ESC201T : Introduction to Electronics

Lecture 33: Digital Circuits-3

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Obtaining Boolean expressions from truth Table

x_1	x_2	x_3	y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$$y = \overline{x_1}.\overline{x_2}.x_3 + \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.x_3 + x_1.x_2.x_3$$

Sum of Products (SOP) form

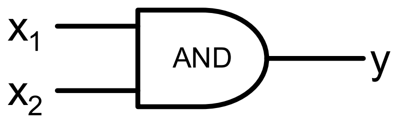
$$y = (x_1 + x_2 + x_3).(x_1 + \overline{x_2} + x_3).(\overline{x_1} + x_2 + x_3).(\overline{x_1} + \overline{x_2} + x_3)$$

Product of Sum (POS) form

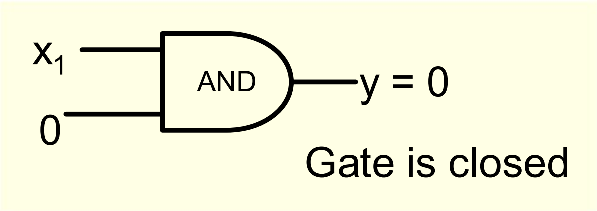
Implementing Boolean expressions

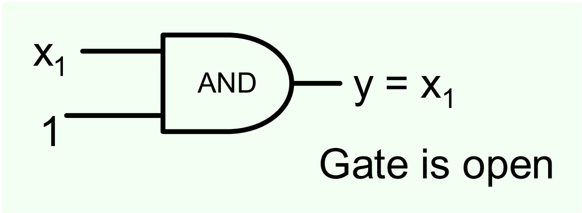
Elementary Gates

AND: $y = x_1 . x_2$

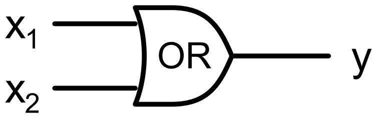


Why call it a gate?

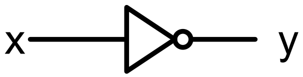




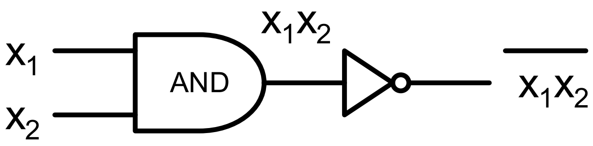
OR: $y = x_1 + x_2$

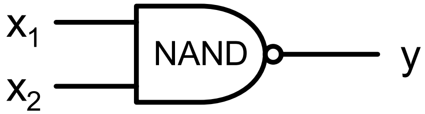


NOT: $y = \overline{x}$

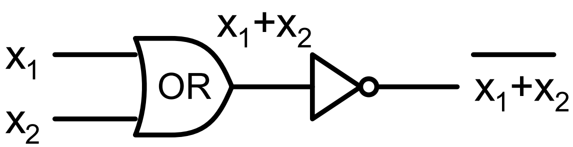


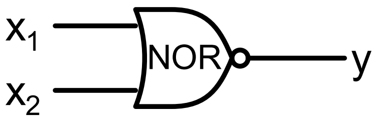
NAND: $y = \overline{x_1 . x_2}$





NOR: $y = \overline{x_1 + x_2}$

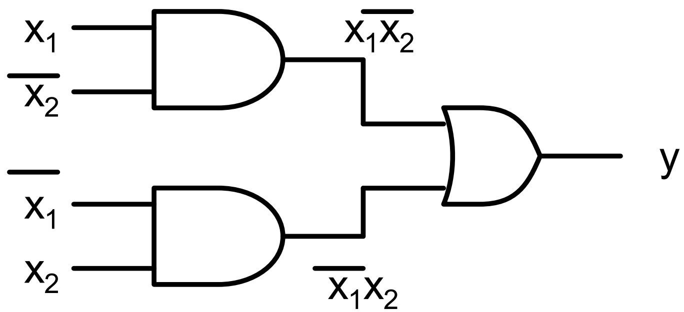


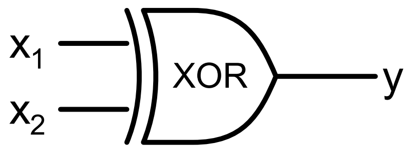


XOR: $y = x_1 \oplus x_2 = x_1.\overline{x_2} + \overline{x_1}.x_2$

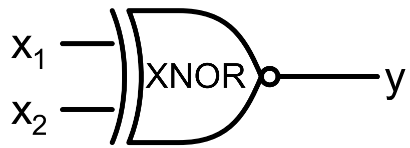
x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

Y is 1 if only one variable is 1 and the other is zero





XNOR: $y = x_1 \odot x_2 = x_1.x_2 + \overline{x_1}.\overline{x_2}$

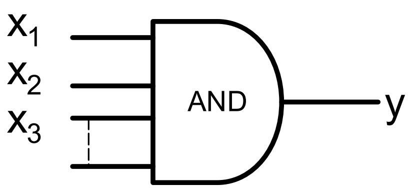


Y is 1 if only both variables are either 0 or 1

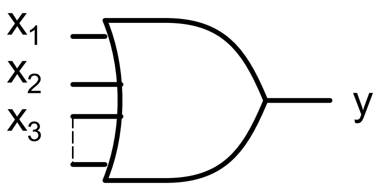
$$y = x_1 \odot x_2 = \overline{x_1 \oplus x_2}$$

Gates with more than 2 inputs

$$\text{AND:} \quad y = x_1 . x_2 . x_3 \ldots$$



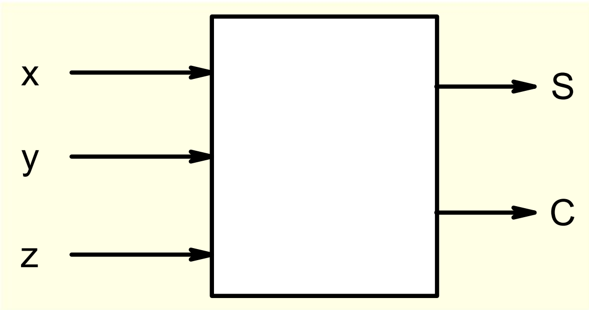
$$\text{OR:} \quad y = x_1 + x_2 + x_3 + \ldots.$$



$$\text{XOR:} \quad y = x_1 \oplus x_2 \oplus x_3 = x_1 . \overline{x_2}\,\overline{x_3} + \overline{x_1} . x_2 . \overline{x_3} + \overline{x_1} . \overline{x_2} . x_3 + x_1 . x_2 . x_3$$

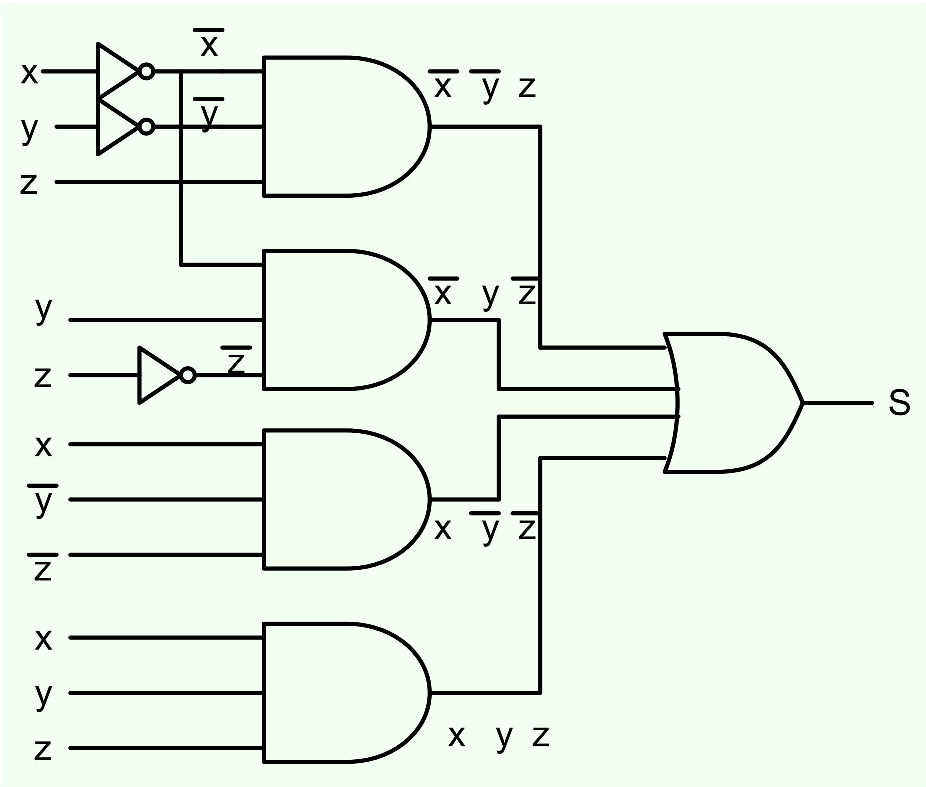
Y = 1 only if odd number of inputs is 1

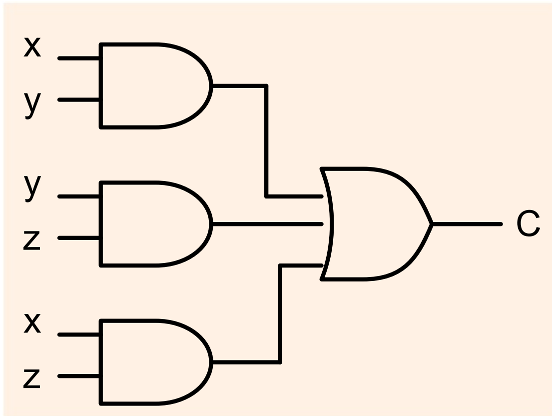
Implementing Boolean expressions using gates



$$S = \overline{x}.\overline{y}.z + \overline{x}.y.\overline{z} + x.\overline{y}.\overline{z} + x.y.z$$

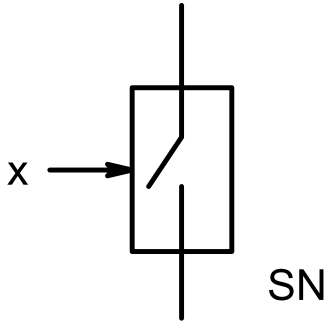
$$C = x.y + x.z + y.z$$





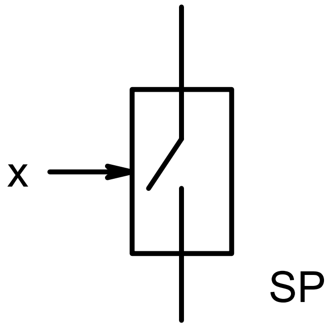
Implementing gates using Switches

Voltage controlled Switch SN:



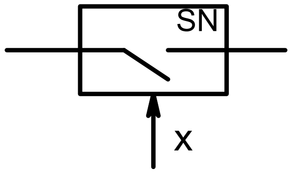
Switch is closed if voltage x is HIGH
Switch is open if voltage x is LOW

Voltage controlled Switch SP:

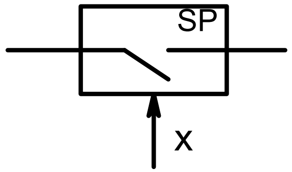


Switch is closed if voltage x is LOW
Switch is open if voltage x is HIGH

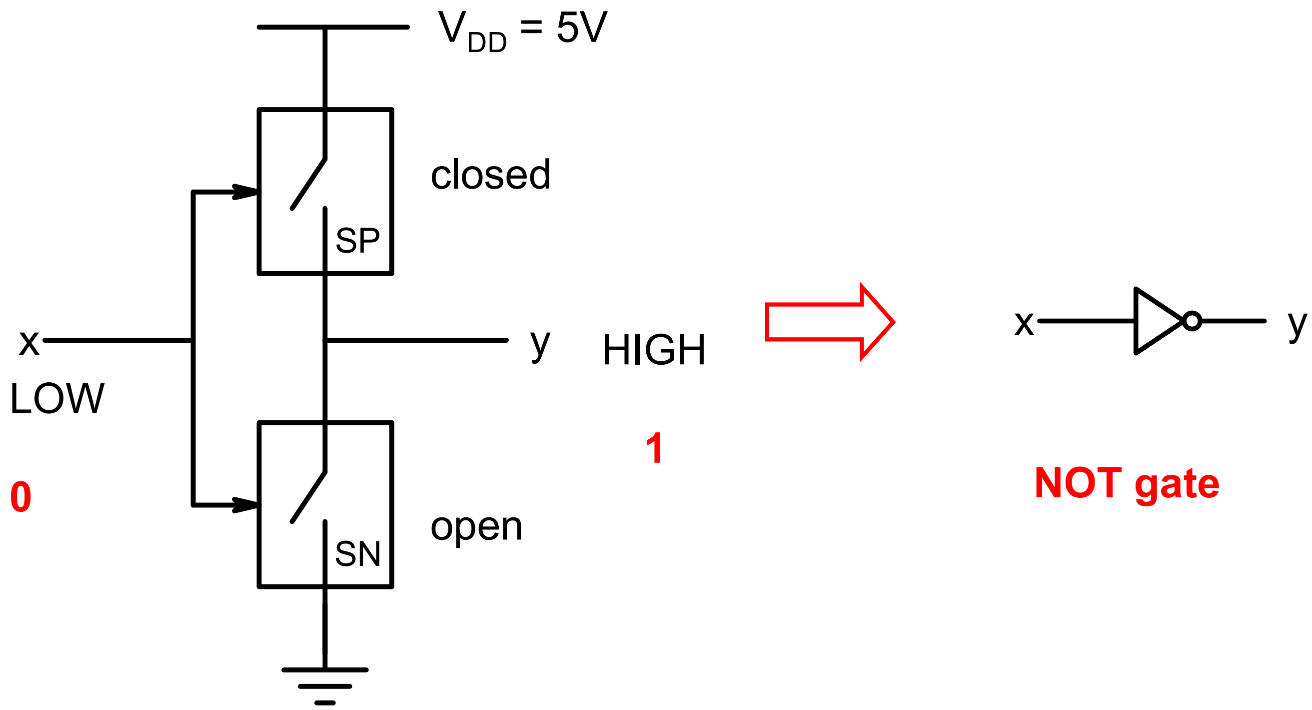
We have seen earlier that transistors act as switches !



Switch is closed if voltage x is HIGH
Switch is open if voltage x is LOW

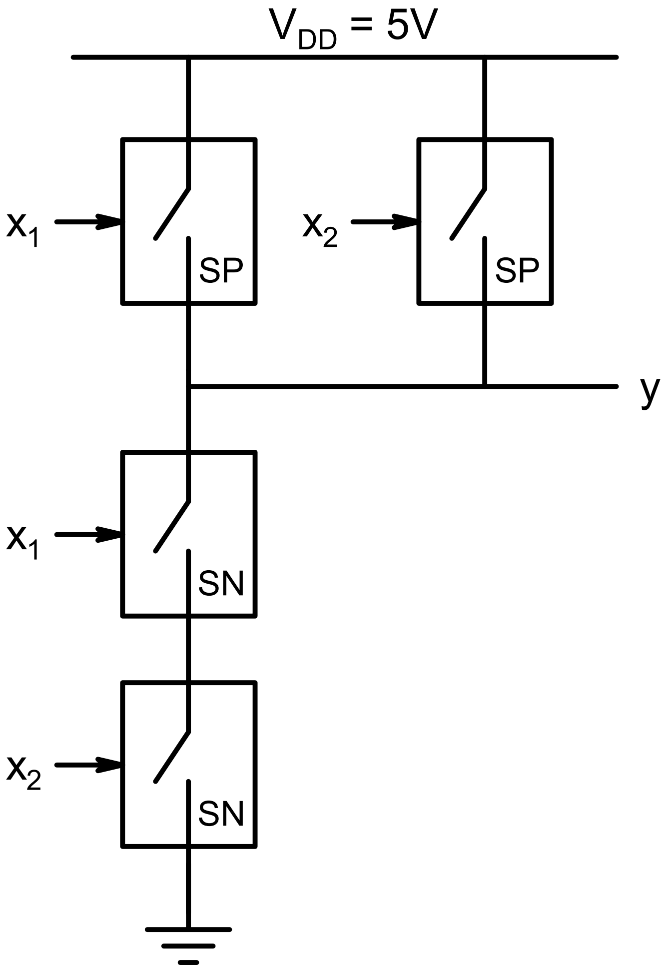


Switch is closed if voltage x is LOW
Switch is open if voltage x is HIGH



NAND Gate

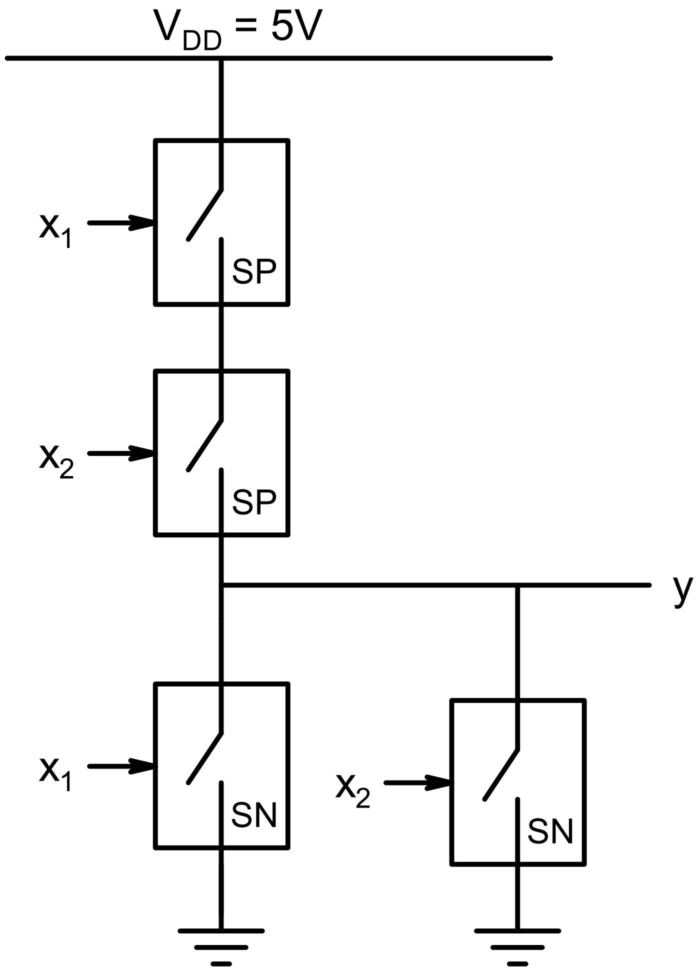
$$\text{NAND:} \quad y = \overline{x_1 . x_2}$$



x_1	x_2	y
LOW	LOW	HIGH
LOW	HIGH	HIGH
HIGH	LOW	HIGH
HIGH	HIGH	LOW

NOR Gate

$$\text{NOR:} \quad y = \overline{x_1 + x_2}$$



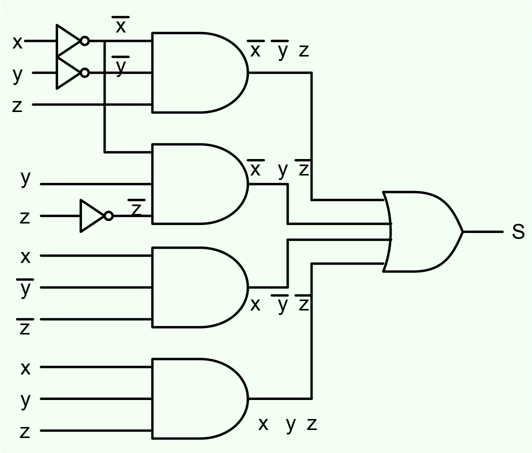
x_1	x_2	y
LOW	LOW	HIGH
LOW	HIGH	LOW
HIGH	LOW	LOW
HIGH	HIGH	LOW

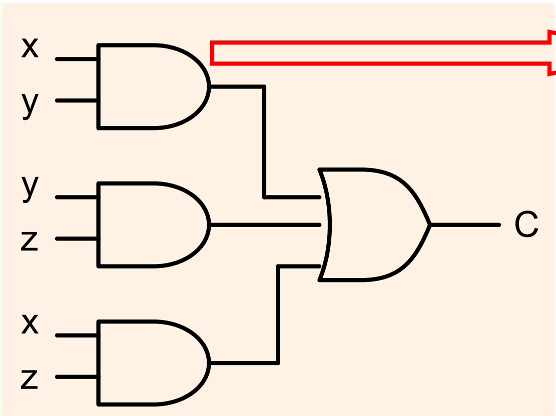
Design Overview

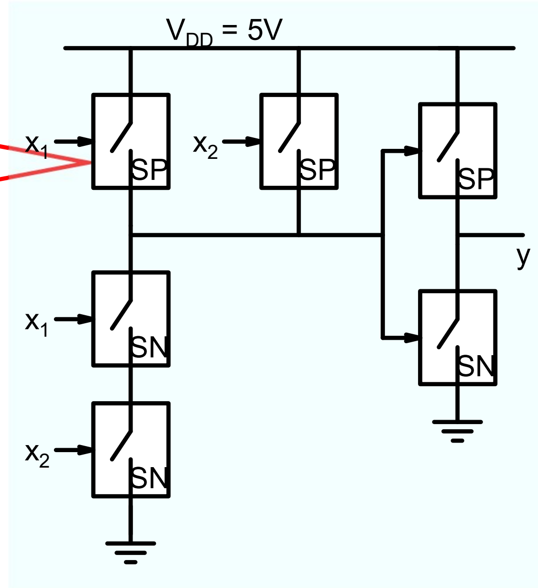
a	b	c	S	CY
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = \overline{x}.\overline{y}.z + \overline{x}.y.\overline{z} + x.\overline{y}.\overline{z} + x.y.z$$

$$C = x.y + x.z + y.z$$







Representation of Boolean Expressions

x	y	f_1
0	0	0
0	1	1
1	0	1
1	1	0

x	y	min term	
0	0	$\bar{x}\,.\,\bar{y}$	m0
0	1	$\bar{x}\,.\,y$	m1
1	0	$x\,.\,\bar{y}$	m2
1	1	$x\,.\,y$	m3

$$\mathrm{f}_1 = \bar{x}.\ y + x.\bar{y}$$

$$\mathrm{f}_1 = m_1 + m_2$$

$$\mathrm{f}_1 = \sum(1,2)$$

$$\mathrm{f}_2 = \sum(0,2,3) = ?$$

$$\mathrm{f}_2 = \bar{x}.\ \bar{y} + x.\bar{y} + x\,.\,y$$

A minterm is a product that contains all the variables used in a function

Three variable functions

x	y	z	min terms	
0	0	0	$\overline{x}.\overline{y}.\overline{z}$	m0
0	0	1	$\overline{x}.\overline{y}.z$	m1
0	1	0	$\overline{x}.y.\overline{z}$	m2
0	1	1	$\overline{x}.y.z$	m3
1	0	0	$x.\overline{y}.\overline{z}$	m4
1	0	1	$x.\overline{y}.z$	m5
1	1	0	$x.y.\overline{z}$	m6
1	1	1	$x.y.z$	m7

$$f_2 = \sum(1,4,7) = ?$$

$$f_2 = \overline{x}.\overline{y}.z + x.\overline{y}.\overline{z} + x.y.z$$

Product of Sum Terms Representation

x	y	f_1
0	0	1
0	1	0
1	0	0
1	1	1

x	y	Max term	
0	0	$x + y$	M0
0	1	$x + \overline{y}$	M1
1	0	$\overline{x} + y$	M2
1	1	$\overline{x} + \overline{y}$	M3

$$f_1 = (x + \overline{y}).(\overline{x} + y)$$

$$f_1 = M_1.M_2$$

$$f_1 = \Pi(1,2)$$

$$f_2 = \Pi(0,3) = ?$$

$$f_2 = (x + y).(\overline{x} + \overline{y})$$

x	y	z	Max. terms	
0	0	0	$x + y + z$	M0
0	0	1	$x + y + \overline{z}$	M1
0	1	0	$x + \overline{y} + z$	M2
0	1	1	$x + \overline{y} + \overline{z}$	M3
1	0	0	$\overline{x} + y + z$	M4
1	0	1	$\overline{x} + y + \overline{z}$	M5
1	1	0	$\overline{x} + \overline{y} + z$	M6
1	1	1	$\overline{x} + \overline{y} + \overline{z}$	M7

$$f_1 = \Pi\,(1,5,7) = ?$$

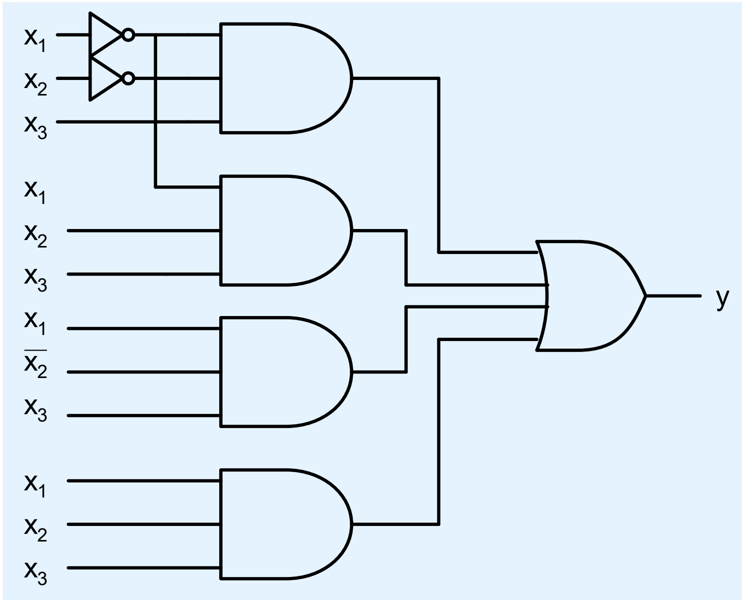
$$f_2 = (x + y + \overline{z}).(\overline{x} + y + \overline{z}).(\overline{x} + \overline{y} + \overline{z})$$

Simplification

x_1	x_2	x_3	y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$$y = \sum (1, 3, 5, 7)$$

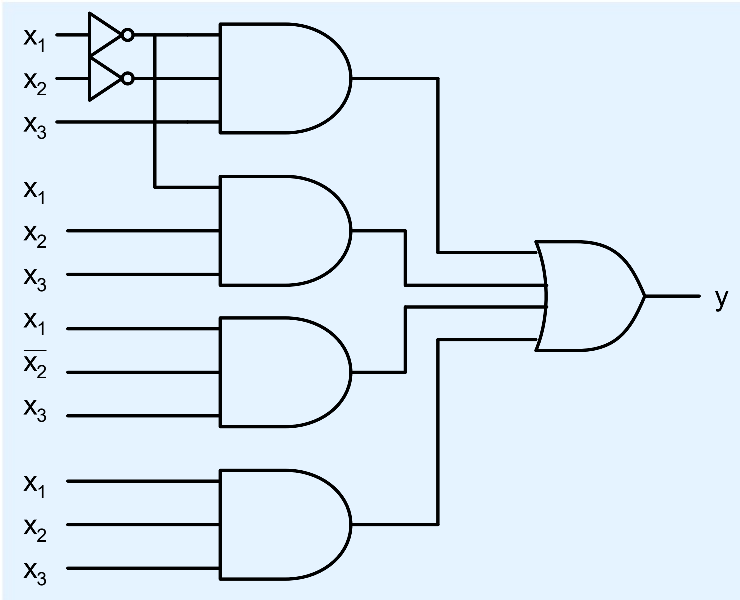
$$y = \overline{x}_1 . \overline{x}_2 . x_3 + \overline{x}_1 . x_2 . x_3 + x_1 . \overline{x}_2 . x_3 + x_1 . x_2 . x_3$$



Simplification of Boolean expression yields : $y = x_3$!! which does not require any gates at all !

Goal of Simplification

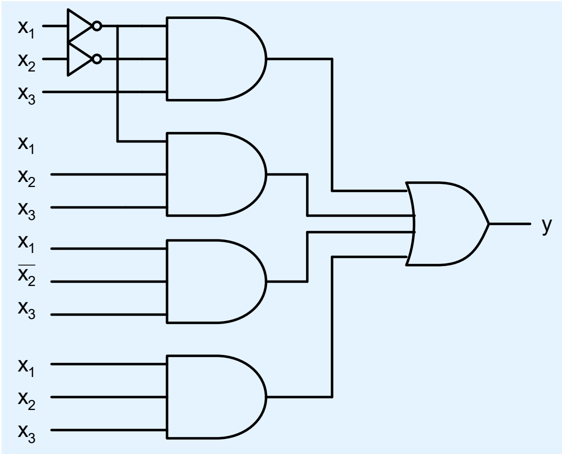
$$y = \overline{x_1}\,.\,\overline{x_2}\,.x_3 + \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.x_3 + x_1.x_2.x_3$$

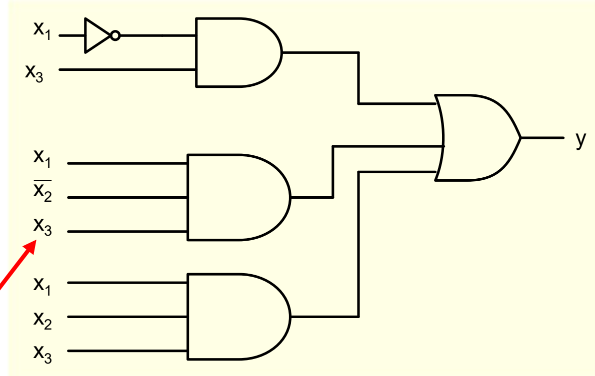


Goal of simplification is to reduce the complexity of gate circuit. This requires that we minimize the number of gates. Since number of gates depends on number of minterms, one of the goals of simplification is to **minimize the number of minterms in SOP expression**

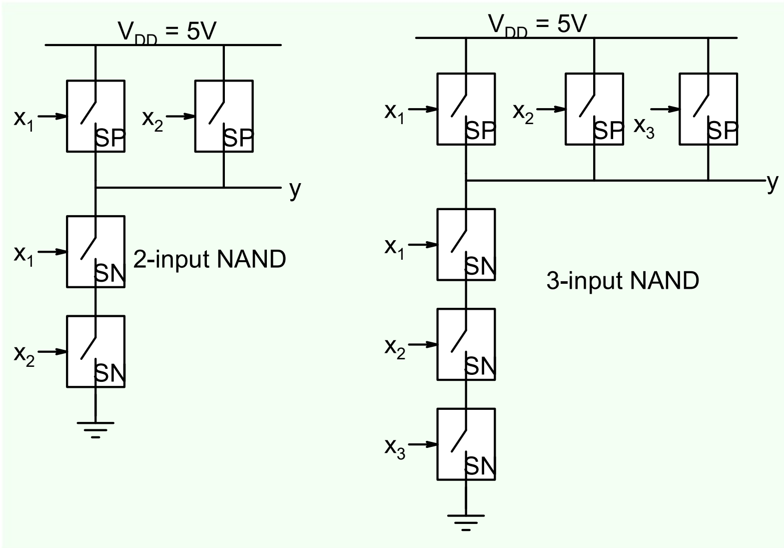
$$y = \overline{x}_1 \,.\, \overline{x}_2 \,.\, x_3 + \overline{x}_1 . x_2 . x_3 + x_1 . \overline{x}_2 . x_3 + x_1 . x_2 . x_3$$

$$\Rightarrow \quad y = \overline{x}_1 \,.\, x_3 + x_1 . \overline{x}_2 . x_3 + x_1 . x_2 . x_3$$





This circuit is simpler not just because it uses 4 gates instead of 5 but also because circuit-2 uses one 2-input and three 3-input gates as compared to five 3-input gates used in circuit-1



Goal of Simplification

$$y = \overline{x_1}.\overline{x_2}.x_3 + \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.x_3 + x_1.x_2.x_3 \quad \Rightarrow \quad y = \overline{x_1}.x_3 + x_1.\overline{x_2}.x_3 + x_1.x_2.x_3$$

In the SOP expression:

1. Minimize number of product terms

2. Minimize number of literals in each term

$$\text{Simplification} \quad \Rightarrow \quad \text{Minimization}$$

Minimization

$$y = \overline{x_1}.\overline{x_2}.x_3 + \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.x_3 + x_1.x_2.x_3$$

$$y = \overline{x_1}.x_3.(\overline{x_2} + x_2) + x_1.x_3.(\overline{x_2} + x_2)$$

$$y = \overline{x_1}.x_3 + x_1.x_3$$

$$y = (\overline{x_1} + x_1).x_3$$

$$y = x_3$$

Principle used: $x + \overline{x} = 1$

$$f = \overline{x}.\overline{y} + \overline{x}.y + x.\overline{y}$$

Apply the Principle: $x + \overline{x} = 1$ to simplify

$$f = \overline{x}.(\overline{y} + y) + x.\overline{y}$$

$$f = \overline{x} + x.\overline{y}$$

How do we simplify further?

$$f = \overline{x}.\overline{y} + \overline{x}.y + x.\overline{y} = \overline{x}.\overline{y} + \overline{x}.\overline{y} + \overline{x}.y + x.\overline{y}$$

Principle used : $x + x = x$

$$f = \overline{x}.\overline{y} + \overline{x}.y + \overline{x}.\overline{y} + x.\overline{y}$$
$$= \overline{x}.(\overline{y} + y) + (\overline{x} + x).\overline{y} = \overline{x} + \overline{y}$$

Simplify

$$f = \overline{x_1}.x_2.\overline{x_3}.x_4 + \overline{x_1}.x_2.x_3.x_4 + x_1.x_2.\overline{x_3}.x_4 + x_1.x_2.x_3.x_4 +$$
$$x_1.\overline{x_2}.\overline{x_3}.x_4 + x_1.\overline{x_2}.x_3.x_4$$

Principle: $x + \overline{x} = 1$ and $x + x = x$

Need a systematic and simpler method for applying these two principles

Karnaugh Map (K map) is a popular technique for carrying out simplification

It represents the information in problem in such a way that the two principles become easy to apply