Work involved in Reversible Steady Flow

Raj Pala,
rpala@iitk.ac.in
Department of Chemical Engineering,
Associate faculty of the Materials Science Programme,
Indian Institute of Technology, Kanpur.

Previously: Entropy analysis in ideal gases

From the first *T ds* relation

$$ds = \frac{du}{T} + \frac{P\,dv}{T} \qquad du = c_v\,dT$$
$$P = RT/v$$

$$s_2 - s_1 = \int_1^2 c_v(T)\frac{dT}{T} + R\ln\frac{v_2}{v_1}$$

$$s_2 - s_1 = c_{v,\mathrm{avg}}\ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$$

$$\ln\frac{T_2}{T_1} = -\frac{R}{c_v}\ln\frac{v_2}{v_1}$$

From the second *T ds* relation

$$ds = \frac{dh}{T} - \frac{v\,dP}{T} \qquad v = RT/P$$

$$s_2 - s_1 = \int_1^2 c_p(T)\frac{dT}{T} - R\ln\frac{P_2}{P_1}$$

$$s_2 - s_1 = c_{p,\mathrm{avg}}\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$Tv^{k-1} = \text{constant}$$
$$TP^{(1-k)/k} = \text{constant}$$
$$Pv^{k} = \text{constant}$$

TD: Cengel & Boles

Work for closed system

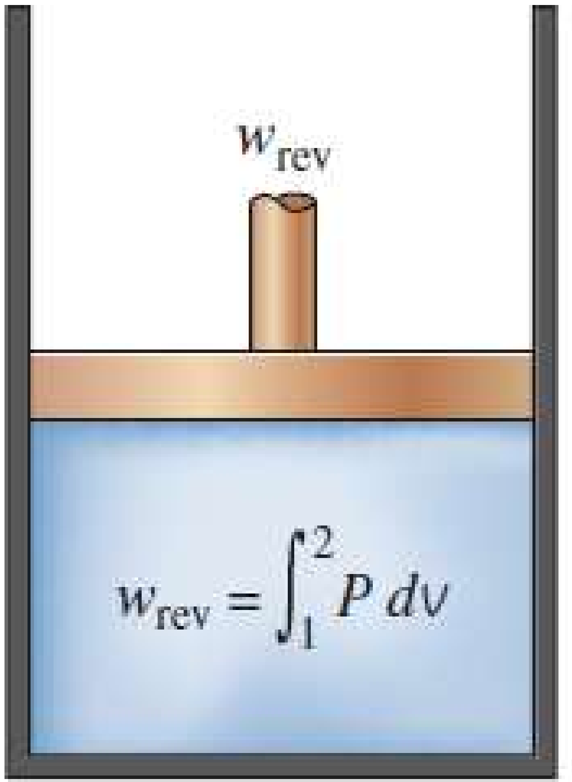


Fig-TD: Cengel & Boles

Work for reversible steady flow device

$$\delta q_{rev} - \delta w_{rev} = dh + dke + dpe$$

$$-\delta w_{rev} = v\, dP + dke + dpe$$

From the second $T\,ds$ relation

$$\delta q_{rev} = T\, ds$$

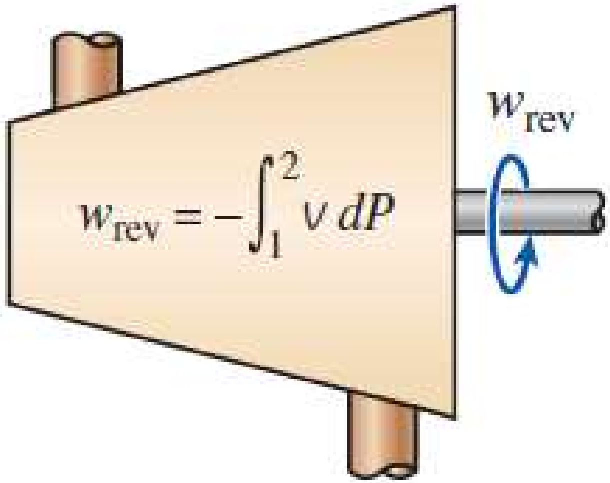
$$T\, ds = dh - v\, dP$$

Work Output

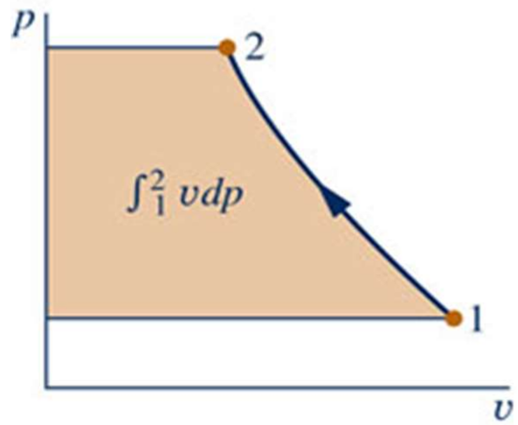
$$w_{rev} = -\int_1^2 v\, dP - \Delta ke - \Delta pe$$

$$w_{rev} = -\int_1^2 v\, dP$$

When kinetic and potential energies are negligible



Work Input $$w_{rev,in} = \int_1^2 v\, dP + \Delta ke + \Delta pe$$



Figs-TD: Cengel & Boles; TD: Moran, Shapiro, Boetner & Bailey

Simplifications for incompressible fluids

$$w_{rev} = -\int_1^2 v \, dP - \Delta ke - \Delta pe$$

For incompressible fluids

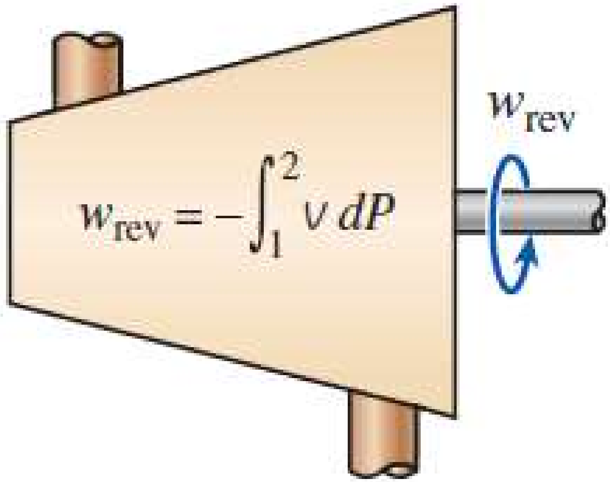
$$w_{rev} = -v(P_2 - P_1) - \Delta ke - \Delta pe$$

No work interaction: Bernoulli Equation

$$v(P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0$$

TD: Cengel & Boles

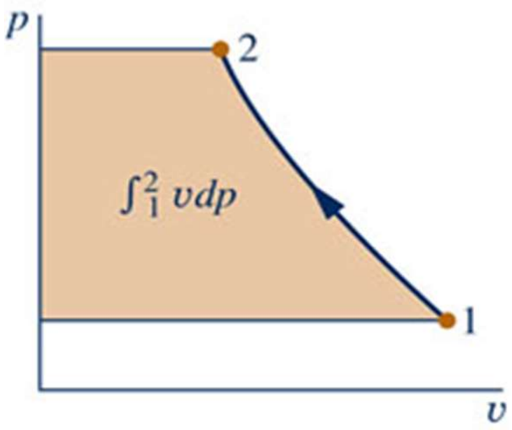
Importance of specific volume



- Work I/P-Small ν (Compression)
- Work O/P-Large ν (Expansion)
- Steam power plant: $\Delta \boldsymbol{P}_{\boldsymbol{rise}}^{\boldsymbol{pump}} \cong \Delta \boldsymbol{P}_{\boldsymbol{drop}}^{\boldsymbol{turbine}}$
- Pump acts on liquid and turbine via vapor; Hence, Work O/P >> Work I/P
- Gas vs. steam power plant: Compression & Cooling

Fig-TD: Cengel & Boles

Minimizing Work Required by Compressor



$$w_{\text{rev,in}} = \int_1^2 v\, dP$$

When kinetic and potential energies are negligible

Isentropic (Pv^k = constant):

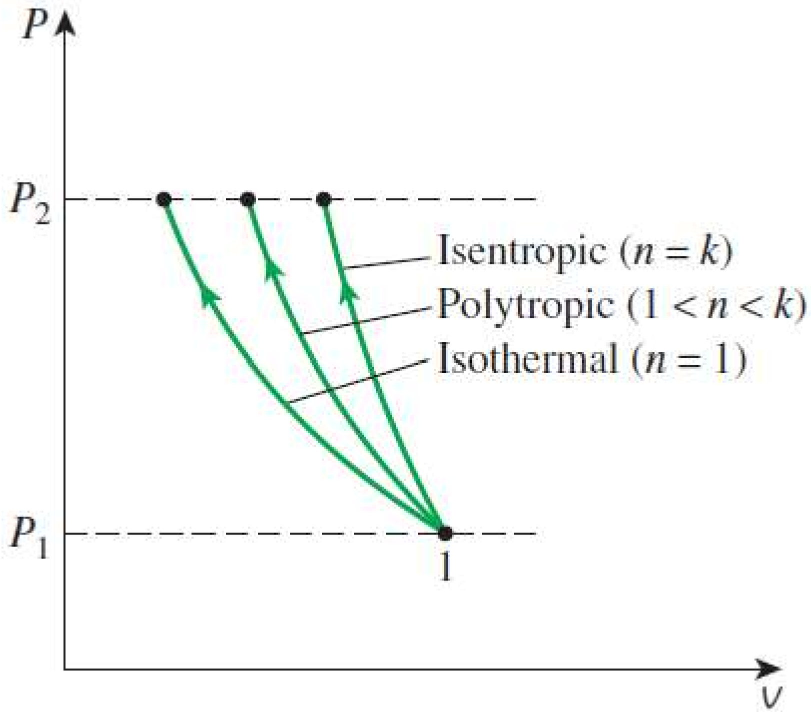
$$w_{\text{comp,in}} = \frac{kR(T_2 - T_1)}{k - 1} = \frac{kRT_1}{k - 1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right]$$

Polytropic (Pv^n = constant):

$$w_{\text{comp,in}} = \frac{nR(T_2 - T_1)}{n - 1} = \frac{nRT_1}{n - 1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

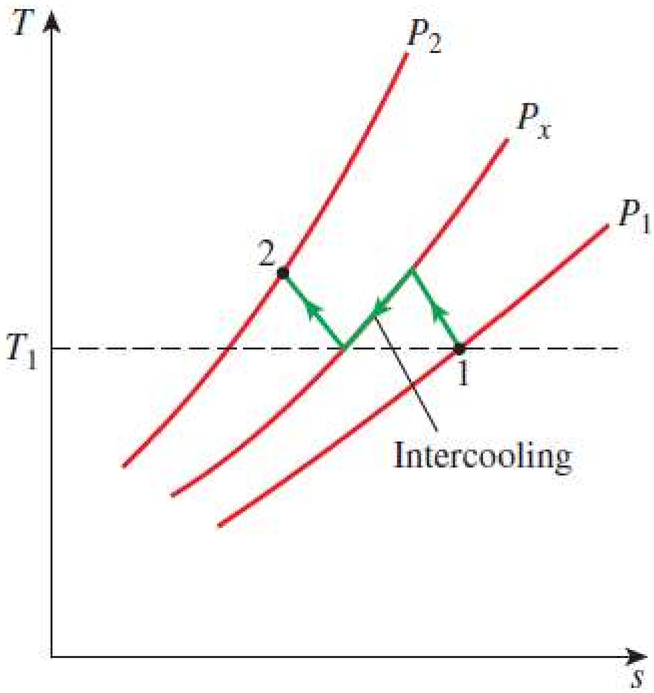
Isothermal (Pv = constant):

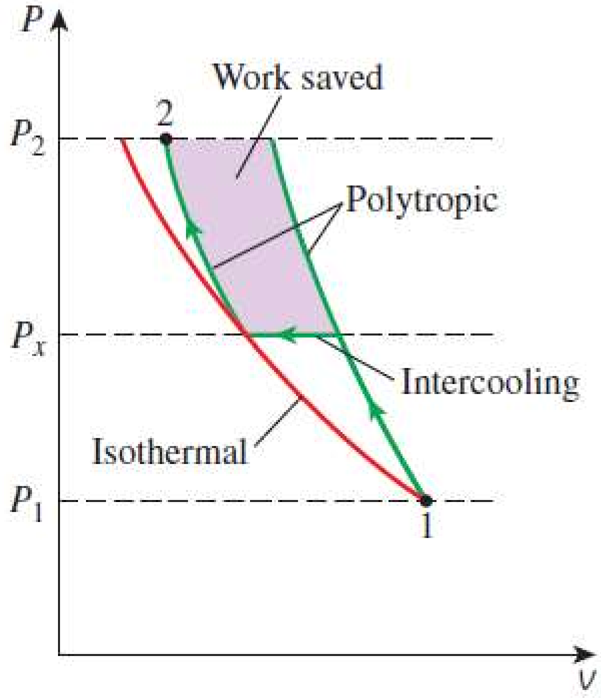
$$w_{\text{comp,in}} = RT \ln \frac{P_2}{P_1}$$



Figs-TD: Cengel & Boles; TD: Moran, Shapiro, Boetner & Bailey

Work in Compressor with intercooling





$$w_{\text{comp,in}} = w_{\text{comp I,in}} + w_{\text{comp II,in}}$$

$$= \frac{nRT_1}{n-1}\left[\left(\frac{P_x}{P_1}\right)^{(n-1)/n} - 1\right] + \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_x}\right)^{(n-1)/n} - 1\right]$$

$$P_x = (P_1P_2)^{1/2} \quad \text{or} \quad \frac{P_x}{P_1} = \frac{P_2}{P_x}$$

Figs-TD: Cengel & Boles

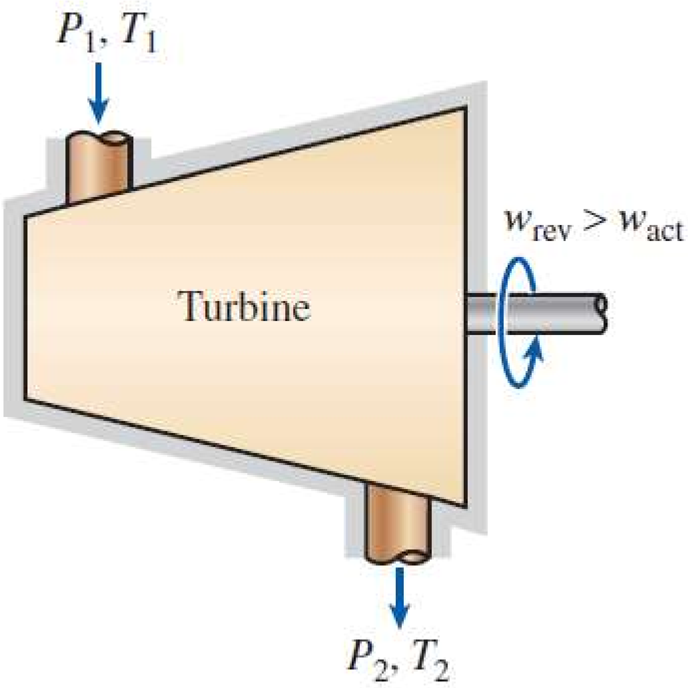
Steady-flow devices deliver (consume) most (least) work when process is reversible

Actual

$$\delta q_{act} - \delta w_{act} = dh + d\text{ke} + d\text{pe}$$

Reversible

$$\delta q_{rev} - \delta w_{rev} = dh + d\text{ke} + d\text{pe}$$



$$\delta q_{act} - \delta w_{act} = \delta q_{rev} - \delta w_{rev}$$

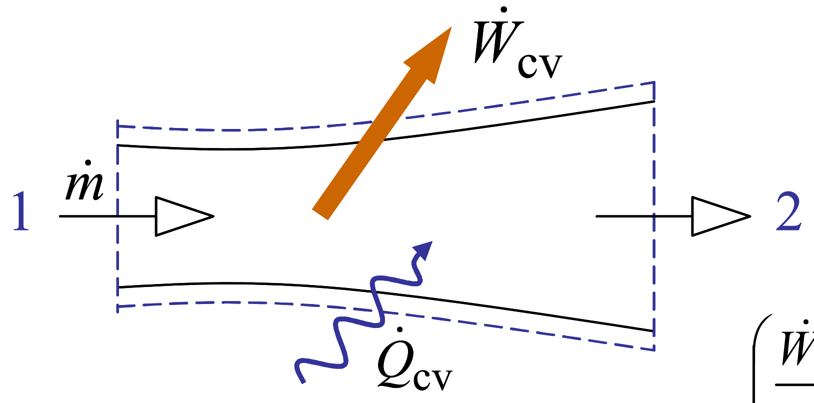
$$\delta w_{rev} - \delta w_{act} = \delta q_{rev} - \delta q_{act}$$

$$\delta q_{rev} = T\, ds \qquad\qquad ds \geq \frac{\delta q_{act}}{T}$$

$$\frac{\delta w_{rev} - \delta w_{act}}{T} = ds - \frac{\delta q_{act}}{T} \geq 0 \qquad\qquad w_{rev} \geq w_{act}$$

Fig-TD: Cengel & Boles

Entropy Changes & Heat Transfer in Reversible Steady Flow process



$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{\text{int}\\\text{rev}}} = \left(\frac{\dot{Q}_{cv}}{\dot{m}}\right)_{\substack{\text{int}\\\text{rev}}} + (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)$$

$$\left(\frac{\dot{Q}_{cv}}{\dot{m}}\right)_{\substack{\text{int}\\\text{rev}}} = \int_1^2 T\,ds$$

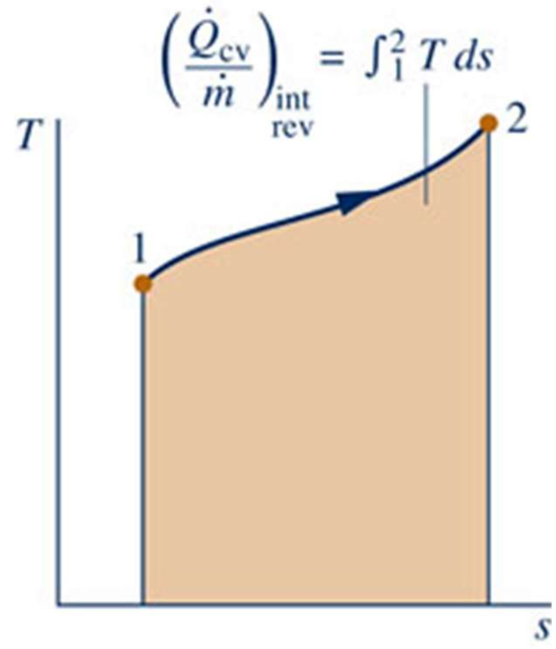


Fig-TD: Moran, Shapiro, Boetner & Bailey

What's next?

- Isoentropic efficiencies of steady-flow devices