ESC201T : Introduction to Electronics

Lecture 14: Sinusoidal Steady State Analysis using Phasors

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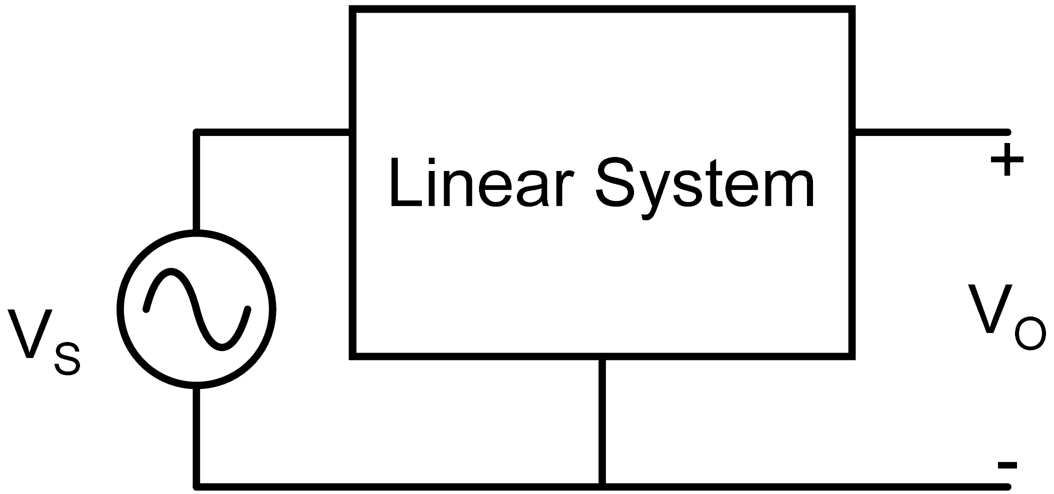
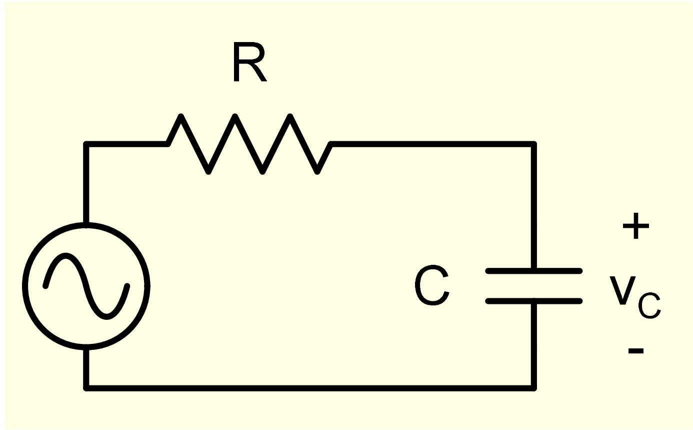
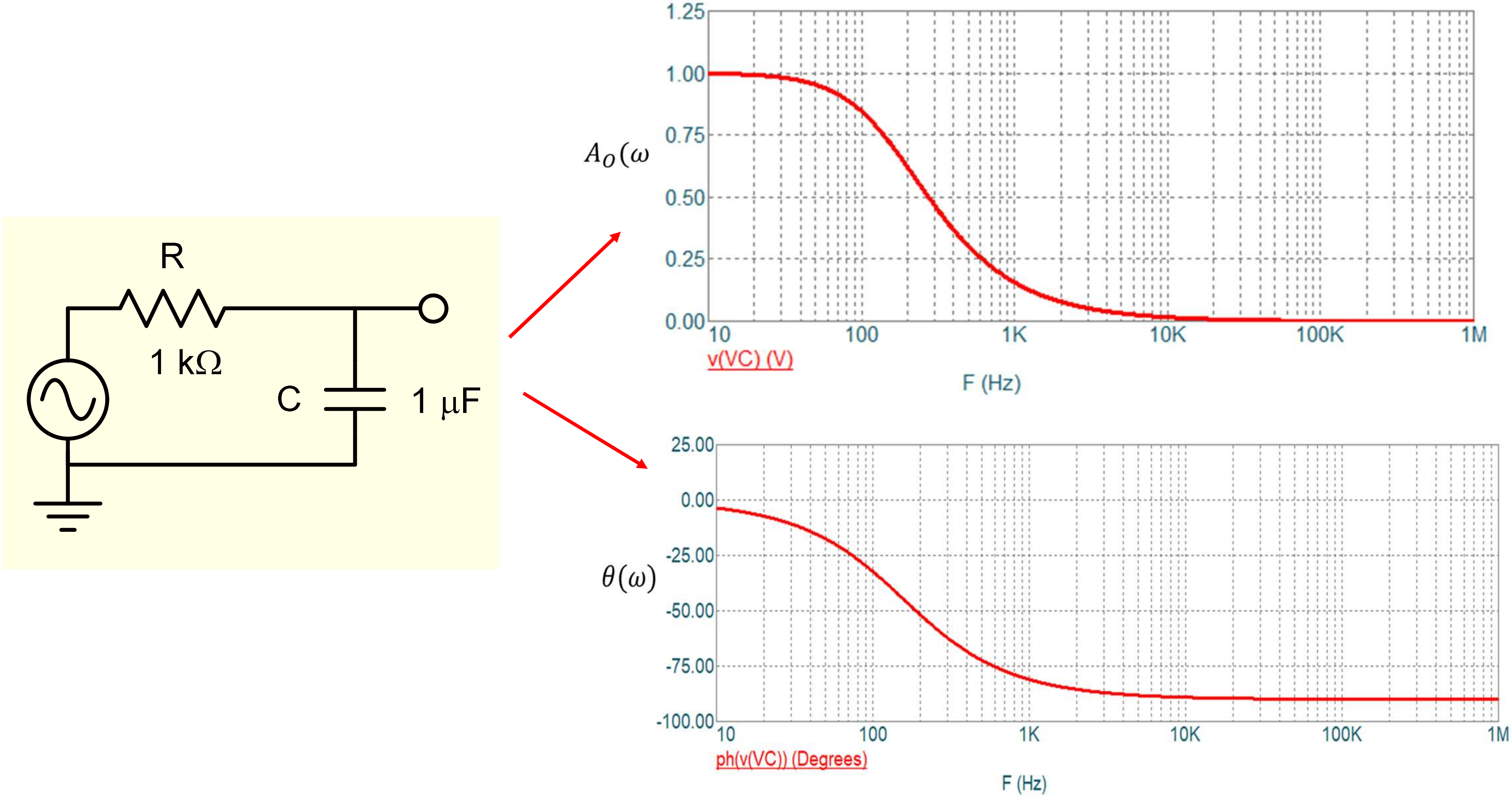


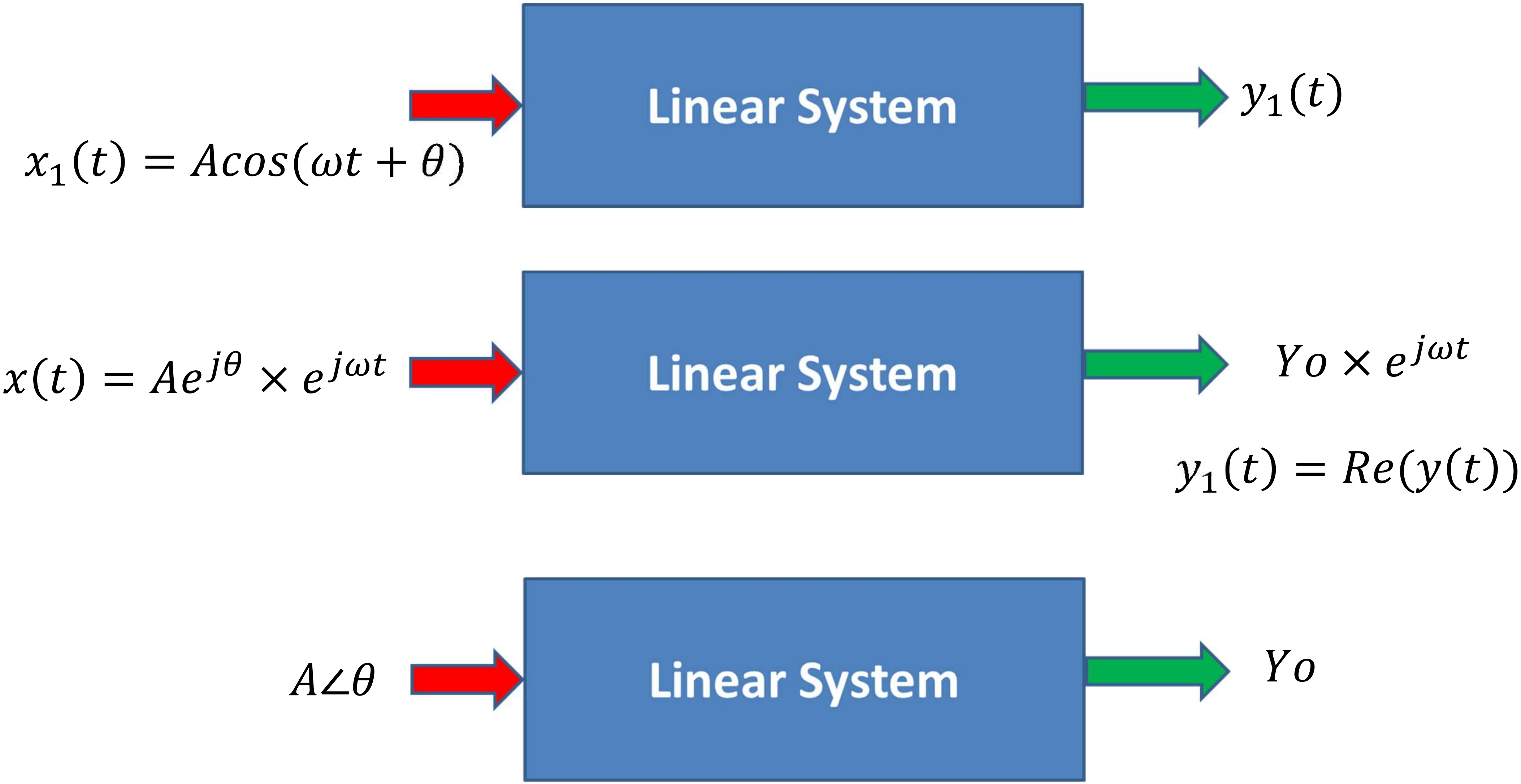


$$V_S = 1 \times \cos(\omega t) \qquad V_O = A_O(\omega) \times \cos(\omega t + \theta(\omega))$$

$A_O(\omega)$ and $\theta(\omega)$ determine the complete characteristics of the system

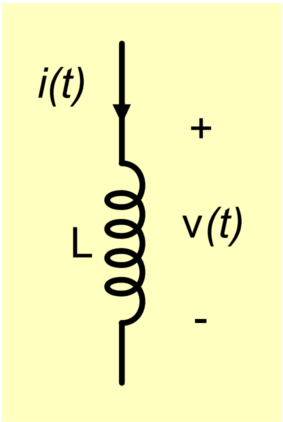


R
1 kΩ
C
1 μF
$A_O(\omega$
1.25
1.00
0.75
0.50
0.25
0.00
10
100
1K
10K
100K
1M
v(VC) (V)
F (Hz)
$\theta(\omega)$
25.00
0.00
-25.00
-50.00
-75.00
-100.00
10
100
1K
10K
100K
1M
ph(v(VC)) (Degrees)
F (Hz)


Linear System
$x_1(t) = Acos(\omega t + \theta)$
$y_1(t)$
Linear System
$x(t) = Ae^{j\theta} \times e^{j\omega t}$
$Yo \times e^{j\omega t}$
$y_1(t) = Re(y(t))$
Linear System
$A\angle\theta$
Yo

Complex Impedances

For the purpose of sinusoidal steady state analysis, circuit elements such as inductors and capacitors can be represented as Complex Impedances



$$I_L = I_m \times e^{j(\omega t+\theta)} \rightarrow I_m\angle\theta$$

$$v_L = L \times \frac{dI_L}{dt} = j\omega L \times I_m \times e^{j(\omega t+\theta)}$$

$$\rightarrow V_L = j\omega L \times I_m \angle\theta$$

$$V_L = \omega L\angle 90^\circ \times I_m \angle\theta$$

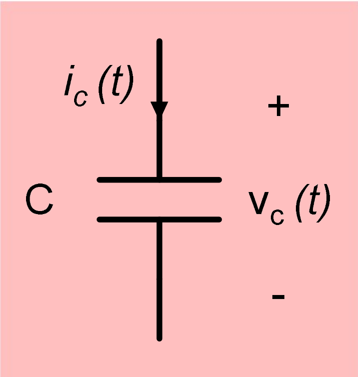
Current through the inductor lags the voltage by 90°

$$V_L = j\omega L \times I_L$$

$$V_L = Z_L \times I_L \qquad Z_L = j\omega L$$

This is like ohms law relationship between phasor voltage and current

Capacitor



$$i_c = C\frac{dv_c}{dt}$$

$$v_C(t) = V_M \times e^{j(\omega t+\ \ )}$$

$$V_C = V_M \angle \theta$$

$$i_C(t) = j\omega C V_M \times e^{j(\omega t+\theta)}$$

$$I_C = j\omega C V_M \angle\theta$$

$$I_C = j\omega C V_M \angle\theta + 90°$$

In a capacitor, current leads voltage by 90^0

$$I_C = j\omega C \times V_M \angle\theta$$

$$V_C = I_C \times Z_C$$

$$Z_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$$

Resistor

$$v_R(t) = V_M \cos(\omega t + \theta)$$

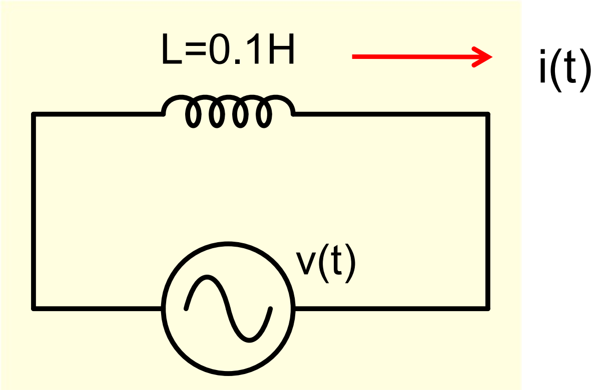
$$i_R(t) = \frac{V_M}{R} \cos(\omega t + \theta)$$

$$V_R = V_M \angle \theta$$

$$I_R = \frac{V_M}{R} \angle \theta$$

$$I_R = \frac{V_R}{R}$$

Example-1



$$v(t) = 2\cos(200t + 45)$$

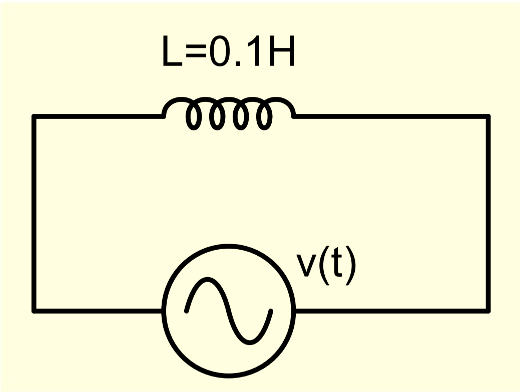
$$\omega = 200$$

$$V_L = 2\angle 45$$

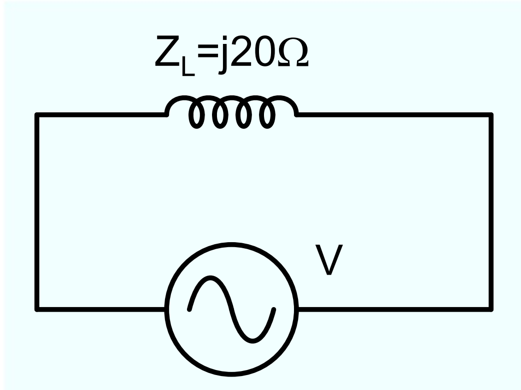
$$V_L = I_L \times j\omega L \Rightarrow I_L = \frac{V_L}{j\omega L}$$

$$I_L = \frac{2\angle 45}{j20} = \frac{2\angle 45}{20\angle 90} = 0.1\angle -45$$

$$i(t) = 0.1\cos(200t - 45)$$



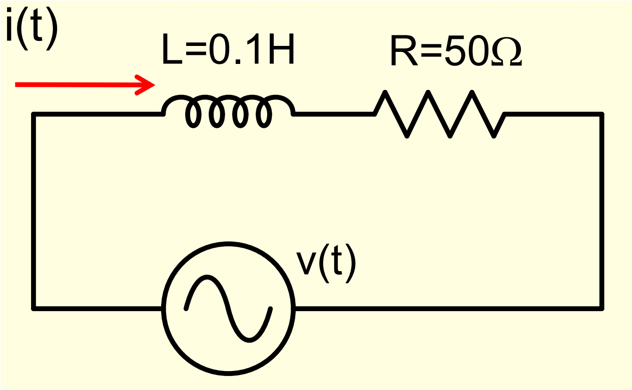
$$Z_L = j\omega L$$



$$I_L = \frac{V_L}{j20}$$

Carry out analysis with phasors keeping in mind that we can always transform phasor to the sinusoidal voltage or current as the case maybe.

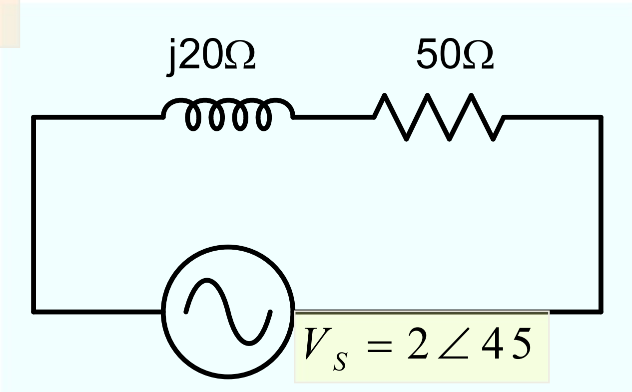
Example-2



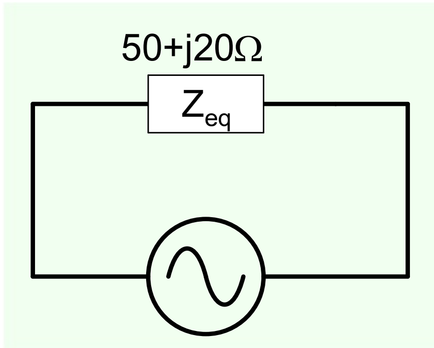
$$v(t) = 2\ \cos(200t + 45)$$

$$\omega = 200$$

$$Z_L = j\omega L$$

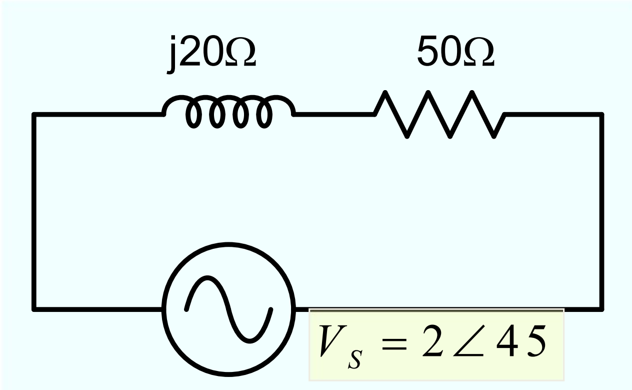


$$V_S = 2\angle 45$$



$$I = \frac{2\angle 45}{50 + j20} = \frac{2\angle 45}{53.85\angle 21.8} = 0.037\angle 23.2$$

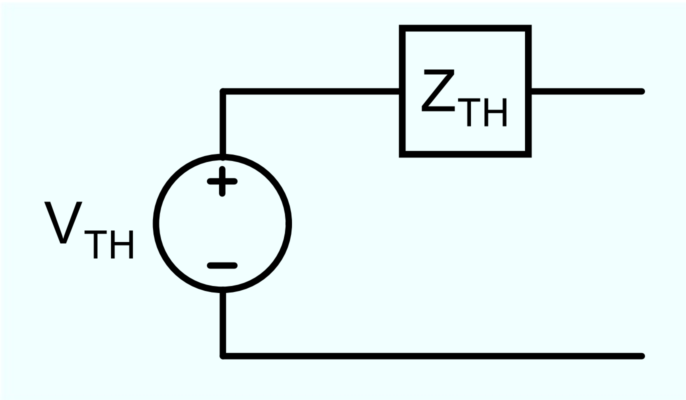
$$i(t) = 0.037\ \cos(200t + 23.2)$$



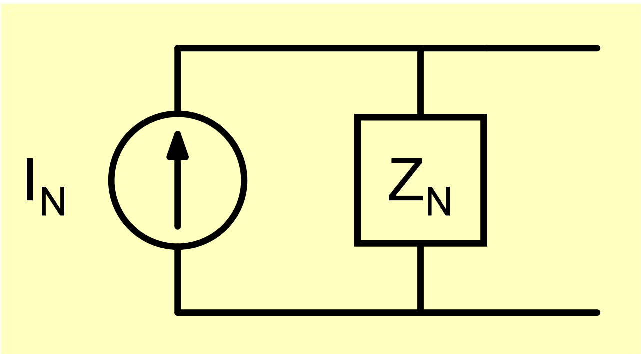
$$V_R = 2\angle 45 \times \frac{50}{50 + j20}$$

Concept of voltage or current division can be used as before

Thevenin and Norton equivalent circuit

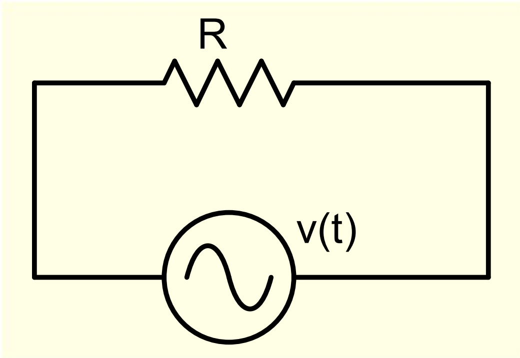


Thevenin voltage is a phasor and Thevenin impedance is in general a complex impedance



Similarly Norton current is a phasor and impedances are complex numbers.

Power dissipation with sinusoidal Voltage



$$p = \frac{v(t)^2}{R}$$

$$p_{avg} = \frac{1}{T}\int_0^T \frac{v(t)^2}{R}\,dt$$

Average

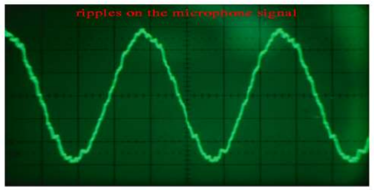
X: $x_1, x_2, x_3, \ldots\ldots\ldots x_N$

$$x_{avg} = \frac{1}{N}\sum x_i$$

If X is continuous,, then its average over a time t_1

$$x_{avg} = \frac{1}{t_1}\int_0^{t_1} x(t)\,dt$$

For periodic signals



$$x_{avg} = \frac{1}{T}\int_0^{T} x(t)\,dt$$

Average Power

$$p_{avg} = \frac{1}{T}\int_0^T \frac{v(t)^2}{R}\,dt$$

We would like to express it like the dc power dissipated in a resistor $p = \dfrac{V^2}{R}$

$$p_{avg} = \frac{\left[\sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}\,\right]^2}{R}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$$

$$p_{avg} = \frac{V_{rms}^2}{R}$$

This is true for any periodic waveform

RMS Value of a Sinusoid

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\, dt}$$

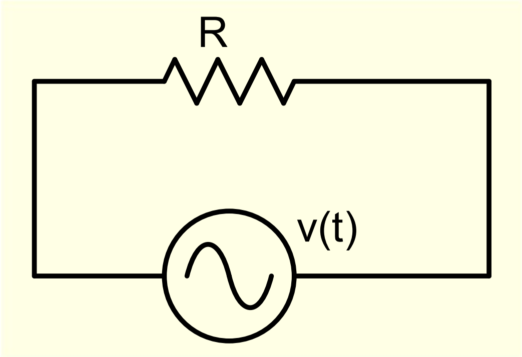
$$v(t) = V_m \cos(\omega t + \theta)$$

$$\int_0^T \cos^2(\omega t + \theta)dt = \int_0^T \frac{1 - \cos(2\omega t + 2\theta)}{2} dt$$

$$= 0.5T - \frac{1}{4\omega}\sin(2\omega t + 2\theta)\Big|_0^T = 0.5T$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

Power dissipation with sinusoidal Voltage



$$v(t) = V_m \cos(\omega t + \theta)$$

$$p_{avg} = \frac{V_{rms}^2}{R}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

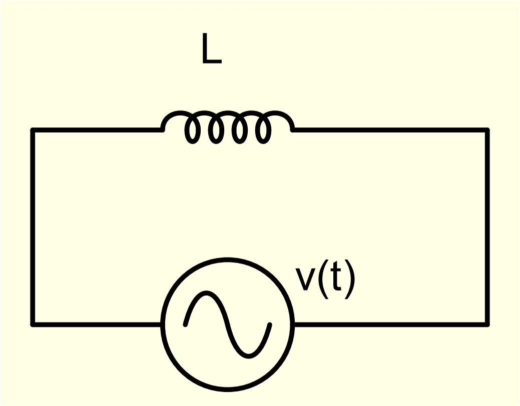
$$p_{avg} = \frac{V_m^2}{2R}$$

$$i(t) = I_m \cos(\omega t + \theta)$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i(t)^2\, dt}$$

$$p_{avg} = 0.5 I_m^2 R$$

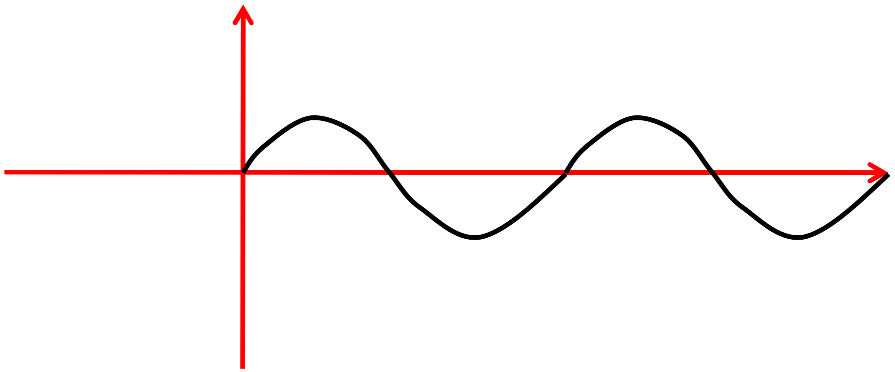


$$v(t) = V_m \cos(\omega t) \qquad i(t) = I_m \cos(\omega t - 90^\circ) = I_m \sin(\omega t)$$

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t)\sin(\omega t)$$

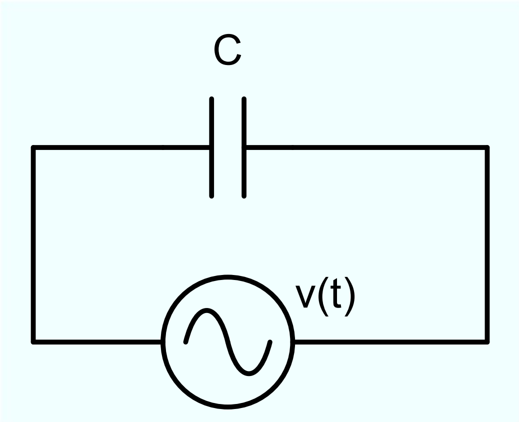
$$= \frac{V_m I_m}{2}\sin(2\omega t) + \frac{V_m I_m}{2}\sin(0)$$

$$= \frac{V_m I_m}{2}\sin(2\omega t)$$



$$p_{avg} = 0$$

- Average power absorbed by an inductor is zero



$$v(t) = V_m \cos(\omega t) \qquad i(t) = I_m \cos(\omega t + 90°) = -I_m \sin(\omega t)$$

$$p(t) = v(t)i(t) = -V_m I_m \cos(\omega t)\sin(\omega t)$$

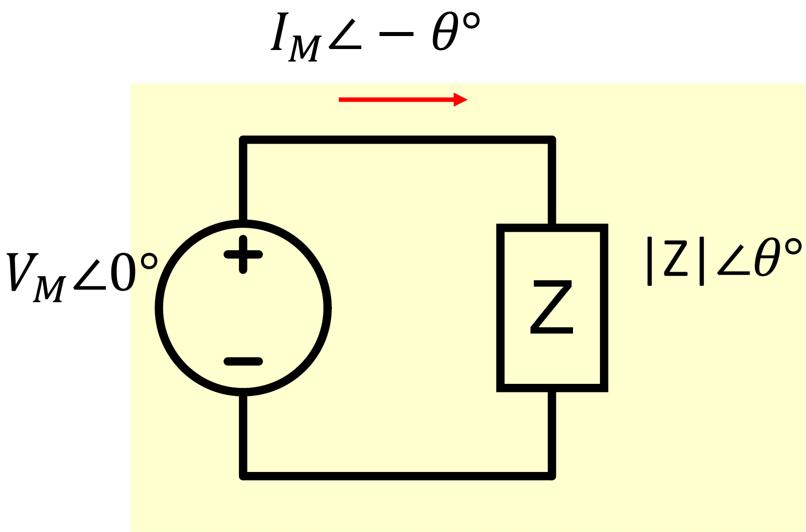
$$= -\frac{V_m I_m}{2}\sin(2\omega t) - \frac{V_m I_m}{2}\sin(0)$$

$$= -\frac{V_m I_m}{2}\sin(2\omega t)$$

$$p_{avg} = 0$$

- Average power absorbed by a capacitor is zero

General Rule



$$v(t) = V_m Cos(\omega t)$$

$$i(t) = I_m Cos(\omega t - \theta)$$

Average Power : $$p = \frac{1}{T}\int_0^T v(t)\times i(t)dt$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos\theta$$

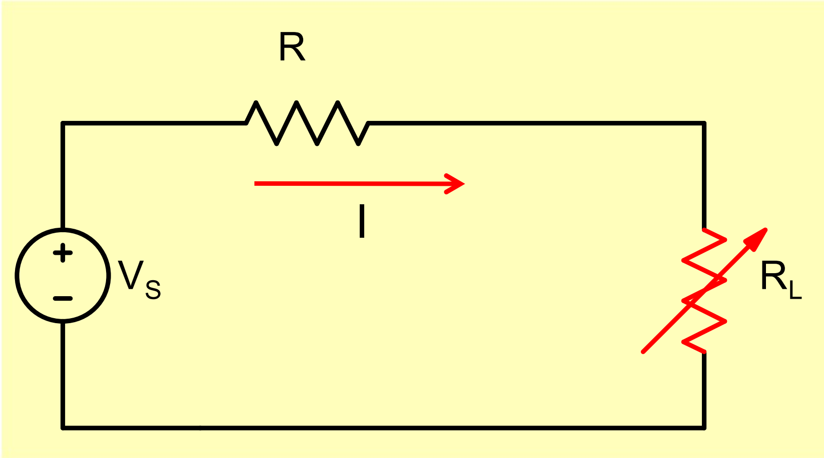
Power Factor $PF = \cos\ \theta$

For a resistor PF = 1, while for inductor and capacitor it is 0

$$j\omega L = \omega L \angle 90 \ ;\ -j\frac{1}{\omega C} = \frac{1}{\omega C}\angle -90$$

Current in phase with voltage gives rise to power dissipation

Maximum Power Transfer for dc circuits



What value of R_L will give rise to maximum load power ?

$$I = \frac{V_S}{R + R_L}$$

$$P_L = I^2 R_L = V_S^2 \times \frac{R_L}{(R + R_L)^2}$$

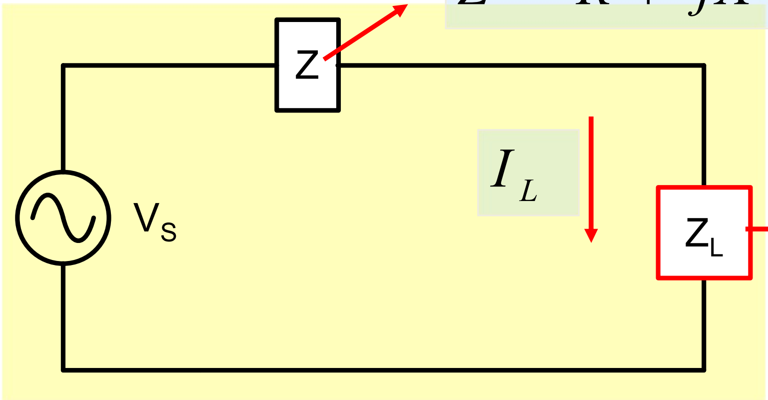
$$\frac{\partial P_L}{\partial R_L} = 0$$

$$R_L = R$$

$$P_{L\max} = \frac{V_S^2}{4 R_L}$$

Maximum Power Transfer for sinusoidal input

$$Z = R + jX$$



$$Z_L = R_L + jX_L$$

$$I_L = \frac{V_S}{R + R_L + j(X + X_L)}$$

$$P_L = \frac{\frac{V_S^2}{2}}{(R + R_L)^2 + (X + X_L)^2} R_L$$

For maximum load power : $X_L = -X$

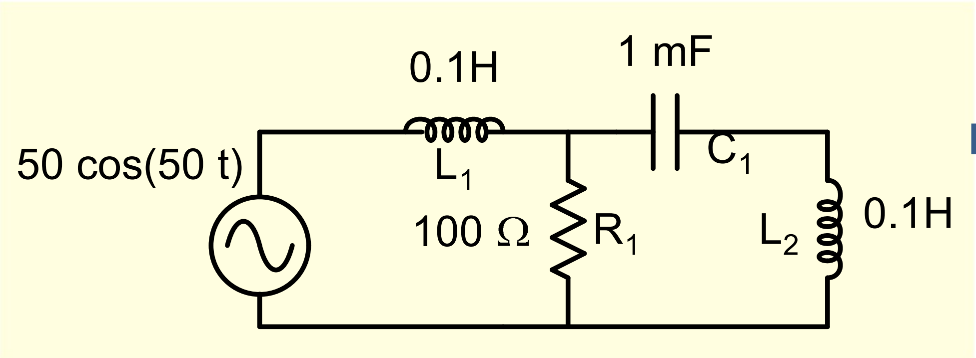
$$P_L = 0.5 \times \frac{V_S^2}{(R + R_L)^2} R_L$$

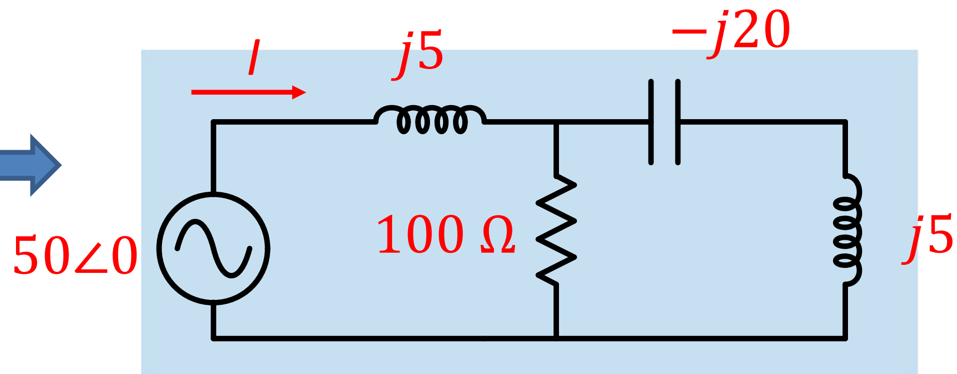
Choose $R_L = R$ to maximize load power

$$Z_L = \overline{Z}$$

Maximum power is transferred to the load when load is complex conjugate of source impedance

Example-3 Determine all you can about the given circuit





$$\omega = 50$$

$$Z = j5 + 100 \| (-j15) = 2.2 - j9.67 \qquad I = \frac{50}{Z} = 1.12 + j4.96 = 5\angle 77.2$$

$$P_S = \frac{50}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times \cos(77.2) = 27.97W$$

Voltage across R1 ?

$$V_{R1} = I \times \left(100 \| (-j15)\right) = 74.8\angle -4.3$$

$$\frac{V_{R1}^2}{2R_1} = 27.97W$$