CS201B: Midsem Examination

October 19, 2020

Submission Deadline: October 21, 2020, 23:55hrs **Maximum Marks**: 50

Question 1. (5 + 10 marks) We have seen generating functions for $\binom{n}{m}$ for variable m keeping n fixed, and for variable n keeping m fixed. If we wish to make both variable then the generating function needs to be over two variables.

- Prove that $\frac{1}{1-y-xy} = \sum_{n\geq 0}\sum_{m\geq 0}\binom{n}{m}x^m y^n$.
- Derive the generating function for $\binom{2n}{n}$ from above two-variable generating function by judicious substitution for one of the two variables.

Question 2. (15 marks) For a fixed number $k > 0$, find the recurrence relation and generating function for the sequence $a_n^k = \lfloor \frac{n}{k} \rfloor$. Use these two to derive the generating function for the sequence $b_n^k = (\lfloor \frac{n}{k} \rfloor)^2$.

Question 3. (10 marks) Given numbers from 0 to $2n-1$ in a sequence, what is the number of permutations of this sequence such that no even number is in its original position (express the number of permutations in terms of derangement numbers d_n)?

Question 4. (10 marks) Let A be a set containing non-empty sets and define $A_\times = \prod_{B\in A} B$. Prove that Axiom of Choice is equivalent to the statement that for every set A as above, $A_\times \neq \emptyset$.