ESC 201T: Introduction to Electronics

Lecture 6: Toolbox For Circuit Analysis-3

Equivalent Resistance and Superposition

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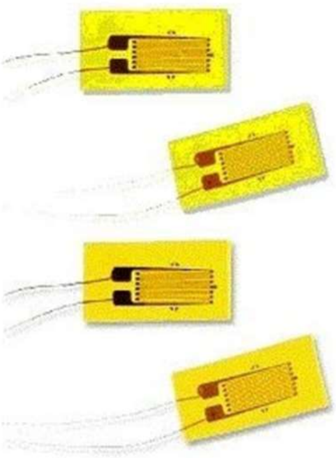
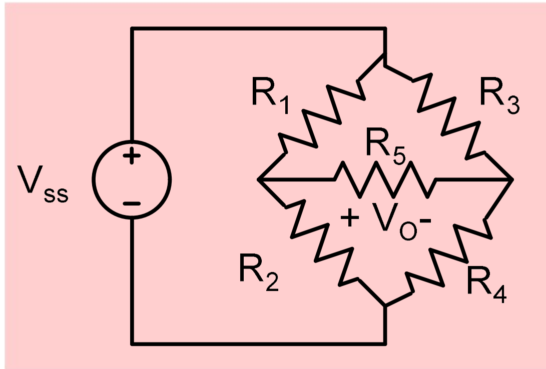
The Five laws of library science by S. R. Ranganathan:

- ❖ Books are for use.
- ❖ Every reader his or her book.
- ❖ Every book its reader.
- ❖ Save the time of the reader.
- ❖ The library must be a growing organism

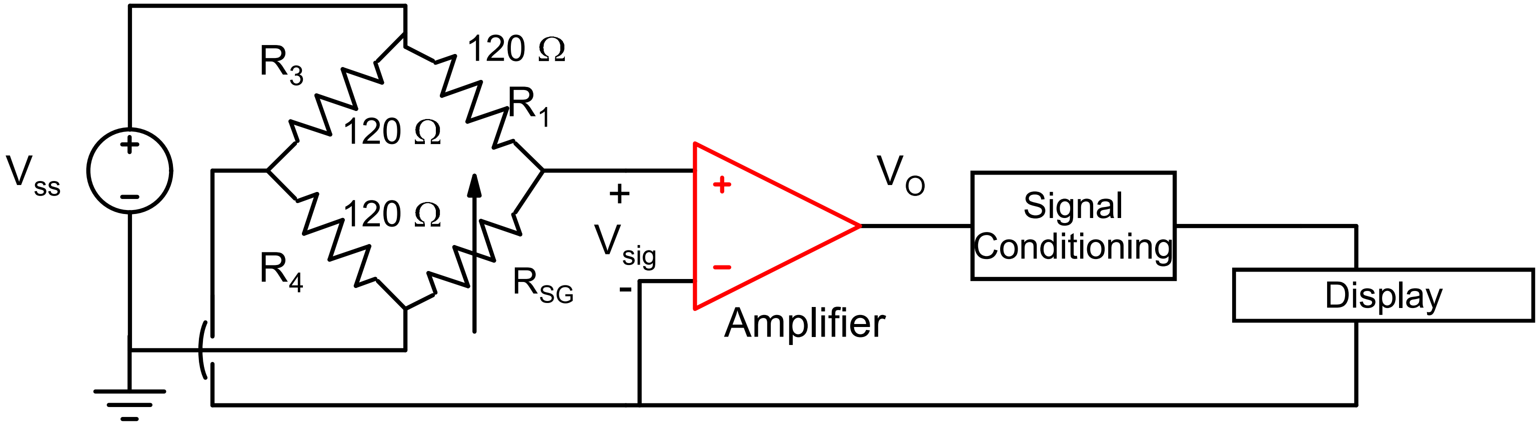
“Analysis is for understanding”

Strain Gauge

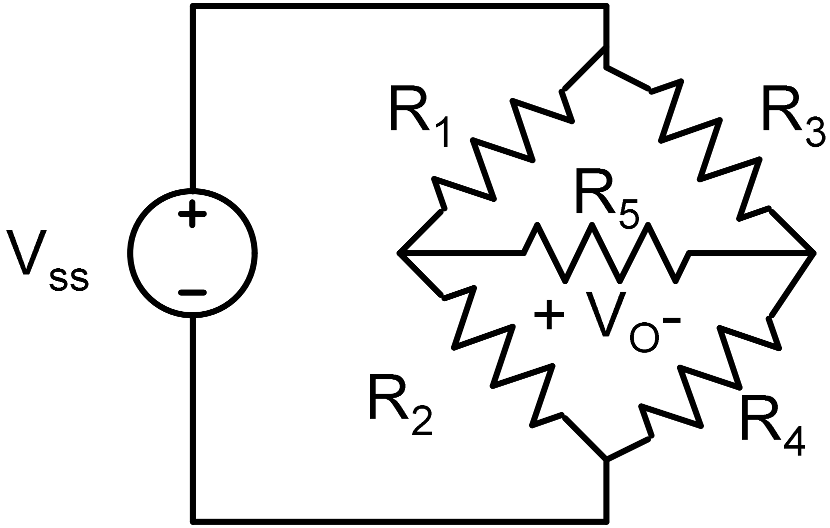


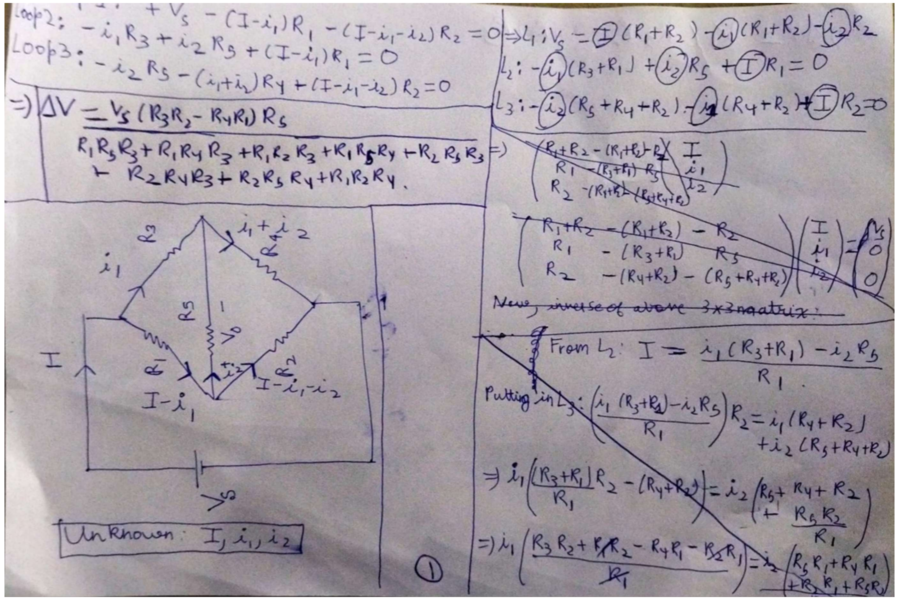


NIE Strain Gauge, 120±0.3Ω,10.0 mm,G.F.-2.11±1%



$$\frac{\Delta R_{SG}}{R_{SG}} = \frac{1}{50} \cdot E \; (\%)$$





$+V_s - (I-i_1)R_1 - (I-i_1-i_2)R_2 = 0 \Rightarrow L_1: V_s = I(R_1+R_2) - i_1(R_1+R_2) - i_2R_2$

Loop2: $-i_1R_3 + i_2R_5 + (I-i_1)R_1 = 0$ $\quad L_2: -i_1(R_3+R_1) + i_2R_5 + IR_1 = 0$

Loop3: $-i_2R_5 - (i_1+i_2)R_4 + (I-i_1-i_2)R_2 = 0$ $\quad L_3: -i_2(R_5+R_4+R_2) - i_1(R_4+R_2) + IR_2 = 0$

$$\Rightarrow \boxed{\Delta V = \frac{V_s(R_3R_2 - R_4R_1)R_5}{R_1R_5R_3 + R_1R_4R_3 + R_1R_2R_3 + R_1R_5R_4 + R_2R_5R_3 + R_2R_4R_3 + R_2R_5R_4 + R_1R_2R_4}}$$

$$\Rightarrow \cancel{\begin{pmatrix} R_1+R_2 & -(R_1+R_2) & -R_2 \\ R_1 & -(R_3+R_1) & R_5 \\ R_2 & -(R_4+R_2) & -(R_5+R_4+R_2) \end{pmatrix}\begin{pmatrix} I \\ i_1 \\ i_2 \end{pmatrix}}$$

$$\cancel{= \begin{pmatrix} R_1+R_2 & -(R_1+R_2) & -R_2 \\ R_1 & -(R_3+R_1) & R_5 \\ R_2 & -(R_4+R_2) & -(R_5+R_4+R_2) \end{pmatrix}\begin{pmatrix} I \\ i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} V_s \\ 0 \\ 0 \end{pmatrix}}$$

~~Now, inverse of above 3x3 matrix:~~

From L_2: $I = \dfrac{i_1(R_3+R_1) - i_2R_5}{R_1}$.

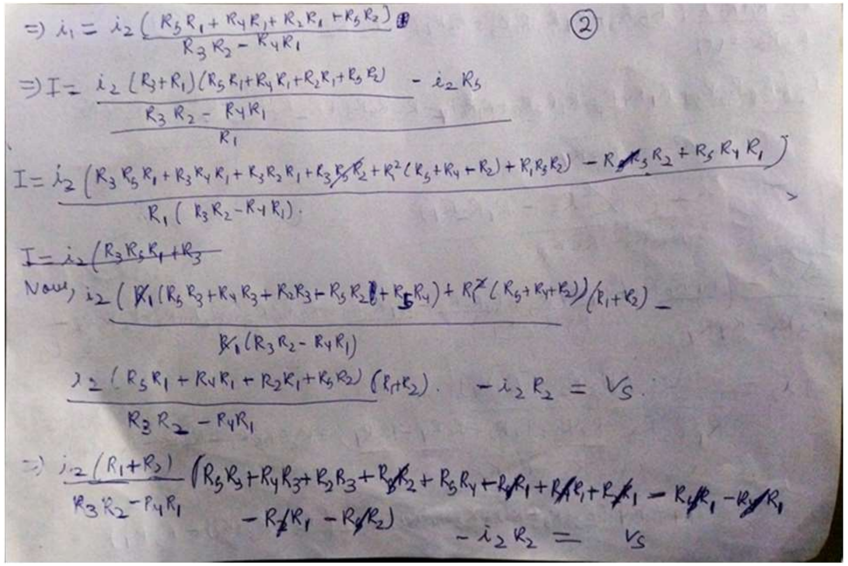
Putting in L_3: $\left(\dfrac{i_1(R_3+R_1) - i_2R_5}{R_1}\right)R_2 = i_1(R_4+R_2) + i_2(R_5+R_4+R_2)$

$$\Rightarrow i_1\left(\frac{(R_3+R_1)}{R_1}R_2 - (R_4+R_2)\right) = i_2\left(R_5 + R_4 + R_2 + \frac{R_5R_2}{R_1}\right)$$

$$\Rightarrow i_1\left(\frac{R_3R_2 + \cancel{R_1R_2} - R_4R_1 - \cancel{R_2R_1}}{\cancel{R_1}}\right) = i_2\left(R_5R_1 + R_4R_1 + R_2R_1 + R_5R_2\right)$$

Unknown: I, i_1, i_2

①



②

$$\Rightarrow i_1 = i_2\frac{(R_5R_1 + R_4R_1 + R_2R_1 + R_5R_2)}{R_3R_2 - R_4R_1}$$

$$\Rightarrow I = \frac{\dfrac{i_2(R_3+R_1)(R_5R_1 + R_4R_1 + R_2R_1 + R_5R_2)}{R_3R_2 - R_4R_1} - i_2R_5}{R_1}$$

$$I = \frac{i_2(R_3R_5R_1 + R_3R_4R_1 + R_3R_2R_1 + R_3\cancel{R_5}R_2 + R_1^2(R_5+R_4+R_2) + R_1R_5R_2) - R_5R_3R_2 + R_5R_4R_1)}{R_1(R_3R_2 - R_4R_1)}$$

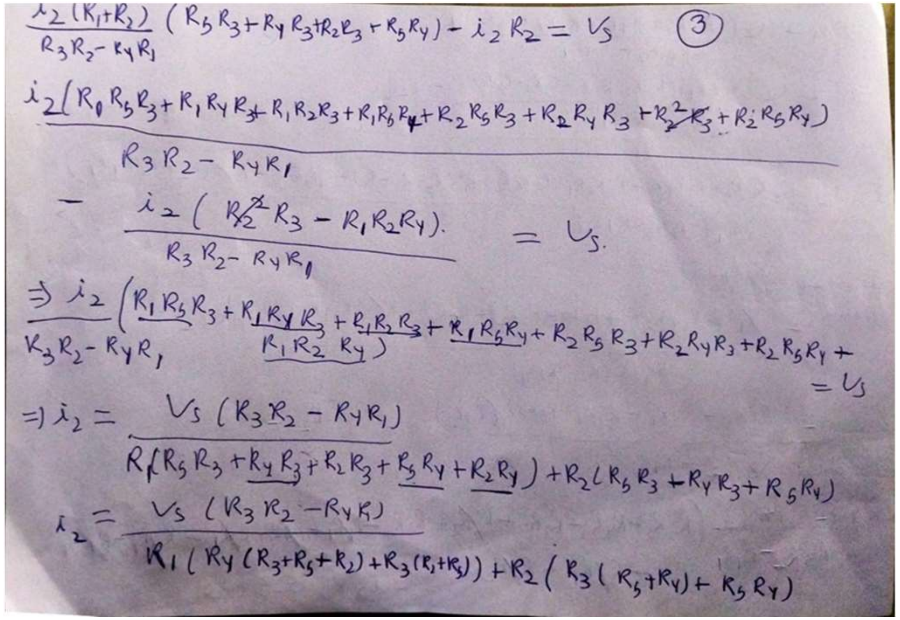
~~$I = i_2(R_3R_5R_1 + R_3$~~

Now, $$\frac{i_2(\cancel{R_1}(R_5R_3 + R_4R_3 + R_2R_3 + R_5R_2 + R_5R_4) + R_1^{\cancel{2}}(R_5+R_4+R_2))}{\cancel{R_1}(R_3R_2 - R_4R_1)}(R_1+R_2) -$$

$$\frac{i_2(R_5R_1 + R_4R_1 + R_2R_1 + R_5R_2)}{R_3R_2 - R_4R_1}(R_1+R_2). \quad - i_2R_2 = V_s.$$

$$\Rightarrow \frac{i_2(R_1+R_2)}{R_3R_2 - R_4R_1}(R_5R_3 + R_4R_3 + R_2R_3 + \cancel{R_5R_2} + R_5R_4 + \cancel{R_5R_1} + \cancel{R_4R_1} + \cancel{R_2R_1} - \cancel{R_5R_1} - \cancel{R_4R_1} - \cancel{R_2R_1} - \cancel{R_5R_2})$$

$$- i_2R_2 = V_s$$



③

$$\frac{i_2(R_1+R_2)}{R_3R_2 - R_4R_1}(R_5R_3 + R_4R_3 + R_2R_3 + R_5R_4) - i_2R_2 = V_s$$

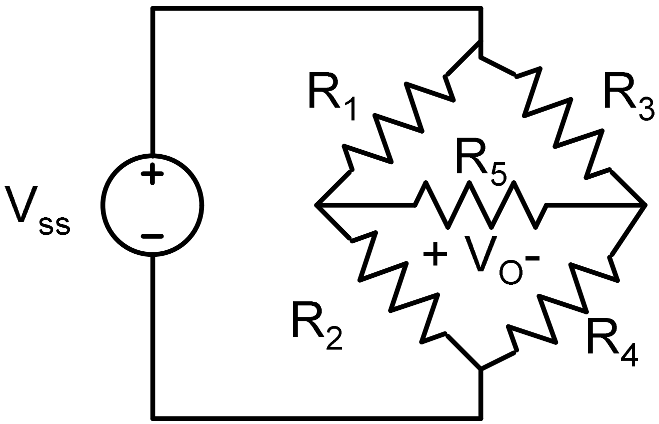
$$\frac{i_2(R_1R_5R_3 + R_1R_4R_3 + R_1R_2R_3 + R_1R_5R_4 + R_2R_5R_3 + R_2R_4R_3 + \cancel{R_2^2R_3} + R_2R_5R_4)}{R_3R_2 - R_4R_1}$$

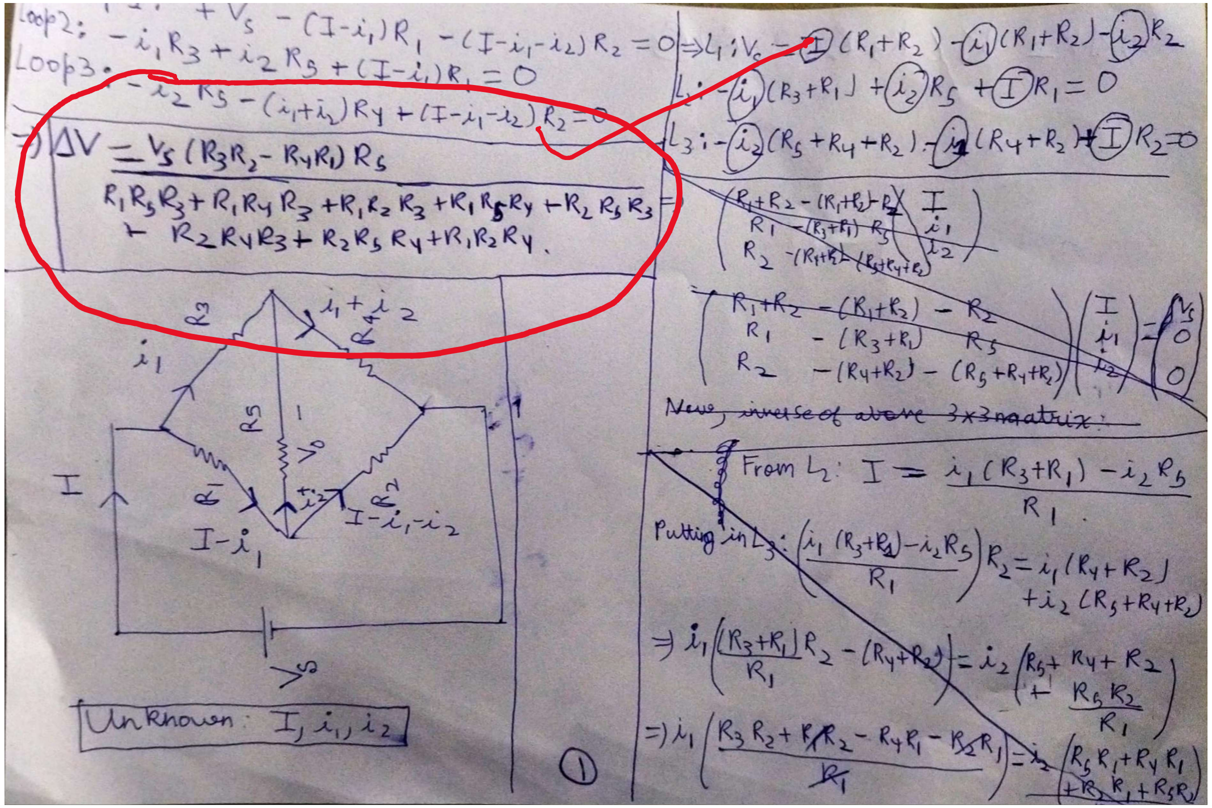
$$- \frac{i_2(\cancel{R_2^2}R_3 - R_1R_2R_4).}{R_3R_2 - R_4R_1} = V_s.$$

$$\Rightarrow \frac{i_2}{R_3R_2 - R_4R_1}(R_1R_5R_3 + R_1R_4R_3 + R_1R_2R_3 + R_1R_5R_4 + R_2R_5R_3 + R_2R_4R_3 + R_2R_5R_4 + R_1R_2R_4) = V_s$$

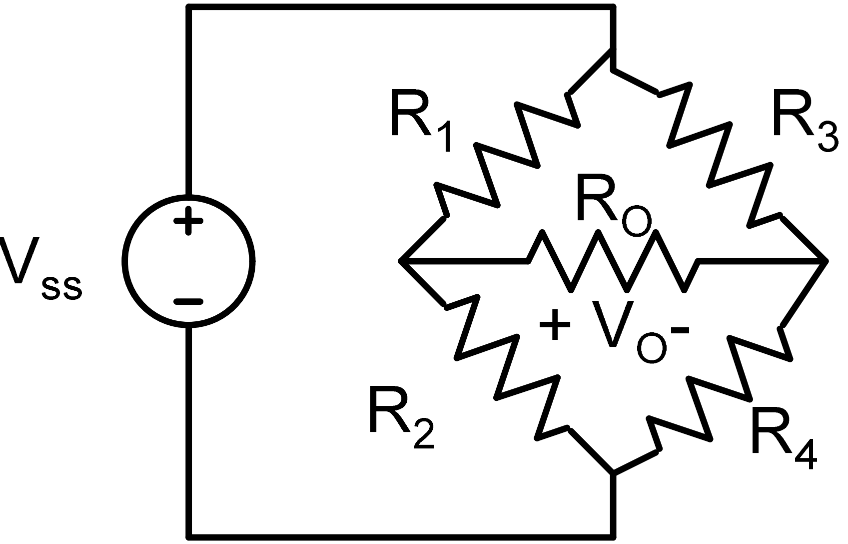
$$\Rightarrow i_2 = \frac{V_s(R_3R_2 - R_4R_1)}{R_1(R_5R_3 + R_4R_3 + R_2R_3 + R_5R_4 + R_2R_4) + R_2(R_5R_3 + R_4R_3 + R_5R_4)}$$

$$i_2 = \frac{V_s(R_3R_2 - R_4R_1)}{R_1(R_4(R_3+R_5+R_2) + R_3(R_2+R_5)) + R_2(R_3(R_5+R_4) + R_5R_4)}$$





$$V_O = \frac{V_{SS} \times R_5 \times (R_2R_3 - R_1R_4)}{R_2R_4R_5 + R_2R_3R_5 + R_1R_4R_5 + R_1R_3R_5 + R_2R_3R_4 + R_1R_3R_4 + R_1R_2R_4 + R_1R_2R_3}$$

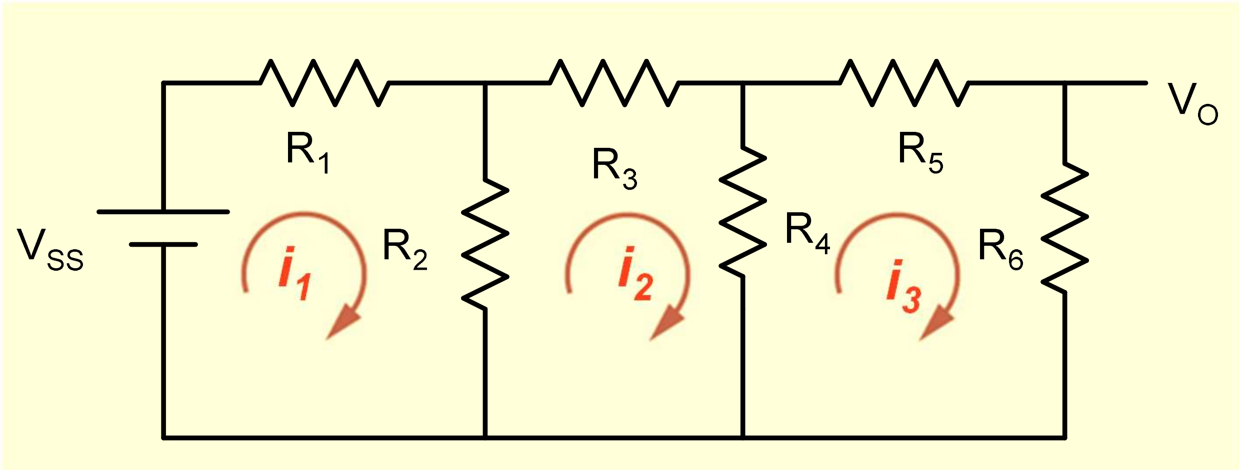


$$V_O = V_{SS} \times \left(\frac{R_2}{R_1+R_2} - \frac{R_4}{R_3+R_4}\right) \times \left\{\frac{R_O}{R_O+\{(R_1\|R_2)+(R_3\|R_4)\}}\right\}$$

A Structured expression that reveals role of each element

$$V_O = \frac{V_{SS}\times R_O\times(R_2R_3-R_1R_4)}{R_2R_4R_0+R_2R_3R_0+R_1R_4R_0+R_1R_3R_0+R_2R_3R_4+R_1R_3R_4+R_1R_2R_4+R_1R_2R_3}$$

A Disordered expression that does not provide insight



$$V_o = \frac{V_{SS} \times R_2\ R_4 R_6}{R_2R_4R_6 + R_2R_4R_5 + R_2R_3R_6 + R_2R_3R_5 + R_2R_3R_4 + R_1R_4R_6 + R_1R_4R_5 + R_1R_3R_6 + R_1R_3R_5 + R_1R_3R_4 + R_1R_2R_6 + R_1R_2R_5 + R_1R_2R_4}$$

- Mesh and Nodal analysis are “brute-force” techniques that are not only time consuming and error prone for us to use but also yield **unstructured** expressions that are often unsuitable for gaining **insight** into operation of circuits and **modifying or designing** them.

- Need techniques that yield relatively simpler structured expressions that reveal the role of different components and that require less effort and is less error prone

Analysis using REUSE Methodology

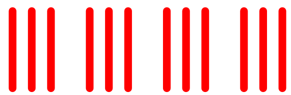
Do not carry out analysis from scratch !

Analyze, Remember and Reuse

Example: we do not carry out multiplication from scratch using repeated addition !

$$\begin{array}{r} 3\ \ 4 \\ \times\ \ 3 \\ \hline 10\ 2 \\ \hline \end{array}$$

$3 \times 4 = 12$

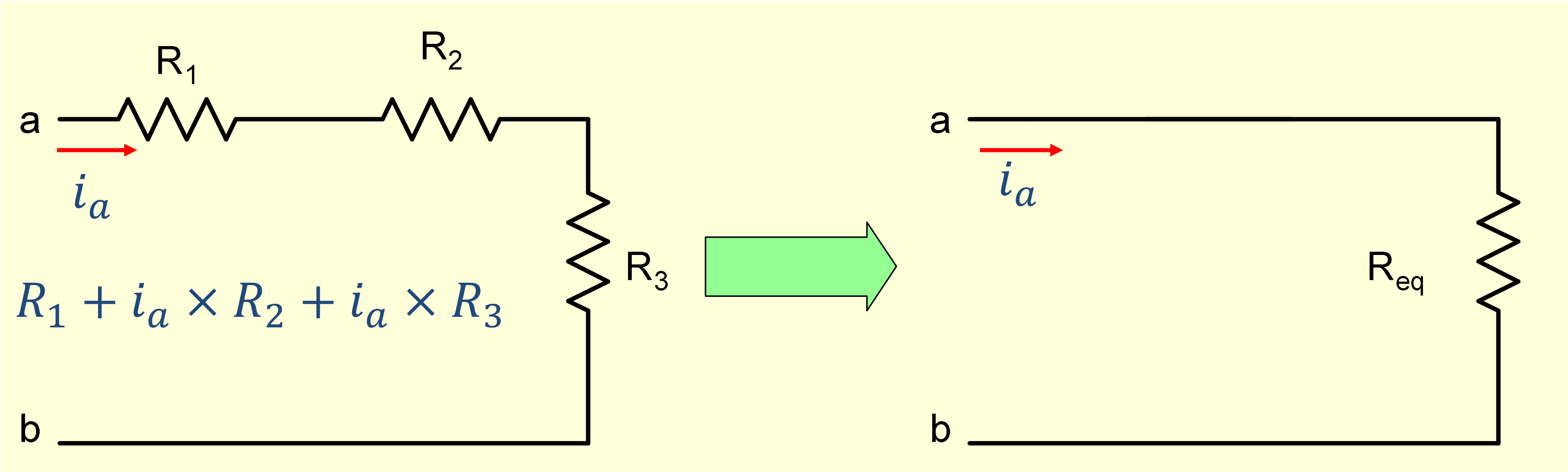


You cannot carry out complex multiplication with ease using the first principle

$4 \times 1 = 4$
$4 \times 2 = 8$
$4 \times 3 = 12$
$4 \times 4 = 16$
....

Memorize multiplication table and use it again and again

Equivalent Series Resistances



$$v_{ab} = i_a \times R_1 + i_a \times R_2 + i_a \times R_3$$

$$v_{ab} = i_a \times (R_1 + R_2 + R_3)$$

$$v_{ab} = i_a \times R_{eq}$$

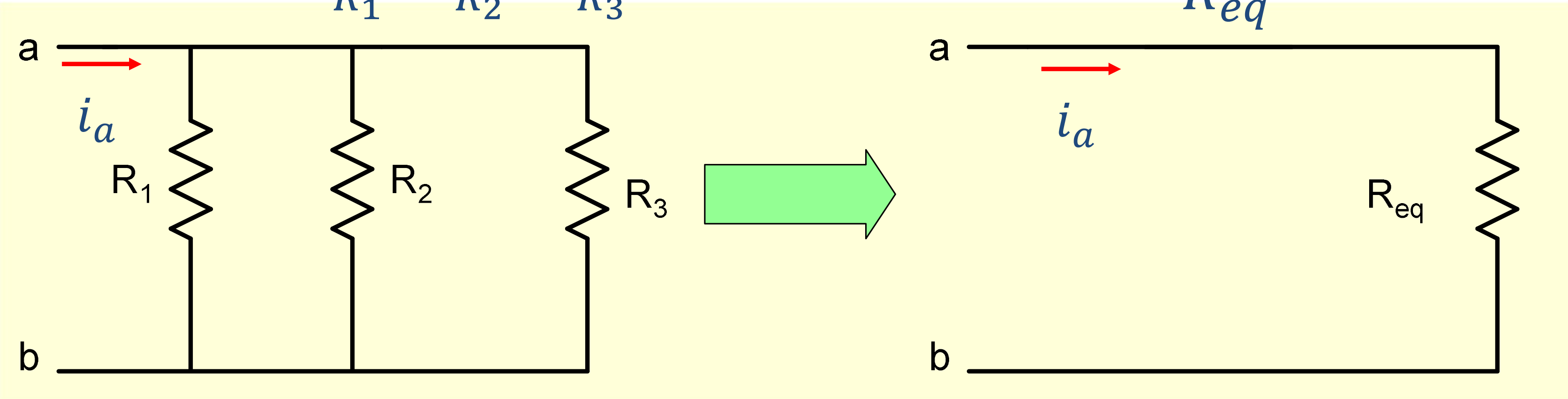
$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

Both circuits are equivalent as far as terminal ***V* vs. *I*** relation is concerned.

Once derived, this is a useful result that can be Re-Used at many places

Parallel Resistances

$$i_a = v_{ab} \times \{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\} \Longleftrightarrow i_a = \frac{v_{ab}}{R_{eq}}$$



$$i_a = \frac{v_{ab}}{R_1} + \frac{v_{ab}}{R_2} + \frac{v_{ab}}{R_3}$$

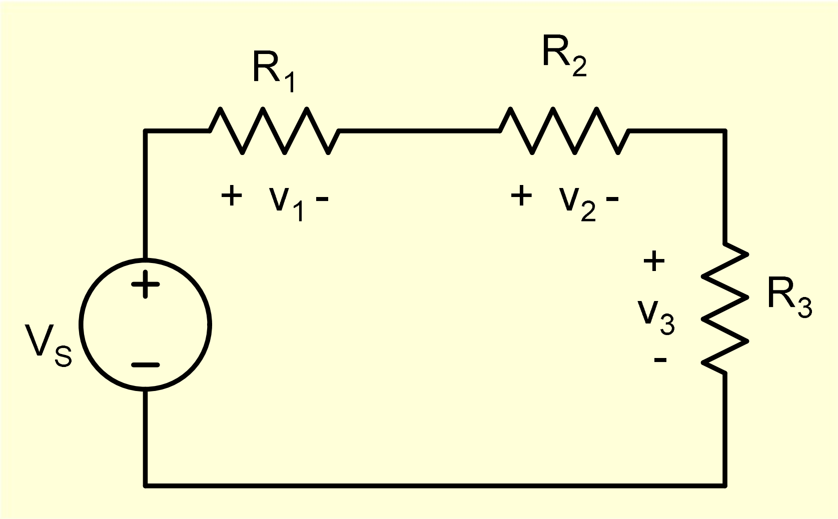
$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Once derived, this is a useful result that can be Re-Used at many places

Voltage division

Another example of Analysis Reuse

A voltage applied to resistors connected in series will be divided among them



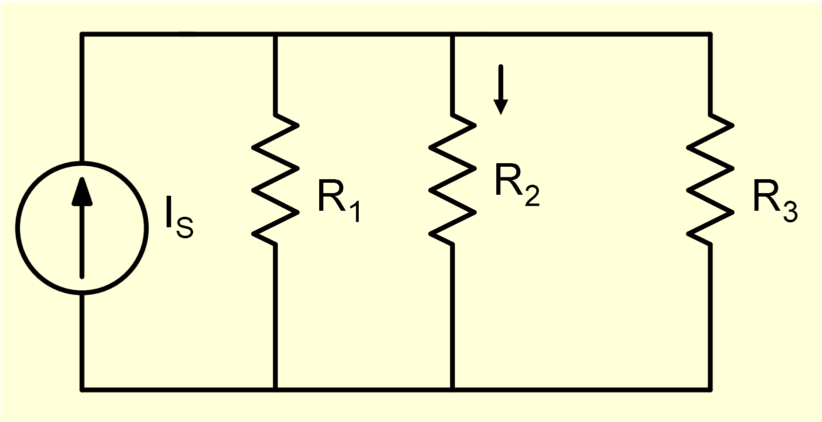
$$v_J = v_S \times \frac{R_J}{\Sigma \mathrm{R_i}}$$

$$v_2 = v_S \times \frac{R_2}{R_1 + R_2 + R_3}$$

Once derived using first principles, this is a useful result that can be Re-Used at many places

Current Division

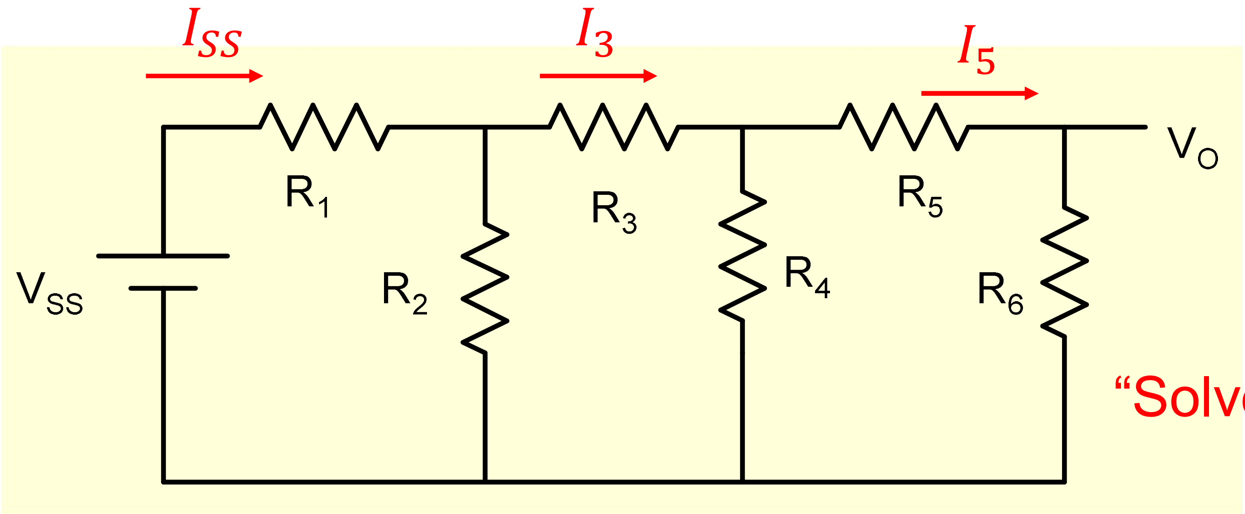
The total current flowing into a parallel combination of resistors will be divided among them



$$I_2 = I_S \times \frac{1/R_2}{1/R_1 + 1/R_2 + 1/R_3}$$

$I_2 = I_S \times \frac{R_1}{R_1 + R_2}$ for two resistors

Example

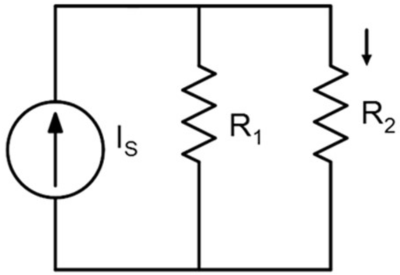


$$V_O = I_5 \times R_6$$

"Solve circuits by inspection"

Re-use series and parallel results

$$R_{eq} = (\{\{(R_5 + R_6)\|R_4\} + R_3\}\|R_2) + R_1 \qquad I_{SS} = \frac{V_{SS}}{R_{eq}}$$

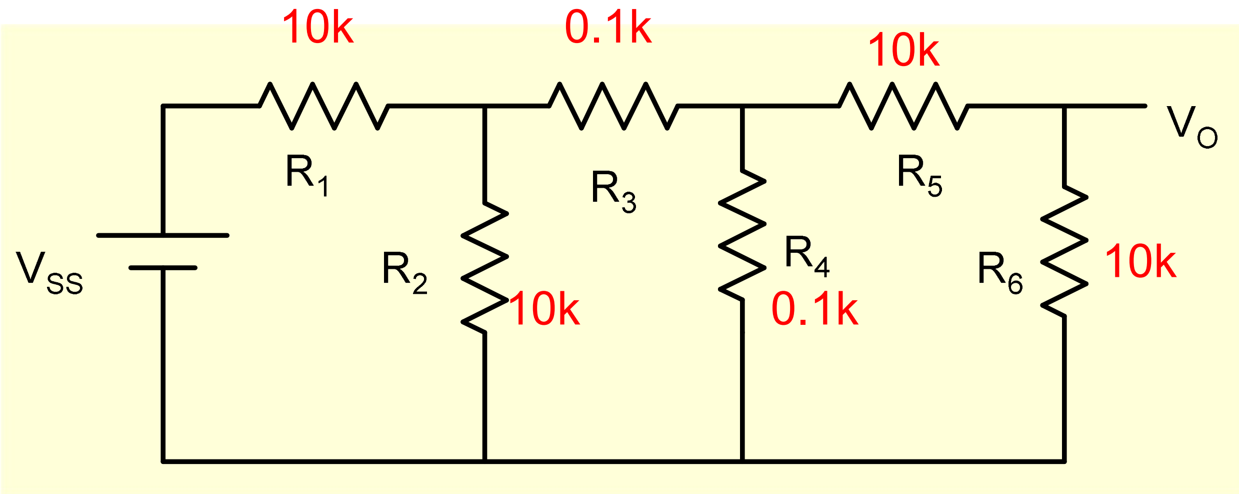


$$I_2 = I_S \times \frac{R_1}{R_1 + R_2}$$

Re-use current division result to calculate I_3 and I_5

$$I_3 = I_{SS} \times \frac{R_2}{R_2 + \{(R_5 + R_6)\|R_4\} + R_3\}} \qquad I_5 = I_3 \times \frac{R_4}{R_4 + R_5 + R_6}$$

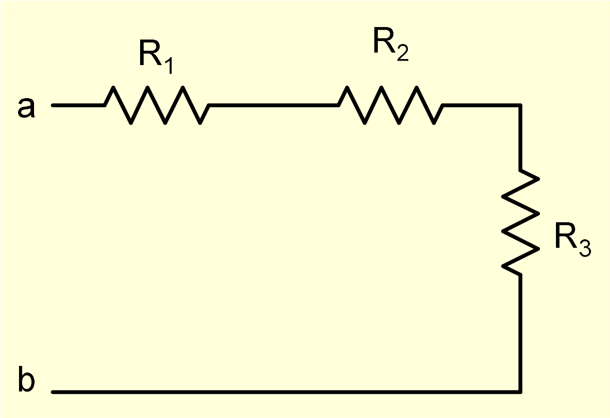
$$V_O = \frac{V_{SS}}{(\{\{(R_5+R_6)\|R_4\}+R_3\}\|R_2)+R_1} \times \frac{R_2}{R_2+\{(R_5+R_6)\|R_4\}+R_3\}} \times \frac{R_4}{R_4+R_5+R_6} \times R_6$$

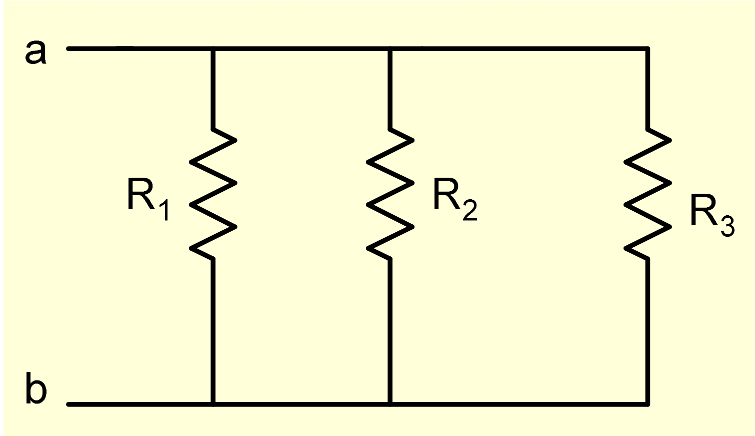


$$V_o = \frac{V_{SS} \times R_2\ R_4 R_6}{R_2R_4R_6 + R_2R_4R_5 + R_2R_3R_6 + R_2R_3R_5 + R_2R_3R_4 + R_1R_4R_6 + R_1R_4R_5 + R_1R_3R_6 + R_1R_3R_5 + R_1R_3R_4 + R_1R_2R_6 + R_1R_2R_5 + R_1R_2R_4}$$

$$V_O = \frac{V_{SS}}{(\{\{(R_5+R_6)\|R_4\}+R_3\}\|R_2)+R_1} \times \frac{R_2}{R_2+\{(R_5+R_6)\|R_4\}+R_3\}} \times \frac{R_4}{R_4+R_5+R_6} \times R_6$$

$$V_O \approx \frac{V_{SS}}{R_1} \times 1 \times \frac{R_4}{2}$$





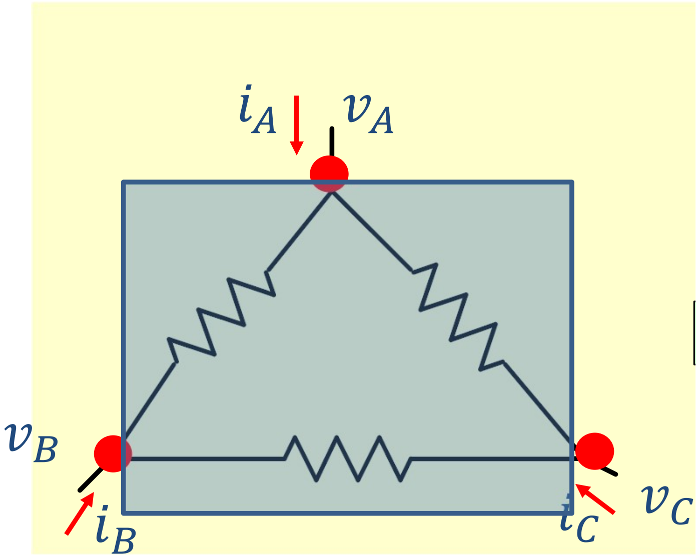
$$R_{eq} = R_1 + R_2 + R_3$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$v_2 = v_S \times \frac{R_2}{R_1 + R_2 + R_3}$$

$$I_2 = I_S \times \frac{1/R_2}{1/R_1 + 1/R_2 + 1/R_3}$$

Equivalent Circuits

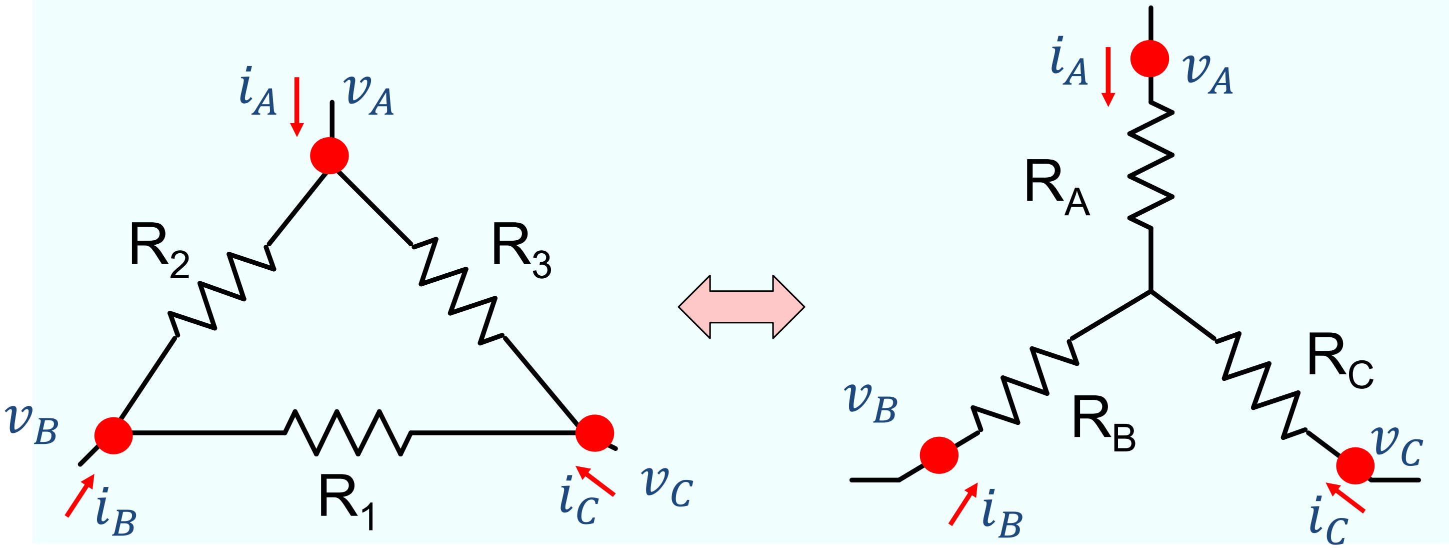


Circuit-A

Circuit-B

B is equivalent to A as far as terminal current voltage characteristics is concerned

Example



$$R_A = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_2 R_1}{R_1 + R_2 + R_3}$$

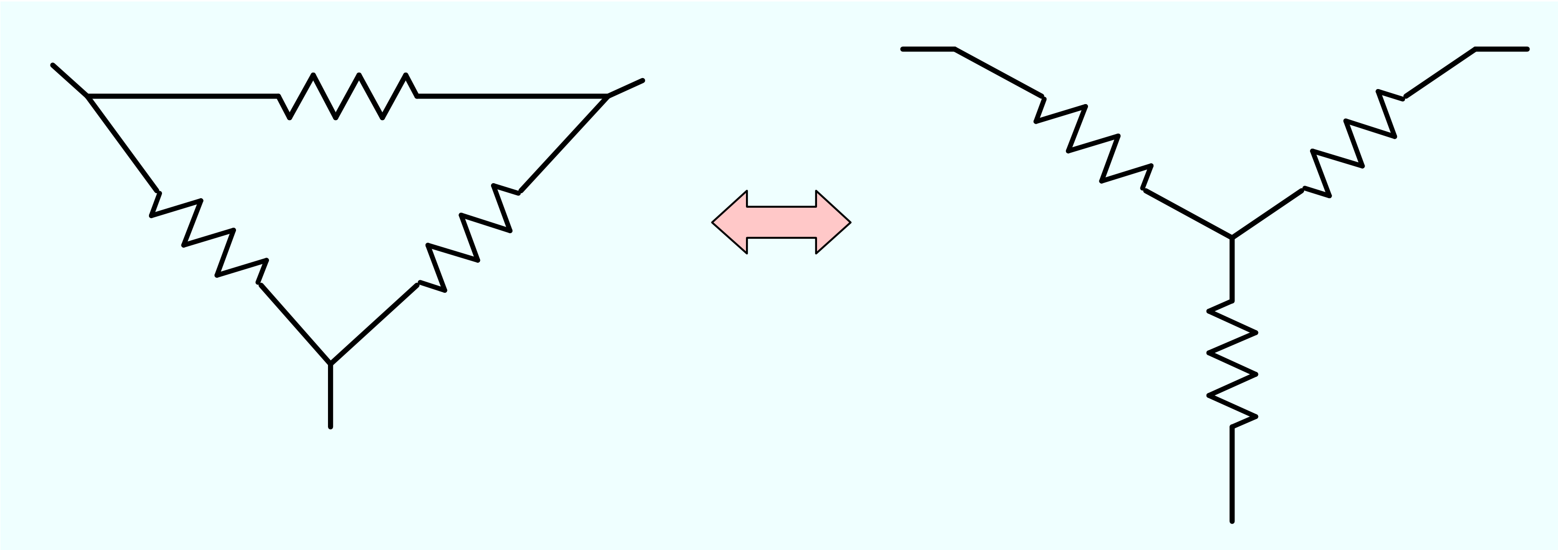
$$R_C = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

$$R_2 = R_A + R_B + \frac{R_A R_B}{R_C}$$

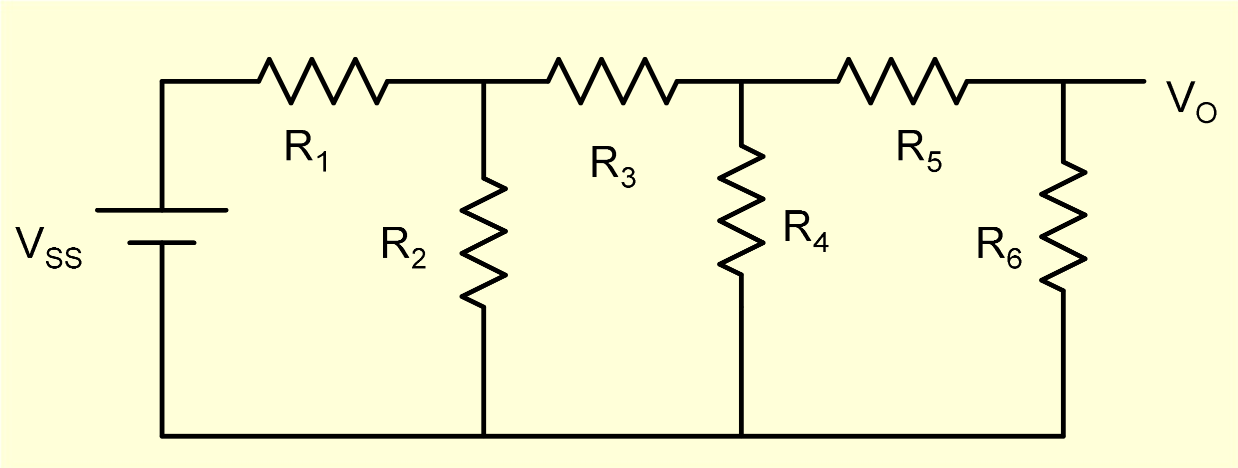
$$R_1 = R_C + R_B + \frac{R_C R_B}{R_A}$$

$$R_3 = R_A + R_C + \frac{R_A R_C}{R_B}$$

$\Delta - Y$ Transformation



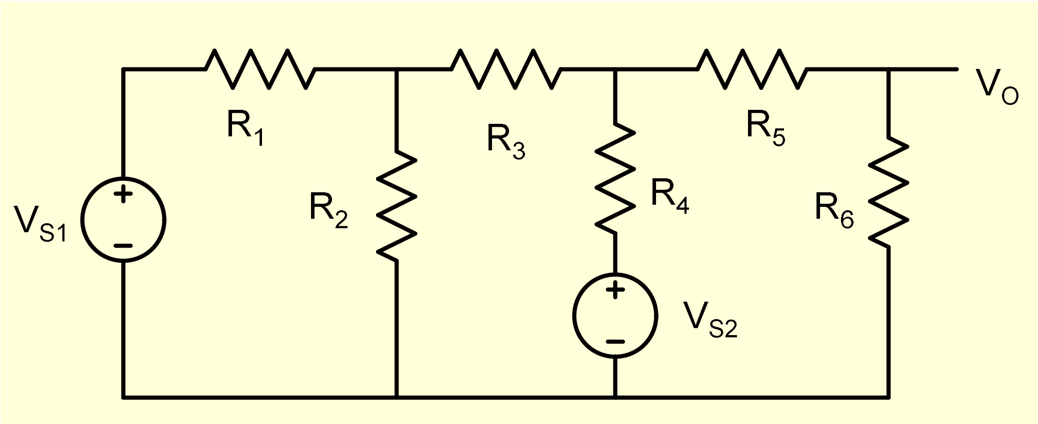
Solve a Complex Problem by Breaking it into Simpler problems.



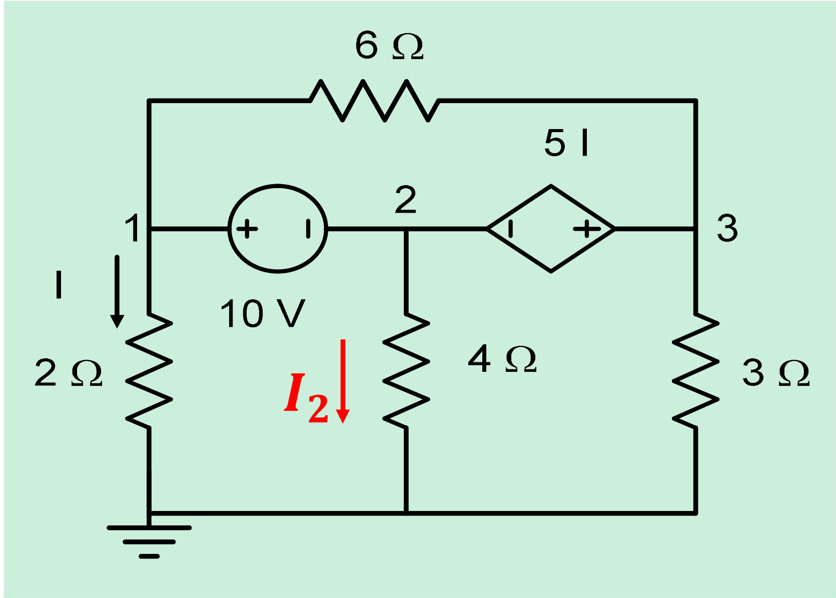
$$V_O = \frac{V_{SS}}{(\{\{(R_5+R_6)\|R_4\}+R_3\}\|R_2)+R_1} \times \frac{R_2}{R_2+\{(R_5+R_6)\|R_4\}+R_3\}} \times \frac{R_4}{R_4+R_5+R_6} \times R_6$$

$$V_O = K \times V_{SS}$$

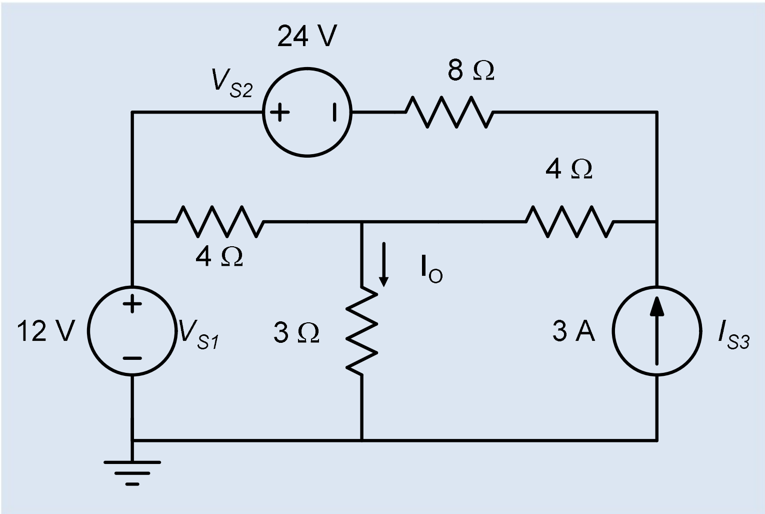
Circuit made of linear components is a linear circuit



$$V_O = K_{O1} \times V_{S1} + K_{O2} \times V_{S2}$$



$$I_2 = K_2 \times 10$$

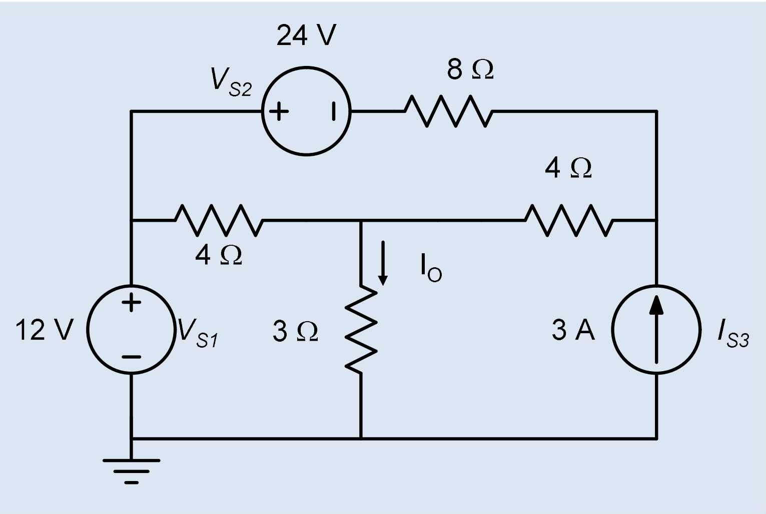


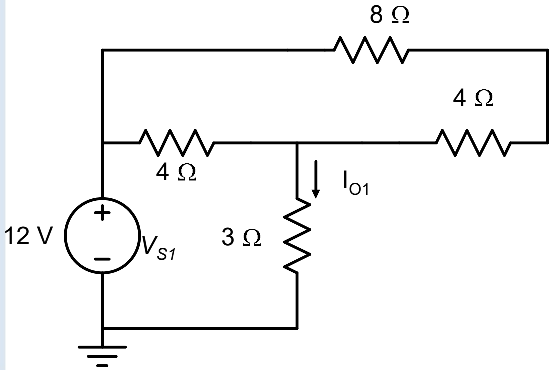
$$I_O = K_1 \times V_{S1} + K_2 \times V_{S2} + K_3 \times I_{S3}$$

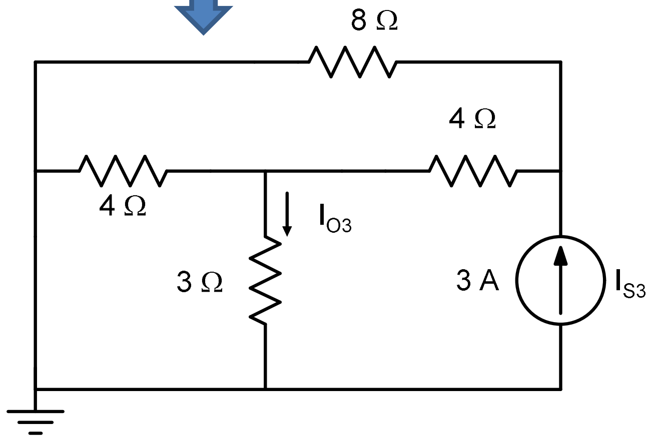
The **superposition principle** states that the total response is the sum of the responses to each of the independent sources acting individually.

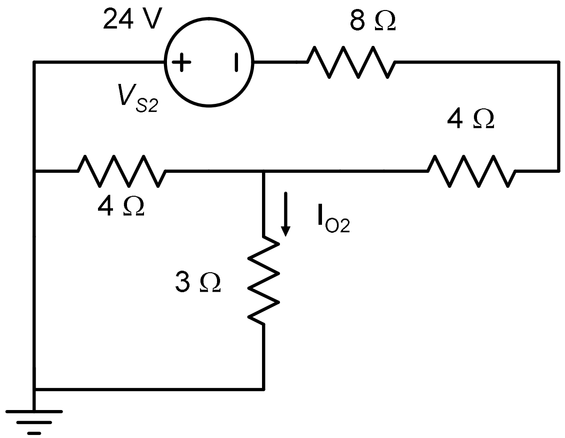
The Superposition principle allows a complex circuit to be decomposed into simpler circuits

$$I_O = K_1 \times V_{S1} + K_2 \times V_{S2} + K_3 \times I_{S3}$$









$$I_O = I_{O1} + I_{O2} + I_{O3}$$