

Practice: Extend the number evaluation scheme to support grammar which has real numbers  
(rule  $\text{number} \rightarrow \text{sign list} . \text{list}$  replaces  $\text{number} \rightarrow \text{sign list}$  )

$\text{number} \rightarrow \text{sign list}$	$\text{list.position} \leftarrow 0$ if $\text{sign.negative}$ then $\text{number.value} \leftarrow - \text{list.value}$ else $\text{number.value} \leftarrow \text{list.value}$
--	--

$\text{sign} \rightarrow +$	$\text{sign.negative} \leftarrow \text{false}$
$\text{sign} \rightarrow -$	$\text{sign.negative} \leftarrow \text{true}$

$\text{list} \rightarrow \text{bit}$	$\text{bit.position} \leftarrow \text{list.position}$ $\text{list.value} \leftarrow \text{bit.value}$
$\text{list}_0 \rightarrow \text{list}_1 \text{ bit}$	$\text{list}_1.\text{position} \leftarrow \text{list}_0.\text{position} + 1$ $\text{bit.position} \leftarrow \text{list}_0.\text{position}$ $\text{list}_0.\text{value} \leftarrow \text{list}_1.\text{value} + \text{bit.value}$

$\text{bit} \rightarrow 0$	$\text{bit.value} \leftarrow 0$
$\text{bit} \rightarrow 1$	$\text{bit.value} \leftarrow 2^{\text{bit.position}}$



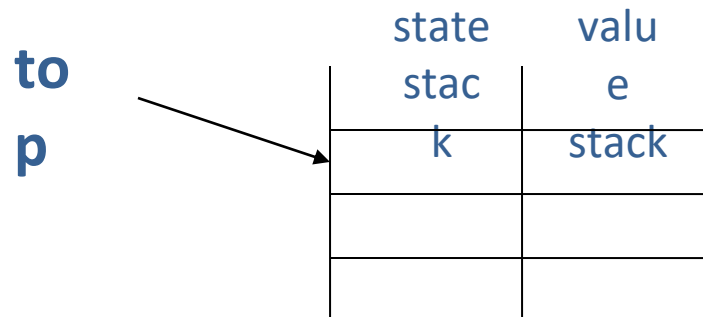
# Compiler Design

## Attribute Evaluation

Amey Karkare  
Department of Computer Science and Engineering  
IIT Kanpur  
[karkare@iitk.ac.in](mailto:karkare@iitk.ac.in)

# Bottom-up evaluation of S-attributed definitions

- Can be evaluated while parsing
- Whenever reduction is made, value of new synthesized attribute is computed from the attributes on the stack
- Extend stack to hold the values also
- The current top of stack is indicated by **top** pointer



# Bottom-up evaluation of S-attributed definitions

- Suppose semantic rule
$$A.a = f(X.x, Y.y, Z.z)$$
is associated with production
$$A \rightarrow XYZ$$
- Before reducing  $XYZ$  to  $A$ , value of  $Z$  is in  $\text{val}(\text{top})$ , value of  $Y$  is in  $\text{val}(\text{top}-1)$  and value of  $X$  is in  $\text{val}(\text{top}-2)$
- If symbol has no attribute then the entry is undefined
- After the reduction,  $\text{top}$  is decremented by 2 and state covering  $A$  is put in  $\text{val}(\text{top})$

# Example: desk calculator

$L \rightarrow E \$$   
 $E \rightarrow E + T$   
 $E \rightarrow T$   
 $T \rightarrow T * F$   
 $T \rightarrow F$   
 $F \rightarrow (E)$   
 $F \rightarrow \text{digit}$

Print (E.val)  
 $E.\text{val} = E.\text{val} + T.\text{val}$   
 $E.\text{val} = T.\text{val}$   
 $T.\text{val} = T.\text{val} * F.\text{val}$   
 $T.\text{val} = F.\text{val}$   
 $F.\text{val} = E.\text{val}$   
 $F.\text{val} = \text{digit.lexval}$

# Example: desk calculator

$L \rightarrow E\$$

$E \rightarrow E + T$

$E \rightarrow T$

$T \rightarrow T * F$

$T \rightarrow F$

$F \rightarrow (E)$

$F \rightarrow \text{digit}$

Before reduction  $\text{ntop} = \text{top} - r + 1$

After code reduction  $\text{top} = \text{ntop}$

$r$  is the #symbols on RHS

## Example: desk calculator

$L \rightarrow E\$$       `print(val(top))`  
 $E \rightarrow E + T$     `val(ntop) = val(top-2) + val(top)`  
 $E \rightarrow T$   
 $T \rightarrow T * F$     `val(ntop) = val(top-2) * val(top)`  
 $T \rightarrow F$   
 $F \rightarrow (E)$       `val(ntop) = val(top-1)`  
 $F \rightarrow \text{digit}$

Before reduction       $\text{ntop} = \text{top} - r + 1$

After code reduction  $\text{top} = \text{ntop}$

$r$  is the #symbols on RHS

INPUT	STATE	Val	PROD
3*5+4\$			
*5+4\$	digit	3	
*5+4\$	F	3	$F \rightarrow \text{digit}$
*5+4\$	T	3	$T \rightarrow F$
5+4\$	T*	3 □	
+4\$	T*digit	3 □ 5	
+4\$	T*F	3 □ 5	$F \rightarrow \text{digit}$
+4\$	T	15	$T \rightarrow T * F$
+4\$	E	15	$E \rightarrow T$
4\$	E+	15 □	
\$	E+digit	15 □ 4	
\$	E+F	15 □ 4	$F \rightarrow \text{digit}$
\$	E+T	15 □ 4	$T \rightarrow F$
\$	E	19	$E \rightarrow E + T$

□ is a dummy placeholder.



# YACC Terminology

$E \rightarrow E + T$      $\text{val}(\text{ntop}) = \text{val}(\text{top}-2) + \text{val}(\text{top})$

In YACC

$E \rightarrow E + T$      $\$\$ = \$1 + \$3$

$\$\$$  maps to  $\text{val}[\text{top} - r + 1]$

$\$k$  maps to  $\text{val}[\text{top} - r + k]$

$r$  = #symbols on RHS ( here 3)

$\$\$ = \$1$  is the *default* action in YACC

# L-attributed definitions

- When translation takes place during parsing, order of evaluation is linked to the order in which nodes are created
- In S-attributed definitions parent's attribute evaluated after child's.
- A natural order in both top-down and bottom-up parsing is depth first-order
- **L-attributed** definition: where attributes can be evaluated in depth-first order

# L attributed definitions ...

- A syntax directed definition is L-attributed if each inherited attribute of  $X_j$  ( $1 \leq j \leq n$ ) at the right hand side of  $A \rightarrow X_1 X_2 \dots X_n$  depends only on
  - Attributes of symbols  $X_1 X_2 \dots X_{j-1}$  and
  - Inherited attribute of  $A$
- Examples (i inherited, s synthesized)

$A \rightarrow LM$

$L.i = f_1(A.i)$   
 $M.i = f_2(L.s)$   
 $A.s = f_3(M.s)$



$A \rightarrow QR$

$R.i = f_4(A.i)$   
 $Q.i = f_5(R.s)$   
 $A.s = f_6(Q.s)$



# Translation schemes

- A CFG where semantic actions occur within the rhs of production
- Example: A translation scheme to map infix to postfix

$E \rightarrow T R$

$R \rightarrow \text{addop } T R \mid \epsilon$

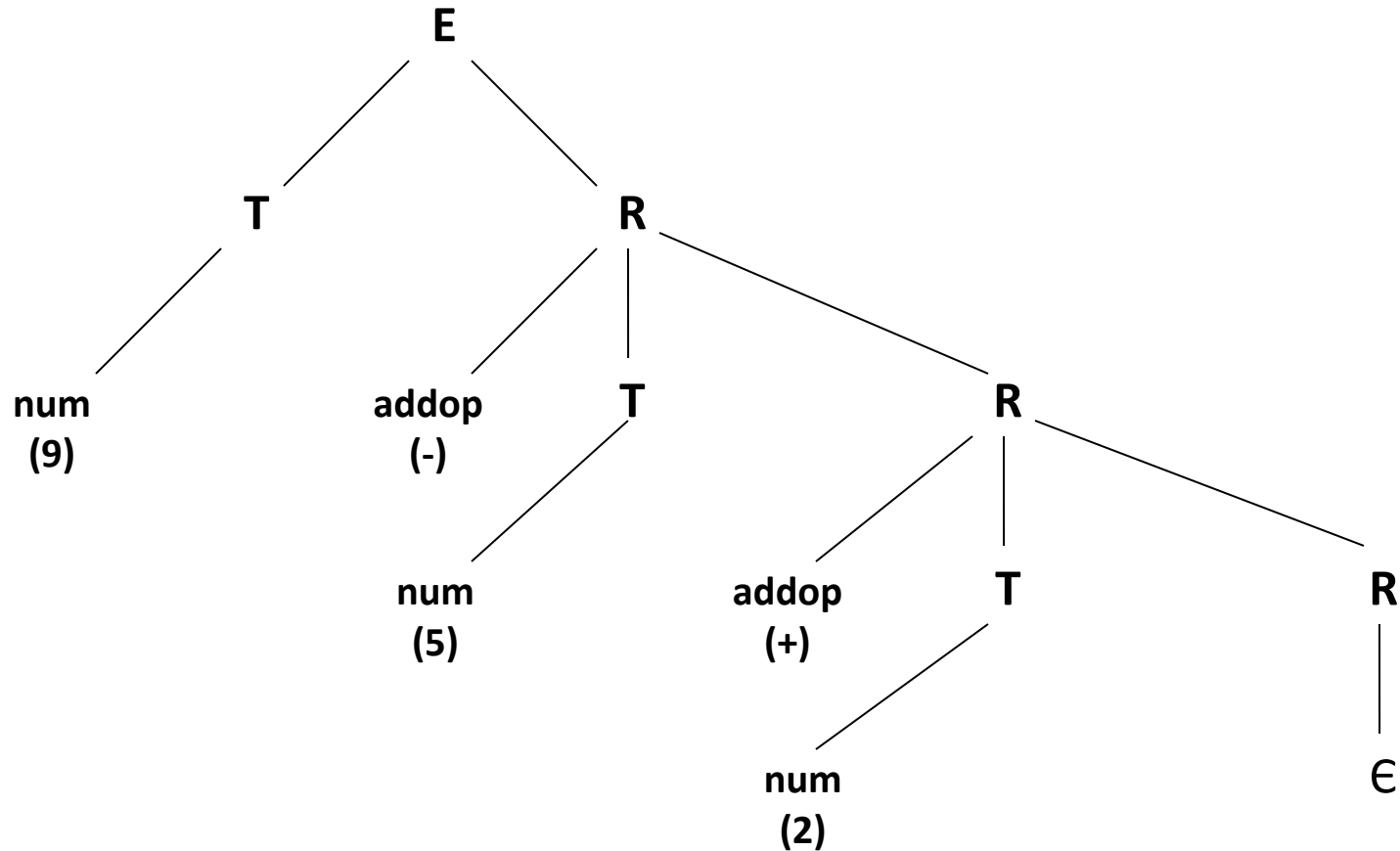
$T \rightarrow \text{num}$

$\text{addop} \rightarrow + \mid -$

Exercises: 1) Create Parse Tree for  $9 - 5 + 2$

2) Add actions to convert infix to postfix

# Parse tree for 9-5+2



# Translation schemes

- A CFG where semantic actions occur within the rhs of production
- Example: A translation scheme to map infix to postfix

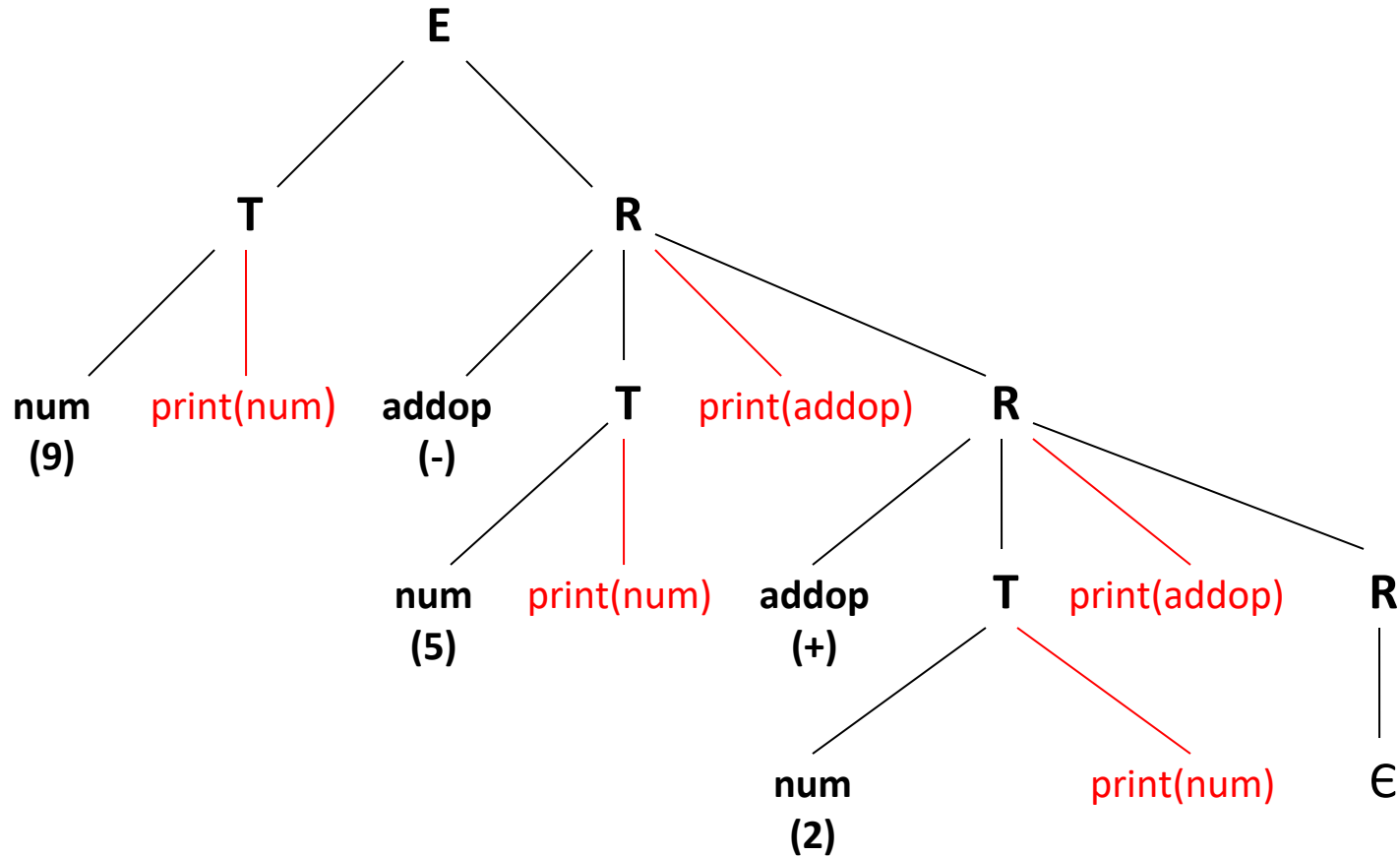
$E \rightarrow T R$

$R \rightarrow \text{addop } T \{\text{print}(\text{addop})\} R \mid \epsilon$

$T \rightarrow \text{num} \{\text{print}(\text{num})\}$

$\text{addop} \rightarrow + \mid -$

# Parse tree for 9-5+2



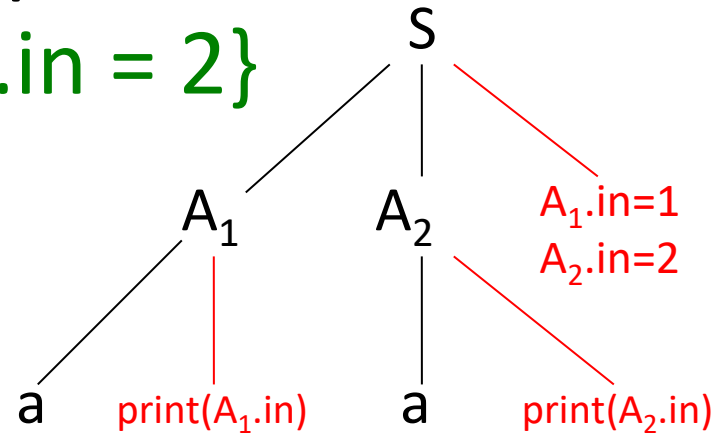
# Evaluation of Translation Schemes

- Assume actions are terminal symbols
- Perform depth first order traversal to obtain  $9 \ 5 - 2 +$
- When designing translation scheme, **ensure** attribute value is available when referred to
- In case of synthesized attribute it is trivial (**why ?**)



- An inherited attribute for a symbol on RHS of a production must be computed in an action before that symbol

$S \rightarrow A_1 A_2 \quad \{A_1.in = 1, A_2.in = 2\}$   
 $A \rightarrow a \quad \{\text{print}(A.in)\}$



depth first order traversal gives error (*undef*)

- A synthesized attribute for the non terminal on the LHS is computed after all attributes it references, have been computed. **The action normally should be placed at the end of RHS.**

# Example: Translation scheme for EQN (LaTeX like equations)

$$S \rightarrow B$$

$$B \rightarrow B_1 B_2$$

$$B \rightarrow B_1 \text{ sub } B_2$$

$$B \rightarrow \text{text}$$

# Example: Translation scheme for EQN (LaTeX like equations)

$S \rightarrow B$

$B.\text{pts} = 10$   
 $S.\text{ht} = B.\text{ht}$

$B \rightarrow B_1 B_2$

$B_1.\text{pts} = B.\text{pts}$   
 $B_2.\text{pts} = B.\text{pts}$   
 $B.\text{ht} = \max(B_1.\text{ht}, B_2.\text{ht})$

$B \rightarrow B_1 \text{ sub } B_2$

$B_1.\text{pts} = B.\text{pts};$   
 $B_2.\text{pts} = \text{shrink}(B.\text{pts})$   
 $B.\text{ht} = \text{disp}(B_1.\text{ht}, B_2.\text{ht})$

$B \rightarrow \text{text}$

$B.\text{ht} = \text{text.h} * B.\text{pts}$

## After putting actions in the right place

$S \rightarrow \begin{array}{l} \{B.\text{pts} = 10\} \\ \{S.\text{ht} = B.\text{ht}\} \end{array} \quad B$

$B \rightarrow \begin{array}{l} \{B_1.\text{pts} = B.\text{pts}\} \quad B_1 \\ \{B_2.\text{pts} = B.\text{pts}\} \quad B_2 \\ \{B.\text{ht} = \max(B_1.\text{ht}, B_2.\text{ht})\} \end{array}$

$B \rightarrow \begin{array}{l} \{B_1.\text{pts} = B.\text{pts}\} \quad B_1 \text{ sub} \\ \{B_2.\text{pts} = \text{shrink}(B.\text{pts})\} \quad B_2 \\ \{B.\text{ht} = \text{disp}(B_1.\text{ht}, B_2.\text{ht})\} \end{array}$

$B \rightarrow \text{text } \{B.\text{ht} = \text{text.h} * B.\text{pts}\}$

# Bottom up evaluation of inherited attributes

- Remove embedded actions from translation scheme
- Make transformation so that embedded actions occur only at the ends of their productions
- Replace each action by a distinct marker non terminal  $M$  and attach action at end of  $M \rightarrow \varepsilon$

$E \rightarrow T R$

$R \rightarrow + T \{\text{print (+)}\} R$

$R \rightarrow - T \{\text{print (-)}\} R$

$R \rightarrow \epsilon$

$T \rightarrow \text{num} \{\text{print(num.val)}\}$

transforms to

$E \rightarrow T R$

$R \rightarrow + T M R$

$R \rightarrow - T N R$

$R \rightarrow \epsilon$

$T \rightarrow \text{num} \quad \{\text{print(num.val)}\}$

$M \rightarrow \epsilon \quad \{\text{print(+)}\}$

$N \rightarrow \epsilon \quad \{\text{print(-)}\}$

## Inheriting attribute on parser stacks

- bottom up parser reduces rhs of  $A \rightarrow XY$  by removing  $XY$  from stack and putting  $A$  on the stack
- Suppose synthesized attributes of  $X$  is inherited by  $Y$  by using the copy rule  $Y.i = X.s$
- $X.s$  is already on the parser stack before any reductions take place in the sub-tree below  $Y$ 
  - $X.s$  can be used easily

# Recall: SDD for Inherited Attributes

$D \rightarrow T L$                        $L.in = T.type$

$T \rightarrow \text{real}$                        $T.type = \text{real}$

$T \rightarrow \text{int}$                        $T.type = \text{int}$

$L \rightarrow L_1, \text{id}$                        $L_1.in = L.in;$   
    $\text{addtype}(\text{id.entry}, L.in)$

$L \rightarrow \text{id}$                        $\text{addtype}(\text{id.entry}, L.in)$

Exercise: Convert to Translation Scheme



# Inherited Attributes: Translation Scheme

$$D \rightarrow T \{L.in = T.type\} L$$
$$T \rightarrow \text{int} \quad \{T.type = \text{integer}\}$$
$$T \rightarrow \text{real} \quad \{T.type = \text{real}\}$$
$$L \rightarrow \{L_1.in = L.in\} L_1, \text{id} \{addtype(\text{id.entry}, L_{in})\}$$
$$L \rightarrow \text{id} \{addtype(\text{id.entry}, L_{in})\}$$

**Example:** take string      real p,q,r

State stack	INPUT	PRODUCTION
	real p,q,r	
real	p,q,r	
T	p,q,r	$T \rightarrow \text{real}$
Tp	,q,r	
TL	,q,r	$L \rightarrow \text{id}$
TL,	q,r	
TL,q	,r	
TL	,r	$L \rightarrow L, \text{id}$
TL,	r	
TL,r	-	
TL	-	$L \rightarrow L, \text{id}$
D	-	$D \rightarrow TL$

**Observation:** Every time a string is reduced to L, T is just below it on the stack

## Example ...

- Every time a reduction to L is made, the value of T type is just below it
- Use the fact that T.type is at a known place in the value stack
- When production  $L \rightarrow id$  is applied, id.entry is at the top of the stack and T.type is just below it, therefore,

$addtype(id.entry, L.in) \Leftrightarrow$   
 $addtype(val[top], val[top-1])$

- Similarly when production  $L \rightarrow L_1$ , id is applied id.entry is at the top of the stack and T.type is three places below it, therefore,

$addtype(id.entry, L.in) \Leftrightarrow$   
 $addtype(val[top], val[top-3])$

## Example ...

Therefore, the translation scheme becomes

$D \rightarrow T L$

$T \rightarrow \text{int}$

$\text{val}[\text{top}] = \text{integer}$

$T \rightarrow \text{real}$

$\text{val}[\text{top}] = \text{real}$

$L \rightarrow L, \text{id}$

$\text{addtype}(\text{val}[\text{top}], \text{val}[\text{top}-3])$

$L \rightarrow \text{id}$

$\text{addtype}(\text{val}[\text{top}], \text{val}[\text{top}-1])$

# Simulating the evaluation of inherited attributes

- The scheme works only if grammar allows position of attribute to be predicted.

- Consider the grammar

$$\begin{array}{ll} S \rightarrow aAC & C_i = A_s \\ S \rightarrow bABC & C_i = A_s \\ C \rightarrow c & C_s = g(C_i) \end{array}$$

- C inherits  $A_s$
- there may or may not be a B between A and C on the stack when reduction by rule  $C \rightarrow c$  takes place
- When reduction by  $C \rightarrow c$  is performed the value of  $C_i$  is either in [top-1] or [top-2]

# Simulating the evaluation ...

- Insert a marker M just before C in the second rule and change rules to

$$S \rightarrow aAC$$

$$S \rightarrow bABMC$$

$$C \rightarrow c$$

$$M \rightarrow \varepsilon$$

$$C_i = A_s$$

$$M_i = A_s; C_i = M_s$$

$$C_s = g(C_i)$$

$$M_s = M_i$$

- When production  $M \rightarrow \varepsilon$  is applied we have  $M_s = M_i = A_s$
- Therefore value of  $C_i$  is always at  $\text{val}[\text{top}-1]$

## Simulating the evaluation ...

- Markers can also be used to simulate rules that are **not copy rules**

$$S \rightarrow aAC$$

$$C_i = f(A.s)$$

- using a marker

$$S \rightarrow aANC$$

$$N \rightarrow \varepsilon$$

$$N_i = A_s; C_i = N_s$$

$$N_s = f(N_i)$$

- **Algorithm:** Bottom up parsing and translation with inherited attributes
- **Input:** L attributed definitions
- **Output:** A bottom up parser
- Assume every non terminal has one inherited attribute and every grammar symbol has a synthesized attribute
- For every production  $A \rightarrow X_1 \dots X_n$  introduce n markers  $M_1 \dots M_n$  and replace the production by

$$A \rightarrow M_1 X_1 \dots M_n X_n$$

$$M_1 \rightarrow \epsilon, \dots, M_n \rightarrow \epsilon$$

- Synthesized attribute  $X_{j,s}$  goes into the value entry of  $X_j$
- Inherited attribute  $X_{j,i}$  goes into the value entry of  $M_j$



# Algorithm ...

- If the reduction is to a marker  $M_j$  and the marker belongs to a production

$$A \rightarrow M_1 X_1 \dots M_n X_n$$

Then for computation of  $X_{j,i}$

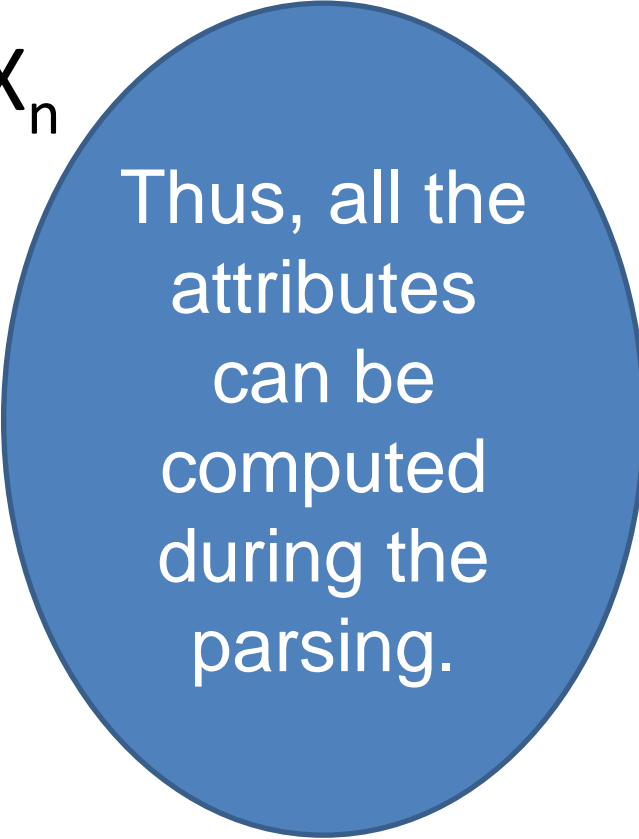
$X_{2,s}$  is in position  $\text{top}-2j+6$

$X_{2,i}$  is in position  $\text{top}-2j+5$

$X_{1,s}$  is in position  $\text{top}-2j+4$

$X_{1,i}$  is in position  $\text{top}-2j+3$

$A_i$  is in position  $\text{top}-2j+2$



Thus, all the attributes can be computed during the parsing.

# Space for attributes at compile time

- Lifetime of an attribute begins when it is first computed
- Lifetime of an attribute ends when all the attributes depending on it, have been computed
- Space can be conserved by assigning space for an attribute only during its lifetime

# Example

- Consider following definition

$D \rightarrow T L$

$L.in := T.type$

$T \rightarrow \text{real}$

$T.type := \text{real}$

$T \rightarrow \text{int}$

$T.type := \text{int}$

$L \rightarrow L_1, l$

$L_1.in := L.in; l.in = L.in$

$L \rightarrow l$

$l.in = L.in$

$l \rightarrow l_1[\text{num}]$

$l_1.in = \text{array}(\text{num}, l.in)$

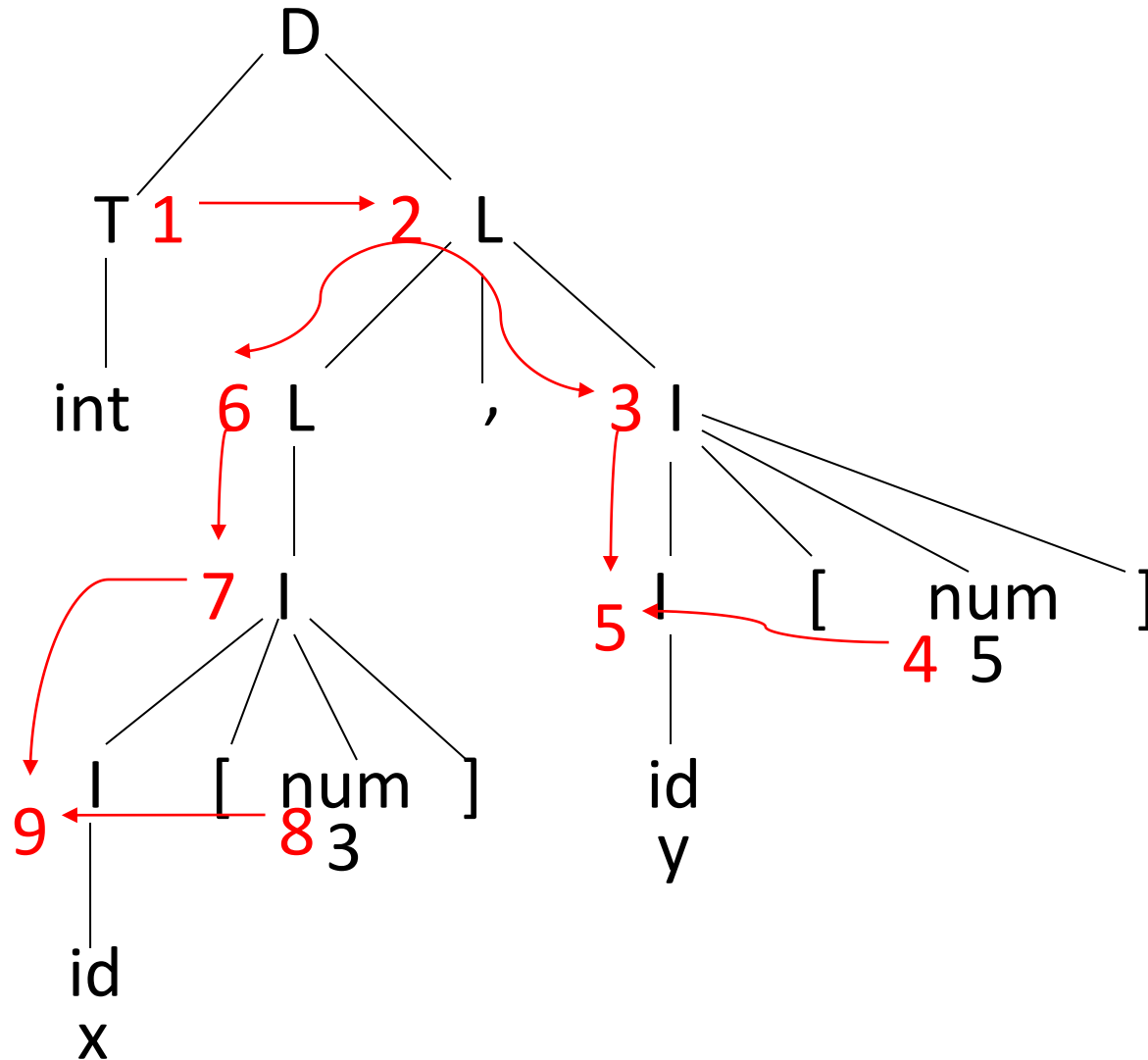
$l \rightarrow \text{id}$

$\text{addtype}(\text{id.entry}, l.in)$

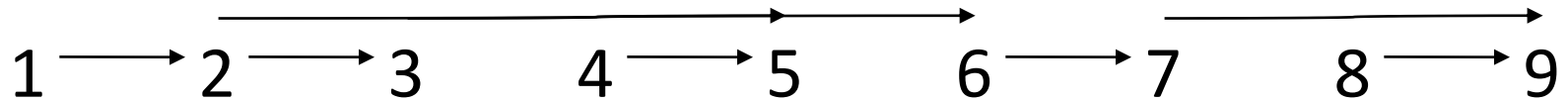
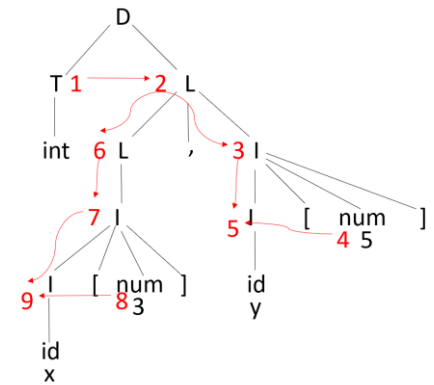
Consider string `int x[3], y[5]`

its parse tree and dependence graph

Consider string `int x[3], y[5]`



# Resource requirement



Allocate resources using  
life time information

R1 R1 R2 R3 R2 R1 R1 R2 R1

Allocate resources using lifetime and copy information

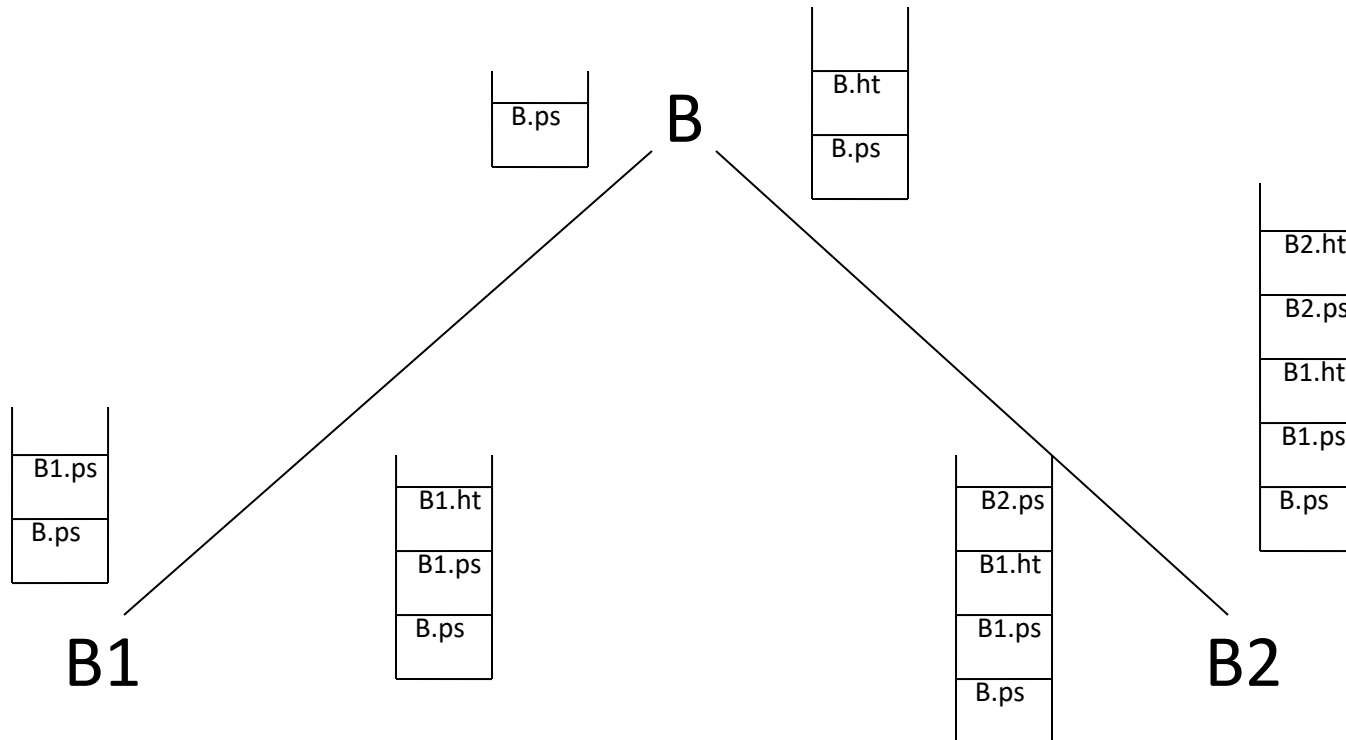
R1 =R1 =R1 R2 R2 =R1 =R1 R2 R1

# Space for attributes at Compiler Construction time

- Attributes can be held on a single stack. However, lot of attributes are copies of other attributes
- For a rule like  $A \rightarrow B C$  stack grows up to a height of five (assuming each symbol has one inherited and one synthesized attribute)
- Just before reduction by the rule  $A \rightarrow B C$  the stack contains  $I(A) I(B) S(B) I(C) S(C)$
- After reduction the stack contains  $I(A) S(A)$
- Using multiple stacks can help in reducing space

# Example

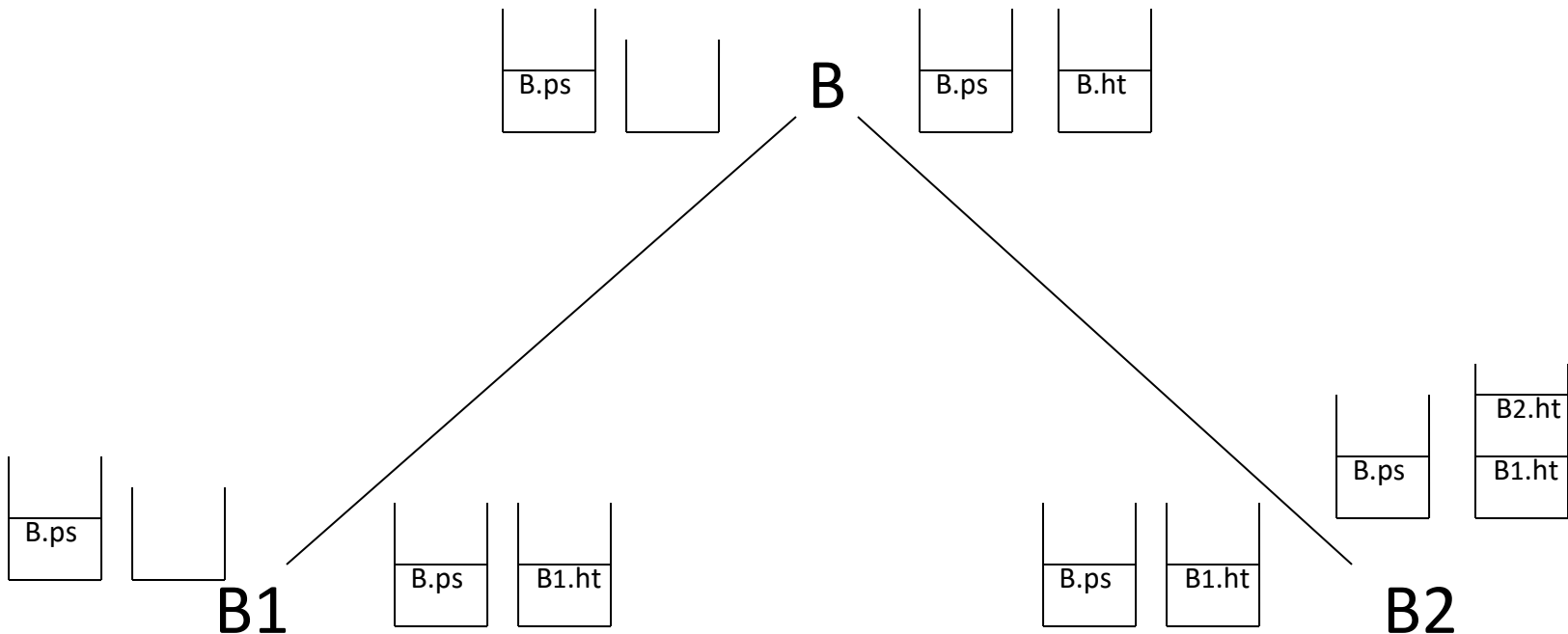
- Consider rule  $B \rightarrow B1 B2$  with inherited attribute *ps* and synthesized attribute *ht*
- The parse tree for this string and a snapshot of the stack at each node appears as





# Example ...

- However, if different stacks are maintained for the inherited and synthesized attributes, the stacks will normally be smaller



# Reading Assignment

Section 5.5, 5.9 of the OLD Dragonbook  
(3 authors: Aho, Sethi, Ullman).