Modelling Volatility and Time-varying Dependence

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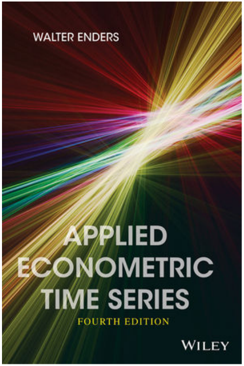
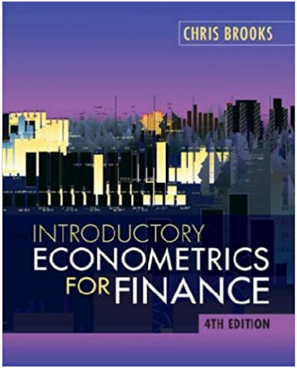
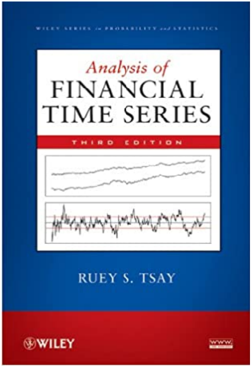
Major references







Futures and Spot Markets

Stylized facts

- Asset and commodity prices are volatile
- Hedging has been used to mitigate price risks. For example, futures contract prices have been used as a hedging tool to reduce risks involving spot transactions.
- In order for futures contract to be used effectively in managing spot price risk, optimal hedge ratio needs to be estimated.

Futures and Spot Markets

Stylized facts

- Majority of the futures contracts are settled in cash.
- Cash-settled future contracts have no physical delivery of the underlying asset for the buyer or seller; rather, the counter-parties agree to accept a cash credit or debit resulting from their trading price relative to the settlement price of a futures contract.
- At the end of each day, the exchange declares the daily settlement prices, and profit/loss amounts are calculated for each account. If there is a profit, the investors might withdraw this excess cash over its required margin, and, if there is a loss, the amount is deducted from the account.

- It is possible to hedge a spot portfolio by shorting futures contracts in the futures market?
- The question is, how much spot exposure will be hedged by the futures contract?
- The optimal hedge ratio is the proportion of futures to spot positions, which minimizes both the variance for the entire portfolio and price change risk.

How to calculate the hedge ratios

- The Optimal Hedge ratios can be calculated in two ways:
  - The conventional constant OLS
  - Time-varying volatility models
- A sample portfolio is constructed with a certain amount of spot underlying and futures contracts.
- Short futures contracts are used to hedge the spot exposure.

- We calculate the returns of Futures and Spot using their first Ln difference:
- Model 1: **Ederington (1979)**
  - Step 1: Run the OLS regression: $R_s = \alpha + \beta \times R_f + \epsilon$
  - Step 2: Check the value of β, represents the hedge ratio.
- For example, if the slope coefficient is one, then the hedge ratio is one, and the portfolio is **naively hedged**.
- In other words, one unit of a spot portfolio is hedged with exactly one unit of a futures portfolio.

- Hedge effectiveness can be measured by R^2, which is the coefficient of determination of the regression of futures price returns (the independent variable) on cash price returns (the dependent variable).
- The R^2 statistic is an indication of the maximum risk reduction potential of a hedge.
- In this case, R^2 represents the percentage reduction in the variance of unhedged cash price changes that is explained by futures price changes.
- A high R^2 value indicates better hedging effectiveness.

Futures and Spot Markets

How to calculate the hedge ratios

- Model 2: Time-varying volatility models

- Commodity prices are better represented with a time-varying covariance matrix, so the OLS assumption of homoskedasticity is not achieved; therefore, the Bivariate-GARCH model allows for a time-varying covariance matrix. .

- It involves following steps
  - Step 1: Estimate the bivariate GARCH model
  - Step 2: Generate the var-cov matrix of residuals
  - Step 3: Compute the time-varying hedge ratios as: $h_t = \frac{h_{12,t}}{h_{22,t}}$
- where $h_{12,t}$ is the estimated conditional covariance between the spot and futures returns.
- $h_{22,t}$ is the estimated conditional variance of futures returns. Because the conditional covariance is time varying, the optimal hedge will also be time varying.

- The most effectively hedged portfolio is the one with the lowest variance; in other words, hedging effectiveness is calculated by the reduction in variance in the hedged portfolio, compared to that of the unhedged portfolio.
- The returns of unhedged and hedged portfolios are estimated by the following equations, respectively.
    - $R_{unhedged} = S_{t-1} - S_t$
    - $R_{hedged} = (S_{t-1} - S_t) + h_t \times (F_{t-1} - F_t)$
- where $R_{unhedged}$ is the daily return on the unhedged portfolio and R_{hedged} is the daily return on the hedged portfolio, using constant and time-varying optimum hedge ratios.
- The term h_t is the optimum hedge ratio calculated for day t.

Hedging Effectiveness

- The risk of the position is then defined in terms of the variance in the returns of the whole portfolio (hedged and unhedged)
  - Step 1: $var_u = \sigma_s^2$
  - Step 2: $var_h = \sigma_s^2 + h_t^2 + \sigma_f^2 - 2 \times h_t \times \sigma_{sf}$
- Ederington (1979) proposes the percentage reduction in the variances of the hedged and unhedged portfolios as a measure of hedging effectiveness. The following EHE equation is used.
  - Hedging Effectiveness (HE) =

$$\frac{var_u - var_h}{var_u}$$

- The HE is the percentage reduction in the return variance of the hedged portfolio compared with the return variance of the unhedged portfolio.

- **Thick tails**: Mandelbrot (1963): Leptokurtic (thicker than Normal)
- **Volatility clustering** - Mandelbrot (1963): large changes tend to be followed by large changes of either sign.
- **Leverage Effects**: Black (1976), Christie (1982): Tendency for changes in stock prices to be negatively correlated with changes in volatility.
- **Non-trading Effects, Weekend Effects**: Fama (1965), French and Roll (1986) : When a market is closed information accumulates at a different rate to when it is open for example, the weekend effect, where stock price volatility on Monday is three times the volatility on Friday.

- **Expected events**: Cornell (1978), Patell and Wolfson (1979), etc: Volatility is high at regular times such as news announcements or other expected events, or even at certain times of day for example, less volatile in the early afternoon.
- **Volatility and serial correlation**: LeBaron (1992) - Inverse relationship between the two.
- **Co-movements in volatility**: Ramchand and Susmel (1998): Volatility is positively correlated across markets/assets.

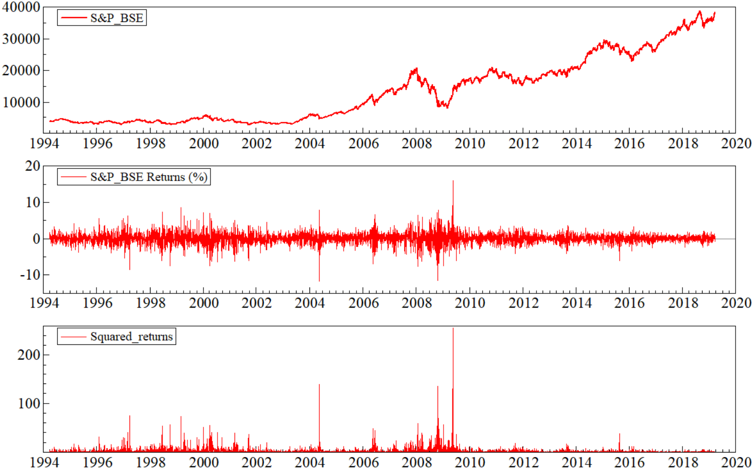
Why Volatility?

- Recent developments in financial econometrics have led to the use of models and techniques that can model the attitude of investors not only towards expected returns but also towards risk (or uncertainty).
- These require models that are capable of dealing with the volatility (variance) of the series.
- Financial and economic time series often exhibit the periods of unusually high volatility followed by more tranquil periods of low volatility.
- The expected value of the magnitude of the disturbance terms may be greater at certain periods compared with others.

Financial Time-Series: Stylized Facts

- Trend in level (BSE30)
- A high level of persistence (rBSE30)
- Volatility is not constant over time (rBSE30)
- Series may have a random walk with drift component (rBSE30)
- Series share co-movements (rBSE)
- Skewed (rBSE30)
- High excess Kurtosis (rBSE30)

S&P BSE 30: Daily Stock Index, Returns and Squared Returns



Heteroskedasticity over time

- These graphs suggest heteroskedasticity over time.
  - Time-varying volatility of returns
    - Of interest in itself to characterize returns
    - Matters for prices of derivatives and some other financial instruments
    - Risk of holding assets VaR
  - Volatility clustering

Realized Volatility

- One can use daily variance in the month to calculate variance for the month, called **realized volatility**.
- The realized variance is the estimate of volatility calculated from the high-frequency data.
- If we divide the time interval from time t to time t+1 into M sub-intervals and denote the corresponding returns as $r_{m,m+1}$, then then the realized variance is calculated as the sum of squared returns.
- $Var[r_{m,m+1}] = \sum_0^{M-1} r_{m,m+1}^2$
- To calculate the realized variance for a given day, first divide the day into many short intervals, calculate returns over those intervals, square these returns and sum these squares up.

Realized Volatility

- Theoretically, the shorter are the intervals, the more precise is the final estimate.
- However, due to market microstructure effects (mostly the bid-ask spread), very short time intervals cannot be chosen.
- Intervals of the length of 5 to 30 minutes are typically used.
- The realized variance provides quite precise estimates of volatility during a particular day.
- The largest limitation of the realized variance is the data availability.
- The high-frequency data are typically costly to obtain and work with.

- This method uses the high, low, opening, and closing prices to estimate variance
  - It can estimate daily variance just knowing opening, high, low and closing prices
  - It assumes that the price follows a random walk
  - Assume that the c_t be the logarithm of the closing price so $r_t = c_t - c_{t-1}$
  - Conventional estimator is based on closing price $\sigma_t^2 = E[(c_t - c_{t-1})^2]$
  - Using only closing price, High H_t, low L_t, and open O_t also often are available
  - Can estimate daily variance of price (not log price) from $\sigma_{GK}^2 = 0.12\frac{(O_t - C_t)^2}{f} + 0.88\frac{0.5(h_t - l_t)^2 + 0.386(c_t - o_t)^2}{1-f}$

    where f is the fraction of the day that the market is closed.

- Volatilities are sometimes annualized
  - Multiply variance by T (T is number of trading days per year)
  - Daily returns often not annualized
    - Will want to annualize returns when comparing them to annual volatility
    - A 1 percent return in one day is a 252 percentage point log return per year.
    - A 2 percent return in one day is a 504 percentage point log return per year.

ARCH Process

- A stochastic process is called ARCH (Autoregressive Conditional Heteroscedasticity) if its time-varying conditional variance is heteroscedastic with autoregression:
  $r_t = \alpha_0 + \beta_1 r_{t-1} + \epsilon_t,\ \epsilon_t \sim N(0, \epsilon_t^2)$ (1)
  $\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 ..... + \alpha_q \epsilon_{t-q}^2$ (2)

- Equation (1) is the mean equation where regressors can be generally added to the right hand side alongside ϵ_t.
- Equation (2) is the variance equation, which is an ARCH(q) process where auto regression is its squared residuals has an order of q, or has q lags..

ARCH Process

- A stochastic process is called GARCH (Generalized Autoregressive Conditional Heteroscedasticity) if its time-varying conditional variance is heteroscedastic with both autoregressive and moving average: $r_t = \epsilon_t,\ N(0, \epsilon_t^2)$
- $\sigma_t^2 = \alpha_0 + \alpha_1\epsilon_{t-1}^2 \ldots.. + \alpha_q\epsilon_{t-q}^2 + \beta_1\sigma_{t-12} + \ldots + \beta_p\sigma_{t-p} \quad (4)$

  $= \alpha_0 + \sum_{i=1}^{q} \alpha_i\epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j\sigma_{t-j}^2$

- Equation (4) is a GARCH(p,q) process where auto regression is its squared residuals has an order of q, and the moving average component has an order of p.

GARCH Process

- GARCH model is parsimonious as compared to ARCH i.e less lags are required.
- GARCH(1,1) model is more simpler to implement than the ARCH because of the lag length
- Suppose we have a GARCH(1,1) model. Extending the variance process backwards yields:
  $\epsilon_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$
  $= \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1(\alpha_0 + \alpha_1 \epsilon_{t-2}^2 + \beta_1 \sigma_{t-2}^2)$
  $= \dots\dots\dots\dots\dots$
  $= \frac{\alpha_0}{1-\beta_1} + \alpha_1 \sum_{m=1}^{\infty} \beta_1^{n-1} \epsilon_{t-n}^2$
- Indeed, only the first few terms would have noteworthy influence since

$$\lim_{n\to\infty} \beta_1^n \to 0$$

- This shows how a higher-order ARCH specification can be approximated by a GARCH(1,1) process.

GARCH Process

- Stationarity condition:
- The unconditional variance of GARCH would be of interest to the property of the model.
- Applying the expectations operator to both sides of equation (4), we have:
  $E(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{q} \alpha_i E(\epsilon_{t-i}^2) + \sum_{j=1}^{p} \beta_j E(\epsilon_{t-j}^2)$
  Noting $E(\sigma_t^2) = E(\epsilon_{t-1}^2) = E(\sigma_{t-j}^2)$ is the unconditional variance of the residual, which is solved as:
  $$\sigma^2 = E(\sigma_t^2) = \frac{\alpha_0}{1 - \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j}$$
  It is clear that for the process a finite variance, the following condition must be met:
  $$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$$
  In commonly used GARCH(1,1) model, the condition is simply $\alpha_1 + \beta_1 < 1$.

GARCH Properties

- Many financial time series have persistent volatility, i.e the sum of α_1 and β_1 is close to being unity.
- A unity sum of α_1 and β_1 leads to so-called Integrated GARCH or IGARCH as the process is not covariance stationary. Nevertheless, this does not pose as serious a problem as it appears.

Variations of the ARCH/ GARCH model

- Variations are necessary to adapt the standard GARCH model to the need arising from examining the time series properties of specific issues in finance and economics.

- There are different versions of time-varying volatility: ARCH-M, GARCH-M and the models of asymmetry- Exponential GARCH(EGARCH) and Threshold GARCH(TGARCH).

The ARCH-M model

- When the conditional variance enters the mean equation for an ARCH process, then ARCH-in-Mean or simply the ARCH-M is derived :
- $r_t = \gamma_1 x_1 + ... + \gamma_m x_m + \phi\sigma_t^2 + \epsilon_t \qquad \epsilon_t \sim N(0, \epsilon_t^2)$
  $\sigma_t^2 = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + ... + \alpha_q\epsilon_{t-q}^2$

  where $x_k, k = 1, 2, ...m$ are exogenous variables which could include lagged y_t.

- In the sense of asset pricing, if r_t is the return on an asset of a firm, then $x_k, k = 1, ...m$ would generally include on the market and possibly other explanatory variables such as the price earning ration and the size.

The ARCH-M model

- The parameter ϕ captures the sensitivity of the return to the time-varying volatility, or in other words, links the return to a time-varying risk premium.

- The ARCH-M model is generalized from the standard ARCH by Engle et al.(1987) and can be further generalized that the conditional variance is GARCH instead of ARCH, and that the conditional standard deviation, instead of the conditional variance, enters the mean equation.

The EGARCH model

- It is given by Nelson(1991) with the following specification:

$$\log(h_t) = \gamma + \sum_{j=1}^{q} \zeta_j \left| \frac{u_{t-j}}{\sqrt{h_{t-j}}} \right| + \sum_{j=1}^{q} \xi_j \frac{u_{t-j}}{\sqrt{h_{t-j}}} + \sum_{i=1}^{p} \delta_i \log(h_{t-i})$$

  where ξ is asymmetric response parameter or leverage parameter.

- The sign of ξ is expected to be positive in most empirical cases so that a negative shock increases the future volatility or uncertainty while a positive shocks of the same eases the effect on future uncertainty.

- This is in contrast to the standrad GARCH model where shocks of the same magnitude, positive or negative, have the same effect on future volatility.

The Asymmetry Explanation

- In macroeconomics analysis, financial markets and corporate finance, a negative shock usually implies bad news, leading to a more uncertain future.
- Consequently, for example, shareholders would require a higher expected return to compensate for bearing increased risk in their investment.
- A statistical asymmetry is, under various circumstances, also a reflection of the real world asymmetry, arising from the nature, process or organization of economic and business activity, e.g. the change in financial leverage is asymmetric to shocks to the share price of a firm.

The Threshold GARCH model

- It is known as the GJR model, named Glosten, Jagannathan and Runkle (GJR, 1993).

- In contrast, the GJR model is much simpler than, though not as elegant as, EGARCH.
- A general GJR model is specified as follows:
$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \{ \alpha_i \epsilon_{t-1}^2 + \delta_i \epsilon_{t-1}^2 \} + \sum_{j=1}^{p} \beta_j \epsilon_{t-1}^2$
there $\delta_i = 0$ if $\epsilon_{t-i} > 0$.

- So, δ_i is greater than zero, we conclude that there is asymmetry.

Multivariate GARCH model

- A bivariate GARCH model expressed in matrices takes the form:
  $r_t = \epsilon_t$, ...(1)
  $\epsilon_{t-1} \,|\Omega_{t-1} \sim N(0, H_t) \quad (5)$
  where vectors

$$r_t = \begin{bmatrix} r_{1t} & r_{2t} \end{bmatrix}$$

,

$$\epsilon_t = \begin{bmatrix} \epsilon_{1t} & \epsilon_{2t} \end{bmatrix}$$

$$H_t = \begin{bmatrix} h_{11t} & h_{12t} \\ h_{21t} & h_{22t} \end{bmatrix}$$

  is the covariance matrix which can be designed in a numver of ways.
- Commonly used specifications of the covariance include VEC, VECH (full parameterusation), BEKK(positive definite parameterization) named after Baba, Engle, Kraft and Kroner (1990) and constant correlation.

Multivariate GARCH model

- Suppose we have multivariate return series
$$r_t = \mu_t + \epsilon_t$$
where the vectors are simple generalizations of a univariate process.

- The vectors are m by one with m dependent variables, often asset returns
$$\mu_t = E[r_t|F_{t-1}]$$

- ϵ_t the innovation in the returns in period t with
$E(\epsilon_t) = 0, \Sigma = Cov[\epsilon|F_{t-1}]$
Σ_t is m by m matrix with $m(m+1)/2$ distinct elements.

- The number of distinct elements in Σ increases with the square of m because there are $(m^2 + m)/2$ distinct elements.
- The number of observations is mT and increases linearly with m

- **Full parameterization**
  - The full parameterization, or VECH, converts the covariance matrix to a vector of variance and covariance. As $\sigma_{ij} = \sigma_{ji}$ the dimension of the vector converted from an $m \times m$ matrix in $\frac{m(m+1)}{2}$.
  - Thus, is a bivariate GARCH process, the dimension of the variance/covariance vector is three.
  - With a trivariate GARCH, the dimension of vector is six, i.e. there are six equations to describe the time varying varinace/covariance. Therefore, it is unlikely to be feasible when more than two variables are involved in a system.

- **Curse of Dimensionality**
  - The implication of the number of parameters in Σ_t increasing with the square of m.
  - Five assets implies $m(m+1)/2 = 15$ distinct elements.
  - Twenty assets implies $m(m+1)/2 = 210$ distinct elements.
  - One hundred assets implies $m(m+1)/2 = 5{,}050$ distinct elements
- Suppose we have 250 observations on each asset
  - Five assets implies 1,250 observations
  - Twenty assets implies 5,000 observations
  - One hundred assets implies 25,000 observations.
- Even one hundred assets is not a lot
- If you have 1000 assets
  - Then 500, 500 distict elements in the covariance matrix
  - Only 250,000 observations
  - May make more sense to look at volatility of portfolio in some circumtances, but not all.

VECH GARCH

- What do vec and vech mean?
- Stacking of matrix elements in a vector by stacking columns

$$\begin{bmatrix} h_{11t} \\ h_{12t} \\ h_{21t} \\ h_{22t} \end{bmatrix} = vec \begin{bmatrix} h_{11,1} & h_{12,1} \\ h_{21,1} & h_{22,1} \end{bmatrix}$$

- Symmetric matrix

$$\begin{bmatrix} h_{11t} \\ h_{12t} \\ h_{21t} \\ h_{22t} \end{bmatrix} = vech \begin{bmatrix} h_{11,1} & h_{12,1} \\ h_{12,1} & h_{22,1} \end{bmatrix}$$

VECH SPECIFICATION

- The VECH specificaton is presented as:

$$vech(H_t) = vech(A_0) + \sum_{i=1}^{q} A_i vech(\epsilon_{t-1}\epsilon'_{t-1}) + \sum_{j=1}^{p} B_j vech(H_{t-j})$$

  where H_T, A_0, A_i, B_j and $\epsilon_{t-i}\epsilon'_{t-i}$ are matrices in their conventional form and vech(.) means the procedure of coversion of a matrix into a vector, as described above.

- For p=q=1, the above equation can be written as:
  $H_t =$
  $$\begin{bmatrix} h_{11t} \\ h_{12t} \\ h_{22t} \end{bmatrix} = \begin{bmatrix} \alpha_{11,0} \\ \alpha_{12,0} \\ \alpha_{22,0} \end{bmatrix} + \begin{bmatrix} \alpha_{11,1} & \alpha_{12,1} & \alpha_{13,1} \\ \alpha_{21,1} & \alpha_{22,1} & \alpha_{23,1} \\ \alpha_{31,1} & \alpha_{32,1} & \alpha_{33,1} \end{bmatrix} \begin{bmatrix} \epsilon^2_{1,t-1} \\ \epsilon_{1,t-1}\epsilon_{2,t-1} \\ \epsilon^2_{2,t-1} \end{bmatrix} + \begin{bmatrix} \beta_{11,1} & \beta_{12,1} & \beta_{13,1} \\ \beta_{21,1} & \beta_{22,1} & \beta_{23,1} \\ \beta_{31,1} & \beta_{32,1} & \beta_{33,1} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix}$$

So, the simplest multivariate model has 21 parameters to estimate.

VECH GARCH

- VECH GARCH model has each term in the variance-covariance matrix evolved independently according to a GARCH (1,1)
- Two variables example as mentioned in the previous slide:

$$h_{11,t} = \alpha_{11,0} + \alpha_{11,1}\epsilon^2_{1,t-1} + \alpha_{12,1}\epsilon_{1,t-1}\epsilon_{2,t-1} + \alpha_{13,1}\epsilon^2_{1,t-1} + \beta_{11,1}h_{11,t-1} + \beta_{12}h_{12,t-1} + \beta_{13}h_{22,t-1}$$

$$h_{12,t} = \alpha_{12,0} + \alpha_{21,1}\epsilon^2_{1,t-1} + \alpha_{22,1}\epsilon_{1,t-1}\epsilon_{2,t-1} + \alpha_{23,1}\epsilon^2_{1,t-1} + \beta_{21,1}h_{11,t-1} + \beta_{22}h_{12,t-1} + \beta_{23}h_{22,t-1}$$

$$h_{22,t} = \alpha_{13,0} + \alpha_{31,1}\epsilon^2_{1,t-1} + \alpha_{32,1}\epsilon_{1,t-1}\epsilon_{2,t-1} + \alpha_{33,1}\epsilon^2_{1,t-1} + \beta_{31,1}h_{11,t-1} + \beta_{32}h_{12,t-1} + \beta_{33}h_{22,t-1}$$

- The conditional variance of each variable $h_{11,t}$ and $h_{22,t}$ depend on its own past, the past of the other variable. There is a rich interaction between the variables.
- Still no guarantee that the estimated variance-covariance matrix will be positive definite in every period.

VECH GARCH

- In a VECH GARCH (1,1) in which all variances and covariances depend on all lagged variances and cross-products of errors
  - The number of parameters in each equation is 3m+1
  - The number of equations is m(m+1)/2
  - The total number of parameters is [m(m+1)/2](3m+1)
  - Two assets imply 21 parameters
  - Five assets imply 240 parameters
  - Ten assets imply 1705 parameters

Diagonal VECH GARCH model

- In order to reduce the curse of dimensionality, the model imposes restrictions, called as diogonal Vech model.

  $\mathrm{h}_{11,t} = \alpha_{11,0} + \alpha_{11,1}\epsilon^2_{1,t-1} + \beta_{11}h_{11,t-1}$
  $h_{12,t} = \alpha_{12,0} + \alpha_{22,1}\epsilon_{1,t-1}\epsilon_{2,t-1} + \beta_{22}h_{12,t-1}$
  $\mathrm{h}_{22,t} = \alpha_{13} + \alpha_{33}\epsilon^2_{2,t-1} + \beta_{33}h_{22,t-1}$

- Far fewer parameters. The diagonalized version is called **Diagonal vech**.

- The model consider only diogonal elements such that h_{ijt} contains only lags of itself and the cross products of $\epsilon_{it}\epsilon_{jt}$

BEKK MODEL

- Positive definite parameterization
  - Baba, Engle, Kraft and Kroner (1991 and Kroner (1995) popularized what is now called as the BEKK model which ensures that the conditional variances are positive.
  - The idea is to force all of the parameters to enter the model via quadratic forms ensuring that all the variances are positive.
  - The BEKK specification takes the following form:

    $$H_t = A_0 A_0' + A_i' \epsilon_{t-i} \epsilon_{t-i}' A_i + B_j H_{t-j} B_j'$$

    where A_0 is a symmetric $(N \times N)$ parameter matrix and A_i and B_j are unrestricted $(N \times N)$ parameter matrices.

- In the bi-variate system with $p = q = 1$, the above equation can be written as:

$$\begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} = \begin{bmatrix} \alpha_{11,0} & \alpha_{12,0} \\ \alpha_{21,0} & \alpha_{22,0} \end{bmatrix} +$$

$$\begin{bmatrix} \alpha_{11,1} & \alpha_{12,1} \\ \alpha_{21,1} & \alpha_{22,1} \end{bmatrix} \begin{bmatrix} \epsilon^2_{1,t-1} & \epsilon_{1,t-1}\epsilon_{2,t-1} \\ \epsilon_{1,t-1}\epsilon_{2,t-1} & \epsilon^2_{2,t-1} \end{bmatrix} \begin{bmatrix} \alpha_{11,1} & \alpha_{12,1} \\ \alpha_{21,1} & \alpha_{22,1} \end{bmatrix}^T +$$

$$\begin{bmatrix} \beta_{11,1} & \beta_{12,1} \\ \beta_{21,1} & \beta_{22,1} \end{bmatrix} \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} \beta_{11,1} & \beta_{12,1} \\ \beta_{21,1} & \beta_{22,1} \end{bmatrix} T$$

- We can examine the sources of uncertainty and, moreover,assess the effect of signs of shocks by Writing the variances and covariance explicitly. Matrix multiplications can be:
$h_{11,t} = (\alpha_{11,0}^2 + \alpha_{12,0}^2) + (\alpha_{11,1}^2\epsilon_{1,t-1}^2 + 2\alpha_{11,1}\alpha_{21,1}\epsilon_{1,t-1}\epsilon_{2,t-1} + \alpha_{21,1}^2\epsilon_{2,t-1}^2) + (\beta_{11,1}^2 h_{11,t-1} + 2\beta_{11,1}\beta_{21,1}h_{12,t-1} + \beta_{21,1}^2 h_{22,t-1})$

$h_{12,t} = h_{21,t} = \alpha_{12,0} + (\alpha_{11,1}\alpha_{12,1}\epsilon_{1,t-1}^2 + (\alpha_{12,1}\alpha_{21,1} + \alpha_{11,1}\alpha_{22,1})\epsilon_{1,t-1}\epsilon_{2,t-1} + \alpha_{21,1}\alpha_{22,1}\epsilon_{2,t-1}^2) + (\beta_{11,1}\beta_{21,1}h_{11,t-1} + (\beta_{12,1}\beta_{21,1} + \beta_{11,1}\beta_{22,1})h_{12,t-1} + (\beta_{21,1}\beta_{22,1}h_{22,t-1})$

$h_{22,t} = (\alpha_{21,0}^2 + \alpha_{22,0}^2) + (\alpha_{12,1}^2\epsilon_{1,t-1}^2 + 2\alpha_{12,1}\alpha_{22,1}\epsilon_{1,t-1}\epsilon_{2,t-1} + \alpha_{22,1}^2\epsilon_{2,t-1}^2) + (\beta_{12,1}^2 h_{11,t-1} + 2\beta_{12,1}\beta_{22,1}h_{12,t-1} + \beta_{22,1}^2 h_{22,t-1})$

- Looking at the diagonal elements in the above matrix i.e $h_{11,t}$ and $h_{22,t}$ we can assess the impact of the shock in one series on the uncertainty or volatility of the other, and the impact could be asymmetric or only be one one way effective.
- In particular, one might also be interested in assessing the effect of the signs of shocks in the two series.
- The merit of the model is that the there are only 11 parameters to be estimated compared to 21 parameters in the VECH representation.
- The demerit of this model is that it has a large number of parameters that are not globally identified.
- Changing the signs of all elements of A, B and C(alpha coefficients) will have no effect on the value of the likelihood function.
- Convergence becomes an issue sometime.

- It is clear that $\alpha_{11,1}$ and $\alpha_{22,1}$ represent the effect of the shock on the future uncertainty of the same time series and $\alpha_{21,1}$ and $\alpha_{12,1}$ represent the cross effect, i.e the effect of the shock of the second series on the future uncertainty of the first series, and vice versa.
- The interesting point is that if $\alpha_{11,1}$ and $\alpha_{21,1}$ have different signs, then the shocks with different signs in the two time series tend to increase the future uncertainty in the first time series.
- Similarly, if $\alpha_{22,1}$ and $\alpha_{12,1}$ have different signs, the future uncertainty of the second time series might increase if the two shocks have different signs.
- The model is appropriately fit to investigate volatility spillovers between two financial markets.

- Bollerslev (1990) suggested that the time-varying conditional variances be parametrized in order to be proportional to the product of the corresponding conditional standard deviation.

- The intuition for this model is the following:
  - Assume that $h_{ij,t}$ is the covariance between two assets i and j to be modeled. Also let $h^2_{i,t}$ be the conditional variance modeled by some univariate GARCH model.
  - Under the assumption of keeping correlation constant, denoting ρij the constant correlation
    between the assets i and j. It follows that $\rho_{ij} = \frac{h_{ij,t}}{h_{i,t}, h_{j,t}}$, $h_{ij,t} = \rho_{ij} h_{i,t}, h_{j,t}$
  - ρ_{ij} is computed using standardized residuals.

A two-variable Constant CC Model matrix form is

- Two variable Constant conditional-correlation GARCH model can be written as:

$$\begin{bmatrix} h_{11t} \\ h_{22t} \end{bmatrix} = \begin{bmatrix} \alpha_{10} \\ \alpha_{20} \end{bmatrix} + \begin{bmatrix} \alpha_{11,1} & \alpha_{12,1} \\ \alpha_{21,1} & \alpha_{22,1} \end{bmatrix} \begin{bmatrix} \epsilon^2_{1,t-1} \\ \epsilon^2_{2,t-1} \end{bmatrix} + \begin{bmatrix} \beta_{11,1} & \beta_{12,1} \\ \beta_{21,1} & \beta_{22,1} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{22,t-1} \end{bmatrix}$$

- The Constant conditional-correlation GARCH model

$$\mathrm{h}_{11,t} = \alpha_{10} + \alpha_{11}\epsilon^2_{1,t-1} + \alpha_{12}\epsilon^2_{2,t-1} + \beta_{11} h_{11,t-1} + \beta_{12} h_{22,t-1}$$
$$h_{22,t} = \alpha_{20} + \alpha_{21}\epsilon^2_{1,t-1} + \alpha_{22}\epsilon^2_{2,t-1} + \beta_{21} h_{11,t-1} + \beta_{22} h_{22,t-1}$$
$$h_{12,t} = \rho_{12}(h_{11,t}, h_{22,t})^{0.5}$$

- Bollerslev (1990) introduces a time-invariant (n,n) correlation matrix with unit diagonal elements:

$$R = \begin{pmatrix} 1 & \rho_{12} & \dots & \rho_{1n} \\ \rho_{12} & 1 & \dots & \dots \\ \dots & \dots & \dots & \rho_{n-1,n} \\ \rho_{1n} & \dots & \rho_{n-1,n} & 1 \end{pmatrix},$$

- The temporal variation in $h_t = D_t^{0.5}$ R $D_t^{0.5}$

  where Dt is the diagonal matrix of the conditional variances.
- We only need to model the dynamics of the n conditional variances and to estimate the CC matrix, so that the number of parameters to estimate reduces itself to $n(1+p+q)+n(n+1)/2$.

- An advantage of this approach is that the if the conditional variances in the Dt are all positive and conditional correlation matrix R is positive definite, the sequence of the conditional covariance matrices Σ_t is guranteed to be positive definite for all t.

- The CC model was later challenged by Engle (2002), Engle and Sheppard (2001) and Tse and Tsui (2002).

- The basic idea is that the conditional correlation matrix R_t is in fact time-varying, so that the conditional covariance matrix becomes

$$h_t = D_t^{0.5}\ \mathrm{R}_t \mathrm{D}_t^{0.5}$$

Engle(2002) proposed DCC model which follows two-steps estimation procedure. In the first step, GARCH parameters are estimated followed by correlations in the second step

Suppose we have 12 assets in a model Mean equation $r_t = \gamma_1 r_{t-1} + \epsilon_t$......
where $r_t = (r_{1,t}, r_{2,t}, ..., r_{12,t})^{'}$, $(\epsilon_{1,t}, \epsilon_{2,t}, ...\epsilon_{12,t})^{'}$,
$\epsilon_T / I_{T-1} \sim N(0, H_t)$
Variance equation: $h_t = D_t^{0.5}\ \mathrm{R}_t \mathrm{D}_t^{0.5}$

Where $D_t = \mathrm{D} \begin{bmatrix} \sqrt{h_{11,t}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{h_{12,12,t}} \end{bmatrix}$

$$R_t = \begin{bmatrix} 1 & \rho_{1,2,t} & \cdots & \rho_{1,12,t} \\ \vdots & \ddots & \vdots & \vdots \\ \rho_{12,1,t} & \cdots & \cdots & 1 \end{bmatrix}$$

R_t is the conditional correlation matrix of residuals or standardized residuals and $h_{11,t}, h_{22,t}, ...h_{12,12,t}$ are variance equations for futures and spot returns series.

- The conditional correlation matrix is defined by:
  - $R_t = \text{diag}(Q_t)^{-0.5} \times Q_t \times \text{diag}(Q_t)^{-0.5}$

  - $Q_t = (1-\delta_1-\delta_2)\bar{Q} + \delta_1(u_{t-1}u_{t-1}^{'}) + \delta_2 Q_{t-1}$

    where $\bar{Q}$ is the unconditional covariance matrix of $u_t = \epsilon_{it}/_{it}{}_{i=1,\ldots,n}$ and $diag(Q_t)$ is the (n,n) matrix with the diagonal of Q_t on the diagonal and zero off-diagonal.

  - The parameters δ_1 and δ_2 are assumed to satisfy $0 \leq \delta_1$ and $\delta_2 \geq 0$ and $\delta_1 + \delta_2 \leq 1$

- Expanding the variance-co-variance matrices into individual equations will get:

$h_{i,i,t} = c_t + \alpha_i h_{i,i,t-1} + b_i \epsilon^2_{i,t-1} \qquad i = 1, 2, ..., 12$

$h_{i,j,t} = \rho_{i,j,t} \sqrt{h_{i,i,t}} \sqrt{h_{j,j,t}} \qquad i, j = 1, 2, ...12 \text{ and } i \neq j$

$\rho_{i,j,t} = q_{i,j,t} / \sqrt{q_{ii,t}} \sqrt{q_{jj,t}} \qquad i, j = 1, 2, ...12 \text{ and } i \neq j$

where the conditional covariance $q_{i,j,t}$ between standardized residuals $\eta_{i,t}$ and $\eta_{j,t}$ can be expressed in the following two ways. The first is the mean reverting approach, which is given by

$q_{i,j,t} = \rho \bar{i,j}_{,t} q_{i,j,t} (1 - \alpha - \beta) + \alpha q_{ij,t-1} + \beta \eta_{i,t-1} \eta_{j,t-1}$

$i, j = 1, 2, ...12 \text{ and } i \neq j$

with $\eta_{i,t} = \epsilon_{i,t} / \sqrt{h_{ii,t}}, \eta_{j,t} = \epsilon_{j,t} / \sqrt{h_{jj,t}}$ and $\rho \bar{i}_{,j}$ as the unconditional correlation between $\epsilon_{1,t}$ and $\epsilon_{22,t}$.

- The log likelihood of this estimator can be written:

$$L = -\frac{1}{2}\sum_{t=1}^{T}\left(klog(2\pi) + log|H_t| + r_t' H_t^{-1} r_t\right)$$
$$= -\frac{1}{2}\sum_{t=1}^{T}\left(klog(2\pi) + log|D_t R_t D_t| + r_t' D_t^{-1} R_t^{-1} D_t^{-1} r_t\right)$$
$$= -\frac{1}{2}\sum_{t=1}^{T}\left(klog(2\pi) + 2log|D_t| + log|R_t| + Z_t' R_t^{-1} Z_t\right)$$

- Let the parameters in D_t be denoted θ and the additional parameters in R_t be denoted ϕ.

- The log likelihood can written as the sum of a volatility part and a correlation part.
  $L(\theta, \phi) = L_v(\theta) + L_c(\theta, \phi)$
  The volatility term is:
  $L_v(\theta) = -\frac{1}{2}\sum_t \left(nlog(2\pi) + log|D_t|^2 + r_t' D_t^{-2} r_t \right)$
  and the correlation component is:
  $L_c(\theta, \phi) = -\frac{1}{2}\sum_t \left(log|R_t| + Z_t' R_t^{-1} Z_t - Z_t' Z_t \right)$

ADCC Model

- Cappiello, Engle and Sheppard (2006) introduce asymmetric effects in conditional asset correlations.
- For high-frequency financial time-series data, asymmetric effects play a central role in revealing the role of negative returns during crisis period compared to stable one.
- The ADCC model is a special case of the DCC model.
  $Q_t = (1 - \theta_1 - \theta_2)\bar{Q} - \lambda\bar{Z} + \theta_1\epsilon_{t-1}\epsilon_{t-1}^{'} + \theta_2 Q_{t-1} + \lambda\zeta_{t-1}\zeta_{t-1}^{'}$
  where the coefficient λ explains the asymmetric effect in the model.
- $\zeta_{t-1} = I[\epsilon_t < 0] \otimes \epsilon_t$ where $\otimes$ is the Hamadard product and I[.]shows function that takes value 1 when residuals are negative and 0 otherwise.
- $z = \zeta_{t-1}\zeta_{t-1}^{'}$ is the covariance matrix of $[E(\zeta_{t-1}\zeta_{t-1}^{'})]$.
- The ADCC helps understand the increases in the conditional correlation during crisis period or market downturn period.

R commands for GARCH modelling

Function	Description
`coef()`	Extract estimated coefficients
`infocriteria()`	Calculate information criteria for fit
`likelihood()`	Extract likelihood
`nyblom()`	Calculate Hansen-Nyblom coefficient stability test
`signbias()`	Calculate Engle-Ng sign bias test
`newsimpact()`	Calculate news impact curve
`as.data.frame()`	Extract data, fitted data, residuals and conditional vol
`sigma()`	Extract conditional volatility estimates
`residuals()`	Extract residuals
`fitted()`	Extract fitted values
`getspec()`	Extract model specification
`gof()`	Compute goodness-of-fit statistics
`uncmean()`	Extract unconditional mean
`uncvariance()`	Extract unconditional variance
`plot()`	Produce various plots
`persistence()`	Calculate persistence of fitted model

Maximum Likelihood

$r_t = \mu + \epsilon_t$
$\epsilon_t \sim N(0, H_t)$, conditional on the past
$L = -\frac{1}{2}\sum_{t=1}^{T}[log\,|H_t| + \epsilon_t' H_t^{-1}\epsilon_t]$

Diagnostic checking

- Tsst standardized residuals:
  $\epsilon_t = H_t^{-1/2}(r_t - E_{t-1}(r_t))$
- Test for own autocorrelation
- Test for cross asset autocorrelation
- Test for product autocorrelation
- Test for asymmetries

- **Objective**: To examine the volatility spillover of crude oil prices with stock prices of technology and clean energy stock index.
- **Method**: VARMA-MGARCH model of Ling and McAleer (2003)
- WilderHill Clean Energy Index (ECO), NYSE Arca Technology Index (PSE) and West Texas Intermediate (WTI) for the period 02 May 200530 April 2015.

$$\sigma_{it}^2 = 0.511(H_{it} - L_{it})^2 - 0.019[(C_{it} - O_{it})(H_{it} + L_{it} - 2O_{it}) - 2(H_{it} - O_{it})(L_{it} - O_{it})] - 0.383(C_{it} - O_{it})^2$$

MGARCH Results

	BEKK			CCC			DCC		
Variable	Coeff.	T-Stat.	Signif.	Coeff.	T-Stat.	Signif.	Coeff.	T-Stat.	Signif.
Mean									
μ_{10}	0.000	1.065	0.287	0.000	1.289	0.197	0.000	0.794	0.427
μ_{11}	0.049b	1.790	0.073	0.030	0.930	0.352	0.040	1.405	0.160
μ_{12}	0.018	1.183	0.237	0.017	1.203	0.229	0.022	1.249	0.212
μ_{13}	0.003	0.068	0.946	0.062	1.147	0.251	0.036	0.727	0.467
μ_{20}	0.000	1.180	0.238	0.001b	1.683	0.092	0.000	0.948	0.343
μ_{21}	0.009	0.394	0.694	−0.003	−0.143	0.886	−0.006	−0.253	0.800
μ_{22}	−0.030	−1.441	0.150	−0.026	−1.141	0.254	−0.031	−1.319	0.187
μ_{23}	0.039	1.023	0.307	0.056	1.401	0.161	0.056	1.341	0.180
μ_{30}	0.001a	4.852	0.000	0.001a	4.202	0.000	0.001a	4.233	0.000
μ_{31}	−0.016	−1.076	0.282	−0.018	−0.995	0.320	−0.018	−1.044	0.297
μ_{32}	0.013	1.439	0.150	0.004	0.468	0.640	0.011	0.932	0.351
μ_{33}	−0.016	−0.601	0.548	−0.009	−0.290	0.772	−0.006	−0.212	0.832

MGARCH Results

Variance									
c_{11}	0.002a	8.836	0.000	0.000	3.966	0.000	0.000a	3.403	0.001
c_{21}	0.001a	2.920	0.003						
c_{22}	0.001a	5.499	0.000	0.000	1.264	0.206	0.000a	2.863	0.004
c_{31}	0.001a	4.207	0.000						
c_{32}	0.000	0.147	0.883						
c_{33}	0.002a	9.398	0.000	0.000a	4.725	0.000	0.000a	3.890	0.000
α_{11}	0.263a	11.870	0.000	0.087a	2.506	0.012	0.063a	2.878	0.004
α_{12}	0.049a	2.356	0.018	−0.006	−0.465	0.642	0.025a	2.097	0.036
α_{13}	0.053a	2.909	0.004	−0.033	−0.666	0.506	0.000	0.001	0.999
α_{21}	0.023	1.294	0.196	0.056a	3.111	0.002	0.067a	2.624	0.009
α_{22}	0.216a	12.593	0.000	0.071a	3.575	0.000	0.056a	4.265	0.000
α_{23}	0.023b	1.919	0.055	-0.125a	−3.528	0.000	-0.085b	−1.876	0.061
α_{31}	−0.027	−0.636	0.525	0.018	0.772	0.440	0.025	1.128	0.259
α_{32}	−0.065	−1.302	0.193	0.007	0.703	0.482	0.013a	1.846	0.065
α_{33}	0.204a	6.333	0.000	0.042	1.127	0.260	0.038	1.078	0.281
β_{11}	0.957a	98.673	0.000	0.908a	13.212	0.000	0.929a	26.682	0.000
β_{12}	−0.011	−1.287	0.198	0.039	1.099	0.272	-0.040a	−2.032	0.042
β_{13}	−0.004	−0.633	0.527	−0.071	−0.514	0.608	0.001	0.013	0.990
β_{21}	−0.005	−0.978	0.328	−0.097	−1.113	0.266	−0.044	−1.070	0.284
β_{22}	0.974a	225.609	0.000	0.896a	28.652	0.000	0.935a	56.494	0.000
β_{23}	−0.005	−1.461	0.144	0.450b	1.865	0.062	0.039	0.698	0.485
β_{31}	0.008	0.447	0.655	−0.005	−0.128	0.898	0.002	0.052	0.959
β_{32}	0.010	0.449	0.653	0.091a	3.580	0.000	−0.014	−1.067	0.286
β_{33}	0.951a	84.792	0.000	0.845a	15.374	0.000	0.903a	13.752	0.000
ρ_{21}				0.338a	19.834	0.000			
ρ_{31}				0.765a	77.667	0.000			
ρ_{32}				0.238a	15.023	0.000			
θ_1							0.028a	3.225	0.001
θ_2							0.963a	79.527	0.000
Log L	21507.9			21418.3			21523.3		
AIC	−17.372			−17.300			−17.385		
SBC	−17.287			−17.215			−17.303		

Note: Model estimated using QMLE robust standard errors. The sample variables in the order are ECO (1), WTI (2) and PSE (3). In the variance equations, c indicates constant term, α and β denote ARCH and GARCH terms, respectively. ARCH and GARCH terms are interpreted as short-term and long-term volatility spillovers between two variables. Using Schwartz information criterion, we have used one lag for VAR.

DCC Plot

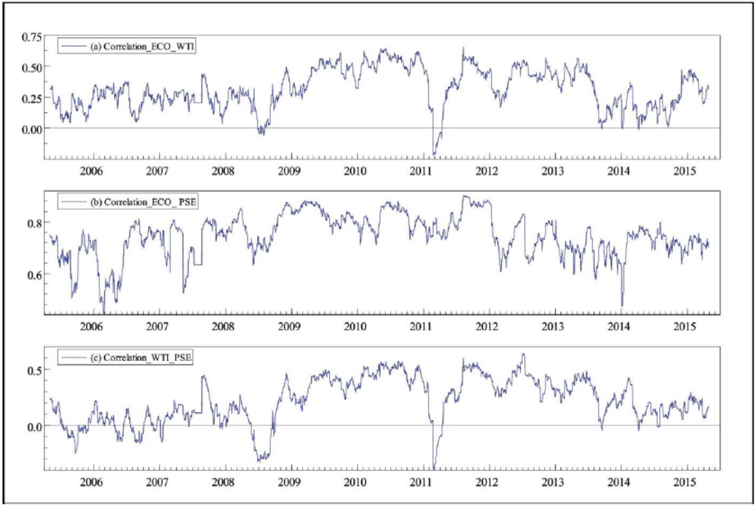


Fig. 6. Time-varying conditional correlations from the DCC model.

Title: On the dynamic dependence and investment performance of crude oil and clean energy stocks

- Kroner and Sultan (1993) Chkili (2016) used conditional volatility estimates to calculate hedge ratios.
- A typical hedging strategy is implemented by taking long (buy) in one asset (say asset 1) and short (sell) position in second asset (say asset 2). The time-varying hedge ratio between asset (1) and asset (2) $\mathrm{H}_{12,t} = \mathrm{h}_{12,t}/h_{22,t}$

Application 1: Clean Energy Hedge Ratio

Title: On the dynamic dependence and investment performance of crude oil and clean energy stocks

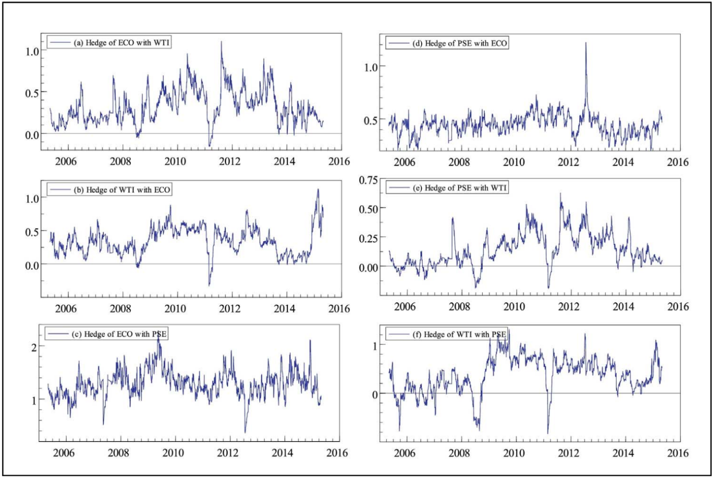
- The daily data of WilderHill Clean Energy Index (ECO), NYSE Arca Technology Index (PSE) and the futures contracts (nearest contract to maturity) of West Texas Intermediate (WTI).

Hedging results

Hedge ratio long/short summary statistics.

	Mean	Std. Dev.	Min.	Max.
ECO/WTI	0.32	0.20	−0.16	1.10
ECO/PSE	1.29	0.25	0.36	2.29
WTI/ECO	0.35	0.20	−0.33	1.12
WTI/PSE	0.37	0.37	−0.84	1.31
PSE/ECO	0.45	0.10	0.20	1.22
PSE/WTI	0.14	0.14	−0.19	0.63

Dynamic Hedging Results



Dynamic portfolio Weights

- Suppose we have two assets: asset (p) and asset (q) at time t. $h_{pq,t}$ is the conditional covariance between assets p and q. $h_{qq,t}$ is the conditional variance of asset q.

$$w_{pq,t} = \frac{h_{qq,t} - h_{pq,t}}{h_{pp,t} - 2h_{pq,t} + h_{qq,t}}$$

$$w_{pq,t} = \begin{cases} 0 & if w_{pq,t} < 0 \\ w_{pq,t} & if 0 \leq w_{pq,t} \leq 1 \\ 1 & if w_{pq,t} > 1 \end{cases}$$

Portfolio Weigths

Portfolio weights summary statistics.

	Mean	Std. Dev.	Min.	Max.
ECO/WTI	0.52	0.22	0.00	1.00
ECO/PSE	0.01	0.08	0.00	1.00
WTI/PSE	0.20	0.12	0.00	0.68

Application 2: India Commodity Derivatives

Objective: To examine the process of information transmission in Indias agriculture commodity futures market by investigating the price discovery and directional volatility spillovers between futures and spot prices of nine agricultural commodities viz., Barley, Cardamom, Castor seed, Chana (Chickpea), Chili, Mentha oil, Pepper, Soybean and Refined Soya, traded on Multi-Commodity Exchange (MCX) and National Commodity Derivatives Exchange (NCDEX). The study uses the daily data from January 01, 2009 to May 31, 2013.

MGARCH model: GARH-BEKK estimation results

		μ_1	μ_2	$c_{(1,1)}$	$c_{(2,1)}$	$c_{(2,2)}$	$\alpha_{(1,1)}$	$\alpha_{(1,2)}$	$\alpha_{(2,1)}$	$\alpha_{(2,2)}$	$\beta_{(1,1)}$	$\beta_{(1,2)}$	$\beta_{(2,1)}$	$\beta_{(2,2)}$
Barley	***Coeff.***	-0.015	0.009	0.223	-0.246	0.000	0.590	-0.011	-0.087	0.161	0.763	0.042	0.119	0.939
		[-0.823]	[0.377]	[4.812]*	[-1.442]	[0.000]	[9.950] *	[-0.153]	[-2.064] *	[3.81] *	[12.92] *	[0.554]	[2.172] *	[13.568] *
Castor seed	***Coeff***	-0.003	0.029	0.632	0.414	0.000	0.105	0.090	0.470	0.080	0.303	-0.426	0.158	1.070
		[-0.136]	[1.017]	[9.495]*	[7.977] *	[0.000]	[1.134]	[1.487]	[6.688] *	[1.583]	[3.57] *	[-7.427]*	[5.041] *	[73.734] *
Chana	***Coeff***	-0.018	-0.040	0.560	0.184	0.000	0.328	0.049	0.412	0.122	0.469	-0.226	0.164	1.058
		[-0.739]	[-1.480]	[11.42] *	[5.377] *	[0.010]	[7.403] *	[1.562]	[7.403] *	[2.87] *	[6.30] *	[-4.362] *	[4.110] *	[63.400] *
Pepper	***Coeff***	0.002	0.000	0.174	0.824	0.000	-0.007	-0.281	-0.374	0.296	0.539	-0.061	0.562	0.490
		[0.073]	[-0.021]	[1.139] *	[4.866] *	[0.000]	[-0.125]	[-2.947] *	[-2.741] *	[3.07] *	[3.47] *	[-0.725]	[3.839] *	[2.488] *
Refined soya	***Coeff***	-0.001	-0.018	0.138	-0.027	0.000	-0.091	-0.363	0.220	0.373	1.083	0.217	-0.241	0.794
		[-0.025]	[-0.711]	[3.936] *	[-0.771]	[0.000]	[-1.078]	[-9.017] *	[4.799] *	[8.01] *	[77.20] *	[17.592] *	[-9.581] *	[45.781] *
Soybean	***Coeff***	0.032	0.030	0.158	-0.049	0.000	-0.166	0.167	0.261	0.300	1.049	0.198	-0.172	0.774
		[1.215]	[1.619]	[3.344] *	[-0.867]	[0.000]	[-1.564]	[1.695]	[3.097] *	[1.948]	[53.90] *	[4.267] *	[-2.998] *	[7.298] *
Chilli	***Coeff***	-0.041	-0.026	0.176	0.655	0.101	0.161	0.103	-0.020	0.595	0.984	0.090	-0.031	0.378
		[-0.942]	[-0.795]	[4.525] *	[0.501]	[0.014]	[2.276] *	[1.802]	[-0.561]	[3.53] *	[62.01] *	[0.394]	[-0.580]	[2.860] *
Moil	***Coeff***	0.008	-0.017	0.822	0.027	0.000	0.421	0.184	-0.032	0.307	-0.199	-0.133	0.432	0.964
		[0.514]	[-1.107]	[22.03] *	[1.275]	[0.000]	[9.034] *	[9.046] *	[-1.259]	[9.60] *	[-2.74] *	[-2.960] *	[7.863] *	[59.78] *
Cardamom	***Coeff***	0.011	0.014	0.620	0.080	0.000	0.945	0.128	-0.658	0.113	-0.121	-0.262	0.337	1.045
		[0.282]	[0.560]	[19.74] *	[2.883] *	[0.000]	[9.232] *	[2.930] *	[-4.200] *	[2.18] *	[-1.25]	[-4.671] *	[16.62] *	[149.72] *

Note: * denotes the level of significance at 5% and better. Values in parentheses are t-values

Thanking you!!

Dynamic conditional-correlation matrix form is

- An alternative to the constant conditional-correlation GARCH model

$\sigma_{11,t} = \gamma_{10} + \gamma_{11}\epsilon^2_{1,t-1} + \delta_{11}\sigma_{11,t-1}$
$\sigma_{22,t} = \gamma_{20} + \gamma_{22}\epsilon^2_{2,t-1} + \delta_{22}\sigma_{22,t-1}$
$\sigma_{12,t} = \rho_{12,t}(\sigma_{11,t}, \sigma_{22,t})^{0.5}$