ESC201T : Introduction to Electronics

Discussion-7 : 04/11/20

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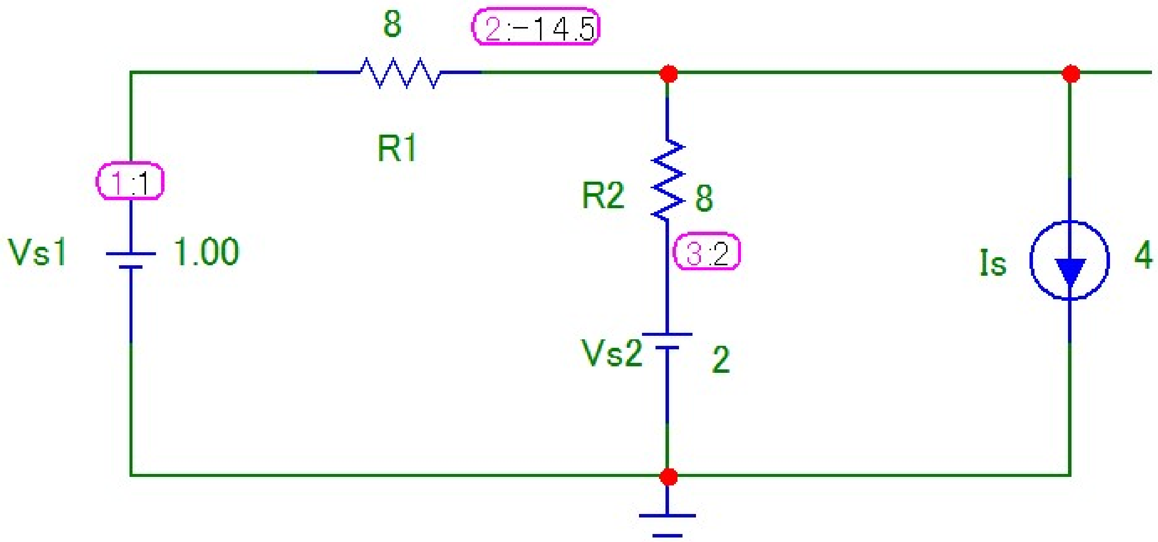
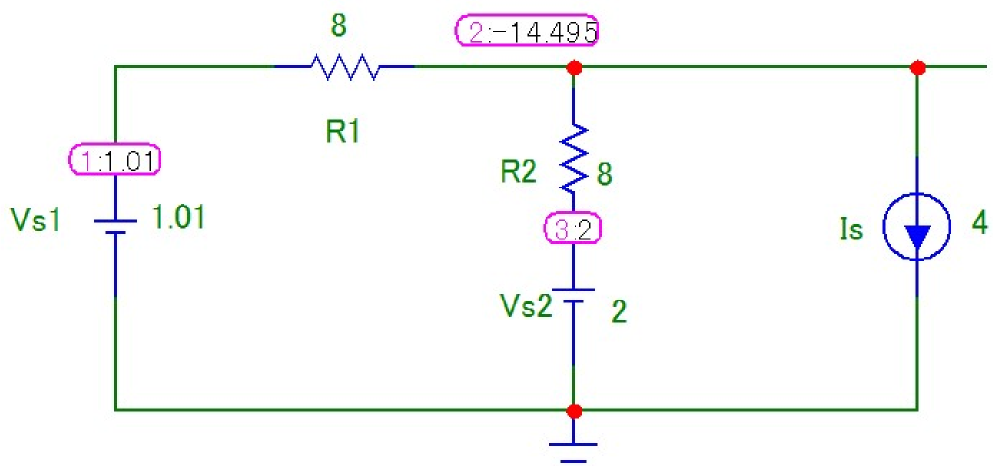
Announcements

Discussion Quiz from 6.45-6.55 pm
Topic : Diode Circuit

Major Quiz-3 on 11/11/2020 during Discussion hour.
Topic : 2 port parameters, semiconductors, diode circuits till Lecture 23
Non-Objective Type

Major Quiz-4 on 27/11/2020 during Tutorial hour.
Topic : Amplifiers and Digital Circuits
Objective type

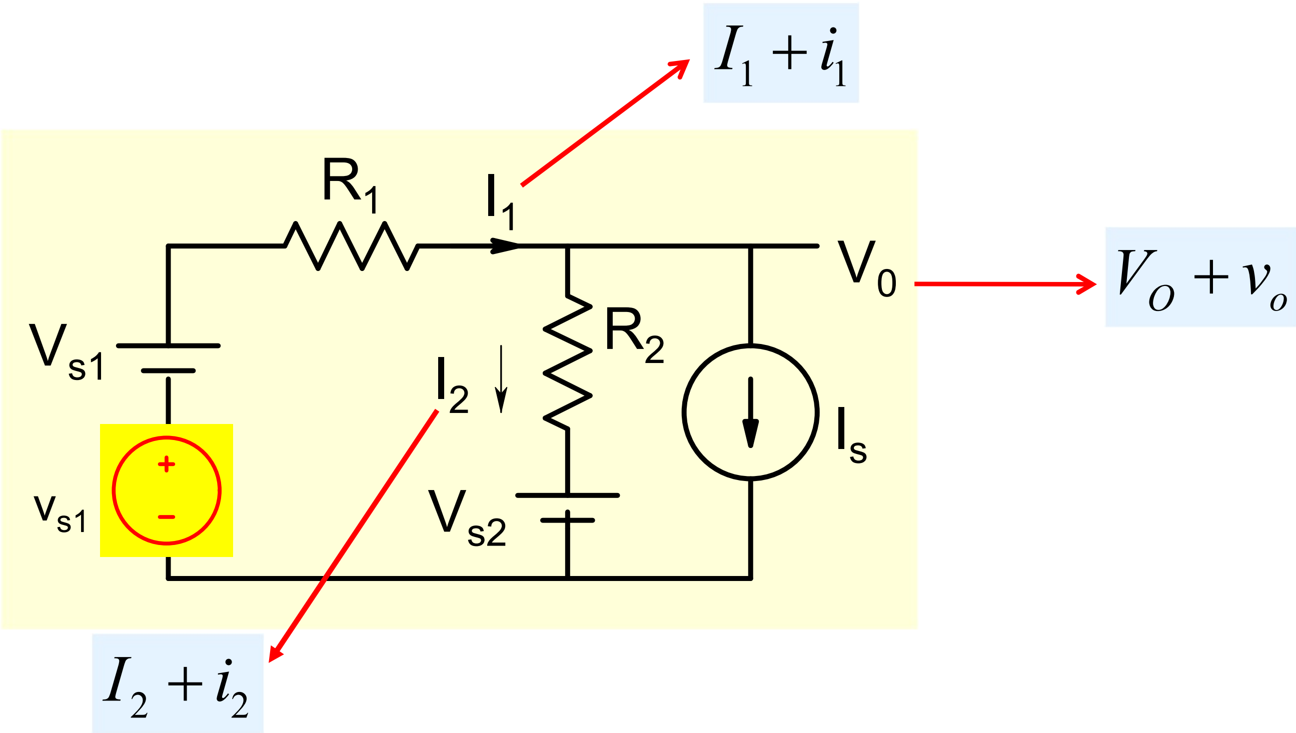




ΔV_0=5mV

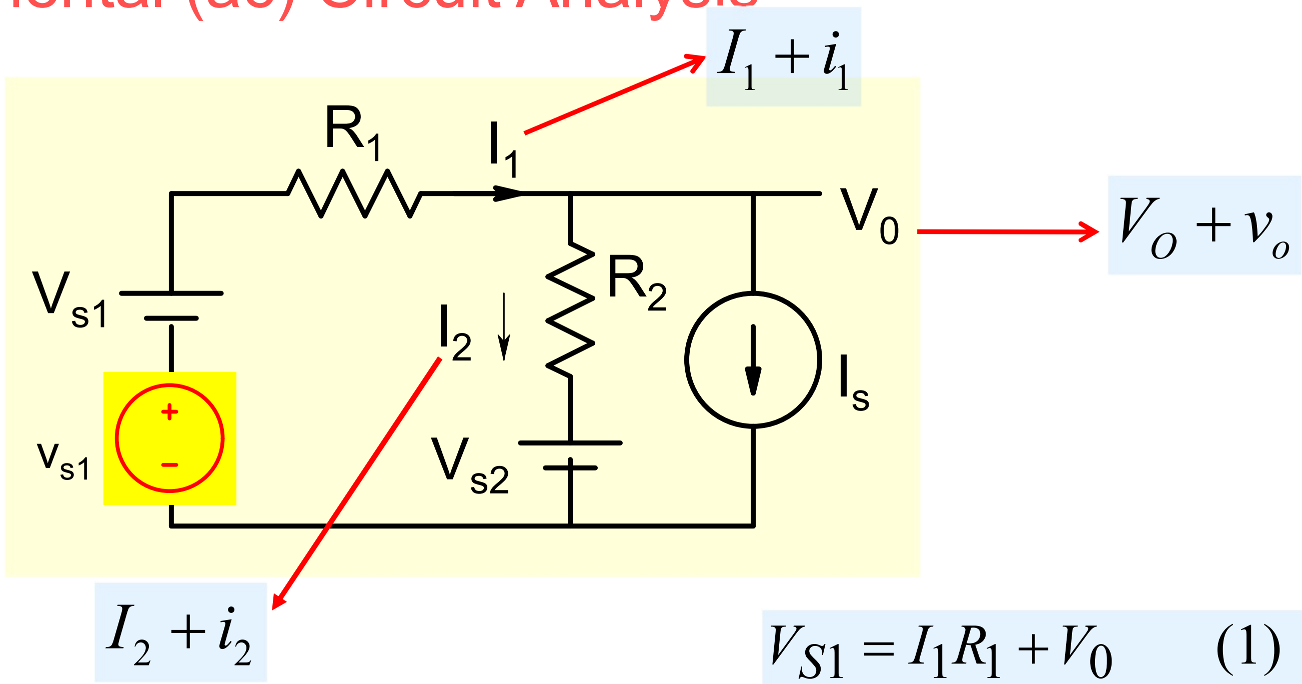
G-Number

Incremental (ac) Circuit Analysis



Incremental circuit analysis attempts to find the relationships between incremental voltages and currents v_{s1}, i_1, v_o…….

Incremental (ac) Circuit Analysis



$$V_{S1} = I_1 R_1 + V_0 \qquad (1)$$

$$I_1 = I_S + (V_0 - V_{S2})/R_2 \qquad (2)$$

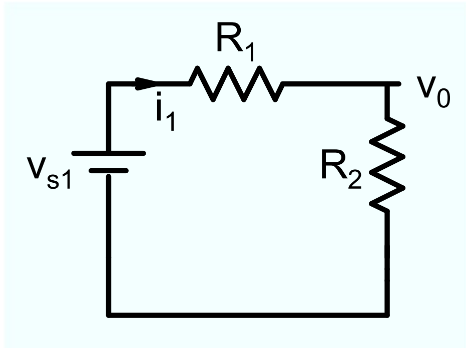
$$V_{S1} + v_{s1} = (I_1 + i_1) \times R_1 + V_0 + v_o \qquad (3)$$

$$I_1 + i_1 = I_S + (V_0 + v_o - V_{S2})/R_2 \qquad (4)$$

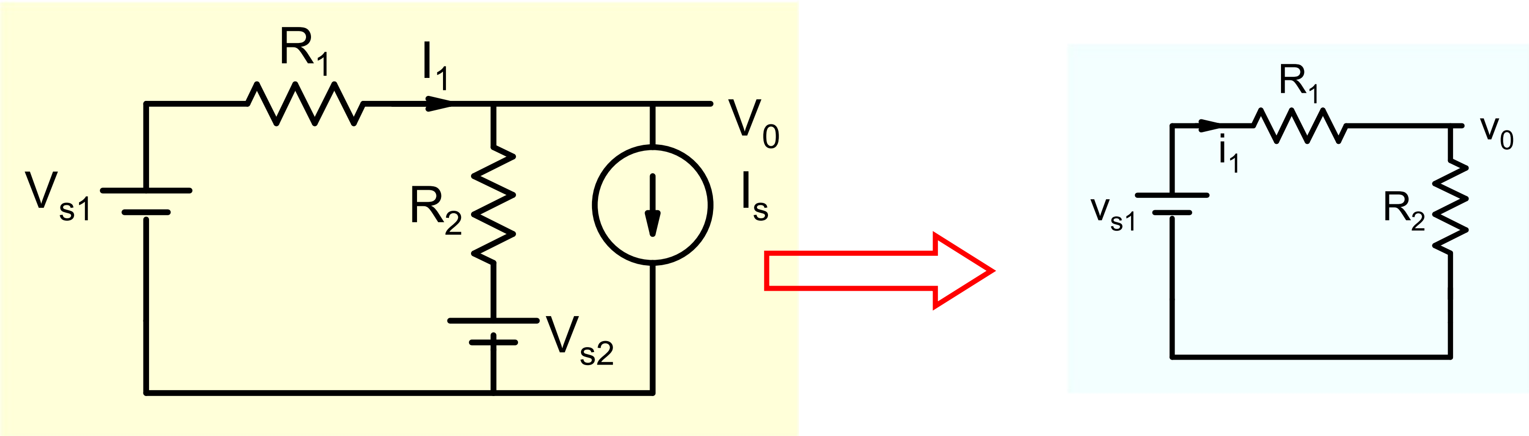
$$\text{(3)-(1)}: v_{s1} = i_1 R_1 + v_o$$

$$\text{(4)-(2)}: i_1 = v_o / R_2$$

$$v_{s1} = i_1 R_1 + i_1 R_2$$



Method: Incremental equivalent circuit

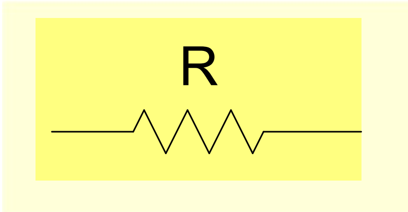


Analyze incremental equivalent circuit obtained by replacing each circuit element by its increment circuit model (sometimes called ac model).

Incremental model

$I = f(V)$ $I + i = f(V + v)$ Relation between *i and v*

Incremental (ac) Models: Resistor



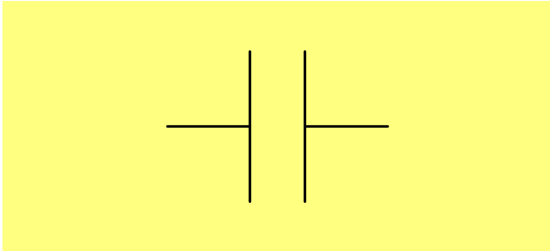
$$V = I \times R$$

$$V + v = (I + i) \times R$$

$$\Rightarrow \quad v = i \times R$$

Incremental model of a resistor is a resistor of the same magnitude

Incremental (ac) Models: Capacitor



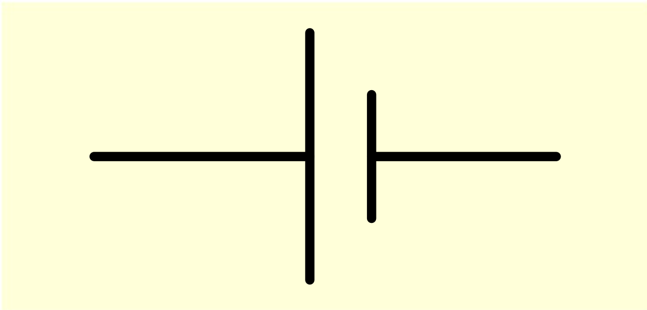
$$I = C\frac{dV}{dt}$$

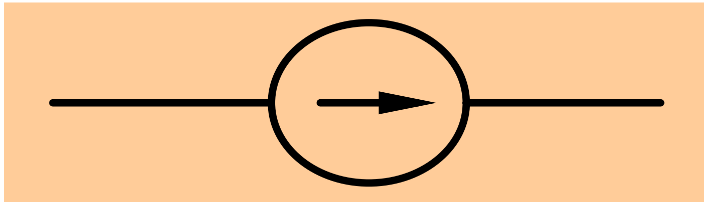
$$I + i = C\frac{d(V + v)}{dt}$$

$$i = C \times \frac{dv}{dt}$$

Incremental model of a capacitor is a capacitor of the same magnitude. The same holds for an inductor as well.

Incremental (ac)   Models





$$V = cons\tan t$$

$$V + v = cons\tan t$$

$$\Rightarrow v = 0$$

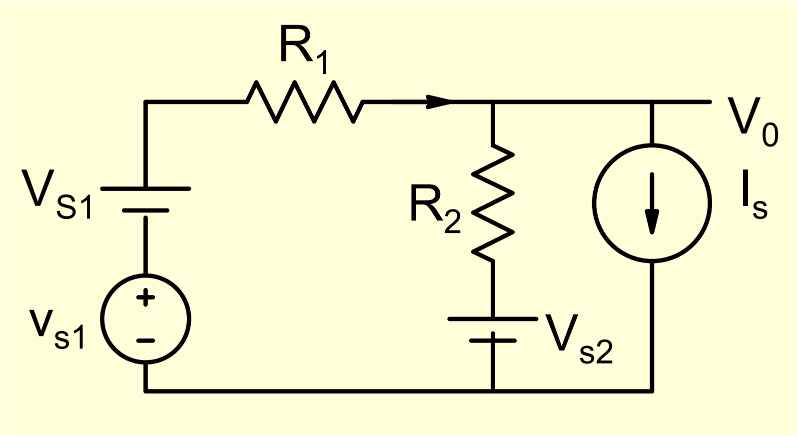
$$I = cons\tan t$$

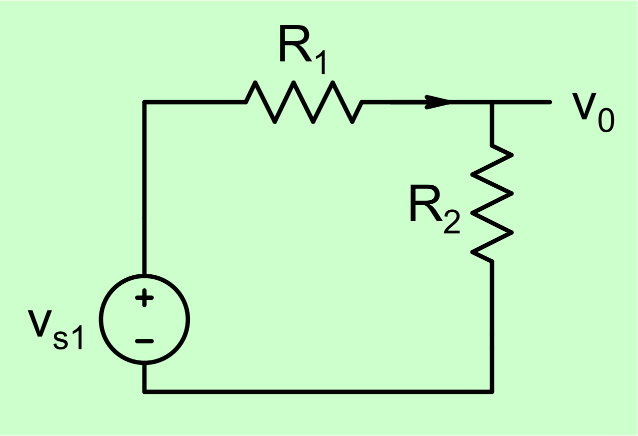
$$\Rightarrow i = 0$$

Incremental model of a constant Voltage Source is a short circuit

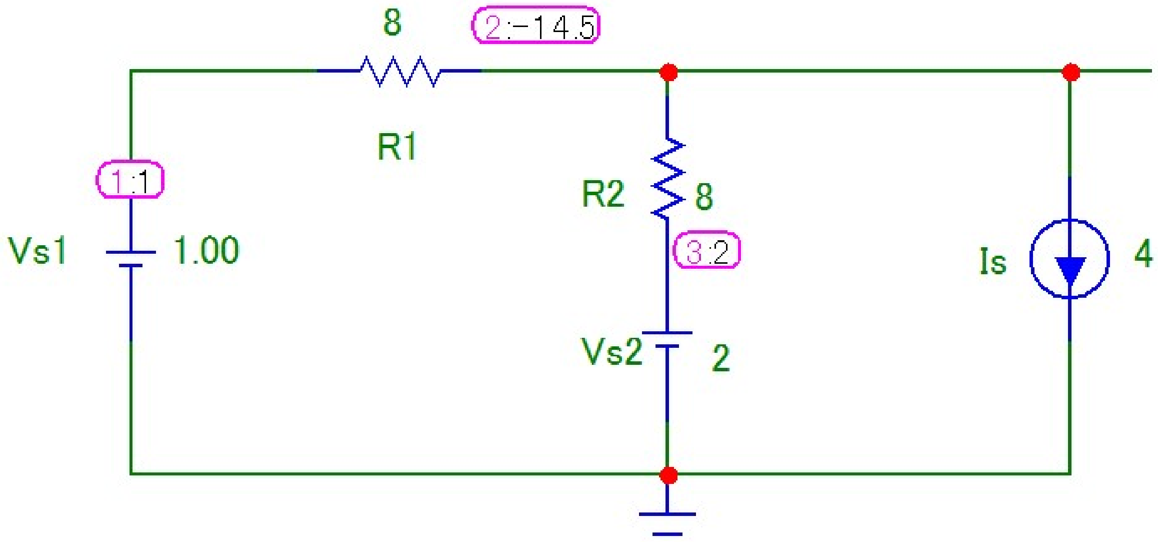
Incremental model of a constant current Source is an open circuit

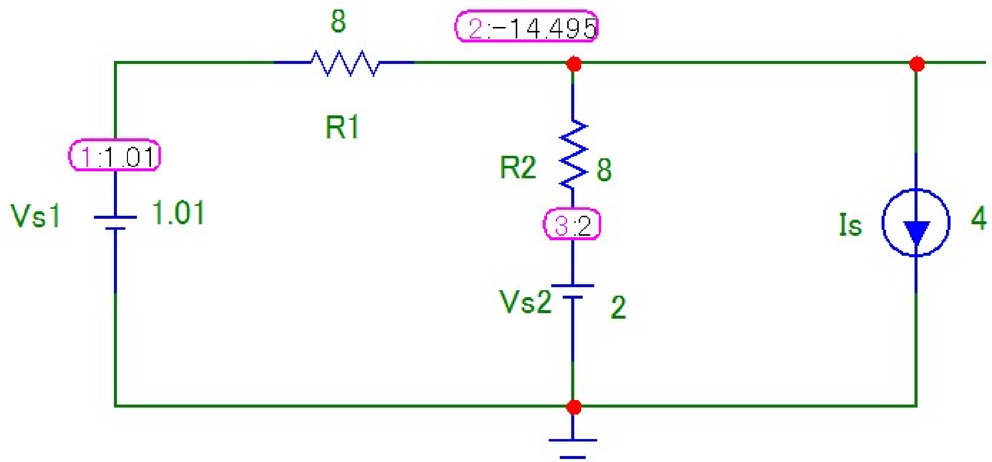
Incremental (ac) circuit Analysis

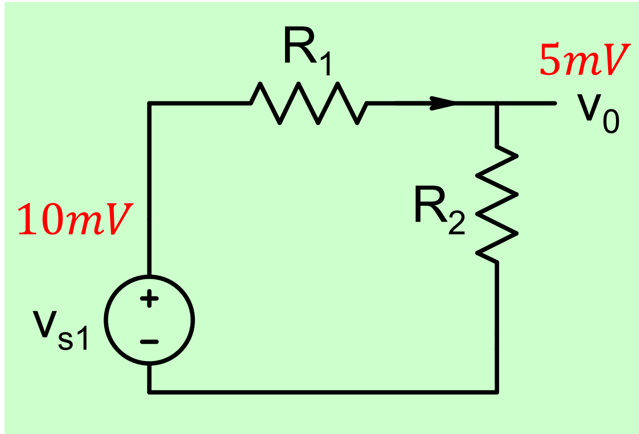




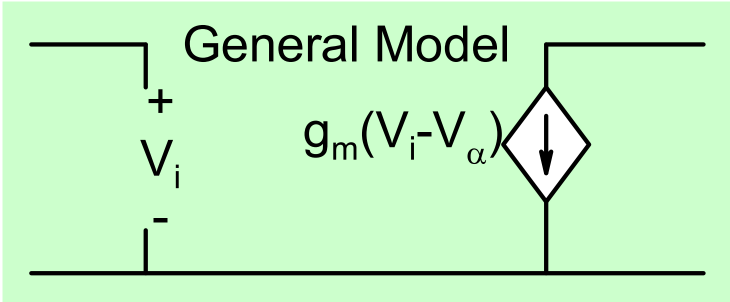
$$v_o = v_{s1} \times \frac{R_2}{R_1 + R_2}$$







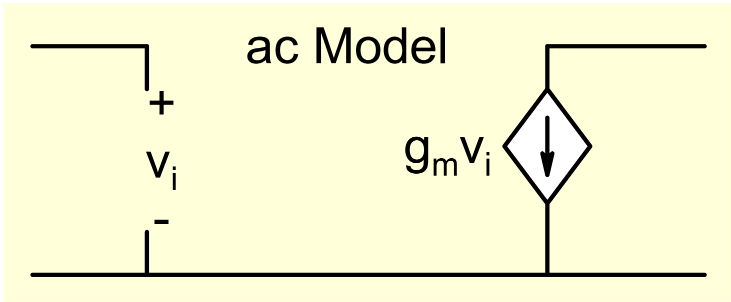
$\Delta\ V_0 = 5\text{mV}$



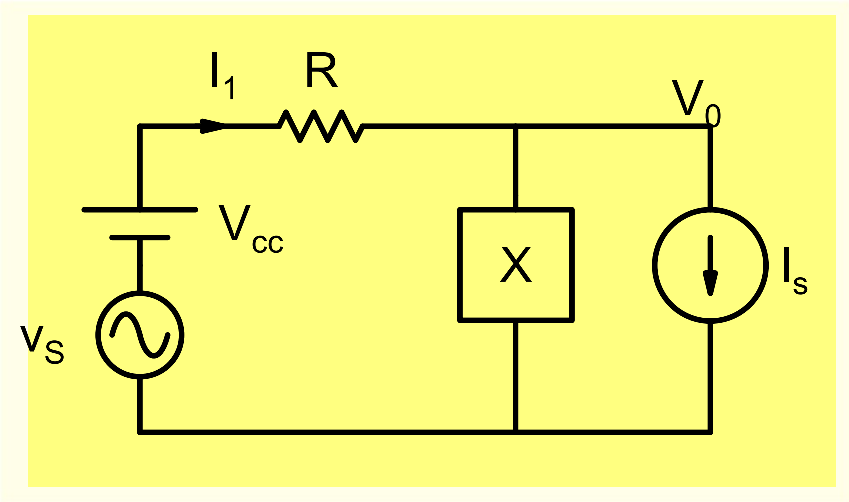
$$I_o = g_m(V_i - V_\alpha)$$

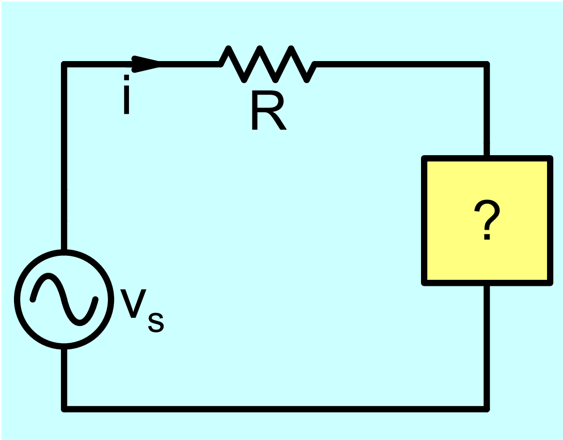
$$I_O + i_o = g_m(V_I + v_i - V_\alpha)$$

$$i_o = g_m v_i$$



Nonlinear element





$$I_x = k \times V_x^2$$

$$I_x + i_x = k \times (V_x + v_x)^2$$

Incremental Model

$$I_x + i_x = k \times (V_x + v_x)^2$$

$$i_x = k\{(V_X + v_x)^2 - V_X^2\}$$

$$i_x = kV_X^2\{(1 + v_x/V_X)^2 - 1\}$$

Problem: model is <u>NONLINEAR</u>

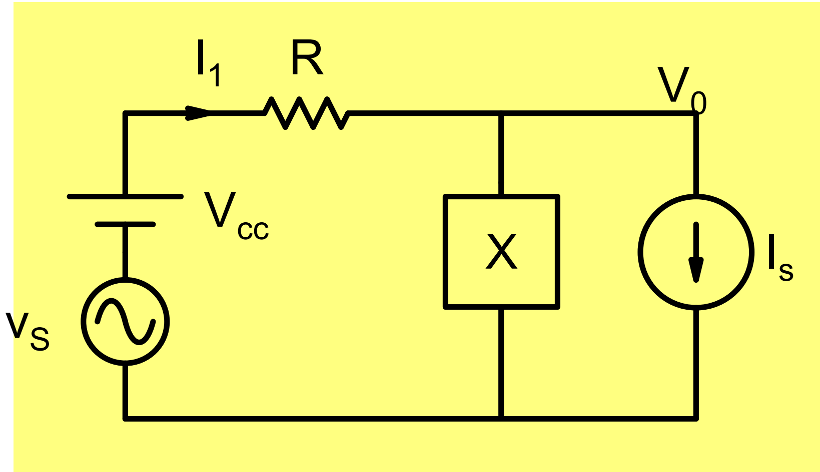
Small signal approximation: $v_x << V_X$

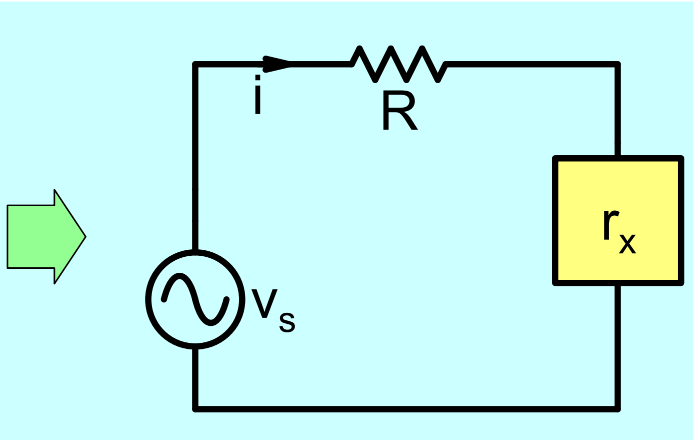
$$(1 + \frac{v_x}{V_X})^2 \cong 1 + 2\frac{v_x}{V_X}$$

$$i_x \cong 2kV_X \times v_x$$

$$i_x = v_x / r_x \quad ; r_x = \frac{1}{2kV_X}$$

Nonlinear element





$$I_x = k \times V_x^2$$

$$r_x = \frac{1}{2\,kV_o}$$

$$v_o = v_s \times \frac{r_x}{R + r_x}$$

How small is small?

Depends on how much error we can tolerate!

$$i_x = kV_X^2\{(1+v_x/V_X)^2 - 1\}$$

$$i'_x \cong 2k \times V_X \times v_x$$

$V_X = 1V$

v_x (V)	Error (%)
0.02	1
0.22	10
0.5	20
1.0	33.3

$$I_x = k \times V_x^4$$

$V_X = 1V$

v_x (V)	Error (%)
0.007	1
0.071	10
0.5	50.77
1.0	73.3

Stronger nonlinearity implies smaller voltage for same error

One of the strongest non-linearity is exponential

$$I_x = k \times \exp(\frac{V_x}{.026})$$

$$I_X + i_x = k \times \exp(\frac{V_X + v_x}{.026})$$

$$i_x = k \times \exp(\frac{V_X}{.026}) \times \{\exp(\frac{v_x}{.026}) - 1\}$$

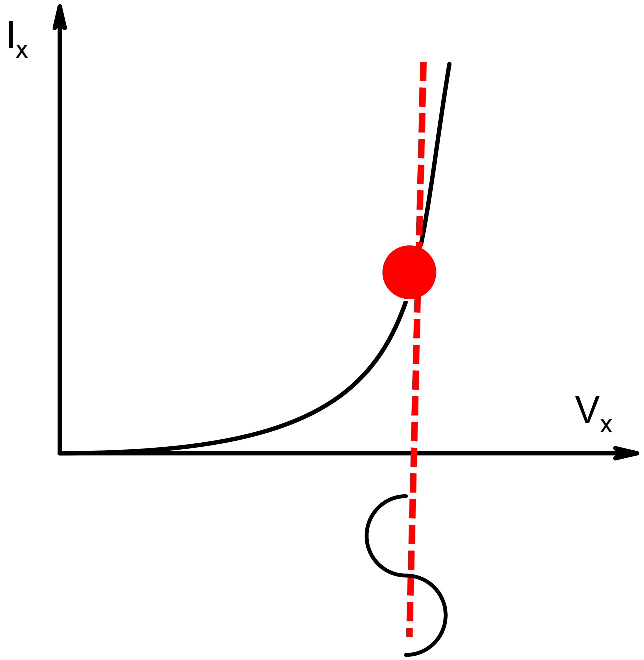
$$i_x \cong k \times \exp(\frac{V_X}{.026}) \times (\frac{v_x}{.026})$$

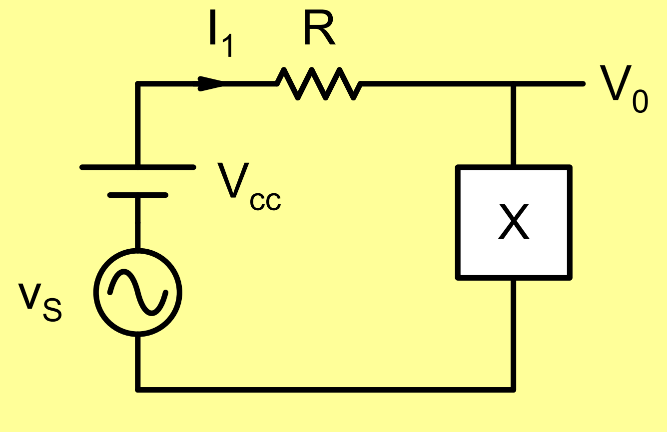
$V_X = 0.7V$

v_x (mV)	Error (%)
0.53	1
5.4	10
10	18
26	41.8

Small Signal Models

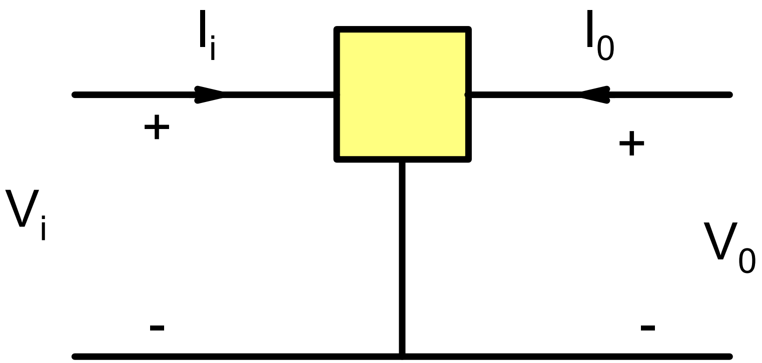
Narrowing the magnitude of applied voltage to very small values allows development of model which are linear





A SS model is a linear model which is valid for small changes around a nominal value

General approach for obtaining small **signal model** for a 3-terminal device



$$I_i = f_i(V_i)$$

$$I_I + i_i = f_i(V_I + v_i)$$

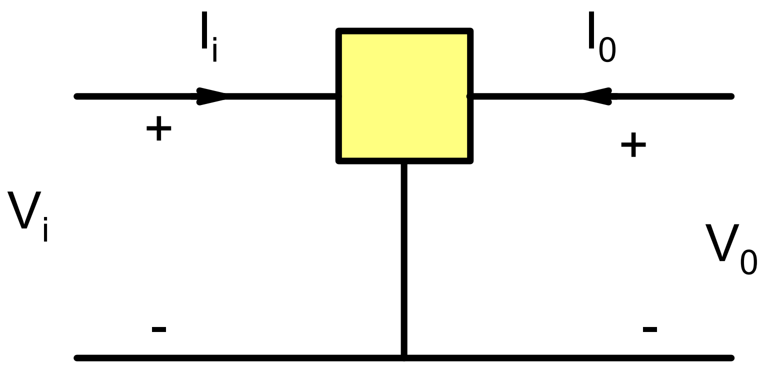
Assumption: unilateral

(Output does not affect the input)

$$\Rightarrow i_i = v_i \left.\frac{\delta f_i}{\delta V_i}\right|_{V_I} = \frac{v_i}{r_i}$$

$$r_i = \frac{1}{\left.\frac{\delta f_i}{\delta V_i}\right|_{V_I}}$$

From the input port, the device appears as a resistance



Output port:

$$I_O = f_O(V_i, V_O)$$

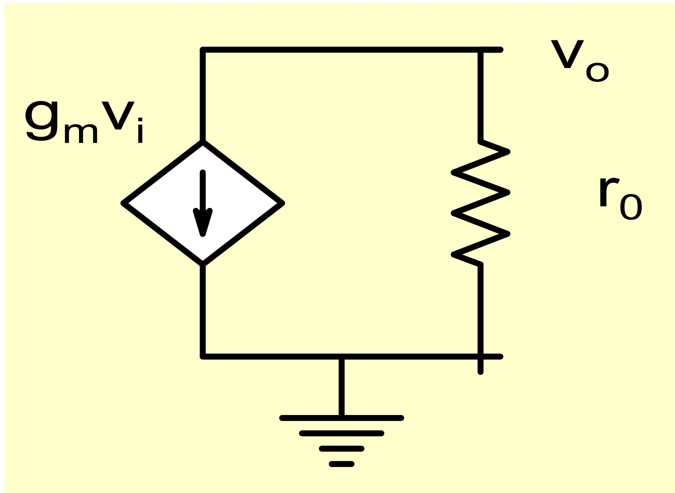
$$I_O + i_o = f_O(V_i + v_i, V_O + v_o)$$

$$I_O + i_o = f_o(V_i, V_o) + v_i \frac{\delta f_0}{\delta V_i}\Big|_{V_O} + v_o \frac{\delta f_0}{\delta V_0}\Big|_{V_I} + \ldots$$

$$i_o \cong v_i \frac{\delta f_0}{\delta V_i}\Big|_{V_O} + v_o \frac{\delta f_0}{\delta V_0}\Big|_{V_I}$$

$$i_o = g_m v_i + \frac{v_o}{r_o}$$

$$g_m = \frac{\delta f_0}{\delta V_i}\Big|_{V_O} \qquad r_0 = \frac{1}{\frac{\delta f_0}{\delta V_0}\Big|_{V_I}}$$



Complete small signal model (dc) for a 3-terminal unilateral device.