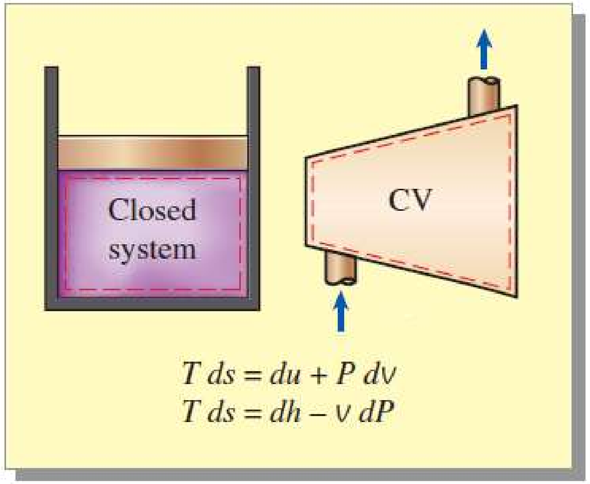
Entropy Changes in Ideal Gases

Raj Pala,
rpala@iitk.ac.in
Department of Chemical Engineering,
Associate faculty of the Materials Science Programme,
Indian Institute of Technology, Kanpur.

Previously: Beyond $\Delta s = \left(\frac{Q}{T}\right)_{rev}$, *TdS relationships & Entropy chanes in liquids/Solids*

$$ds = \frac{du}{T} + \frac{P\,dv}{T}$$

$$ds = \frac{dh}{T} - \frac{v\,dP}{T}$$



- Computational procedure dependent on the changes in state properties

$$dv \cong 0 \qquad ds = \frac{du}{T} = \frac{c\,dT}{T}$$

Liquids, solids: $$s_2 - s_1 = \int_1^2 c(T)\,\frac{dT}{T} \cong c_{avg} \ln \frac{T_2}{T_1} \qquad \text{(kJ/kg} \cdot \text{K)}$$

Isentropic: $$s_2 - s_1 = c_{avg} \ln \frac{T_2}{T_1} = 0 \quad \rightarrow \quad T_2 = T_1$$

Fig-TD: Cengel & Boles

Entropy analysis in ideal gases

From the first $T\,ds$ relation

$$ds = \frac{du}{T} + \frac{P\,dv}{T} \qquad du = c_v\,dT \qquad P = RT/v$$

$$ds = c_v \frac{dT}{T} + R\frac{dv}{v}$$

$$s_2 - s_1 = \int_1^2 c_v(T)\frac{dT}{T} + R\ln\frac{v_2}{v_1}$$

From the second $T\,ds$ relation

$$ds = \frac{dh}{T} - \frac{v\,dP}{T}$$

$$dh = c_p\,dT \qquad v = RT/P$$

$$s_2 - s_1 = \int_1^2 c_p(T)\frac{dT}{T} - R\ln\frac{P_2}{P_1}$$

Constant specific heat approximation

$$s_2 - s_1 = \int_1^2 c_v(T)\frac{dT}{T} + R\ln\frac{v_2}{v_1} \longrightarrow s_2 - s_1 = c_{v,\mathrm{avg}}\ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$$

$$s_2 - s_1 = \int_1^2 c_p(T)\frac{dT}{T} - R\ln\frac{P_2}{P_1} \longrightarrow s_2 - s_1 = c_{p,\mathrm{avg}}\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$(\mathrm{kJ/kg\cdot K})$

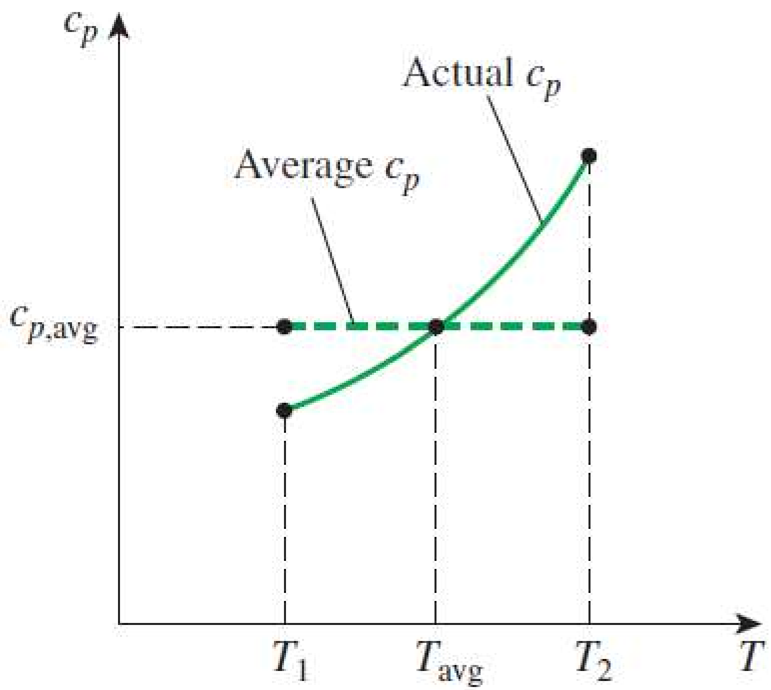


Fig-TD: Cengel & Boles

Isoentropic processes: Constant specific heat

$$s_2 - s_1 = c_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

$$\ln \frac{T_2}{T_1} = -\frac{R}{c_v} \ln \frac{v_2}{v_1}$$

$$\ln \frac{T_2}{T_1} = \ln \left(\frac{v_1}{v_2}\right)^{R/c_v}$$

$$R = c_p - c_v,\ k = c_p/c_v$$

and thus $R/c_v = k - 1$

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$\left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \left(\frac{v_1}{v_2}\right)^{k}$$

$$Tv^{k-1} = \text{constant}$$

$$TP^{(1-k)/k} = \text{constant}$$

$$Pv^{k} = \text{constant}$$

What's next?

- Reversible steady-flow work