1.a. Two gamblers A and B agree to play as follows. They throw two fair dice. If the sum S of the outcomes is < 8, then B receives Rs. S from A, otherwise B pays A Rs. x. Determine x so that the game is fair. [3]

1.b. Consider the following mgf:

$$M_X(t) = \frac{3(e^{2t} - e^t)}{4t} + (1 - \alpha)\frac{3}{2\pi^2}\sum_{n=1}^{\infty}\frac{e^{t/n}}{n^2}$$

for $t \neq 0$, and $M_X(0) = 1$.
Find (i) $P(X = 1/2)$, and (ii) $P(X \in (1.1, 1.25))$. [1.5+1.5]

1.a.

The probability distribution of S:

$S = s$	2	3	4	5	6	7	8	9	10	11	12
$P[S=s]$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

If $S < 8 \equiv S \leq 7$

For fair game we must have

(1 mark)

$$2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + \cdots + 7 \cdot \frac{6}{36}$$

$$= x \cdot \frac{5}{36} + x \cdot \frac{4}{36} + x \cdot \frac{3}{36} + x \cdot \frac{2}{36} + x \cdot \frac{1}{36}$$

(1 mark)

$$\Rightarrow 2 + 6 + 12 + 20 + 30 + 42 = 15x$$

$$\Rightarrow 112 = 15x$$

$$\Rightarrow \boxed{x = \frac{112}{15} = 7.467}$$

(1 mark)

1.b. Here $M_X(t) = \frac{3}{4}\left(\frac{e^{2t} - e^{t}}{t}\right) + \left(1 - \frac{3}{4}\right)\frac{6(1-\alpha)}{\pi^2}\sum_{n=1}^{\infty}\frac{e^{t/n}}{n^2}$

$\therefore F = \alpha F_c + (1-\alpha) F_D$ with $\alpha = \frac{3}{4}$.

Where F_c is $U(1,2)$

and F_D has pmf: $X = \frac{1}{n}$ w.p. $\frac{6(1-\alpha)}{\pi^2}\frac{1}{n^2}$

$\therefore X \sim F$

(i). $P(X = \frac{1}{2}) = \left(1 - \frac{3}{4}\right)\frac{6(1-\alpha)}{\pi^2}\frac{1}{4} = \frac{3(1-\alpha)}{8\pi^2}$ (1.5 mark)

(ii). $P[X \in (1.1, 1.25)] = \frac{3}{4} \times (0.15) = 0.1125$ (1.5 mark)