Exergy Balance Over Control Volume

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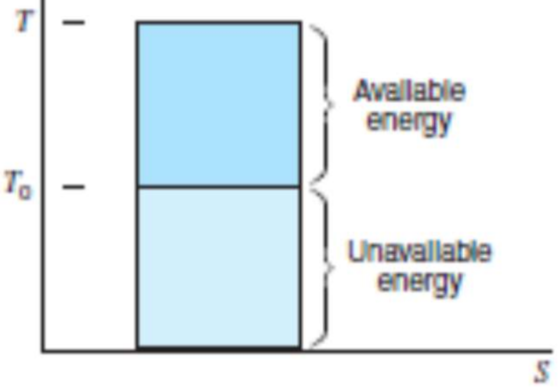
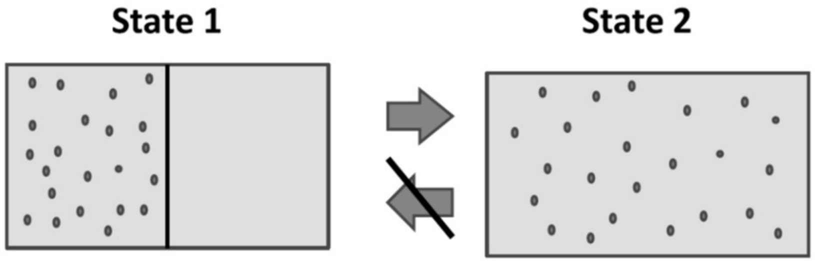
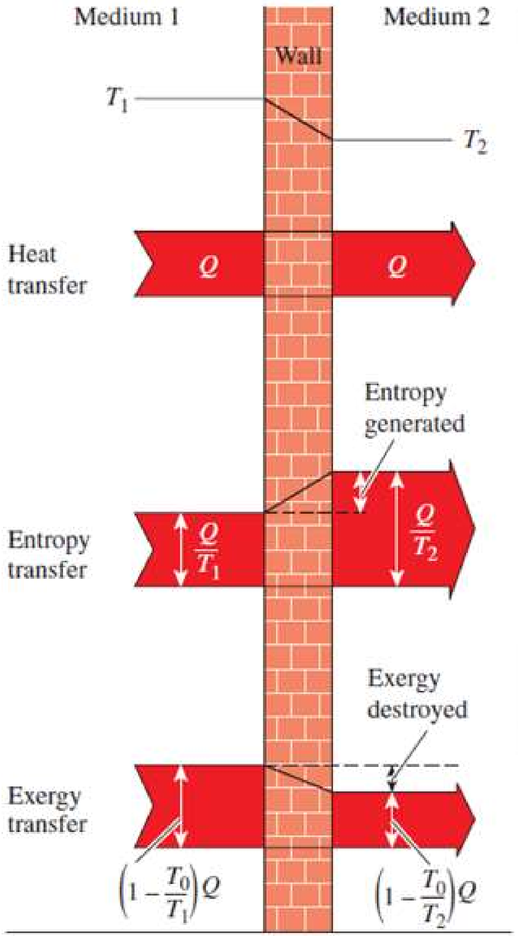
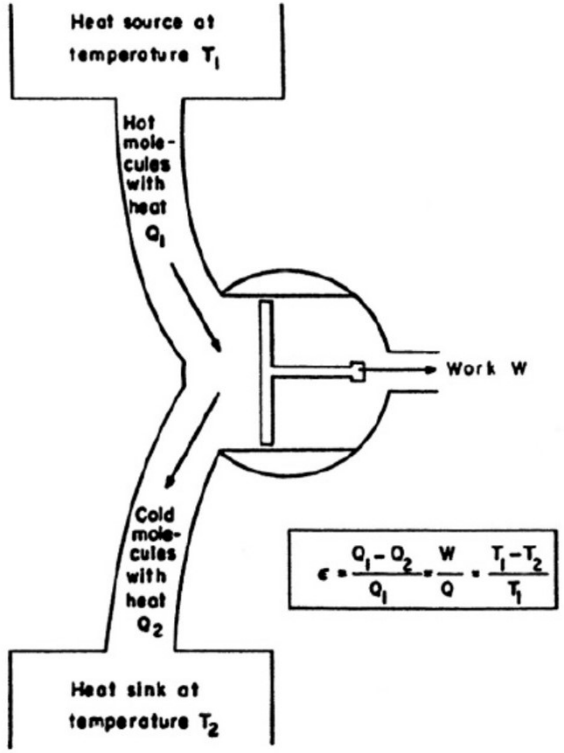
Previously: Exergy Destruction & Balance

$$W_{\text{rev HE}} = Q - Q_0$$





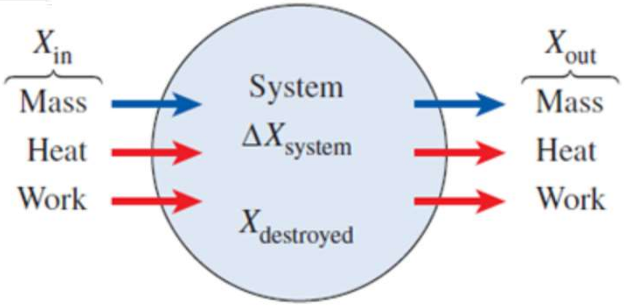




$$X_{\text{destroyed}} = T_0 S_{\text{gen}} \geq 0$$

$$X_{\text{destroyed}} \begin{cases} > 0 & \text{Irreversible process} \\ = 0 & \text{Reversible process} \\ < 0 & \text{Impossible process} \end{cases}$$

$$\underbrace{X_{\text{in}} - X_{\text{out}}}_{\substack{\text{Net exergy transfer} \\ \text{by heat, work, and mass}}} - \underbrace{X_{\text{destroyed}}}_{\substack{\text{Exergy} \\ \text{destruction}}} = \underbrace{\Delta X_{\text{system}}}_{\substack{\text{Change} \\ \text{in exergy}}}$$



$$\dot{X}_{\text{heat}} = (1 - T_0/T)\dot{Q},\ \dot{X}_{\text{work}} = \dot{W}_{\text{useful}},\ \text{and}\ \dot{X}_{\text{mass}} = \dot{m}\psi$$

Figs-TD-Borgnakke & Sonntag; Cengel & Boles Modern Electrochemistry 2B, Bockris & Reddy

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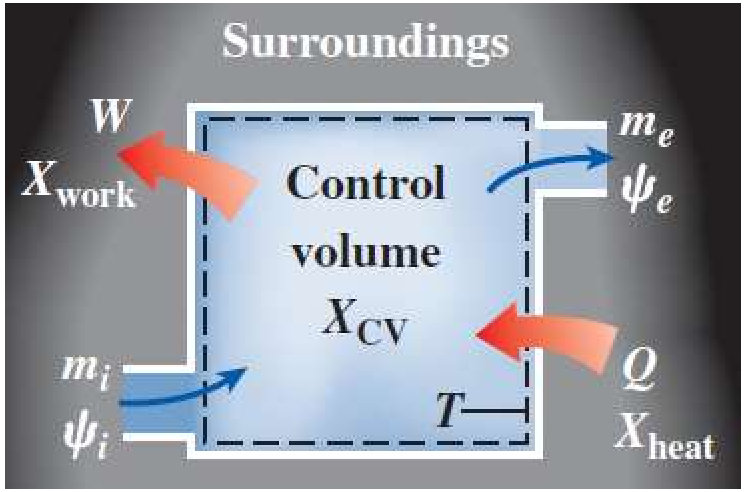


$$X_{\text{heat}} - X_{\text{work}} + X_{\text{mass,in}} - X_{\text{mass,out}} - X_{\text{destroyed}} = (X_2 - X_1)_{\text{CV}}$$

$$\sum\left(1 - \frac{T_0}{T_k}\right)Q_k - [W - P_0(V_2 - V_1)] + \sum_{\text{in}} m\psi - \sum_{\text{out}} m\psi - X_{\text{destroyed}} = (X_2 - X_1)_{\text{CV}}$$

$$\sum\left(1 - \frac{T_0}{T_k}\right)\dot{Q}_k - \left(\dot{W} - P_0\frac{dV_{\text{CV}}}{dt}\right) + \sum_{\text{in}} \dot{m}\psi - \sum_{\text{out}} \dot{m}\psi - \dot{X}_{\text{destroyed}} = \frac{dX_{\text{CV}}}{dt}$$

Figs-TD: Cengel & Boles

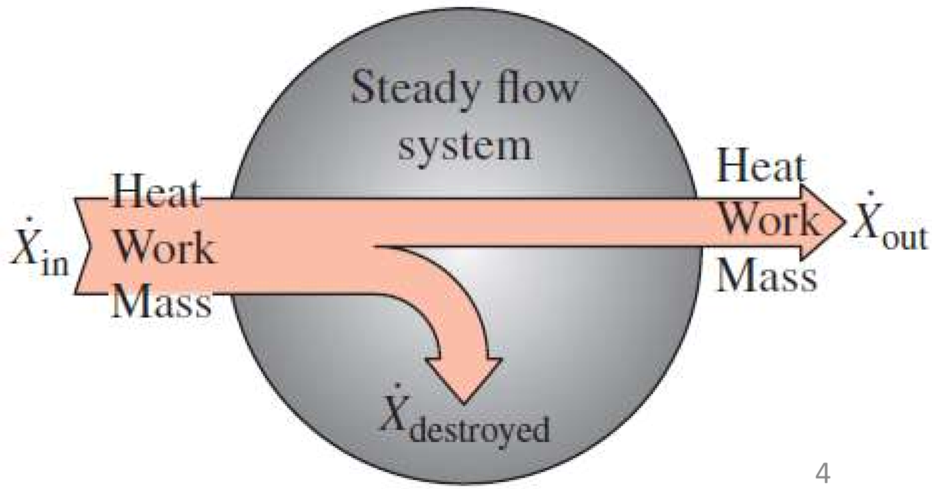
Exergy Balance For Steady Flow Systems

Steady-flow: $$\sum\left(1-\frac{T_0}{T_k}\right)\dot{Q}_k-\dot{W}+\sum_{\text{in}}\dot{m}\psi-\sum_{\text{out}}\dot{m}\psi-\dot{X}_{\text{destroyed}}=0$$

Single-stream: $$\sum\left(1-\frac{T_0}{T_k}\right)\dot{Q}_k-\dot{W}+\dot{m}(\psi_1-\psi_2)-\dot{X}_{\text{destroyed}}=0$$

$$\psi_1-\psi_2=(h_1-h_2)-T_0(s_1-s_2)+\frac{V_1^2-V_2^2}{2}+g(z_1-z_2)$$

Per-unit mass: $$\sum\left(1-\frac{T_0}{T_k}\right)q_k-w+(\psi_1-\psi_2)-x_{\text{destroyed}}=0$$



Figs-TD: Cengel & Boles

Reversible Work

Single-stream: $$\sum\left(1-\frac{T_0}{T_k}\right)\dot{Q}_k-\dot{W}+\dot{m}(\psi_1-\psi_2)-\dot{X}_{\text{destroyed}}=0$$

General: $$W=W_{\text{rev}} \qquad \text{when } X_{\text{destroyed}}=0$$

Single stream: $$\dot{W}_{\text{rev}}=\dot{m}(\psi_1-\psi_2)+\sum\left(1-\frac{T_0}{T_k}\right)\dot{Q}_k \qquad \text{(kW)}$$

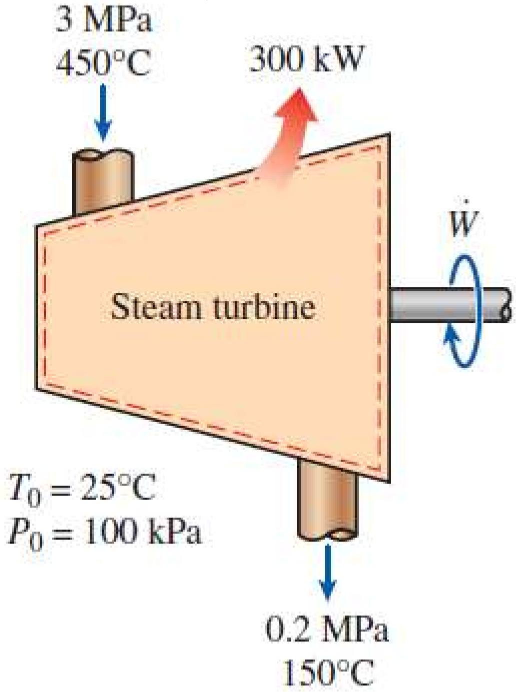
Adiabatic, single stream: $$\dot{W}_{\text{rev}}=\dot{m}(\psi_1-\psi_2)$$

Figs-TD: Cengel & Boles

Second law efficiency for turbines

$$\eta_{\text{II,turb}} = \frac{w}{w_{\text{rev}}} = \frac{h_1 - h_2}{\psi_1 - \psi_2} \quad \text{or} \quad \eta_{\text{II,turb}} = 1 - \frac{T_0 s_{\text{gen}}}{\psi_1 - \psi_2}$$

(Adiabatic turbine)



$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt^{\nearrow 0\,\text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}h_1 = \dot{W}_{\text{out}} + \dot{Q}_{\text{out}} + \dot{m}h_2 \qquad (\text{since ke} \cong \text{pe} \cong 0)$$

$$\dot{W}_{\text{out}} = \dot{m}(h_1 - h_2) - \dot{Q}_{\text{out}}$$

$$\underbrace{\dot{X}_{\text{in}} - \dot{X}_{\text{out}}}_{\substack{\text{Rate of net exergy transfer}\\ \text{by heat, work, and mass}}} - \underbrace{\dot{X}_{\text{destroyed}}^{\nearrow 0\,\text{(reversible)}}}_{\substack{\text{Rate of exergy}\\ \text{destruction}}} = \underbrace{dX_{\text{system}}/dt^{\nearrow 0\,\text{(steady)}}}_{\substack{\text{Rate of change}\\ \text{in exergy}}} = 0$$

$$\dot{X}_{\text{in}} = \dot{X}_{\text{out}}$$

$$\dot{m}\psi_1 = \dot{W}_{\text{rev,out}} + \dot{X}_{\text{heat}}^{\nearrow 0} + \dot{m}\psi_2$$

$$\dot{W}_{\text{rev,out}} = \dot{m}(\psi_1 - \psi_2)$$

$$= \dot{m}[(h_1 - h_2) - T_0(s_1 - s_2) - \Delta \text{ke}^{\nearrow 0} - \Delta \text{pe}^{\nearrow 0}]$$

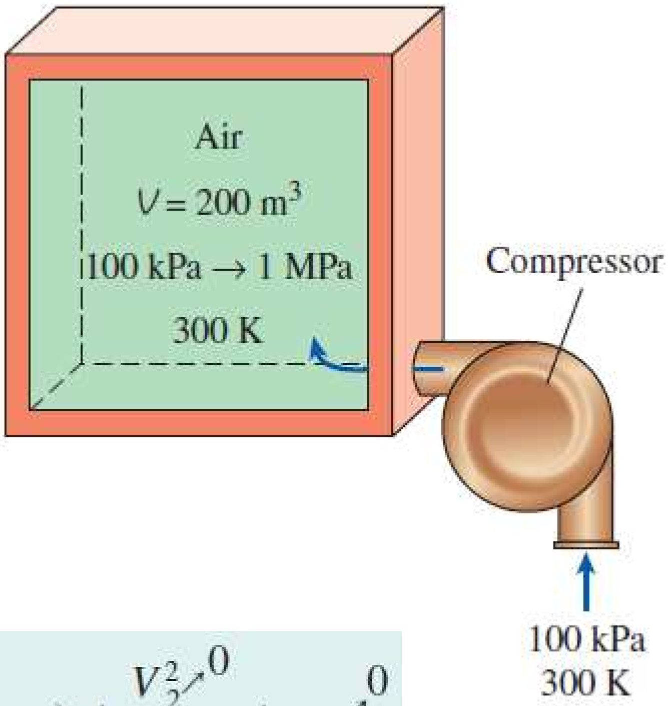
$$\eta_{\text{II}} = \frac{\dot{W}_{\text{out}}}{\dot{W}_{\text{rev,out}}}$$

$$\dot{X}_{\text{destroyed}} = \dot{W}_{\text{rev,out}} - \dot{W}_{\text{out}}$$

Figs-TD: Cengel & Boles

Second law efficiency for compressor

$$\eta_{\text{II,comp}} = \frac{w_{\text{rev,in}}}{w_{\text{in}}} = \frac{\psi_2 - \psi_1}{h_2 - h_1} \quad \text{or} \quad \eta_{\text{II,comp}} = 1 - \frac{T_0 s_{\text{gen}}}{h_2 - h_1}$$



$$\underbrace{X_{\text{in}} - X_{\text{out}}}_{\substack{\text{Net exergy transfer}\\ \text{by heat, work, and mass}}} \underbrace{- X_{\text{destroyed}}^{\nearrow 0\,(\text{reversible})}}_{\substack{\text{Exergy}\\ \text{destruction}}} = \underbrace{\Delta X_{\text{system}}}_{\substack{\text{Change}\\ \text{in exergy}}}$$

$$X_{\text{in}} - X_{\text{out}} = X_2 - X_1$$

$$W_{\text{rev,in}} + m_1\psi_1^{\nearrow 0} = m_2\phi_2 - m_1\phi_1^{\nearrow 0}$$

$$W_{\text{rev,in}} = m_2\phi_2$$

$$\phi_2 = (u_2 - u_0)^{\nearrow 0\,(\text{since } T_2 = T_0)} + P_0(v_2 - v_0) - T_0(s_2 - s_0) + \frac{V_2^2}{2}^{\nearrow 0} + gz_2^{\nearrow 0}$$
$$= P_0(v_2 - v_0) - T_0(s_2 - s_0)$$

$$P_0(v_2 - v_0) = P_0\left(\frac{RT_2}{P_2} - \frac{RT_0}{P_0}\right) = RT_0\left(\frac{P_0}{P_2} - 1\right) \quad (\text{since } T_2 = T_0)$$

$$T_0(s_2 - s_0) = T_0\left(c_p \ln\frac{T_2}{T_0}^{\nearrow 0} - R\ln\frac{P_2}{P_0}\right) = -RT_0 \ln\frac{P_2}{P_0} \quad (\text{since } T_2 = T_0)$$

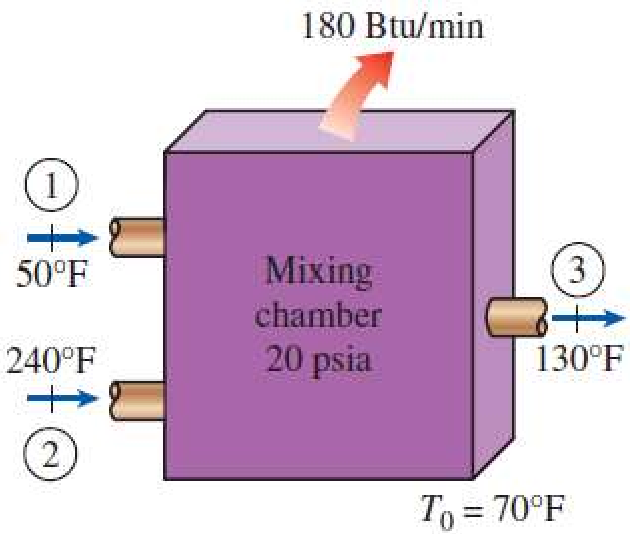
Figs-TD: Cengel & Boles

Second law efficiency for other flow devices

$$\eta_{\text{II,mix}} = \frac{\dot{m}_3\psi_3}{\dot{m}_1\psi_1 + \dot{m}_2\psi_2}$$

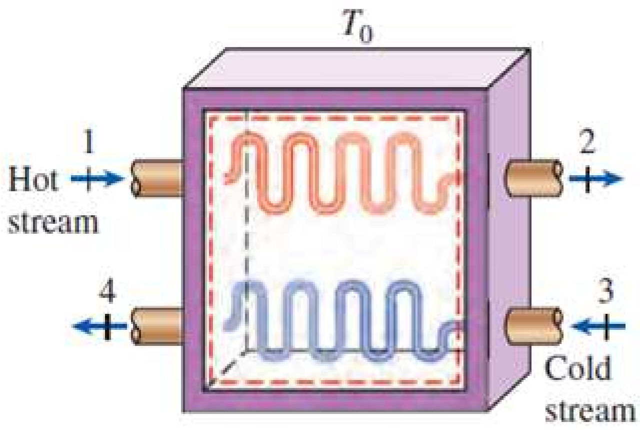
$$\eta_{\text{II,mix}} = 1 - \frac{T_0\dot{S}_{\text{gen}}}{\dot{m}_1\psi_1 + \dot{m}_2\psi_2}$$

$$\dot{S}_{\text{gen}} = \dot{m}_3 s_3 - \dot{m}_2 s_2 - \dot{m}_1 s_1$$



$$\eta_{\text{II,HX}} = \frac{\dot{m}_{\text{cold}}(\psi_4 - \psi_3)}{\dot{m}_{\text{hot}}(\psi_1 - \psi_2)}$$

$$\eta_{\text{II,HX}} = 1 - \frac{T_0\dot{S}_{\text{gen}}}{\dot{m}_{\text{hot}}(\psi_1 - \psi_2)}$$



Figs-TD: Cengel & Boles

What's next?

- Thermodynamic property relationships