ESC201T : Introduction to Electronics

Lecture 17: LCR Filters (Resonance)

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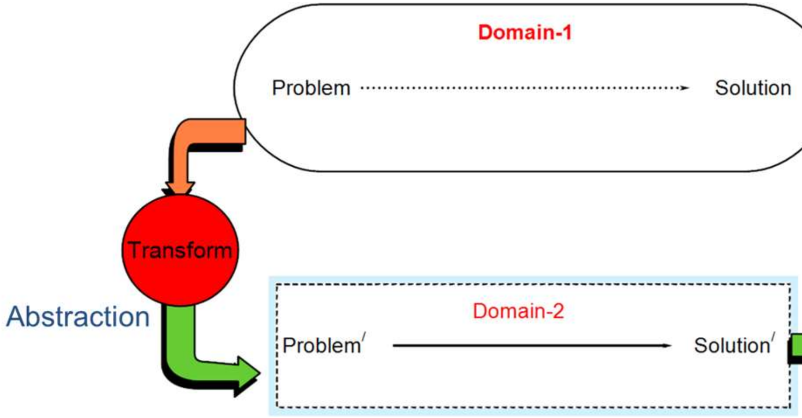
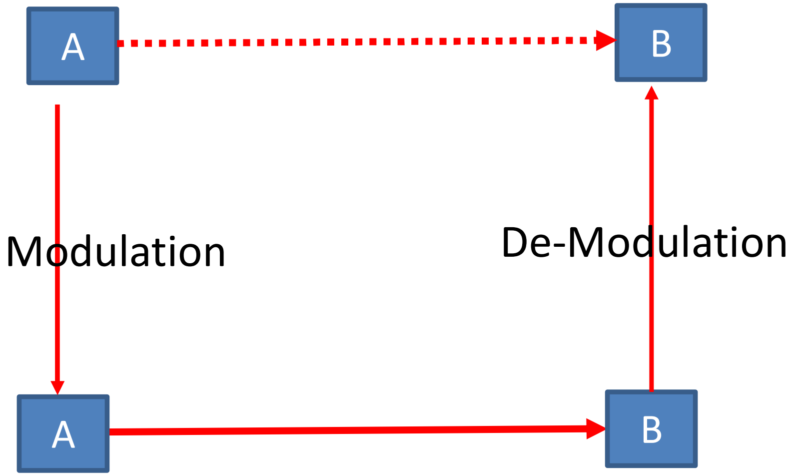
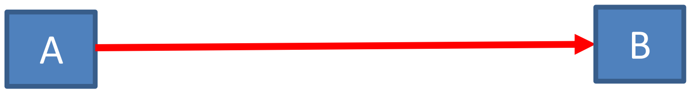
Wireless Transmission of Speech signal



$$x(t) = x_m \sin(2\pi \times 10^3 t)$$

$$x(t) = x_m \sin(2\pi \times 10^3 t) \qquad x(t) = x_m \sin(2\pi \times 10^3 t)$$





$$y(t) = y_m \sin(2\pi \times 450 \times 10^3 t)$$

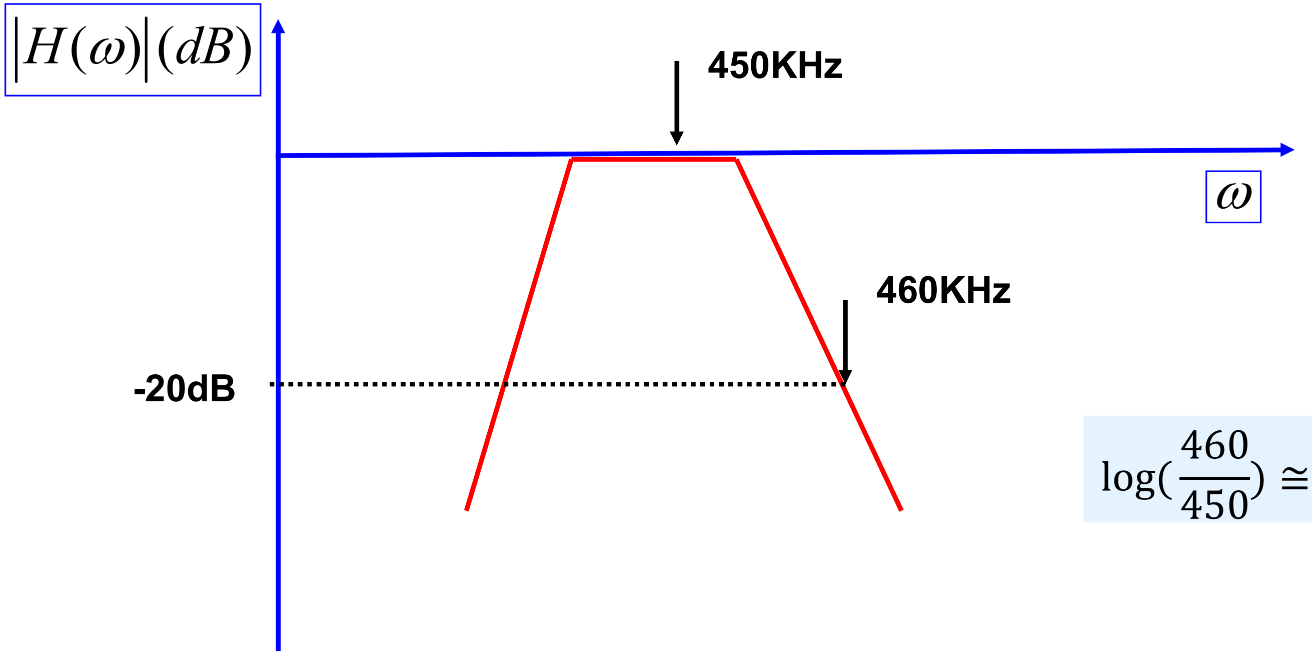
Carrier wave

$$Z(t) = (y_m + m \times x(t)) \sin(2\pi \times 450 \times 10^3 t)C$$

Amplitude Modulated (AM) Radio

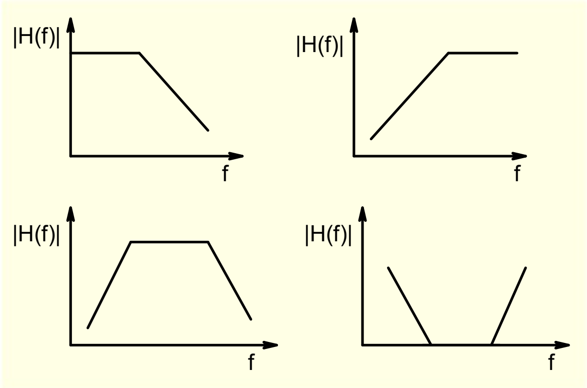
Different radio channels are separated by very narrow frequency interval.

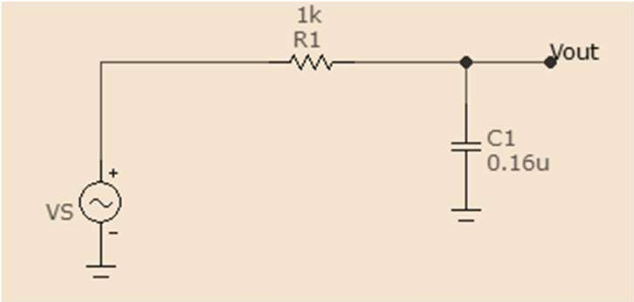
For example, one may want to receive a 450KHz signal but reject 460KHz or 440KHz

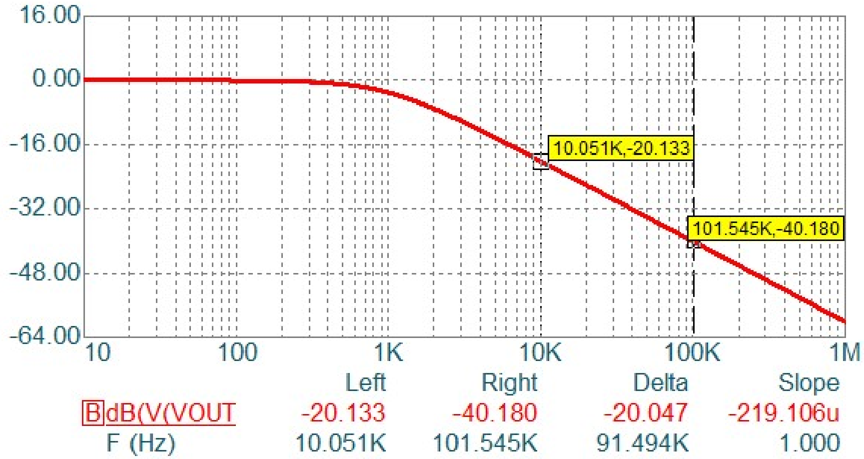


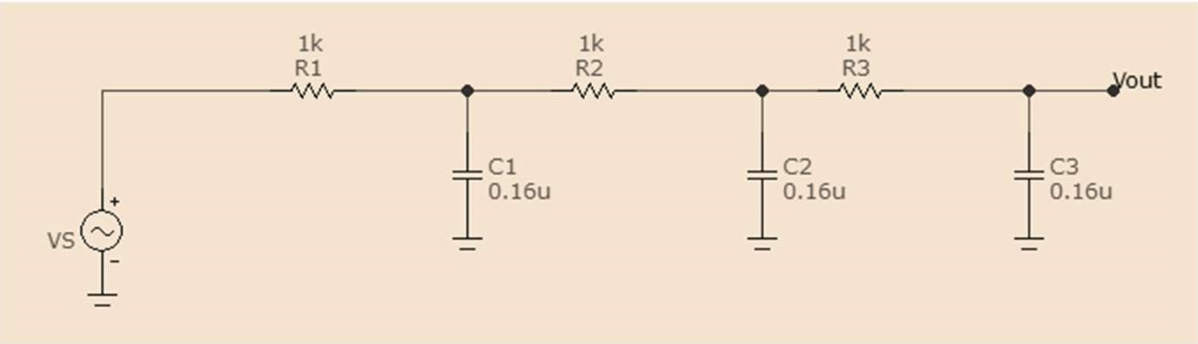
$$\log\left(\frac{460}{450}\right) \cong 9.5 \times 10^{-3} decades$$

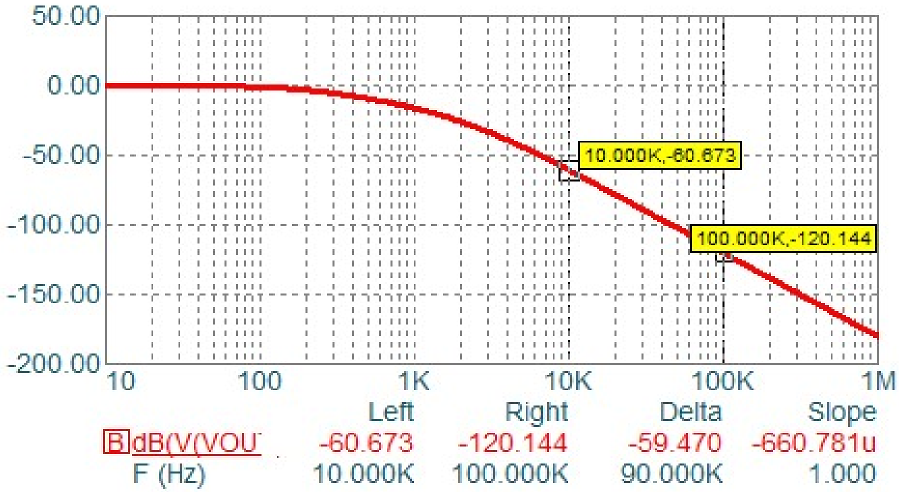
This implies an attenuation of ~ -2000 dB/decade


|H(f)|
f
|H(f)|
f
|H(f)|
f
|H(f)|
f


1k
R1
Vout
C1
0.16u
VS


16.00
0.00
-16.00
-32.00
-48.00
-64.00
10
100
1K
10K
100K
1M
10.051K,-20.133
101.545K,-40.180
Left
Right
Delta
Slope
B dB(V(VOUT
-20.133
-40.180
-20.047
-219.106u
F (Hz)
10.051K
101.545K
91.494K
1.000

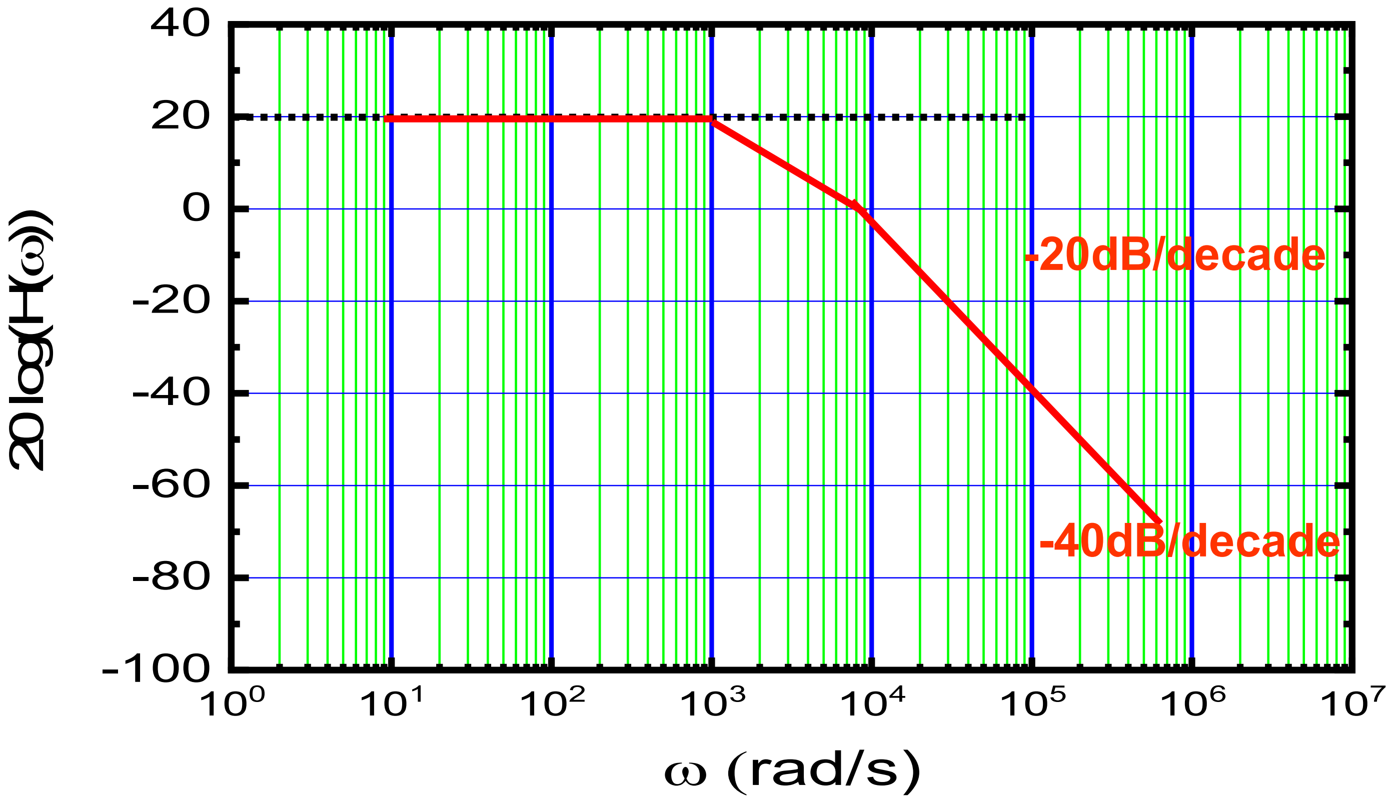

1k
R1
1k
R2
1k
R3
Vout
C1
0.16u
C2
0.16u
C3
0.16u
VS


50.00
0.00
-50.00
-100.00
-150.00
-200.00
10
100
1K
10K
100K
1M
10.000K,-60.673
100.000K,-120.144
Left
Right
Delta
Slope
B dB(V(VOU
-60.673
-120.144
-59.470
-660.781u
F (Hz)
10.000K
100.000K
90.000K
1.000

Second Order System

$$H(\omega)=\frac{10}{1+j\dfrac{\omega}{10^3}}\times\frac{1}{1+j\dfrac{\omega}{10^4}}$$

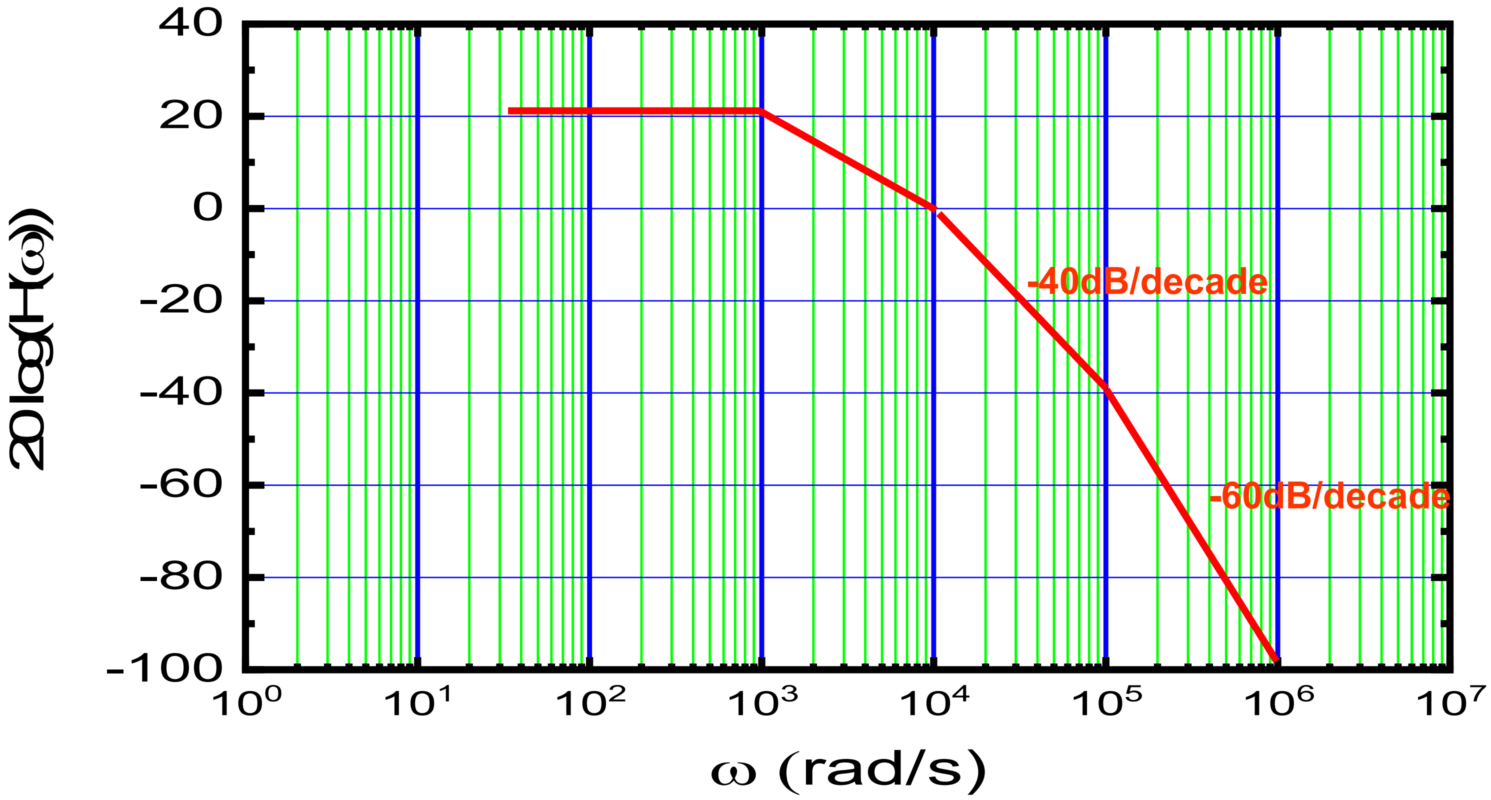
$$20\mathrm{Log}_{10}(|H(\omega)|)=20-10Log_{10}(1+(\frac{\omega}{10^3})^2)-10Log_{10}(1+(\frac{\omega}{10^4})^2)$$



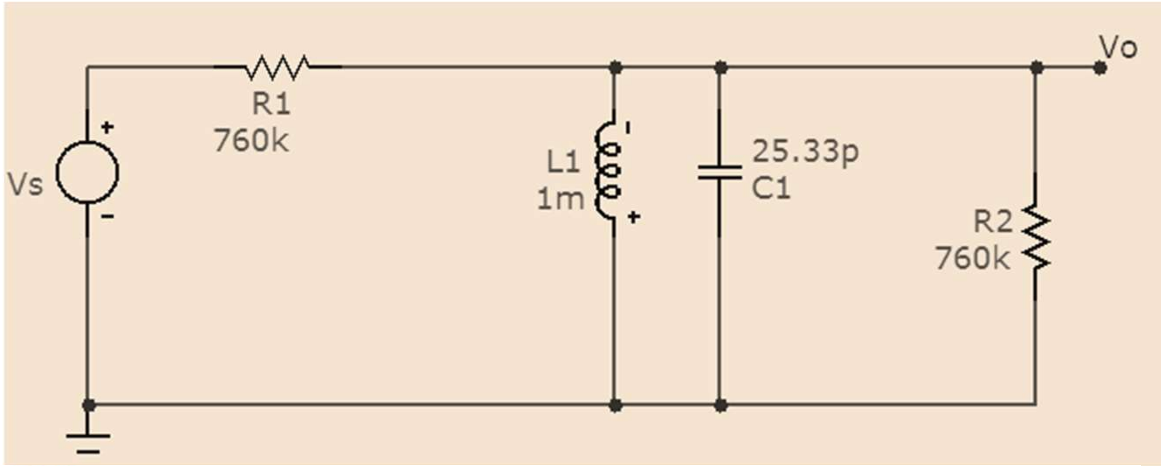
Third Order System

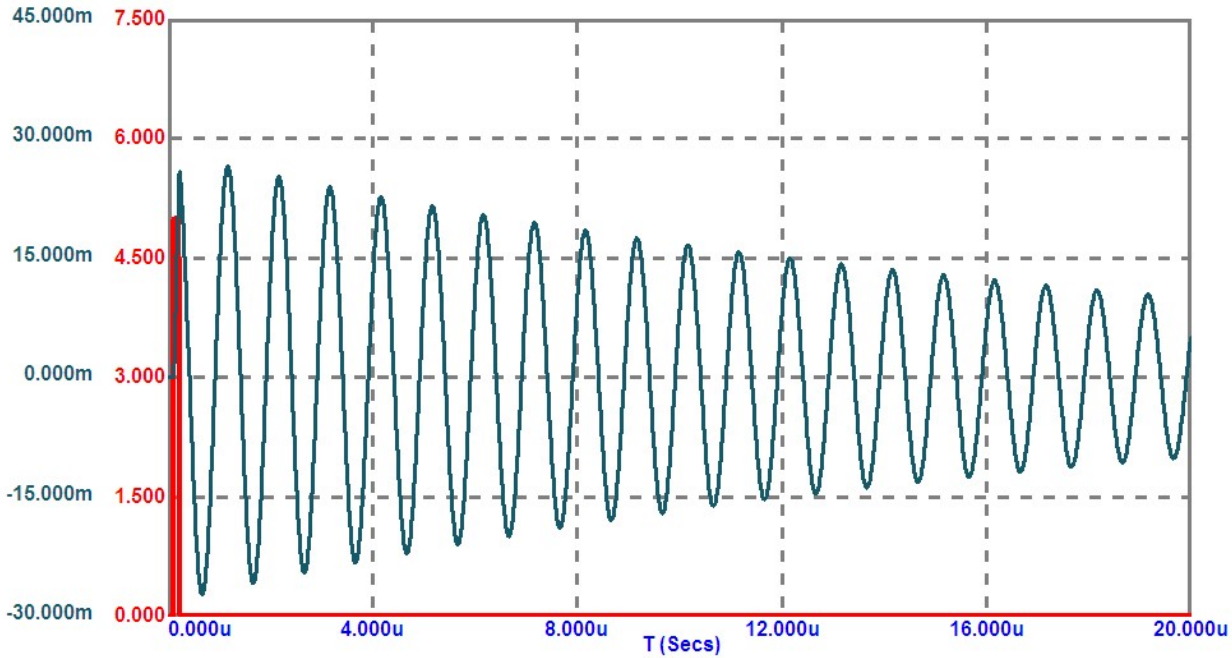
$$H(\omega)=\frac{10}{1+j\frac{\omega}{10^3}}\times\frac{1}{1+j\frac{\omega}{10^4}}\times\frac{1}{1+j\frac{\omega}{10^5}}$$

$$20\mathrm{Log}_{10}(|H(\omega)|)=20-10Log_{10}(1+(\frac{\omega}{10^3})^2)-10Log_{10}(1+(\frac{\omega}{10^4})^2)-10Log_{10}(1+(\frac{\omega}{10^5})^2)$$

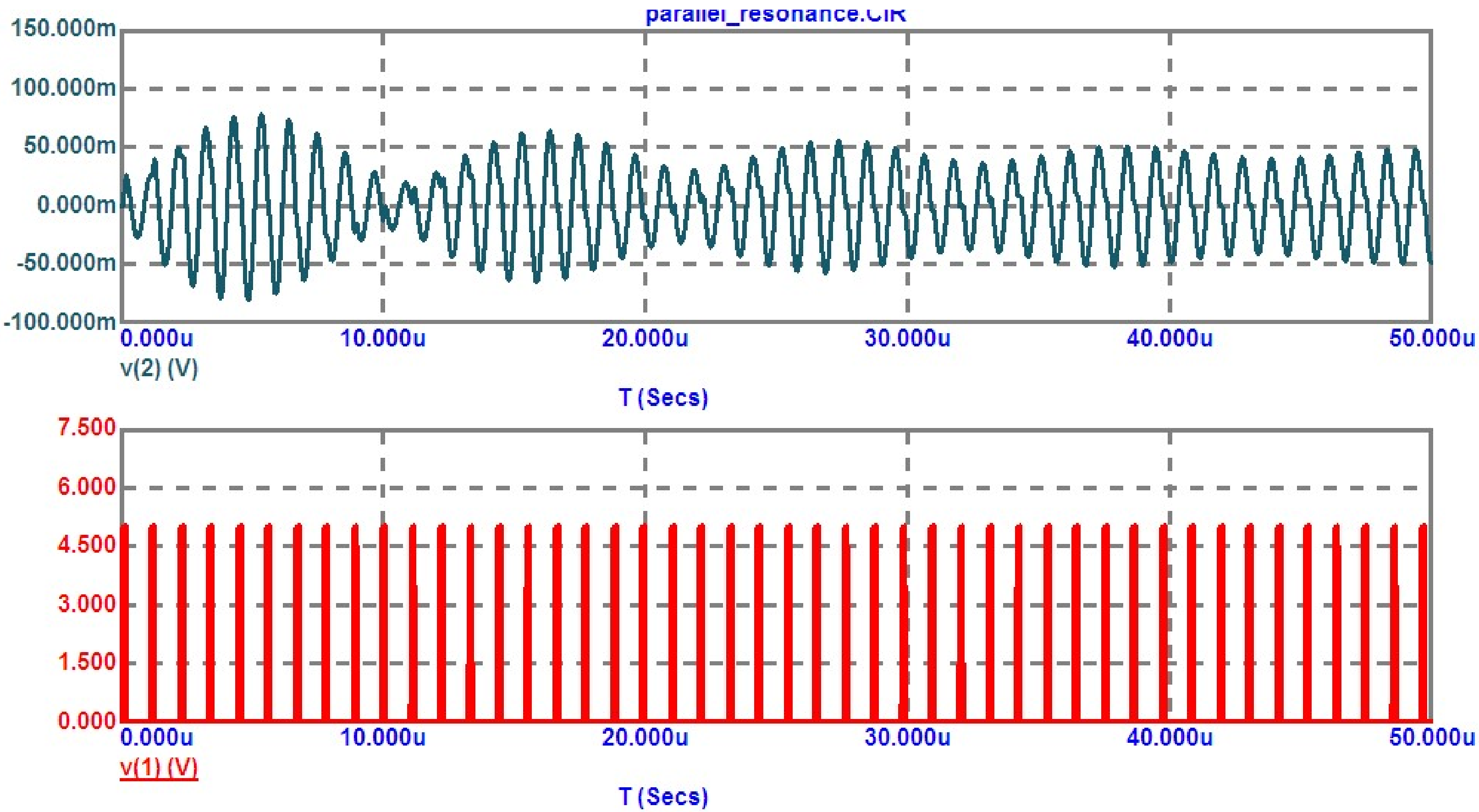


Resonance



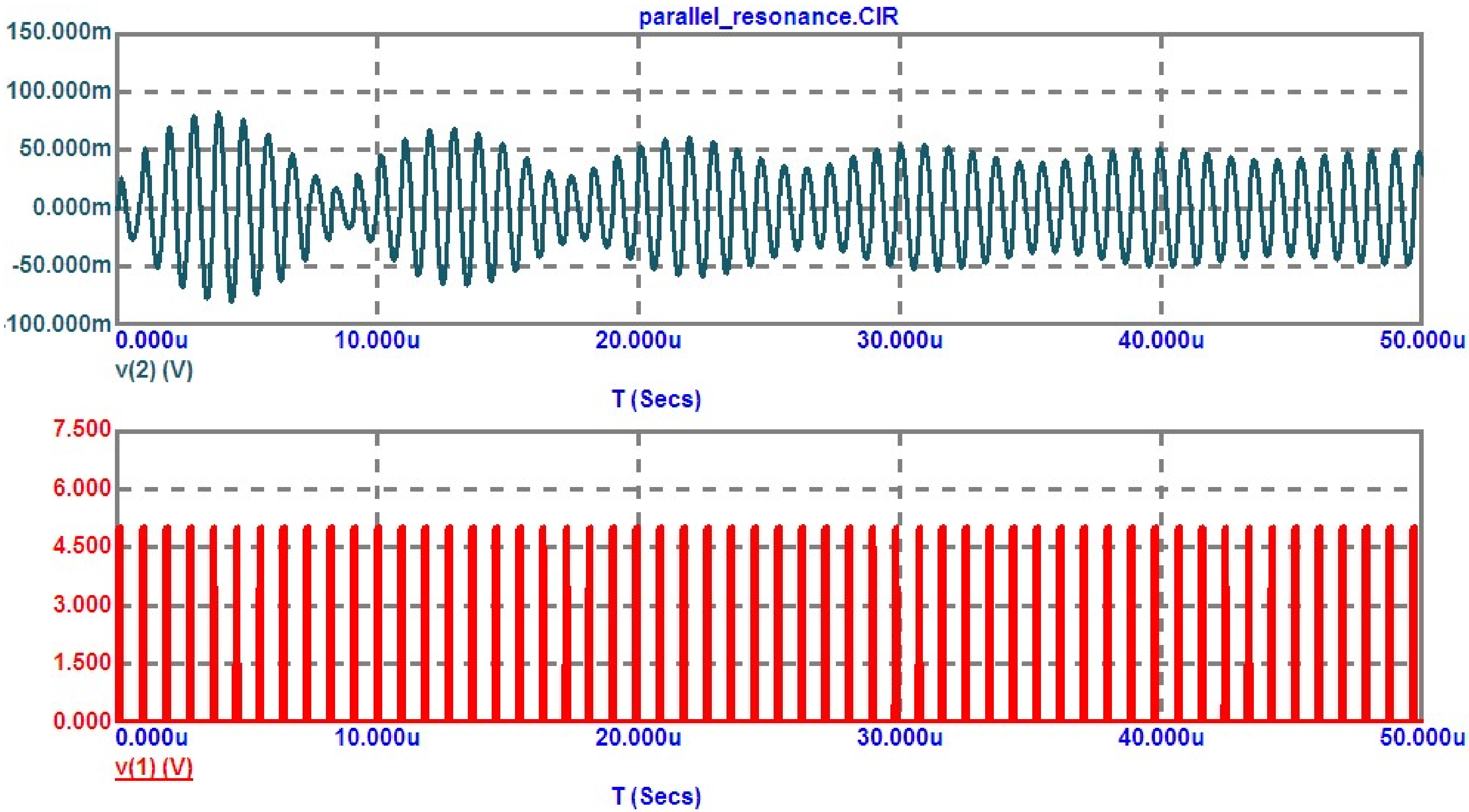


A small disturbance leads to oscillatory behavior

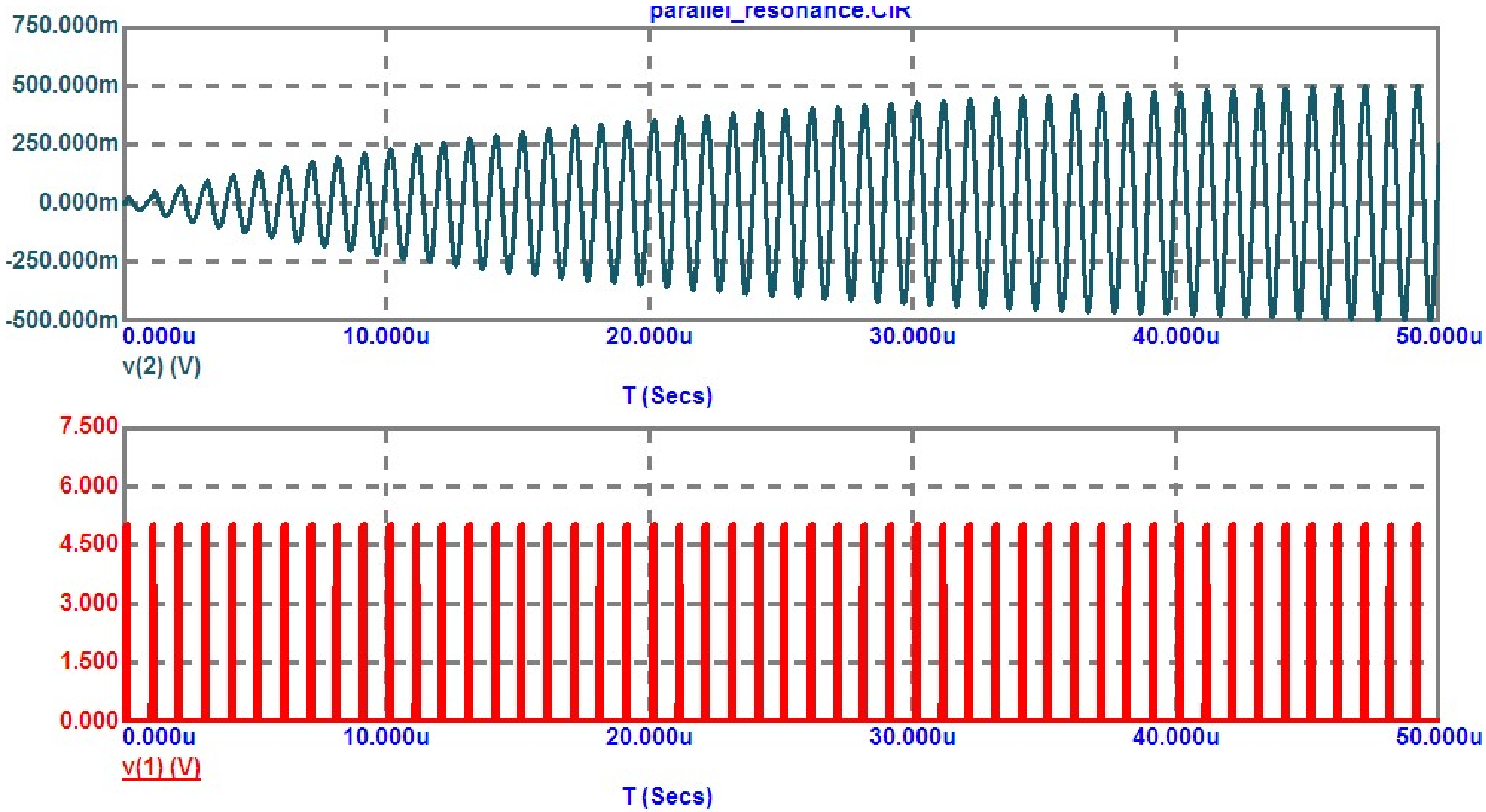


parallel_resonance.CIR
150.000m
100.000m
50.000m
0.000m
-50.000m
-100.000m
0.000u
10.000u
20.000u
30.000u
40.000u
50.000u
v(2) (V)
T (Secs)
7.500
6.000
4.500
3.000
1.500
0.000
0.000u
10.000u
20.000u
30.000u
40.000u
50.000u
v(1) (V)
T (Secs)

T = 1.1μs

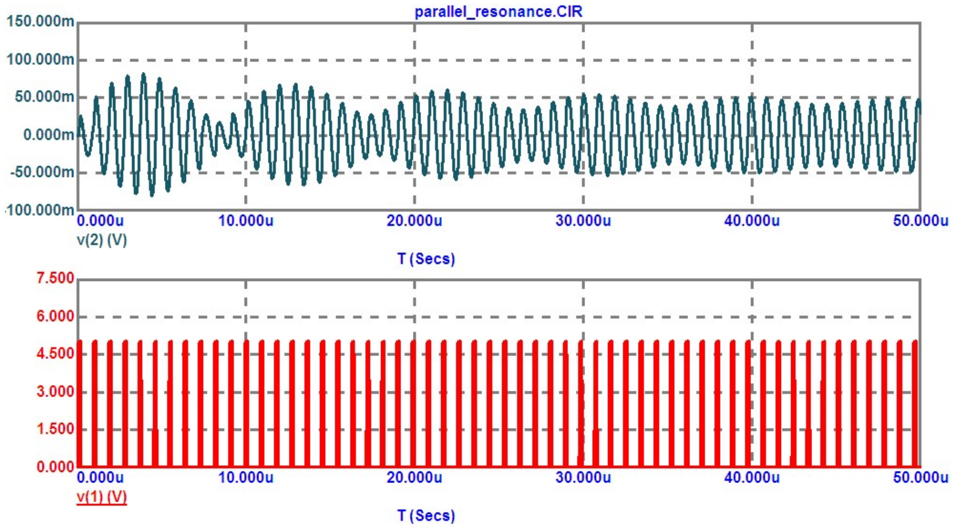

parallel_resonance.CIR
150.000m
100.000m
50.000m
0.000m
-50.000m
-100.000m
0.000u
10.000u
20.000u
30.000u
40.000u
50.000u
v(2) (V)
T (Secs)
7.500
6.000
4.500
3.000
1.500
0.000
0.000u
10.000u
20.000u
30.000u
40.000u
50.000u
v(1) (V)
T (Secs)

T = 0.9μs

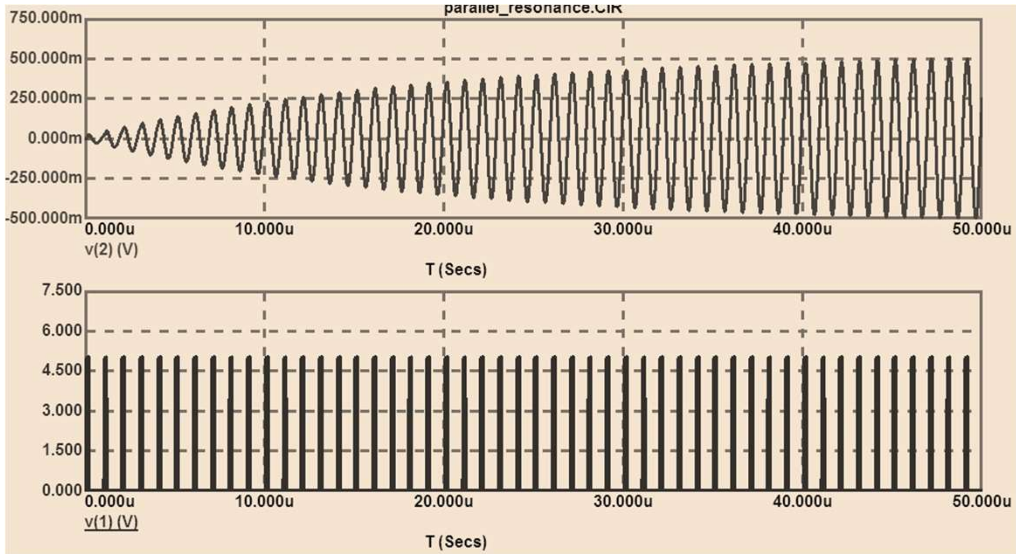


T = 1μs

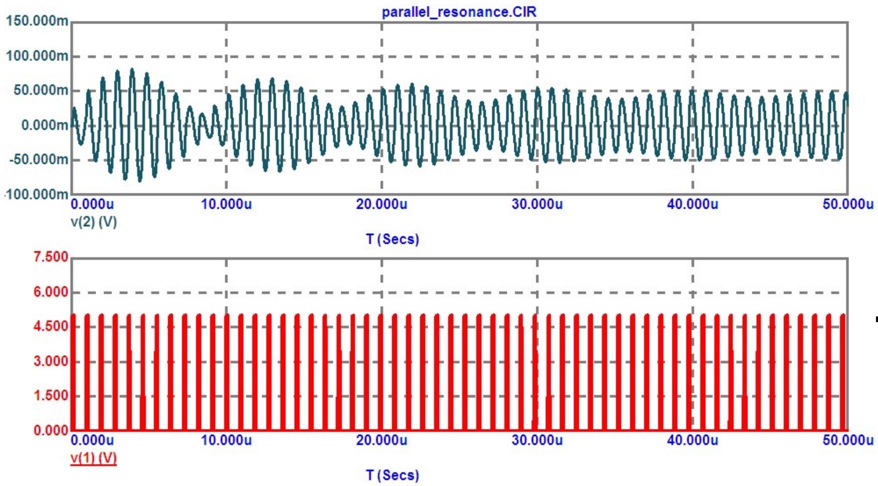
The amplitude is 10 times larger even though input magnitude is same !



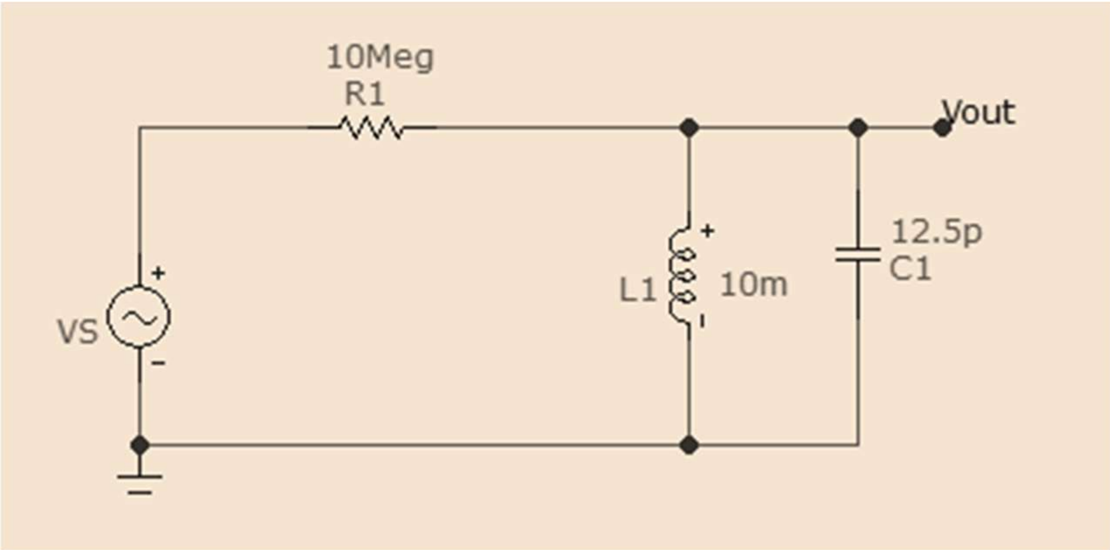
T = 0.9μs

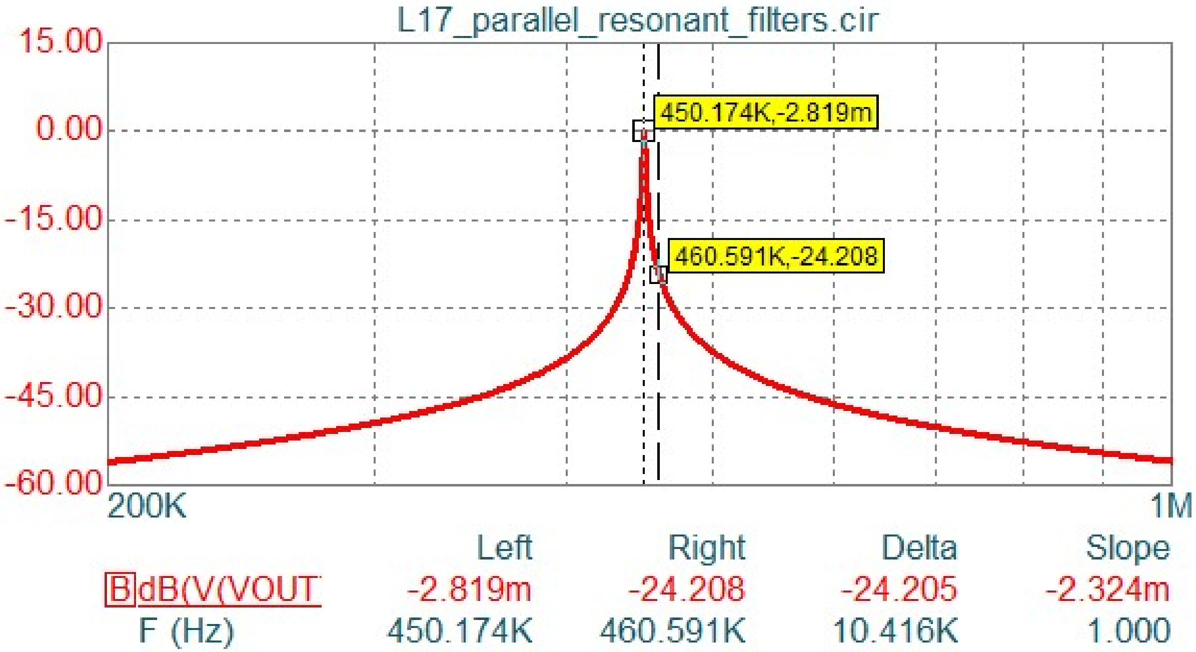


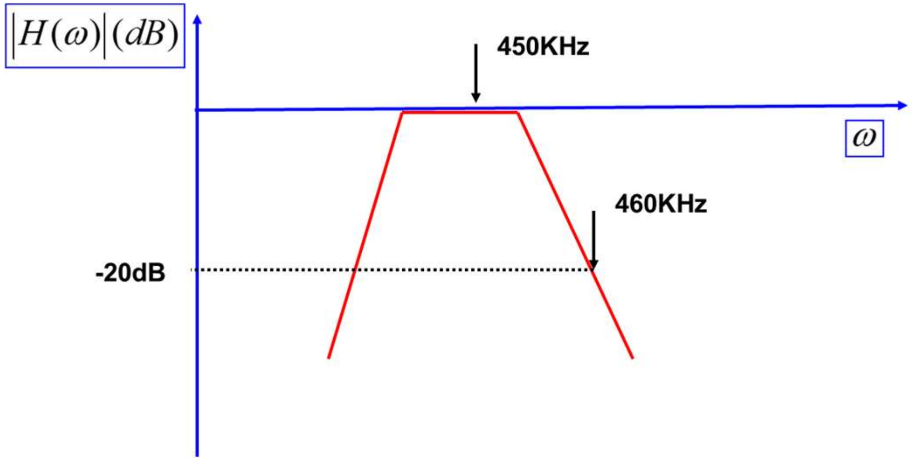
T = 1μs

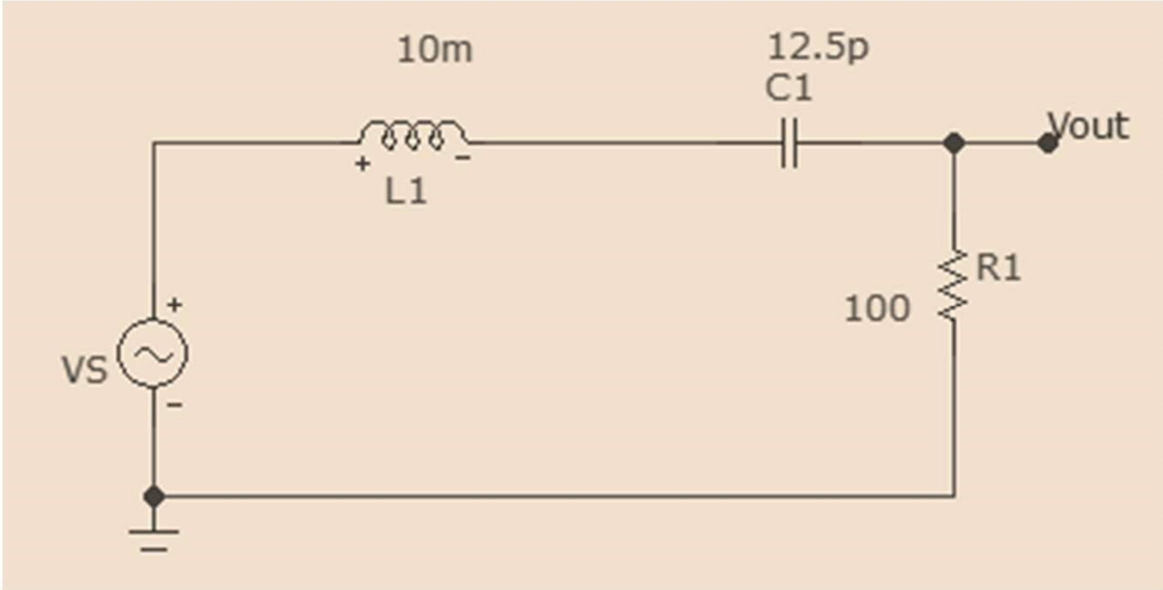


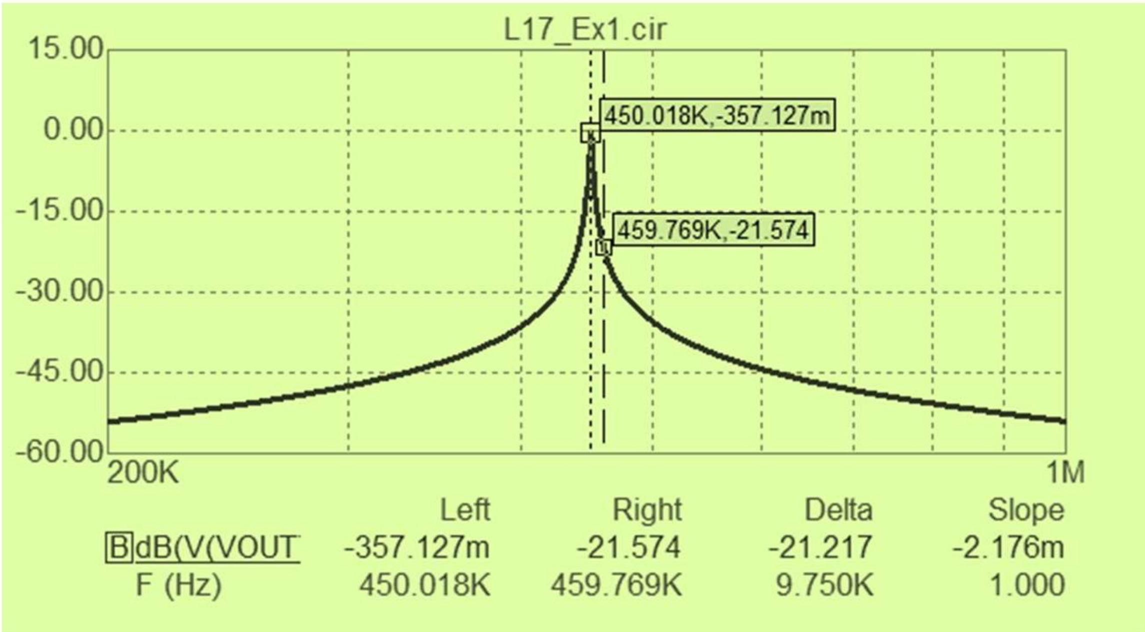
T = 0.9μs


10Meg
R1
Vout
L1
10m
12.5p
C1
VS


L17_parallel_resonant_filters.cir
15.00
0.00
-15.00
-30.00
-45.00
-60.00
200K
1M
450.174K,-2.819m
460.591K,-24.208
Left
Right
Delta
Slope
B dB(V(VOUT
-2.819m
-24.208
-24.205
-2.324m
F (Hz)
450.174K
460.591K
10.416K
1.000

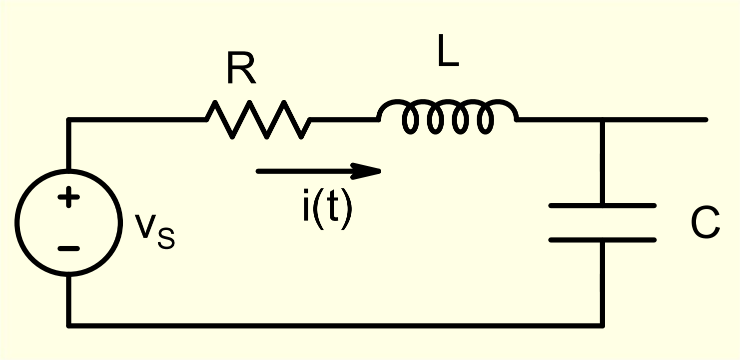

$|H(\omega)|(dB)$
450KHz
460KHz
-20dB
ω


10m
12.5p
C1
Vout
L1
R1
100
VS


L17_Ex1.cir
15.00
0.00
-15.00
-30.00
-45.00
-60.00
450.018K,-357.127m
459.769K,-21.574
200K
1M
Left Right Delta Slope
B dB(V(VOUT -357.127m -21.574 -21.217 -2.176m
F (Hz) 450.018K 459.769K 9.750K 1.000

Series Resonant Circuit

In this series resonant circuit, current reaches a peak at a condition in which capacitive and inductive reactance cancel each other to give rise to a purely resistive circuit



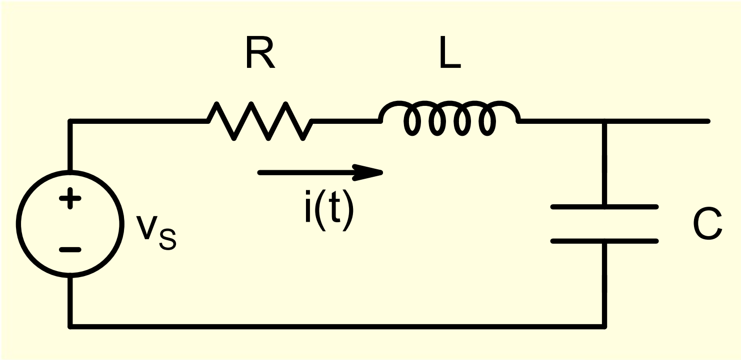
$$Z_{eq} = R + j\omega L - j\frac{1}{\omega C}$$

Resonant frequency: $$j\omega_O L - j\frac{1}{\omega_O C} = 0 \Rightarrow \omega_O = \frac{1}{\sqrt{LC}}$$

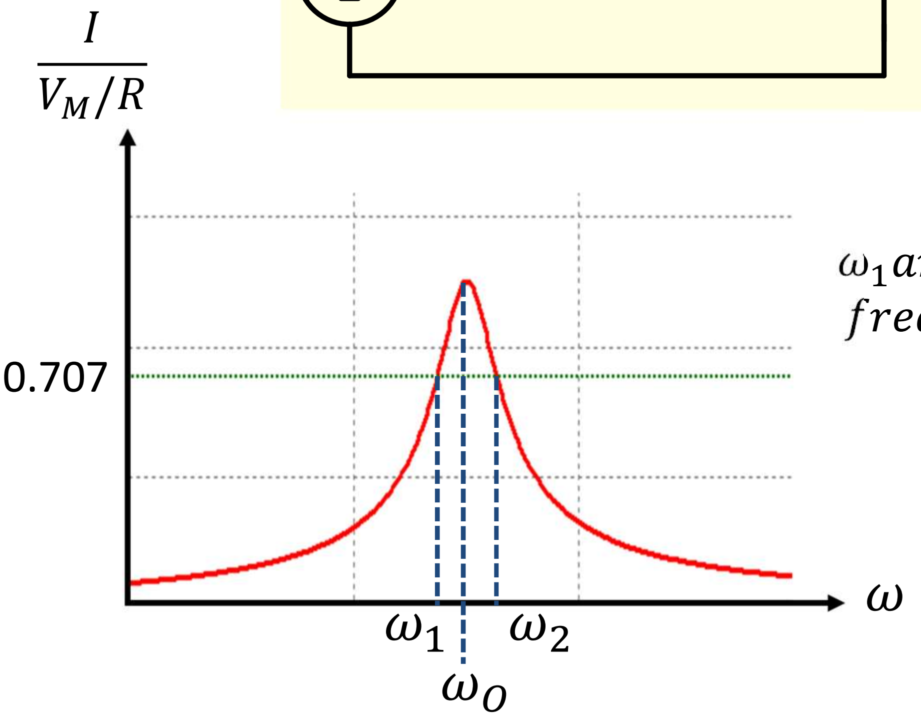
$$f_O = \frac{1}{2\pi\sqrt{LC}}$$

$$Z_{eq} = R$$

Current and voltage are in phase (power factor is unity) and current is maximum !



$$|I(\omega)| = \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$



ω_1 and ω_2 are called half power frequencies since $\mathrm{P} \propto I^2$

$$|I(\omega_1)| = \frac{V_m}{\sqrt{R^2 + (\omega_1 L - \frac{1}{\omega_1 C})^2}} = \frac{V_m}{\sqrt{2}R}$$

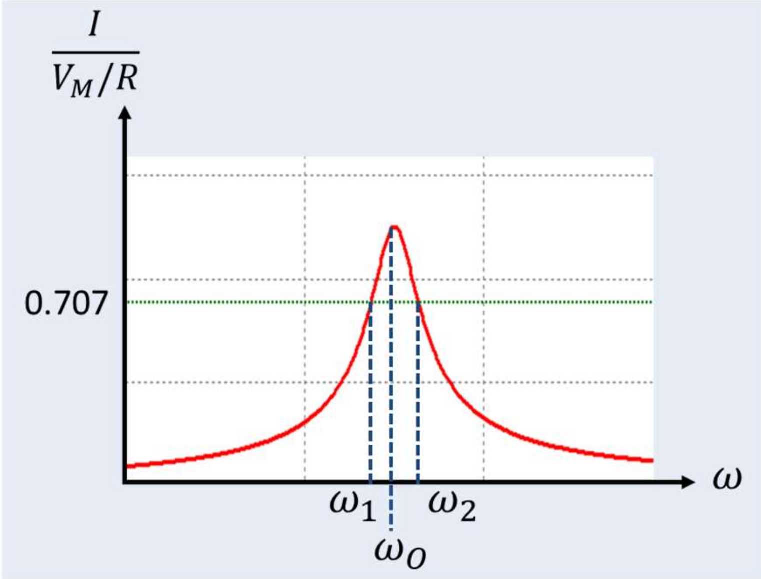
$$|I(\omega_2)| = \frac{V_m}{\sqrt{R^2 + (\omega_2 L - \frac{1}{\omega_2 C})^2}} = \frac{V_m}{\sqrt{2}R}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$



$$\omega_O = \sqrt{\omega_1\omega_2}$$

$$B = \omega_2 - \omega_1 = \frac{R}{L}$$

Quality (Q) factor: Sharpness of resonance

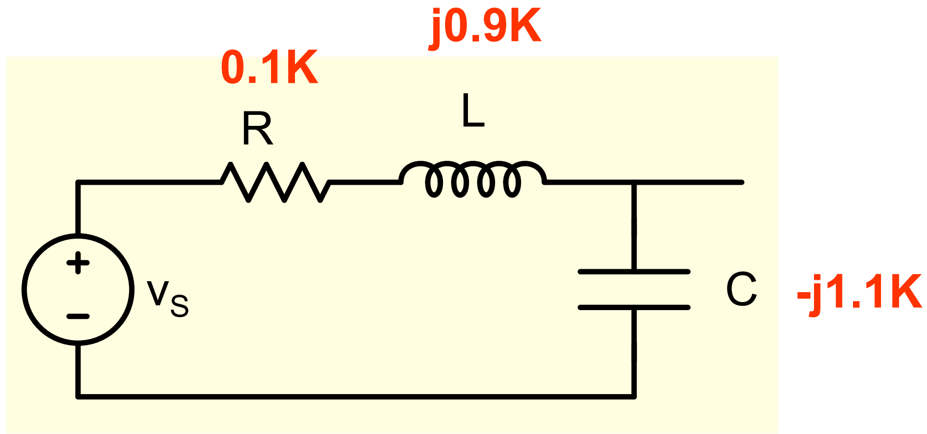
$$Q = \frac{\omega_O}{\mathrm{B}} = \frac{\omega_O}{\Delta\omega} = \frac{\omega_O L}{R}$$

$$Q = 2\pi \frac{\text{Peak Stored Energy}}{\text{Energy dissipated in one period at resonance}}$$

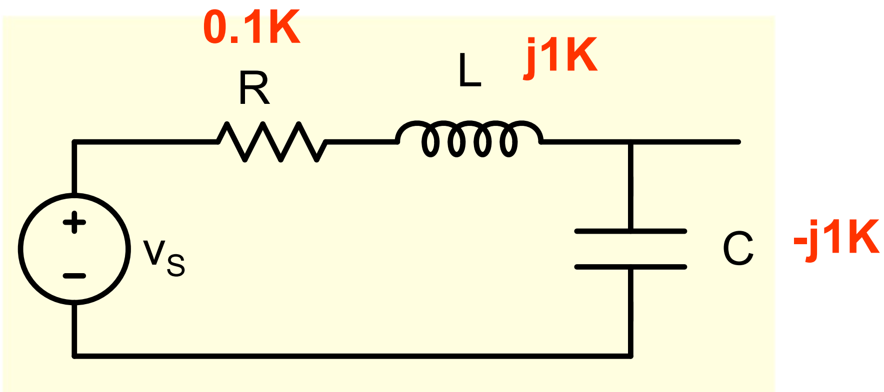
$$Q = 2\pi \times \frac{\frac{1}{2} L \times I_m^2}{\frac{1}{2} I_m^2 R \times T_O} = \frac{\omega_O L}{R}$$

$$\omega_O = \frac{1}{\sqrt{LC}} \Rightarrow Q = \frac{1}{\omega_O CR}$$

$$Q = \frac{\omega_O L}{R} = \frac{1}{\omega_o \mathrm{RC}}$$



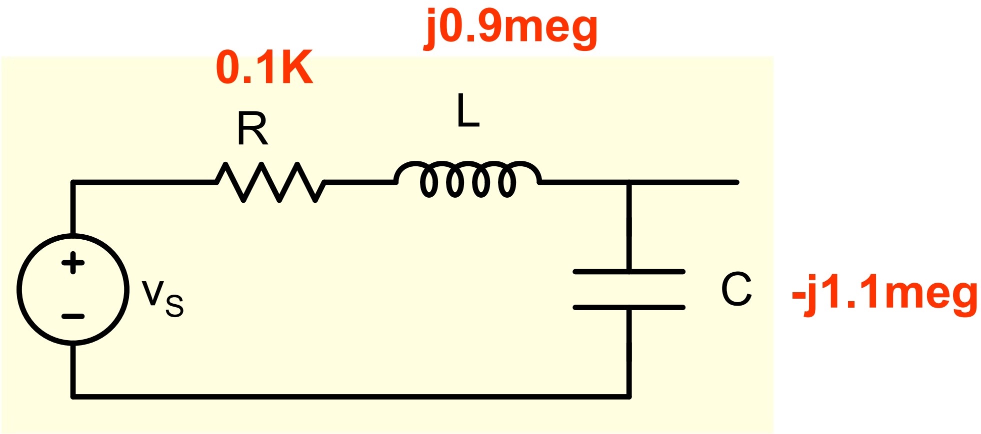
Z=0.1K-j0.1K



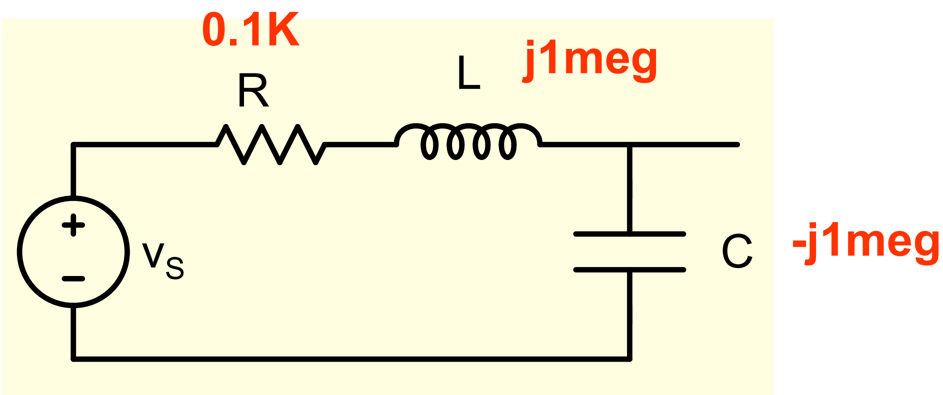
Z=0.1K

Not very large change in impedance as we approach resonance !

$$Q = \frac{\omega_O L}{R} = \frac{1}{\omega_o \mathrm{RC}}$$



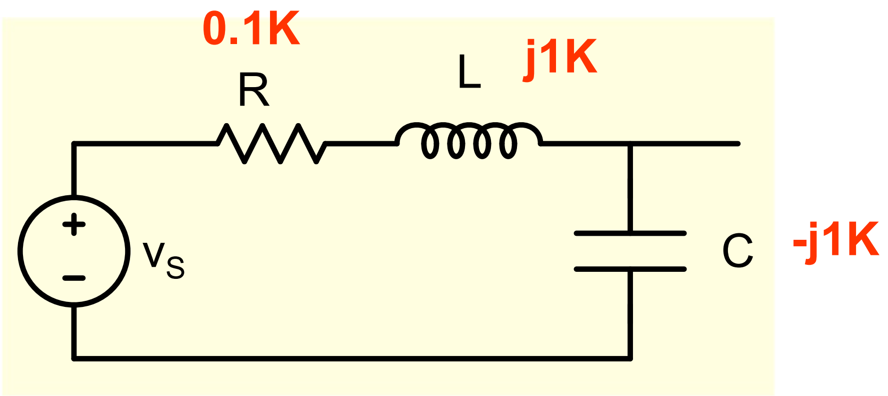
Z=0.1K-j0.1meg



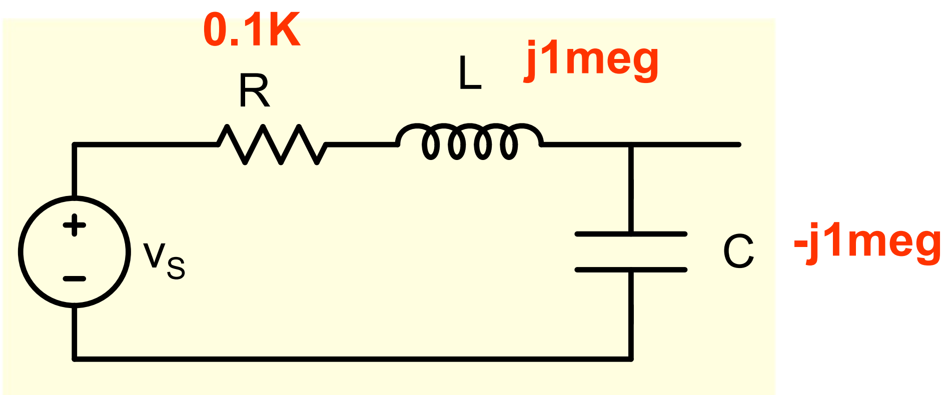
Z=0.1K

very large change in impedance as we approach resonance ! Implying high quality factor

Quality factor Q

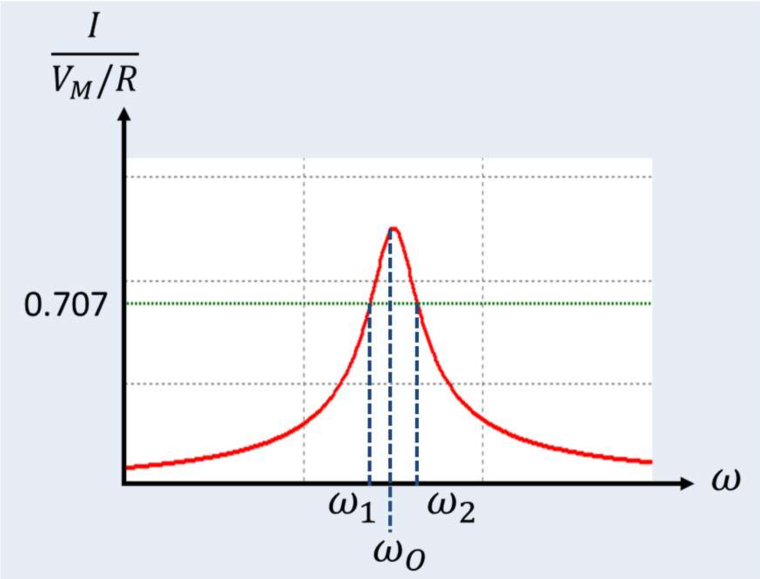


Z=0.1K



Z=0.1K

$$Q = \frac{\omega_O L}{R} \quad or \quad Q = \frac{1/\omega_O C}{R}$$



$$Q = \frac{\omega_O L}{R}$$

$$B = \omega_2 - \omega_1 = \frac{R}{L}$$

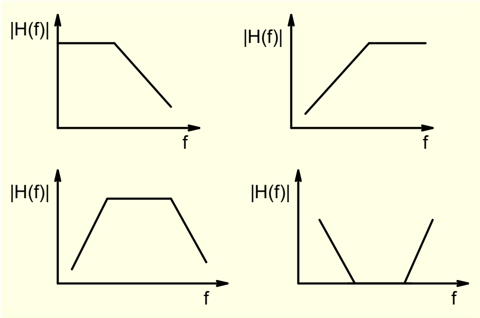
$$Q = \frac{\omega_O}{B} = \frac{\omega_O}{\Delta\omega}$$

Hence Q represents sharpness of resonance

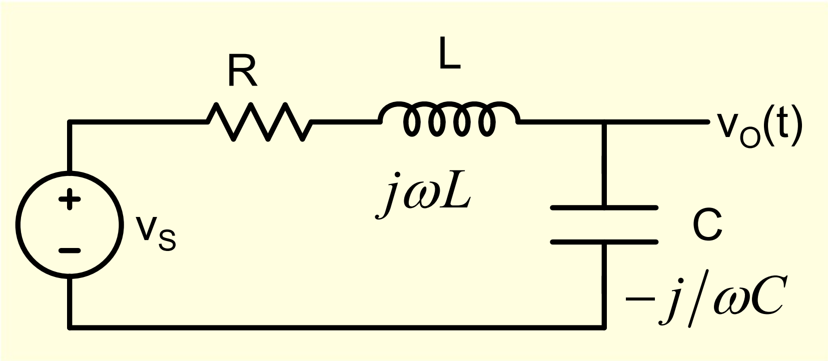
For high Q circuits:

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

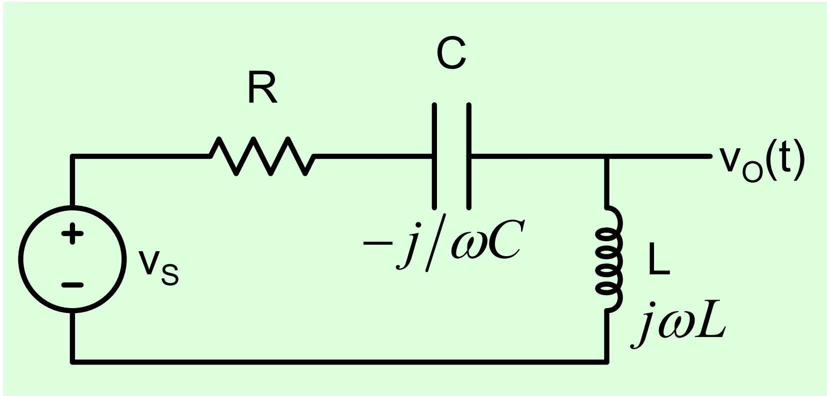
R-L-C filters



|H(f)|
f
|H(f)|
f
|H(f)|
f
|H(f)|
f

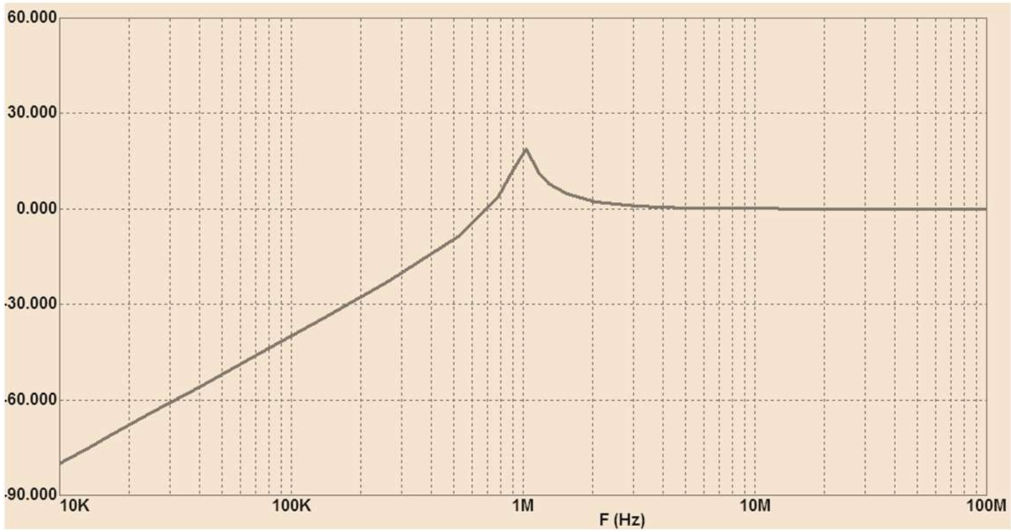


R
L
$v_O(t)$
$j\omega L$
+
–
v_S
C
$-j/\omega C$

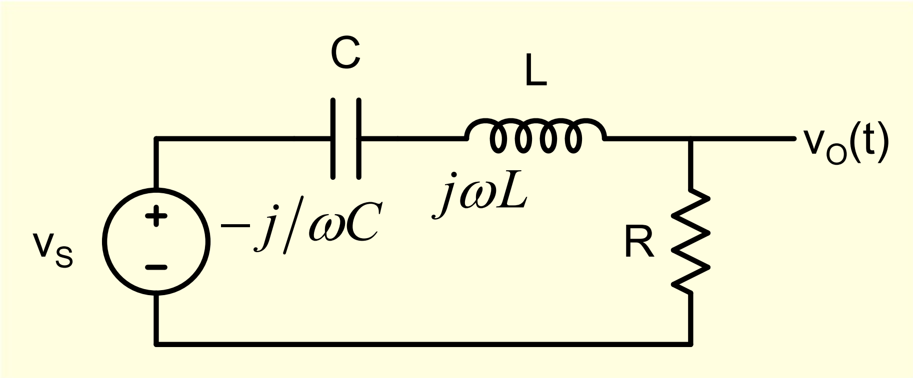


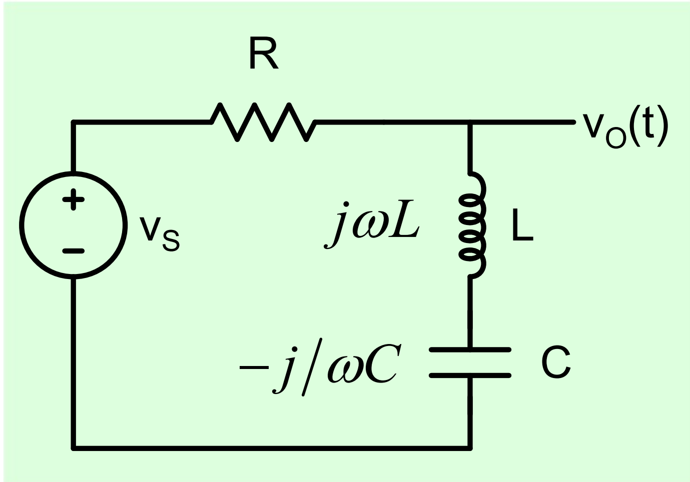
R
C
$v_O(t)$
+
–
v_S
$-j/\omega C$
L
$j\omega L$

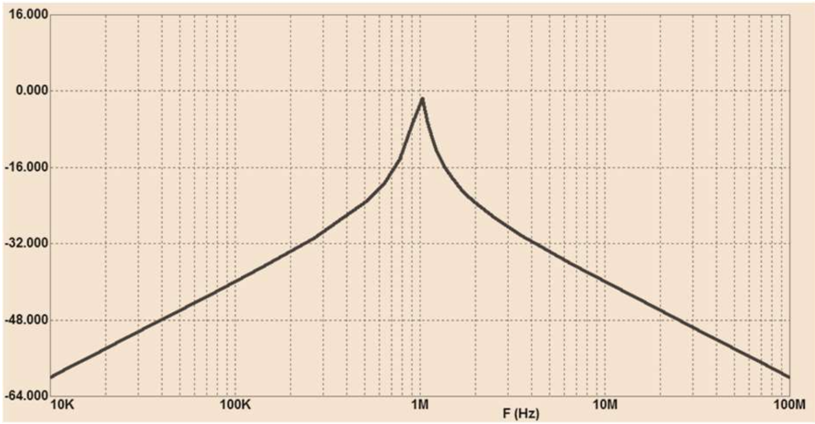
60.000
30.000
0.000
-30.000
-60.000
-90.000
10K
100K
1M
10M
100M

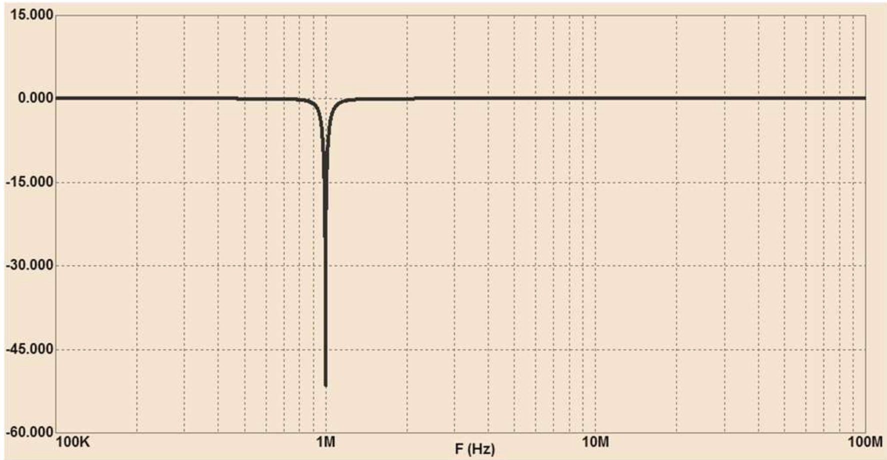


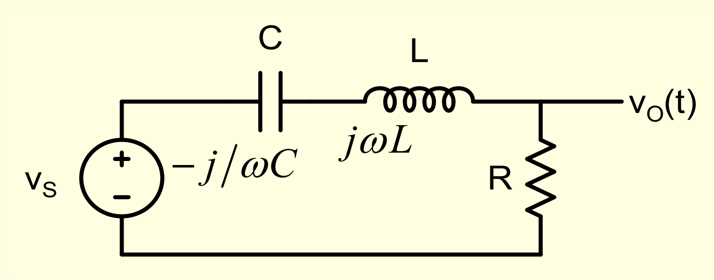
60.000
30.000
0.000
-30.000
-60.000
-90.000
10K
100K
1M
F (Hz)
10M
100M

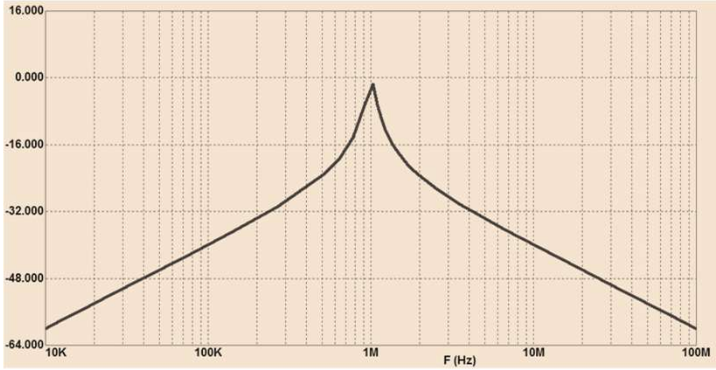

C
L
$j\omega L$
$-j/\omega C$
v_S
R
$v_O(t)$


R
$v_O(t)$
v_S
$j\omega L$
L
$-j/\omega C$
C


16.000
0.000
-16.000
-32.000
-48.000
-64.000
10K
100K
1M
10M
100M
F (Hz)


15.000
0.000
-15.000
-30.000
-45.000
-60.000
100K
1M
10M
100M
F (Hz)





How much Q do we need to pass 450KHz but reject 460KHz by 20dB?

$$\left|H(\omega)\right| = \left|\frac{V_O(\omega)}{V_{IN}(\omega)}\right| = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

Assuming V_{IN} = 1V and noting that Q = $\omega_O L/R$

$$\left|V_O(\omega)\right| = \frac{1}{\sqrt{1 + \frac{\omega_O^2}{\omega^2} Q^2 (\frac{\omega^2}{\omega_O^2} - 1)^2}}$$

For $\omega = \omega_O$, V_O = 1 so the signal simply passes through !

$$\omega_O = 2 \times \pi \times 450 \times 10^3 = 2.8 \times 10^6\, rad/s$$

$$\left|V_O(\omega)\right| = \frac{1}{\sqrt{1+\frac{\omega_O^2}{\omega^2}Q^2(\frac{\omega^2}{\omega_O^2}-1)^2}}$$

$$\left|V_O(\omega)\right| \cong \frac{\omega_O^2}{Q\times(\omega^2-\omega_O^2)}$$

$$\left|V_O(\omega)\right| \cong \frac{\omega_O}{2Q\times(\omega-\omega_O)}$$

For an attenuation of -20dB or 10^{-1} at ω-ωo = 62.8 Krad/s

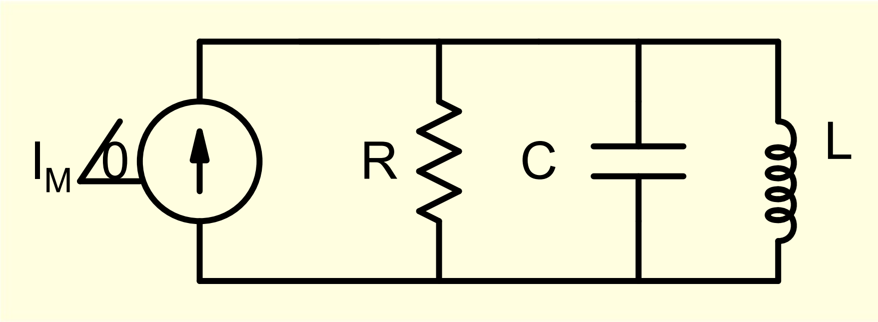
$$Q \cong 226.3$$

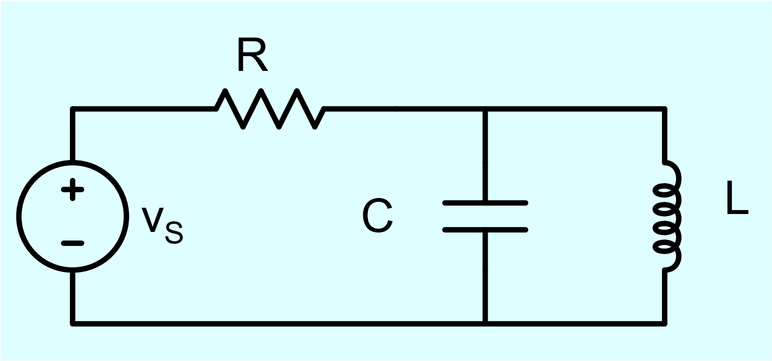
Example: for Q = 226.3 at 450KHz

$$Q = \frac{\omega_o L}{R}$$

Suppose $L = 10^{-3}H; \Rightarrow R = 12.5\ \Omega; \Rightarrow C = 125pF$

Parallel Resonance



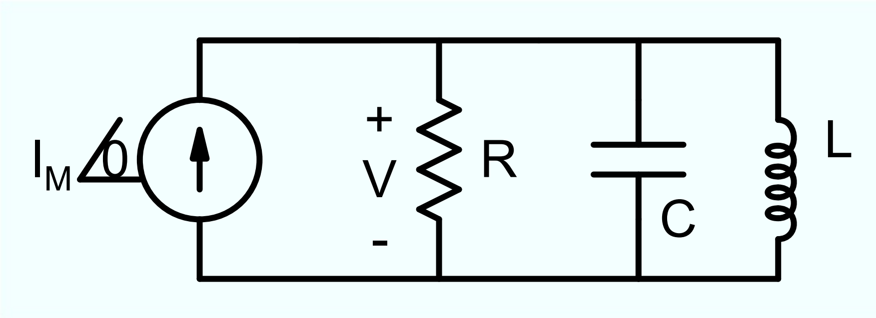


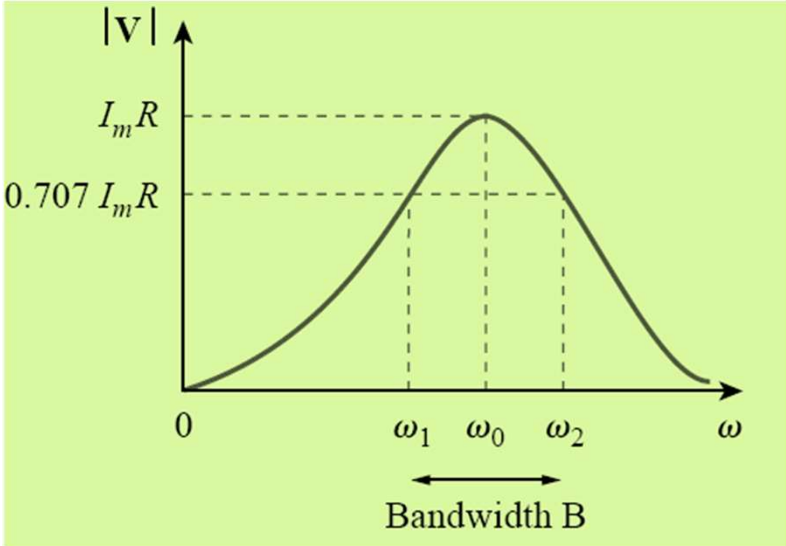
$$Y_{eq} = \frac{1}{R} + j\omega C - j\frac{1}{\omega L}$$

Resonant frequency: $$j\omega_O C - j\frac{1}{\omega_O L} = 0 \Rightarrow \omega_O = \frac{1}{\sqrt{LC}}$$

$$f_O = \frac{1}{2\pi\sqrt{LC}}$$

$$Z_{eq} = R$$





$$|V(\omega)| = \frac{I_m R}{\sqrt{1 + \frac{R^2 C^2}{L^2}(\omega L - \frac{1}{\omega C})^2}}$$

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

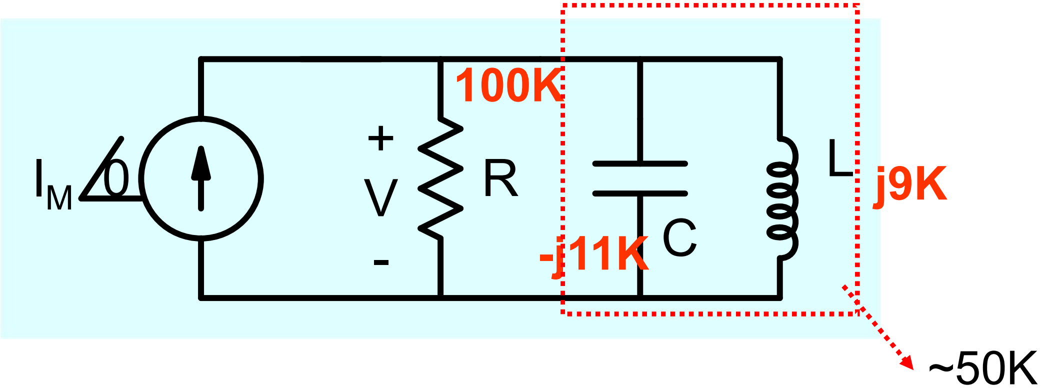
$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

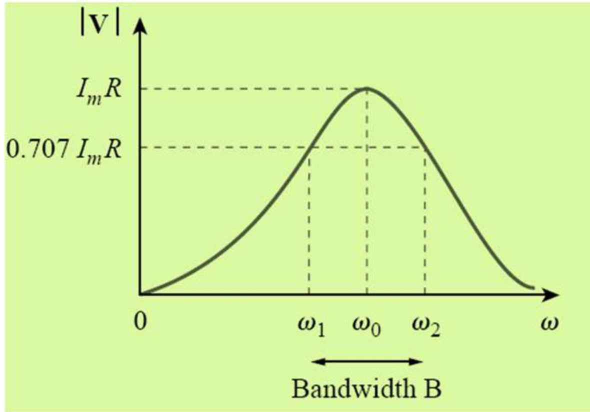
$$\omega_1 = \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} - \frac{\omega_0}{2Q}, \qquad \omega_2 = \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} + \frac{\omega_0}{2Q}$$

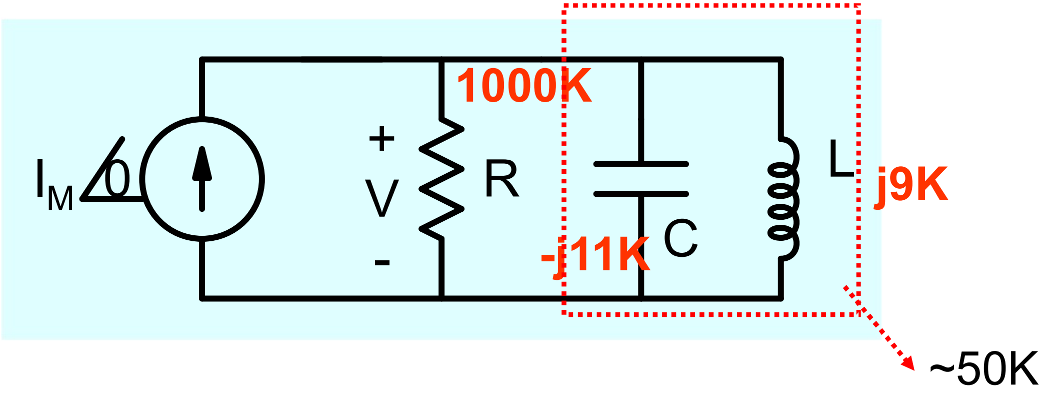
For high Q:

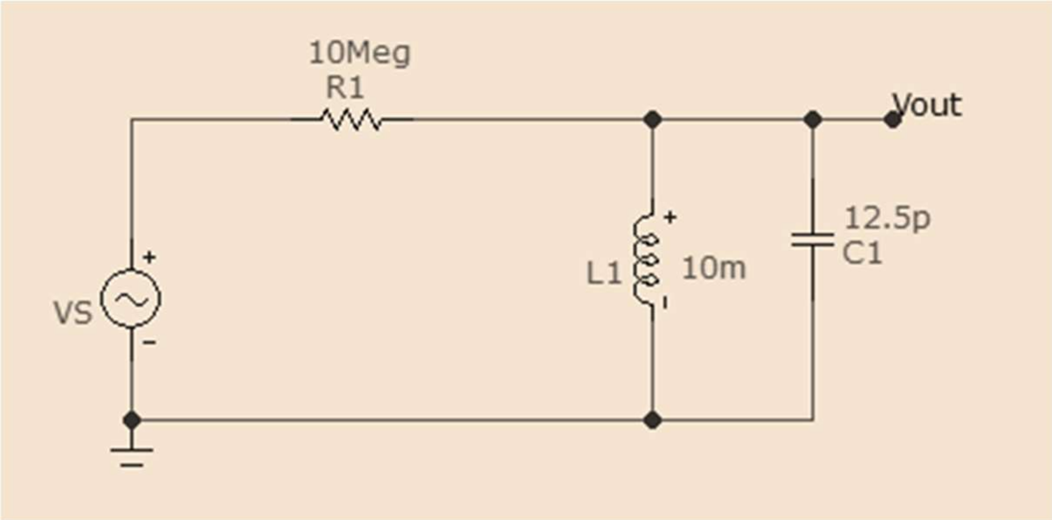
$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

Why is $Q = \frac{R}{\omega L}$ for parallel resonance ?



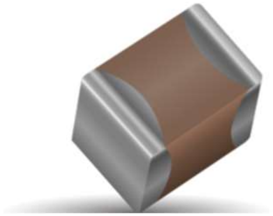






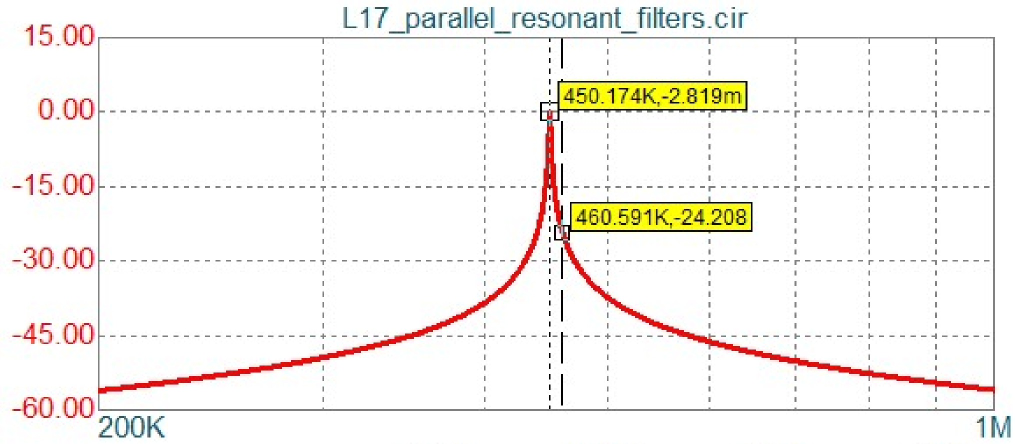
C0G (NP0) Dielectric

General Specifications

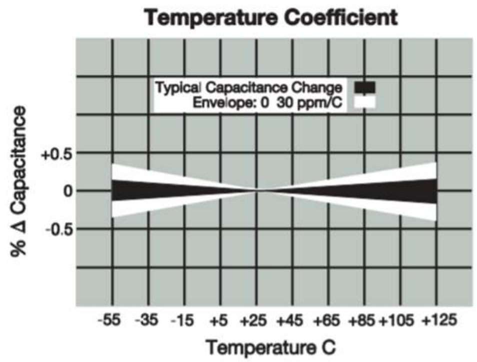


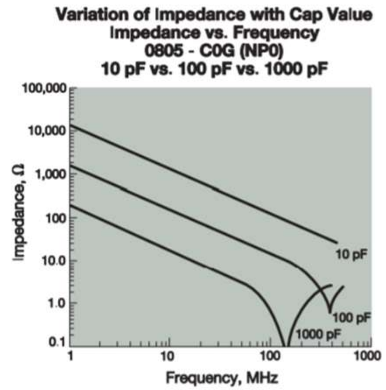
C0G (NP0) is the most popular formulation of the "temperature-compensating," EIA Class I ceramic materials. Modern C0G (NP0) formulations contain neodymium, samarium and other rare earth oxides.

C0G (NP0) ceramics offer one of the most stable capacitor dielectrics available. Capacitance change with temperature is 0 ±30ppm/°C which is less than ±0.3% C from -55°C to +125°C. Capacitance drift or hysteresis for C0G (NP0) ceramics is negligible at less than ±0.05% versus up to ±2% for films. Typical capacitance change with life is less than ±0.1% for C0G (NP0), one-fifth that shown by most other dielectrics. C0G (NP0) formulations show no aging characteristics.

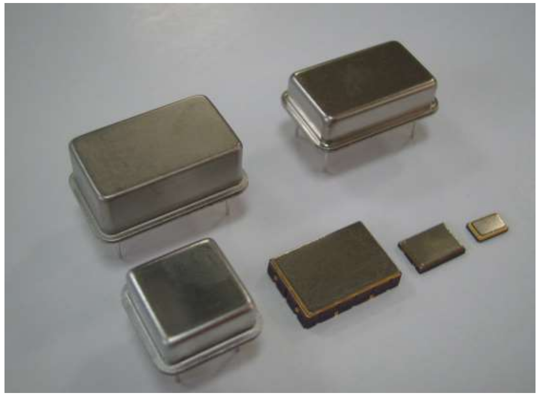


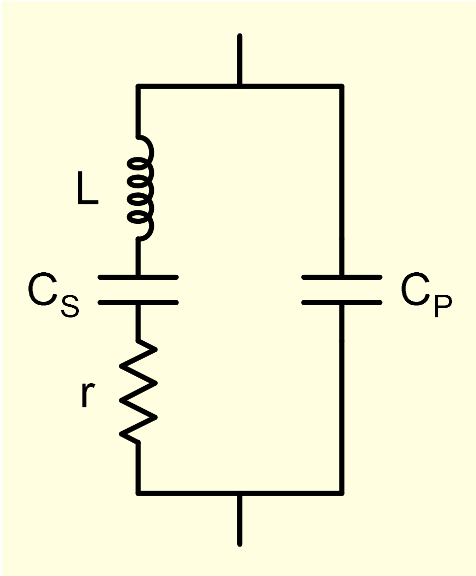
	Left	Right	Delta	Slope
B dB(V(VOUT	-2.819m	-24.208	-24.205	-2.324m
F (Hz)	450.174K	460.591K	10.416K	1.000



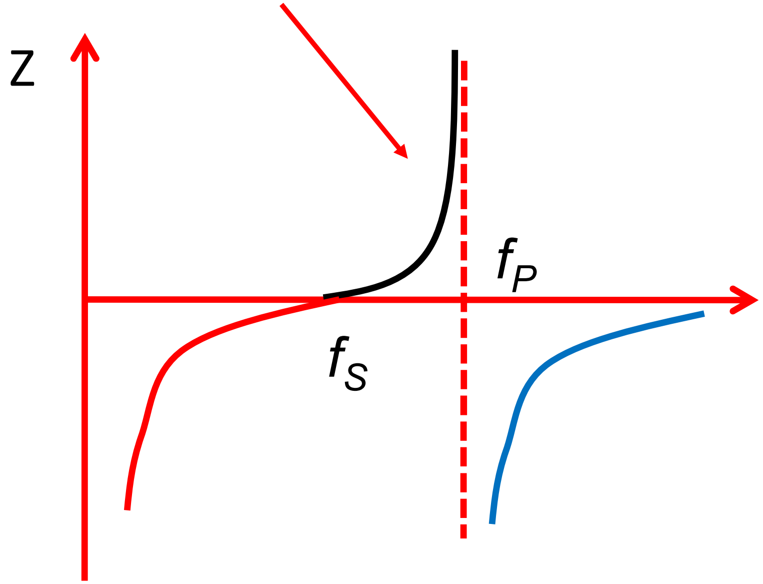


Piezoelectric Quartz Crystal





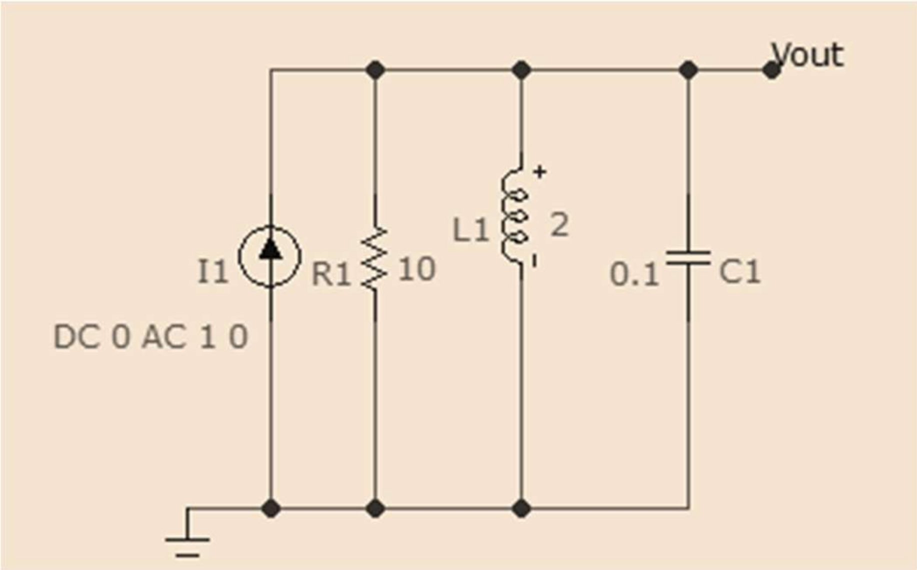
$C_S >> C_P$



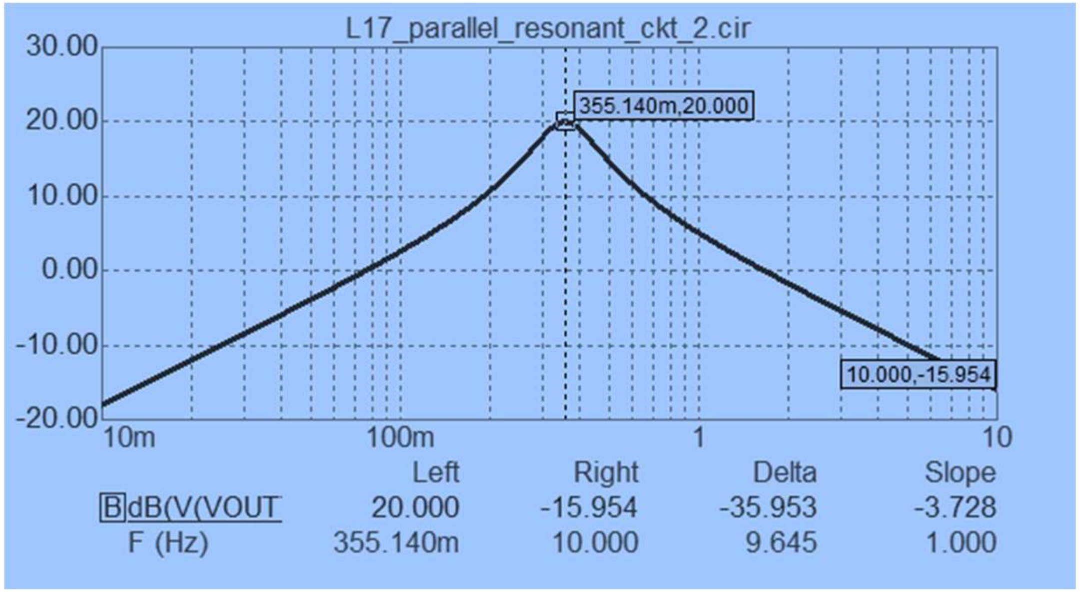
Q can be very high ~10^4 -10^6

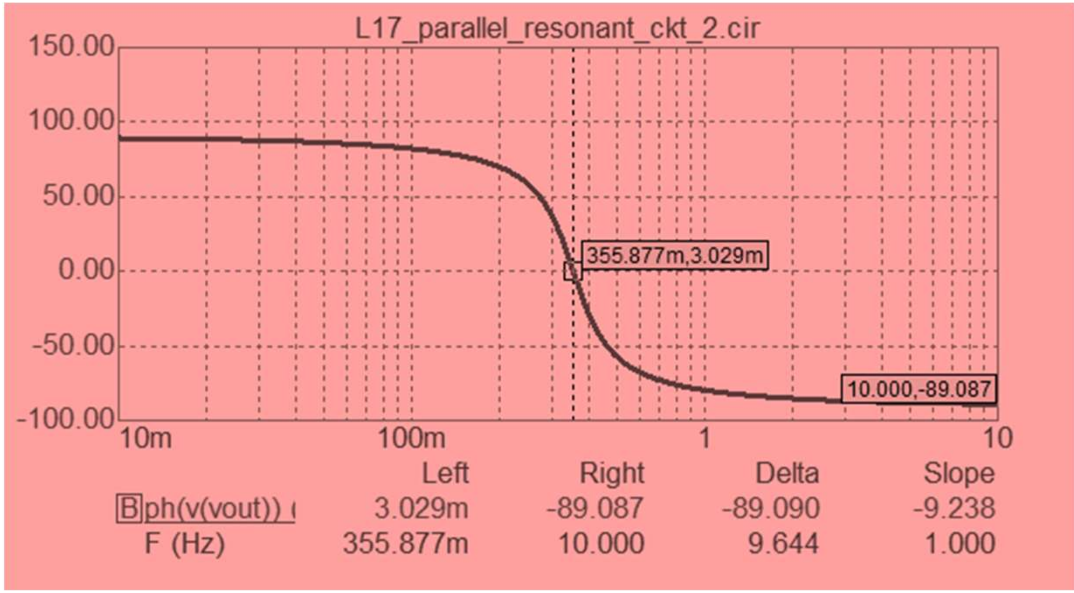
PARAMETERS	
ABRACON P/N	ABLS Series
Frequency	3.579545 MHz to 75 MHz
Operation Mode	AT cut (Fundamental or 3rd OT) or BT cut (See options) 3.579545MHz - 24.0MHz (Fundamental) (Standard) 24.01 - 75.00MHz (3rd- Overtone) (Standard) 24.01MHz - 50.00MHz (Fund. AT or BT) (Option. See 2.1)
Operating Temperature	0°C to + 70°C (see options)
Storage Temperature	- 55°C to + 125°C
Frequency Tolerance at +25°C	± 50 ppm max. (see options)
Frequency Stability over the Operating Temp. (Ref to +25°C)	± 50 ppm max. (see options)
Equivalent Series Resistance	See Table 1
Shunt Capacitance C_0	7pF max.
Load Capacitance C_L	18pF (see options)
Drive Level	1 mW max., 100µW typical
Aging at 25°C ± 3°C Per Year	± 5ppm max.
Insulation Resistance	500 MΩ min at 100Vdc ± 15V
Spurious Responses	-3dB max.
Drive level dependency (DLD)	from 1µW to 500µW (minimum 7 points tested)

Resonant Frequency

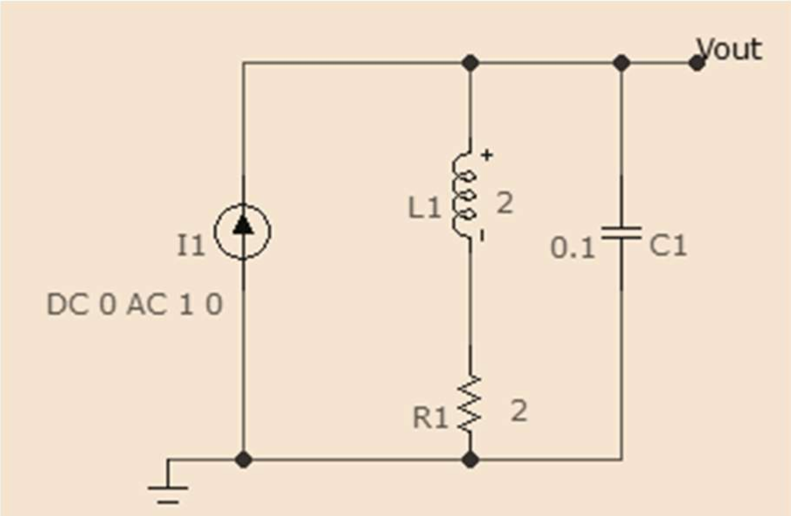


$$f_O = \frac{1}{2\pi\sqrt{LC}}$$





What is the resonant frequency (unity power factor) ?

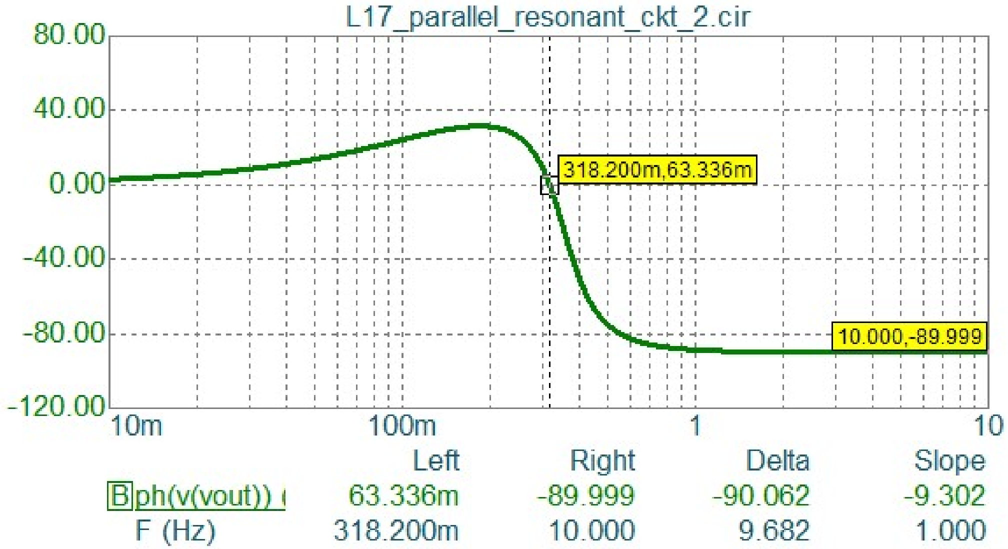


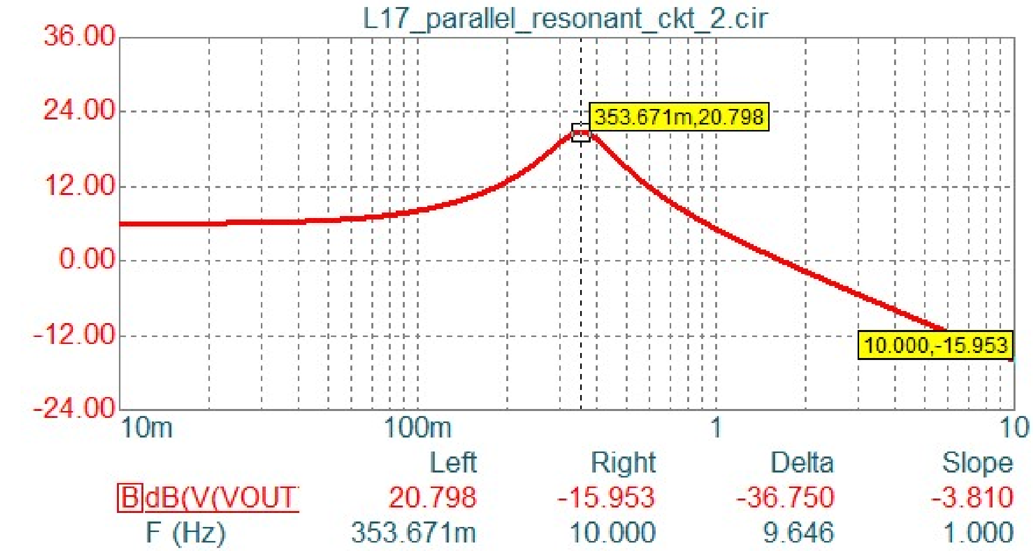
$$Y = j\omega 0.1 + \frac{1}{2 + j2\omega}$$

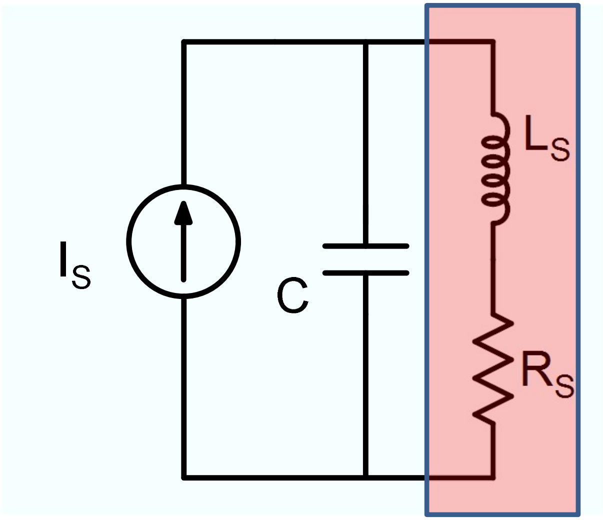
For unity power factor, imaginary part = 0

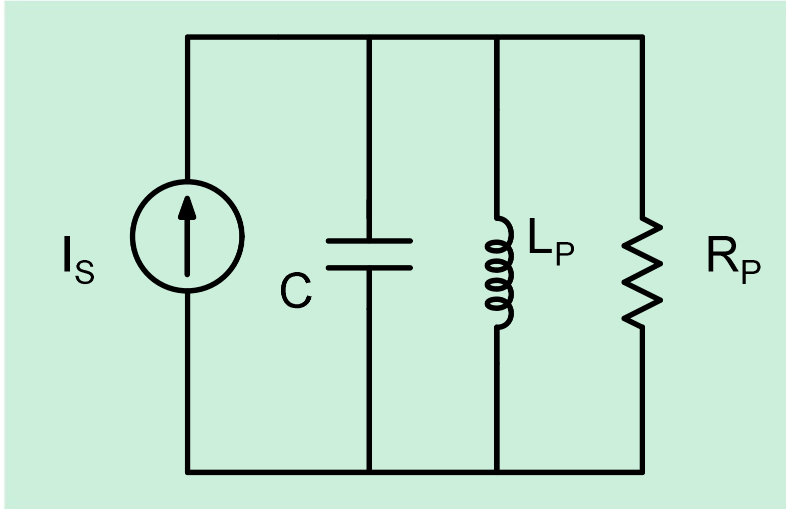
$$Y = \frac{2}{4 + 4\omega^2} + j\omega 0.1 - \frac{j2\omega}{4 + 4\omega^2}$$

$$0.1\omega_O - \frac{2\omega_O}{4 + 4\omega_O^2} = 0 \Rightarrow \omega_O = 2\, rad/s$$









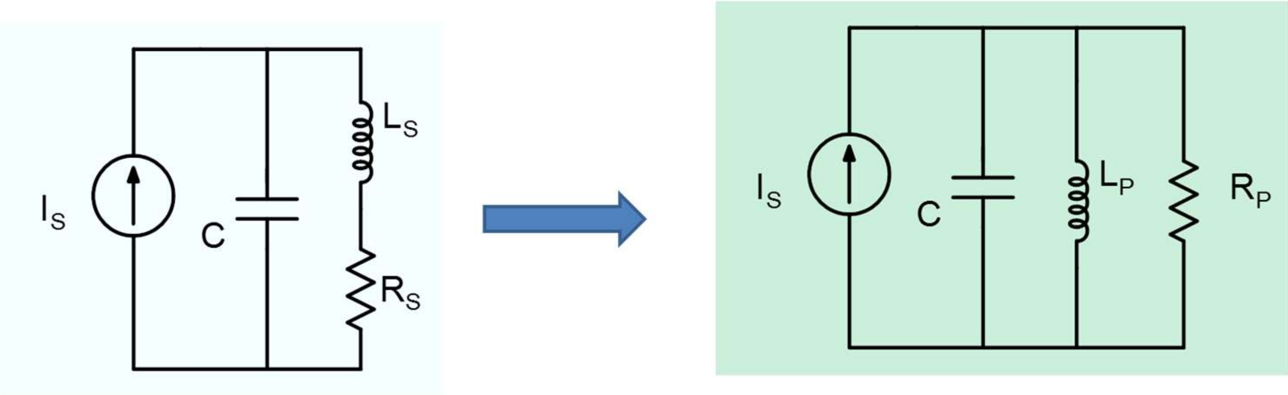
$$R_P = R_S + \frac{\omega^2 L_S^2}{R_S}$$

$$Z_S = R_S + j\omega L_S$$

$$L_P = L_S + \frac{R_S^2}{\omega^2 L_S}$$

$$Y_S = \frac{1}{Z_S} = \frac{1}{R_S + j\omega L_S}$$

$$Y_S = \frac{R_S - j\omega L_S}{R_S^2 + \omega^2 L_S^2} = \frac{R_S}{R_S^2 + \omega^2 L_S^2} + \frac{1}{j\omega} \times \frac{\omega^2 L_S}{R_S^2 + \omega^2 L_S^2} = \frac{1}{R_P} + \frac{1}{j\omega L_P}$$



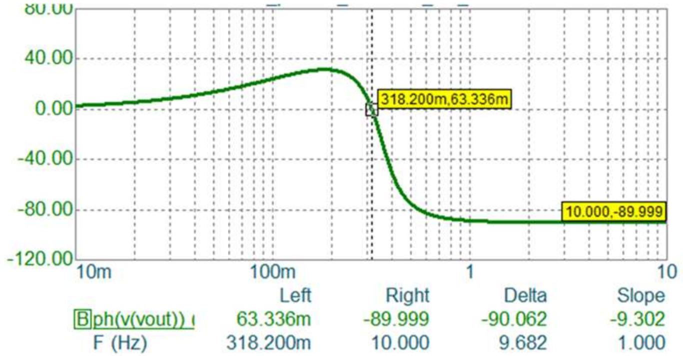
$$R_P = R_S + \frac{\omega^2 L_S^2}{R_S}$$

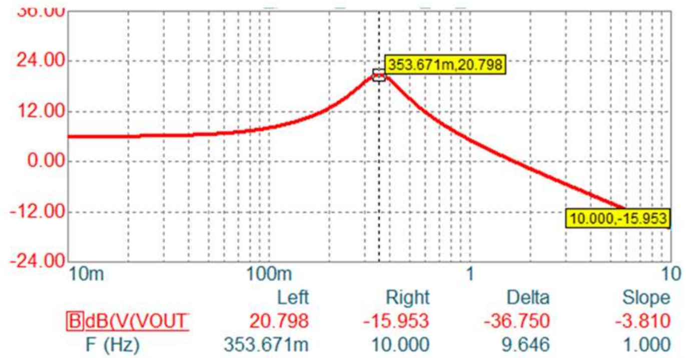
$$L_P = L_S + \frac{R_S^2}{\omega^2 L_S}$$

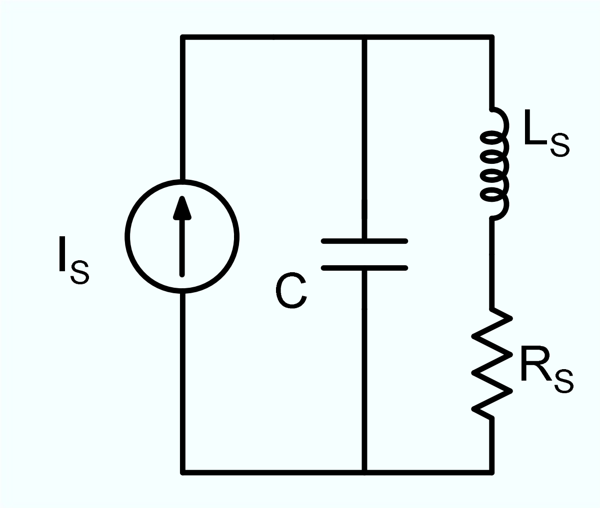
unity power factor frequency : $\omega_P L_P = \dfrac{1}{\omega_P C}$

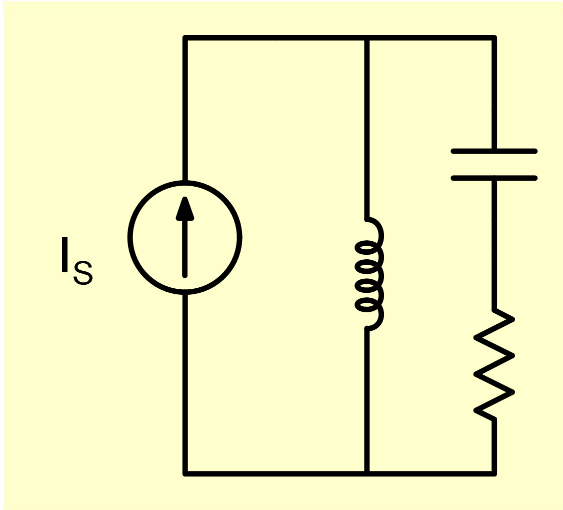
$$Y = \frac{1}{R_P} + j\omega C - \frac{j}{\omega_P L_P}$$

Y_{min} is reached at a higher frequency than f_P

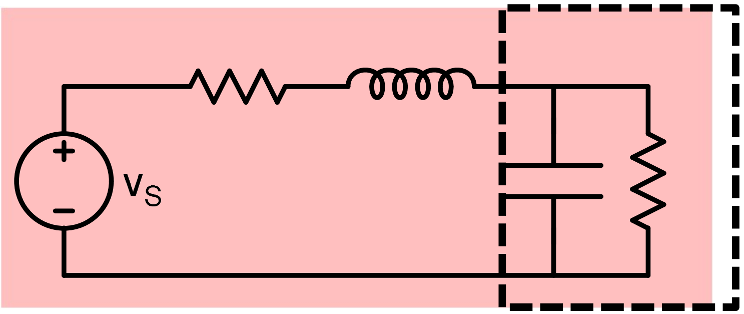


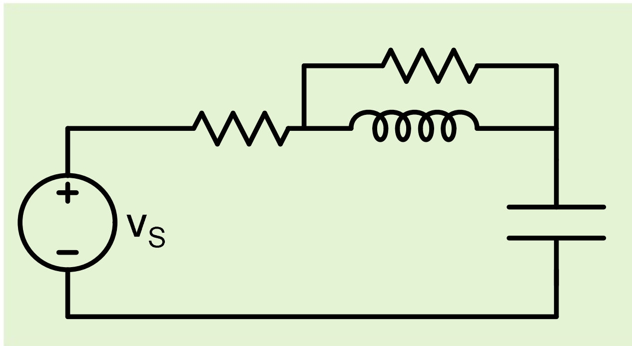






$$Y = R + j\omega C$$





$$Z_S = \frac{1}{Y} = \frac{1}{R + j\omega C} = \frac{R - j\omega C}{R^2 + \omega^2 C^2}$$

Resonance Allows Highly Efficient Implementation Of Highly Selective Filters