Introduction to Forward and Futures

Wasim Ahmad

Department of Economic Sciences
Indian Institute of Technology Kanpur

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- Ways Derivatives are used?
  - To hedge risks
  - To speculate (take a view on future direction of market);
  - To lock in an arbitrage profit (If Law of One Price (LOP) is violated, buy low & sell high).
  - To change the nature of a liability
  - To change the nature & risks of an investment (exposure) without incurring the costs of selling one portfolio and buying another

- A futures contract is a legal agreement between a buyer and a seller in which:
  - The buyer agrees to take delivery of something at a specified price at the end of a designated period of time.
  - The seller agrees to make delivery of something at a specified price at the end of a designated period of time.

- The price at which the parties agree to transact in the future is called the futures price.
- The designated date at which the parties must transact is called the **settlement date** or **delivery date**.
- The "something" that the parties agree to exchange is called the **underlying**.

- Example Futures
  - Suppose a futures contract is traded on an exchange where the underlying to be bought or sold is asset (gold) and the settlement is three months from now.
  - Assume further that trader A buys this futures contract, and trader B sells this futures contract, and the price at which they agree to transact in the future is $100.
  - Then $100 is the futures price.

- Long futures vs. Short futures
  - When an investor takes a position in the market by buying a futures contract (or agreeing to buy at the future date), the investor is said to be in a long position or to be **long futures**.
  - If, instead, the investor's opening position is the sale of a futures contract (which means the contractual obligation to sell something in the future), the investor is said to be in a short position or **short futures**.
  - The buyer of a futures contract will realize a profit if the futures price increases.
  - The seller of a futures contract will realize a profit if the futures price decreases.

- Profit from Futures Contract
  - Suppose a futures contract is traded on an exchange where the underlying to be bought or sold is asset (gold/ounce) and the settlement is three months from now.
  - Assume further that trader A buys this futures contract, and trader B sells this futures contract, and the price at which they agree to transact in the future is $100.
  - Then $100 is the futures price.
  - Suppose that one month after Trader A and Trader B take their positions in the futures contract, the futures price of gold increases to $120 per ounce.
  - Trader A, the buyer of the futures contract, could then sell the futures contract and realize a profit of $20.

- Example Profit from Futures Contract
  - Suppose that one month after Trader A and Trader B take their positions in the futures contract, the futures price of gold increases to \$120 per ounce.
  - A, the buyer of the futures contract, could then sell the futures contract and realize a profit of \$20.
  - Effectively, at the settlement date, he has agreed to buy asset Gold for \$100 and has agreed to sell the underlying asset for \$120.
  - Trader B, the seller of the futures contract, will realize a loss of \$20.

- Example Profit from Futures Contract
  - If the futures price falls to \$40 and Trader B buys back the contract at \$40, he realizes a profit of \$60 because he agreed to sell asset gold for \$100 and now can buy it for \$40.
  - Trader A would realize a loss of \$60.
  - Thus, if the futures price decreases, the buyer of the futures contract realizes a loss while the seller of the futures contract realizes a profit

- Liquidating a position
  - Most financial futures contracts have settlement dates in the months of March, June, September, or December.
  - This means that at a predetermined time in the contract settlement month, the contract stops trading, and a price is determined by the exchange for settlement of the contract.
  - For example, on 13th February, 2020, suppose trader A buys and trader B sells a futures contract that settles on the third Friday of April 2020.
  - Then, on that date, A and B must perform — A agreeing to buy asset gold at \$100, and B agreeing to sell asset gold at \$100.
  - The exchange will determine a settlement price for the futures contract for that specific date.

- Liquidating a position
  - The exchange will determine a settlement price for the futures contract for that specific date.
  - For example, if the exchange determines a settlement price of \$130, then A has agreed to buy asset gold for \$100 but can settle the position for \$130, thereby realizing a profit of \$30. B would realize a loss of \$30.

- Liquidating a position
  - A party to a futures contract has two choices regarding the liquidation of the position.
  - First, the position can be liquidated prior to the settlement date.
  - For this purpose, the party must take an offsetting position in the same contract. For the buyer of a futures contract, this means selling the same number of identical futures contracts; for the seller of a futures contract, this means buying the same number of identical futures contracts.
  - An identical contract means the contract for the same underlying and the same settlement date.

- Liquidating a position
  - So, for example, if trader A buys one futures contract for asset gold with settlement in the third week of April 2020 on March 20, 2020, and wants to liquidate a position on 26 March, 2020, he can sell one futures contract for asset gold with settlement in September 2019
  - Similarly, if trader B sells one futures contract for asset gold with settlement in third week of April 2020 on March 20, 2020, and wants to liquidate a position on 26 March, 2020, he can buy one futures contract for asset gold with settlement in April 2020.

- The Role of the Clearing House
  - Associated with every futures exchange is a clearinghouse, which performs several functions.
  - One of these functions is to guarantee that the two parties to the transaction will perform.
  - Because of the clearinghouse, the two parties need not worry about the financial strength and integrity of the other party taking the opposite side of the contract.

- The Role of the Clearing House
  - After initial execution of an order, the relationship between the two parties ends.
  - The clearinghouse interposes itself as the buyer for every sale and as the seller for every purchase.
  - Thus, the two parties are then free to liquidate their positions without involving the other party in the original contract, and without worry that the other party may default.

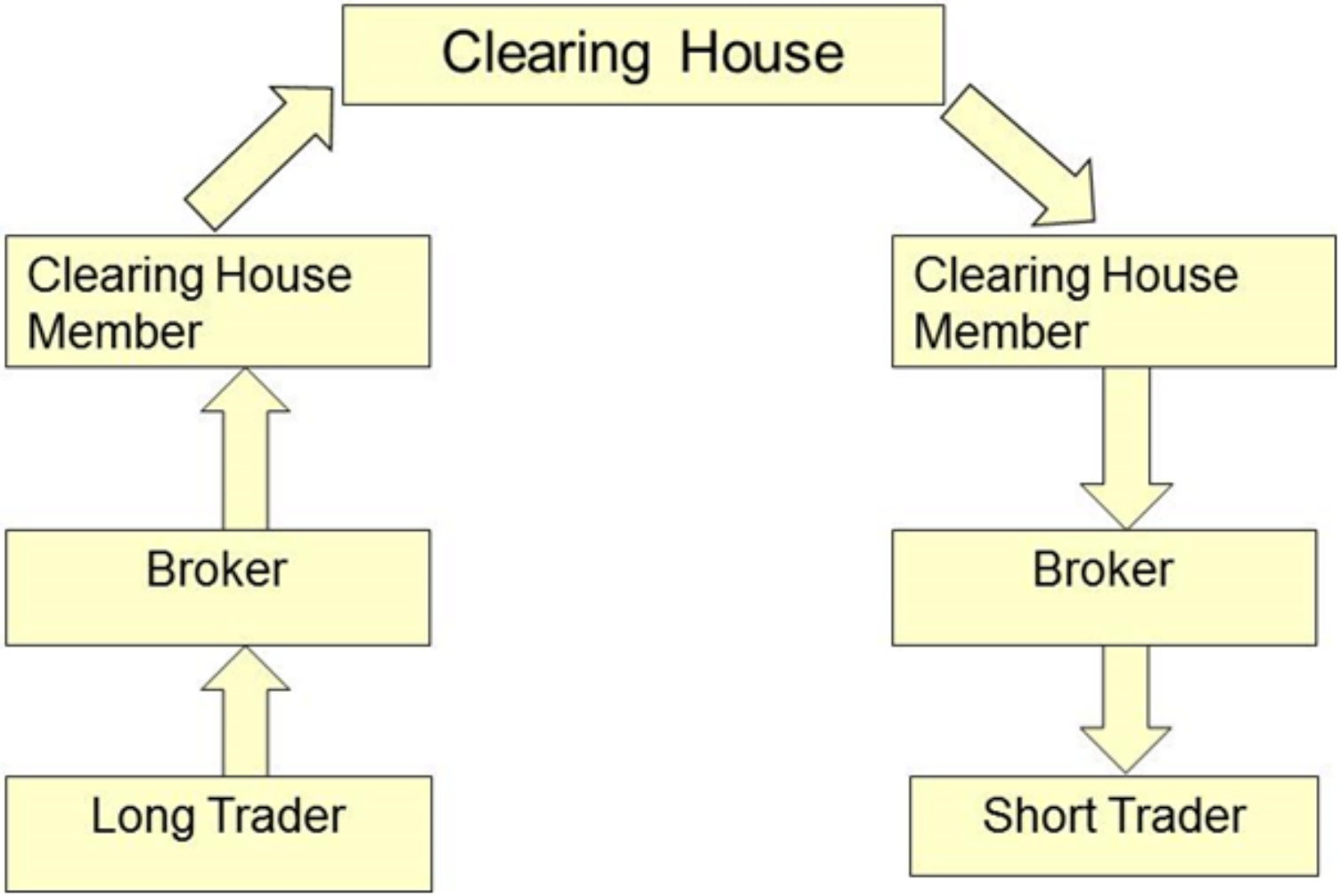
- Margin Requirements
  - When a position is first taken in a futures contract, the investor must deposit a minimum dollar amount per contract as specified by the exchange. This amount, called **initial margin**, is required as a deposit for the contract. The initial margin may be in the form of an interest-bearing security such as a Treasury bill.
  - The initial margin is placed in an account, and the amount in this account is referred to as the investor's equity.
  - As the price of the futures contract fluctuates each trading day, the value of the investor's equity in the position changes.

- Margin Requirements
  - At the end of each trading day, the exchange determines the **settlement price** for the futures contract.
  - The settlement price is different from the closing price, which is the price of the security in the final trade of the day (whenever that trade occurred during the day).
  - By contrast, the settlement price is that value the exchange considers to be representative of trading at the end of the day.
  - The exchange uses the settlement price to mark to market the investor's position, so that any gain or loss from the position is quickly reflected in the investor's equity account.

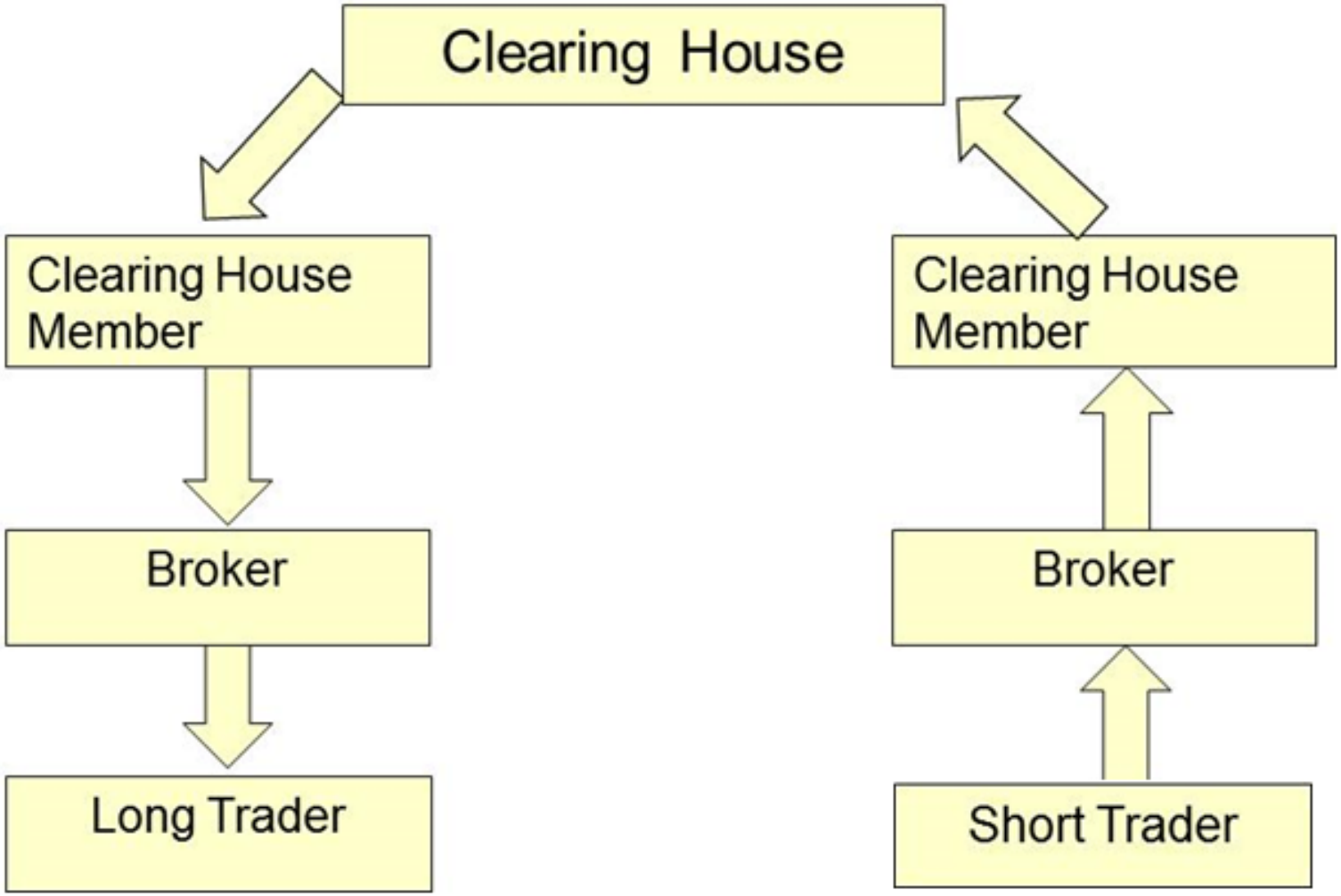
- Margin Requirements
  - A maintenance margin is the minimum level (specified by the exchange) by which an investor's equity position may fall as a result of unfavorable price movements before the investor is required to deposit additional margin.
  - The maintenance margin requirement is a dollar/rupee amount that is less than the initial margin requirement.
  - It sets the floor that the investor's equity account can fall to before the investor is required to furnish additional margin.
  - The additional margin deposited, called **variation margin**, is an amount necessary to bring the equity in the account back to its initial margin level.

- Margin Requirements
  - Unlike initial margin, variation margin must be in cash, not interest-bearing instruments.
  - Any excess margin in the account may be withdrawn by the investor.
  - If a party to a futures contract who is required to deposit a variation margin fails to do so within 24 hours, the futures position is liquidated by the clearing house.

- Margin Cash Flows when Futures **Price Decreases**



- Margin Cash Flows when Futures **Price Increases**



- Margin Requirements Example
  - Let's assume the following margin requirements for asset gold
    - Initial margin \$7 per contract.
    - Maintenance margin \$4 per contract.
  - Suppose that Trader A buys 500 contracts at a futures price of \$100, and Trader B sells the same number of contracts at the same futures price.
  - The initial margin for both A and B is \$3,500, which is determined by multiplying the initial margin of \$7 by the number of contracts, 500.
  - A and B must put up \$3,500 in cash or Treasury bills or other acceptable collateral.
  - At this time, \$3,500 is the equity in the account.

- Margin Requirements Example
  - The maintenance margin for the two positions is $2,000 (the maintenance margin per contract of $4 multiplied by 500 contracts).
  - That means the equity in the account may not fall below $2,000. If it does, the party whose equity falls below the maintenance margin must put up additional margin, which is the variation margin.

- Margin Requirements Example
  - Regarding the variation margin, note two things:
    - First, the variation margin must be cash.
    - Second, the amount of variation margin required is the amount to bring the equity up to the initial margin, not the maintenance margin.

- Mark-to-market procedure
  - Suppose we have following settlement prices at the end of four consecutive trading days after the transaction was entered into:

Trading Day		Settlement Price
1		$99
2		97
3		98
4		95

- Mark-to-market procedure
  - First consider trader A's position.
    - At the end of trading day 1, A realizes a loss of \$1 per contract, or \$500 for the 500 contracts he bought.
    - A's initial equity of \$3,500 is reduced by \$500 to \$3,000.
    - No action is taken by the clearinghouse, because A's equity is still above the maintenance margin of \$2,000.
    - At the end of the second day, A realizes a further loss as the price of the futures contract declines \$2 to \$97, resulting in an additional reduction in his equity position by \$1,000.
    - A's equity is then \$2,000. Despite the loss, no action is taken by the clearinghouse, since the equity is not less than the \$2,000 maintenance margin requirement.

- Mark-to-market procedure
  - First consider trader A's position.
    - At the end of trading day 3, A realizes a profit from the previous trading day of \$1 per contract, or \$500.
    - A's equity increases to \$2,500.
    - The drop in price from 98 to 95 at the end of trading day 4 results in a loss for the 500 contracts of \$1,500 and a reduction of A's equity to \$1,000.
    - Since A's equity is now below the \$2,000 maintenance margin, A is required to put up additional margin of \$2,500 (variation margin) to bring the equity up to the initial margin of \$3,500.
    - If A cannot put up the variation margin, his position will be liquidated. That is, his contracts will be sold by the clearinghouse

- Mark-to-market procedure
  - Trader B's position (seller).
    - Since B sold the futures contract, he benefits if the price of the futures contract declines.
    - As a result, his equity increases at the end of the first 2 trading days.
    - In fact, at the end of trading day 1, he realizes a profit of 500, which increases his equity to \$4,000. he is entitled to remove the \$500 profit and utilize these funds elsewhere.
    - Suppose he does, and, as a result, his equity remains at \$3,500 at the end of trading day 1.
    - At the end of trading day 2, he realizes an additional profit of \$1,000 that he can withdraw.
    - At the end of trading day 3, he realizes a loss of \$500, because the price increased from \$97 to \$98.
    - This results in a reduction of his equity to \$3,000. Finally, on trading day 4, he realizes a profit of \$1,500, making his equity \$4,500. he can withdraw \$1,000.

- Leveraging aspect of futures
  - When taking a position in a futures contract, a party need not put up the entire amount of the investment.
  - Instead, the exchange requires that only the initial margin be invested.
    - suppose A has \$100 and wants to invest in asset gold because he believes its price will appreciate.
    - If asset gold is selling for \$100, he can buy one ounce of the gold in the cash market, the market where goods are delivered upon purchase.
    - His payoff will then be based on the price action of one unit of asset gold.

- Leveraging aspect of futures
  - Suppose that the exchange where the futures contract for asset gold is traded requires an initial margin of only 5%, which in this case would be $5.
  - Then the trader A can purchase 20 contracts with his $100 investment.
  - His payoff will then depend on the price action of 20 units of asset gold.Thus, he can leverage the use of his funds.
  - (The degree of leverage equals 1/margin rate. In this case, the degree of leverage equals 1/0.05, or 20.)
  - While the degree of leverage available in the futures market varies from contract to contract, as the initial margin requirement varies, the leverage attainable is considerably greater than in the cash market.

- Basic Features of Forward Contracts
  - A forward contract, just like a futures contract, is an agreement for the future delivery of the underlying at a specified price at the end of a designated period of time.
  - Futures contracts are standardized agreements as to the delivery date (or month) and quality of the deliverable, and are traded on organized exchanges.
  - A forward contract differs in that it is usually nonstandardized (that is, the terms of each contract are negotiated individually between buyer and seller), there is no clearinghouse, and secondary markets are often nonexistent or extremely thin.
  - Unlike a futures contract, which is an exchangetraded product, a forward contract is an over-the-counter instrument.

- Basic Features of Forward Contracts
  - Because there is no clearinghouse that guarantees the performance of a counterparty in a forward contract, the parties to a forward contract are exposed to **counterparty risk**, the risk that the other party to the transaction will fail to perform.
  - Futures contracts are marked to market at the end of each trading day, while forward contracts usually are not.
  - Consequently, futures contracts are subject to interim cash flows because additional margin may be required in the case of adverse price movements or because cash may be withdrawn in the case of favorable price movements.
  - A forward contract may or may not be marked to market.

- Basic Features of Forward Contracts
  - Where the counterparties are two high-credit-quality entities, the two parties may agree not to mark positions to market.
  - However, if one or both of the parties are concerned with the counterparty risk of the other, then positions may be marked to market.
  - Thus, when a forward contract is marked to market, there are interim cash flows just as with a futures contract.
  - When a forward contract is not marked to market, then there are no interim cash flows.

- Institutional Features of Futures Markets
  - **Volume** - amount of trading activity in those contracts over some period of time.
  - **Open Interest** total number of outstanding contracts at a given point in time in a contract's life cycle.

- How to calculate Open Interest?
  - The below table shows that 4 trades were executed on the first day. Each contract has a lot size of 200 units. At the end of the day open interest is 7000. A and C have 10 and 25 short contract positions respectively and B and D have 20 and 15 long contracts, respectively. E has no open position at the end of the day.

- How to calculate Open Interest?

Trade No.	Trade details	No. of contracts	Trade description	Open Interest
1	A short - B long	20	A and B are both new traders	20
2	C short - D long	25	C and D are both new traders	45
3	A long - E short	10	A squares up open position w.r.t Trade No. 1 and E is the new seller	45
4	D short - E long	10	D and E square up open positions w.r.t Trade No. 2 and 3 respectively.	35

- Calculate open interest for the data given in Table below

Trade No.	Trade details	No. of contracts	Trade description	Open Interest
1	A short-B long	200	A and B are both new traders	?
2	B short- C long	50	B squares up and C is new trader	?
3	A short - D long	90	A is a old trader but takes short position. D is a new trader	?
4	D short - E long	75	D square up open positions E is the new trader	?

- Calculate open interest for the data given in Table below

Trade No.	Trade details	No. of contracts	Trade description	Open Interest
1	A short-B long	200	A and B are both new traders	200
2	B short- C long	50	B squares up and C is new trader	200
3	A short - D long	90	A is a old trader but takes short position. D is a new trader	290
4	D short - E long	75	D square up open positions E is the new trader	290

- Futures price patterns
  - Futures prices can be:
  - a. An increasing function of maturity: $F > S$; Contango; Normal.
  - b. A decreasing function of maturity: $F < S$; Backwardation; Inverted.
  - Futures Price converges to Spot Price at maturity, if the spot asset is the same asset underlying the futures contract.

- Forward price of any contract is based on cost of carry.
  - Cost of carry includes all cost an investor would incur if the investor would buy the asset today, and carries it till the maturity minus any carrying return.
  - Carrying cost includes financing cost, storage and insurance charges, etc.
  - Carrying return is the dividend or bonus an investor would receive during the maturity period by virtue of owning the underlying assets.
  - Net carry cost is the difference between carrying cost and carrying return.

- Notation and background:
  - T = Time until delivery of the forward contract (fraction of year)
  - S = Spot price of underlying asset at time t (today)
  - S_T: Spot price of underlying asset at time T (maturity), a random variable
  - K: Delivery price promised in forward contract at time T
  - F: Forward price prevailing in market at time t
  - f: Value of a long forward contract at time t
  - r: Risk free rate per annum at time t, for investment maturing at T (LIBOR)

- Forward price is the spot price plus the net carrying cost of the asset from today till the maturity of the contract.
  - Underlying asset does not provide any return/income $F_{(0,T)} = S_0 e^{RT}$
  - Underlying asset provides known cash return/income $F_{(0,T)} = (S_0 - I)e^{RT}$
  - Underlying asset provides known yield $F_{0,T} = S_0 e^{(R-q)T}$
  - These three formulas give the theoretical value of the contract.

  - where, $F_{(0,T)}$ = Forward price of the contract on day) for a contract maturing on day T.
  - T = time to maturity in years (T = 0.5 means six months forward contract).
  - R = continuously compounded risk free rate of return.
  - I = present value of the known income (dividend) the underlying asset provides during the life of the contract.
  - q = known yield the underlying asset provides during the life of the contract.

- If actual value of the contract in the forwards market $F_{(Actual)}$ is different than the $F_{(Theoretical)}$, then the arbitrageurs undertake cash-and-carry or reverse-cash-and-carry arbitrage.

$F_{(Actual)} > F_{(Thoeretical)}$	
Cash and carry arbitrage	
On day 0	On maturity day T
Step 1: Borrow S_0	Step 1: Deliver underlying
Step 2: Buy underlying	Step 2: Receive $F_{(Actual)}$
Step 3: Sell forward	Step 3: Return S_0 with interest

$F_{(Actual)} < F_{(Thoeretical)}$	
Reverse-cash-and-carry-arbitrage	
Step 1: Short sell underlying asset at S_0	Step 1: Receive S_0 and interest
Step 2: Lend S_0 for a period equal to the maturity of the forward contract	Step 2: Take delivery of the underlying and pay F_0
Step 3: Buy forward	Step 3: Deliver the underlying asset to make up for the short sale.

- Example 1: An investor would like to buy shares of a company after 6 months. The shares are quoted at Rs. 887 today. The investor can borrow money at continuously compounded rate of 8.16% per annum. The six months forward contract is quoted as Rs. 945. Find out what would be the arbitrage profit if arbitrage is possible.

- Example 1: An investor would like to buy shares of a company after 6 months. The shares are quoted at Rs. 887 today. The investor can borrow money at continuously compounded rate of 8.16% per annum. The six months forward contract is quoted as Rs. 945. Find out what would be the arbitrage profit if arbitrage is possible.
- Solution
  - $F_{(Theoretical)} = F_{(0,T)} = S_0 e^{RT} = 887 e^{(8.16\% \times 0.5)} =$ Rs. 924
  - $F_{(Actual)} =$ Rs. 945
  - as $F_{(Actual)}$ is greater than $F_{(Theoretical)}$, cash and carry arbitrage is possible.

- On day 0
  - Party borrows S_0 = Rs. 887 for 6 months at 8.16%
  - Buy underlying i.e. share of company
  - Sell forward at F_{Actual} = Rs. 945.
- On maturity (after 4 months)
  - Deliver the underlying
  - Receive Rs. 945 for selling the forward
  - Return Rs. 924 for borrowing Rs. 887 for 6 months
  - Profit = Rs. (945- 924) = Rs. 21.

- **Example 2**: Spot price of gold (per 10 gram) is Rs. 10,550. The storage and insurance cost for 2 months is Rs. 275 for every 10 gram of gold. The investors can borrow/lend at Rs. 7.75% continuously compounded rate. If the two months gold forward is trading either at: (a) Rs. 11,230 (b) Rs. 10, 453
- Find out how arbitrageurs will be the able to make arbitrage profit

- Solution
  - Spot price is given as Rs. 10, 550 $= S_0$
  - R = 7.75%, U = Rs. 275 paid at the begining of the storage period, T = 0.1667 (2 months)
  - (a) $F_{(Theoretical)} = F_{(0,T)} = (S_0 + U)e^{RT} = (10550 + 275)e^{(0.0775 \times 0.1667)}$ = Rs. 10, 965.75
  - CASE A: two months forward price on gold (per 10 gram) = Rs. 11,230 $= F_0$
  - Two months futures price on gold = Rs. 10,453 $= F_0$
- $F_{(Actual)} > F_{(Theoretical)}$, cash and carry arbitrage will be undertaken by the trader.
- $F_{(Actual)} < F_{(Theoretical)}$, reverse cash and carry arbitrage will be undertaken by the trader.

- **Solution**
  - **Cash and carry transactions**
  - On day 0
    - Borrow = Rs. 10,825 (purchase cost + storage cost)
    - Buy gold at Rs. 10,550
    - Pay storage cost of Rs. 275
    - Sell forward at Rs. 11,230
  - On maturity (after 4 months)
    - Pay gold to forward counter party and receive Rs. 11, 230
    - Return Rs. 10, 965.73 for borrowing Rs. 10, 825
    - Earn a net profit of Rs. 264.27

- **Solution**
  - **Reverse Cash and carry transactions**
  - On day 0
    - Sell gold (short sell)
    - Receive = Rs. 10, 550
    - Save Rs. 275 as a storage cost
    - Invest Rs. 10,550 for two months
    - Buy forward at Rs. 10,453
  - On maturity (after 4 months)
    - Receive gold from long forward position
    - Pay Rs. 10, 453 as payment for long forward
    - Receive Rs. 10, 965.75 from investment of Rs. 10, 825 for 2 months
    - Earn a net profit of Rs. 512. 75

- Example 1: An investor would like to buy shares of a company after 4 months. The shares are quoted at Rs. 127 today. The investor can borrow money at continuously compounded rate of 8.55% per annum.
  - 1. What would be the forward price of the share if there in no dividend income expected in these 4 months?
  - 2. What would be the forward price of the share if there is Rs. 2.75 per share dividend after 2 months?

- Example 1: An investor would like to buy shares of a company after 4 months. The shares are quoted at Rs. 127 today. The investor can borrow money at continuously compounded rate of 8.55% per annum.
  - 1. What would be the forward price of the share if there in no dividend income expected in these 4 months?
  - 2. What would be the forward price of the share if there is Rs. 2.75 per share dividend after 2 months?
- Solution
- Forward price of a company share without any income during the maturity. Maturity period = 4 months = 0.333 years

  $F = Se^{rT} = 127 \times e^{(8.55\% \times 0.333)} = 130.80$

- 2. Futures price of company share with income during the maturity.
- Dividend = 2.75, Dividend date 2 months = 0.1667 years
- Present value of dividend = $2.75/e^{(8.55\% \times 0.1667)} = 2.711$
- With known income, the forward price is :
  $F = (S - I)e^{rT} = (127\text{- } 2.711) \times e^{(8.55\% \times 0.333)} = 128.01$

- Commodity futures
  - In case of consumable commodities like crude oil, copper, coal and agricultural products etc.
  - Users of such commodities trade in commodity spot and forward/futures as they require these commodities to be either used in production processs or for consumption.
  - Pricing of such commodities can be analyzed under two situations:
  - WITH and WITHOUT SUPPLY CONSTRAINTS

- **Without supply constraint**
  - If the commodity is available abundantly, i.e. there is no shortage currently or expected in future, then forward price will confirm to the cost-of-carry model as given below:

$$F_{(0,T)} = S_0 e^{(R+U)T}$$

  - Forward contracts are priced in a manner such that a trader can borrow (S_0) for a period **T** at a rate of **r** to buy the commodity in spot and store it for period **T** by incurring storage/insurance costs at a rate of **U**.

  - Cash and carry and reverse cash carry strategies are dependent upon actual and theoretical prices of forward/futures.

- **With supply constraint**
  - Pricing of forwards for consumption commodities in case of supply restrictions is also undertaken using cost of carry but with some adjustments. This adjustment is known as "Convenience yield".

  - Convenience yield arises from benefits of holding physical asset and such benefits are not available to forward/futures holders

$$F_{(0,T)} = S_0 e^{(R+U-Y)T}$$

where
R = continuously compounded risk free rate of return **U** = present value of all costs, including storage, insurance, expressed as a percentage of the underlying spot price. **Y** = Convenience yield expressed as percentage of spot price **T** = Time to maturity in terms of years.

- Spot price of S_0 Soybean oil = Rs. 93, 500 (per 1000 kg), R = 7.75% per annum continuously compounded. $U = 0.35\%$ per 1000 kg per annum continuously compounded, T = 0.333 (4 months), $F_{0,4}$ = Rs. 95,785 (per 1000 kg)
  - a) Calculate the theoretical futures price based on cost of carry model.
  - b) Find out the convenience yield in absolute as well as percentage term?

- Spot price of S_0 Soybean oil = Rs. 93, 350 (per 1000 kg), R = 7.75% per annum continuously compounded. $U = 0.35\%$ per 1000 kg per annum continuously compounded, T = 0.333 (4 months), $F_{0,4}$ = Rs. 95,785 (per 1000 kg)
  - a) Calculate the theoretical futures price based on cost of carry model?
  - **Solution**: Theoretical price based on cost of carry model

$$F_{(0,T)} = S_0 e^{(R+U)T}$$

$$F_{(0,T)} = 93{,}350 e^{(7.75\%+0.35\%)\times 0.333} = 96{,}110.25$$

  - b) Find out the convenience yield in absolute as well as percentage term?
  - **Convenience yield**
    - The theoretical futures price = 96,110.25
    - Actual futures price = 95,785
    - Convenience yield = 96,110.25 – 95,785 = 325.25
    - in percentage term, it will be 1.01%

Thanking You