CS201

Mathematics For Computer Science

Indian Institute of Technology, Kanpur

Assignment 1

Due by: Sept 27, 2020

Instructions.

- Solutions should be mandatorily LaTeXed using the template shared and submitted through GradeScope before time. Mention Group Numbers and member names in solutions (refer template instructions).
- Clearly express solutions avoiding unnecessary details. Everything discussed in class is not required to be proved again. And anything non-trivial must be proved.
- Write the solutions on your own. Acknowledge the source wherever required. Keep in my mind department's Anti-Cheating Policy.

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1. Let $S = \{(a, b, c) | a, b, c \in \mathbb{Z}\}$ be the set of all triplets of integers. Show that $|S| = \aleph_0$.
2. For any $a, b, c, d \notin \{-\infty, \infty\}$, show that $|[a, b]| = |[c, d]|$ where $[x, y]$ is the set of all real numbers between x and y.
3. Show that $|[0, 1]| = \aleph_1$ where $[0, 1]$ is the set of all real numbers between 0 and 1.
4. Show that $|\{0, 1\}^*| = \aleph_1$ where $\{0, 1\}^*$ is the set of all binary strings of infinite length.
5. Suppose R is a partial order on A and S be a partial order on B. Let L be a binary relation on $A \times B$ defined as $(a, b)L(a', b')$ iff
   - $a \neq a'$ and aRa'
   - $a = a'$ and bSb'.

Show that L is also a partial order on $A \times B$. Is it a total order?

6. Let R be a binary relation on $\mathbb{N}$ defined as aRb if $b = 2^k a$ where k is a non-negative integer. Show that R is a partial order on $\mathbb{N}$.

7. Let n be a positive integer. Consider the relation $\equiv_n$ on $\mathbb{Z}$ such that $a \equiv_n b \iff a = b \mod n$. Show that $\equiv_n$ is an equivalence relation on $\mathbb{Z}$. What are the equivalence classes?

8. Consider the relation S on $\mathbb{N}$ such that $aSb \iff ab$ is a perfect square. Show that S is an equivalence relation on $\mathbb{N}$. What are the equivalence classes?

9. There was an ambiguity in the definition of a well-ordering in the lectures. It is clarified here.

   A well-ordering R on set A is a partial order such that for every subset $B \subseteq A$, B has an element m such that mRb for every $b \in B$.

   In lecture 6, a partial order is shown to be a well-ordering twice: once during proof of the implication that Axiom of Choice implies Zorn's Lemma, and next during proof of the implication that Zorn's Lemma implies Well-Ordering Principle. Redo both these proofs in light of the above clarification.