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A.1.

(a) $L_1 = \{x \in \{0,1\}^* \mid x$ does not contain the substring $101\}$

$$R.E. = 0^* (1^* 000^*)^* 1^* 0^*$$

We see that 1 can be followed by a 1 or 00 and this pattern is repeated. ⇒ $(1^* 000^*)^*$. The string can start or end with 0 ⇒ we have 0^* on both ends. and can contain only 1's so we have 1^*.

(b) $L_2 = \{x \in \{0,1\}^* \mid x$ has atmost two 0's & atmost three 1's$\}$

no of 0's	no. of 1's	R.E.
0	0	ε
0	1	1
0	2	11
0	3	111
1	0	0
1	1	01 + 10
1	2	011 + 110 + 101
1	3	0111 + 1011 + 1101 + 1110
2	0	00
2	1	100 + 010 + 001
2	2	1100 + 1010 + 1001 + 0101 + 0110 + 0011
2	3	111000 + 11010 + 11001 + 00111 + 10110 + 10011 + 01101 + 01011 + 10101 + 0111

A.3.

	q_0	q_1	q_2	q_3
q_1	X			
q_2		X		
q_3	X		X	
q_4		X		X

After 1 step:

$p \notin F$

$q \in F$

where $F = \{q_1, q_3\}$

	q_0	q_1	q_2	q_3
q_1	X			
q_2	X	X		
q_3	X		X	
q_4		X	X	X

$(q_0, 0), (q_2, 0) \rightarrow \{q_1, q_2\}$

and

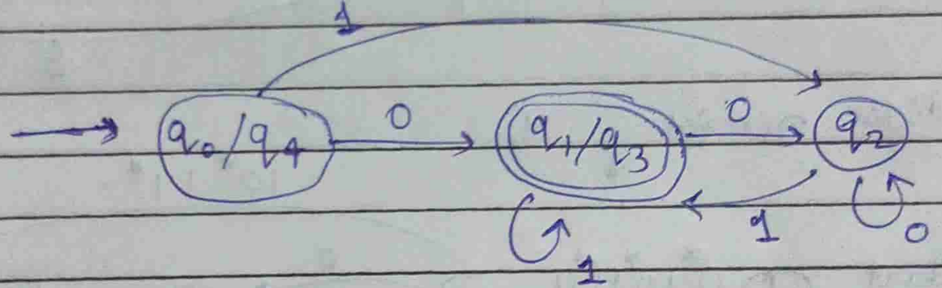
$(q_4, 0), (q_2, 0) \rightarrow \{q_1, q_2\}$

$\{q_1, q_2\}$ is marked

$\Rightarrow$ we mark $\{q_0, q_2\}$ & $\{q_2, q_4\}$

$\Rightarrow$ $q_1 \approx q_3$ and $q_0 \approx q_4$

$\Rightarrow$ Min-DFA



Q. 4 (a) $L_1 = \{ a^m b^n c^n d^{2m} \mid n, m \geq 0 \}$

CFG $\rightarrow$

$S \rightarrow ASD \mid X \mid \varepsilon$
$X \rightarrow BXC \mid \varepsilon$
$A \rightarrow a$ $B \rightarrow b$ $C \rightarrow c$ $D \rightarrow dd$

(b) $L_2 = \{ a^n b^m \mid n \neq m \}$

CFG $\rightarrow$

$S \rightarrow AX \mid XB$
$A \rightarrow aA \mid a$
$B \rightarrow Bb \mid b$
$X \rightarrow aXb \mid \varepsilon$

Q.5. $L = \{ a^n b^m \mid gcd(i,j) = 1 \ \& \ n, m \geq 0 \}$

let this L be regular.
Consider a language $L_1 = a^* b^*$
$\Rightarrow$ by Clousure Property.

$\bar{L}$ is regular, L_1 is regular.
Hence $L_2 = \bar{L} \cap L_1$ is regular.

$L_2 = \{ a^n b^m \mid gcd(n,m) > 1 \ ; \ n, m \geq 0 \}$

Apply Pumping lemma on L_2.

$p \geq 0$

let $w = a^q b^q$ where q is prime $> p$

Now for any partition xyz of w such that $|y| > 0$ it is of the form.

xyz where $x = a^{t-r}$, $y = a^r$
$z = a^{q-t} b^q$

where $t \leq p$ & $r > 0$

$xy^i z = a^{q+r(i-1)} b^q$

for $i = 0$ | $xy^0 z = a^{q-r} b^q$

$\gcd(q, q-r) = 1$ as q is prime & $r > 0$

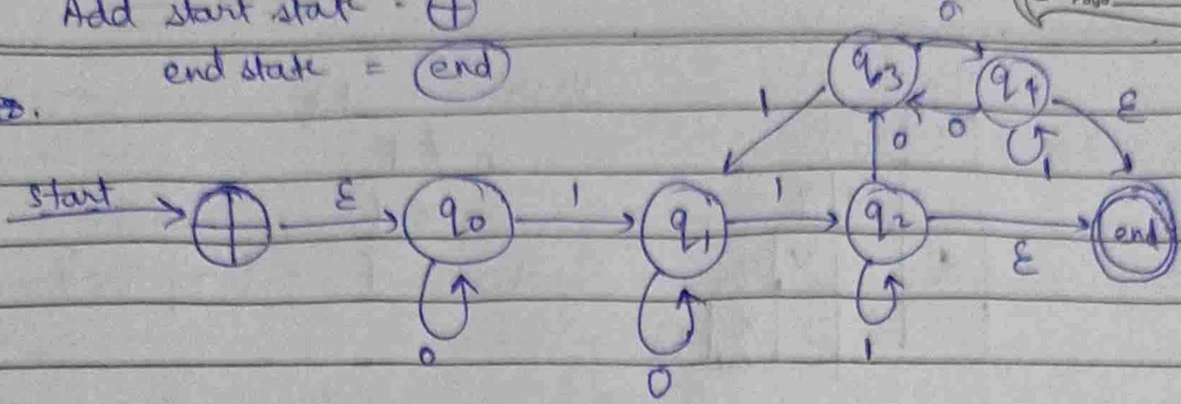
$\Rightarrow a^{q-r} b^q \notin L_2$ &

$\Rightarrow L_2$ is irregular which is a contradication.
Hence L is irregular.

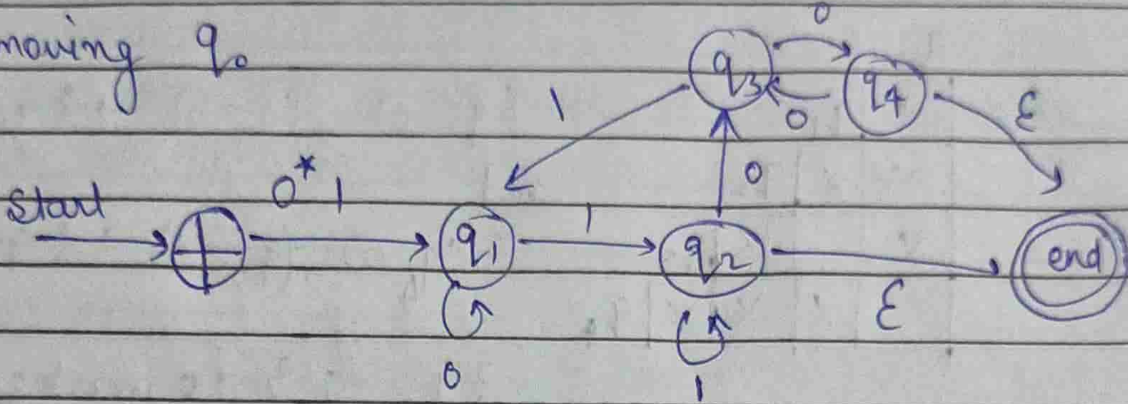
Q.2. Converting DFA to GNFA

Add start state = ⊕

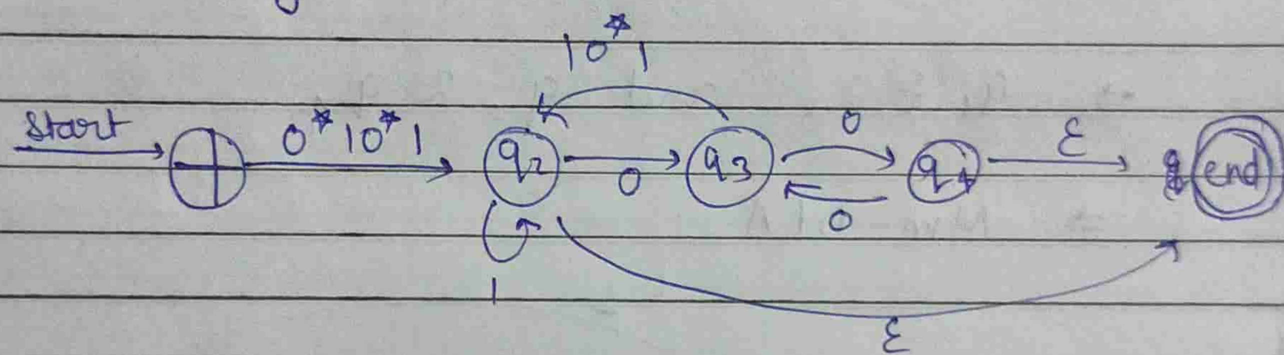
end state = end



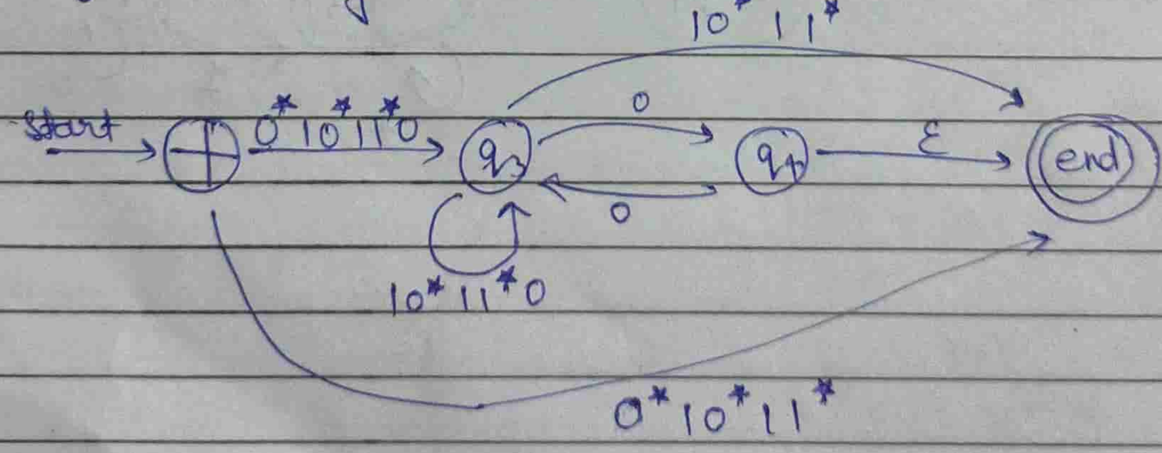
After Removing q_0



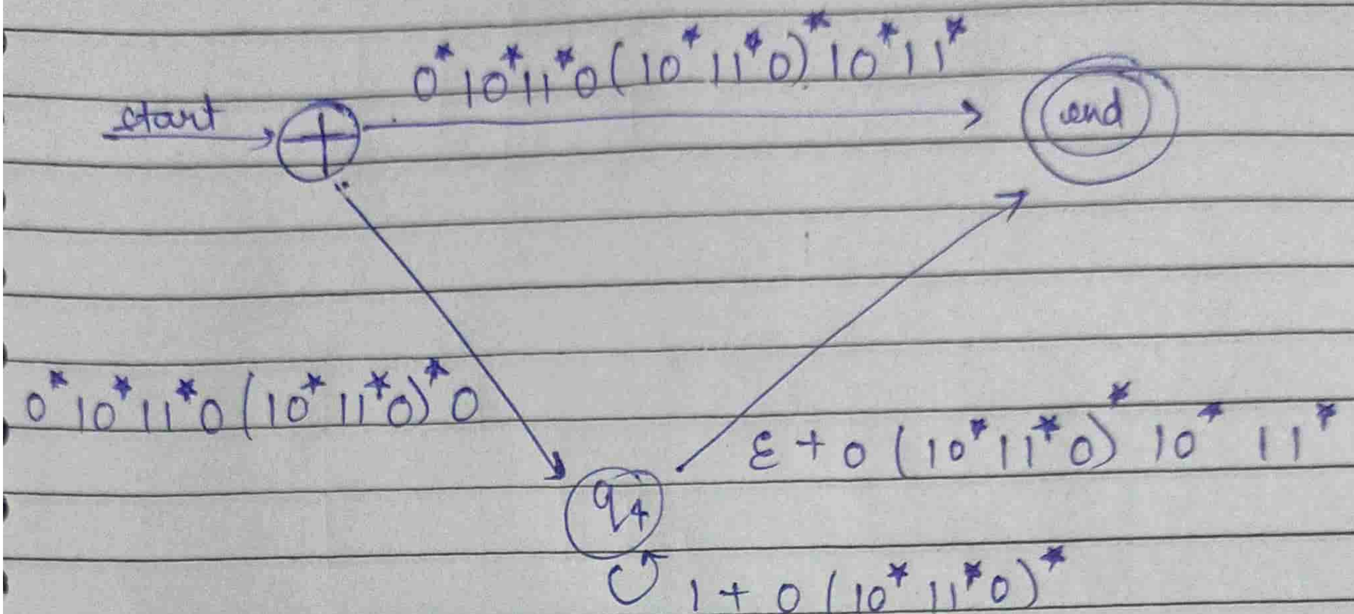
After removing q_1



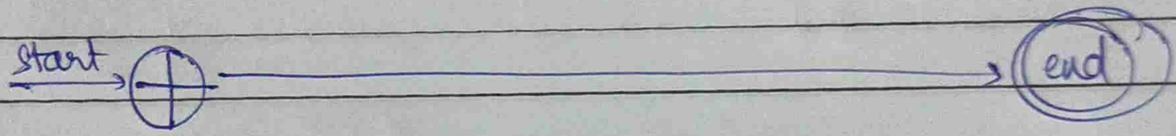
After removing q_2



After removing q_3



After removing q_4,



$$0^*10^*11^*0(10^*11^*0)^*10^*11^* +$$
$$(0^*10^*11^*0(10^*11^*0)^*0)(\varepsilon + 0(10^*11^*0)^*10^*11^*)$$
$$(1+0(10^*11^*0)^*0)^*$$