CAPM and Factor models

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The Origination of Factor model
Why?

- The beta in CAPM model does not completely account for systematic risk and that the cross-section of stock returns is strongly associated with the market capitalization (size), book to market ratio (B/M) and return momentum.

- Fama and French (1992, hereafter FF) find that there is a negative relationship between the size and average return, and there is a stronger positive relation between average return and book-to-market equity.

The Origination of Factor model
Why?

- Therefore, FF in their three factor model, in addition to the market factor, they introduce two other factors: a size factor, SMB (small minus big) and a value factor, HML (high minus low).

- However, Carhart (1997) further introduced the momentum factor constructed by the monthly return difference between the returns on the high and low prior returns portfolios, to capture the cross-sectional return patterns.

The Origination of Factor model
Why?

- **CAPM**

  $R_{pt} - R_{ft} = \alpha_p + \beta_{mkt}(R_{mt} - R_{ft})$

- **FF-3 Factor model**

  $R_{pt} - R_{ft} = \alpha_p + \beta_{mkt}(R_{mt} - R_{ft}) + \beta_{SMB}(SMB) + \beta_{HML}(HML)$

- **Carhart Four Factor model**

  $R_{pt} - R_{ft} =$
  $\alpha_p + \beta_{mkt}(R_{mt} - R_{ft}) + \beta_{SMB}(SMB) + \beta_{HML}(HML) + \beta_{MOM}(MOM)$

- **Five-factor model**

  $R_{pt} - R_{ft} = \alpha_p + \beta_{mkt}(R_{mt} - R_{ft}) + \beta_{SMB}(SMB) + \beta_{HML}(HML) +$
  $\beta_{RMW}(RMW) + \beta_{CMA}(CMA)$

The Origination of Factor model
Why?

- **Five-factor model**
- The model was developed by Fama and French (2017)

$$R_{pt} - R_{ft} = \alpha_p + \beta_{mkt}(R_{mt} - R_{ft}) + \beta_{SMB}(SMB) + \beta_{HML}(HML) + \beta_{RMW}(RMW) + \beta_{CMA}(CMA)$$

where, RMW (robust minus weak) is the profitability factor, formed from operating profit (OP) and CMA (conservative minus aggressive) is the investment factor using year-on-year change in total assets (Investments).

- Fama and French did the analysis for the US market.
- For US market, at the end of June of each year, NYSE, AMEX and NASDAQ stocks are allocated into two groups: Small (S) and big (B) based on whether their market Equity (ME) in June is below or above the median ME.
- NYSE, AMEX and NASDAQ stocks are allocated in an independent sort to three book-to-market equity (BE/ME) groups: Low (L), Medium (M) and High (H) based on the breakpoints for bottom 30 percent, middle (40 percent) and top (30 percent) of the values of BE/ME for NYSE stocks.

- The FF methodology involves a cross classification of stocks on two dimensions – **Size**, measured by market capitalization, and **Value**, measured by the ratio of book value per share to market price per share = B/M ratio.

		Value as measured by B/M ratio		
		Value (V)	Neutral (N)	Growth (G)
Size	Big (B)	BV	BN	BG
	Small (S)	SV	SN	SG

- Six size-BE/ME portfolios (S/L, S/M, S/H, B/L, B/M, B,H) are defined as the intersection of the two ME and the three BE/ME groups.
- FF calculates SMB to mirror the risk factor in returns related to size.
- SMB is calculated as the average return on the three small stock portfolios (S/L,S/M and SH) minus the average return on the three big stock portfolios (B/L, B/M and B/H).

- SMB

$$\text{SMB} = \frac{1}{3}(\text{small value} + \text{small neutral} + \text{small growth}) - \frac{1}{3}(\text{Big value} + \text{big neutral} + \text{big growth})$$

- On the contrary, firms that have a low price-to-book (i.e. Stock price / Book value per share) value tend to have low earnings on assets while a high market to book value is associated with persistently high earnings (Fama and French, 1992).

- FF Calculated the HML as the average return on the two value portfolios minus the average return on the two growth portfolios:

$$\text{HML} = \frac{1}{2}(\text{small value} + \text{big value}) - \frac{1}{2}(\text{small growth} + \text{big growth})$$

- Momentum (Mom) is meant to represent the risk factor in returns related to momentum (Carhart, 1997).
- **Mom** is the average return on the two high prior return portfolios minus the average return on the two low prior return portfolios and thus calculated as:

$$\text{MOM} = \tfrac{1}{2}(\text{small high} + \text{big high}) - \tfrac{1}{2}(\text{small low} + \text{big low})$$

Construction of Factors
Practical Example

- Suppose we work on two indicators: Return on Equity which is defined as
- ROE $= \frac{NetIncome}{Equity}$
- Market capitalization : Total market value of the company (in any currency). This is proxied as the size of the firm.

Construction of Factors
Practical Example: STEPS

- **STEP 1**: Calculate the excess returns. That is, R_i - R_f
- **STEP 2**: Sort the excess returns on Size (market capitalization, denoted as S) and then on value (ROE).
- **STEP 3**: As above mentioned, denote bottom 30 percent, middle (40 percent) and top (30 percent) as Low (1), Medium (2) and High (3).
- For instance: The classification could be S_1, S_2 and S_3 for size and V_1, V_2 and V_3 for value.
- **STEP 4**: We can apply the above formula. That is:

$$\text{SMB} = \tfrac{1}{3}(\text{small value} + \text{small neutral} + \text{small growth}) - \tfrac{1}{3}(\text{Big value} + \text{big neutral} + \text{big growth})$$

$$\text{SMB} = \tfrac{1}{3}(S_1V_1 + S_1V_2 + S_1V_3) - \tfrac{1}{3}(S_3V_1 + S_3V_2 + S_3V_3)$$

- **STEP 5**:

$$\text{HML} = \frac{1}{2}(\text{small value} + \text{big value}) - \frac{1}{2}(\text{small growth} + \text{big growth})$$

$$\text{HML} = \frac{1}{2}(S_1V_1 + S_3V_1) - \frac{1}{2}(S_1V_3 + S_3V_3)$$

- The HML factor is thus designed to capture the effect of value while being largely free of the influence of size.

Construction of Factors
Portfolio formation date

- Fama and French (1993) formed their portfolios in June of each year after considering a 6-month gap from the fiscal year ends (December) to account for the time taken for the publication of accounting data.
- In India, as the fiscal year ends for most Indian firms (89%) is March, assuming a 6-months gap for publication of accounting data, therefore, September of each year is considered appropriate for analysis.
- At the end of September each year, the stocks were classified as Big (B) and Small (S), based on their market capitalisation at September-end.

Construction of Factors
Portfolio formation date: Indian case

- At the end of September each year, the stocks may be classified as Big (B) and Small (S), based on their market capitalisation at September-end.

- At the same time, the stocks were independently classified as Value (V), Neutral (N) and Growth (G) based on their B/M ratio. There were two possibilities here depending on the financial year end:
  - If the firm's financial year ended in March, the B/M ratio is computed in September using the data as at the end of March of the same year.
  - If the firm's financial year ended in any other quarter, the B/M ratio is computed in September of year t using the data as at the firm's financial year end of year t-1.

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Construction of Factors
Portfolio formation date: Indian case

- As per the scheme revealed by **Jagadeesh and Titman (1993) and Carhart (1997)**, the momentum factor is calculated based on the classification of Winners (W) and Losers (L).
- It is done based on their momentum returns at the end of each month.
- The approach uses the daily total returns adjusted for dividends to estimate the 11-month holding period returns.
- Some researchers calculate monthly returns even when a stock is traded only for a part of its first month of trading using the available daily returns.

Construction of Factors
Portfolio formation date: Indian case

- The momentum returns at the end of month t is the 11- month returns from the end of month t- 12 to t-1.
- By using the momentum returns, the stocks were grouped as below:
  - W – group consisted of the top 30% by the momentum return.
  - L – group consisted of the bottom 30% by the momentum return.

Construction of Factors
Data analysis

- Inline with FF scheme, the momentum portfolios were orthogonalized to the size factor.

		Momentum	
		Winners (W)	Losers (L)
Size	Big (B)	WB	LB
	Small (S)	WS	LS

- Based on the size and momentum groups, four size-momentum portfolios – WS, WB, LB, LS, are formed every month.
- The momentum factor WML (Winners minus Losers) is calculated as as the simple average of the differences in the returns of WS - LS and WB - LB.
- The WML factor was thus designed to capture the effect of value while being largely free of the influence of size.

- The WML factor was thus designed to capture the effect of value while being largely free of the influence of size.

$$\text{MOM} = \tfrac{1}{2}(\text{small winner} + \text{big winner}) - \tfrac{1}{2}(\text{small Loser} + \text{big loser})$$

Non-Standard CAPM Model
PERSONAL TAXES

- The simple form of the CAPM ignores the presence of taxes in arriving at an equilibrium solution. The implication of this assumption is that investors are indifferent between receiving income in the form of capital gains or dividends and that all investors hold the same portfolio of risky assets.

- Investors should judge the return and risk on their portfolio after taxes. This implies that, even with homogeneous expectations about the before-tax return on a portfolio, the relevant (after-tax) efficient frontier faced by each investor will be different.

Non-Standard CAPM Model
PERSONAL TAXES

- The return on any asset or portfolio is given by

$$\mathrm{E}(\mathrm{R}_i) = R_f + \beta_i[(E(R_M - R_f) - \tau(\delta_M - R_F)] + \tau(\delta_i - R_F)$$

where,
δ_M = the dividend yield of the market portfolio
δ_i = the dividend yield for stock i
τ = a tax factor that measures the relevant market tax rates on capital gains and income.

  - When dividends are on average taxed at a higher rate than capital gains, τ is positive, and expected return is an increasing function of dividend yield.

Non-Standard CAPM Model
THE MULTI-BETA CAPM

- Merton (1973) has constructed a generalized inter-temporal CAPM in which a number of sources of uncertainty would be priced.
- Merton models investors as solving lifetime consumption decisions when faced with multiple sources of uncertainty.
- In this multi-period setting, uncertainty exists not only about the future value of securities but also about such other influences as future labor income, future prices of consumption goods, future investment opportunities, and so on.

Non-Standard CAPM Model
THE MULTI-BETA CAPM

- The inflation model is the simplest form of a multi-beta CAPM where the expected return on any security can be expressed as a function of two sensitivities,
  $E(R_i) = R_f + \beta_{iM}[E(R_M) - R_f] + \beta_{il1}[E(R_{I1}) - R_F] + \beta_{il2}[E(R_{I2}) - R_F] + ...$
  where,
  In this relationship, all of the R_{ljs} are expected returns on a set of portfolios that allows the investor to hedge a set of risks with which he or she is concerned.

Empirical Issues in Equity Research
Some Hypotheses of the CAPM

- The first is that higher risk (beta) should be associated with a higher level of return.
- The second is that return is linearly related to beta; that is, for every unit increase in beta, there is the same increase in return.
- The third is that there should be no added return for bearing non-market risk.

Empirical Issues in Equity Research
Some Early Empirical Tests

- Most of the early empirical tests of the CAPM involved the use of a time series (first pass) regression to estimate betas and the use of a cross-sectional (second pass) regression to test the hypotheses we derived from the CAPM model.

- Lintner and then reproduced by Douglas (1968) first estimated beta for each of the 301 common stocks in his sample.

- He estimated beta by regressing each stock's yearly return against the average return for all stocks in the sample using data from 1954 to 1963.

- The first-pass regression had the form: $R_{it} = \alpha_i + b_i R_{Mt} + e_{it}$, where b_i (the regression coefficient) was the estimate of the true beta for stock i.

- Lintner then performed the second-pass cross-sectional regression:

- $\overline{R}_i = a_1 + a_2 b_i + a_3 S_{ei}^2 + \epsilon_i$,
where S_{ei}^2 is the residual variance from the first-pass regression.

- Each parameter of this model has a theoretical value: a_3 should be equal to zero, a_1 should be equal to either R_F or any risk free asset. a_2 should be equal to market risk premium.

- However, the estimated values which they obtained were: $a_1 = 0.108$, $a_2 = 0.063$ and $a_3 = 0.237$.

- The term representing residual risk was statistically significant and positive. Both a_2 and a_3 are statistically different from zero at the 0.01 level. These results seem to violate the CAPM.

Thanking You