Q.1. (a) We design the language ~~over~~ with input alphabet
$\Sigma = \{0, 1, x\}$

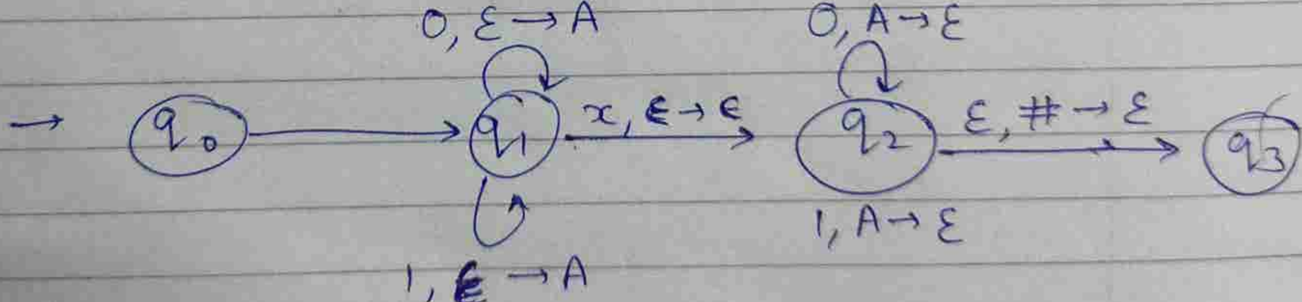
~~$L = \{ a x b$~~
~~$L = \{ m x n$~~
$L = \{ a x b \mid |a| = |b| , a, b \in \{0,1\}^* \}$

We give input of form axb and if it get accepted we get $|a| = |b|$ otherwise $|a| \neq |b|$

PDA which accepts above language L. with input Σ & stack alphabet Γ

PDA be $= (Q, \Sigma\, \Gamma, \delta, q_0, F)$
where $\Sigma = \{0, 1, x\}$, $\Gamma = \{\#, A\}$

$q_0 = q_0$
$F = \{q_3\}$ $Q = \{q_0, q_1, q_2, q_3\}$



(b) langugage will be
$L_2 = \{ a x b , a, b \in \{0,1\}^* \mid a = b \}$

We can check if $a = b$ by checking if L_2 accepts axb as if it accepts axb, then $a = b$ otherwise $a \neq b$.

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(C) PDA for L_2 cannot be given.
~~L_2 defined in~~
we will proove L_2 defined in (b) is not CFL.
using contrapositive form of pumping lemma.
and if L_2 is not CFL it does not have PDA.

Given p, Choose $w = 0^p 1^p x 0^p 1^p$,
$\Rightarrow w \in L_2$ and $|w| \geq p$. Consider partition
$w = abcde$.

Case 1 x lies in either b or d.
if $i=0$, then $ab^0cd^0e = ace$, will not contain
x & hence is not accepted by L_2.

~~Case 2 x lies in either a or e.~~

Case 2 x lies in c.
Since $|bcd| \leq p$, on setting $i=0$
$ab^0cd^0e = ace$ will have form $0^p 1^{p-k_1} x 0^{p-k_2} 1^p$
with
$k_1 > 0$ or $k_2 > 0$ as $bd > 0$.
Hence ace will not be accepted by L_2

Case 3 x lies in either a or e.
let x lies in a, then since $|bd| > 0$ on setting $i=0$
ab^0cd^0e will be alike uxv s.t. $|u| > |v|$
$\Rightarrow u \neq v \Rightarrow uxv$ is not accepted by L_2

Hence by contrapositive statement of pumping lemma.
L_2 is not a CFL so not have a PDA.

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A.2. $L = \{a^n b^j c^k \mid k = jn, j, k, n \geq 0\}$

We shall proove the language is not CFL using pumping lemma.
For $p > 0$, consider ~~$w = a^p b^p c^q$~~ $w = a^p b^p c^q$ where $q = p^2$.
As $w \in L$ & $|w| > p$ $\Rightarrow$ w can be divided into $w = uvxyz$, for this we consider the given cases.

Case I $|vxy| \leq p$ (length of vxy can be atmost p).

vxy ~~contain~~ can't contain both a & c together as the separation between a and c is p letters. It can contain either a or c or neither.
Consider $w = uv^0xy^0z = uxz$.

Case I vxy contain only one type of character.

If vxy contains only a, then $w' = a^{p-k_1} b^p c^{p^2}$ or vxy only contains b. then $w' = a^p b^{p-k_2} c^{p^2}$ or vxy only contains c, then $w' = a^p b^p c^{p^2-k_3}$.

Way in which L accepts $w' := a^i b^j c^k$ if $k = ij$ where $k_1 = 0$, $k_2 = 0$ or $k_3 = 0$ respectively for three cases. But $|vy| > 0 \Rightarrow k_1 > 0, k_2 > 0$ and $k_3 > 0$. Hence w' is not accepted by L.

Case 2. vxy contains of both a & b only.

$w' = uxz$ will be of the form $a^{p-k_1} b^{p-k_2} c^{p^2}$
Since $(p-k_1)(p-k_2) < p^2$
$\Rightarrow w' \notin L$.

~~Case~~
Case 3. vxy contains b & c only.

$w' = uxz$ will be of form $a^p b^{p-k_1} c^{p^2-k_2}$
let say $w' \in L$, then
$pk_1 = k_2$, $p(p-k_1) = p^2 - k_2$, $p^2 - pk_1 = p^2 - k_2$

all are true. Two cases to be consider for which $pk_1 = k_2$ which are k_2 can be 0 or p as $|vxy| \le p$ and k_1 is integer.
if $k_2 = 0 \Rightarrow k_1 = 0$ which is contradiction as $|vy| > 0$.
If $k_2 = p$ then $k_1 = 1$ but it imply that ~~$|vy| = p+1$~~ $|vy| = p+1 > p$ which again is ~~contra~~ wrong. as $|vxy| \le p$

As no value of k_1, k_2 satisfy the condition $pk_1 = k_2$, our assumption is wrong that $w' \in L$. $\Rightarrow \boxed{w' \notin L}$

Hence, by contrapositive statement ~~of p~~ of pumping lemma, L is not a CFL.

Q.3 (a). CFG Production rules are:-

$S \rightarrow S_1 \mid S_2$

$S_1 \rightarrow X C_1$

$X \rightarrow aXb \mid \varepsilon$

$C_1 \rightarrow cC_1 \mid \varepsilon$

$S_2 \rightarrow AY \mid YC_2$

$A \rightarrow aA \mid a$

$C_2 \rightarrow cC_2 \mid c$

$Y \rightarrow aYc \mid B$

$B \rightarrow bB \mid \varepsilon$

languages represented by symbols.

$S = \{ a^i b^i c^k \mid i=j \text{ or } i \neq k \text{ where } i,j,k \geq 0 \}$

$S_1 = \{ a^i b^j c^k \mid i=j \text{ where } i,j,k \geq 0 \}$

$S_2 = \{ a^i b^j c^k \mid i \neq k \text{ where } i,j,k \geq 0 \}$

$X = \{ a^i b^j \mid i=j \text{ where } i,j \geq 0 \}$

$C_1 = \{ c^i \mid i \geq 0 \}$

$A = \{ a^i \mid i \geq 1 \}$

$C_2 = \{ c^i \mid i \geq 1 \}$

$B = \{ b^i \mid i \geq 0 \}$

$Y = \{ a^i b^j c^k \mid i=k \text{ where } i,j,k \geq 0 \}$

A.3(b) Given production rules.

i). $S \to BSB \mid B \mid \varepsilon$

ii) $B \to 00 \mid \varepsilon$

(a) Add new variable S_0

$S_0 \to S$

$S \to BSB \mid B \mid \varepsilon$

$B \to 00 \mid \varepsilon$

(b) removing $B \to \varepsilon$

$S_0 \to S$

$S \to BSB \mid B \mid \varepsilon \mid BS \mid SB$

$B \to 00$

(c) Removing $S \to \varepsilon$

$S_0 \to S \mid \varepsilon$

$S \to BSB \mid BS \mid SB \mid B \mid BB$

$B \to 00$

(d) removing unit ~~rules~~ rule $S \to B$

$S_0 \to S \mid \varepsilon$

$S \to BSB \mid BS \mid SB \mid 00 \mid BB$

$B \to 00$

(e) removing unit rule $S_0 \to S$

$S_0 \to BSB \mid BS \mid SB \mid 00 \mid BB \mid \varepsilon$

$S \to BSB \mid BS \mid SB \mid 00 \mid BB$

$B \to 00$

(f) ~~Soln~~ Shortening $S_0 \to BSB$ ~~to~~ $S_0 \to BC$ and $S \to BSB$ to $S \to BC$ & adding $U \to SB$

$S_0 \to BU \mid BS \mid SB \mid 00 \mid BB \mid \varepsilon$

$S \to BU \mid BS \mid SB \mid 00 \mid BB$

$B \to 00$

$U \to SB$

(g) Adding variable. A & $A \rightarrow 0$ rule.

$$S_0 \rightarrow BU \mid BS \mid SB \mid AA \mid BB \mid \varepsilon$$
$$S \rightarrow BU \mid BS \mid SB \mid AA \mid BB$$
$$B \rightarrow VV$$
$$U \rightarrow SB$$
$$V \rightarrow 0$$

This is the Chomsky Normal form.