Code Generation: Sethi Ullman Algorithm

Amey Karkare

karkare@cse.iitk.ac.in

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- Complexity is linear in the size of the expression tree. Reasonably efficient.

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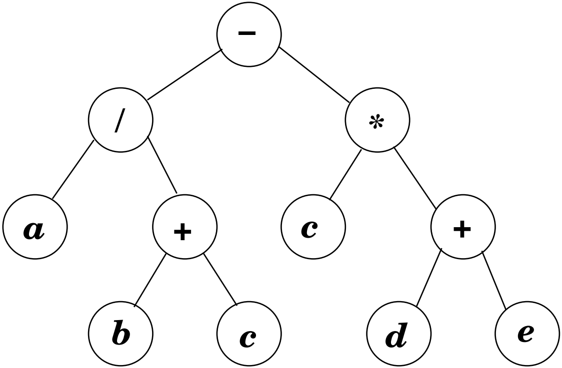
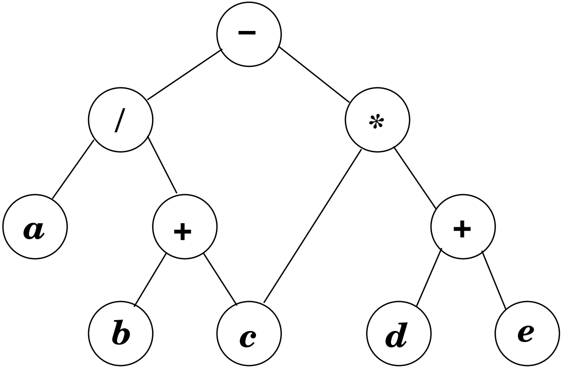


Expression Trees

- We have not identified common sub-expressions; else we would have a directed acyclic graph (DAG):



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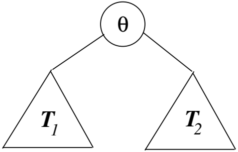
     then 

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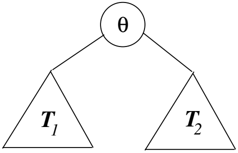
     then 

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- In this example

  $\Sigma = \{a, b, c, d, e, \ldots\}$, and $\Theta = \{+, -, *, /, \ldots\}$

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- Note:
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  2. In instruction 4, the destination register is the same as the left operand register.

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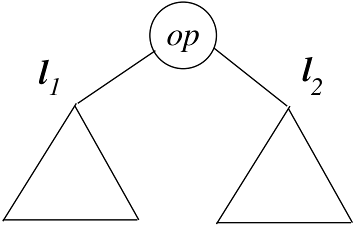
- If the left and right subtrees require l_1, and l_2 ($l_1 < l_2$) registers respectively, what should be the order of evaluation?



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- The maximum register requirement in this case is $max(l_1, l_2 + 1) = l_2 + 1$.

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Therefore the subtree requiring more registers should be evaluated first.

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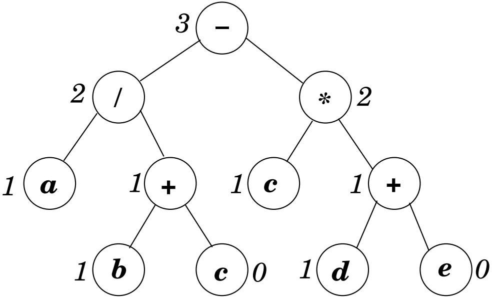

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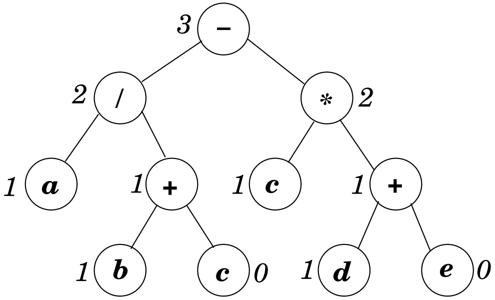

- Left and the right leaves are labeled 1 and 0 respectively, because the left leaf must necessarily be in a register, whereas the right leaf can reside in memory.

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- Visit the tree in post-order. For every node visited do:
  1. Label each left leaf by 1 and each right leaf by 0.
  2. If the labels of the children of a node n are l_1 and l_2 respectively, then

$$\begin{aligned} label(n) &= max(l_1, l_2), if\, l_1 \neq l_2 \\ &= l_1 + 1, otherwise \end{aligned}$$

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5. $swap(rstack)$ swaps the top two registers on the stack.

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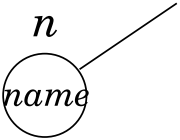

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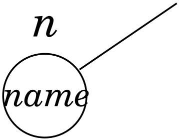

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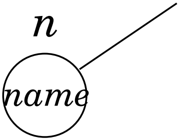
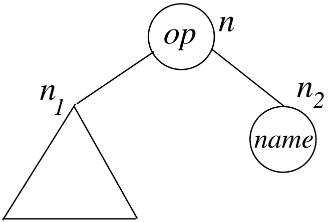


$$gen(top(rstack) \leftarrow name)$$

Comments: n is named by a variable say *name*. Code is generated to load *name* into a register.

The Algorithm

2. *n's right child is a leaf:*



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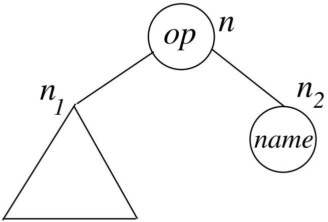

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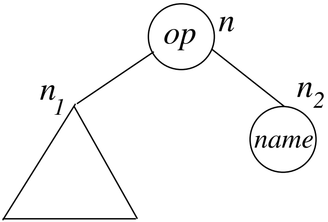

$gencode(n_1)$

$gen(top(rstack) \leftarrow top(rstack)\ op\ name)$

Comments: n_1 is first evaluated in the register on the top of the stack, followed by the operation *op* leaving the result in the same register.

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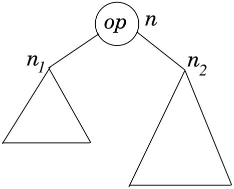
3. *The left child of n requires lesser number of registers. This requirement is strictly less than the available number of registers*



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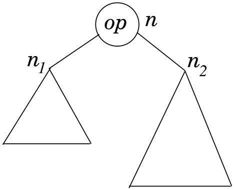

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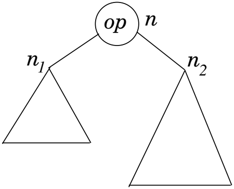

swap(*rstack*); Right child goes into next to top register

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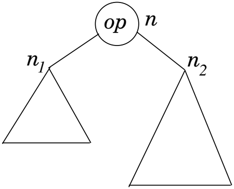

swap(*rstack*);	Right child goes into next to top register
gencode(n_2);	Evaluate right child

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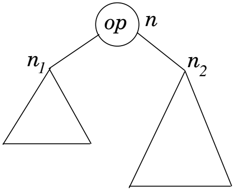

$swap(rstack)$;	Right child goes into next to top register
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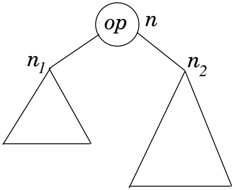

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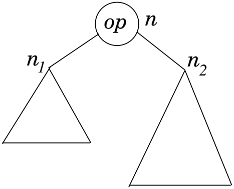

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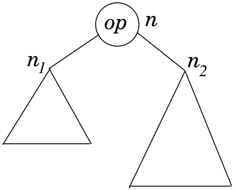

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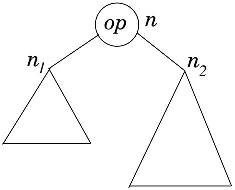

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$swap(rstack)$	Restore register stack

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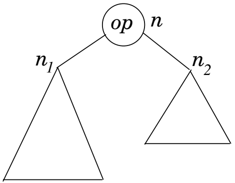

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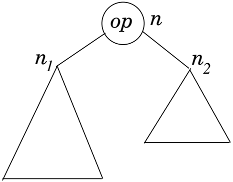

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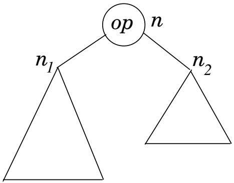

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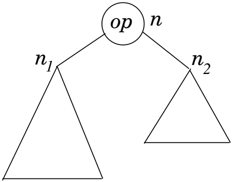

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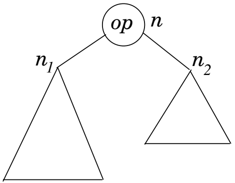

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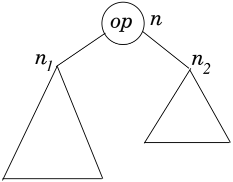

$gencode(n_1);$
$R := pop(rstack);$
$gencode(n_2);$
$gen(R \leftarrow R\ op\ top(rstack));$

The Algorithm

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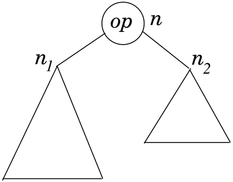

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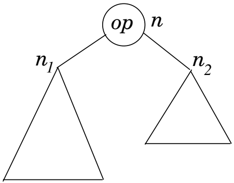

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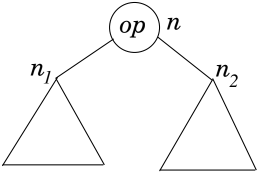
Comments: Same as case 3, except that the left sub-tree is evaluated first.

The Algorithm

5. *Both the children of n require registers greater or equal to the available number of registers.*



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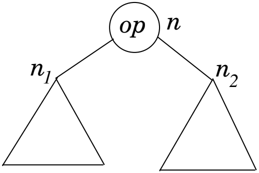

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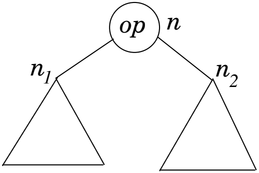

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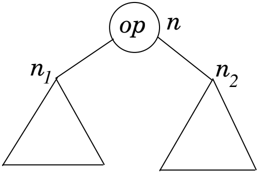

$gencode(n_2)$;

$T := pop(tstack)$;

The Algorithm

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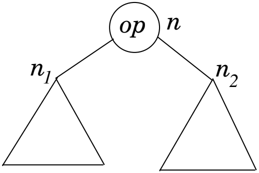

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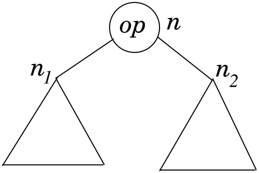

$gencode(n_2);$
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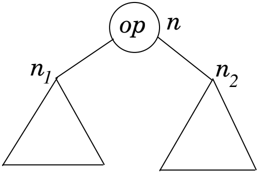

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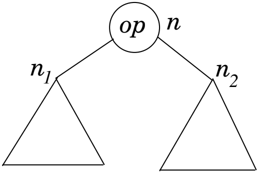

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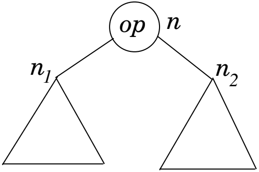

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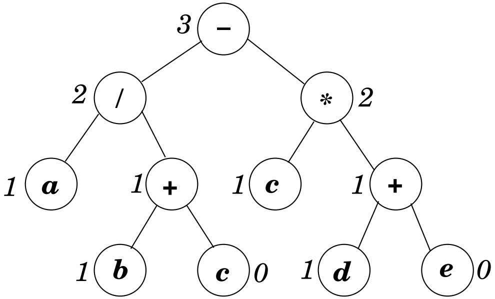
Comments: In this case the right sub-tree is first evaluated into a temporary. This is followed by the evaluations of the left sub-tree and *n* into the register on the top of the stack.

An Example

For the example:



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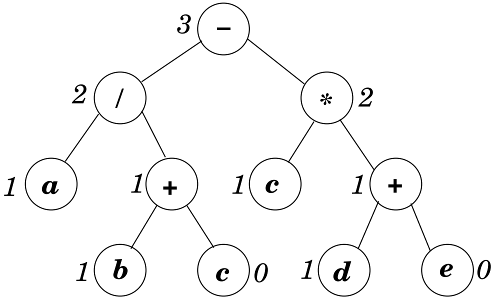
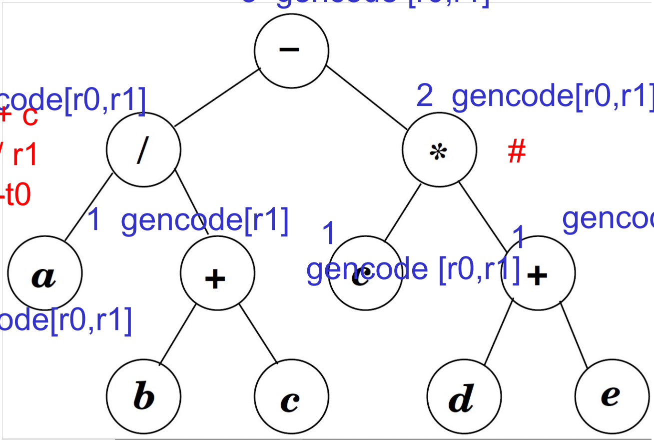


assuming two available registers r_0 and r_1, the calls to gencode and the generated code are shown on the next slide.

```
r0 <- c
r1 <- d
r1 <- r1 + e
r0 <- r0 + r1
t0 <- r0
r0 <- a
r1 <- b
r1 <- r1 + c
r0 <- r0 / r1
r0 <- r0 -t0
```



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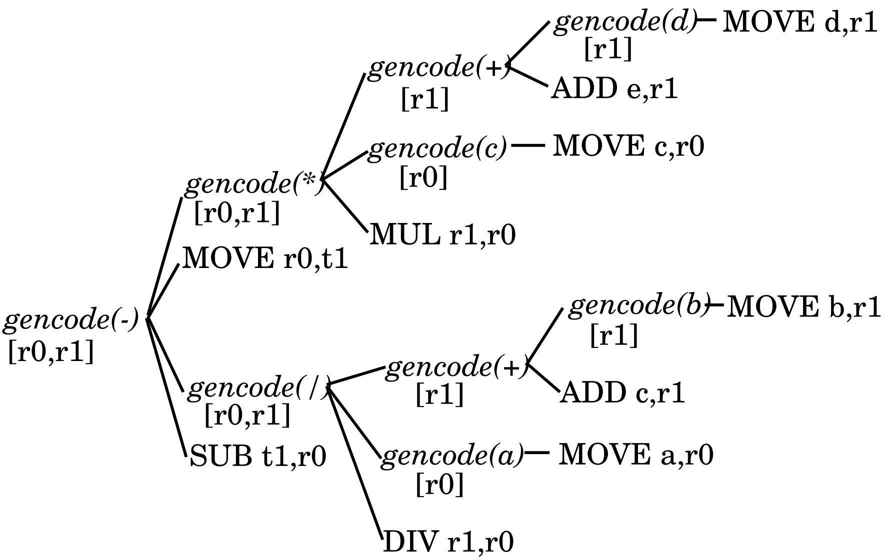
gencode(d)
[r1]
MOVE d,r1
gencode(+)
[r1]
ADD e,r1
gencode(c)
[r0]
MOVE c,r0
gencode(*)
[r0,r1]
MUL r1,r0
MOVE r0,t1
gencode(-)
[r0,r1]
gencode(b)
[r1]
MOVE b,r1
gencode(+)
[r1]
ADD c,r1
gencode(/)
[r0,r1]
gencode(a)
[r0]
MOVE a,r0
SUB t1,r0
DIV r1,r0

SETHI-ULLMAN ALGORITHM: OPTIMALITY

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  1. The number of load instructions generated is optimal.
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  3. The number of stores is optimal.
- We shall now elaborate on each of these.

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1. It is easy to verify that the number of loads required by any program computing an expression tree is at least equal to the number of left leaves. This algorithm generates no more loads than this.

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2. Each node of the expression tree is visited exactly once. If this node specifies a binary operation, then the algorithm branches into steps 2,3,4 or 5, and at each of these places code is generated to perform this operation exactly once.

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SETHI-ULLMAN ALGORITHM: OPTIMALITY

3. The number of stores is optimal: this is harder to show.
   - Define a *major node* as a node, each of whose children has a label at least equal to the number of available registers.
   - If we can show that the number of stores required by *any program* computing an expression tree is at least equal the number of major nodes, then our algorithm produces minimal number of stores (Why?)

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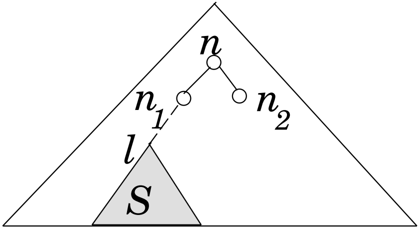

- Let n be the major node in the original tree, just above S, and n_1 and n_2 be its immediate descendants (n_1 could be l itself).

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2. The label of n is $\geq k$.
3. The node n may no longer be a major node *but all other major nodes in the original tree continue to be major nodes in the modified tree.*
4. Therefore the number of major nodes in the modified tree is $M - 1$.
5. If we assume as induction hypothesis that the number of stores for the modified tree is at least $M - 1$, then the number of stores for the original tree is at least M.

SETHI-ULLMAN ALGORITHM: COMPLEXITY

Since the algorithm visits every node of the expression tree twice – once during labeling, and once during code generation, the complexity of the algorithm is $O(n)$.