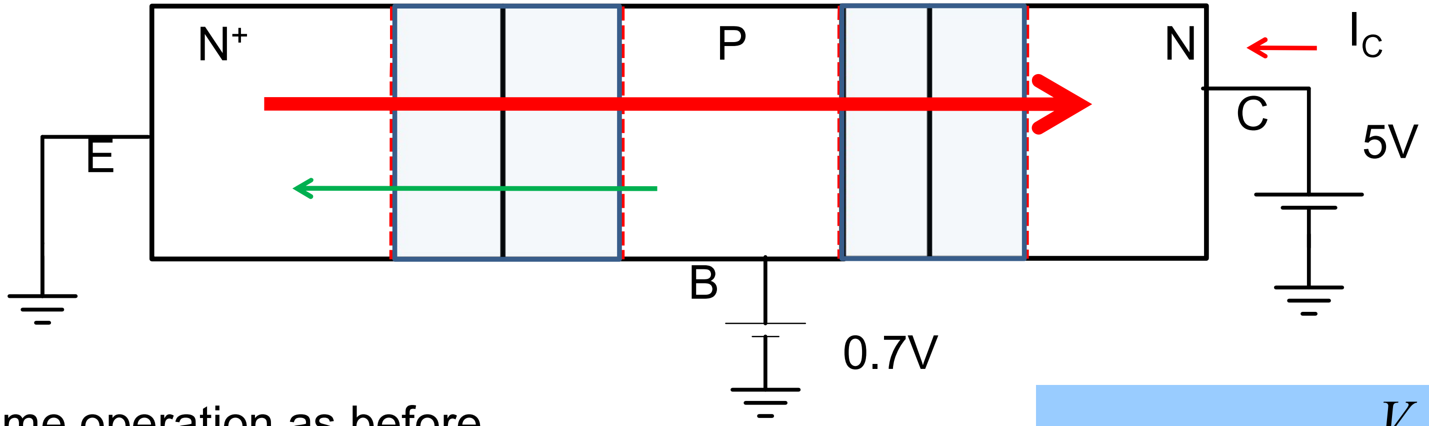


$$I_C \cong I_N$$

$$I_E = I_N + I_P$$

$$I_B \cong I_P$$

$$Current\ Gain\ :\ \beta = \frac{I_C}{I_B} = \frac{I_N}{I_P} >> 1$$

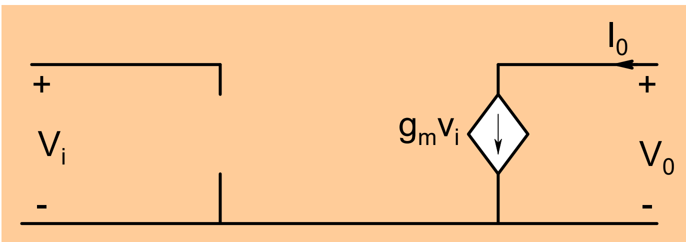


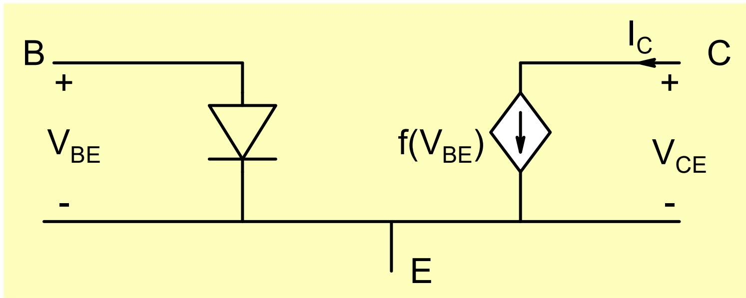
Same operation as before

$$I_C = I_N \propto \exp\left(\frac{V_{BE}}{V_T}\right)$$

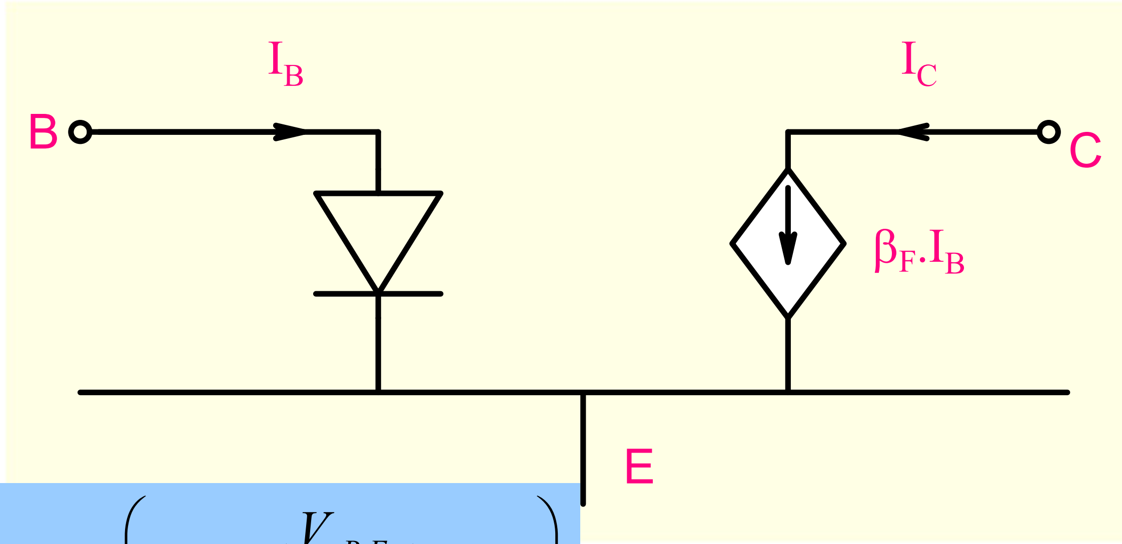
Transistor action

Current is affected by base-emitter voltage and not by collector-base voltage





Alternative representation



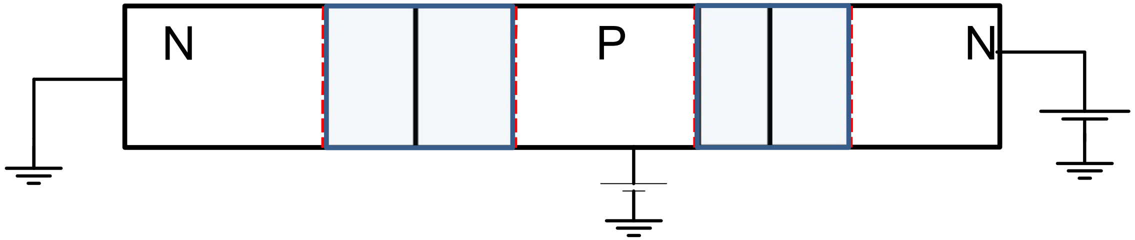
$$\beta = \frac{I_C}{I_B}$$

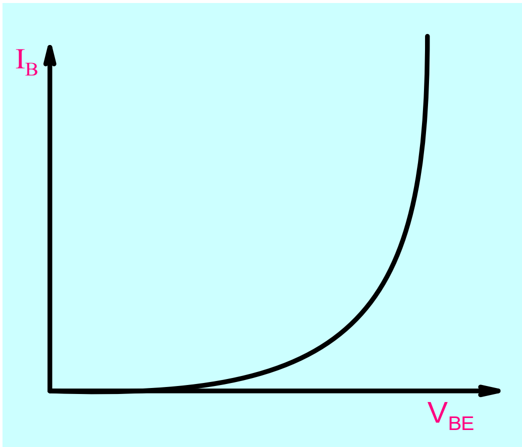
$$I_B = I_{BS}\left(\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)$$

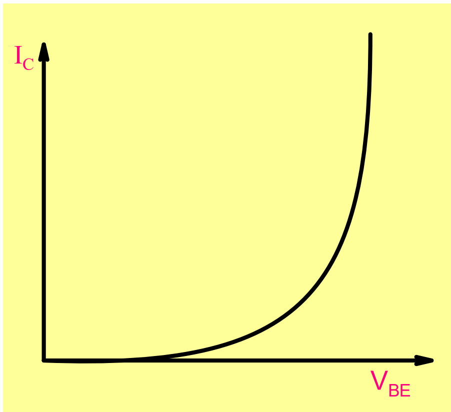
$$I_C = I_S\left(\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)$$

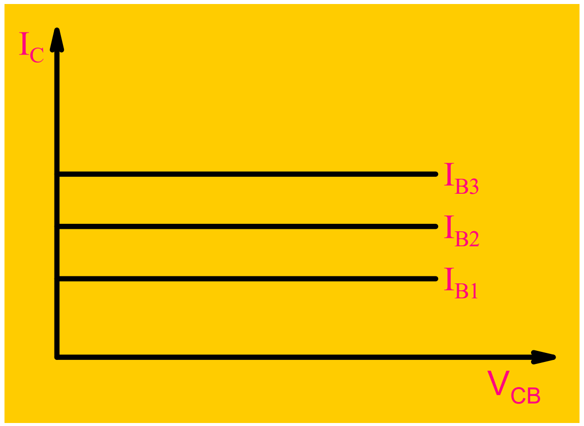
$$I_B = \frac{I_C}{\beta_F}$$

Transistor Characteristics

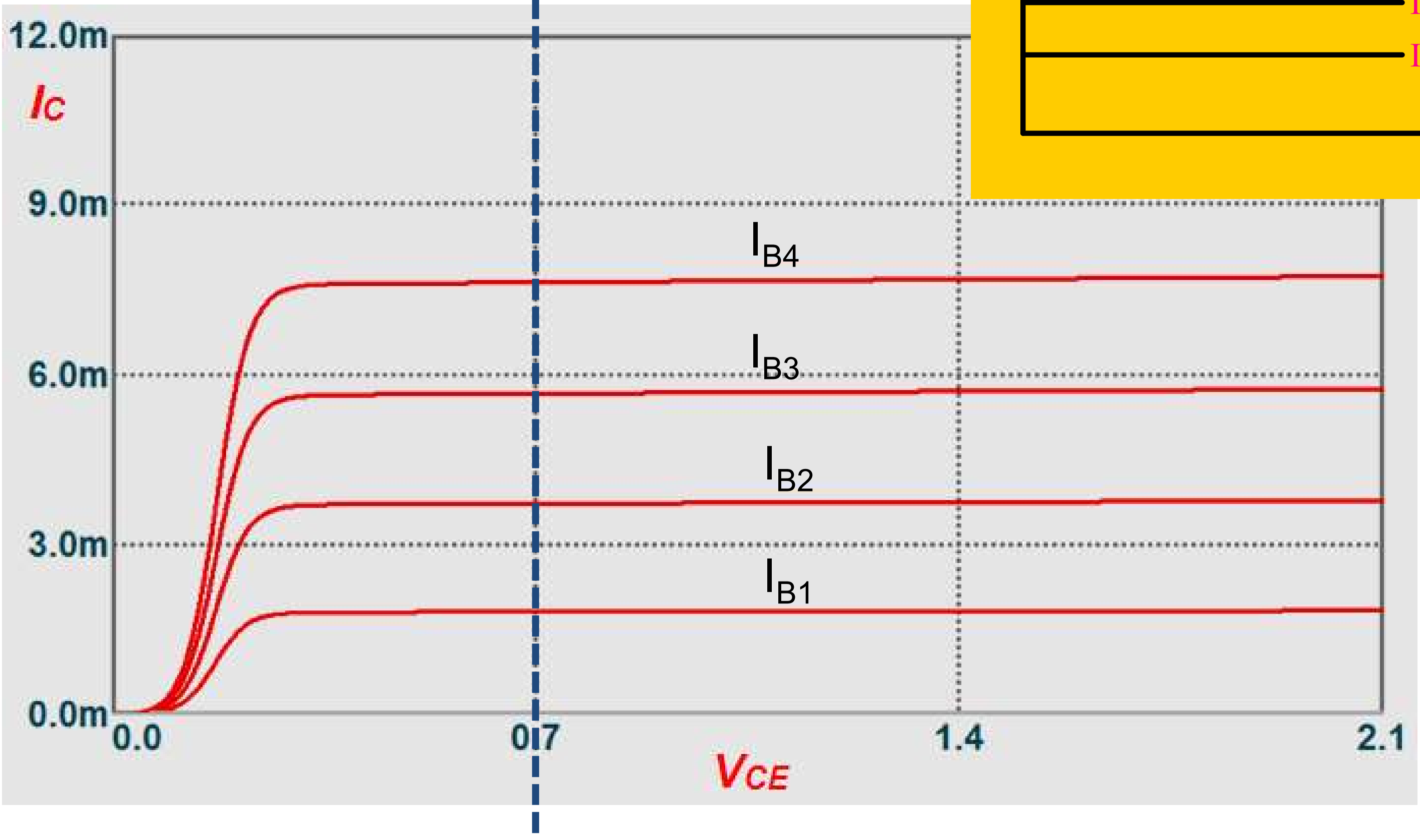


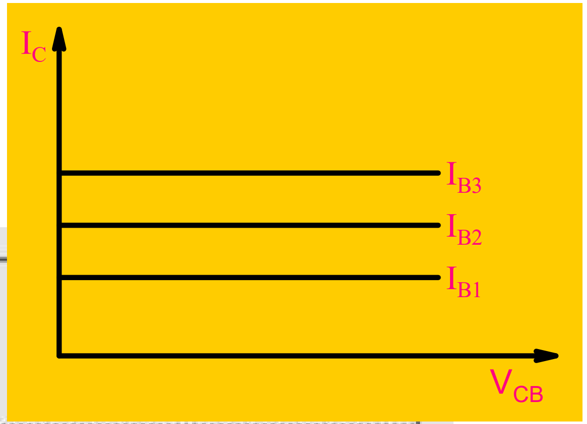






Output Characteristics of the transistor



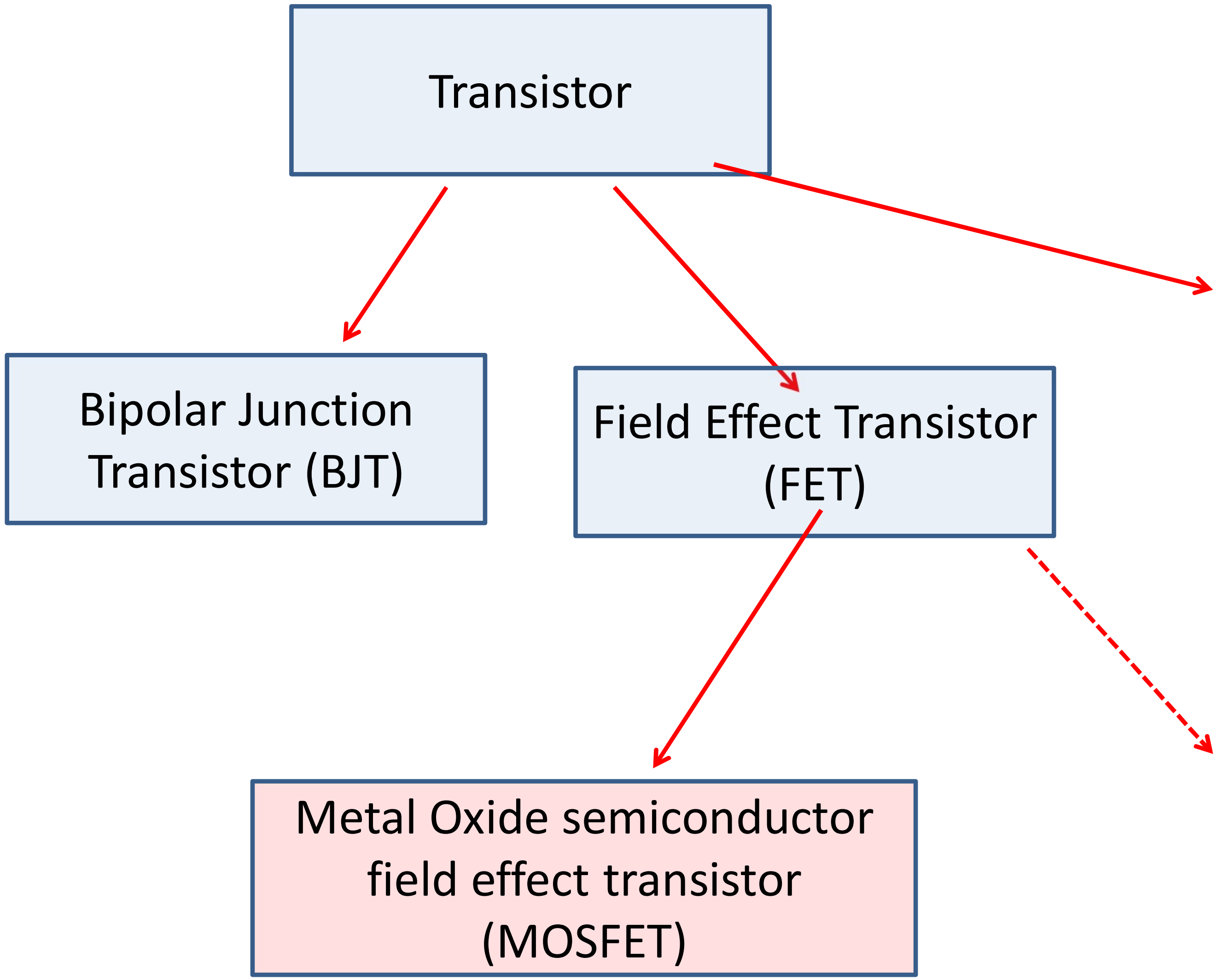


$V_{CE} = V_{CB} + V_{BE}$

$= V_{BE} - V_{BC}$

$$V_{CE} = 0.7 - V_{BC}$$

Transistors



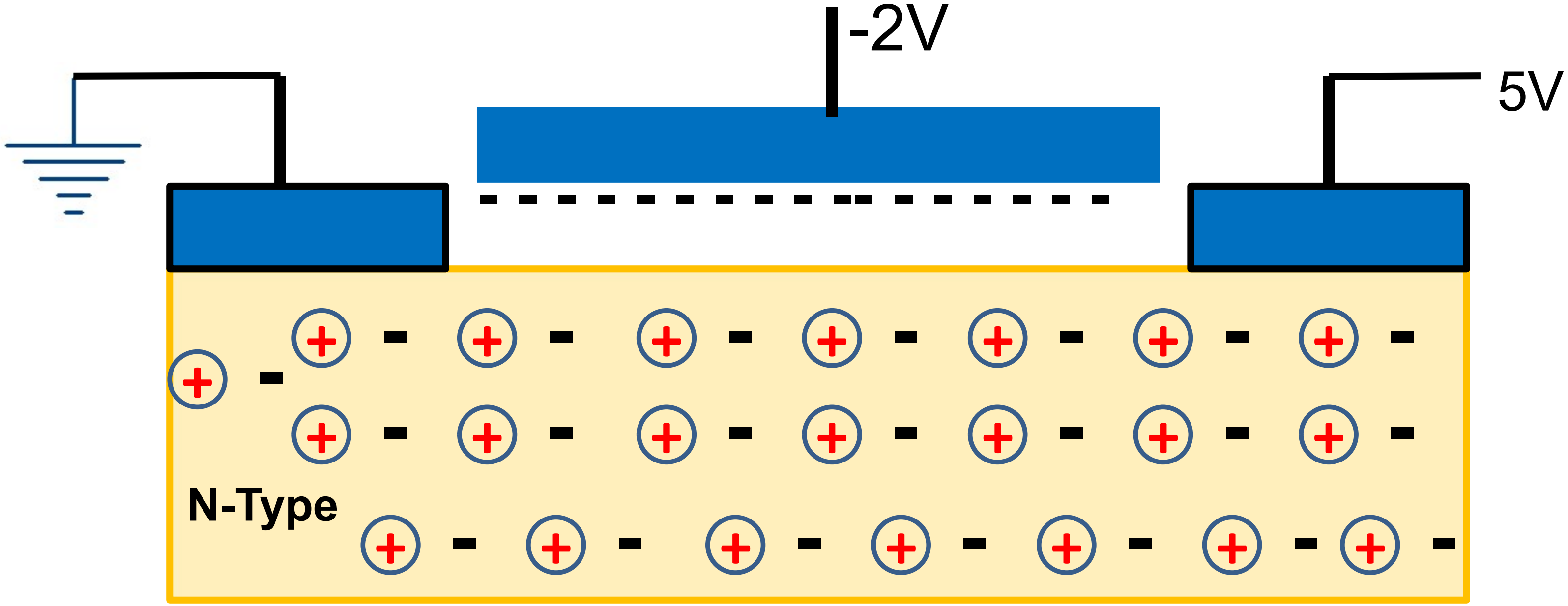
Transistor

Current I_O is much more sensitive to V_{IN} than V_O

$$\frac{\partial I_O}{\partial V_{in}} >> \frac{\partial I_O}{\partial V_o}$$

Field Effect Principle

$$\frac{\partial I_O}{\partial V_{in}} >> \frac{\partial I_O}{\partial V_o}$$



Modulation of conductivity using electric field

Transconductance

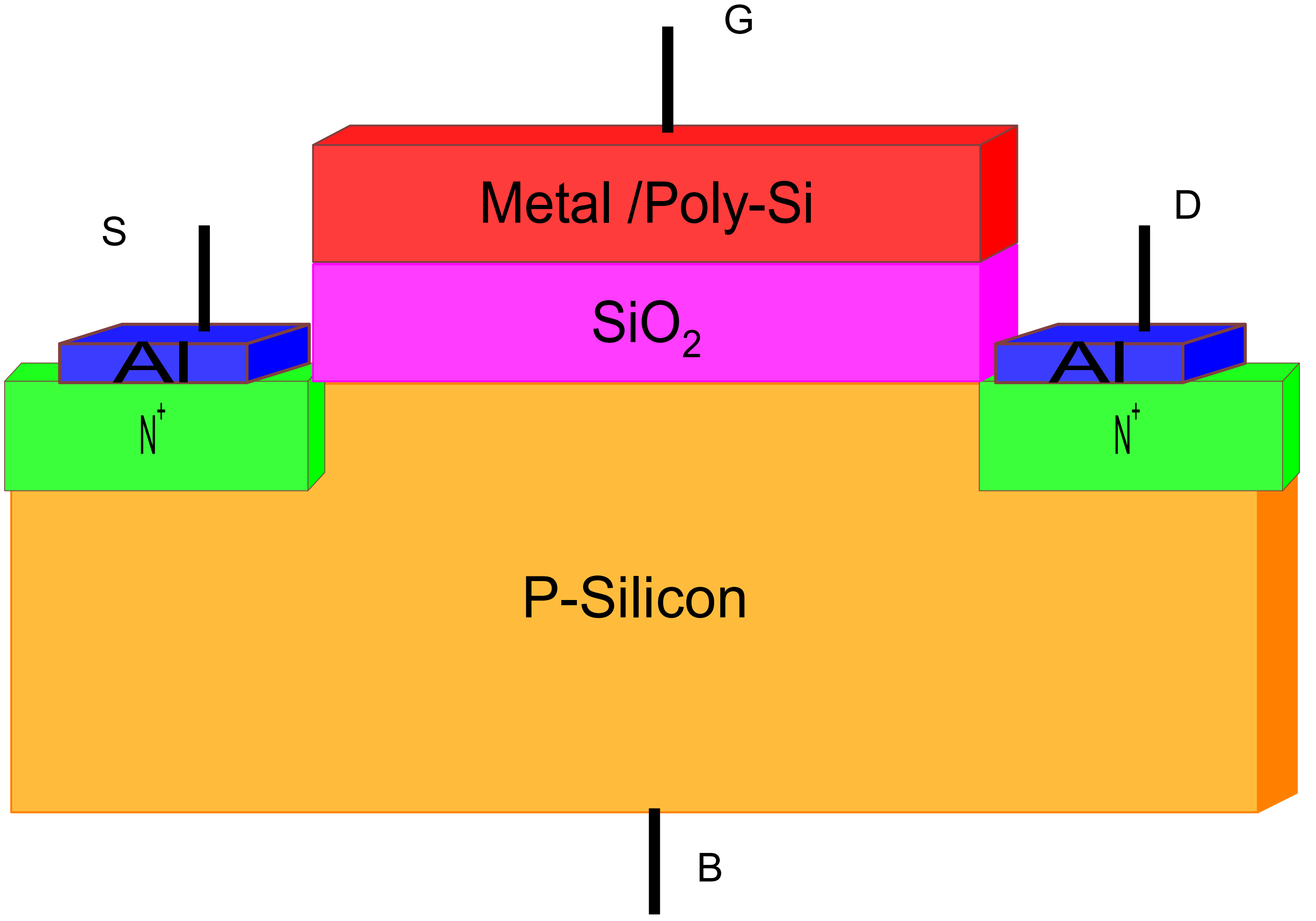
Jan. 28, 1930. J. E. LILIENFELD 1,745,175

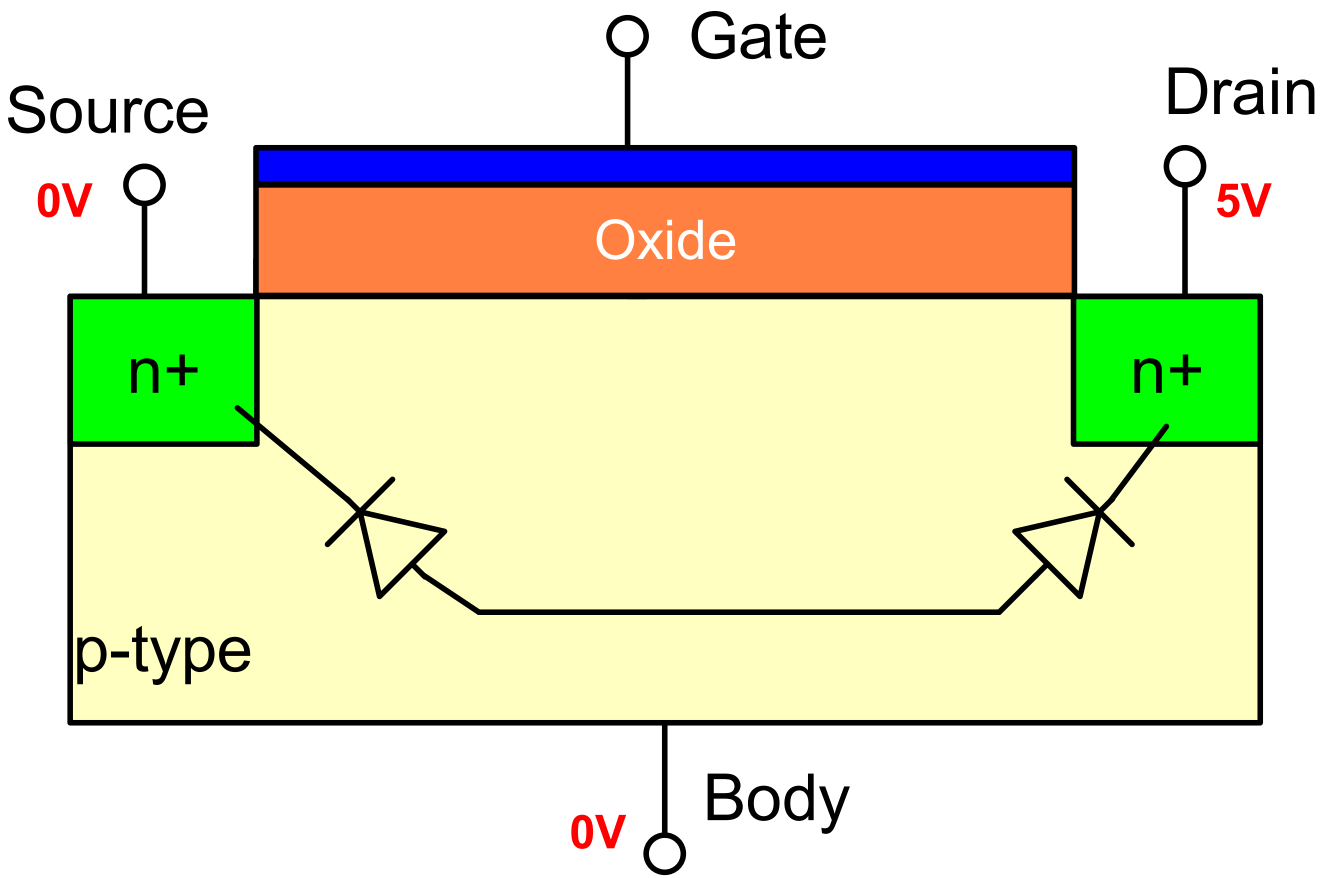
METHOD AND APPARATUS FOR CONTROLLING ELECTRIC CURRENTS

Filed Oct. 8, 1926

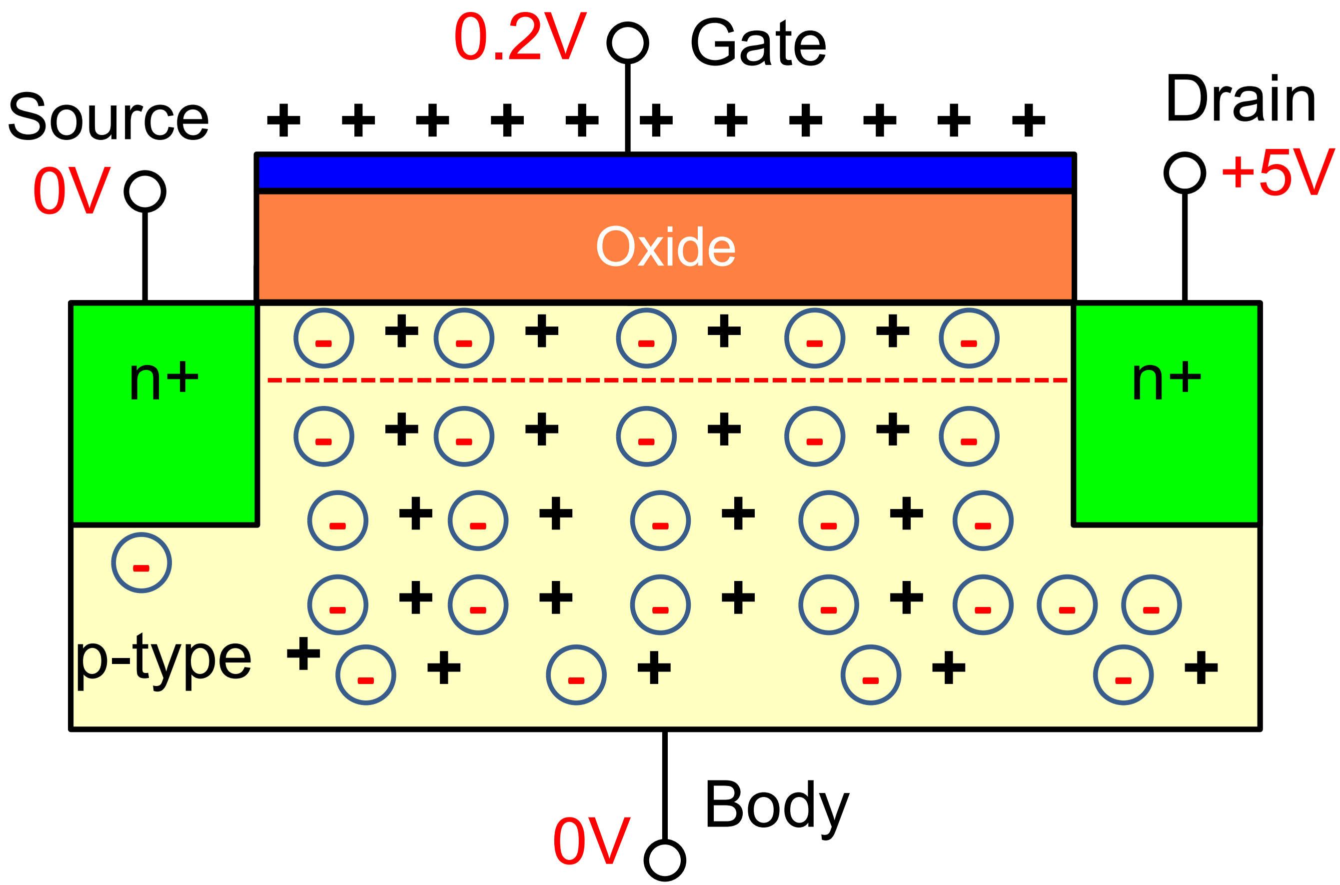
The invention relates to a method of and apparatus for controlling the flow of an electric current between two terminals of an electrically conducting solid by establishing a third potential between said terminals; and is particularly adaptable to the amplification of oscillating currents such as prevail, for example, in radio communication. Heretofore, thermionic tubes or valves have been generally employed for this purpose; and the present invention has for its object to dispense entirely with devices relying upon the transmission of electrons thru an evacuated space and especially to devices of this character wherein the electrons are given off from an incandescent filament. The invention has for a further object a simple, substantial and inexpensive relay or amplifier not involving the use of excessive voltages, and

NMOS Enhancement mode transistor: Inversion Mode Transistor

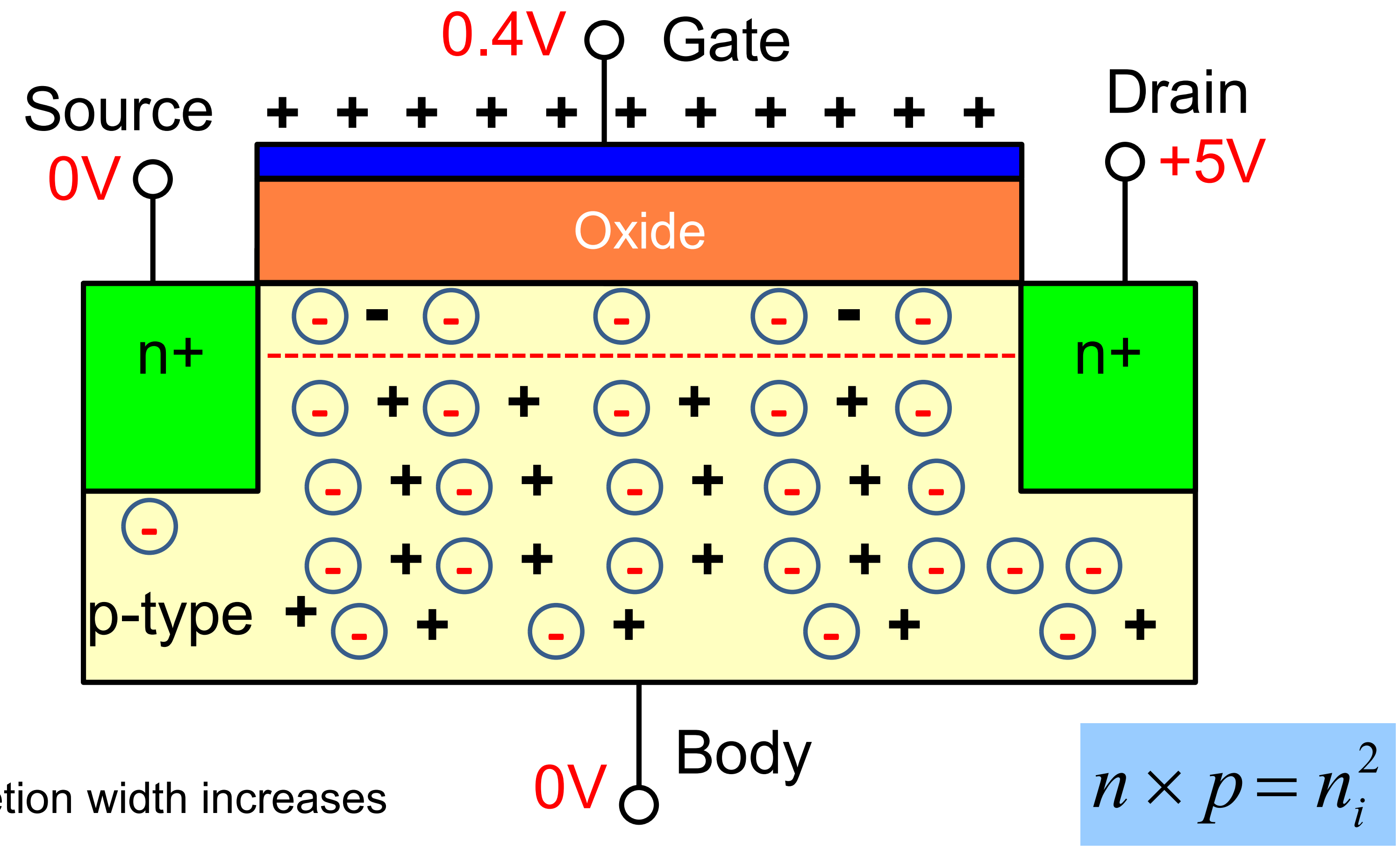




No channel exists when gate voltage is zero and current is zero as well.



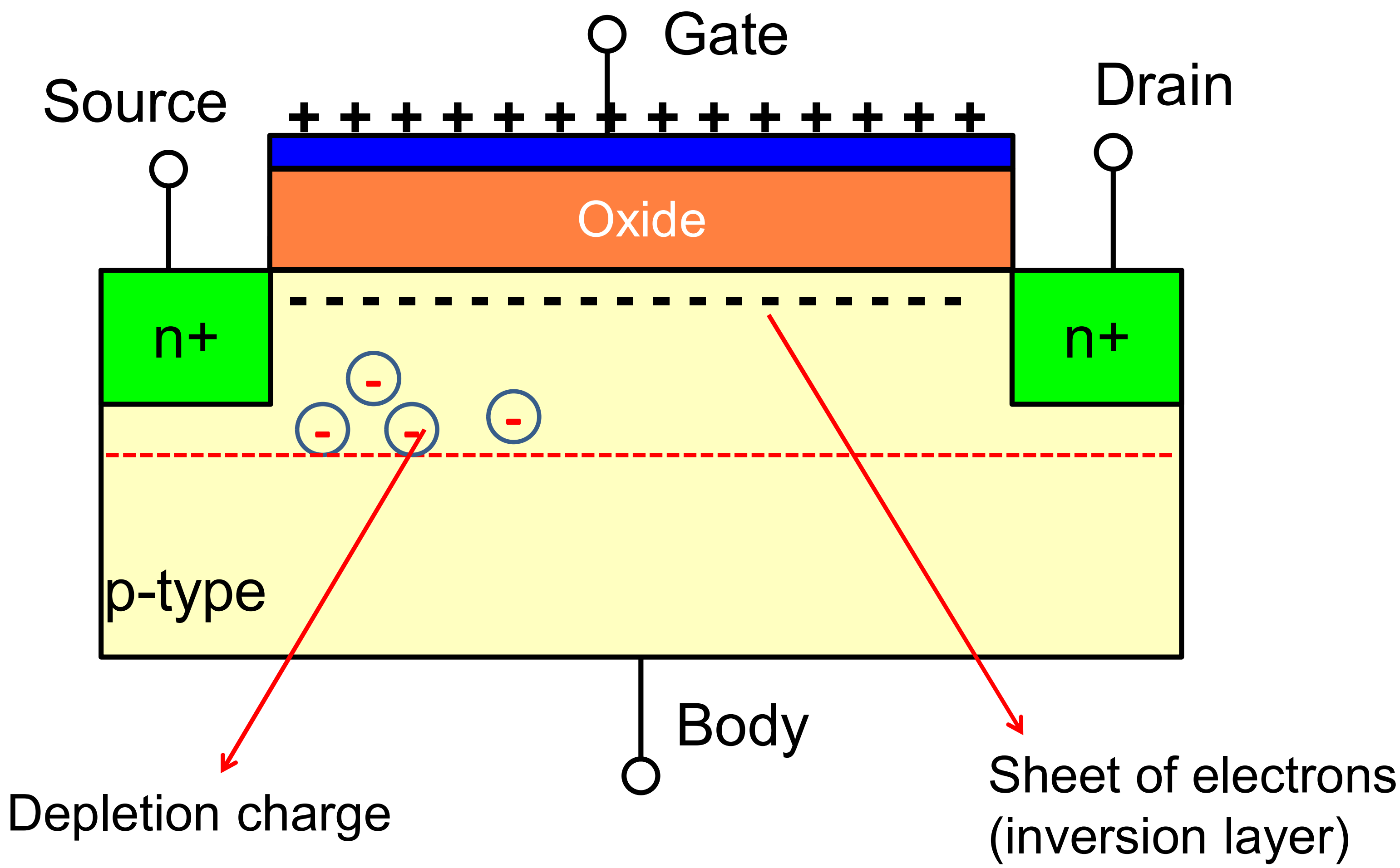
Depletion Region is formed near the Si/SiO_2 interface



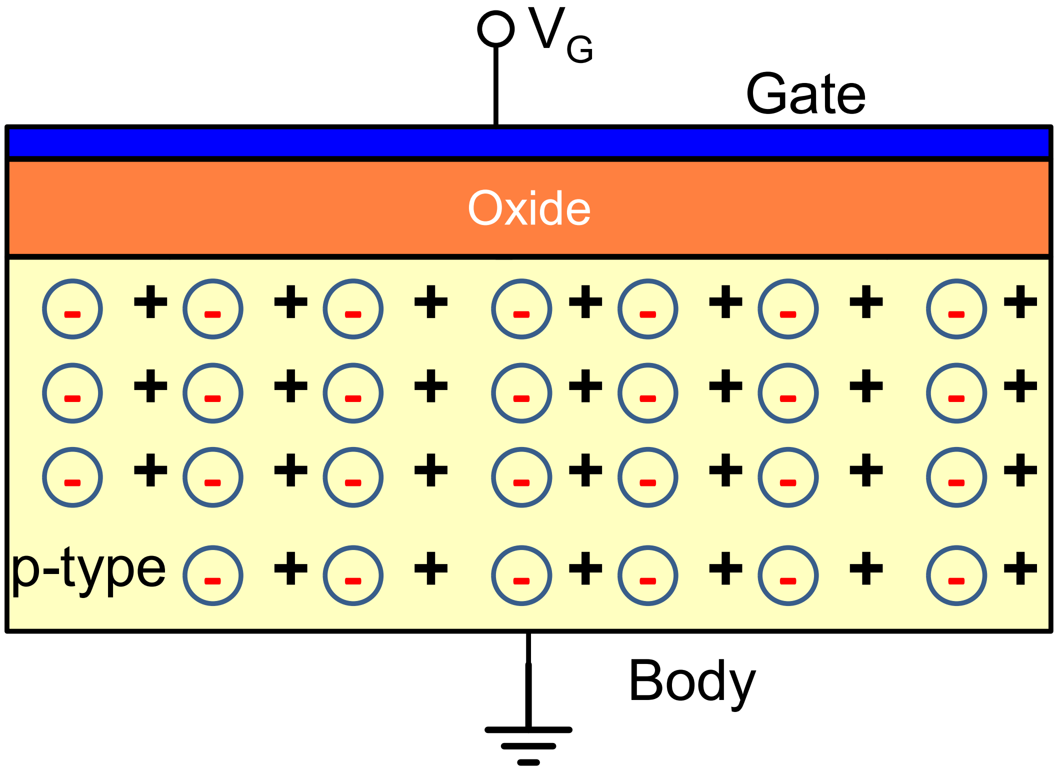
Depletion width increases

But something interesting happens: electron density at the surface also increases

At a sufficiently large voltage (>V_{THN}) a channel of electrons forms at the Si/SiO2 interface.



Conductivity modulation at the surface?

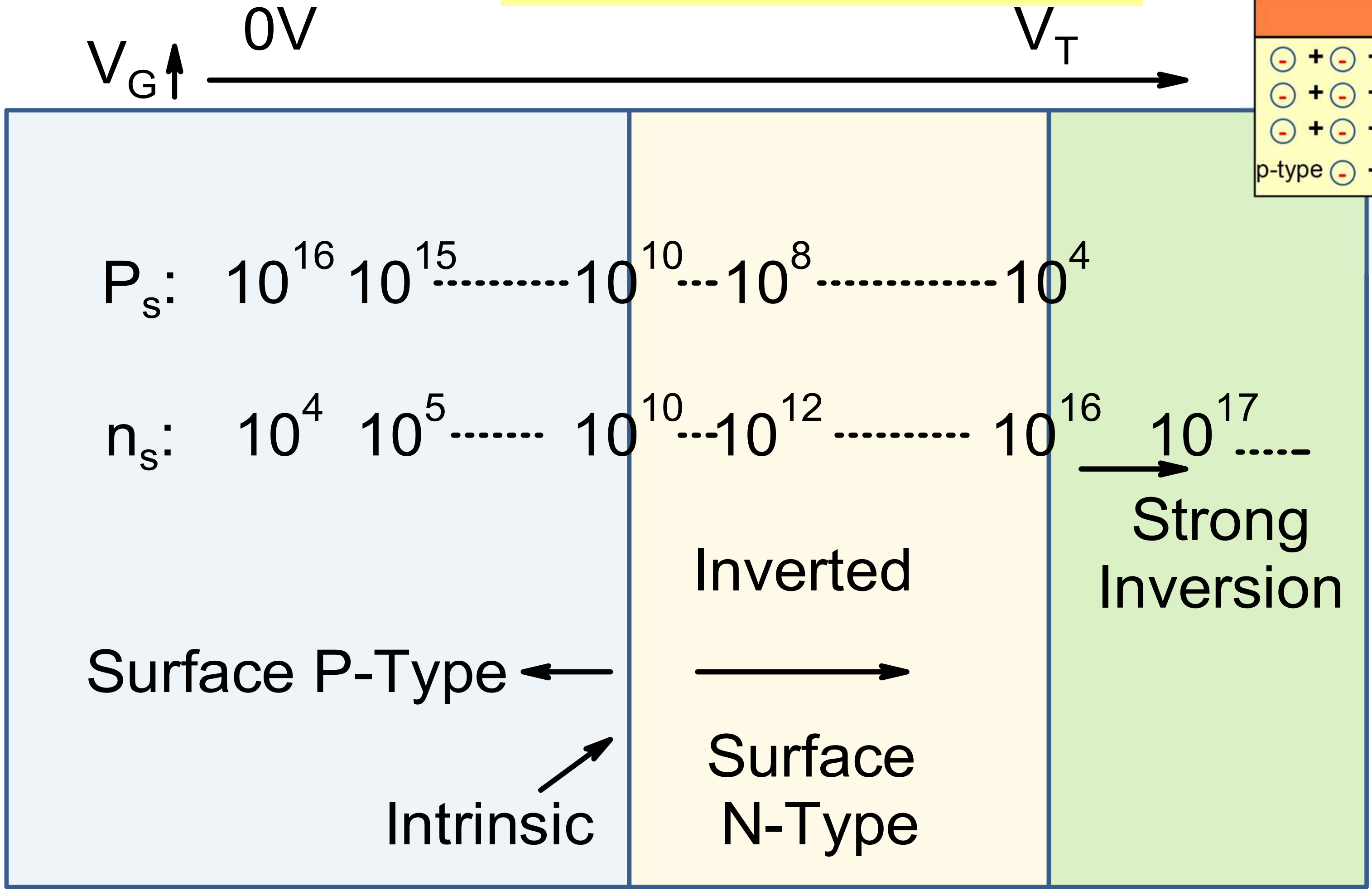


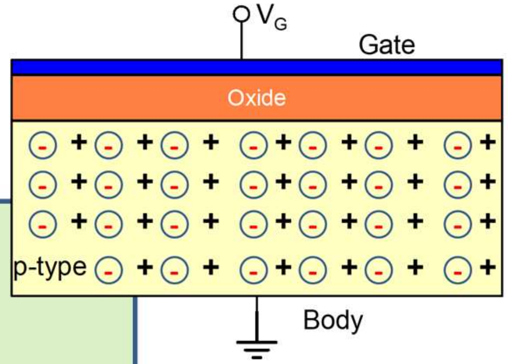
$$p = N_A = 10^{16} cm^{-3}$$

$$n = \frac{n_i^2}{p} \cong 10^4 cm^{-3}$$

Field Effect…

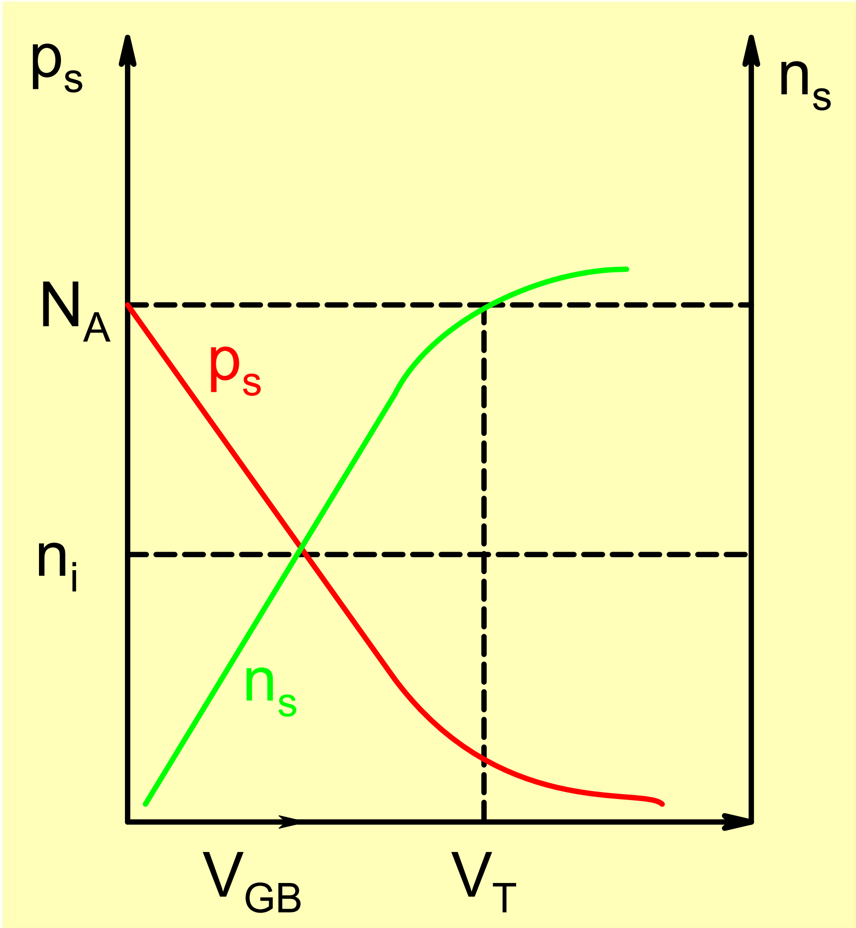
$$n_s \times p_s = n_1^{\,2} \cong 10^{20}\,cm^{-3}$$





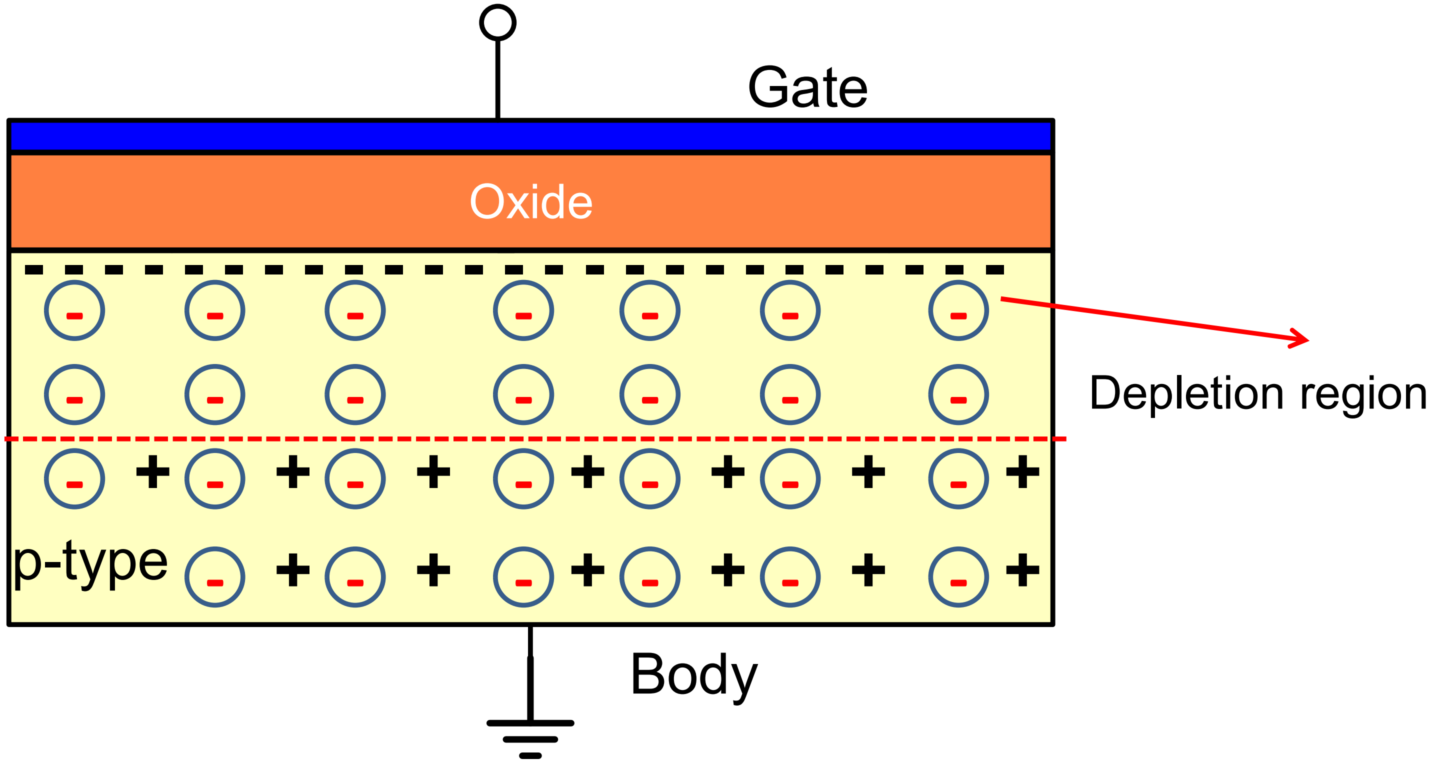
Surface carrier density can be changed from P-type to N-type

Surface Carrier Density



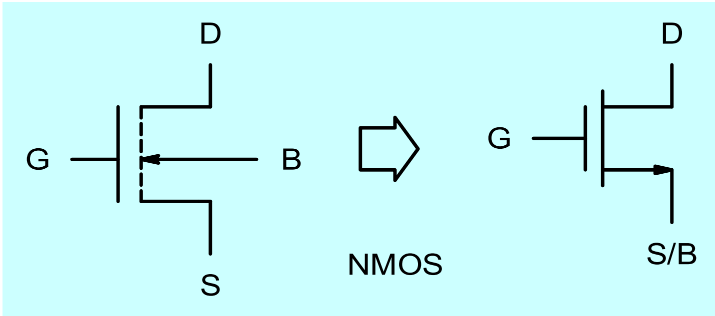
Strong Inversion

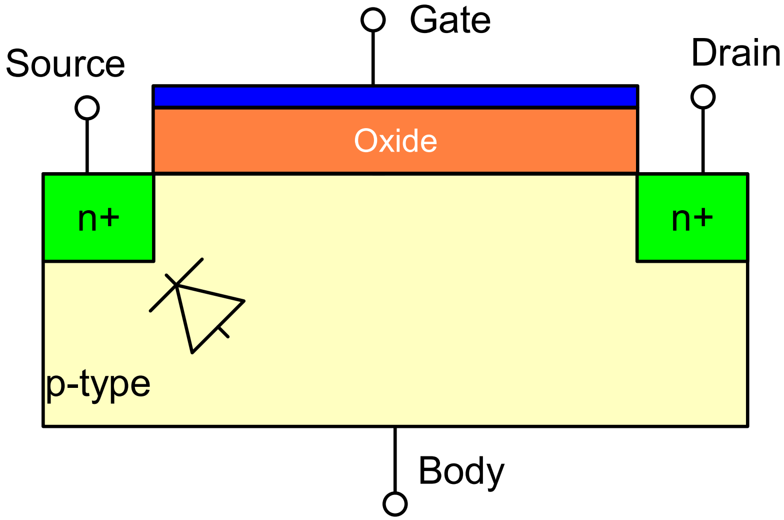
$$V_G > V_{THN}$$

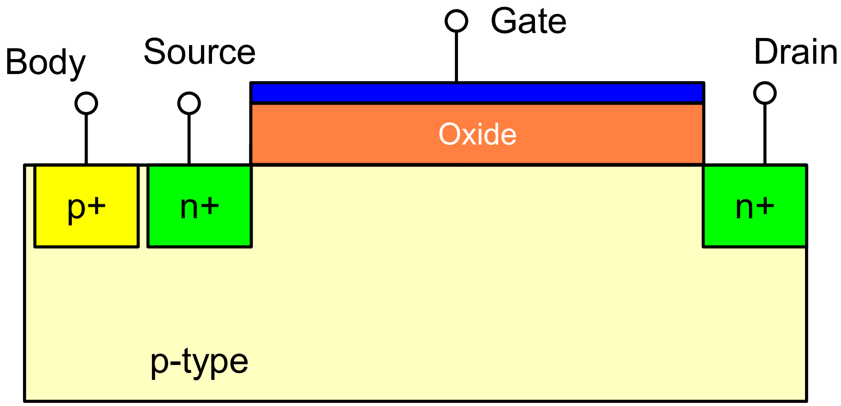


Electrons are accumulated at the surface $n_S >> N_A$

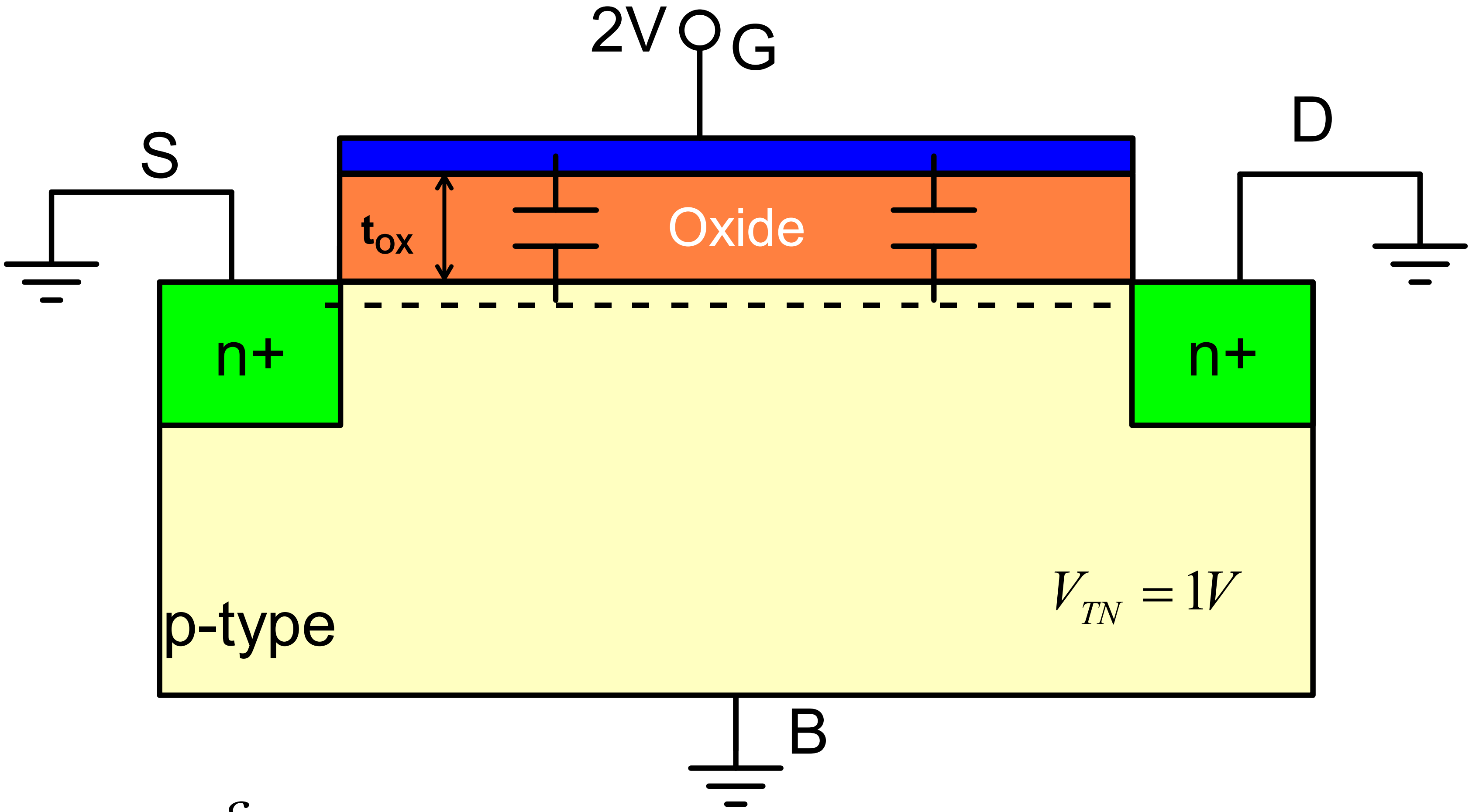
Simplified Symbols and structure





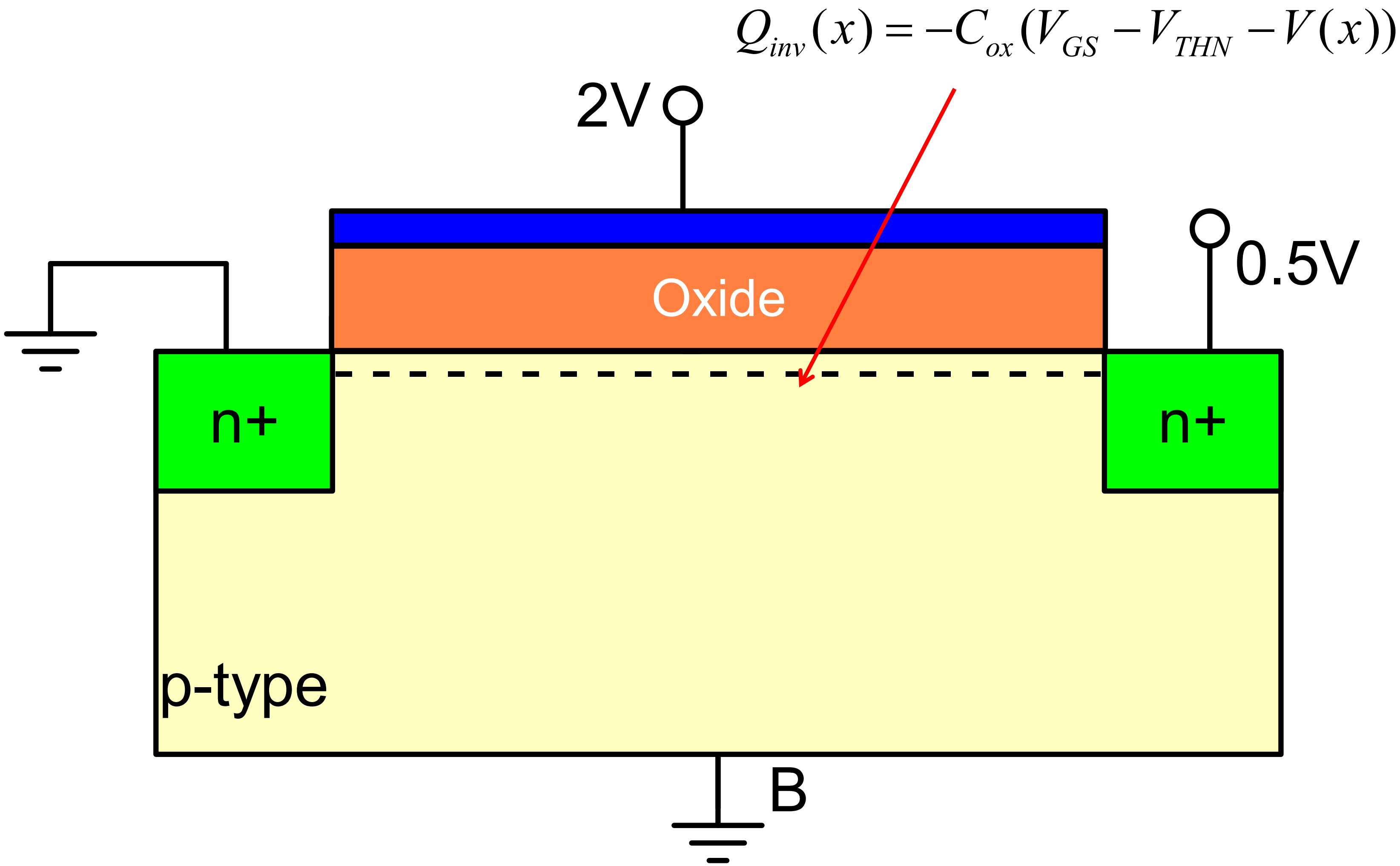


Operation of the MOSFET

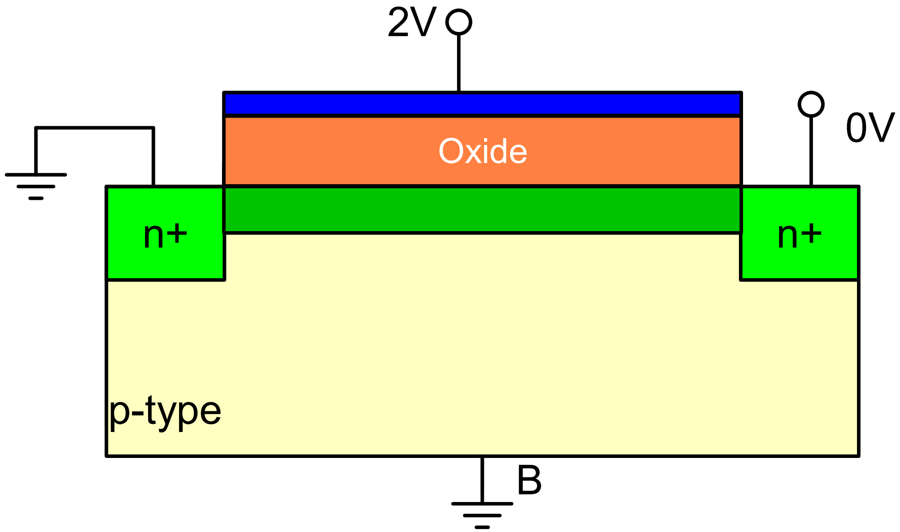


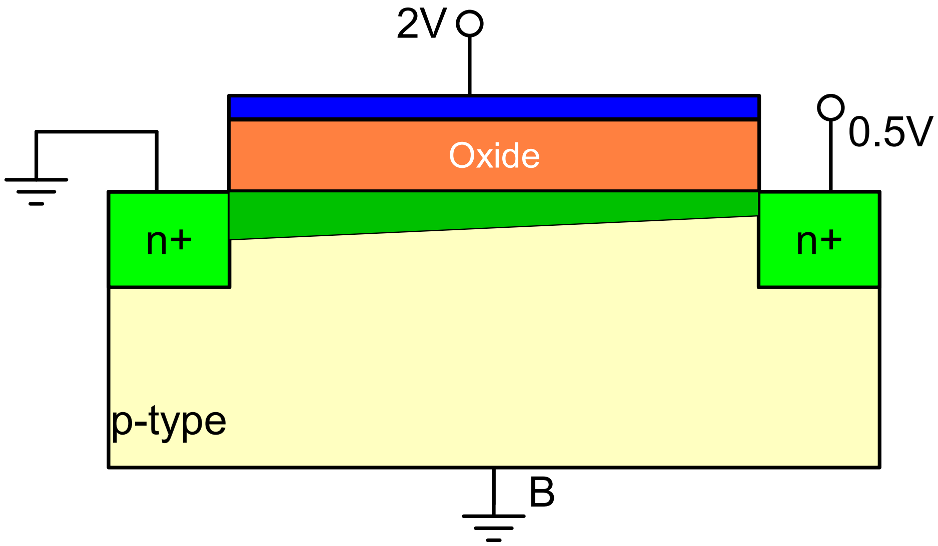
$$C'_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

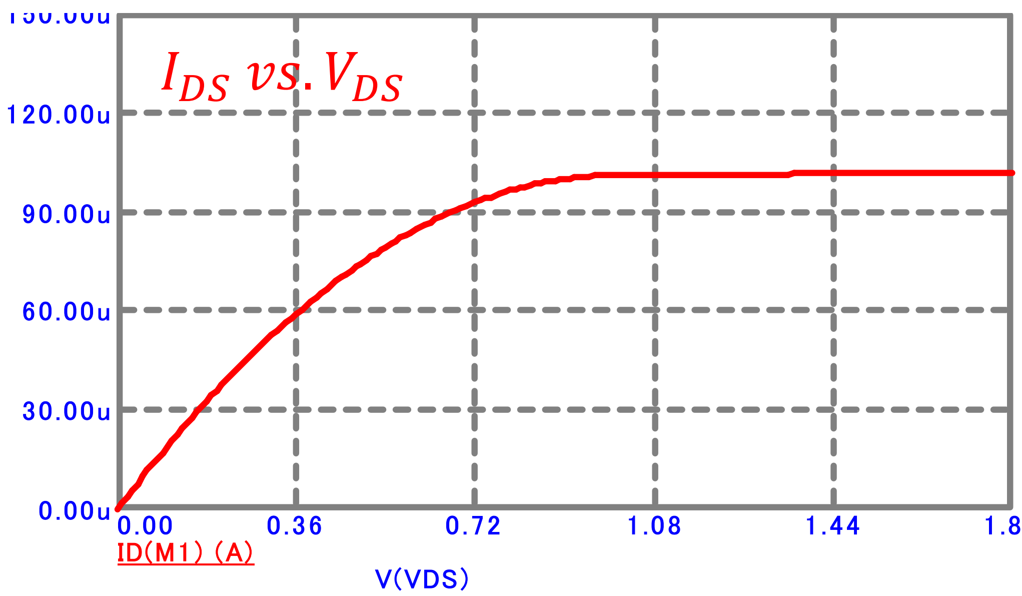
$$\text{Inversion charge/area : } Q_{inv} = -C_{ox}(V_{GS} - V_{THN})$$



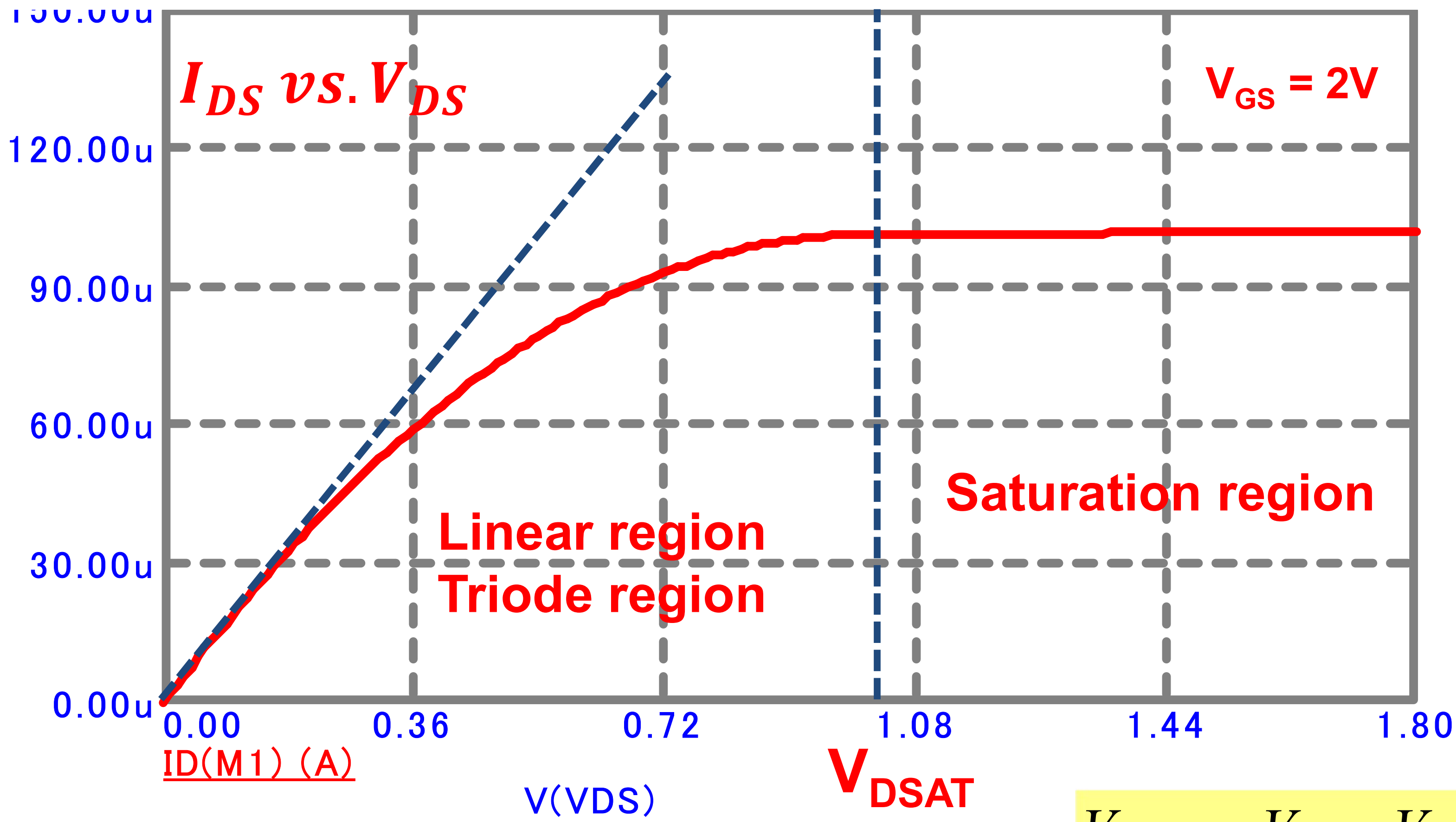
When a positive drain voltage is applied, current flows from drain to source and inversion charge density decreases from source to drain end.


2V
0V
Oxide
n+
n+
p-type
B


2V
0.5V
Oxide
n+
n+
p-type
B


I_{DS} vs. V_{DS}
120.00u
90.00u
60.00u
30.00u
0.00u
0.00
0.36
0.72
1.08
1.44
1.8
ID(M1) (A)
V(VDS)

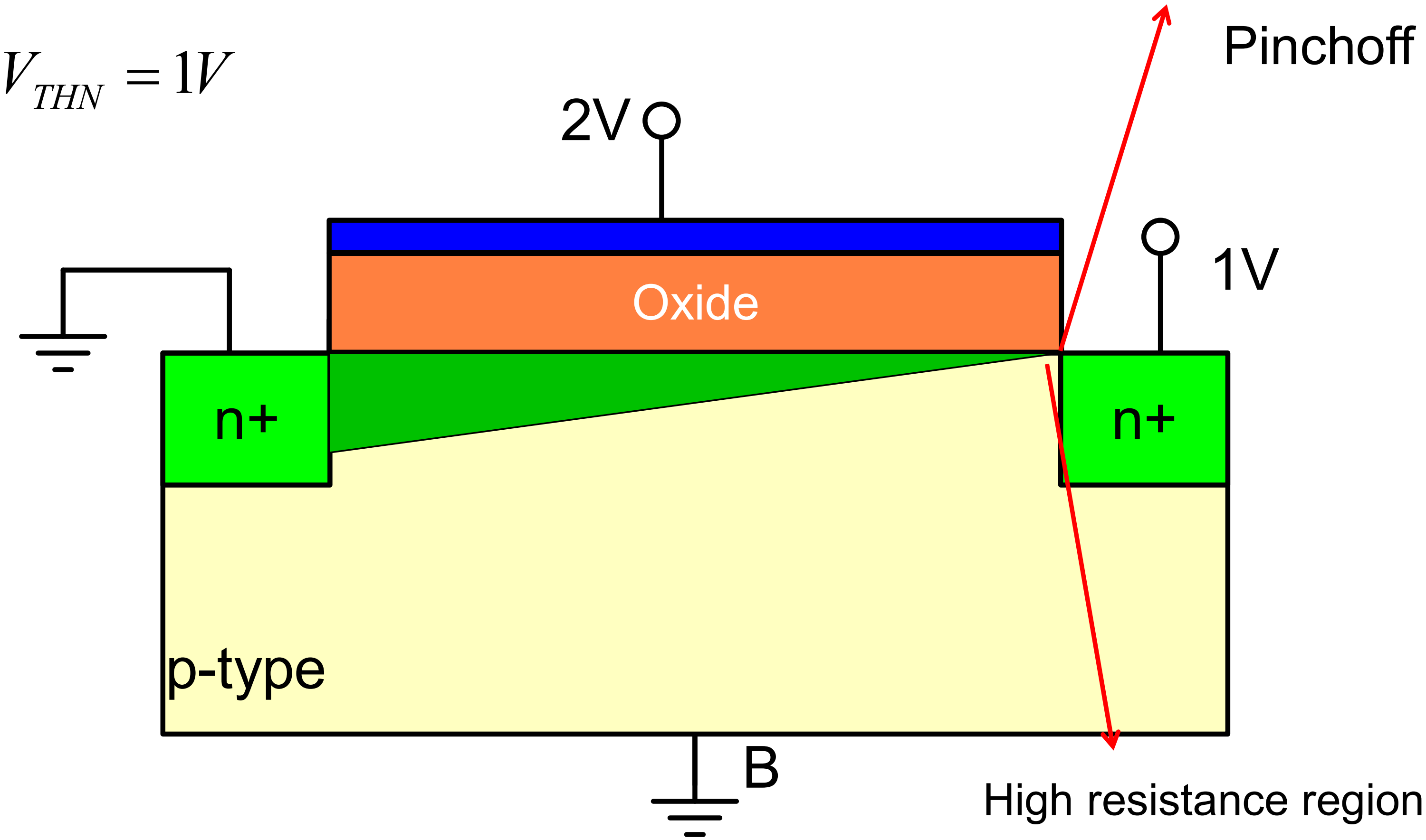
$$I_{DS} = \frac{V_{DS}}{R_{ch}} > 0$$



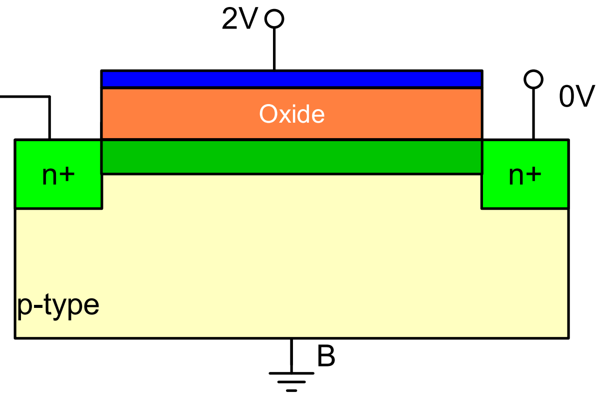
$$V_{DSAT} = V_{GS} - V_{THN}$$

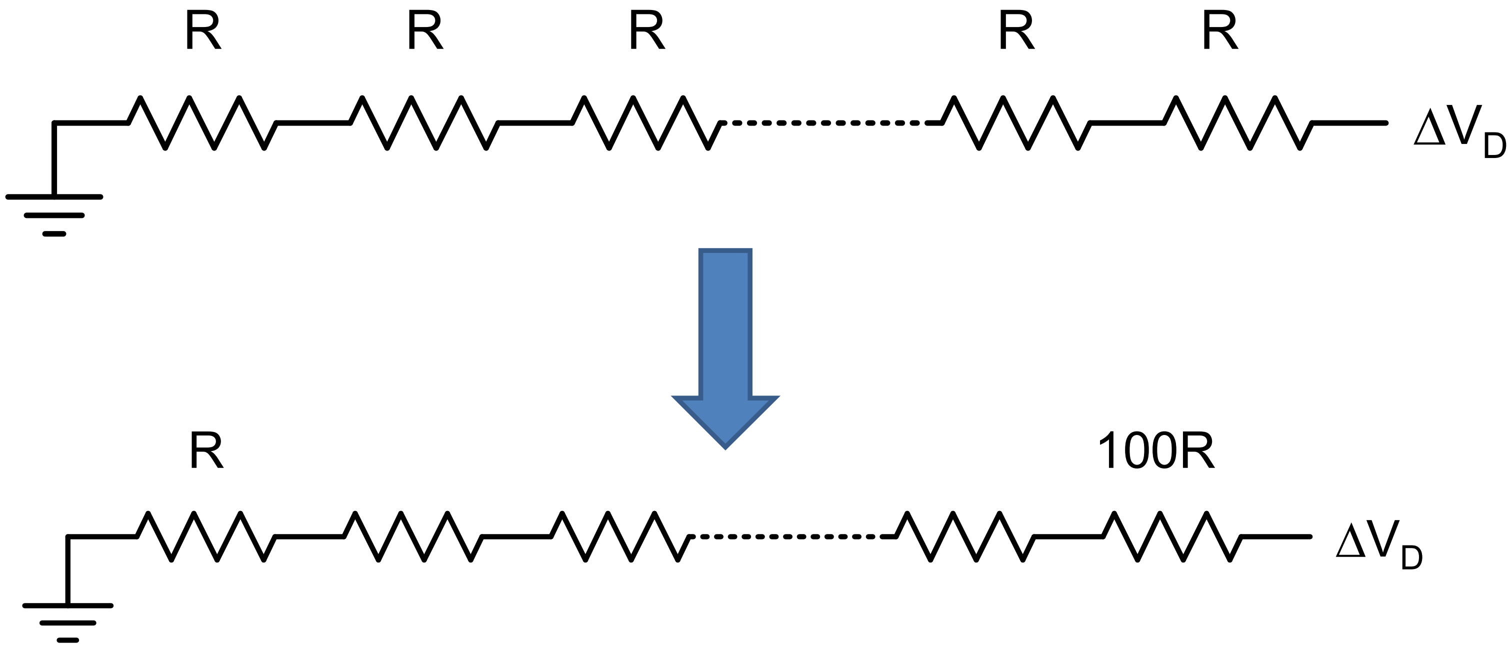
$$Q_{inv}(x) = -C_{ox}(V_{GS} - V_{THN} - V(x)) \cong -C_{ox}(2-1-1) = 0$$

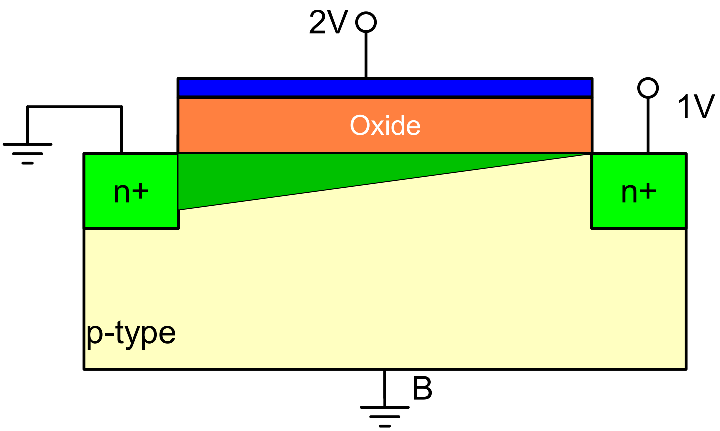
$$V_{THN} = 1V$$

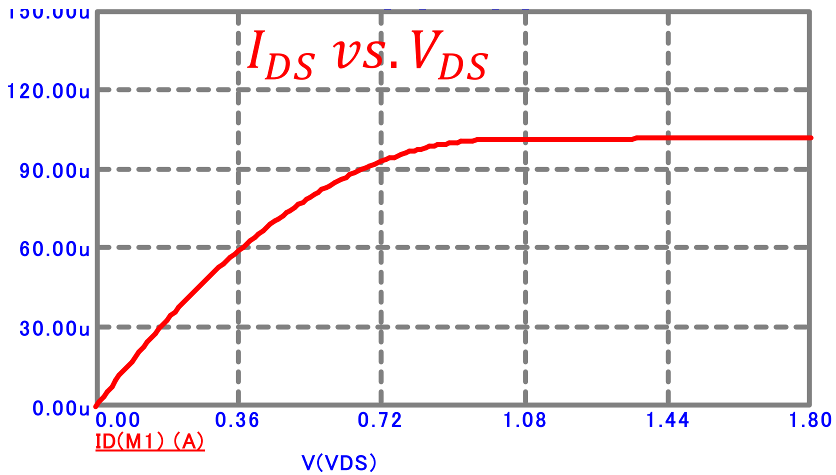


Any further increase in drain bias is absorbed in a small region next to the drain and rest of channel is not much affected and thus current becomes constant.



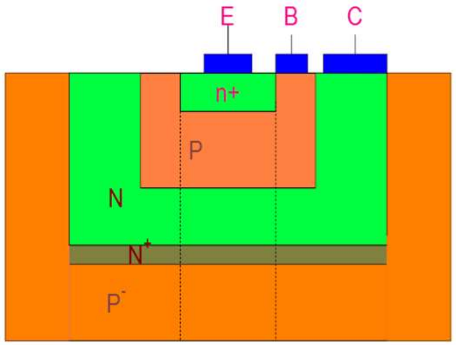




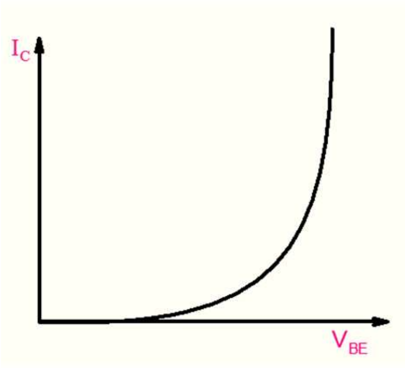


$$\frac{\partial I_{DS}}{\partial V_{GS}} >> \frac{\partial I_{DS}}{\partial V_{DS}}$$

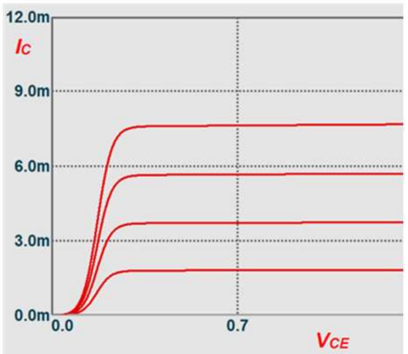
BJT



E
B
C
n+
P
N
N+
P-

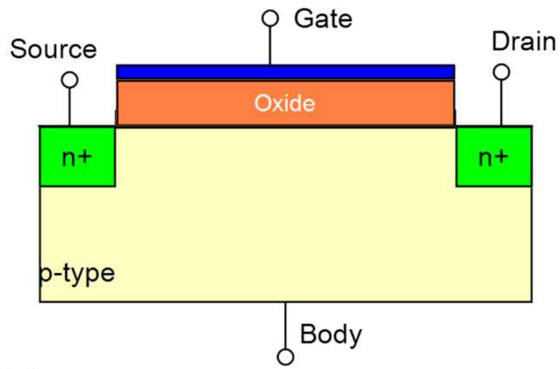


I_C
V_{BE}

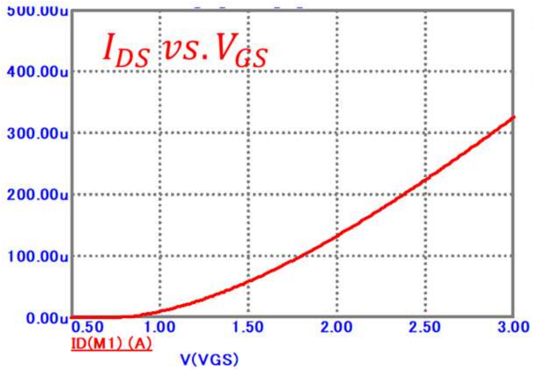


12.0m
I_C
9.0m
6.0m
3.0m
0.0m
0.0
0.7
V_{CE}

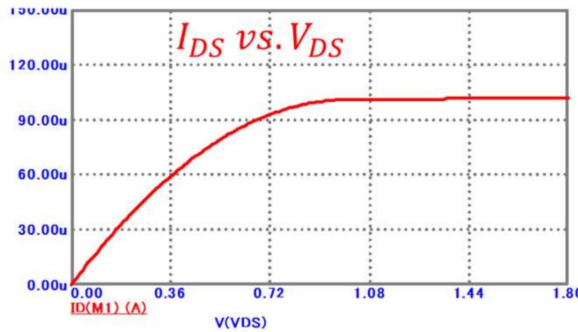
MOSFET



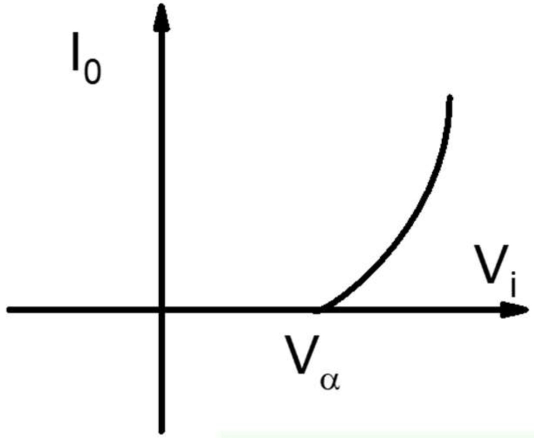
Gate
Source
Drain
Oxide
n+
n+
p-type
Body

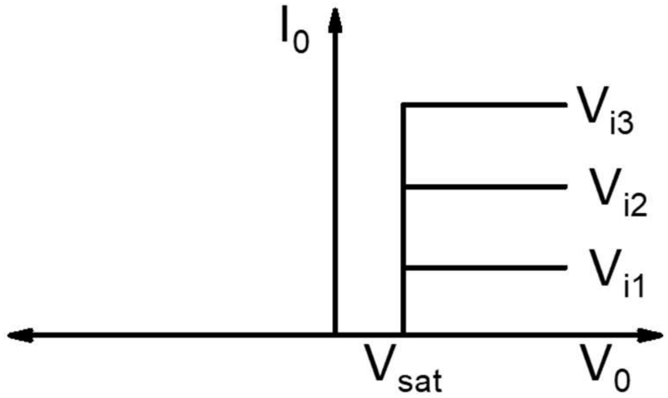


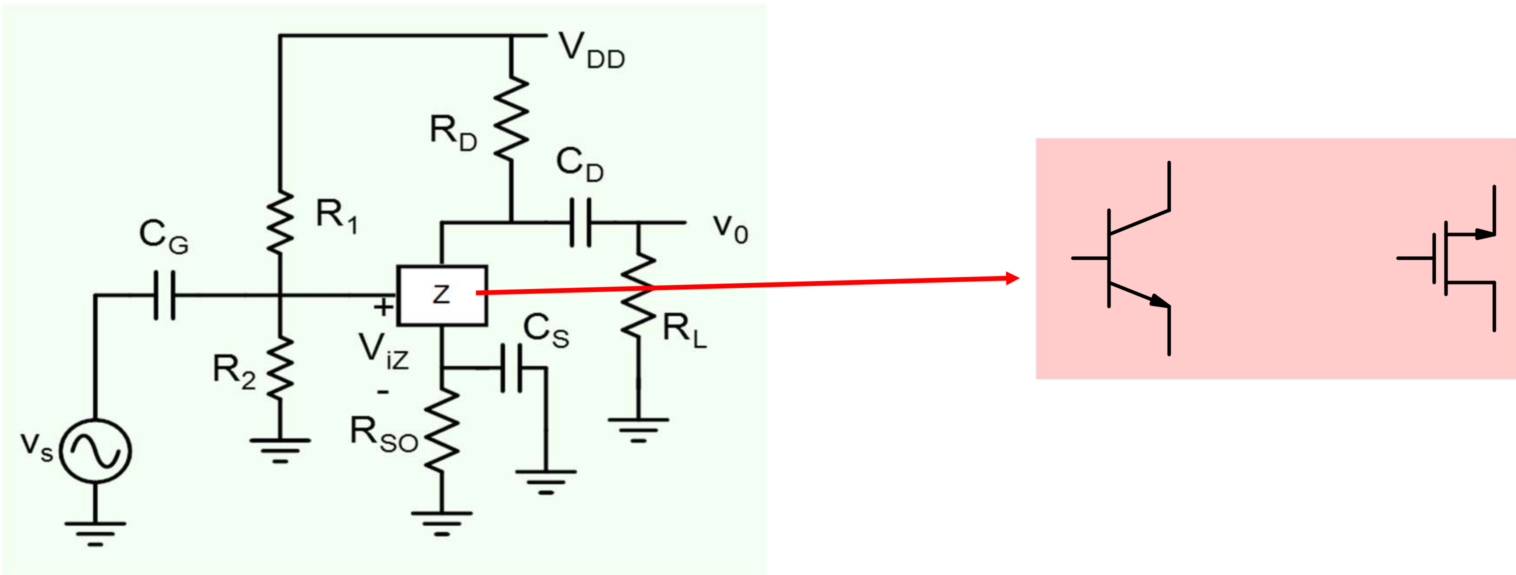
I_{DS} vs. V_{GS}
400.00u
300.00u
200.00u
100.00u
0.00u
0.50
1.00
1.50
2.00
2.50
3.00
ID(M1) (A)
V(VGS)



I_{DS} vs. V_{DS}
120.00u
90.00u
60.00u
30.00u
0.00u
0.00
0.36
0.72
1.08
1.44
1.80
ID(M1) (A)
V(VDS)


I0
Vi
Vα

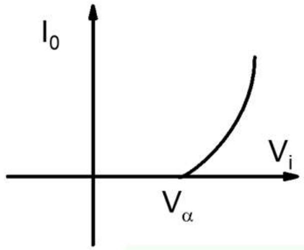

I0
Vi3
Vi2
Vi1
Vsat
V0


VDD
RD
CD
R1
CG
v0
Z
+
ViZ
-
CS
RL
R2
RSO
vs

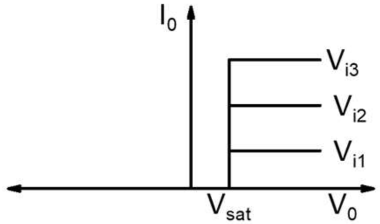
ESC201T: Introduction to Electronics

Lecture 28: Transistor Circuits

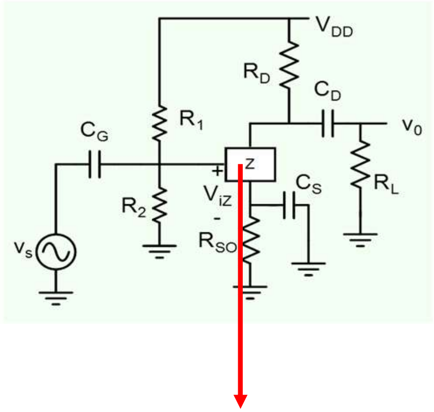
B. Mazhari
Dept. of EE, IIT Kanpur



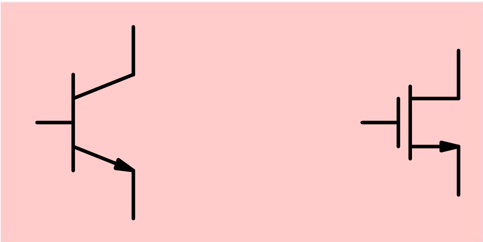
I_0
V_i
V_α

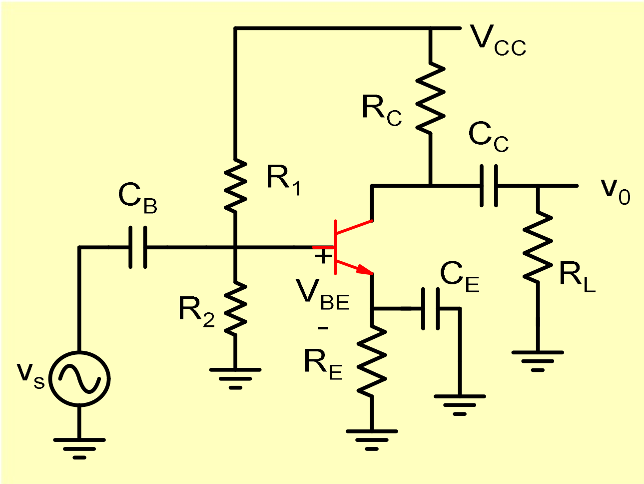


I_0
V_{i3}
V_{i2}
V_{i1}
V_{sat}
V_0

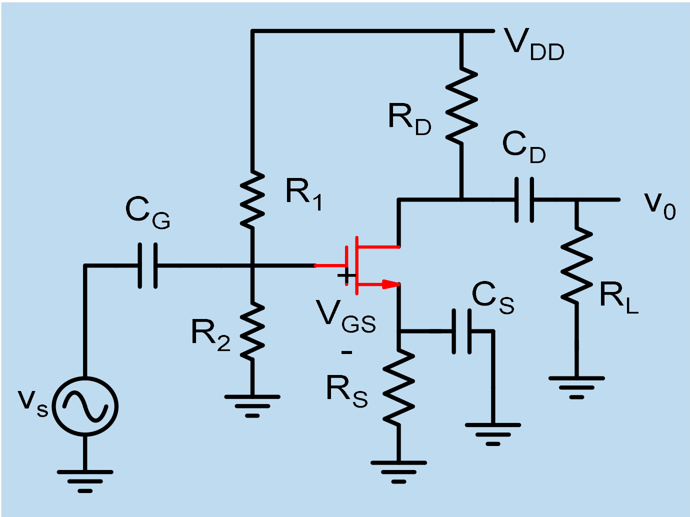


V_{DD}
R_D
C_D
R_1
C_G
v_0
R_L
Z
C_S
V_{iZ}
R_2
R_{SO}
v_s

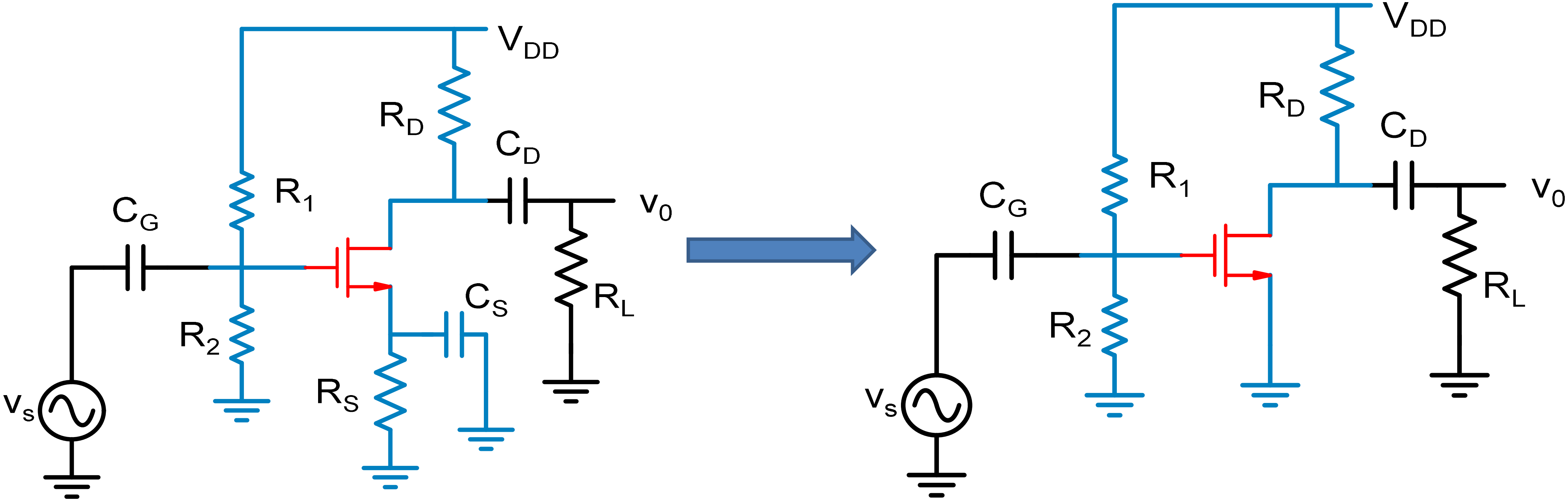




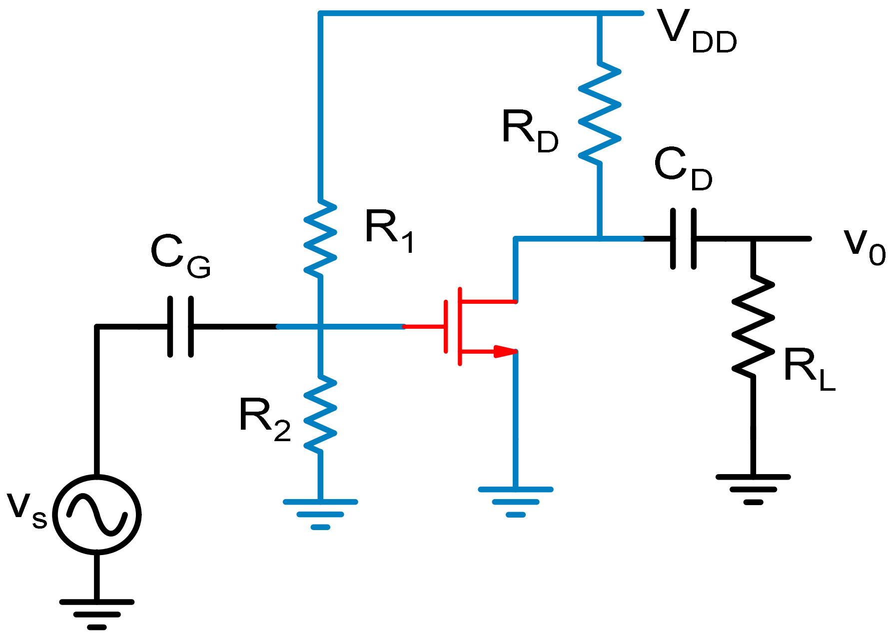
V_{CC}
R_C
C_C
R_1
C_B
v_0
V_{BE}
C_E
R_L
R_2
R_E
v_s



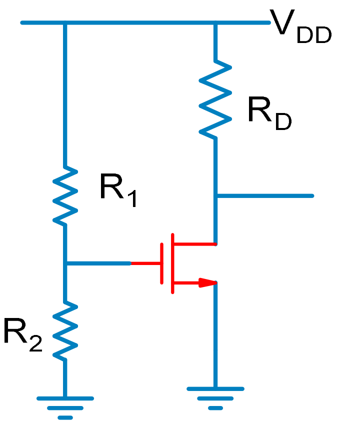
V_{DD}
R_D
C_D
R_1
C_G
v_0
V_{GS}
C_S
R_L
R_2
R_S
v_s



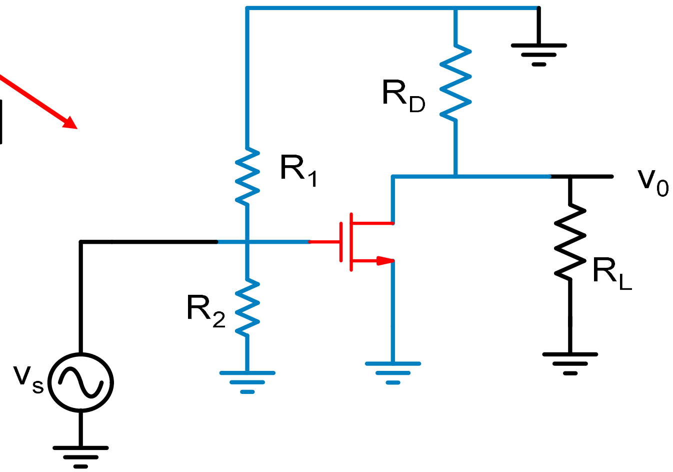
VDD
RD
CD
R1
CG
v0
R2
RL
CS
vs
RS
VDD
RD
CD
R1
CG
v0
R2
RL
vs



dc analysis

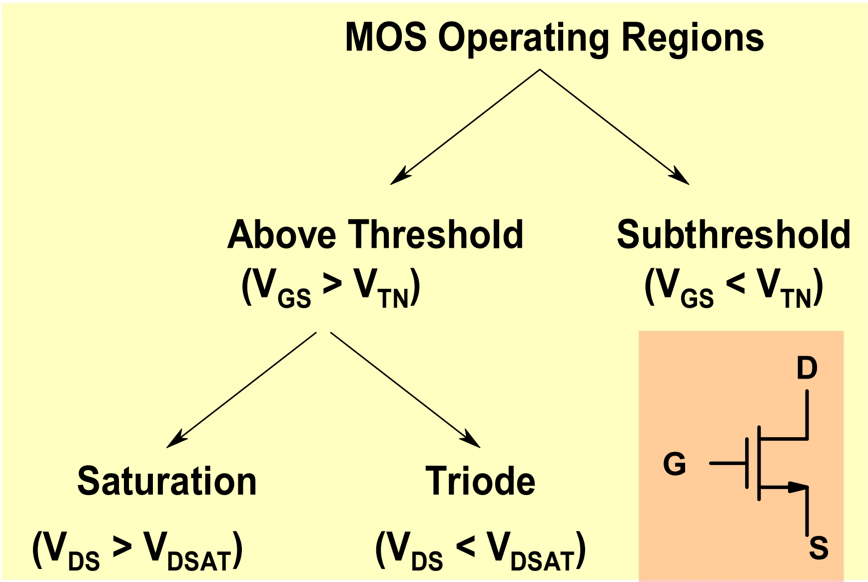


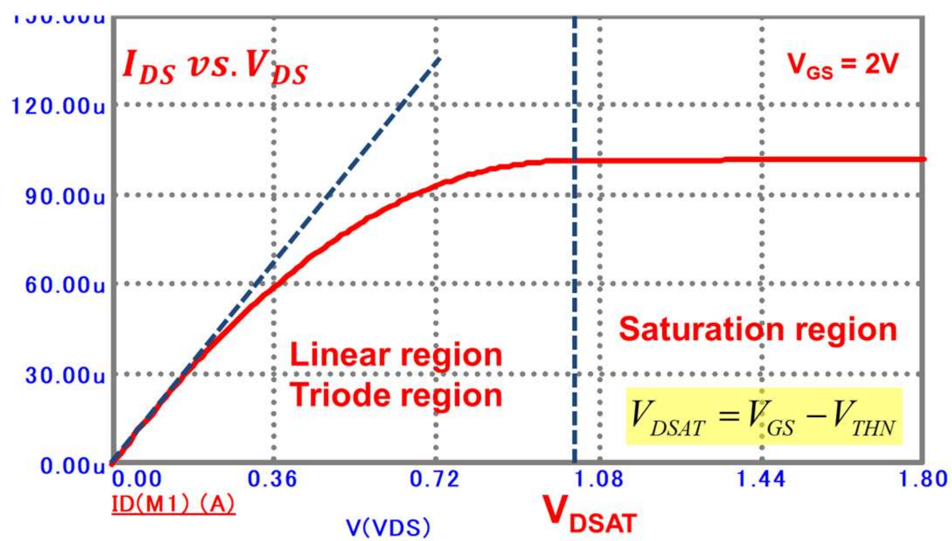
ac or small signal analysis

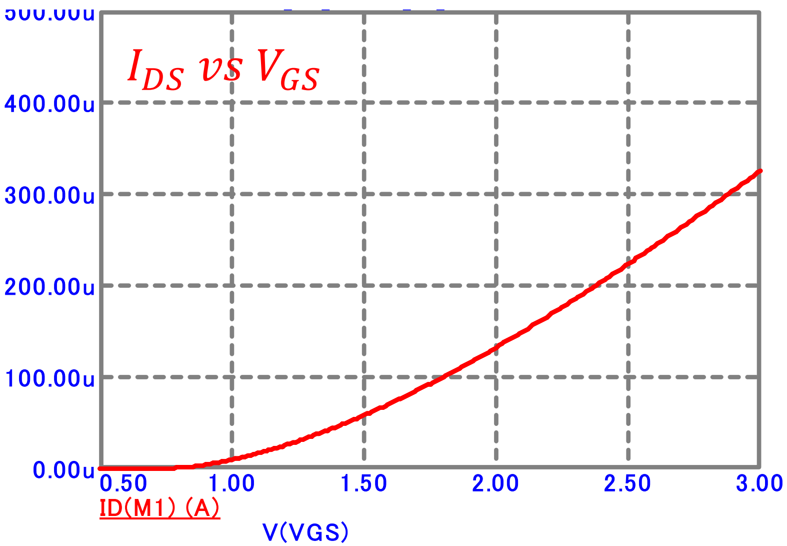


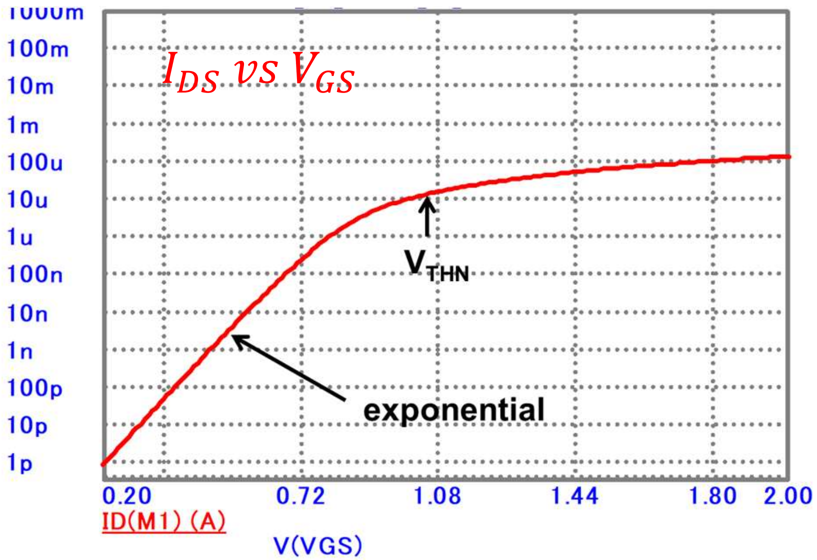
$$A_V = \frac{v_O}{v_S}$$

Need dc and ac (or small signal incremental) model of the transistors

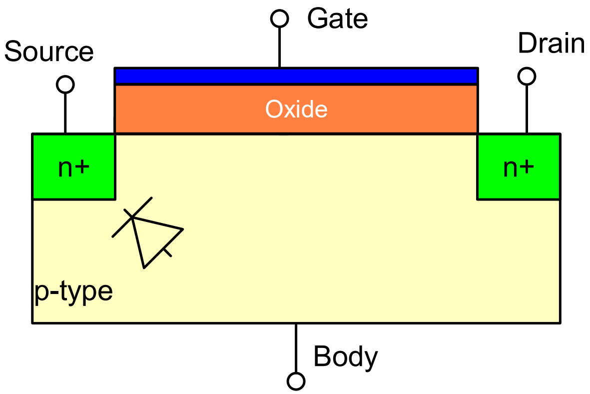

MOS Operating Regions
Above Threshold
($V_{GS} > V_{TN}$)
Subthreshold
($V_{GS} < V_{TN}$)
Saturation
($V_{DS} > V_{DSAT}$)
Triode
($V_{DS} < V_{DSAT}$)
D
G
S

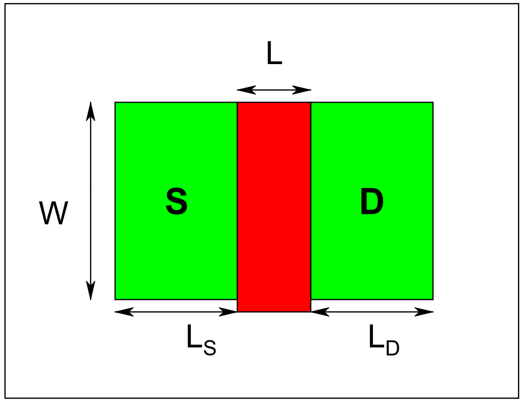

I_{DS} vs. V_{DS}
V_{GS} = 2V
120.00u
90.00u
60.00u
30.00u
0.00u
0.00
0.36
0.72
1.08
1.44
1.80
ID(M1) (A)
V(VDS)
V_{DSAT}
Linear region
Triode region
Saturation region
$V_{DSAT} = V_{GS} - V_{THN}$

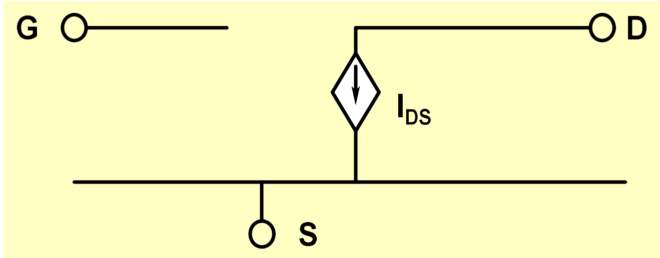

I_{DS} vs V_{GS}
400.00u
300.00u
200.00u
100.00u
0.00u
0.50
1.00
1.50
2.00
2.50
3.00
ID(M1) (A)
V(VGS)


I_{DS} vs V_{GS}
100m
10m
1m
100u
10u
1u
100n
10n
1n
100p
10p
1p
0.20
0.72
1.08
1.44
1.80
2.00
ID(M1) (A)
V(VGS)
V_{THN}
exponential

The dc and ac models of the transistor in saturation region can be represented in the form of an equivalent circuit:

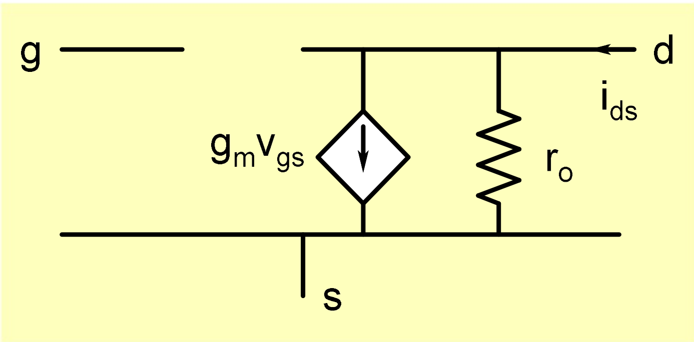






$$I_{DS} = \frac{\beta_N}{2}(V_{GS} - V_{THN})^2 \;\; ; \; \beta_N = KP_N \times \frac{W}{L}$$

kP_N: *Transconductance parameter* $\frac{\mu A}{V^2}$



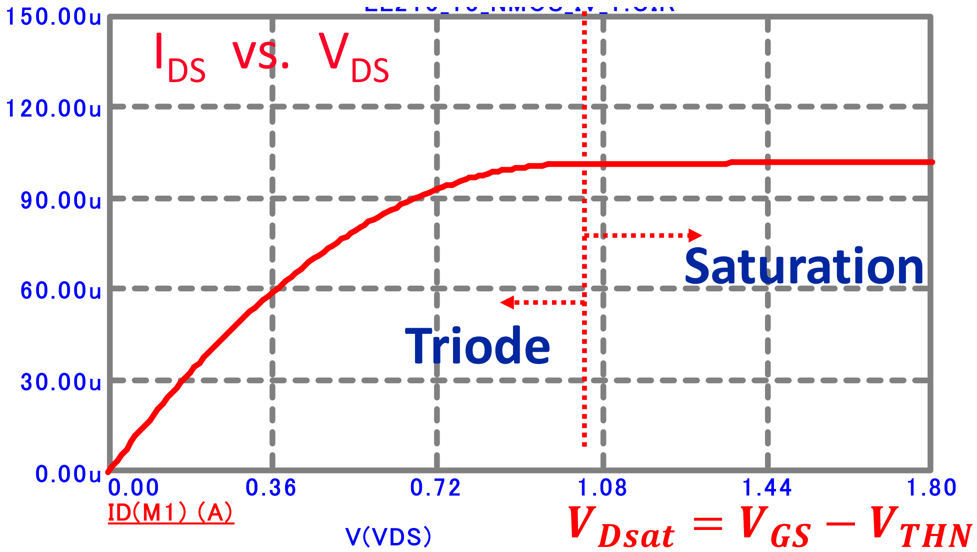
$$g_m = \frac{2I_{DSQ}}{V_{GSQ} - V_{THN}} = \sqrt{2I_{DSQ}\beta}$$

$$r_o = \frac{1}{\lambda_n I_{DSQ}}$$

λ_N *is the channel length modulation parameter*

$$KP_N = 100\mu A/V^2; V_{THN} = 1V; \lambda_n = 0.01V^{-1}$$

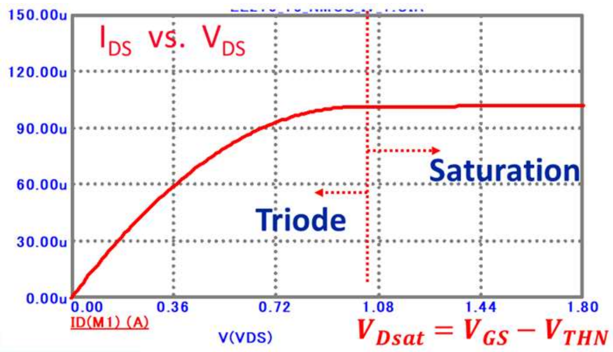
dc Model: Triode (or Linear)

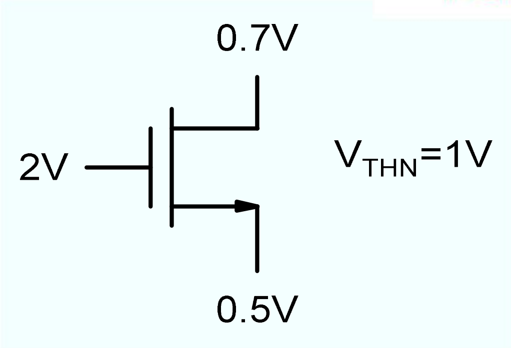


$$I_{DS} = \beta_N \left\{ (V_{GS} - V_{THN}) V_{DS} - \frac{V_{DS}{}^2}{2} \right\}$$

For simplicity we will only consider cases where $I_{DS} \approx KP_N \times \frac{W}{L} \times (\mathrm{V_{GS}} - \mathrm{V_{THN}}) \times V_{DS}$

Which mode is the transistor operating in ?

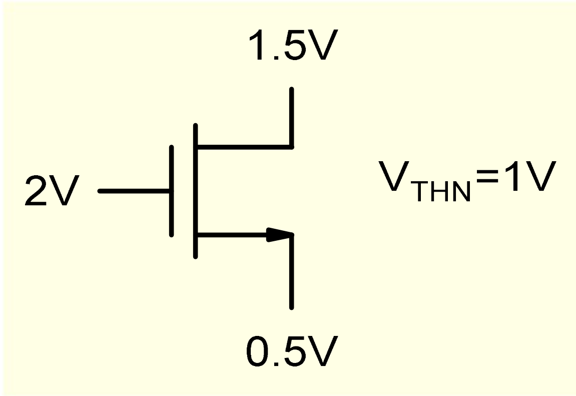




$$V_{GS} = 1.5\ ; V_{DS} = 0.2$$

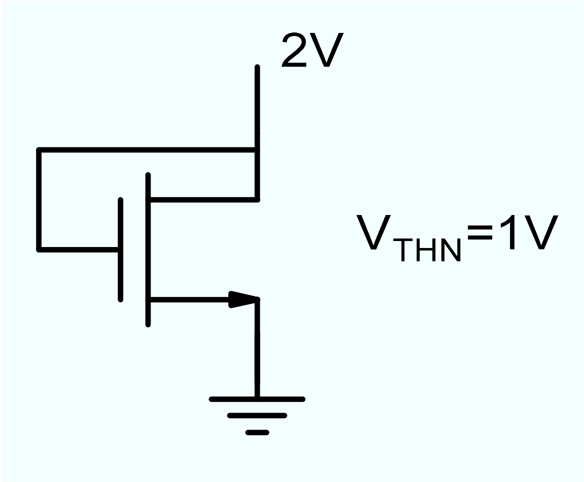
$$V_{DSAT} = V_{GS} - V_{THN} = 0.5$$

$$V_{DS} < V_{DSAT} \Rightarrow Linear$$



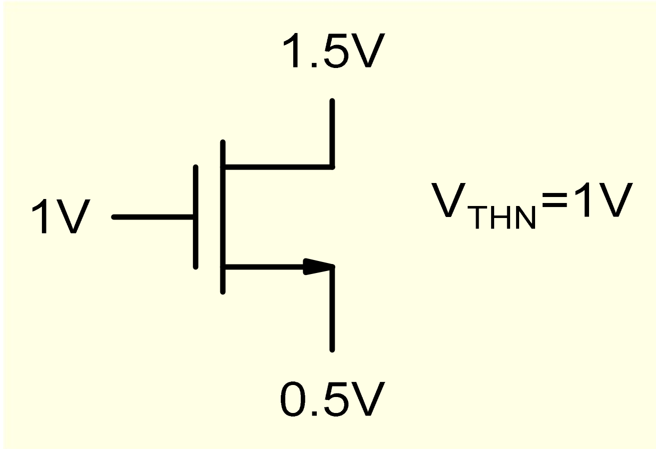
$$V_{DSAT} = 0.5\ ; V_{DS} = 1V$$

Saturation



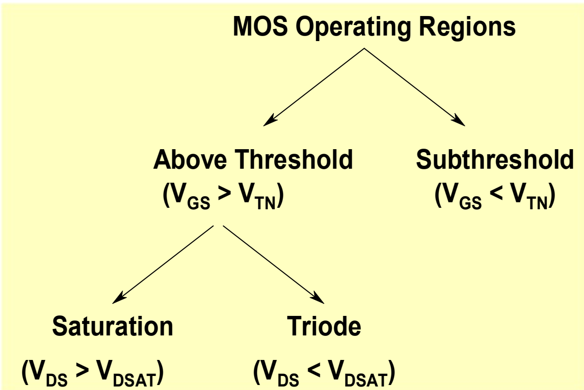
$$V_{GS} = 2\ ;V_{DS} = 2$$

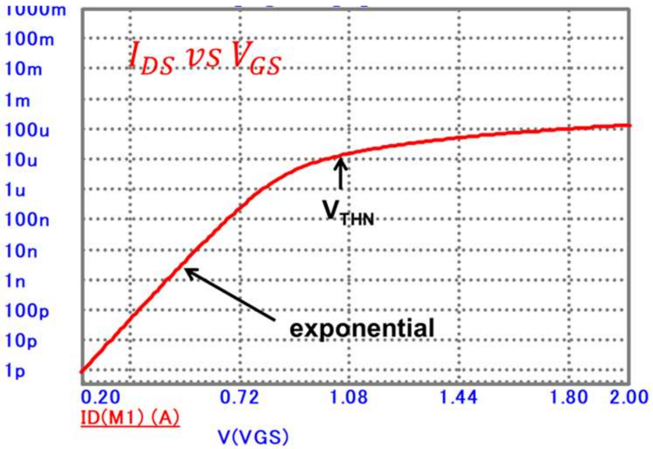
Saturation



$$V_{GS} = 0.5V < V_{THN}$$

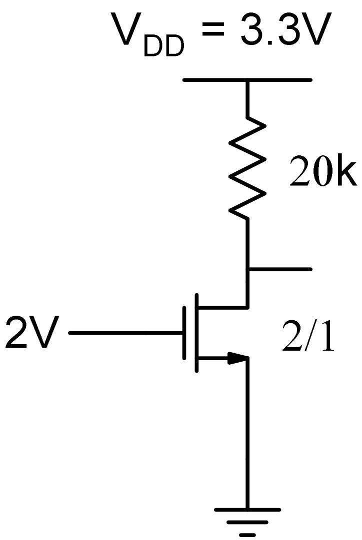
Transistor is in sub-threshold mode of operation





MOSFET Circuits

Example-1



$$KP_N = 100\mu A/V^2; V_{THN} = 1V; \lambda_n = 0.01V^{-1}$$

Determine I_{DS} and V_{DS}

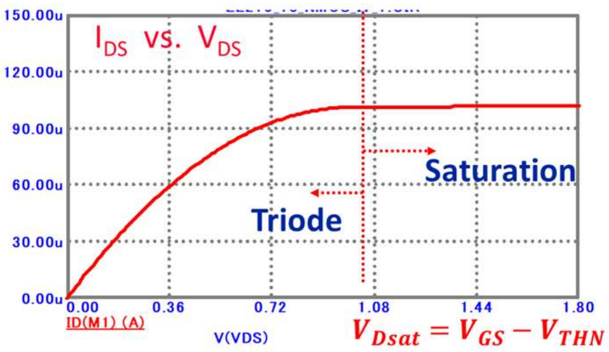
$$Assume\ saturation\ mode\ of\ operation$$

$$I_{DS} = KP_N \times \frac{W}{L} \times \frac{(V_{GS} - V_{THN})^2}{2} = 10^{-4}A$$

$$V_{DS} = V_{DD} - I_{DS} \times R_D = 1.2V$$

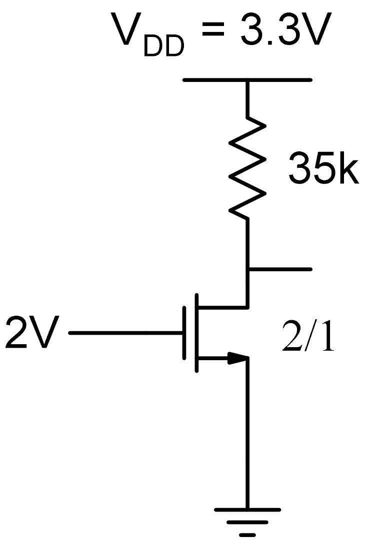
$$V_{Dsat} = V_{GS} - V_{THN} = 1V$$

$$Since\ V_{DS} > V_{Dsat}\ our\ assumption\ is\ correct$$



MOSFET Circuits

Example-2



$$KP_N = 100\mu A/V^2; V_{THN} = 1V; \lambda_n = 0.01V^{-1}$$

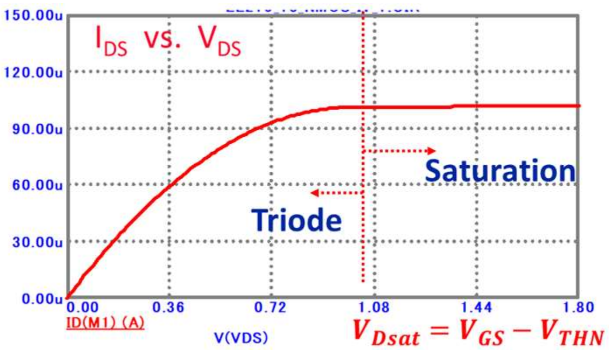
Determine I_{DS} and V_{DS}

$$\textit{Assume saturation mode of operation}$$

$$I_{DS} = KP_N \times \frac{W}{L} \times \frac{(V_{GS} - V_{THN})^2}{2} = 10^{-4}A$$

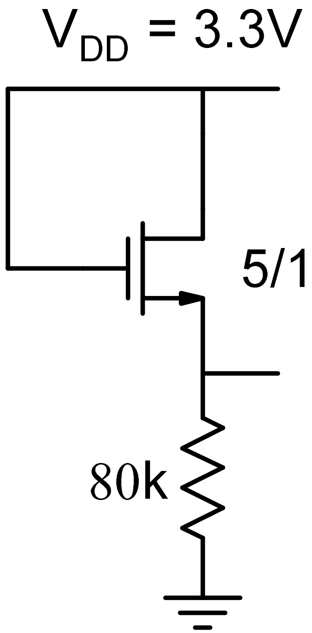
$$V_{DS} = V_{DD} - I_{DS} \times R_D = -0.2V \textit{ so assumption incorrect}$$

$$I_{DS} \approx KP_N \times \frac{W}{L} \times (\mathrm{V_{GS}} - \mathrm{V_{THN}}) \times V_{DS}; \mathrm{V_{DS}} = \mathrm{V_{DD}} - \mathrm{I_{DS}} \times R_D$$
$$\Rightarrow I_{DS} = 8.25 \times 10^{-5}A; V_{DS} = 0.412V$$



$$V_{Dsat} = V_{GS} - V_{THN} = 1V$$

Example-3



$$KP_N = 100\mu A/V^2; V_{THN} = 1V; \lambda_n = 0.01V^{-1}$$

Determine I_{DS} and V_{DS}

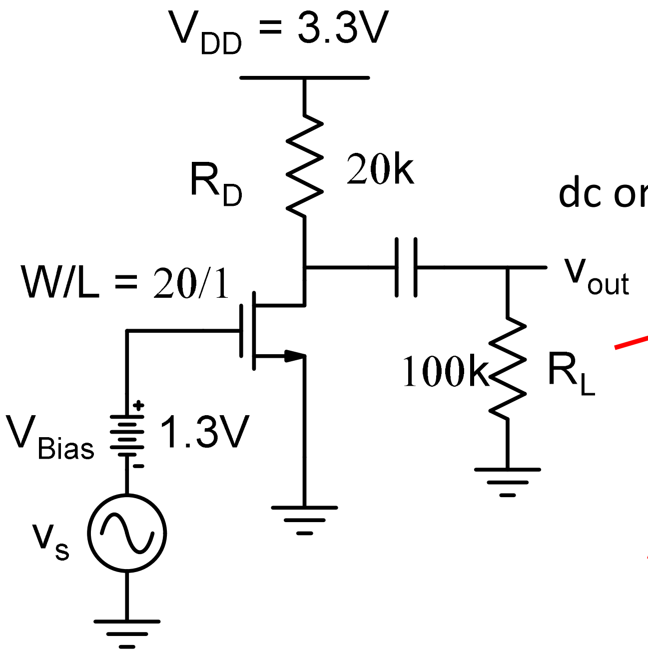
$$V_{DS} = V_{GS}$$
$$\Rightarrow V_{DS} > V_{GS} - V_{THN} = V_{DSAT} \Rightarrow \text{Saturation}$$

$$I_{DS} = KP_N \times \frac{W}{L} \times \frac{(V_{GS} - V_{THN})^2}{2}; V_{GS} = 3.3 - I_{DS} \times 80 \times 10^3$$
$$\Rightarrow I_{DS} = 2.48 \times 10^{-5}A; V_{GS} = 1.315V$$

For the other solution VGS = 0.653V which is not possible since it is less than V_{THN}

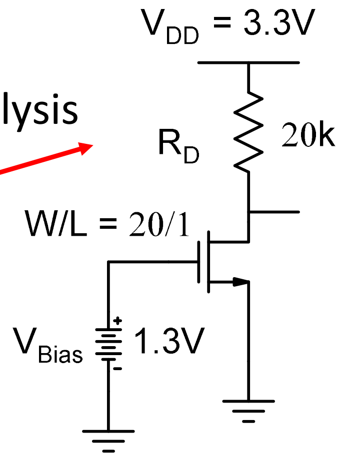
Example-4

Determine the voltage gain of the amplifier $\frac{v_{out}}{v_S}$



$KP_N = 100\mu A/V^2; V_{THN} = 1V; \lambda_n = 0V^{-1}$

dc or Bias point analysis



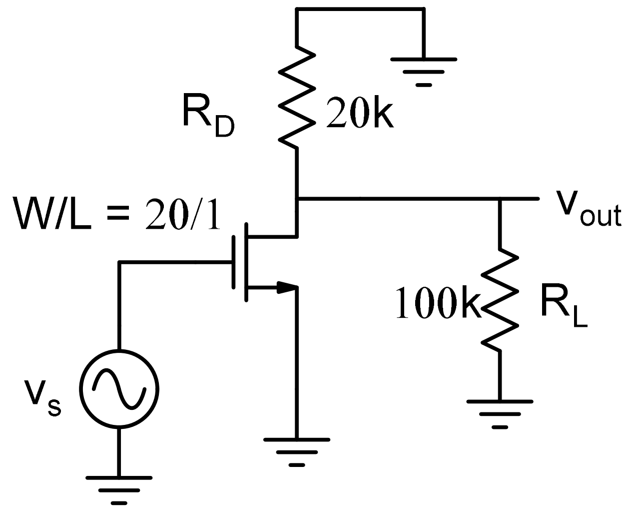
$$I_{DS} = KP_N \times \frac{W}{L} \times \frac{(V_{GS}-V_{THN})^2}{2} = 90\mu A$$

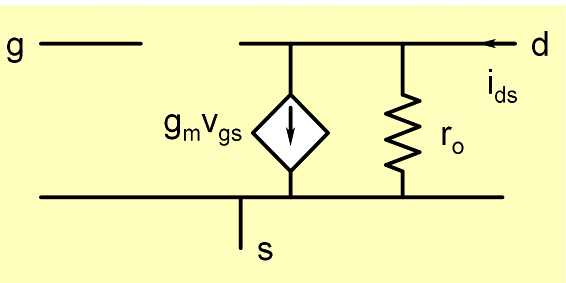
$$V_{DSQ} = V_{DD} - I_{DSQ} \times R_D = 1.5V$$

$$V_{Dsat} = V_{GS} - V_{THN} = 0.3V$$

So Transistor is biased in saturation

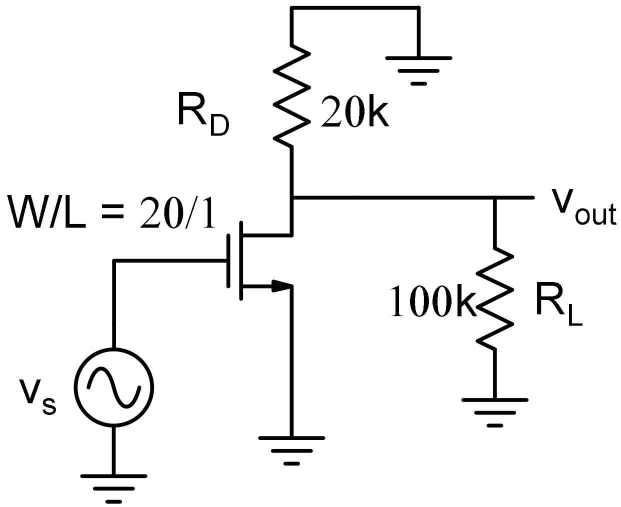
ac or small signal analysis

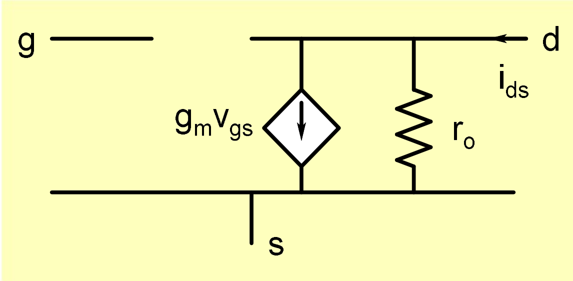




$$g_m = \frac{2I_{DSQ}}{V_{GSQ} - V_{THN}} = \sqrt{2I_{DSQ}\beta} = 6 \times 10^{-4}\Omega^{-1}$$

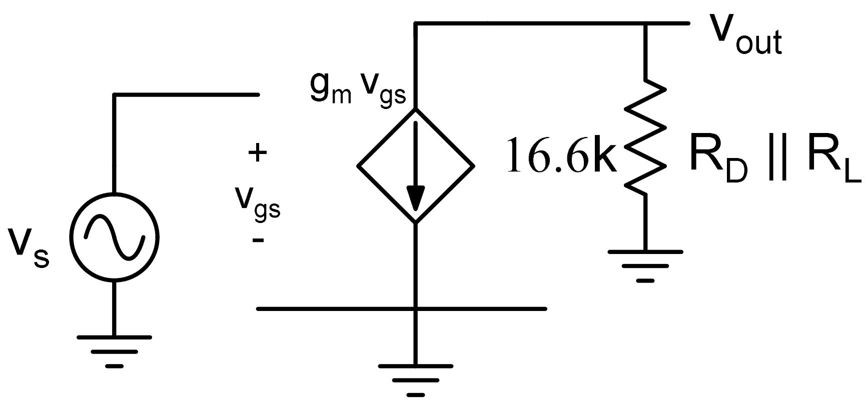
$$r_o = \frac{1}{\lambda_n I_{DSQ}} = \infty$$





$$g_m = \frac{2I_{DSQ}}{V_{GSQ} - V_{THN}} = \sqrt{2I_{DSQ}\beta}$$
$$= 6 \times 10^{-4}\Omega^{-1}$$

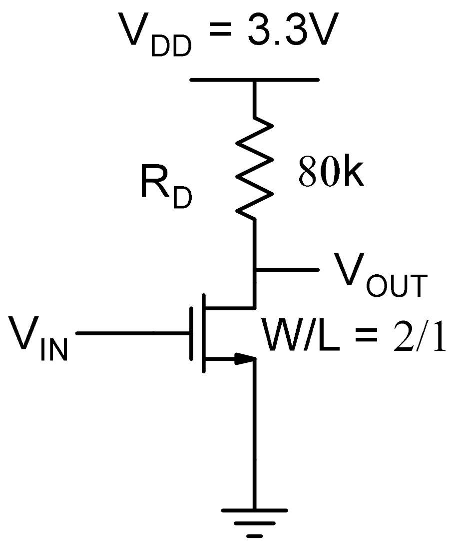
$$r_o = \frac{1}{\lambda_n I_{DSQ}} = \infty$$

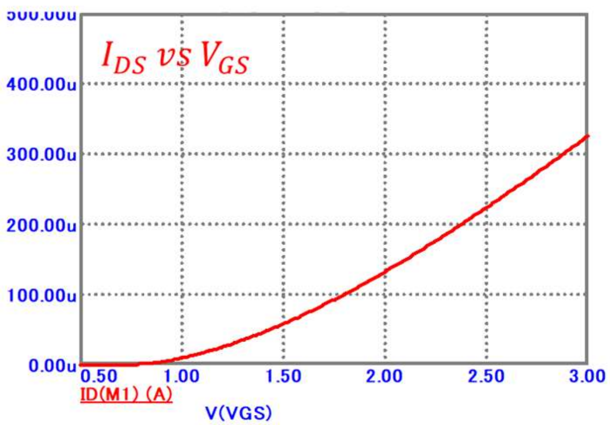


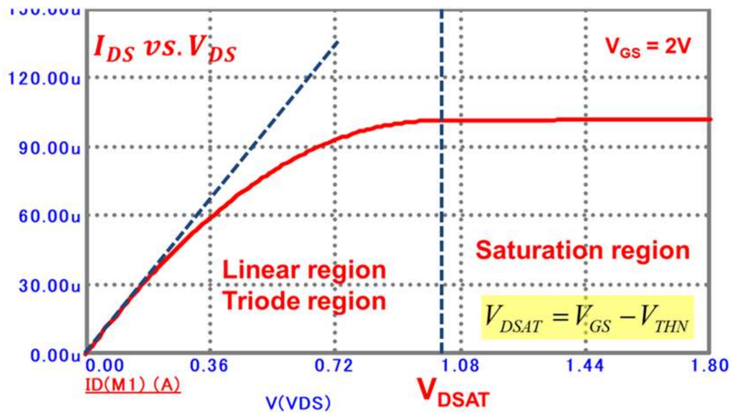
$$\frac{v_{out}}{v_s} = -g_m \times R_D \| R_L = -10$$

Example-5 Plot V_{OUT} vs. V_{IN} curve.

MOS Digital NOT gate



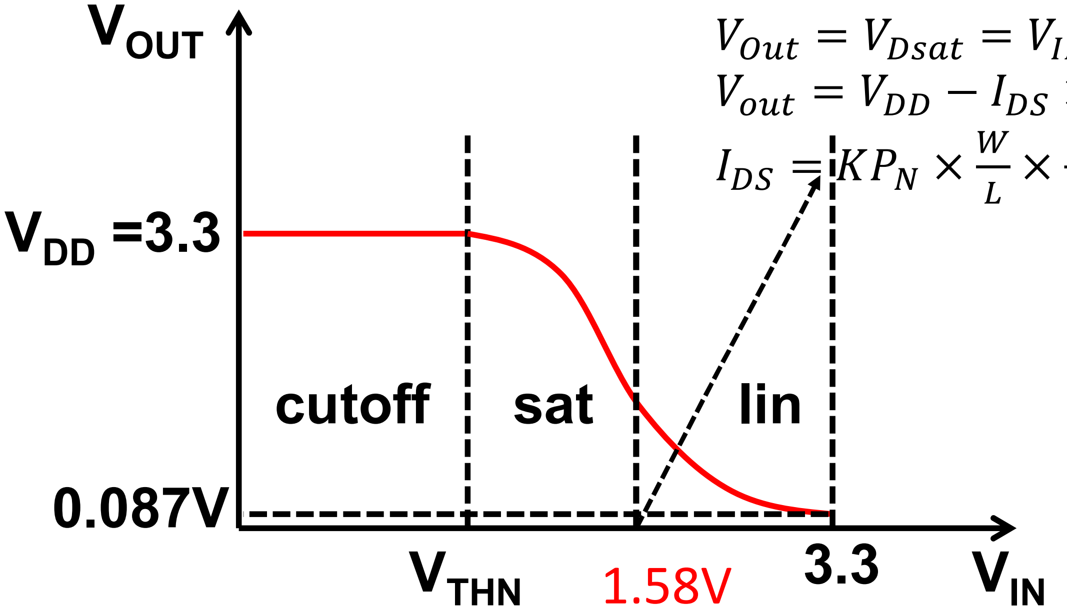




$KP_N = 100\mu A/V^2; V_{THN} = 1V; \lambda_n = 0.01V^{-1}$

$$V_{out} = V_{DD} - I_{DS} \times R_D$$

$$I_{DS} = KP_N \times \frac{W}{L} \times (V_{IN} - V_{THN}) \times V_{out}$$



$$V_{Out} = V_{Dsat} = V_{IN} - V_{THN}$$

$$V_{out} = V_{DD} - I_{DS} \times R_D$$

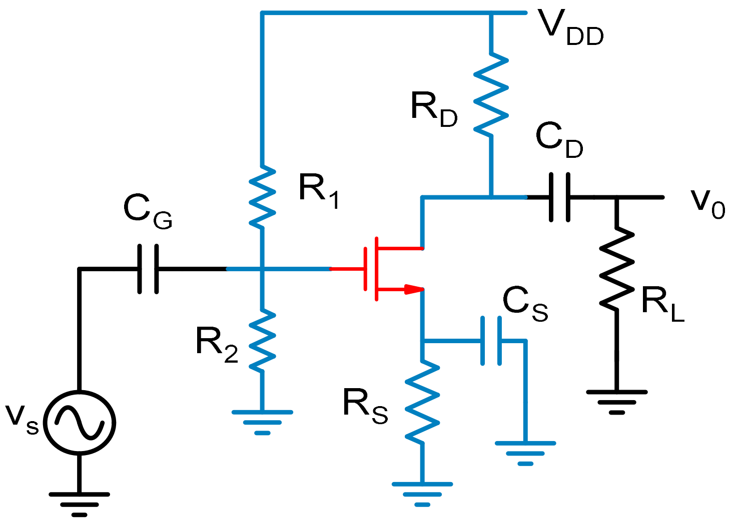
$$I_{DS} = KP_N \times \frac{W}{L} \times \frac{(V_{GS} - V_{THN})^2}{2}$$

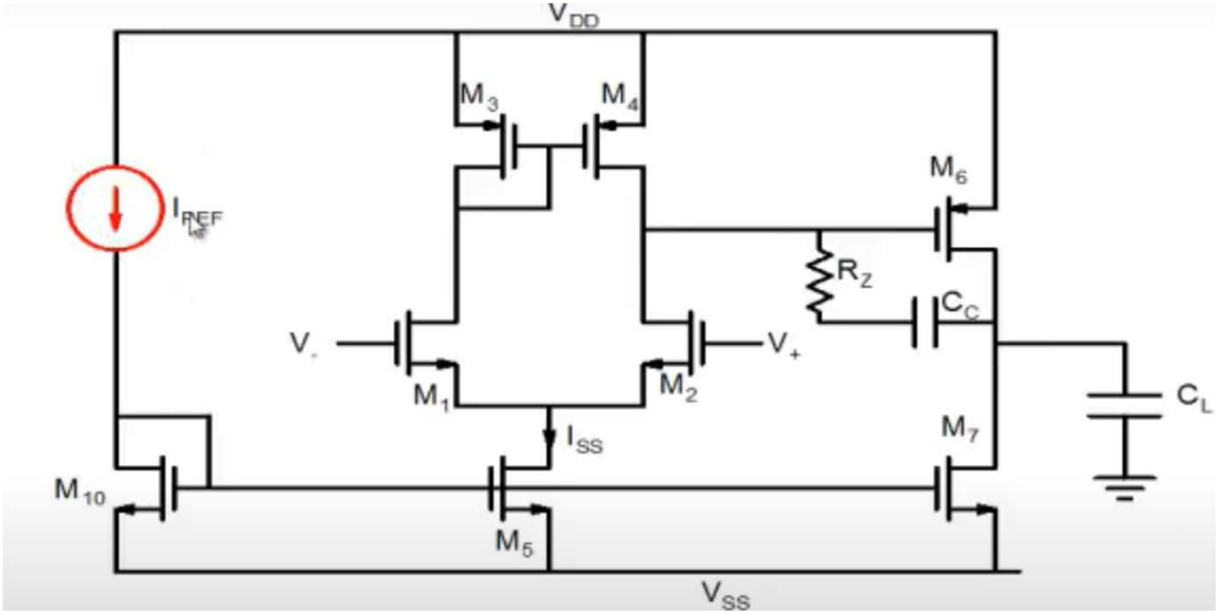
ESC201T : Introduction to Electronics

Lecture 29: Operational Amplifier

B. Mazhari
Dept. of EE, IIT Kanpur

Amplifier Design requires specialized knowledge





It is not possible for every user to design his/her own amplifier !

Why can’t we have experts design and implement amplifiers and make it available to everybody else !

Although this is done, it does not satisfy all the users due to diverse requirements

Custom

Design → **Semi-Custom**

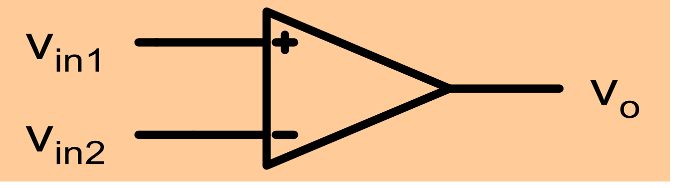
Standard

Semi-custom: partially competed design which is customized by the user

Opamp is a good illustration of the advantages of semi-custom approach

Difference Amplifier

-An amplifier that is sensitive to difference in input voltages and insensitive to what is common.



$$v_{id} = v_{in1} - v_{in2}$$

$$v_{ic} = \frac{v_{in1} + v_{in2}}{2}$$

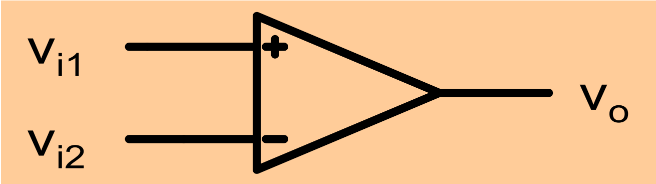
$$v_o = A_d v_{id} + A_{cm} v_{ic}$$

A_d : Differential mode gain

A_{cm} : Common mode gain

$$A_d >> A_{cm}$$

Common Mode Rejection Ratio: $CMRR = \dfrac{A_d}{A_{cm}}$



$$A_d = 100; \quad A_{cm} = 0.01$$

$$v_{i1} = 1V + 5mV \times Sin(\omega t) \; ; \; v_{i2} = 1V - 5mV \times Sin(\omega t)$$

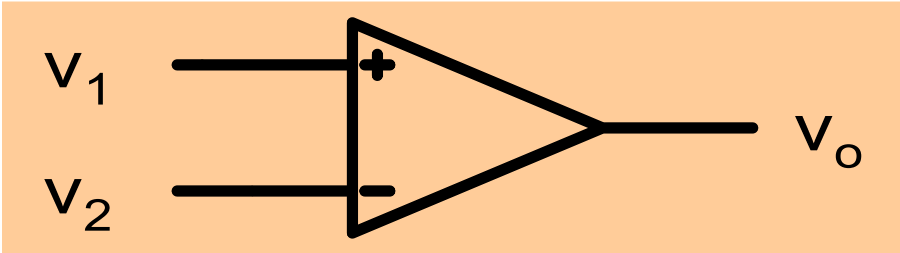
$$v_{id} = v_{in1} - v_{in2} = 10mV \times Sin(\omega t)$$

$$v_{ic} = \frac{v_{in1} + v_{in2}}{2} = 1V$$

$$v_o = A_d v_{id} + A_{cm} v_{ic}$$
$$= 1V \times Sin(\omega t) + 10mV$$

Whatever is common is rejected and whatever is different is amplified !

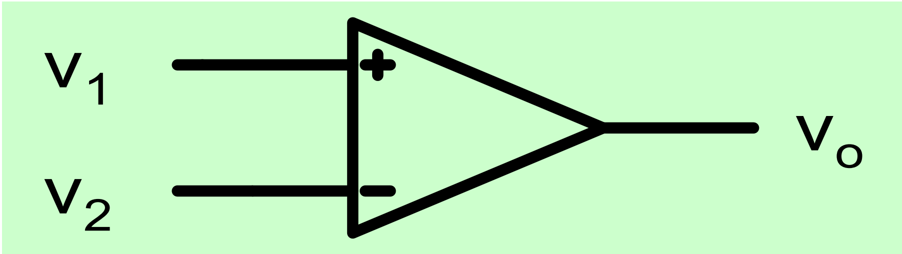
Operational Amplifier



A special kind of difference amplifier

1. Very High Differential-mode voltage gain
2. Very High Common mode Rejection ratio
3. Very High Input Resistance
4. Very Low output Resistance
5. ….

Ideal Operational Amplifier



1. Infinite Differential-mode voltage gain
2. Infinite Common mode Rejection ratio
3. Infinite Input Resistance
4. Zero output Resistance
5. ….

Example: LM 741

LM741
Operational Amplifier

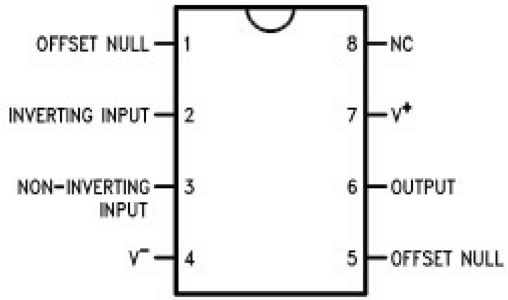
General Description

The LM741 series are general purpose operational amplifiers which feature improved performance over industry standards like the LM709. They are direct, plug-in replacements for the 709C, LM201, MC1439 and 748 in most applications.

The amplifiers offer many features which make their application nearly foolproof: overload protection on the input and output, no latch-up when the common mode range is exceeded, as well as freedom from oscillations.

The LM741C is identical to the LM741/LM741A except that the LM741C has their performance guaranteed over a 0°C to +70°C temperature range, instead of −55°C to +125°C.

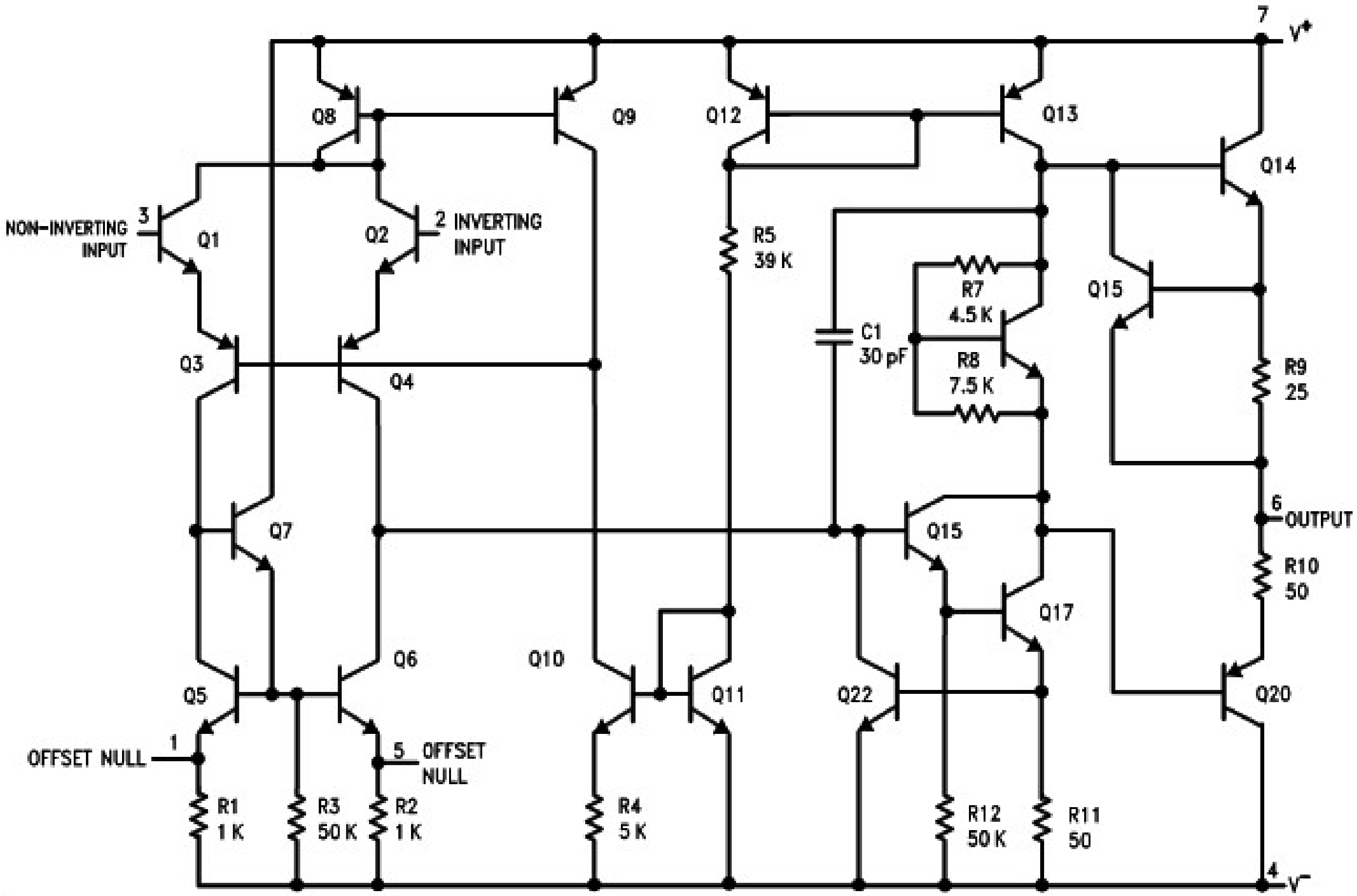
Dual-In-Line or S.O. Package



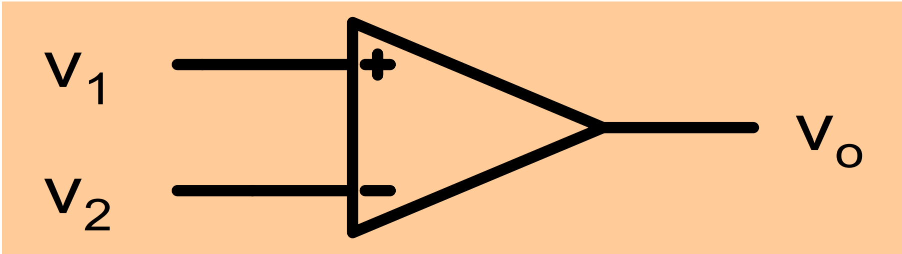
Parameter	Conditions	LM741A			LM741			LM741C			Units
		Min	Typ	Max	Min	Typ	Max	Min	Typ	Max	
Input Resistance	$T_A = 25°C$, $V_S = \pm 20V$	1.0	6.0		0.3	$\boxed{2.0}$		0.3	2.0		MΩ
	$T_{AMIN} \le T_A \le T_{AMAX}$, $V_S = \pm 20V$	0.5									MΩ
Large Signal Voltage Gain	$T_A = 25°C$, $R_L \ge 2\ k\Omega$										
	$V_S = \pm 20V$, $V_O = \pm 15V$	50									V/mV
	$V_S = \pm 15V$, $V_O = \pm 10V$				50	$\boxed{200}$		20	200		V/mV
Common-Mode Rejection Ratio	$T_{AMIN} \le T_A \le T_{AMAX}$										
	$R_S \le 10\ k\Omega$, $V_{CM} = \pm 12V$				70	$\boxed{90}$		70	90		dB
	$R_S \le 50\Omega$, $V_{CM} = \pm 12V$	80	95								dB

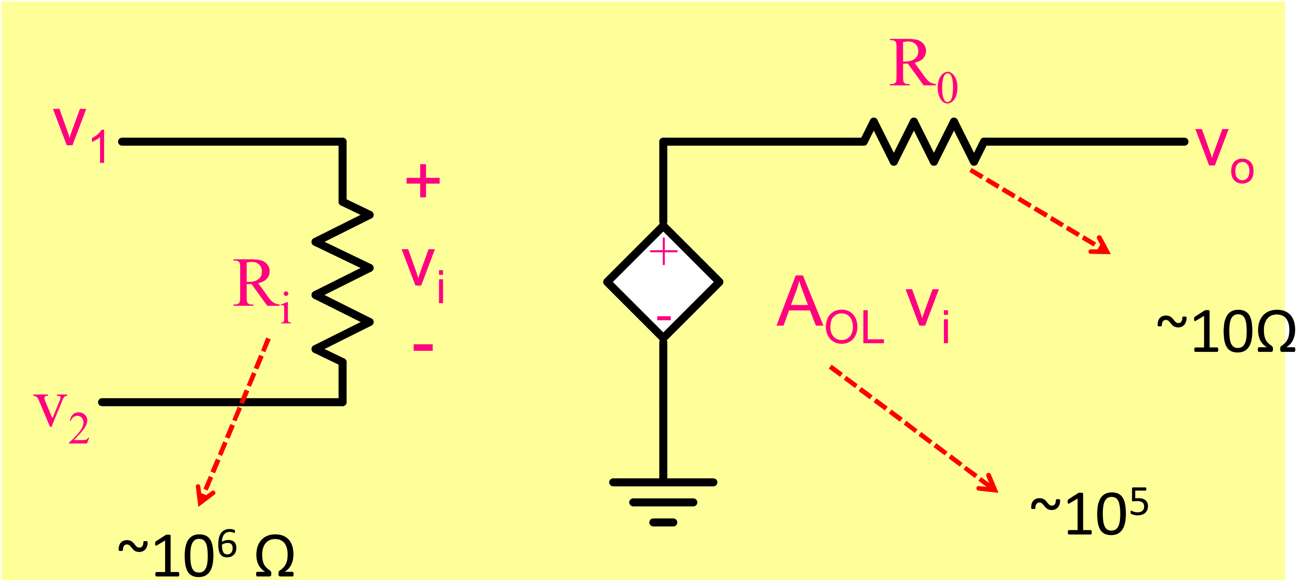
Inside the opamp, there is a complicated circuit containing several transistors and resistors.

Schematic Diagram

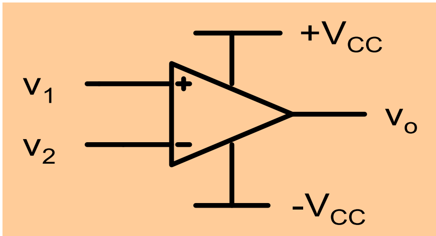


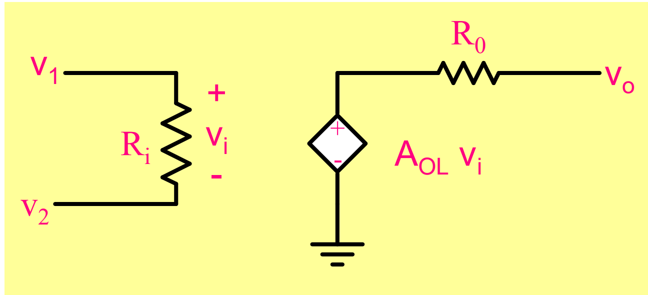
Simple equivalent circuit model of an opamp

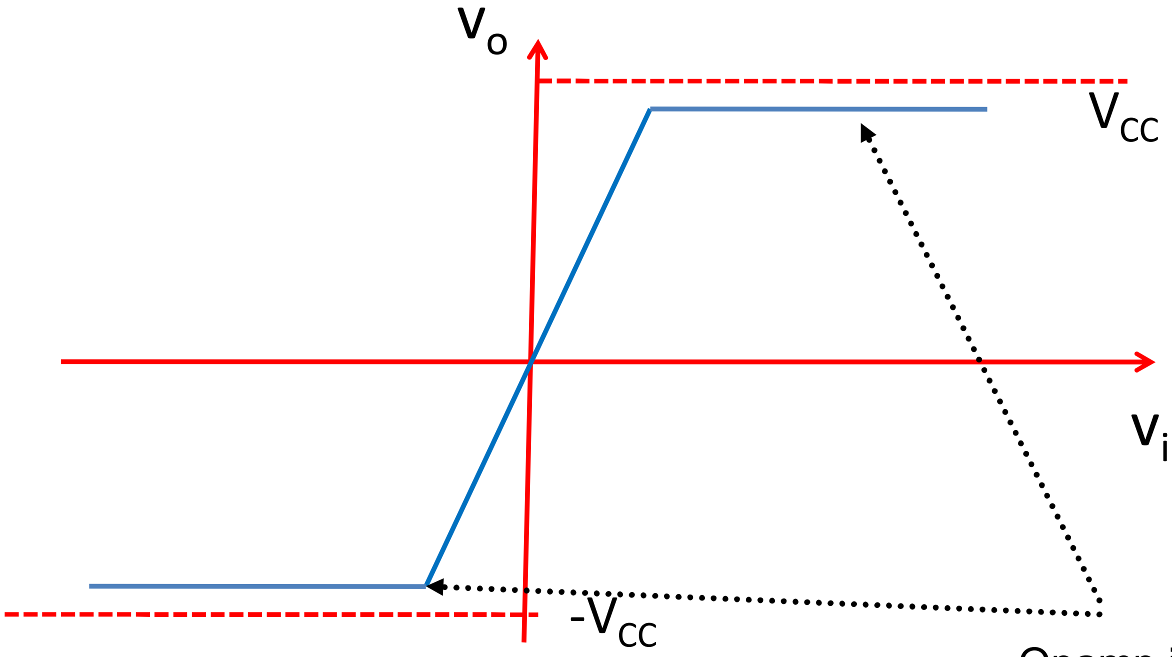




This assumes very high CMRR

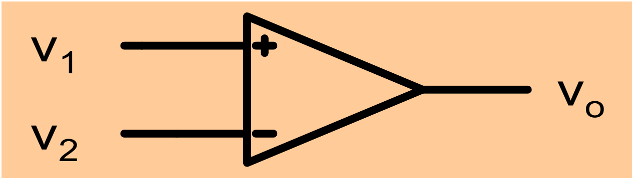


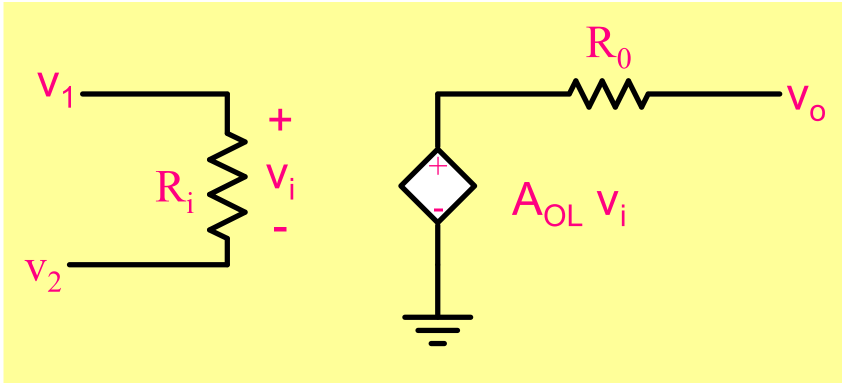


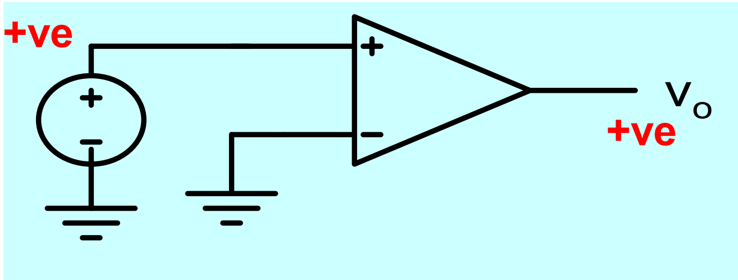


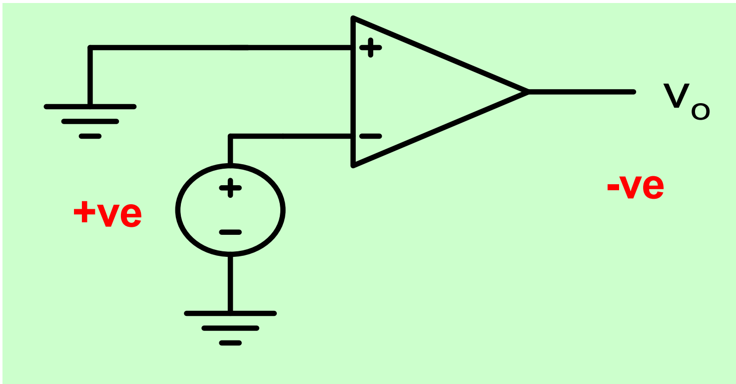
$$v_o = A_{ol} \times v_i$$

Opamp is said to be saturated

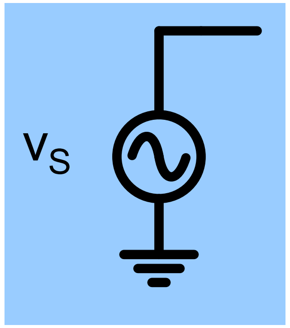

v1
v2
vo


R0
v1
vo
+
Ri
vi
-
AOL vi
v2

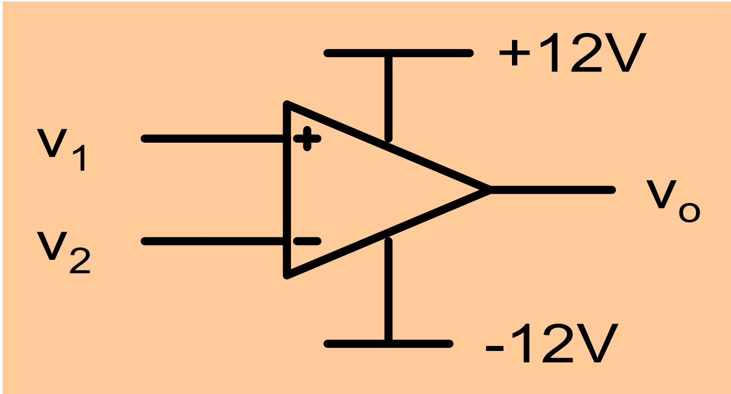

+ve
vo
+ve

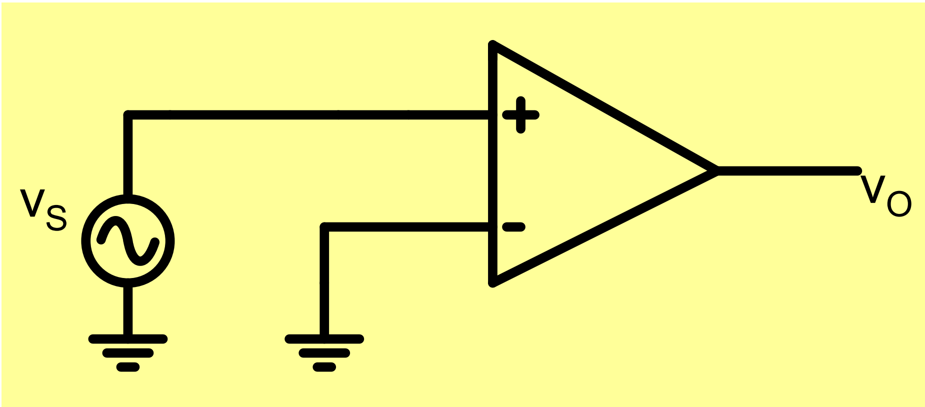

vo
+ve
-ve

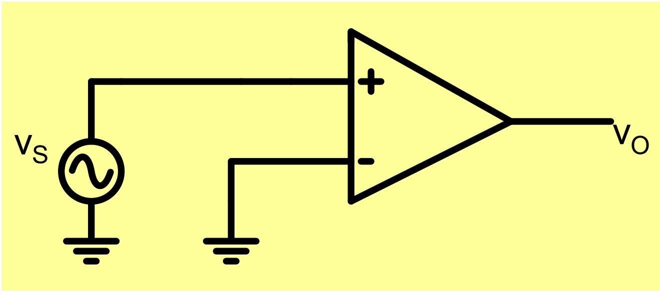
How do we amplify this signal?

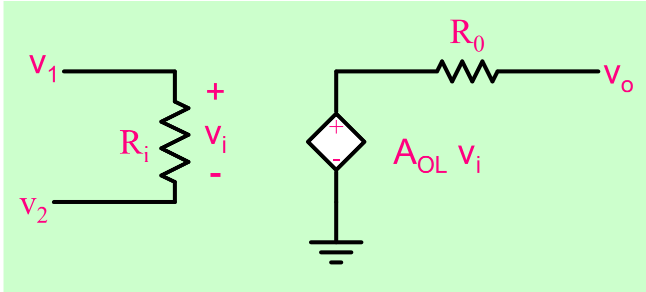


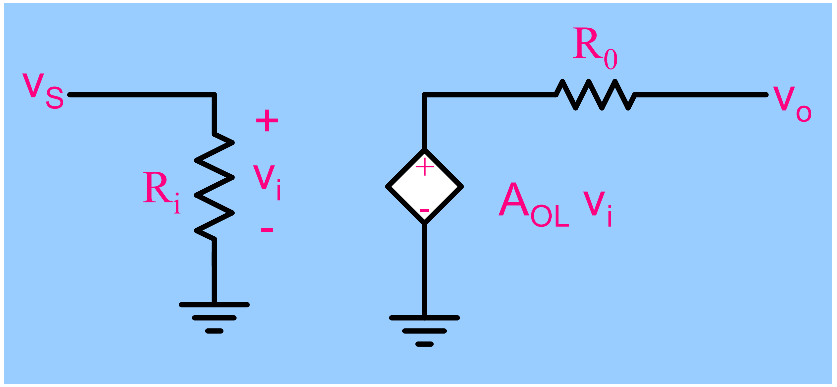
$$v_s = 1mV\ Sin(\omega t)$$







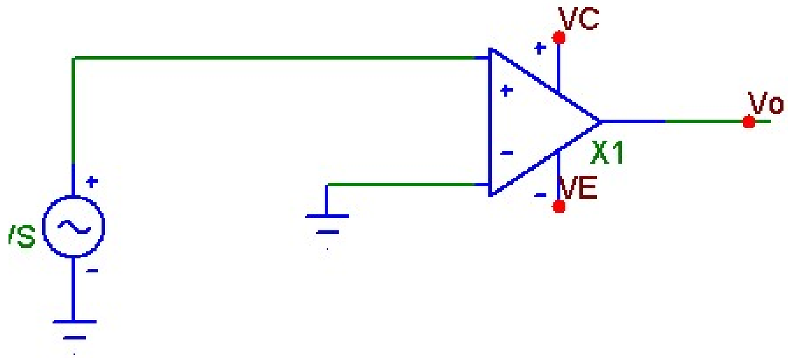


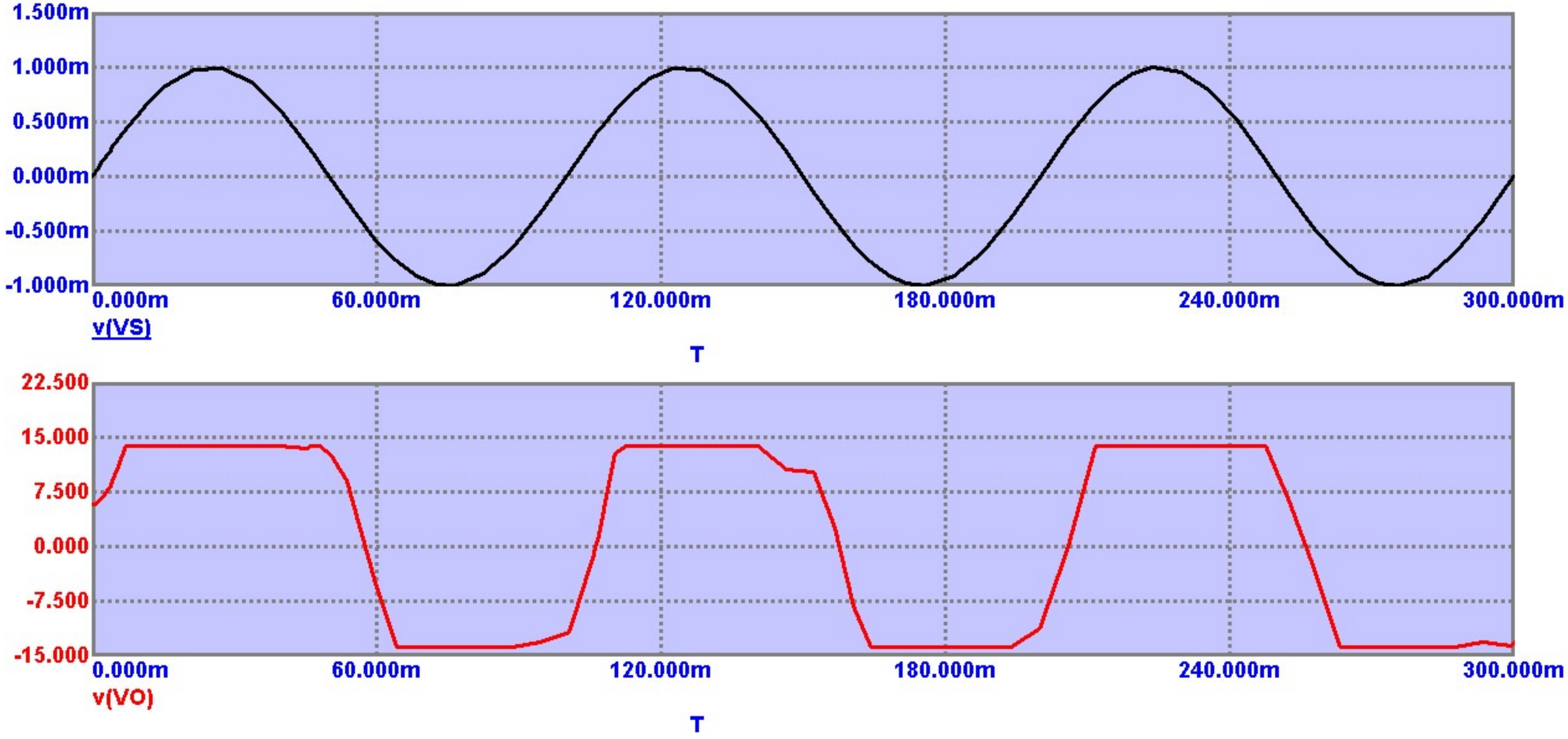


$$v_o = A_{ol} \times v_S = 10^2\, Sin(\omega t)$$

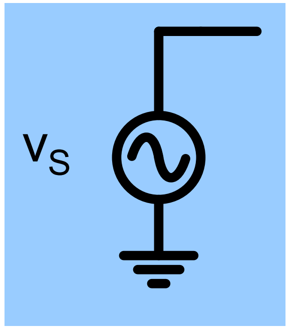
But opamp voltage is limited to $\pm 12V$

Simulation Results

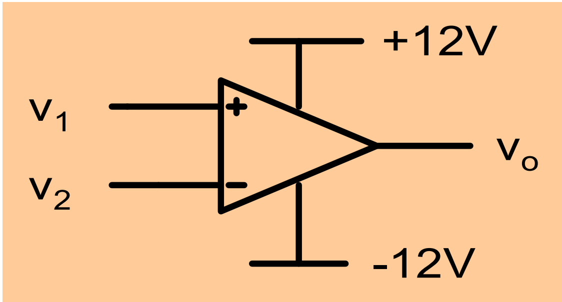


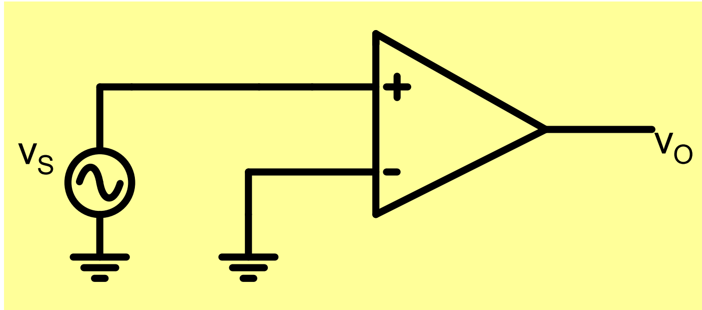


How do we amplify this signal then ?



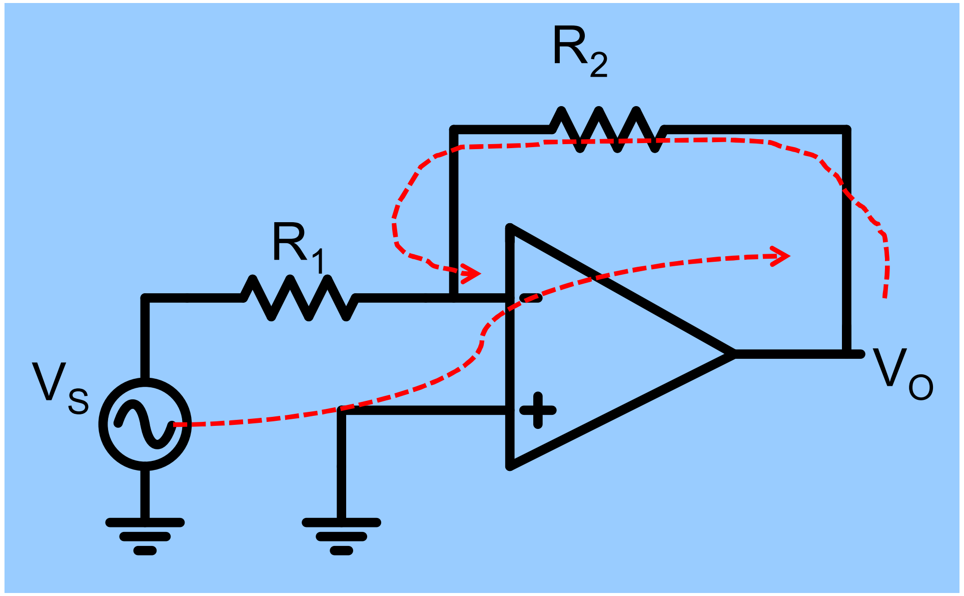
$$v_S = 1mV\ Sin(\omega t)$$





1. Attenuate the signal to 0.1mV and then amplify ?
2. …….

A Better Solution

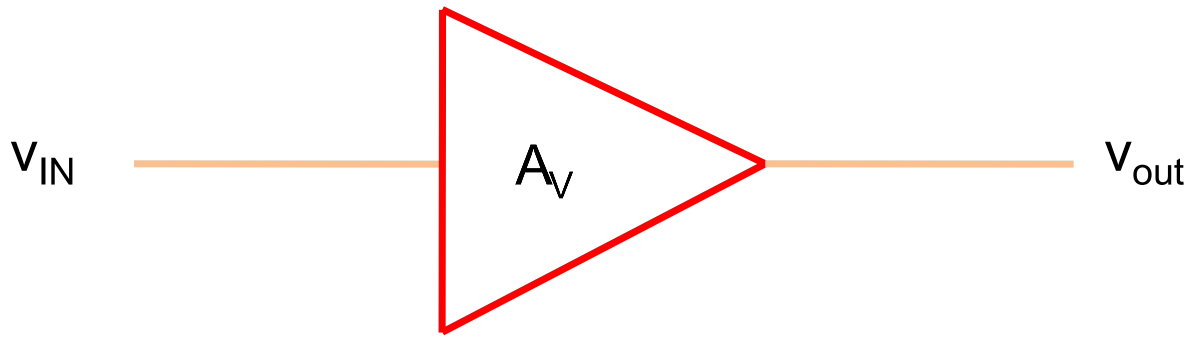


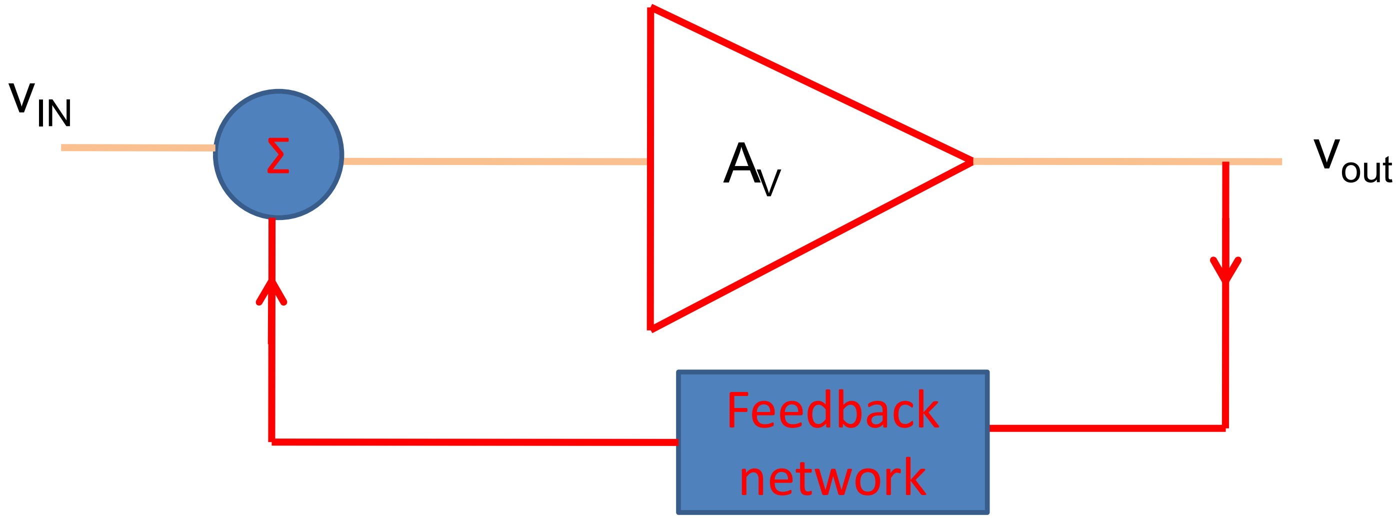
$$\frac{v_O}{v_S} = -\frac{R_2}{R_1}$$

Amplifier has feedback

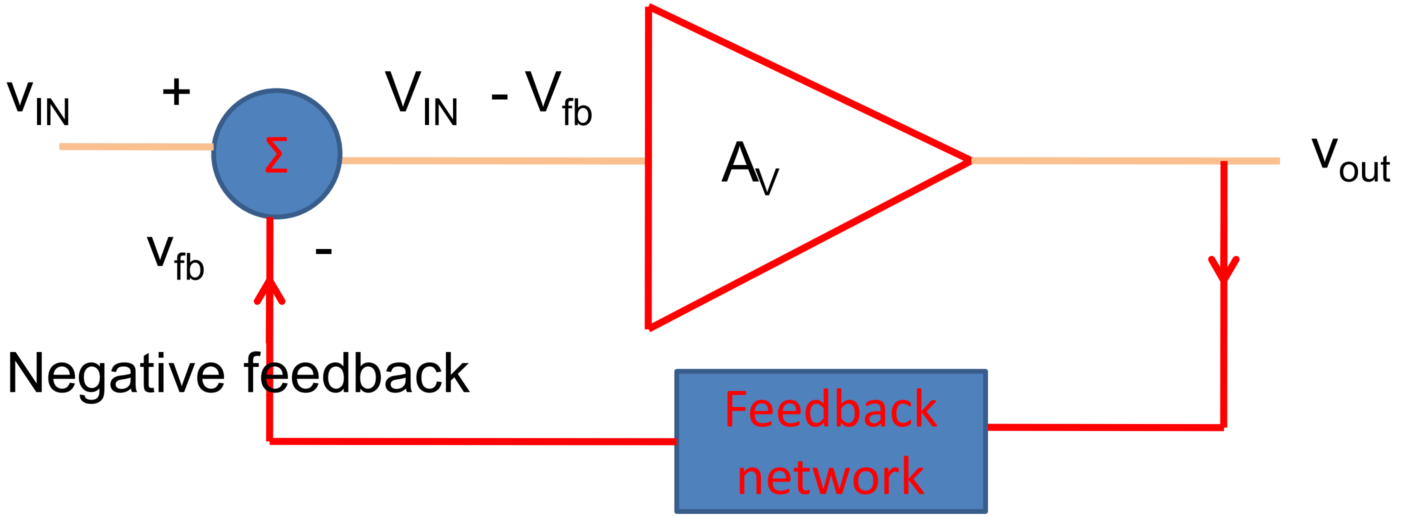
If the feedback signal helps the input voltage we have positive feedback, otherwise negative.

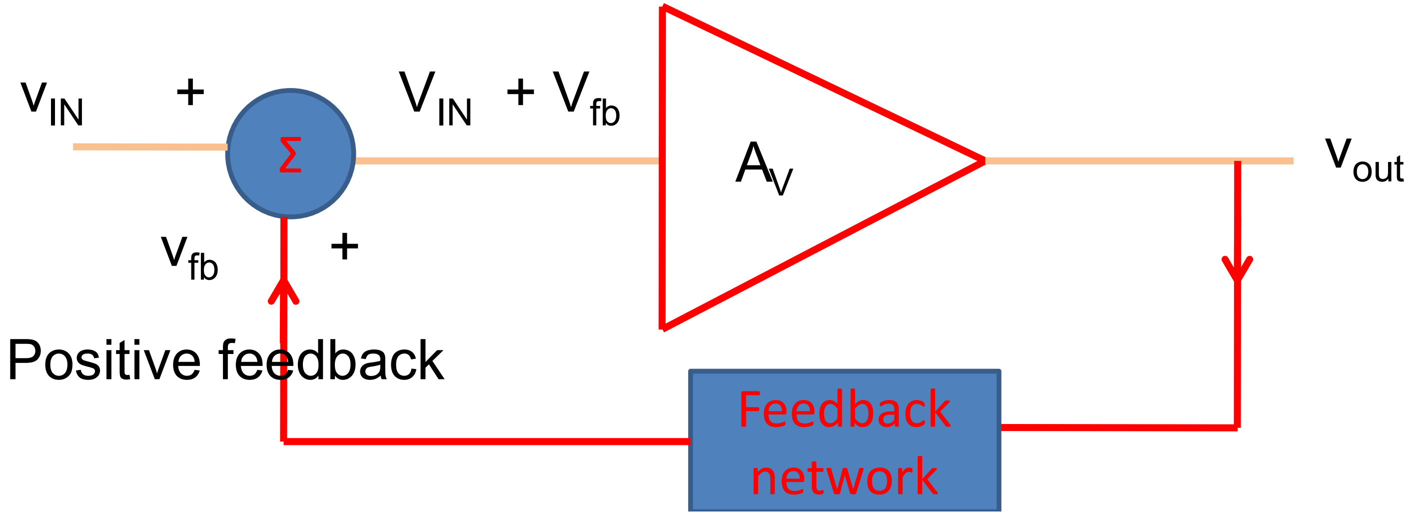
Feedback



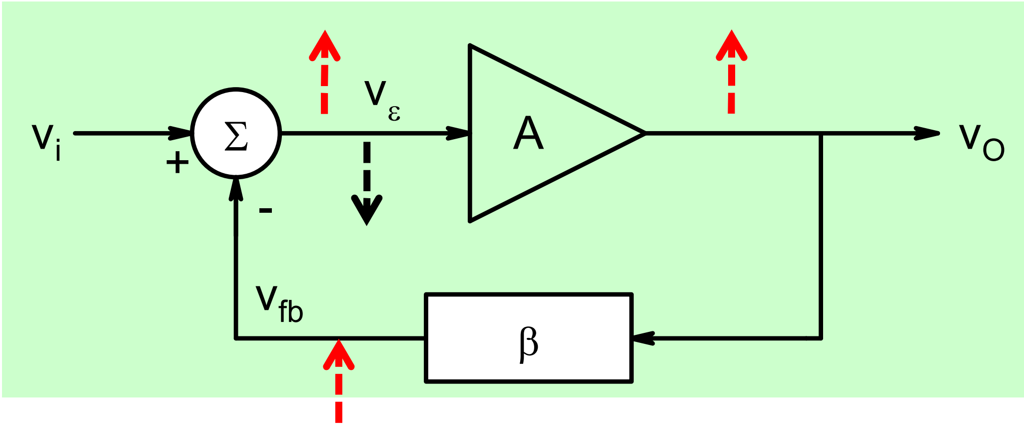


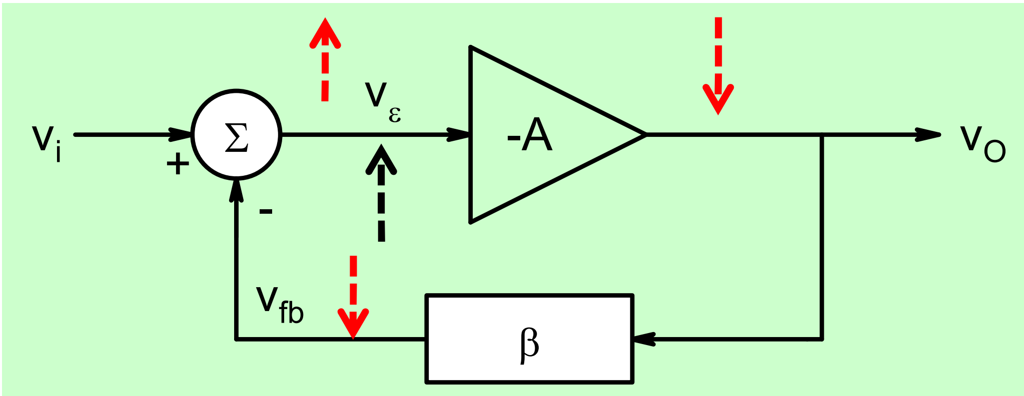
Negative and Positive feedback





Negative and positive feedback





Opamp circuits classification

Negative feedback

Closed loop
Or feedback

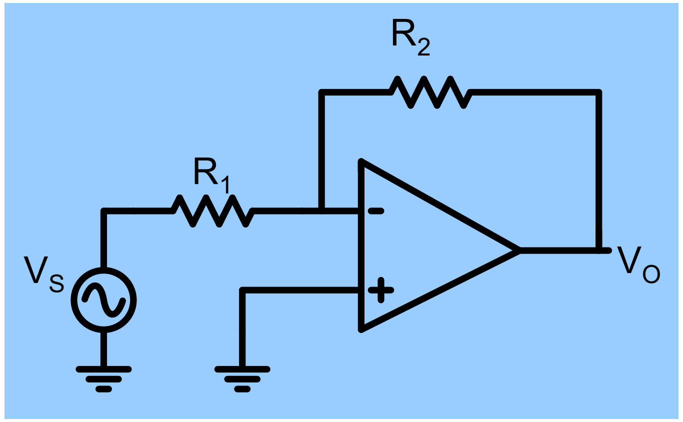
Opamp circuits

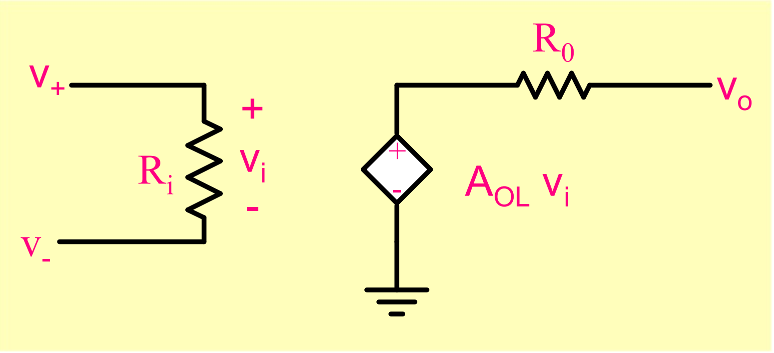
Positive feedback

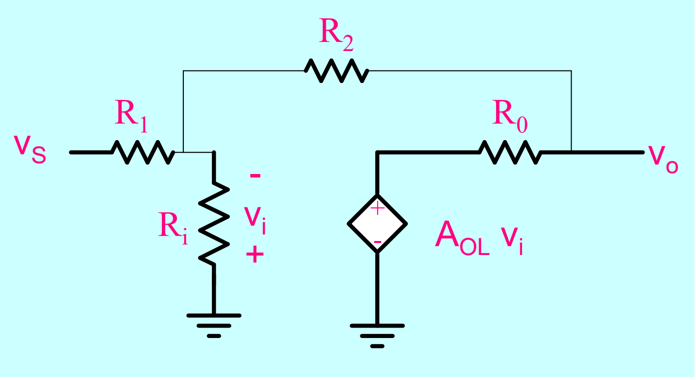
Open loop

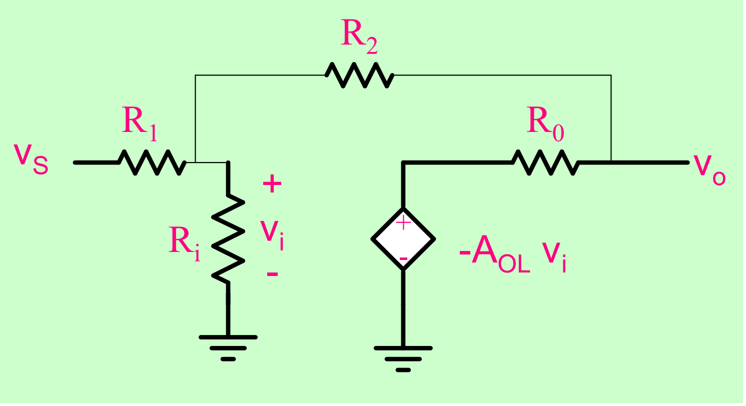
Most Opamp Circuits employ negative feedback

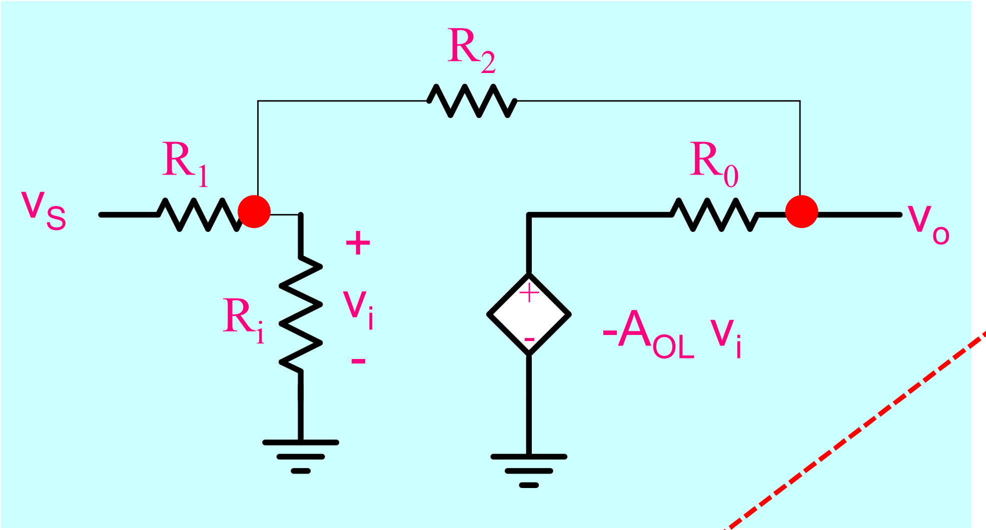
Inverting amplifier











Nodal Analysis

$$\frac{v_S - v_i}{R_1} = \frac{v_i}{R_i} + \frac{v_i - v_o}{R_2} \quad (1)$$

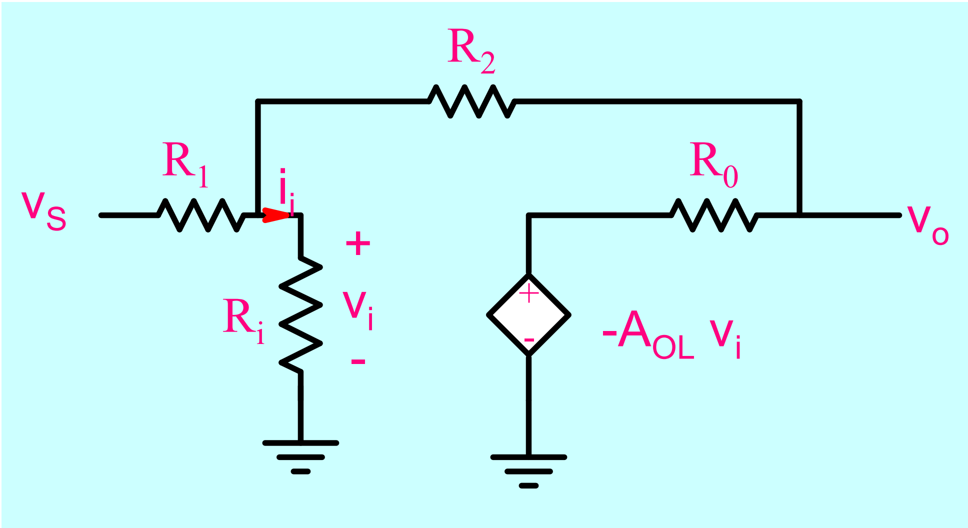
$$\frac{-A_{OL} v_i - v_o}{R_o} = \frac{v_o - v_i}{R_2} \quad (2)$$

$$\frac{v_S}{R_1} = -\frac{v_o}{R_2} + v_i\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_i}\right) \quad (3)$$

$$v_i = v_o \frac{\frac{1}{R_o} + \frac{1}{R_2}}{\frac{-A_{OL}}{R_o} + \frac{1}{R_2}} \quad (4)$$

As $A_{OL} \to \infty \quad v_i \to 0$

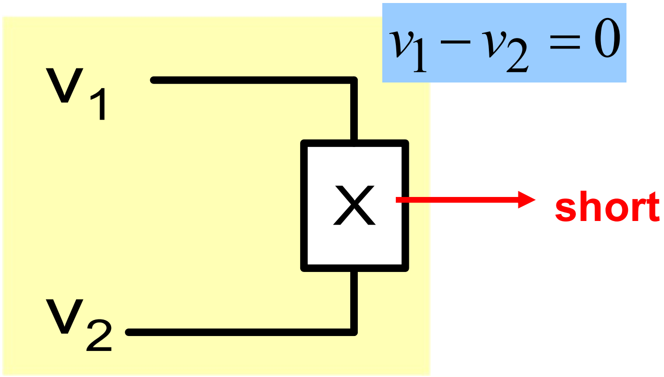
This is called the Virtual Ground property

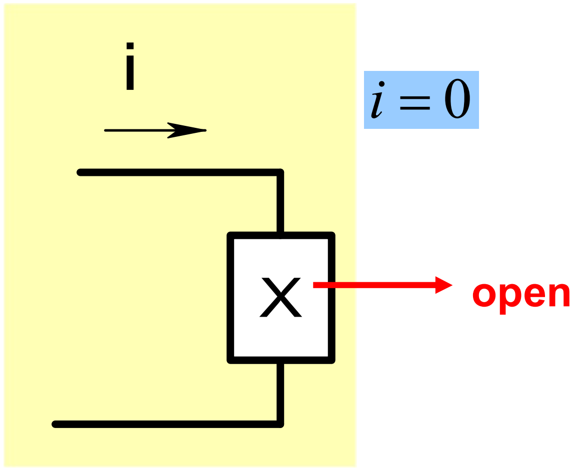


As $A_{OL} \to \infty \quad v_i \to 0$

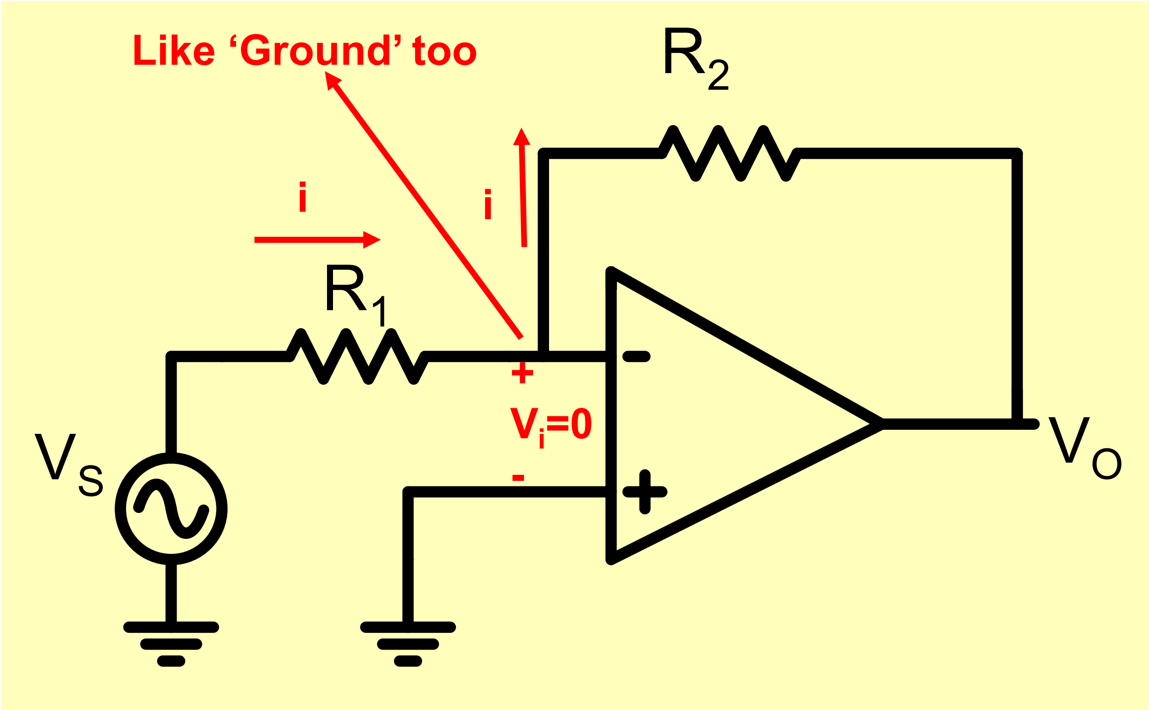
This implies that : $i_i \to 0$

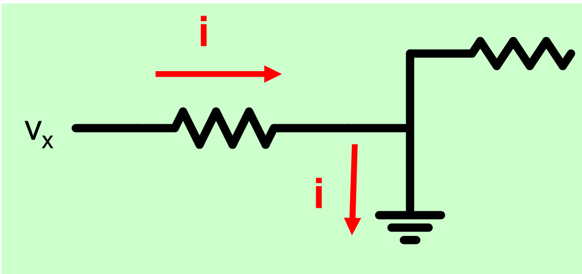
No current flows in or out of either inverting or non-inverting terminals of an ideal opamp





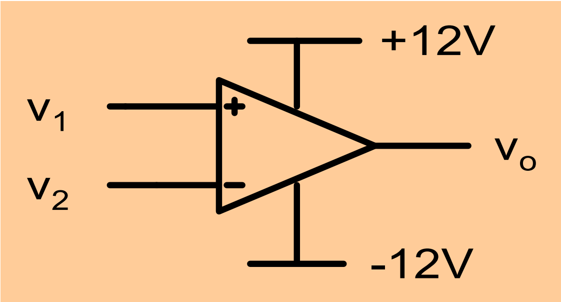
Can something be both a short as well as open circuit ?





Hence the name Virtual ground

Virtual Ground Property



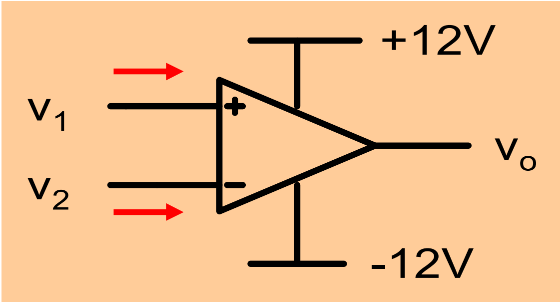
$$v_1 \cong v_2$$

In an opamp with negative feedback, the voltage of the inverting terminal is equal to the voltage of the non-inverting terminal if the gain of the opamp is sufficiently high

This property does not hold under certain conditions such as

- open loop,
- positive feedback
- or if the opamp is saturated.

Two important property for analyzing ideal opamp circuits under negative feedback



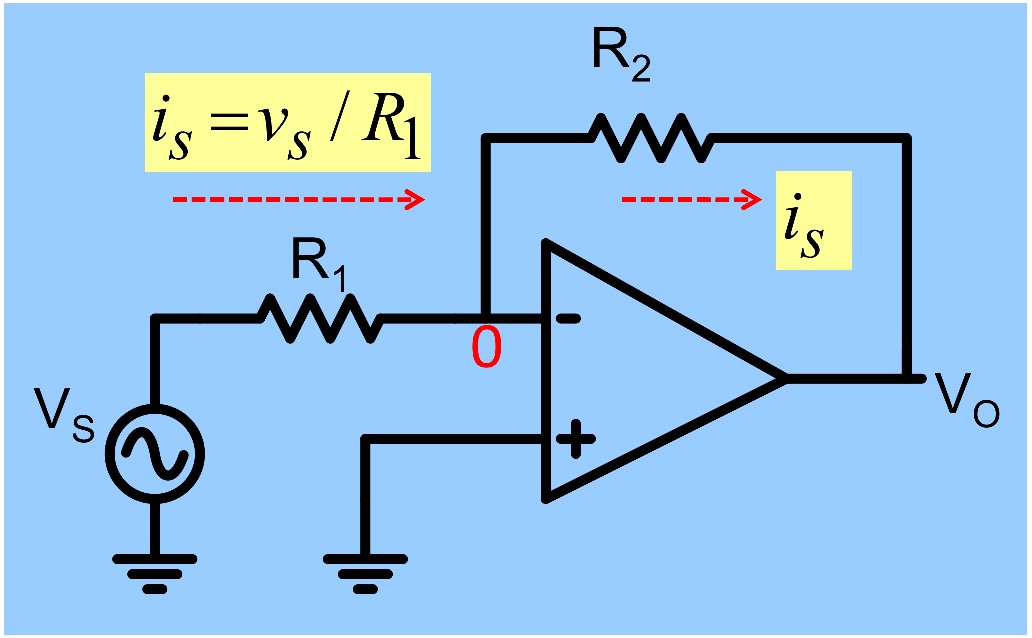
$$1. \quad v_1 = v_2$$

$$2. \quad i_1 = i_2 = 0$$

At the input side opamp appears to be like a short and an open circuit simultaneously !

Inverting amplifier

Re-analyze inverting amplifier with these properties



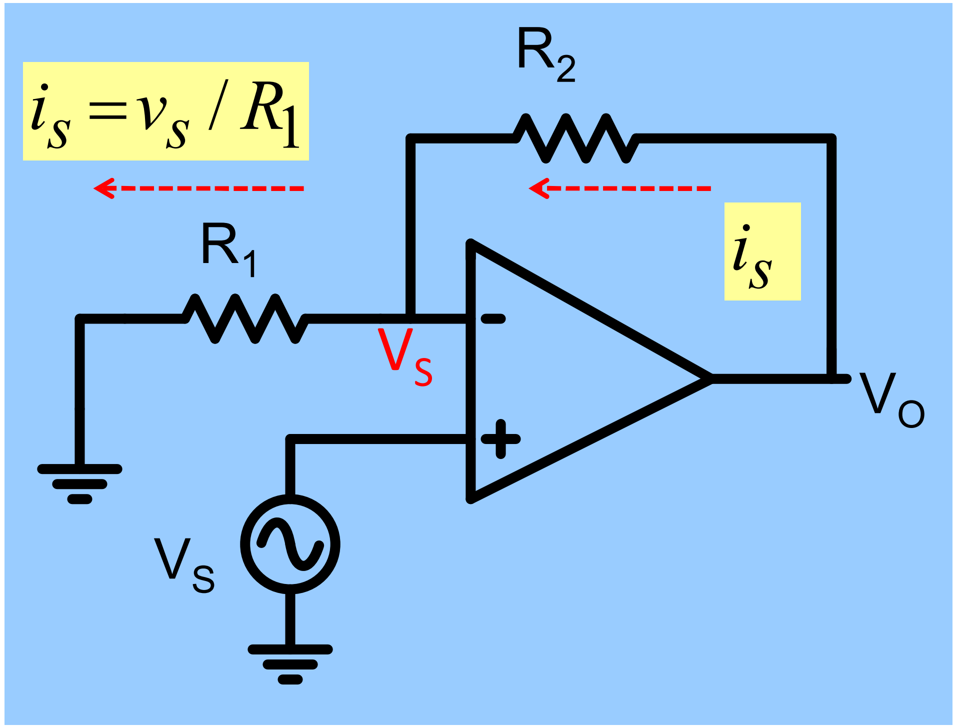
$$\frac{0 - v_o}{R_2} = i_s = \frac{v_s}{R_1}$$

$$\frac{v_o}{v_S} = -\frac{R_2}{R_1}$$

Non-Inverting Amplifier

$$1. \quad v_1 = v_2$$

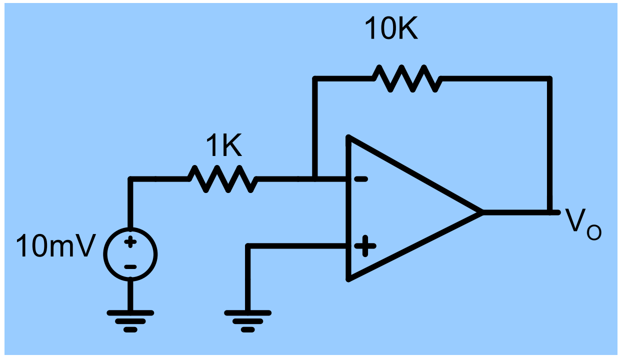
$$2. \quad i_1 = i_2 = 0$$



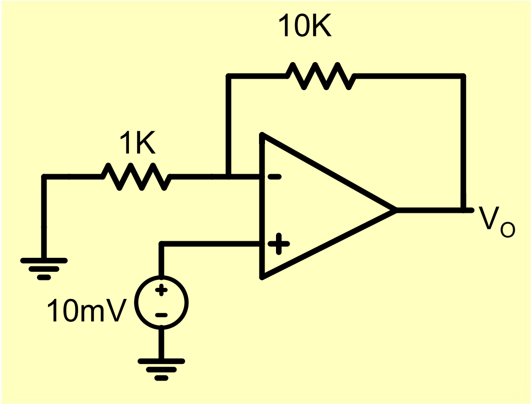
$$\frac{v_o - v_S}{R_2} = i_s = \frac{v_s}{R_1}$$

$$\frac{v_o}{v_S} = 1 + \frac{R_2}{R_1}$$

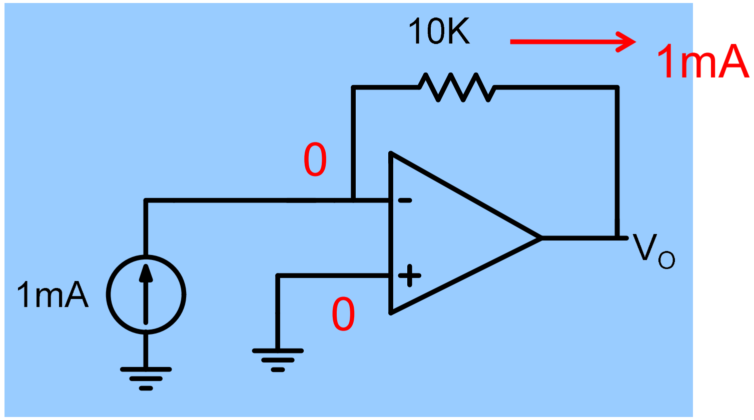
Examples



$$\frac{v_o}{v_S} = -\frac{R_2}{R_1} \Rightarrow v_o = -100mV$$



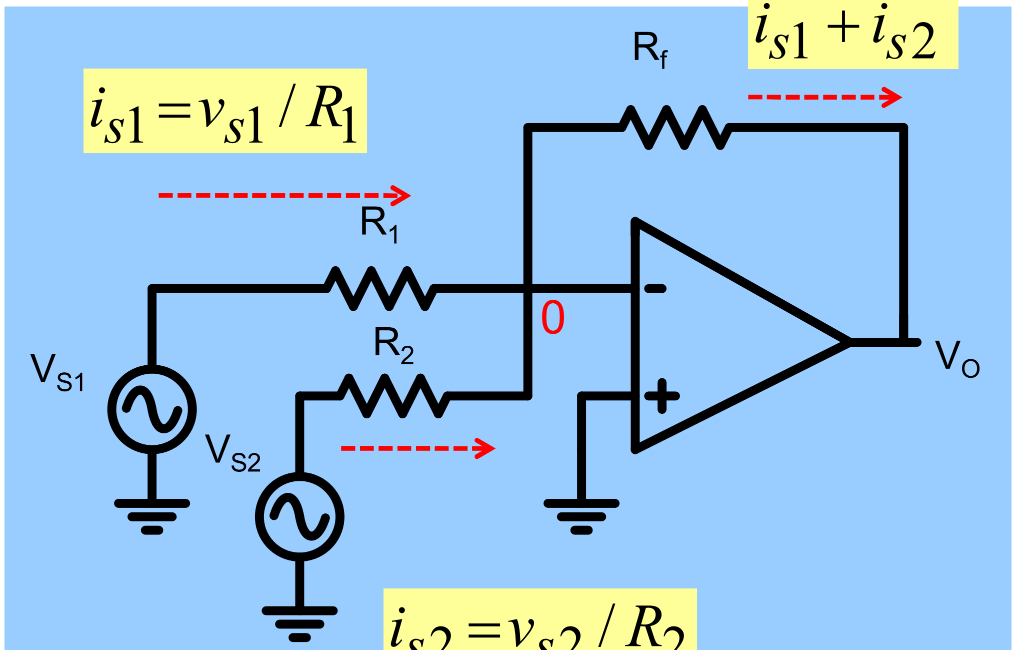
$$\frac{v_o}{v_S} = 1 + \frac{R_2}{R_1} \Rightarrow v_o = 110mV$$



$$\frac{0 - v_o}{10K} = 1mA$$

$$v_o = -10V$$

Adder



$$i_{s1} = v_{s1} / R_1$$

$$i_{s1} + i_{s2}$$

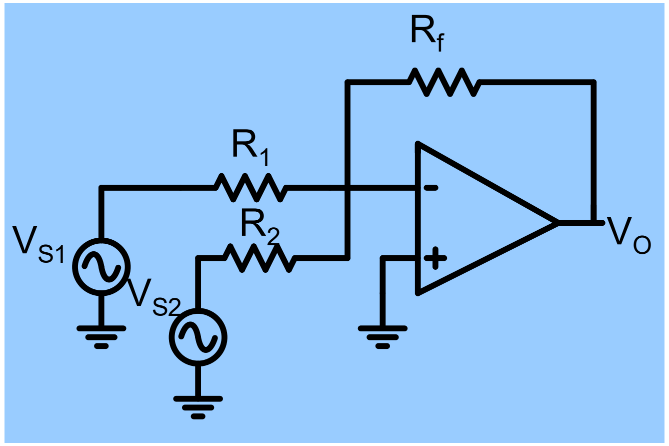
$$i_{s2} = v_{s2} / R_2$$

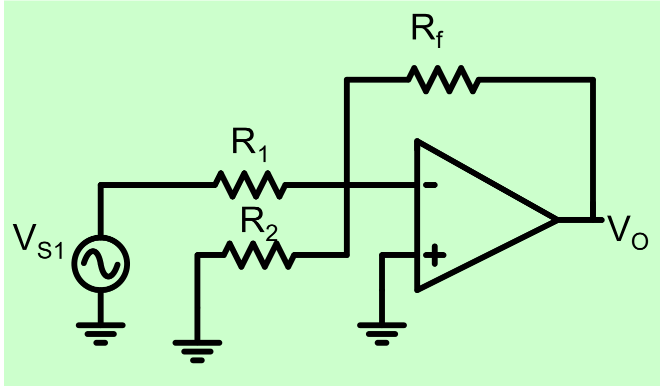
$$\frac{0 - v_o}{R_f} = i_{s1} + i_{s2} = \frac{v_{s1}}{R_1} + \frac{v_{s2}}{R_2}$$

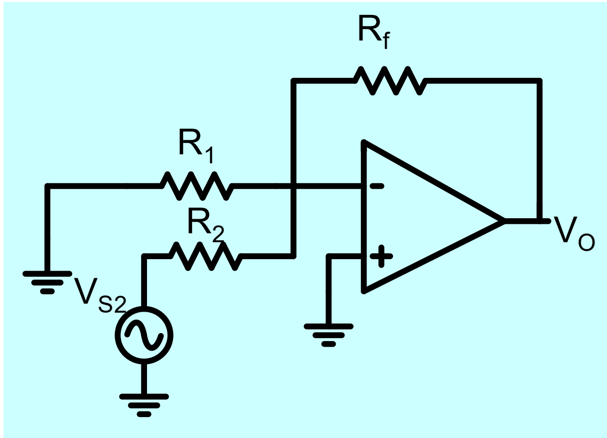
$$v_o = -\left(\frac{R_f}{R_1} v_{s1} + \frac{R_f}{R_2} v_{s2}\right)$$

$$\text{For } \mathrm{R}_1 = \mathrm{R}_2 = \mathrm{R} \quad v_o = -\frac{R_f}{R}(v_{s1} + v_{s2})$$

Alternative Analysis





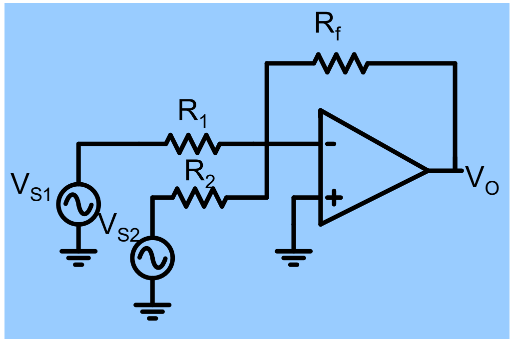


$$v_o = -(\frac{R_f}{R_1})v_{s1} \quad + -(\frac{R_f}{R_2})v_{s2}$$

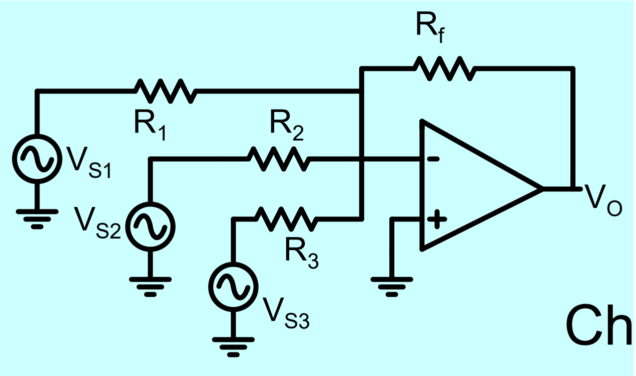
Design Example

Design a circuit that would generate the following output given three input voltages vs1, vs2 and vs3.

$$v_o = -10v_{s1} - 4v_{s2} - 5v_{s3}$$



$$v_o = -\frac{R_f}{R_1}v_{s1} - \frac{R_f}{R_2}v_{s2}$$

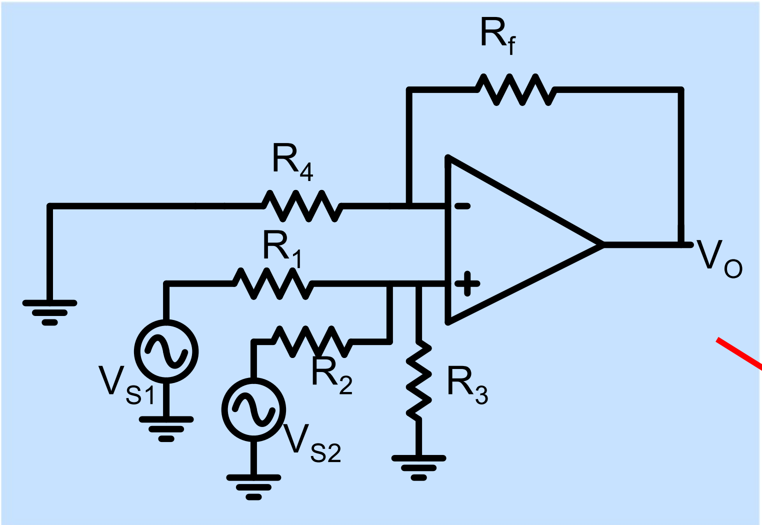


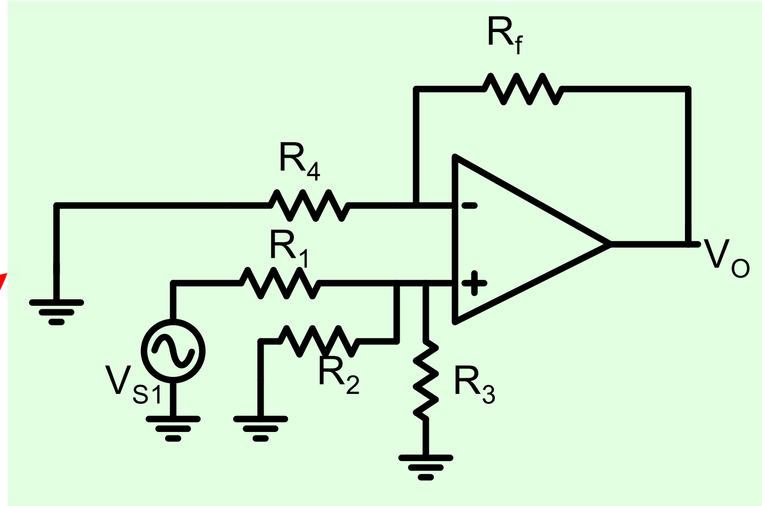
$$v_o = -\frac{R_f}{R_1}v_{s1} - \frac{R_f}{R_2}v_{s2} - \frac{R_f}{R_3}v_{s3}$$

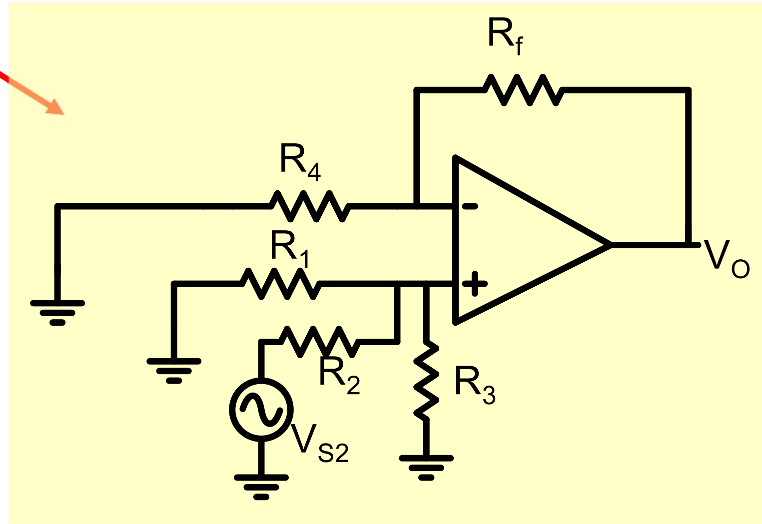
Choose : $R_f = 10K$ $\Rightarrow R_1 = 1K$

$\Rightarrow R_2 = 2.5K$ $\Rightarrow R_3 = 2K$

Adder



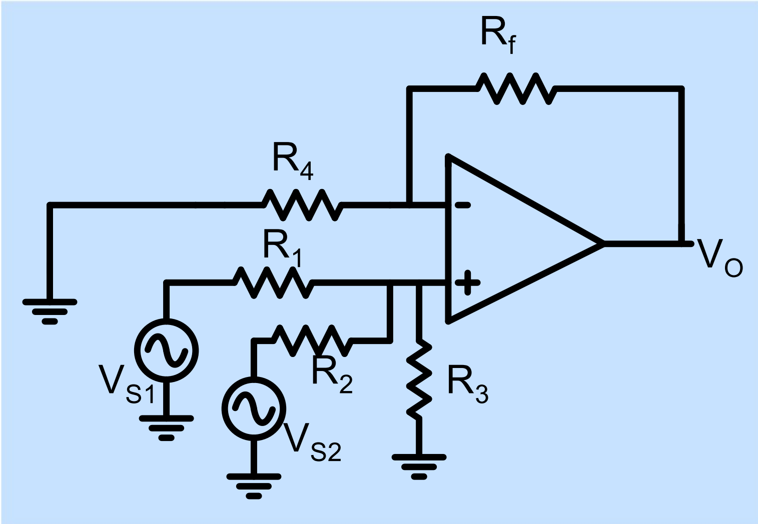




$$v_o = v_{s1} \frac{R_2 \| R_3}{R_2 \| R_3 + R_1} \times \left(1 + \frac{R_f}{R_4}\right)$$

$$+ v_{s2} \frac{R_1 \| R_3}{R_1 \| R_3 + R_2} \times \left(1 + \frac{R_f}{R_4}\right)$$

Adder



$$v_o = v_{s1}\frac{R_2 \| R_3}{R_2 \| R_3 + R_1} \times (1 + \frac{R_f}{R_4})$$

$$+ v_{s2}\frac{R_1 \| R_3}{R_1 \| R_3 + R_2} \times (1 + \frac{R_f}{R_4})$$

High entropy expression !

$$R_P = R_1 \| R_2 \| R_3$$

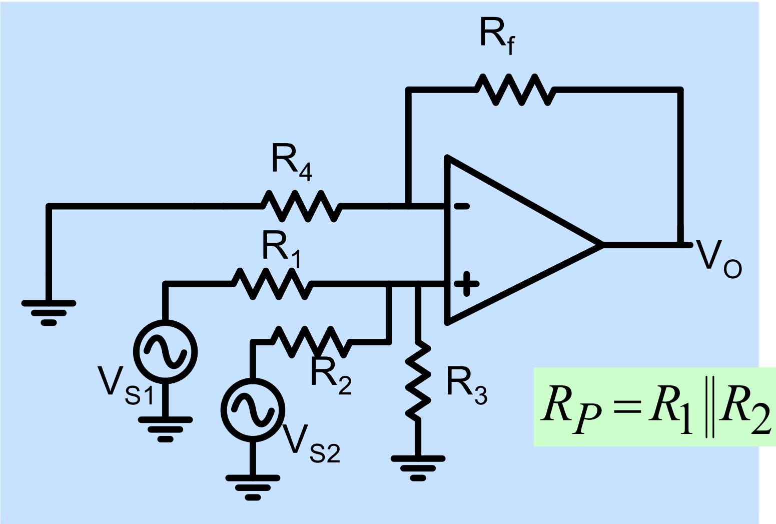
$$v_o = (\frac{R_p}{R_1} v_{s1} + \frac{R_p}{R_2} v_{s2}) \times (1 + \frac{R_f}{R_4})$$

Low entropy expression !

Design Example

Design a circuit that would generate the following output given three input voltages vs1, vs2 and vs3.

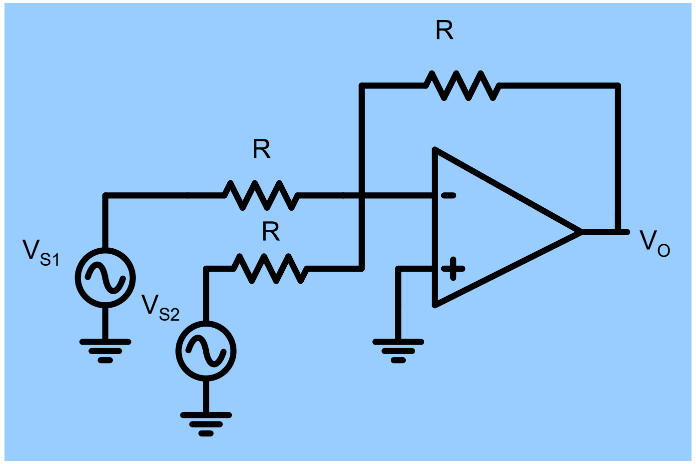
$$v_o = 10v_{s1} + 4v_{s2}$$



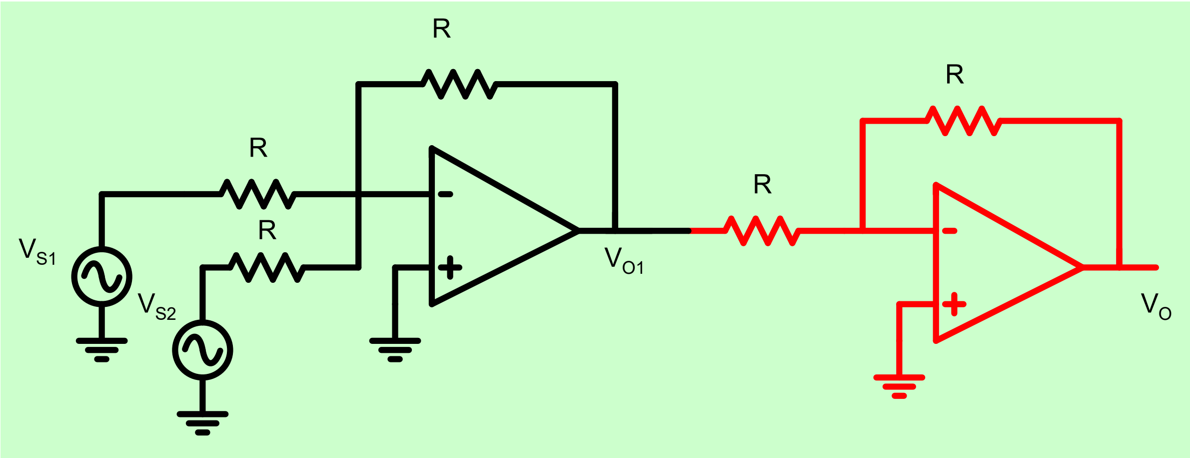
$$v_o = (\frac{R_p}{R_1}v_{s1} + \frac{R_p}{R_2}v_{s2}) \times (1 + \frac{R_f}{R_4})$$

$$R_P = R_1 \| R_2 \| R_3$$

Homework problem !



$$v_o = -(v_{s1} + v_{s2})$$



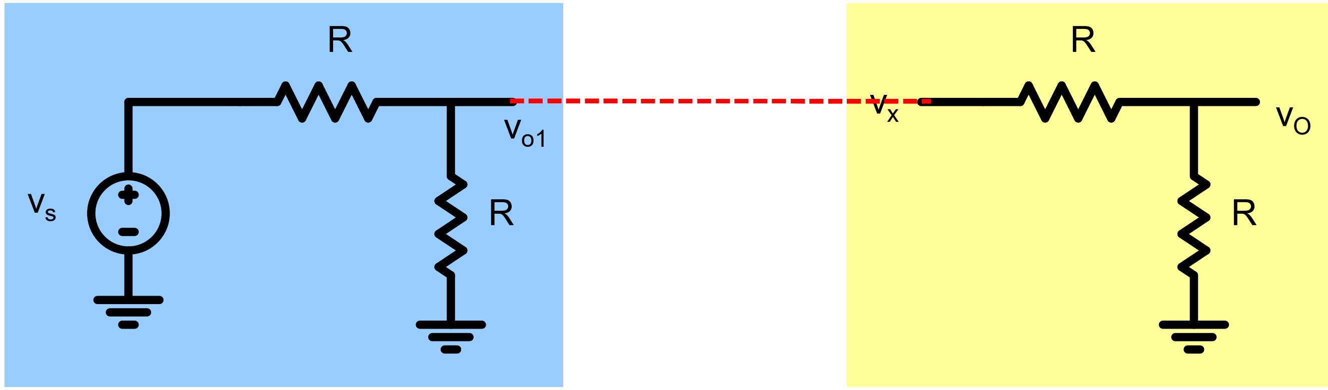
$$v_{o1} = -(v_{s1} + v_{s2})$$

$$v_o = -v_{o1}$$

$$v_o = (v_{s1} + v_{s2})$$

Have we made some assumption here ?

Example



$$\frac{v_{o1}}{v_s} = 0.5$$

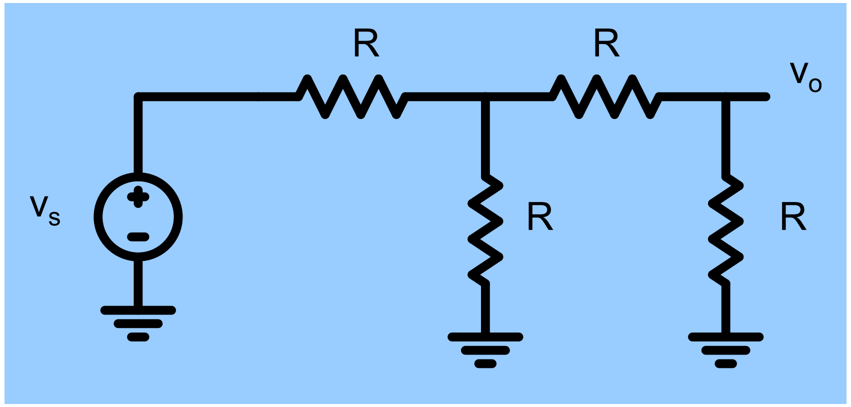
$$\frac{v_o}{v_x} = 0.5$$

$$v_{o1} = v_x$$

$$\frac{v_o}{v_x} = \frac{v_o}{v_{o1}} = 0.5$$

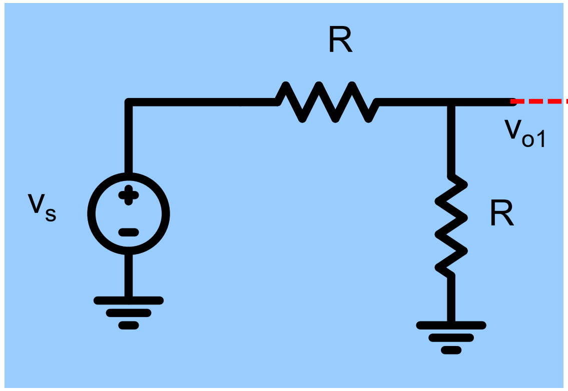
$$\frac{v_o}{v_s} = \frac{v_o}{v_{o1}} \times \frac{v_{o1}}{v_s} = 0.5 \times 0.5 = 0.25$$

BUT

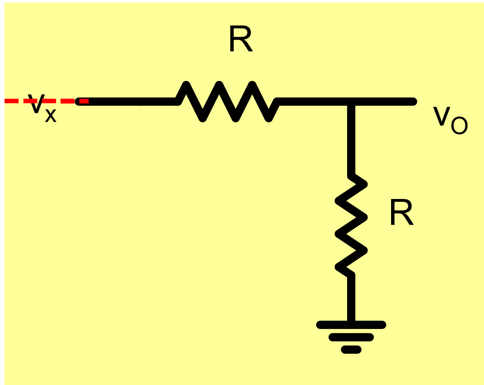


$$\frac{v_o}{v_s} = 0.2$$

Where is the error ?



$$v_{o1} = v_x$$



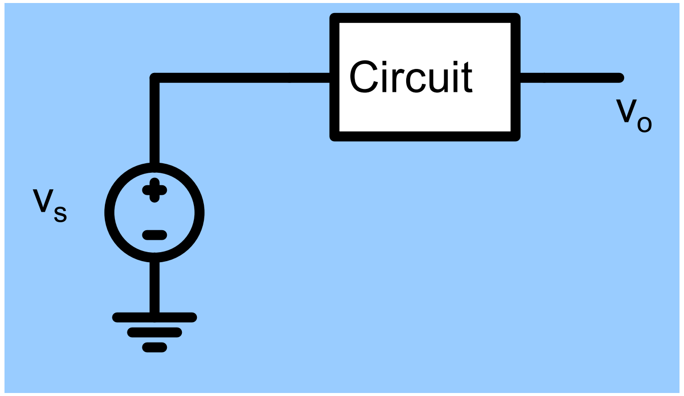
$$\frac{v_{o1}}{v_s} = 0.5$$

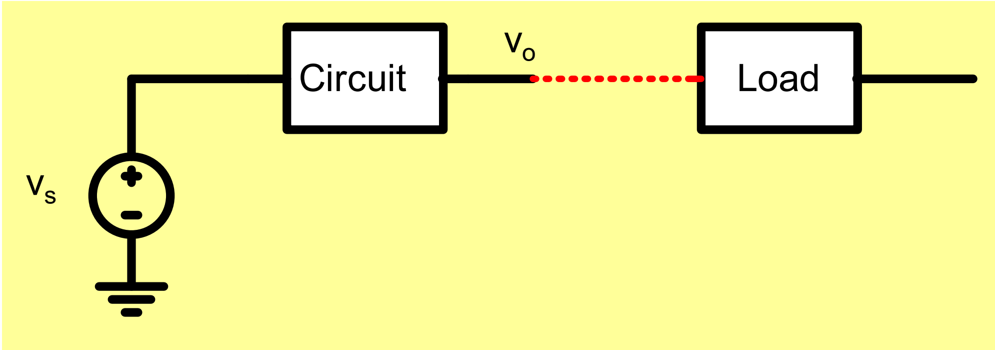
$$\frac{v_{o1}}{v_s} \neq 0.5$$

$$\frac{v_o}{v_x} = 0.5$$

Circuit-1 gets ‘loaded’ by circuit-2 and its output vs. input characteristics get modified.

Loading Effect

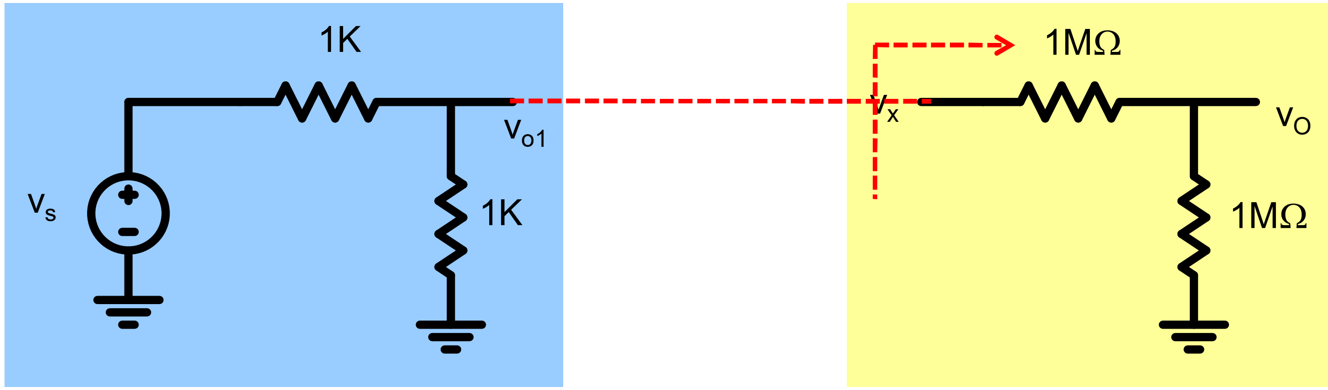




V_o in general gets altered when we connect a load to it

Under what conditions is change in V_O small upon connection of a load ?

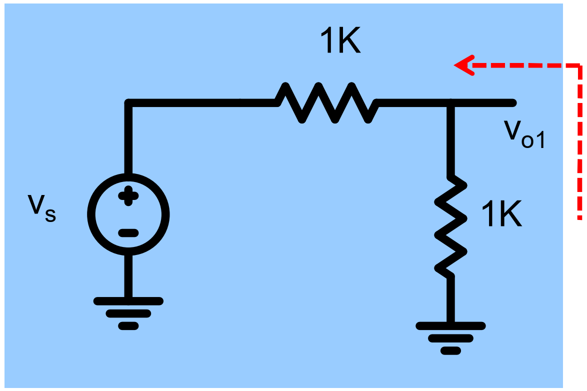
Example



$$\frac{v_{o1}}{v_s} = 0.5$$

$$\frac{v_{o1}}{v_s} \cong 0.5$$

We can describe this effect in terms of output resistance



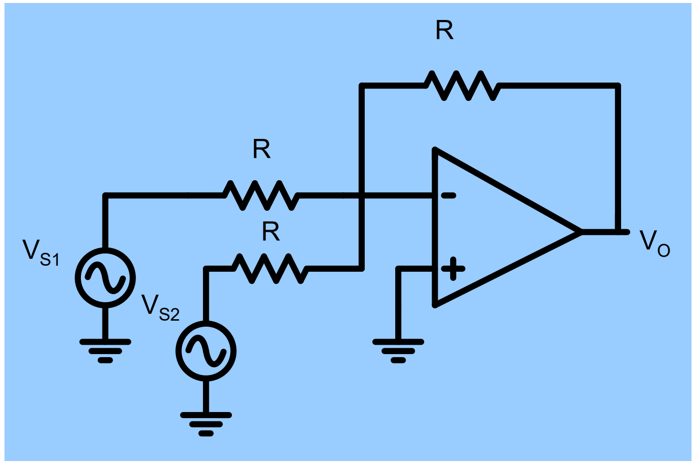
$$R_o = 0.5K$$

$$R_L = 2M\Omega$$

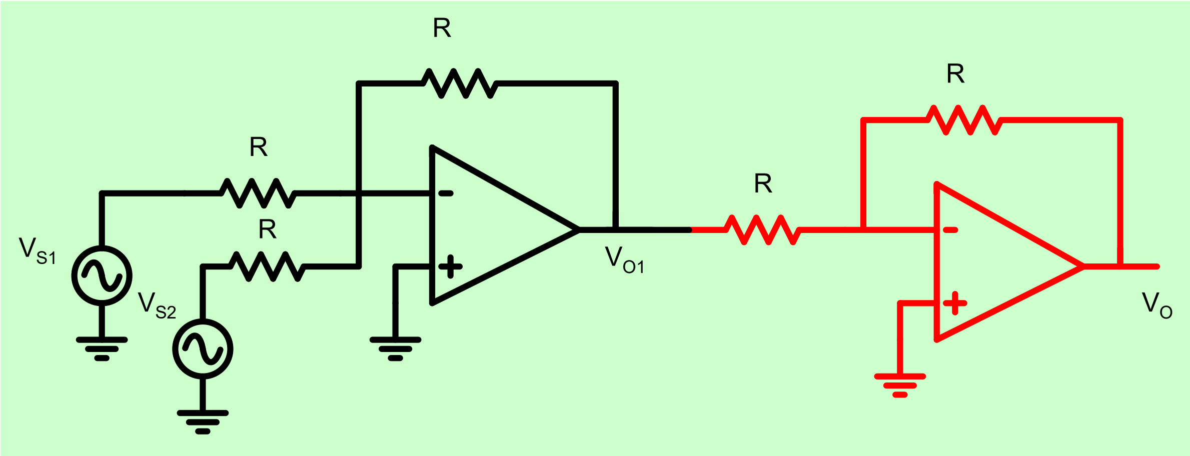
Loading Effect

Whenever output resistance of a circuit is much smaller than the load resistance, the loading effect is minimal.

$$\boxed{R_o << R_L}$$



$$v_o = -(v_{s1} + v_{s2})$$



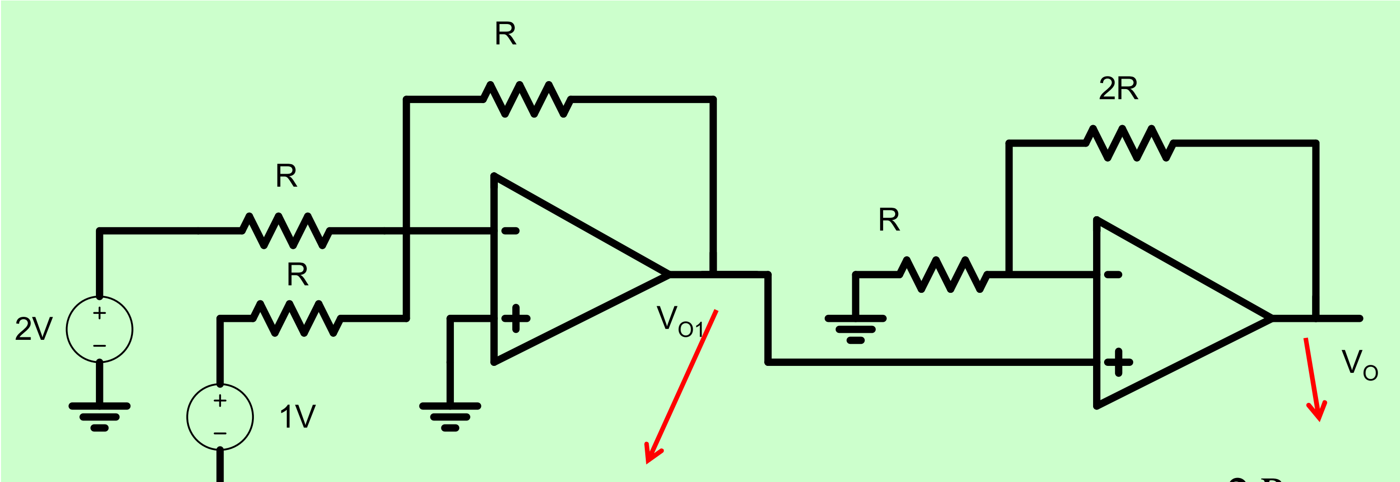
$$v_{o1} = -(v_{s1} + v_{s2})$$

$$v_o = -v_{o1}$$

$$v_o = (v_{s1} + v_{s2})$$

The assumption made here is that there is no loading which is reasonable because opamps have very low resistance

Example



$$v_{o1} = -\{\frac{R}{R}\times 1 + \frac{R}{R}\times 2\} = -3V$$

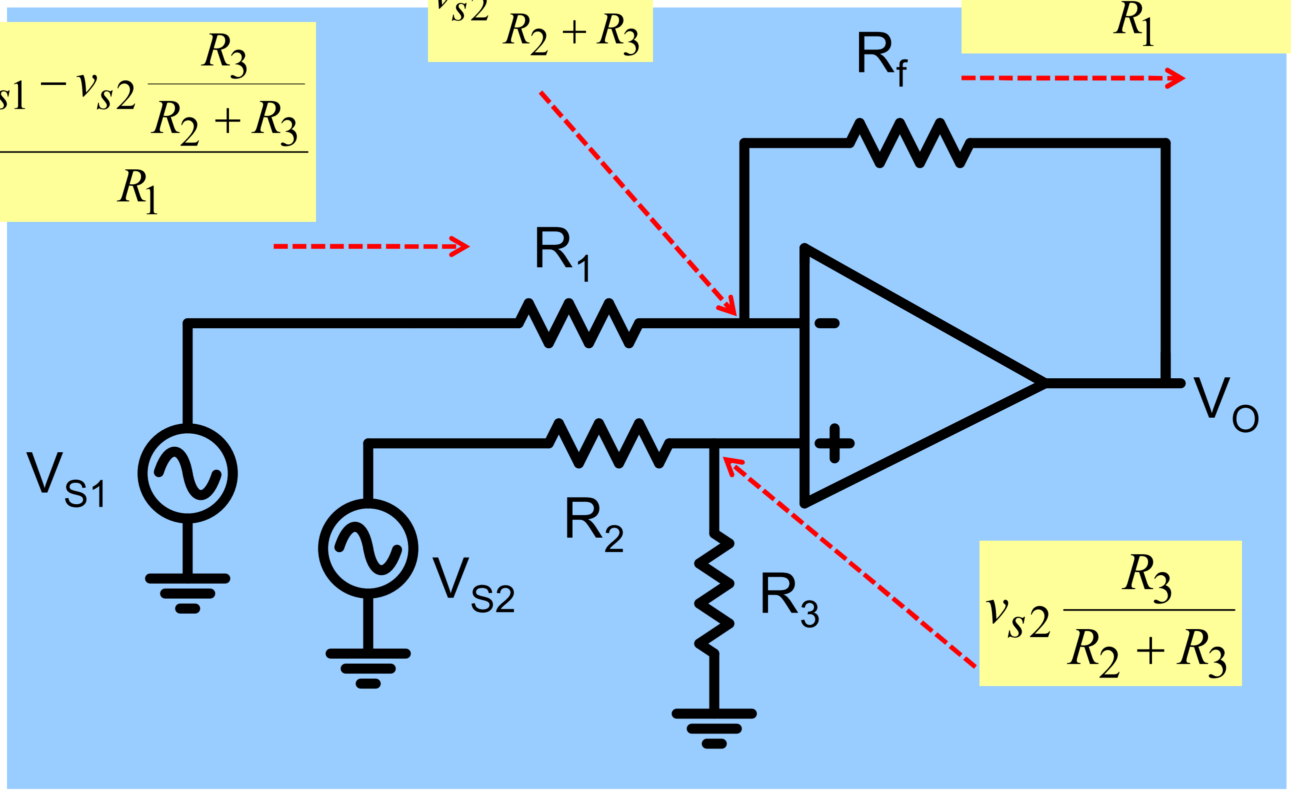
$$\frac{v_o}{v_{o1}} = 1 + \frac{2R}{R} \Rightarrow v_o = -9V$$

Subtractor

$$\frac{v_{s1} - v_{s2}\dfrac{R_3}{R_2 + R_3}}{R_1}$$

$$v_{s2}\frac{R_3}{R_2 + R_3}$$

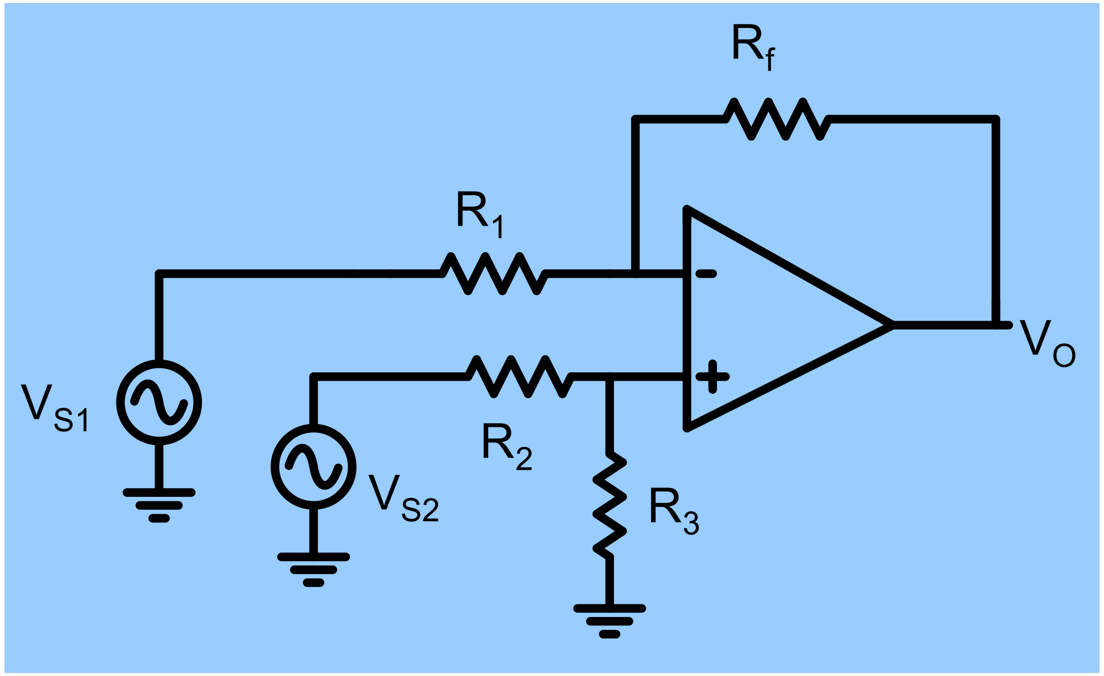
$$\frac{v_{s1} - v_{s2}\dfrac{R_3}{R_2 + R_3}}{R_1}$$



$$v_{s2}\frac{R_3}{R_2 + R_3}$$

$$\frac{v_{s2}\dfrac{R_3}{R_2 + R_3} - v_o}{R_f} = \frac{v_{s1} - v_{s2}\dfrac{R_3}{R_2 + R_3}}{R_1}$$

$$v_o = v_{s2}\frac{\dfrac{R_3}{R_2}}{(1 + \dfrac{R_3}{R_2})}(1 + \frac{R_f}{R_1}) - (\frac{R_f}{R_1})v_{s1}$$

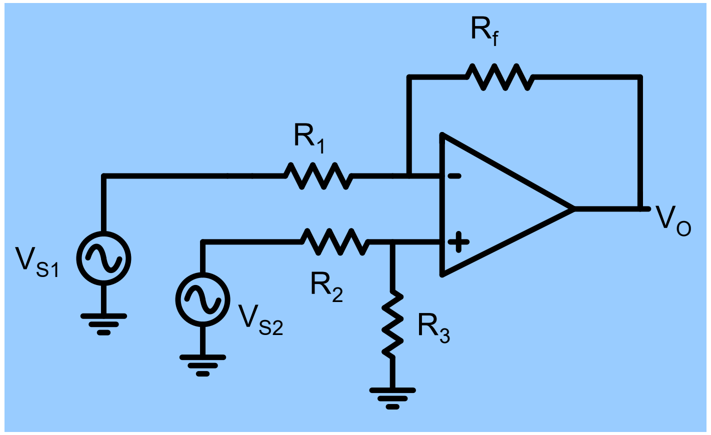


$$v_o = v_{s2}\frac{\frac{R_3}{R_2}}{(1+\frac{R_3}{R_2})}(1+\frac{R_f}{R_1}) - (\frac{R_f}{R_1})v_{s1}$$

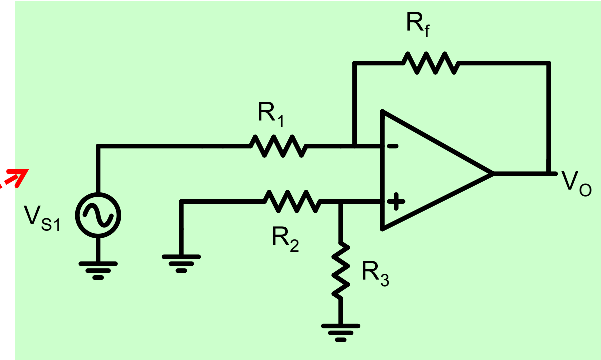
Choose $\frac{R_3}{R_2} = \frac{R_f}{R_1}$

$$v_o = \frac{R_f}{R_1}(v_{s2} - v_{s1})$$

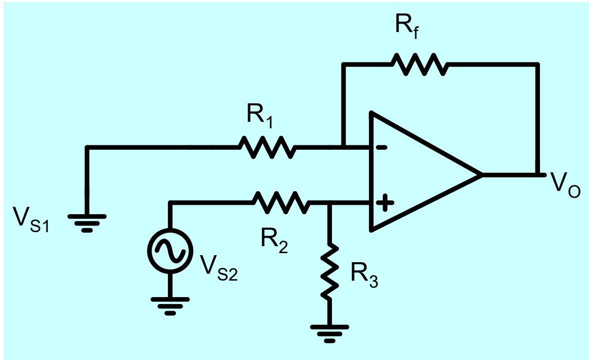
Subtractor: Alternative Analysis



Use superposition theorem



Inverting amplifier

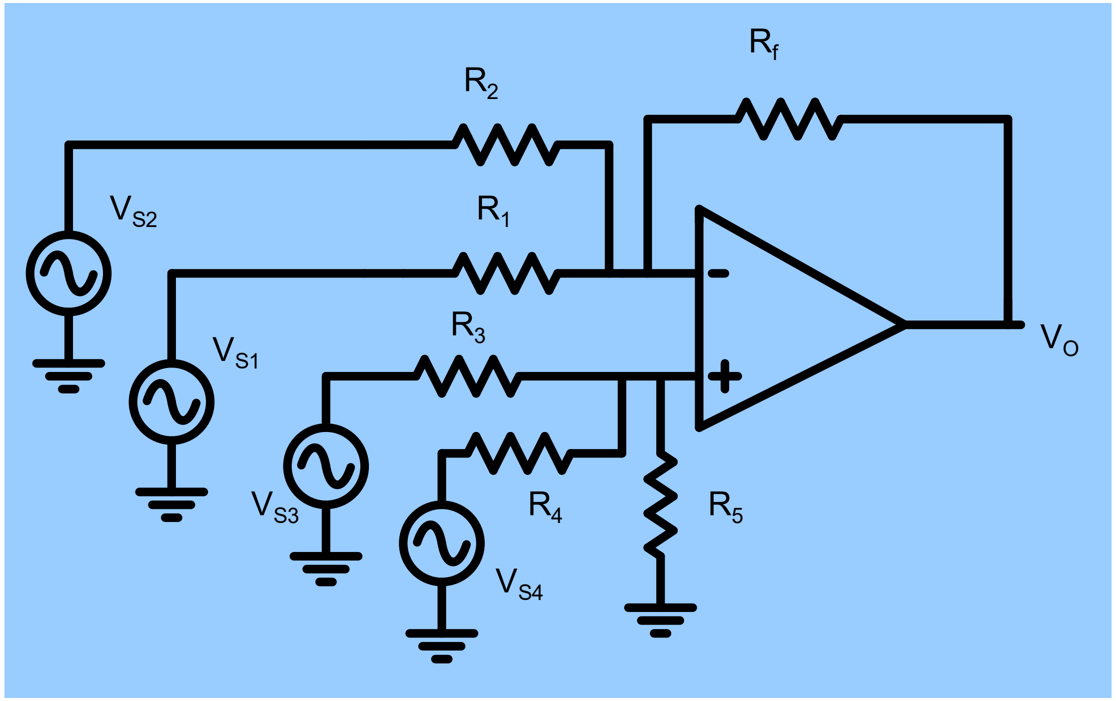


Non-inverting amplifier

$$v_o = -(\frac{R_f}{R_1})v_{s1} \quad + \quad v_{s2}\frac{R_3}{(R_3+R_2)} \times (1+\frac{R_f}{R_1})$$

Analysis is made simpler by **Re-Using** results derived earlier

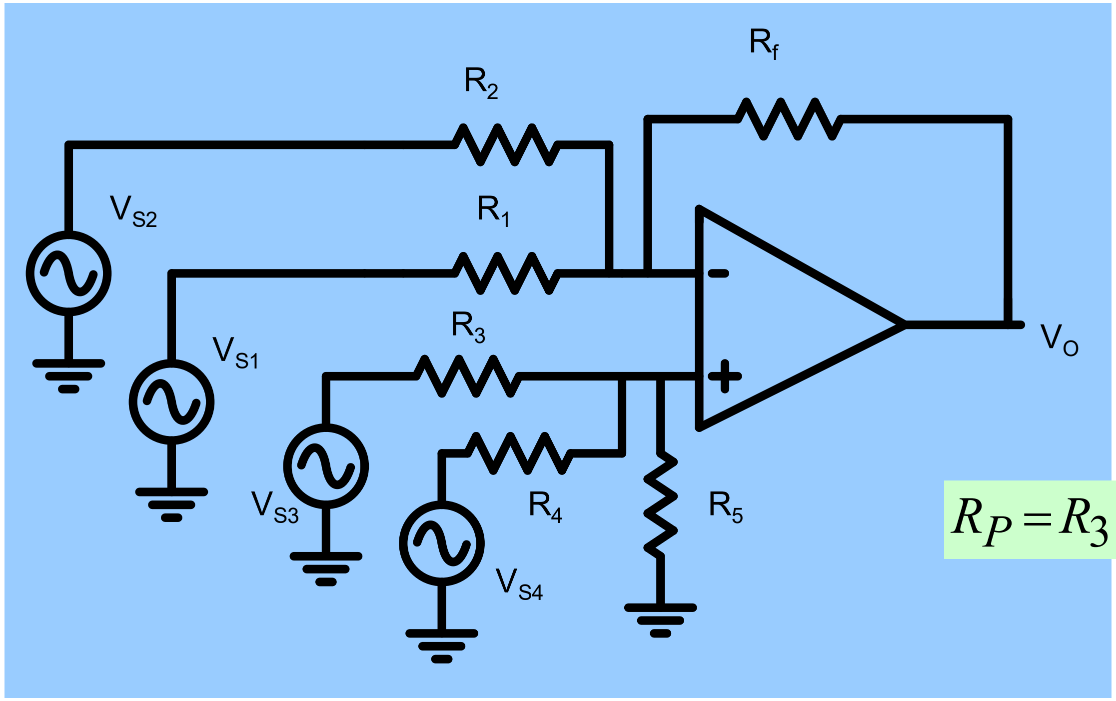
Adder/Subtractor



$$v_o = -\left(\frac{R_f}{R_1}\right)v_{s1} + -\left(\frac{R_f}{R_2}\right)v_{s2} + v_{s3}\frac{R_5 \| R_4}{R_5 \| R_4 + R_3} \times \left(1 + \frac{R_f}{R_1 \| R_2}\right)$$

$$+ v_{s4}\frac{R_5 \| R_3}{R_5 \| R_3 + R_4} \times \left(1 + \frac{R_f}{R_1 \| R_2}\right)$$

Adder/Subtractor

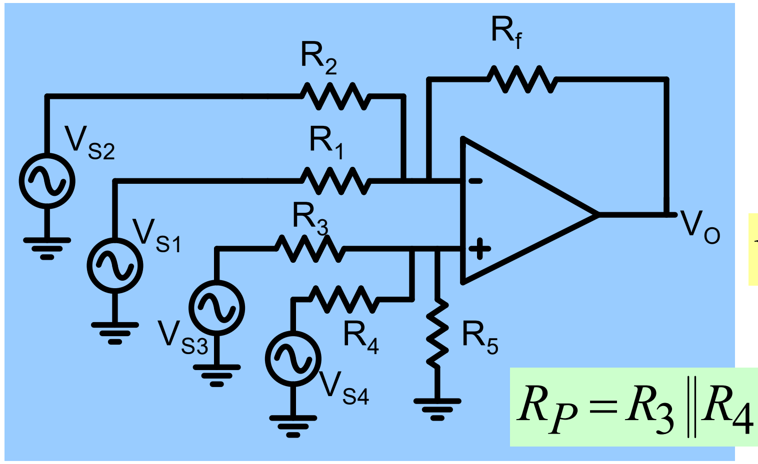


$$R_P = R_3 \| R_4 \| R_5$$

$$v_o = -\left(\frac{R_f}{R_1}\right)v_{s1} + -\left(\frac{R_f}{R_2}\right)v_{s2} + v_{s3}\frac{R_P}{R_3} \times \left(1 + \frac{R_f}{R_1 \| R_2}\right)$$

$$+ \quad v_{s4}\frac{R_P}{R_4} \times \left(1 + \frac{R_f}{R_1 \| R_2}\right)$$

Example



$$v_o = -10v_{s1} - 4v_{s2} + 5v_{s3} + 2v_{s4}$$

$$R_P = R_3 \| R_4 \| R_5$$

$$v_o = -(\frac{R_f}{R_1})v_{s1} - -(\frac{R_f}{R_2})v_{s2} + (1 + \frac{R_f}{R_1 \| R_2}) \times \frac{R_P}{R_3} v_{s3} + (1 + \frac{R_f}{R_1 \| R_2}) \times \frac{R_P}{R_4} v_{s4}$$

Choose : $R_f = 10K$ $\Rightarrow R_1 = 1K$ $\Rightarrow R_2 = 2.5K$

$\Rightarrow \frac{R_P}{R_3} = 0.33$ $\Rightarrow \frac{R_P}{R_4} = 0.133$ $\Rightarrow \frac{R_4}{R_3} = 2.5$

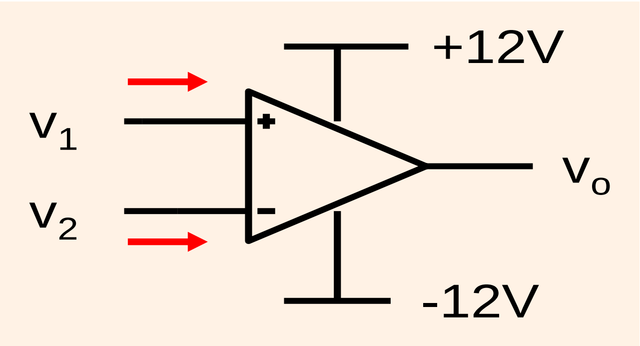
Choose : $R_3 = 1K$ $\Rightarrow R_4 = 2.5K$ $\Rightarrow R_P = 0.33K$ $\Rightarrow R_5 = 0.625K$

ESC201T : Introduction to Electronics

Lecture 30: Operational Amplifier Circuits-2

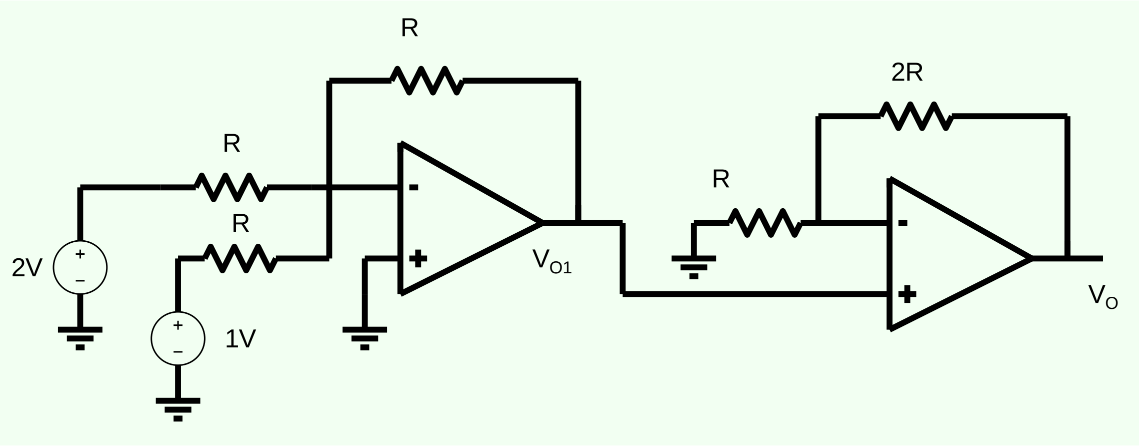
B. Mazhari
Dept. of EE, IIT Kanpur

Important properties for analyzing ideal opamp circuits under negative feedback



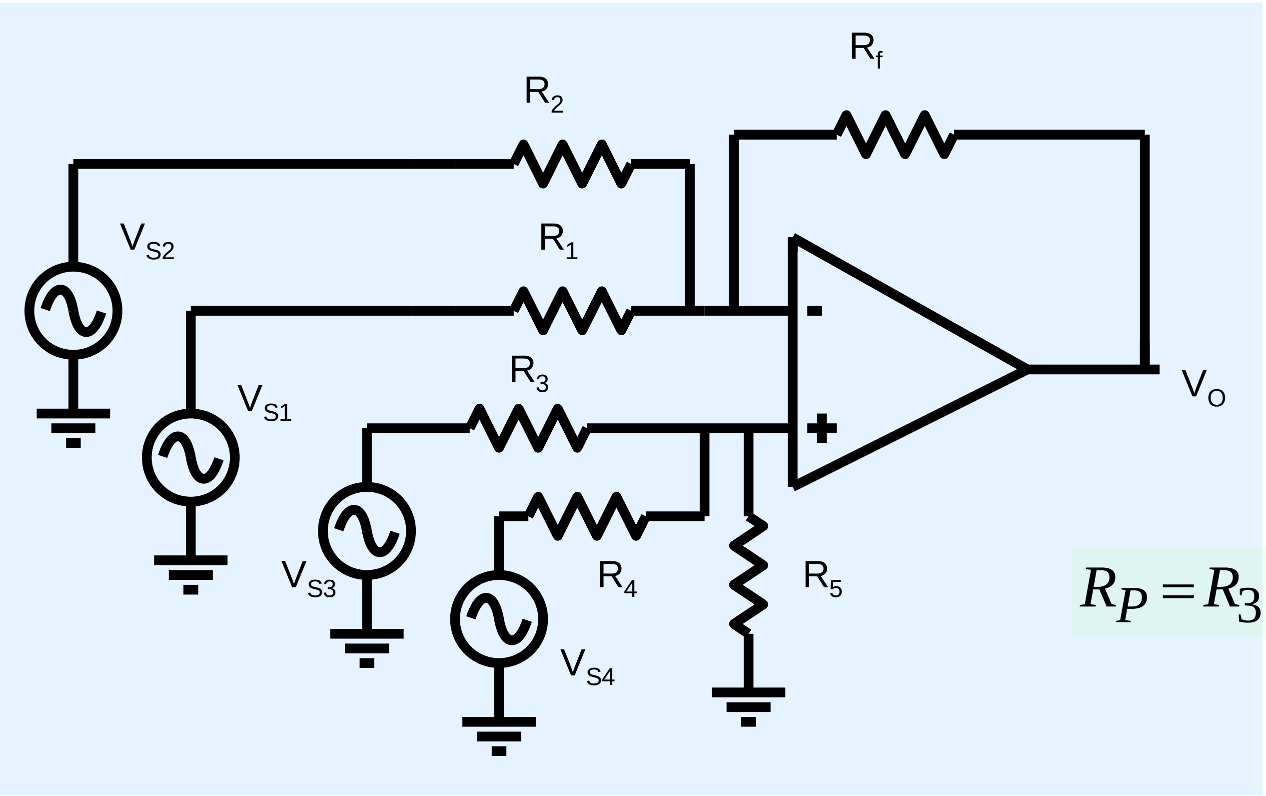
$$1.\quad v_1 = v_2$$

$$2.\quad i_1 = i_2 = 0$$



One stage does not load the preceding stage due to very small output impedance of the opamp

Adder/Subtractor

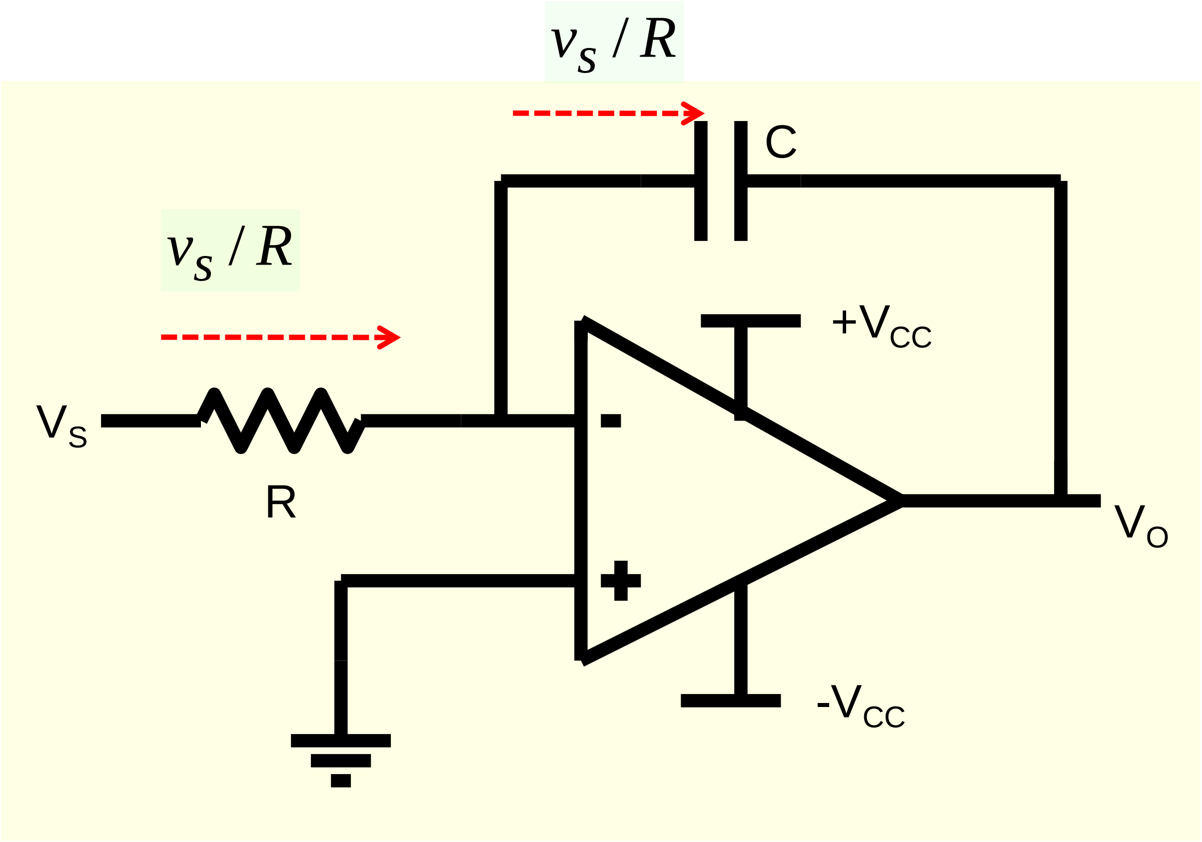


$$R_P = R_3 \| R_4 \| R_5$$

$$v_o = -\left(\frac{R_f}{R_1}\right)v_{s1} \quad + \quad -\left(\frac{R_f}{R_2}\right)v_{s2} \quad + \quad v_{s3}\,\frac{R_P}{R_3} \quad \times\left(1+\frac{R_f}{R_1\|R_2}\right)$$

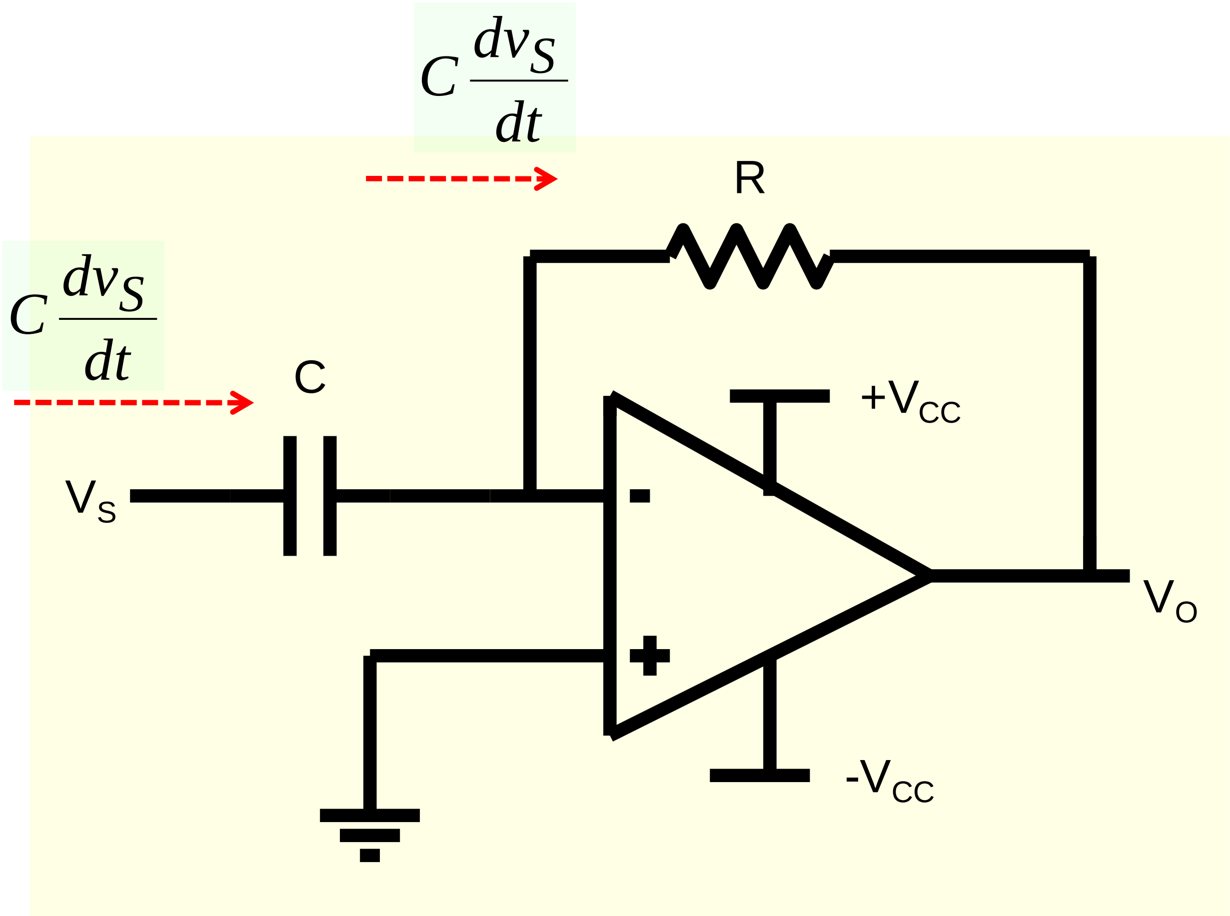
$$+ \quad v_{s4}\,\frac{R_P}{R_4} \quad \times\left(1+\frac{R_f}{R_1\|R_2}\right)$$

Integrator



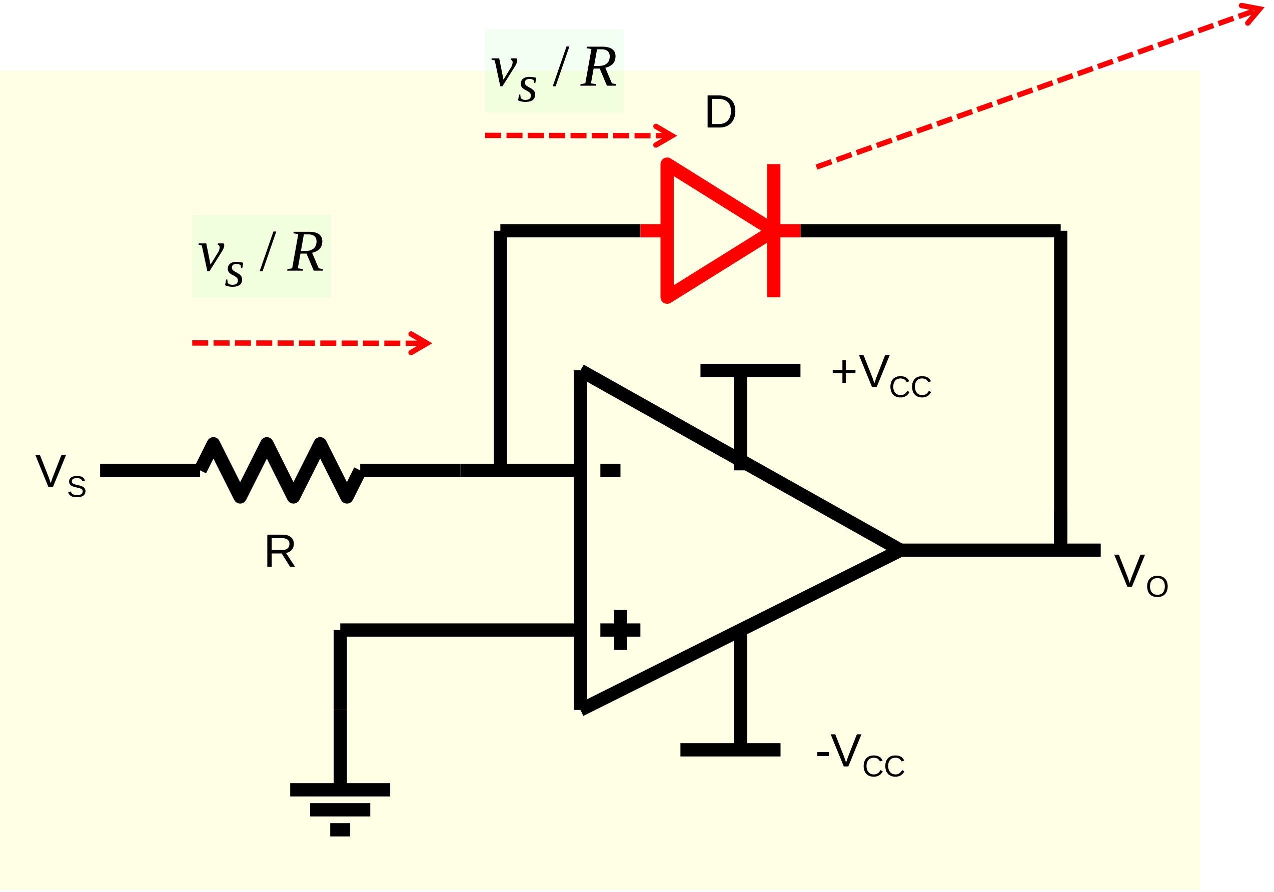
$$\frac{V_S}{R} = -C\frac{dV_O}{dt} \Rightarrow V_O(t) = -\frac{1}{RC}\int V_S dt$$

Differentiator



$$-\frac{V_O}{R} = C\frac{dV_S}{dt} \Rightarrow V_O(t) = -RC\frac{dV_S}{dt}$$

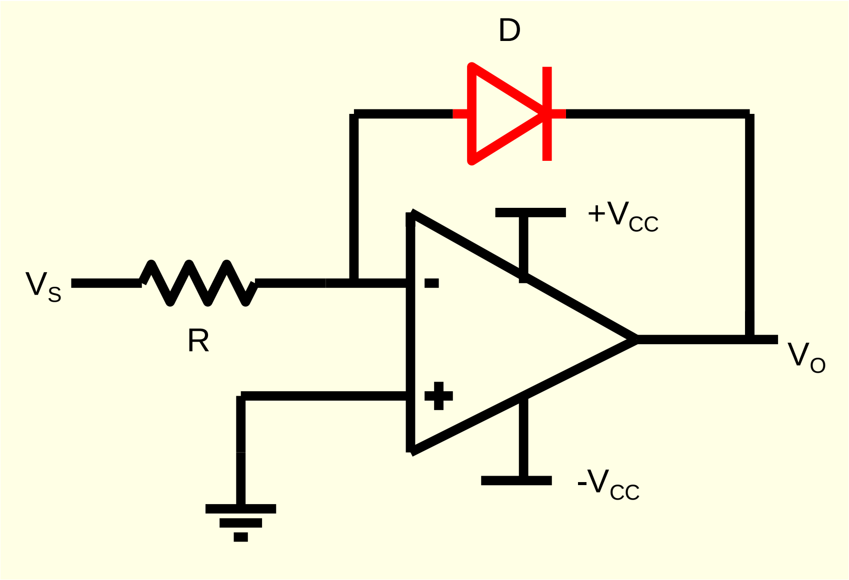
Log Amplifier



$$I = I_S\left(e^{\frac{V}{V_T}} - 1\right)$$

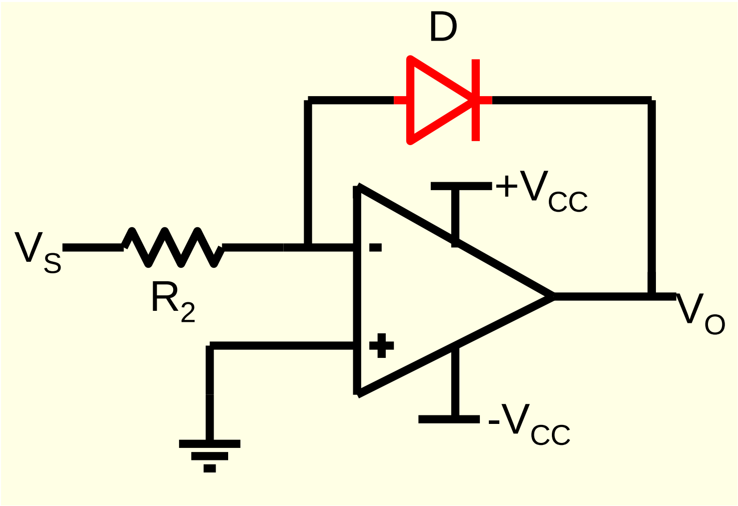
$$\frac{V_S}{R} = I_S\left(e^{-\frac{V_O}{V_T}} - 1\right) \Rightarrow -V_O = V_T \times \ln\left(1 + \frac{V_S}{RI_S}\right) \cong V_T \times \ln\left(\frac{V_S}{RI_S}\right)$$

Temperature Sensor ?

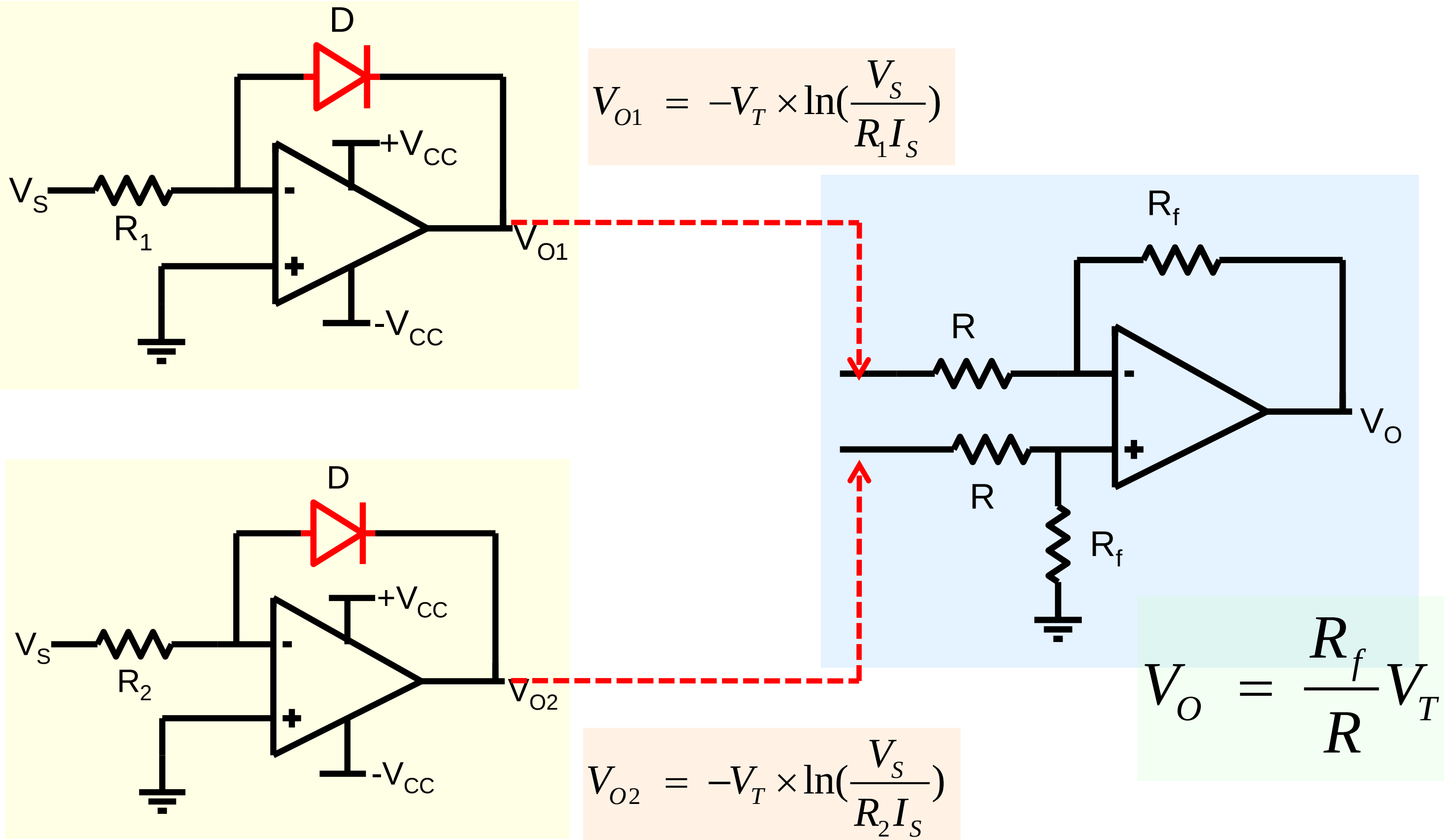


$$V_O = -V_T \times \ln(\frac{V_S}{RI_S}); \; V_T = \frac{k_B T}{q}$$

But I_S is a function of temperature as well.



$$V_O = -V_T \times \ln(\frac{V_S}{R_2 I_S})$$



$$V_{O1} = -V_T \times \ln(\frac{V_S}{R_1 I_S})$$

$$V_{O2} = -V_T \times \ln(\frac{V_S}{R_2 I_S})$$

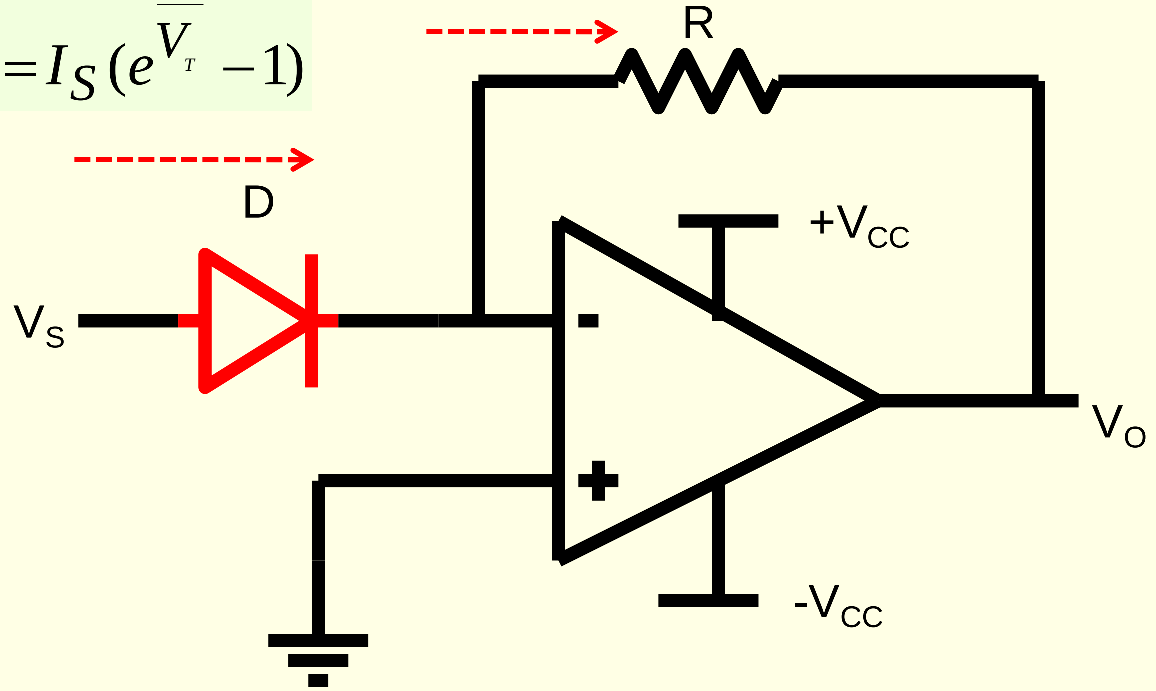
$$V_O = \frac{R_f}{R} V_T \times \ln(\frac{R_2}{R_1})$$

Output voltage is directly proportional to temperature

AntiLog Amplifier

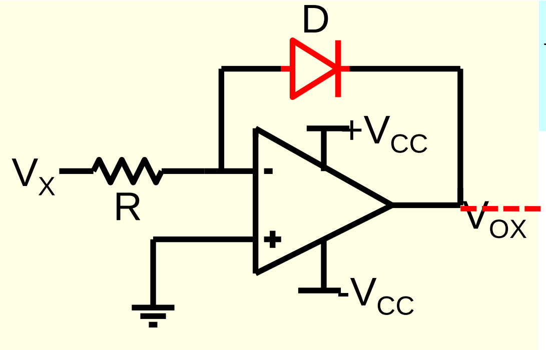
$$I = I_S\left(e^{\frac{V_S}{V_T}} - 1\right)$$

$$I = I_S\left(e^{\frac{V_S}{V_T}} - 1\right)$$



$$-\frac{V_O}{R} = I_S\left(e^{\frac{V_S}{V_T}} - 1\right) \Rightarrow V_O = -RI_S\left(e^{\frac{V_S}{V_T}} - 1\right) \cong -RI_S \times e^{\frac{V_S}{V_T}}$$

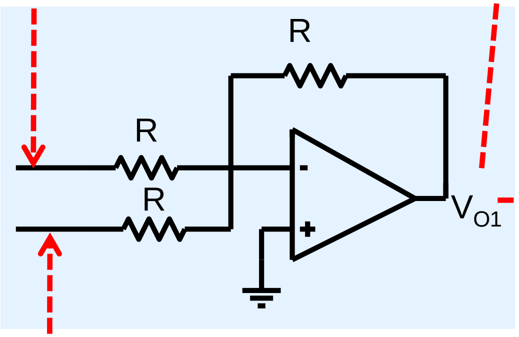
Multiplier

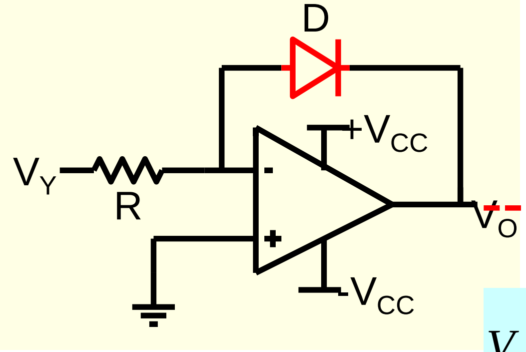


$$V_{OX} = -V_T \times \ln(\frac{V_X}{RI_S})$$

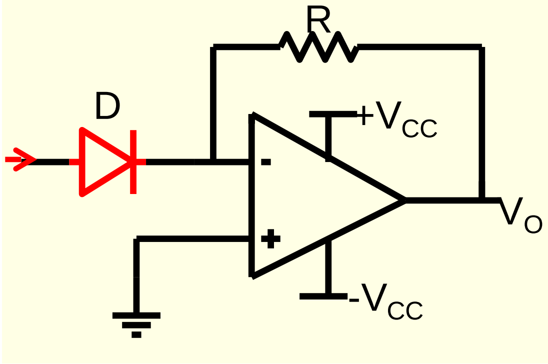
$$V_{O1} = V_T \times (\ln(\frac{V_X}{RI_S}) + \ln(\frac{V_Y}{RI_S}))$$

$$V_{O1} = V_T \times \ln(\frac{V_X V_Y}{R^2 I_S^2})$$





$$V_{OY} = -V_T \times \ln(\frac{V_Y}{RI_S})$$

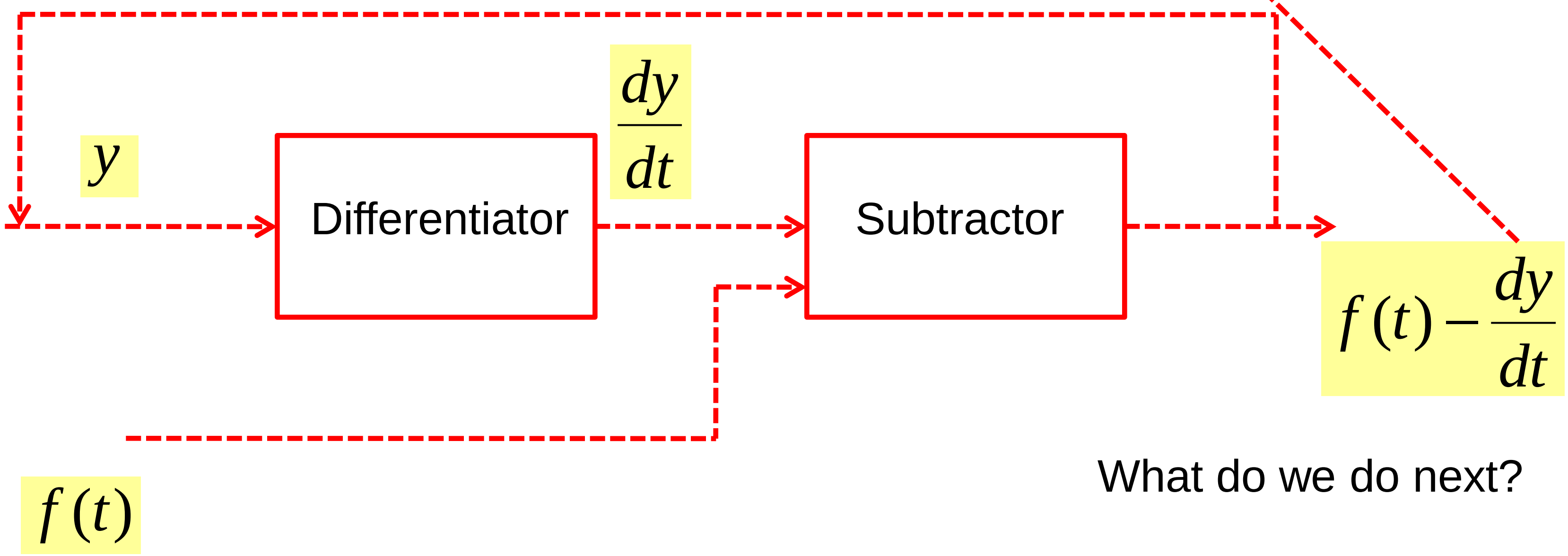


$$V_O \cong -RI_S \times e^{\frac{V_{O1}}{V_T}} = -\frac{V_X V_Y}{RI_S}$$

Solving Differential Equations?

$$\frac{dy}{dt} + y = f(t)$$

$$y = f(t) - \frac{dy}{dt}$$



What do we do next?

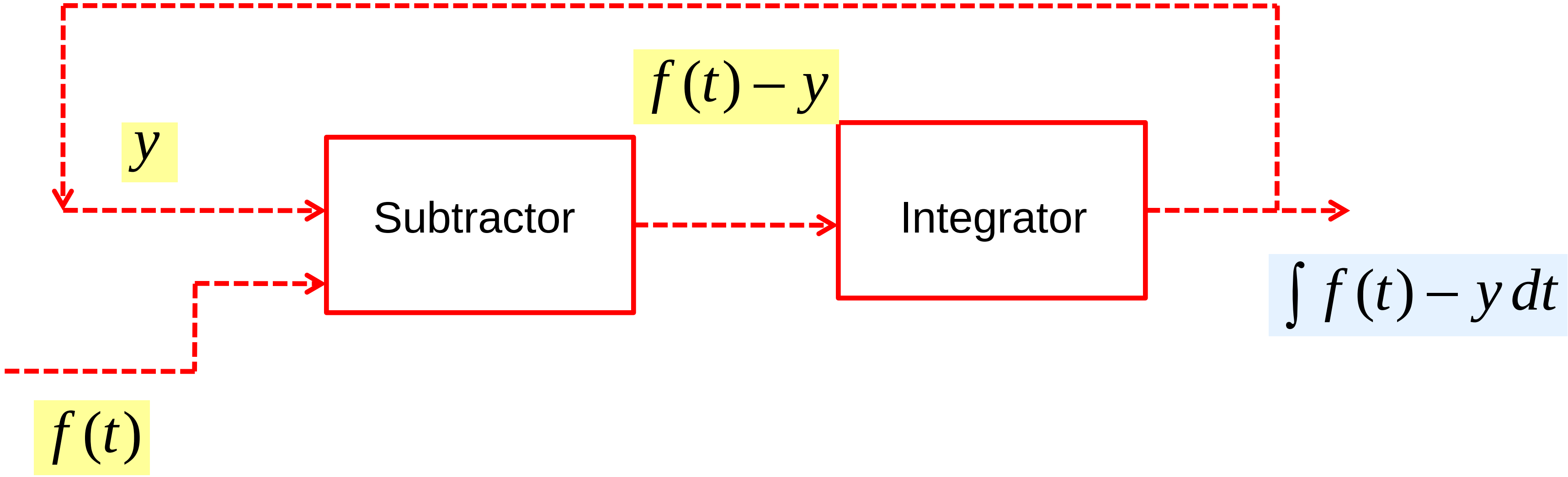
Where do we get y from?

Integrators are preferred over Differentiators because they are less sensitive to noise

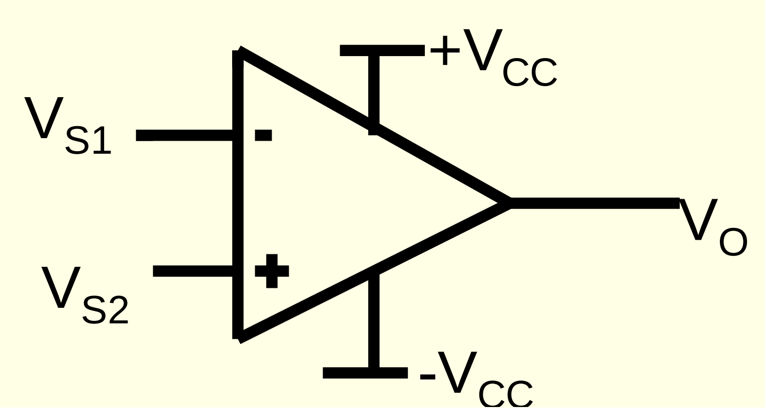
$$\frac{dy}{dt} + y = f(t)$$

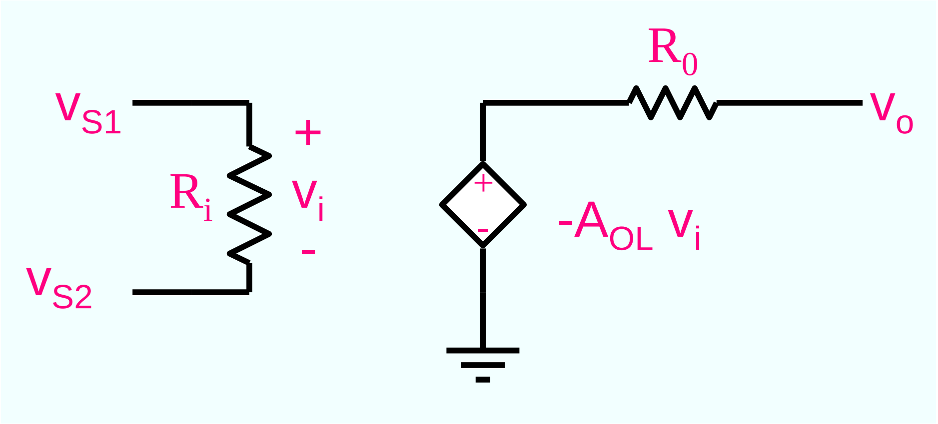
$$\frac{dy}{dt} = f(t) - y \Rightarrow$$

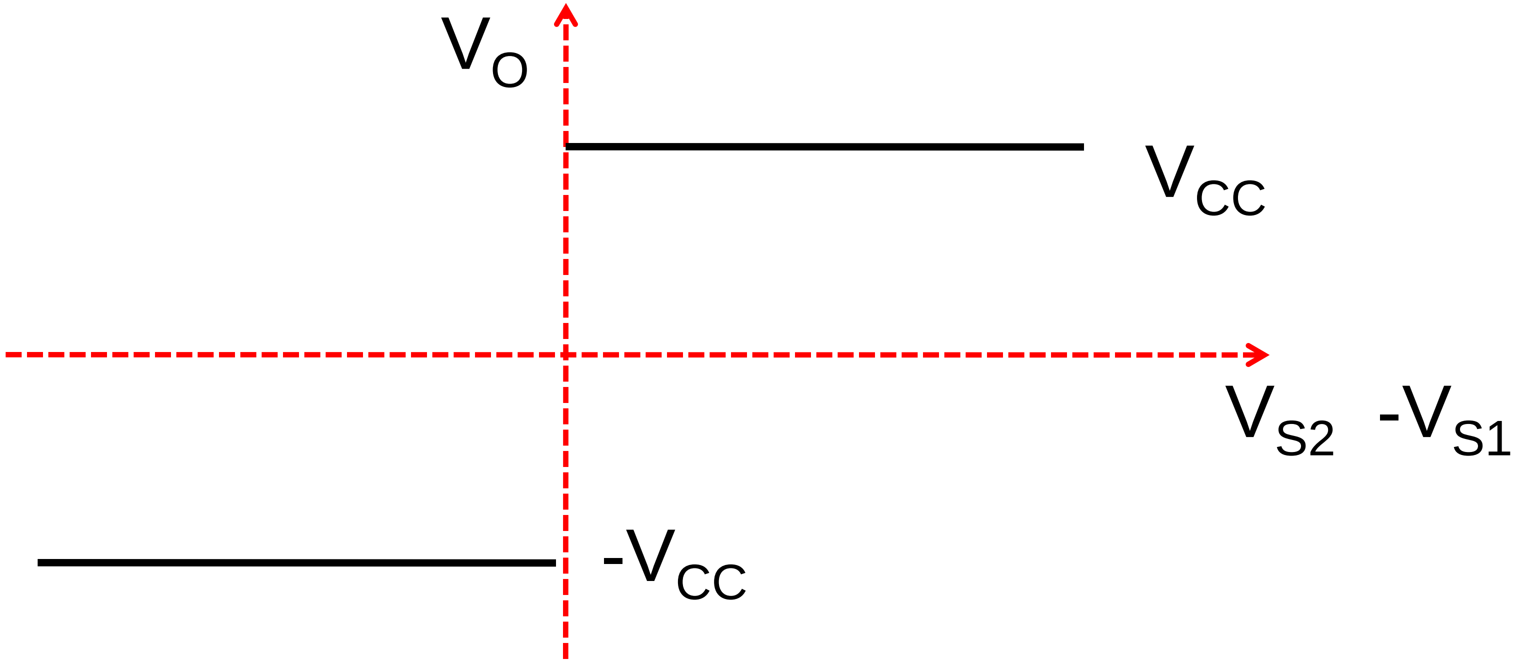
$$y = \int f(t) - y\, dt$$

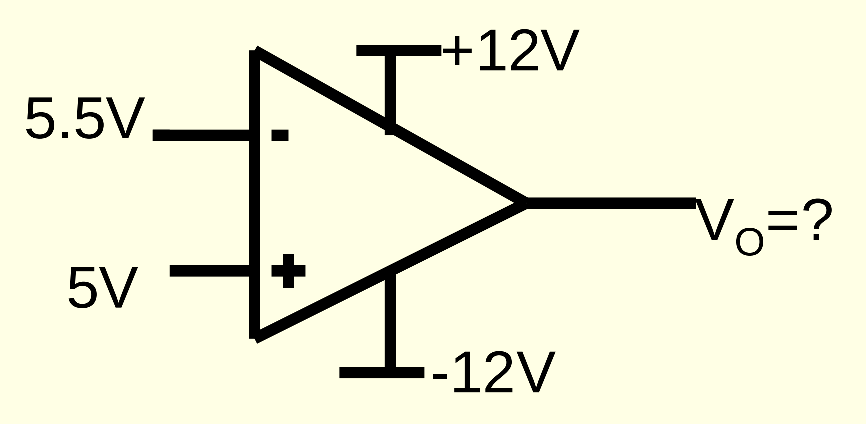


Comparator: Opamp under open Loop condition

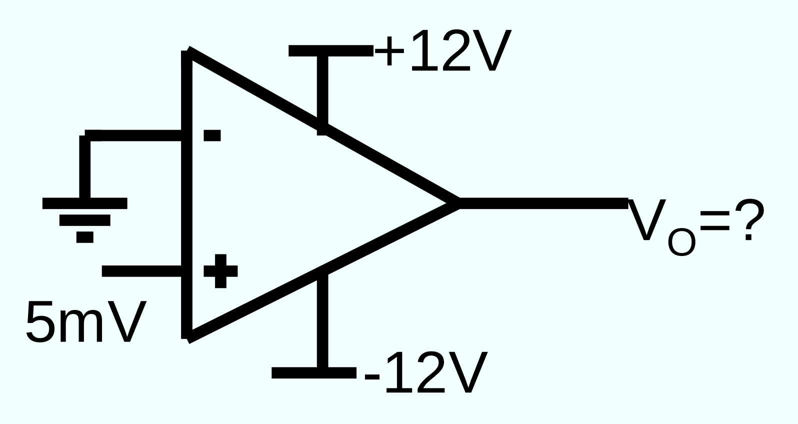








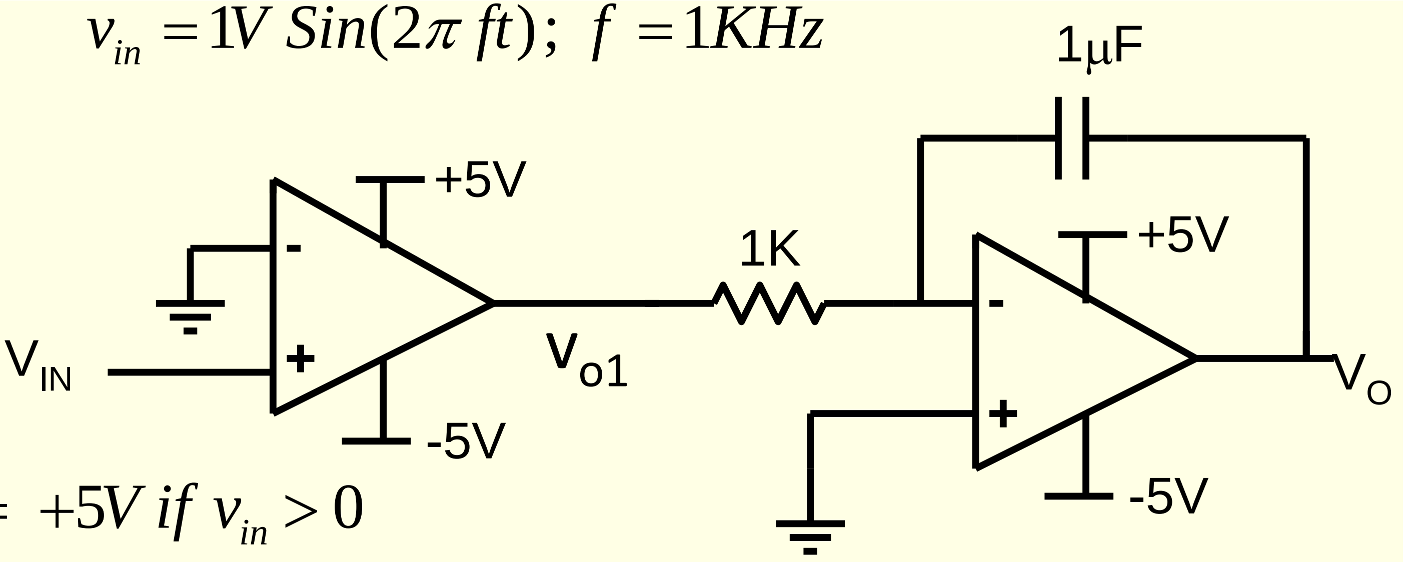
~ -12V



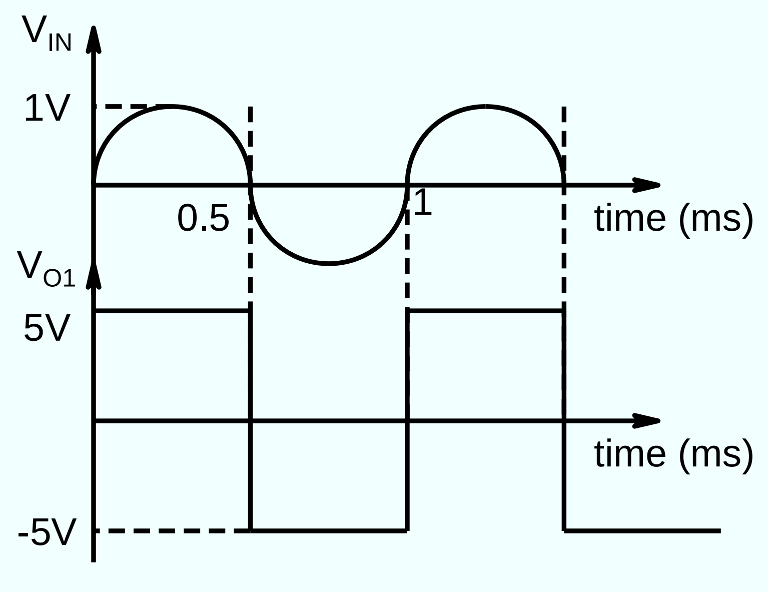
~ +12V

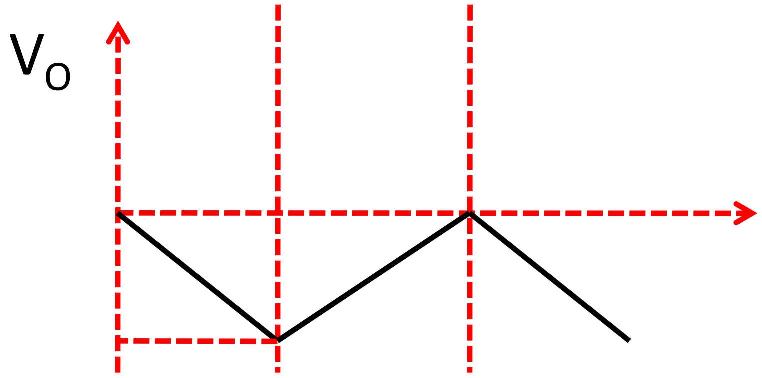
Example

$$v_{in} = 1V\ Sin(2\pi\ ft);\ \ f = 1KHz$$

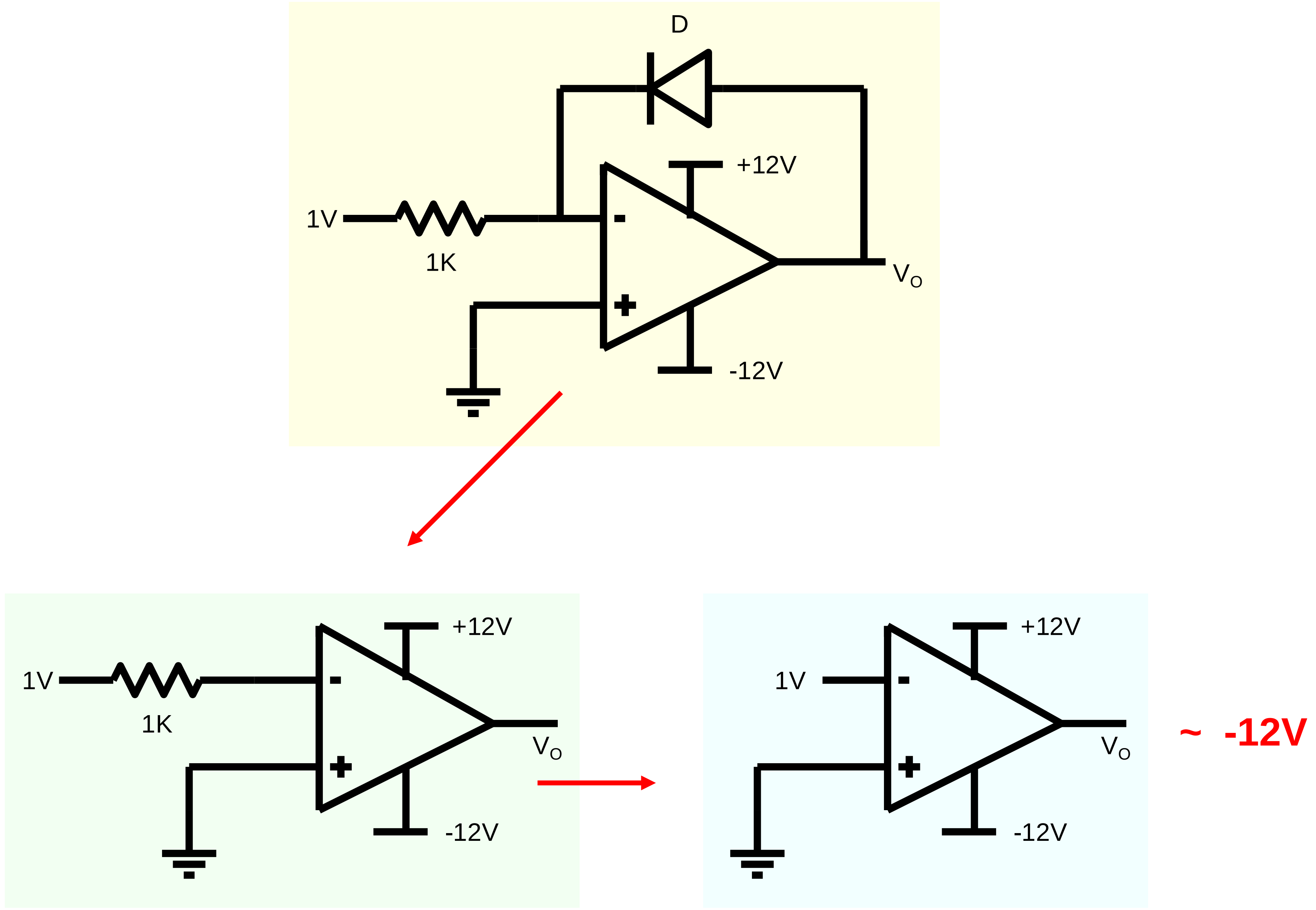


$$V_{O1} = +5V\ if\ v_{in} > 0$$
$$= -5V\ if\ v_{in} < 0$$

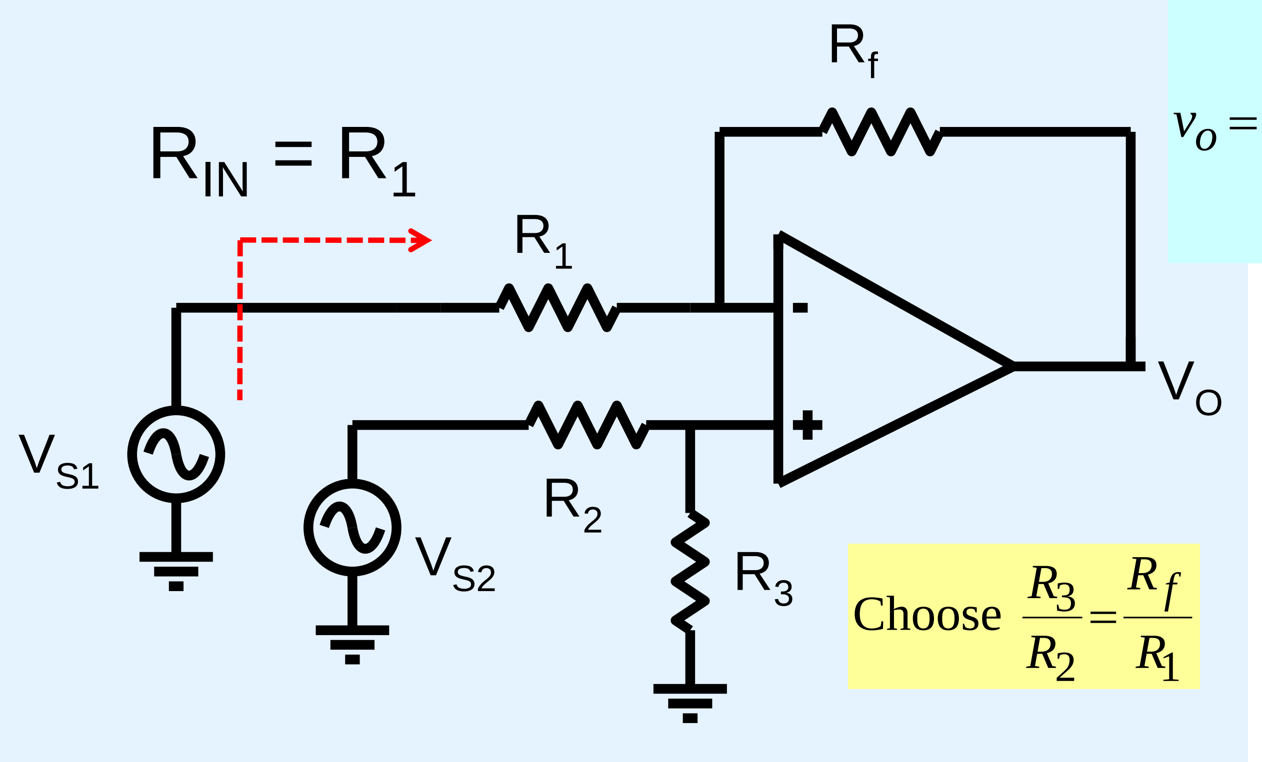




Example



Difference Amplifier



$$v_o = v_{s2}\frac{\frac{R_3}{R_2}}{(1+\frac{R_3}{R_2})}(1+\frac{R_f}{R_1}) - (\frac{R_f}{R_1})v_{s1}$$

Choose $\frac{R_3}{R_2} = \frac{R_f}{R_1}$

$$v_o = \frac{R_f}{R_1}(v_{s2} - v_{s1})$$

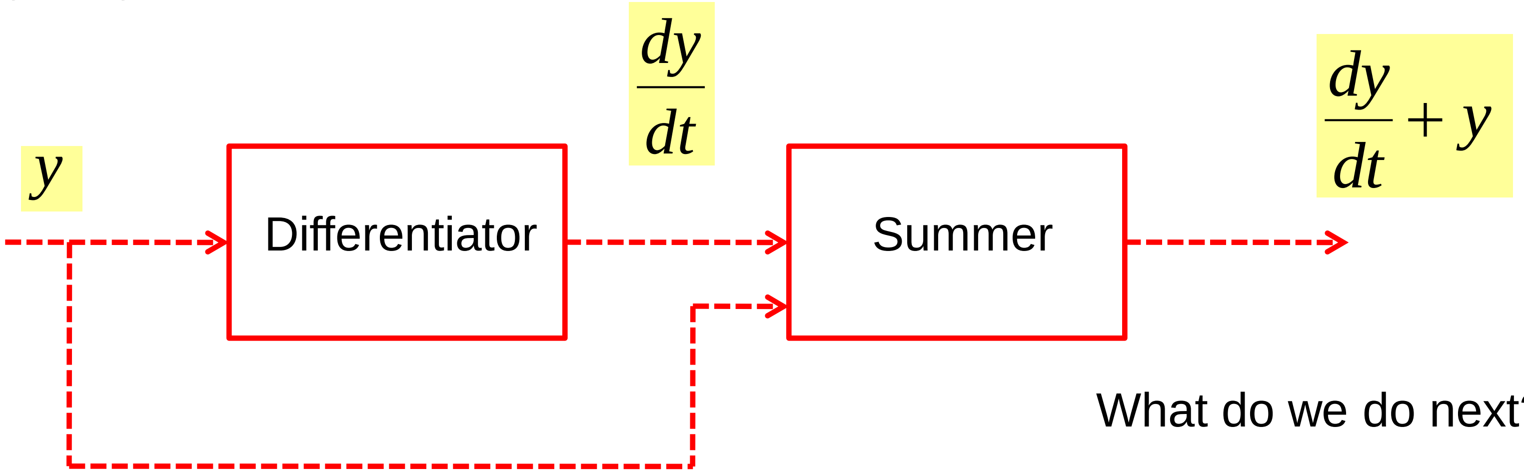
A drawback is that input resistance is relatively Lower

To change gain, we have to change two resistors and a slight mismatch can drastically reduce common mode rejection ratio

Solving Differential Equations?

$$\frac{dy}{dt} + y = f(t)$$

Let us try and solve this equation using opamp circuit blocks that we are familiar with



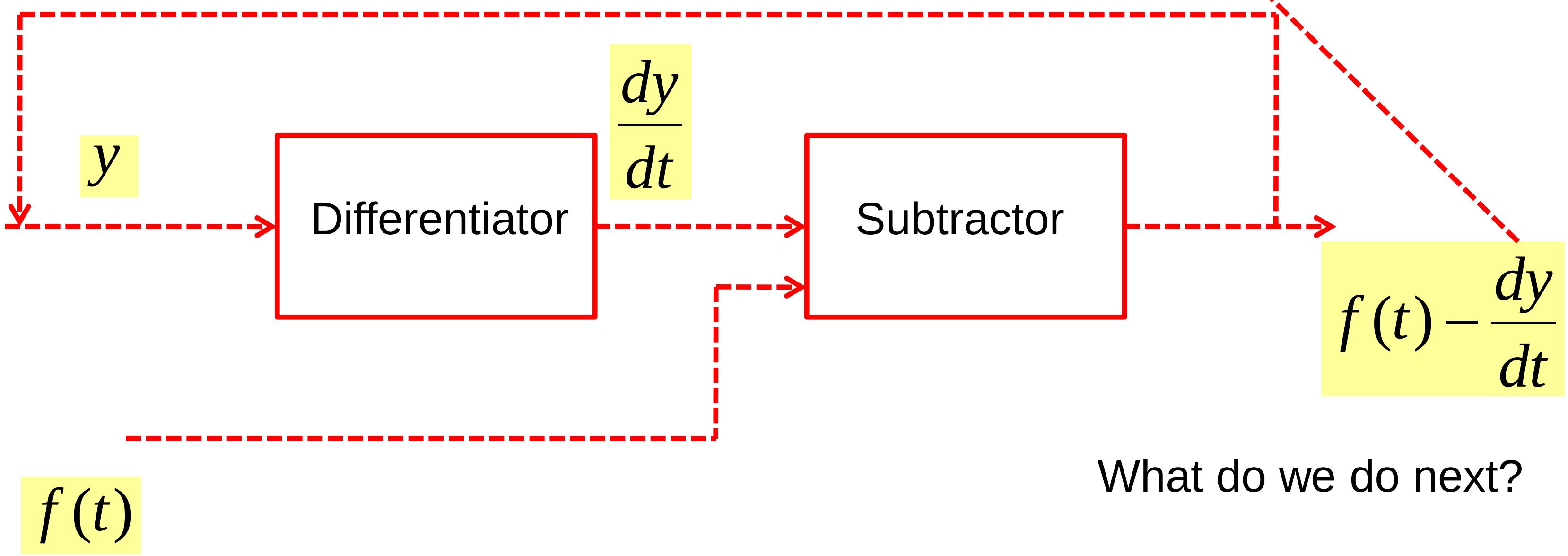
What do we do next?

Where do we get y from?

Solving Differential Equations?

$$\frac{dy}{dt} + y = f(t)$$

$$y = f(t) - \frac{dy}{dt}$$



What do we do next?

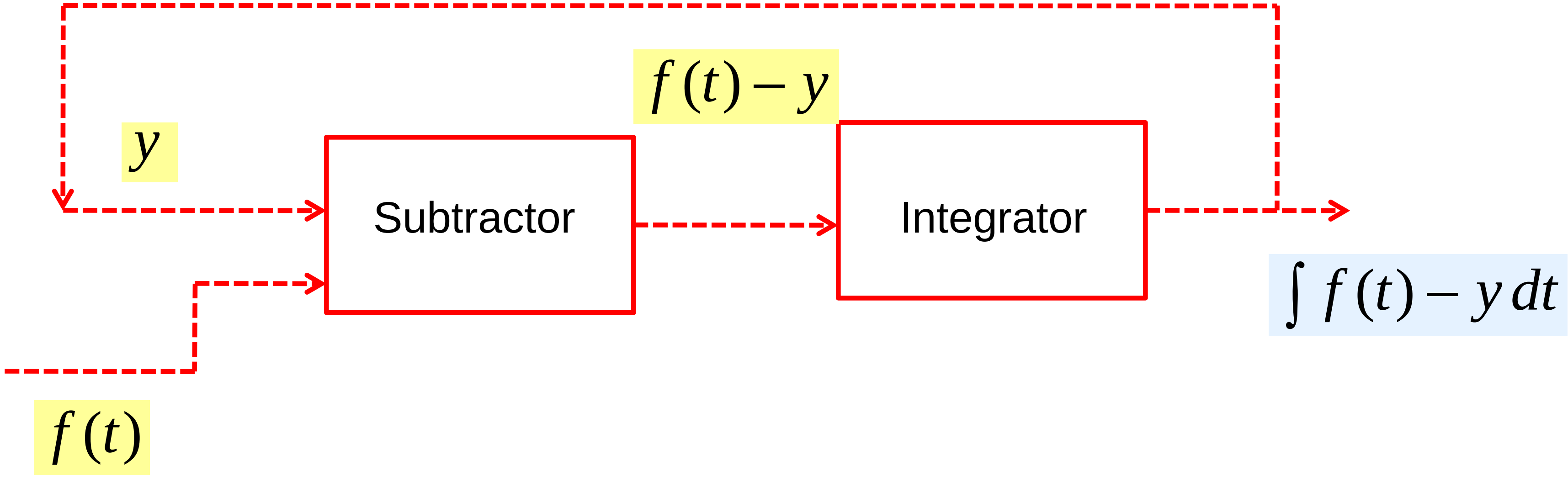
Where do we get y from?

Integrators are preferred over Differentiators because they are less sensitive to noise

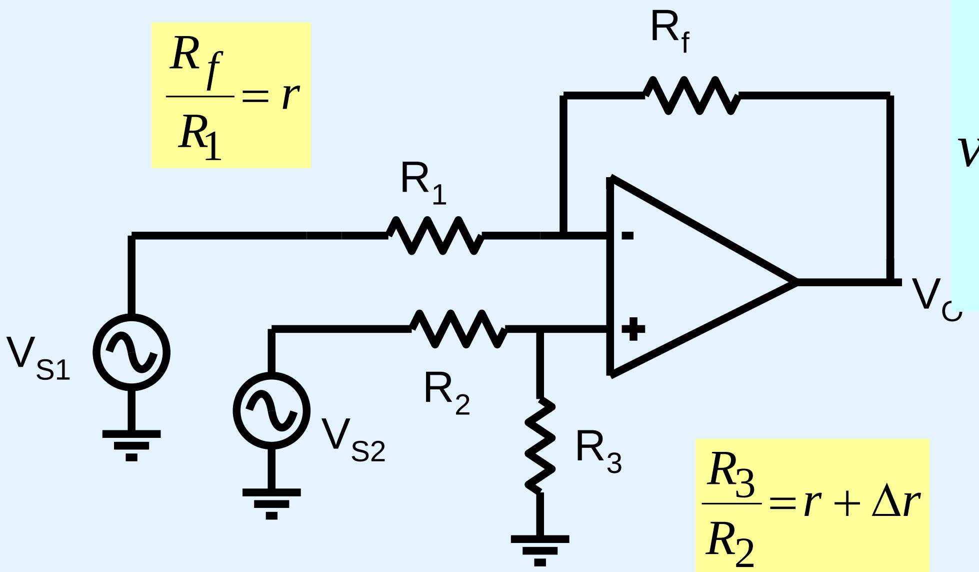
$$\frac{dy}{dt} + y = f(t)$$

$$\frac{dy}{dt} = f(t) - y \Rightarrow$$

$$y = \int f(t) - y \, dt$$



Effect Of Mismatches



$$\frac{R_f}{R_1} = r$$

$$\frac{R_3}{R_2} = r + \Delta r$$

$$v_o = v_{s2} \frac{\frac{R_3}{R_2}}{(1+\frac{R_3}{R_2})}(1+\frac{R_f}{R_1}) - (\frac{R_f}{R_1})v_{s1}$$

$$v_o \cong (v_{s2} - v_{s1})r + v_{s2}\frac{\Delta r}{1+r}$$

Error term

$$v_{id} = v_{S2} - v_{S1}$$

$$v_{ic} = \frac{v_{S1} + v_{S2}}{2}$$

$$v_{S2} = 0.5v_{id} + v_{ic}$$

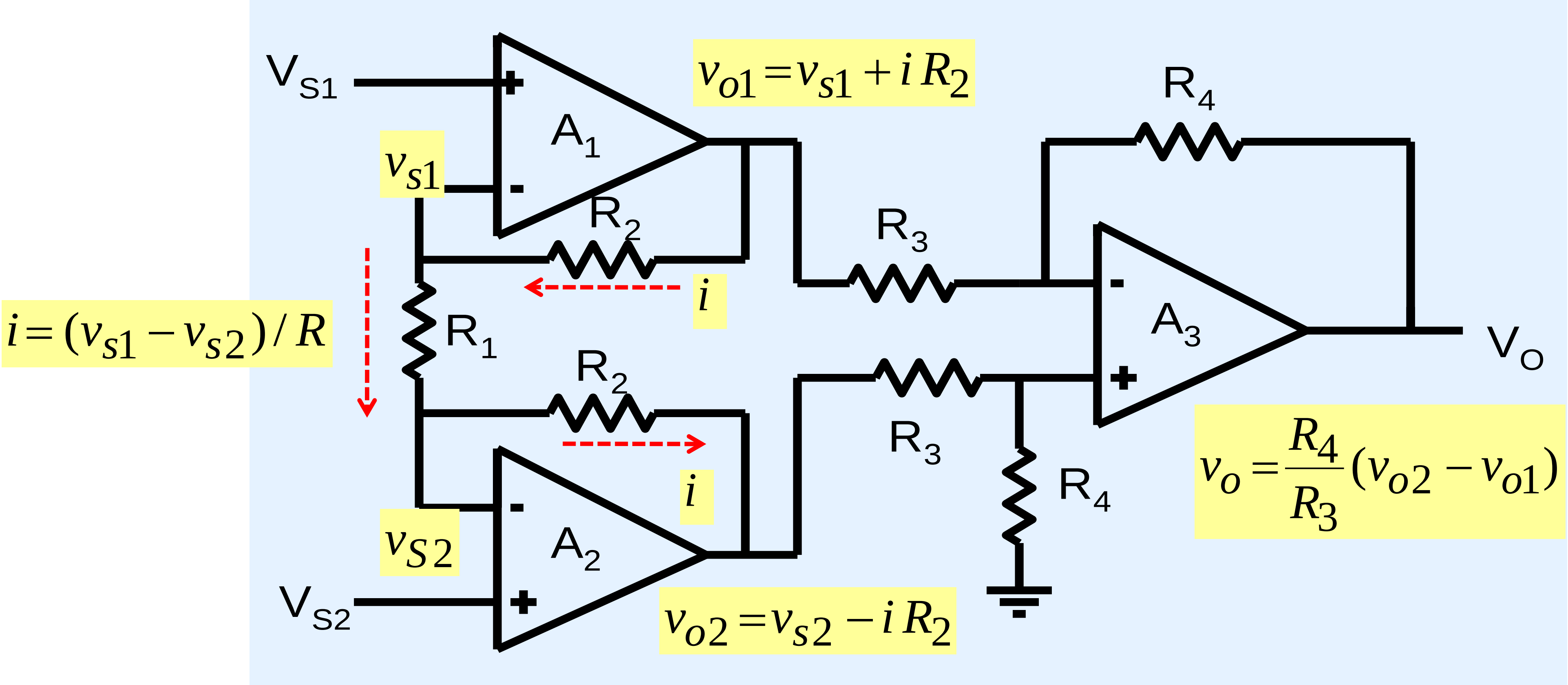
$$v_o = A_d v_{id} + A_{cm} v_{ic}$$

$$A_{dm} = r$$

$$A_{cm} = \frac{\Delta r}{1+r}$$

Common mode gain and CMRR depend on mismatches

Instrumentation Amplifier



$$v_o=\frac{R_4}{R_3}\times(1+\frac{2R_2}{R_1})\times(v_{s2}-v_{S1})$$

Very high input Resistance

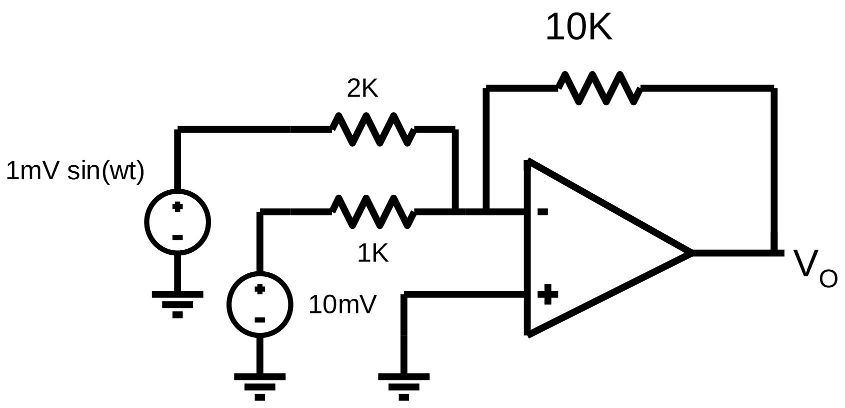
Can change one resistor R_1 and change gain

ESC102 : Introduction to Electronics

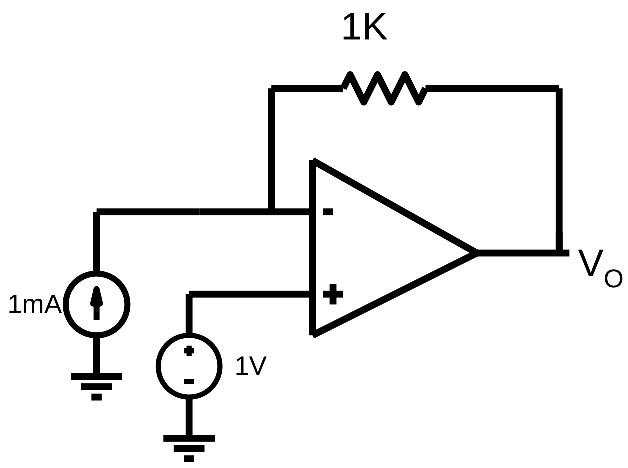
HW10: Solution

B. Mazhari
Dept. of EE, IIT Kanpur

Q.1 Determine the output of the ideal opamp circuits shown below

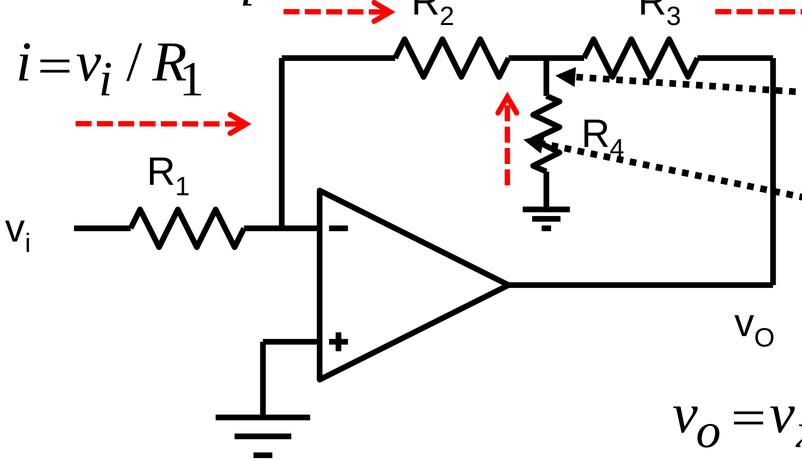


$$v_O = -\{\frac{10K}{1K}\times 10mV + \frac{10K}{2K}\times 1mV\sin(\omega t)\}$$
$$= -\{0.1 + 5\times 10^{-3}\sin(\omega t)\}$$



$$v_+ = v_- = 1V$$

$$\frac{1 - v_O}{1K} = 1mA \qquad v_O = 0V$$



i

$i = v_i / R_1$

$i_o = i + i_x$

$v_x = -iR_2 = -R_2 / R_1 v_i$

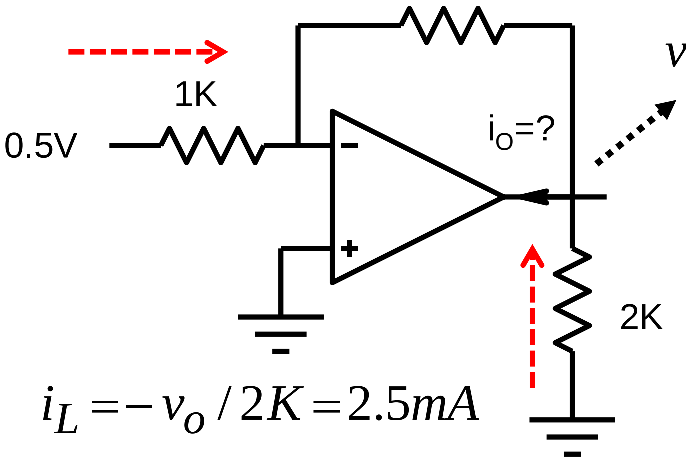
$i_x = -v_x / R_4$

$v_o = v_x - i_o R_3$

$$v_o = -\frac{R_2}{R_1}\left(1 + \frac{R_3}{R_4} + \frac{R_3}{R_2}\right) v_i$$

$i_s = 0.5mA$

$i_s = 0.5 / 1K = 0.5mA$



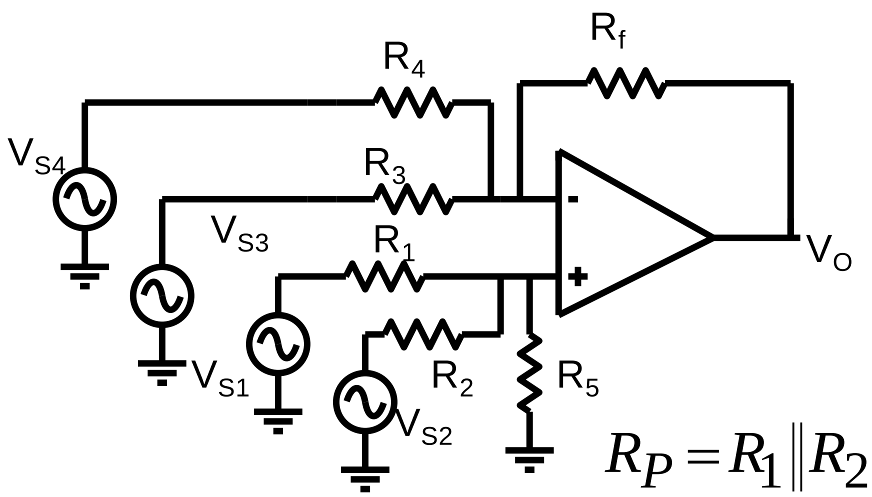
$v_o = -i_s \times 10K = -5V$

$i_o = i_S + i_L = 3mA$

$i_L = -v_o / 2K = 2.5mA$

Q.2 Design an opamp circuit that would generate the following output voltage where Vs1, Vs2, Vs3 and Vs4 are input voltages

$$V_O = 2v_{s1} + 4v_{s2} - 8v_{s3} - 10v_{s4}$$



$$R_P = R_1 \| R_2 \| R_5$$

$$v_o = -(\frac{R_f}{R_3})v_{s3} - -(\frac{R_f}{R_4})v_{s4} + (1 + \frac{R_f}{R_3 \| R_4}) \times \frac{R_P}{R_1} v_{s1} + (1 + \frac{R_f}{R_3 \| R_4}) \times \frac{R_P}{R_2} v_{s2}$$

Choose : $R_f = 10K$ $\Rightarrow R_3 = 1.25K$ $\Rightarrow R_4 = 1K$

$\Rightarrow \frac{R_P}{R_1} = 0.105$ $\Rightarrow \frac{R_P}{R_2} = 0.211$ $\Rightarrow \frac{R_1}{R_2} = 2$

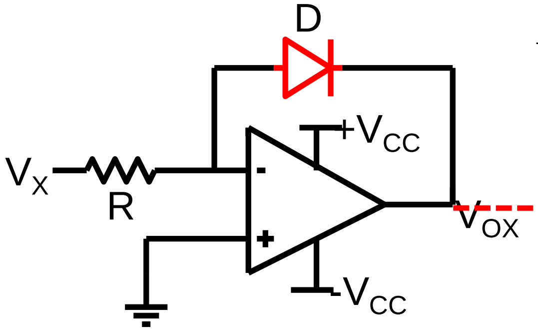
Choose : $R_2 = 1K$ $\Rightarrow R_1 = 2K$ $\Rightarrow R_P = 0.211K$ $\Rightarrow R_5 = 0.308K$

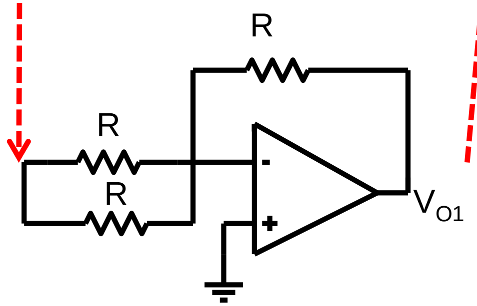
Q.3 Design an opamp circuit that can produce $V_O = K \times V_{IN}^2$ where Vin is the input voltage.

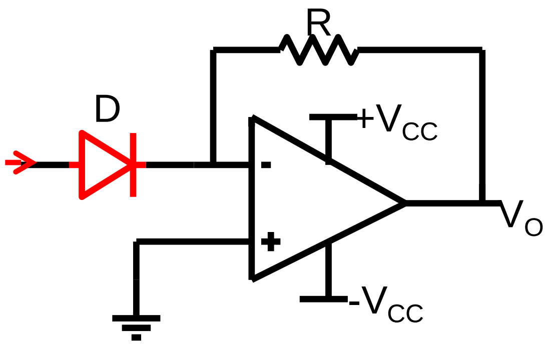
$$V_{O1} = V_T \times (\ln(\frac{V_X}{RI_S}) + \ln(\frac{V_X}{RI_S}))$$

$$V_{OX} = -V_T \times \ln(\frac{V_X}{RI_S})$$

$$V_{O1} = V_T \times \ln(\frac{V_X^2}{R^2 I_S^2})$$

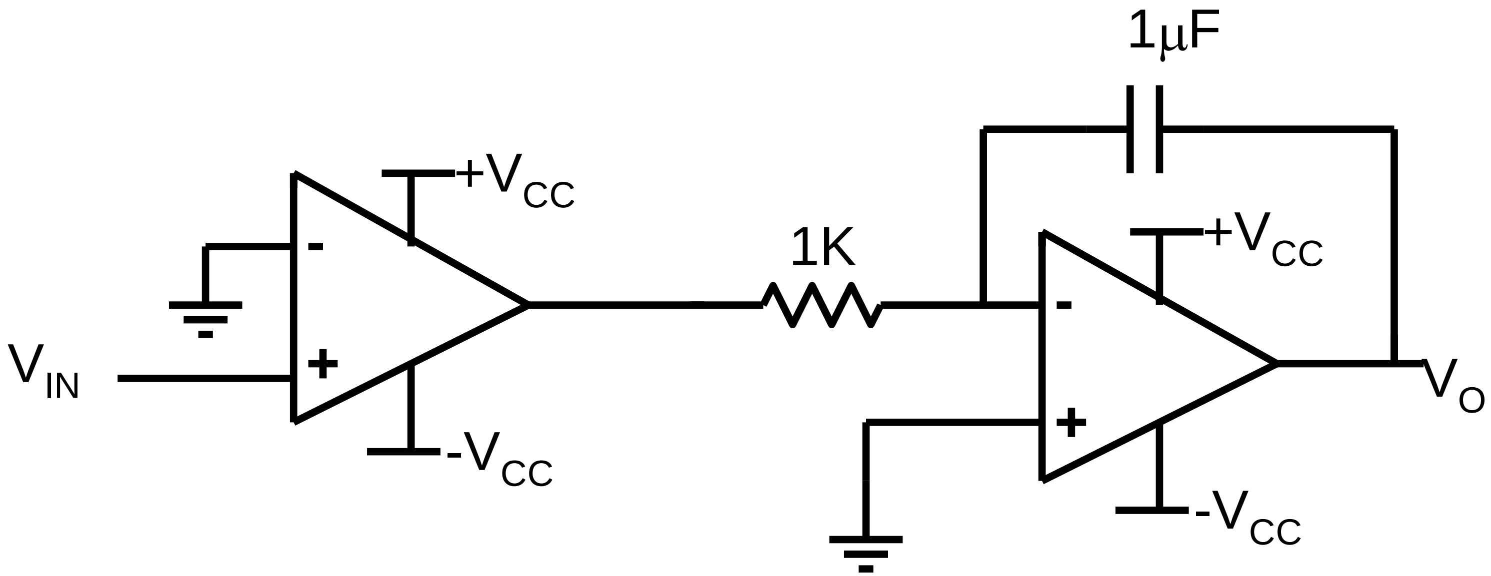


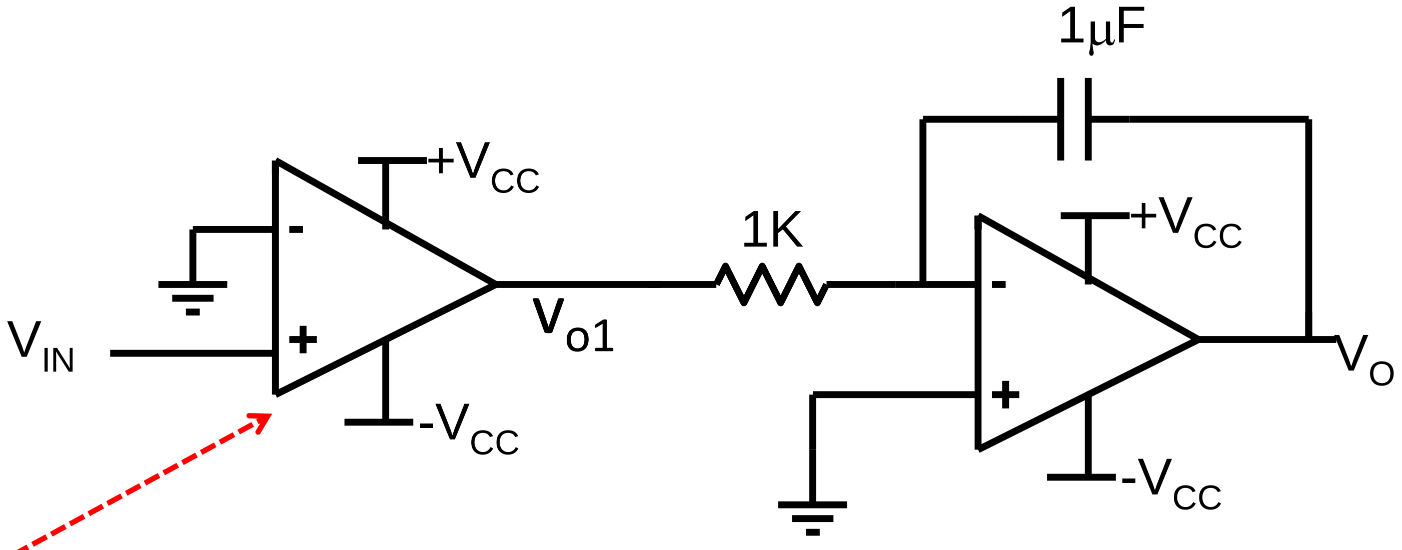




$$V_O \cong -RI_S \times e^{\frac{V_{O1}}{V_T}} = -\frac{V_X^2}{RI_S}$$

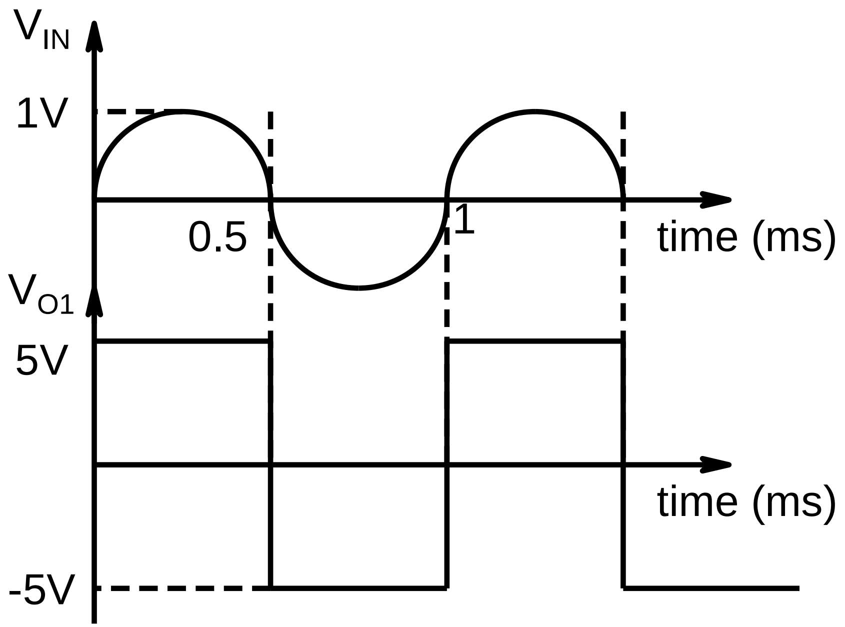
Q.4 Sketch the output voltage of the circuit shown below for $V_{in} = 1V\ Sin(2\pi ft);\ f = 1KHz$ and supply voltages of $\pm 5V$

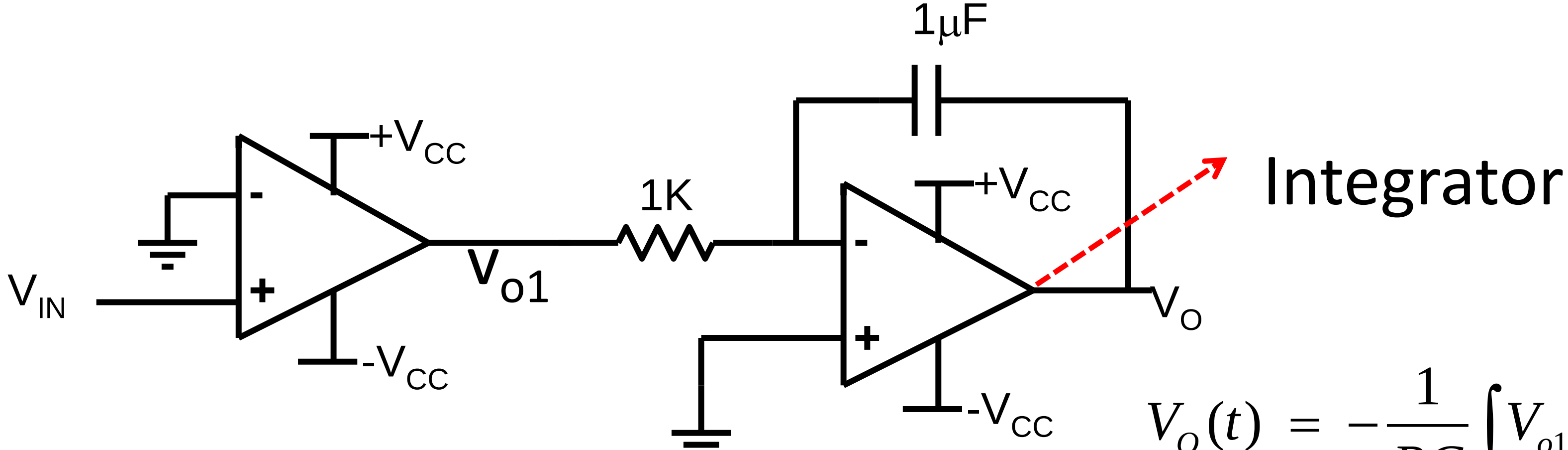




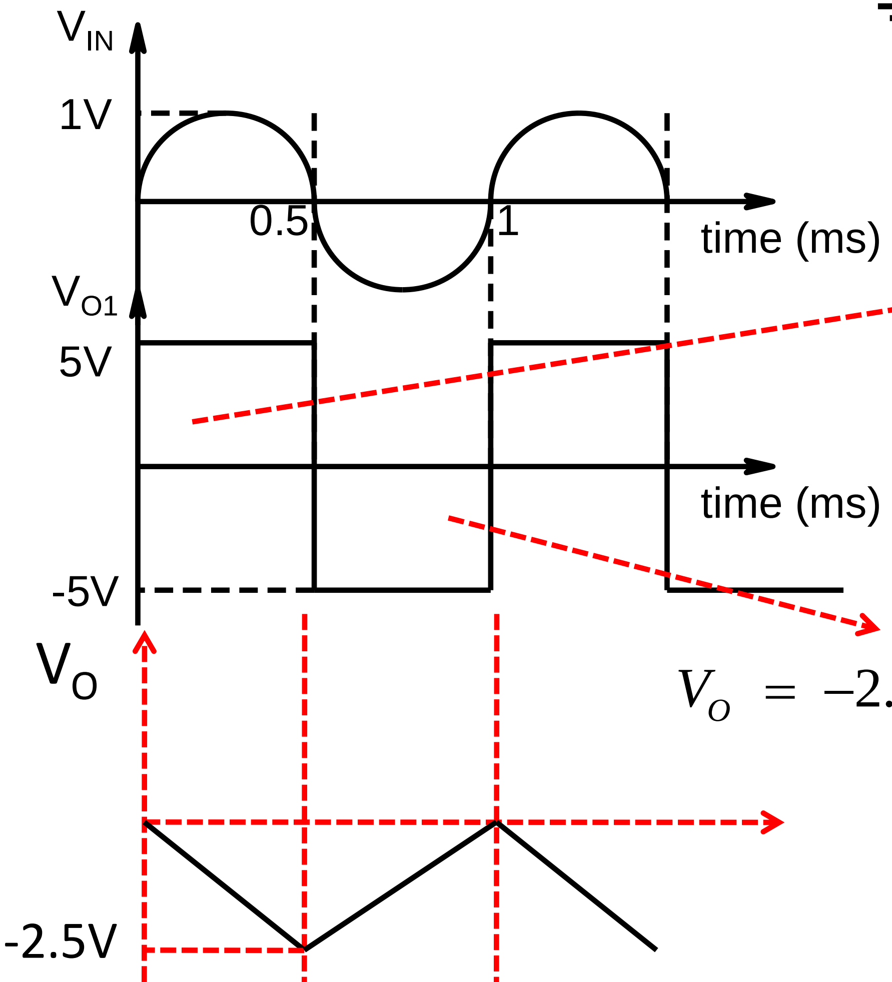
comparator

$$V_{O1} = +5V \; if \; v_{in} > 0$$
$$= -5V \; if \; v_{in} < 0$$





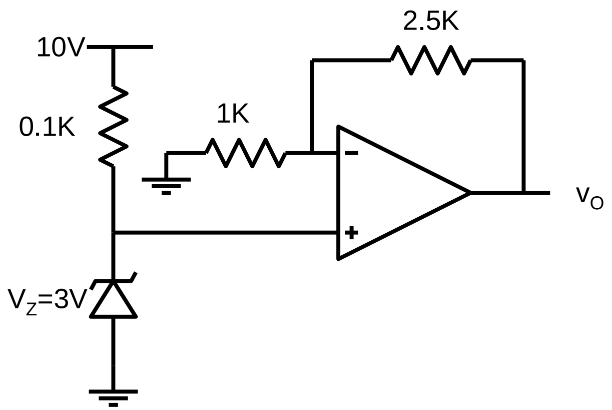
$$V_O(t) = -\frac{1}{RC}\int V_{o1}dt$$
$$= -10^3 \int V_{o1}dt$$

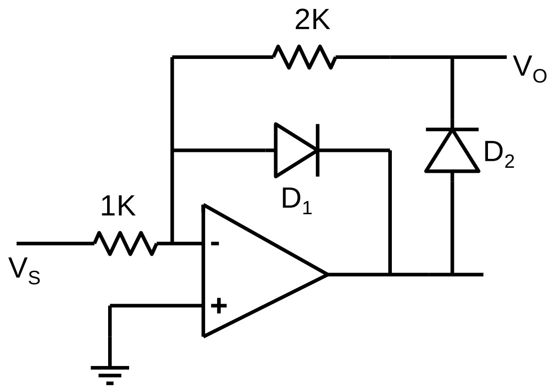


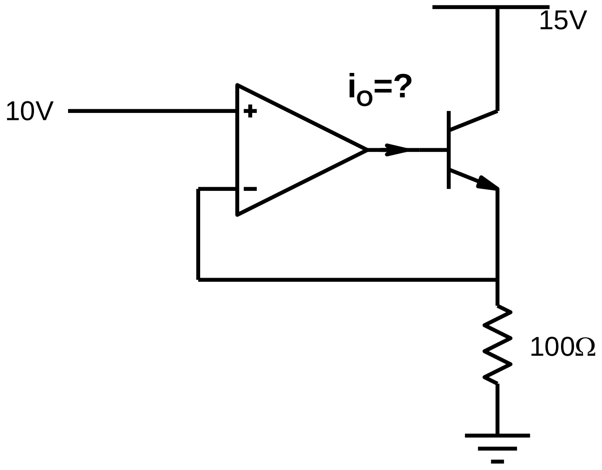
$$V_O = -5\times10^3 \times t \;\; for\; 0 \le t \le 0.5ms$$

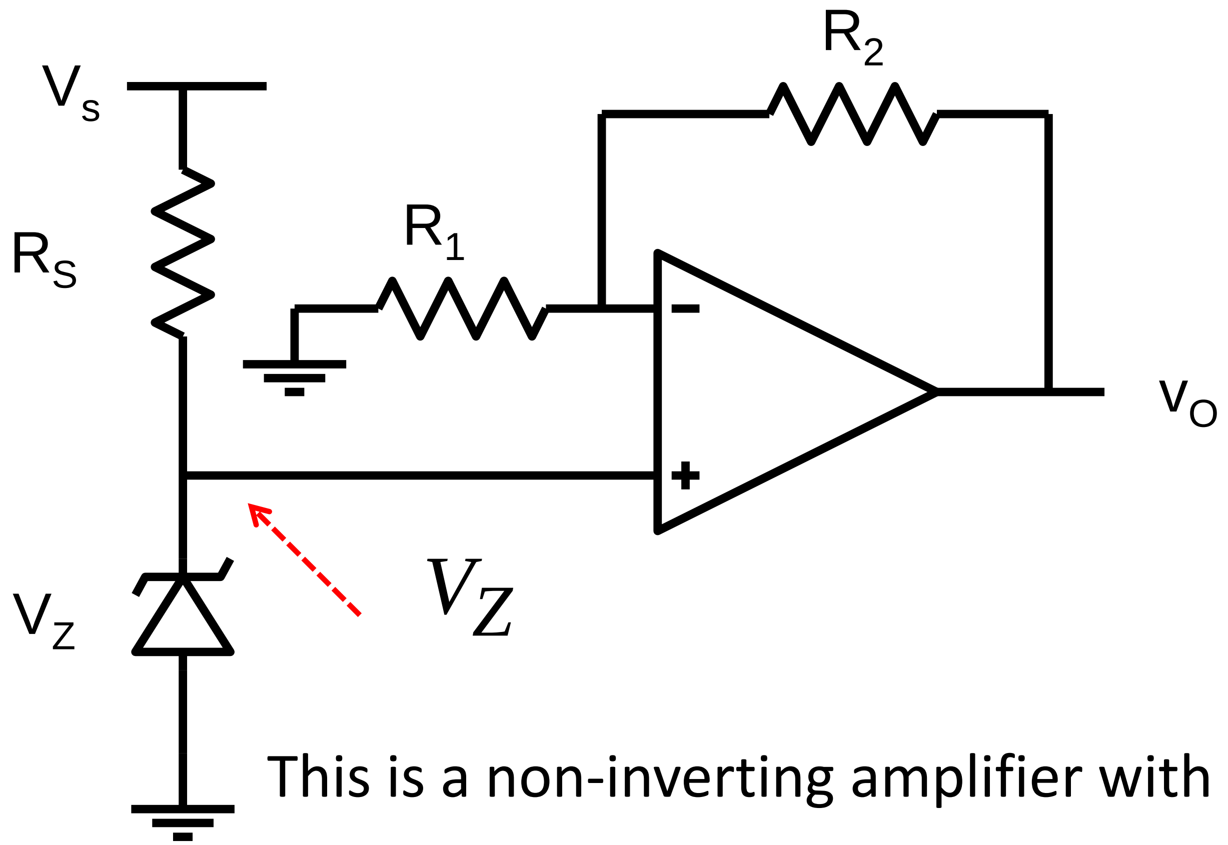
$$V_O = -2.5 + 5\times10^3 \times (t - 0.5ms) \;\; for\; 0.5ms \le t \le 1ms$$

Q.5 Determine the output for the ideal opamp circuits shown below. For the circuit on the right assume that diodes have cut-in voltage of 0.7V. Analyze the circuit for Vs = 1V and Vs = -1V. For the transistor assume a current gain of 100. What is the usefulness of each of the circuits?





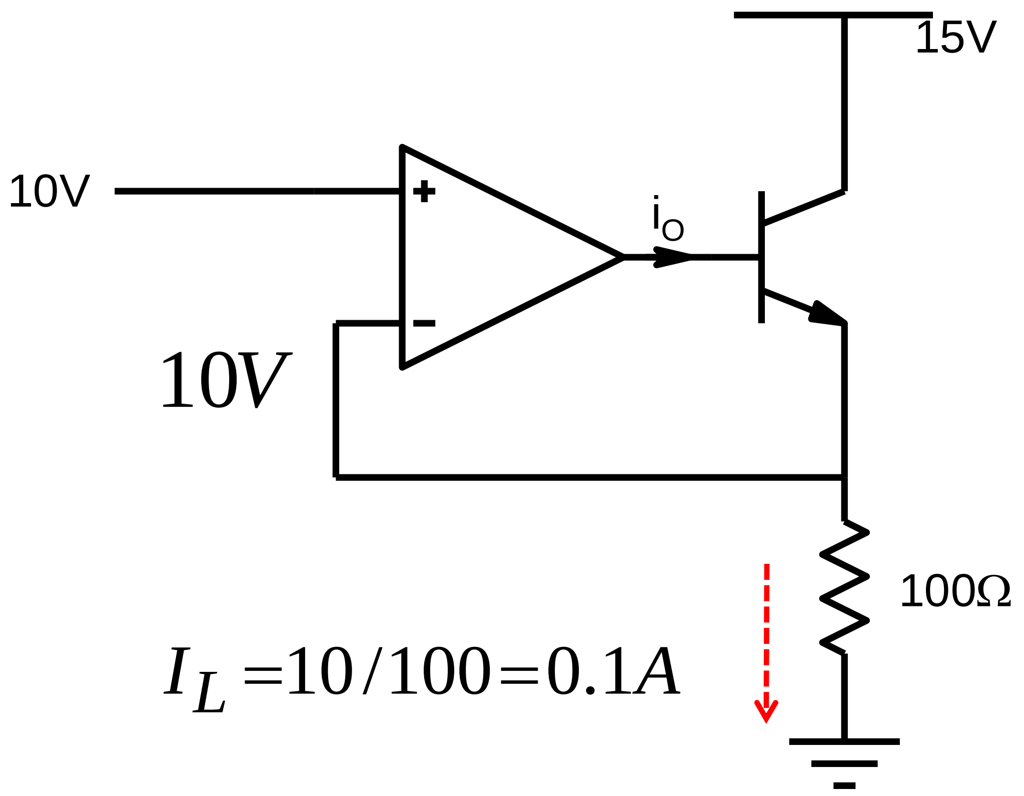




This is a non-inverting amplifier with V_Z as input

$$v_o = V_Z\left(1 + \frac{R_2}{R_1}\right)$$

The circuit produces a constant output voltage Vo even though supply voltage may vary and thus acts like a voltage regulator.

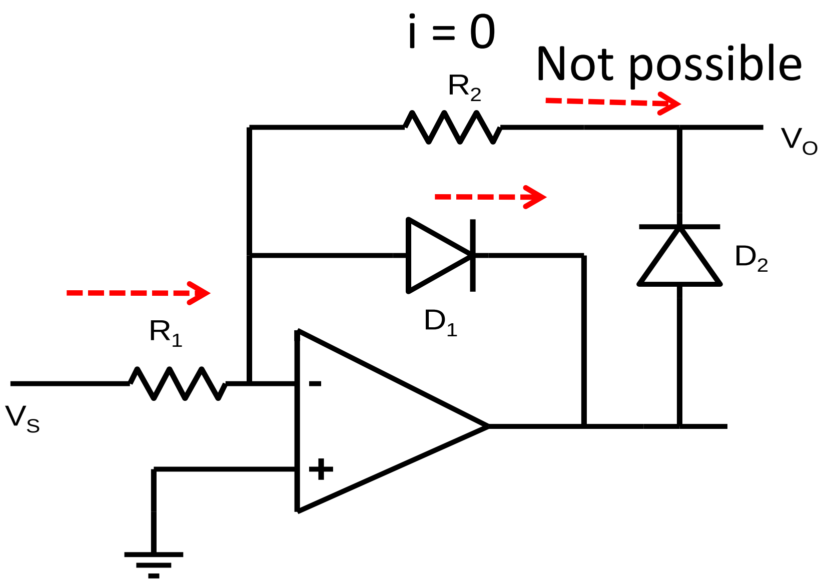


$$I_L = 10/100 = 0.1A$$

$$I_o = I_B = \frac{I_E}{\beta + 1} = 0.99mA$$

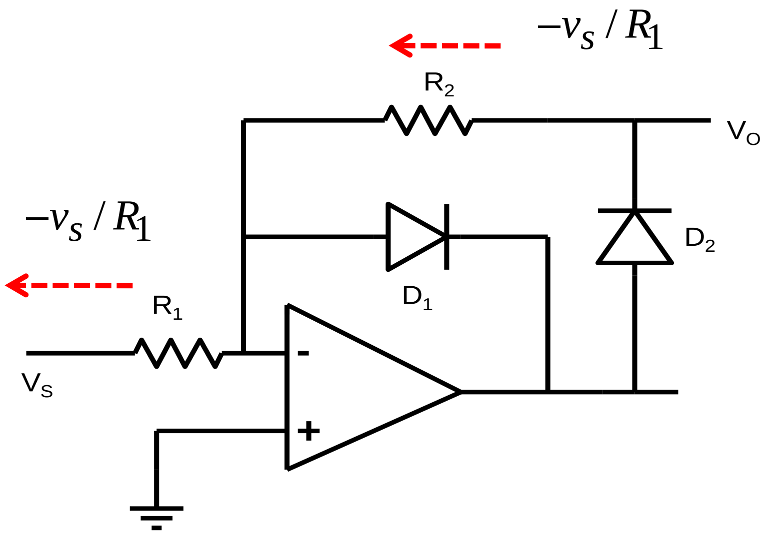
The circuit can supply load current that is much larger than opamp output current

$V_s > 0$



$V_O = 0$

$V_s < 0$



$$\frac{v_o}{v_S} = -\frac{R_2}{R_1}$$

The circuit acts like a rectifier