CS201

Mathematics For Computer Science

Indian Institute of Technology, Kanpur

Assignment 2

Due by: Oct 12, 2020

Instructions.

- Solutions should be mandatorily LaTeXed using the template shared and submitted through GradeScope before time. Mention Group Numbers and member names in solutions (refer template instructions).
- Clearly express solutions avoiding unnecessary details. Everything discussed in class is not required to be proved again. And anything non-trivial must be proved.
- Write the solutions on your own. Acknowledge the source wherever required. Keep in my mind department's Anti-Cheating Policy.

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1. A **partition** of n objects is a collection of its mutually disjoint subsets, called blocks, whose union gives the whole set. Let $S(n; k_1, k_2, \dots, k_n)$ denote the number of all partitions of n objects with k_i i-element blocks (i.e., $k_1 + 2k_2 + \cdots + nk_n = n$). In other words,

   $$k_i = \text{the number of } i\text{-element blocks in a partition}$$

   Show that $S(n; k_1, k_2, \dots, k_n) = \dfrac{n!}{k_1!k_2!\dots k_n!(1!)^{k_1}(2!)^{k_2}\dots(n!)^{k_n}}$.

2. Show that for every k, the product of any k consecutive natural numbers is divisible by $k!$.

3. Show that the number of pairs (A, B) of distinct subsets of $\{1, 2, \dots, n\}$ with $A \subset B$ is $3^n - 2^n$.

4. There is a set of $2n$ people (n males and n females). A good party is a set with the same number of males and females. How many ways are there to build such a good party?

5. (a) Show that the number of integer solution to the equation

$$x_1 + x_2 + \cdots + x_n = k$$

   under the condition that $x_i \geq 0$ for all i is $\binom{n+k-1}{k}$.

   (b) Let n and $k \geq l$ be positive integers. How many different integer solutions are there to the equation $x_1 + x_2 + \cdots + x_n = k$ such that $0 \leq x_i < l$ for all i.