CS201B: Endsem Examination

December 19, 2020

Submission Deadline: 12:00 hrs; December 22, 2020 **Maximum Marks**: 50

Question 1. (10 marks) Define two classes of n-variate polynomials as:

$$P_d(x_1, x_2, \ldots, x_n) = \sum_{\substack{0 \le i_1, i_2, \ldots, i_n \le 1 \\ i_1 + i_2 + \cdots + i_n = d}} x_1^{i_1} x_2^{i_2} \cdots x_n^{i_n}$$

$$Q_d(x_1, x_2, \ldots, x_n) = \sum_{\substack{0 \le i_1, i_2, \ldots, i_n \le d \\ i_1 + i_2 + \cdots + i_n = d}} x_1^{i_1} x_2^{i_2} \cdots x_n^{i_n}$$

for all $d \geq 0$. Prove that:

$$\sum_{0 \le d \le r} (-1)^d P_d \cdot Q_{r-d} = 0$$

for all $r \geq 1$.

Question 2. (5+10 marks) The algorithm in Lecture 18 for finding a perfect matching is wrong. Find a counter example, i.e., a bipartite graph on which the algorithm fails. Fix the algorithm by suitably modifying the definition of subgraph H.

Question 3. (5 marks) Let G be connected graph on $n \geq 4$ vertices with $2n-2$ edges. Prove that G has two cycles of equal length.

Question 4. (5+5 marks) A *completed Sudoku puzzle* is a 9×9 grid filled in with numbers 1 to 9 according to the rules of Sudoku. We say two such puzzles are the same if one can be obtained from other by any of the following operations and their compositions:

- Rotation by 90, 180, and 270 degrees
- Flips along vertical, horizontal, and diagonal axes
- Rotation by 180 degree of each of the 3×3 subgrid simultaneously

Describe the subgroup made up of above three operations. Assuming total number of completed puzzles to be N, calculate the number of distinct completed puzzles.

Question 5. (10 marks) Let $(G, \cdot)$ be a group. A *proper* subgroup of G is a subgroup which is a proper subset of G. H is a *maximal* subgroup of G if H is a proper subgroup of G and there is no other proper subgroup H' such that $H \subset H'$.

Prove that G has a maximal subgroup.