CS340: Theory of Computation Sem I 2021-22

Lecture Notes 13: Deterministic Pushdown Automata

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1 Deterministic Pushdown Automata

Definition 1.1. A *deterministic pushdown automaton* (in short DPDA), $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ is a PDA where the transition function δ is defined as

$$\delta : Q \times \Sigma_\epsilon \times \Gamma_\epsilon \longrightarrow (Q \times \Gamma_\epsilon) \cup \{\emptyset\}$$

and for all $q \in Q$, $a \in \Sigma$ and $X \in \Gamma$, exactly one of the following

$$\delta(q, a, X), \quad \delta(q, \epsilon, X), \quad \delta(q, a, \epsilon), \quad \delta(q, \epsilon, \epsilon)$$

is not the empty set.
The class of languages accepted by DPDAs are known as *deterministic context-free languages* (or in short, DCFLs).

Exercise 1. Construct a DPDA for the language $L_1 = \{0^n 1^n \mid n \geq 0\}$.

- Not all context-free languages have a DPDA. An example is the language PALINDROMES, consisting of all strings that are palindromes. Intuitively this is because the automaton inherently *requires* the use of non-determinism to guess the middle position of a string. The proof to show that PALINDROMES does not have a DPDA accepting it is beyond the scope of this course.
- Every DCFL will have an unambiguous grammar. Is the reverse true?
- Most programming languages can be described using DPDAs.

Exercise 2. Show that there is a CFL that is not a DCFL and has an unambiguous grammar.

2 Closure Properties of DCFLs

- Closed under complement
- Not closed under union.
- Not closed under intersection.