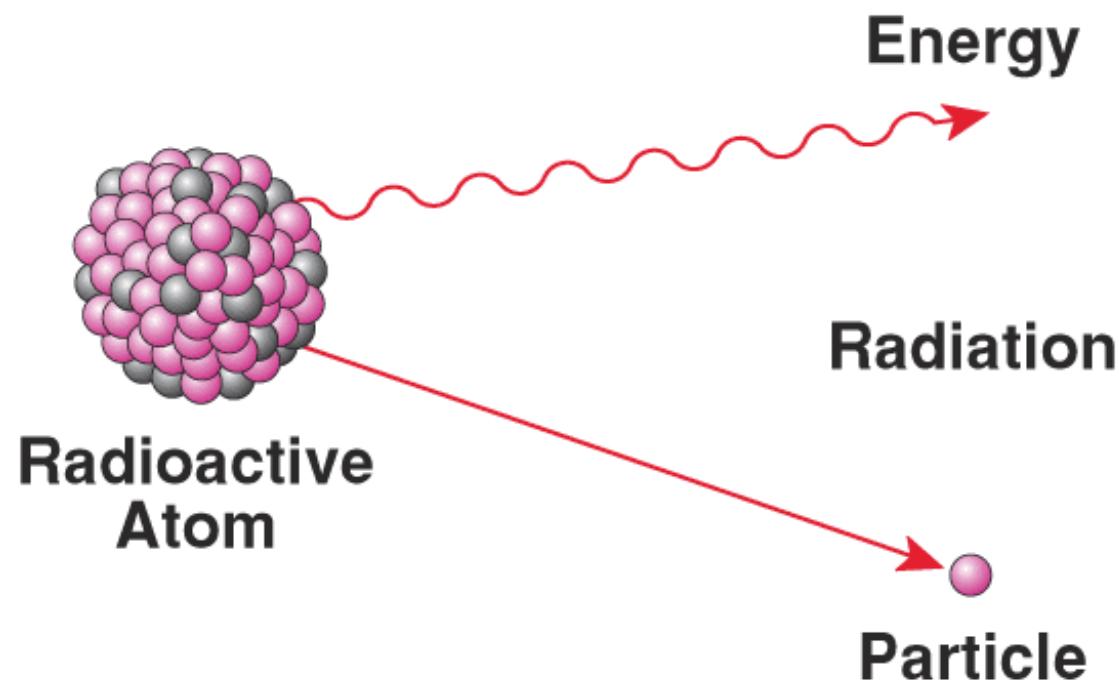
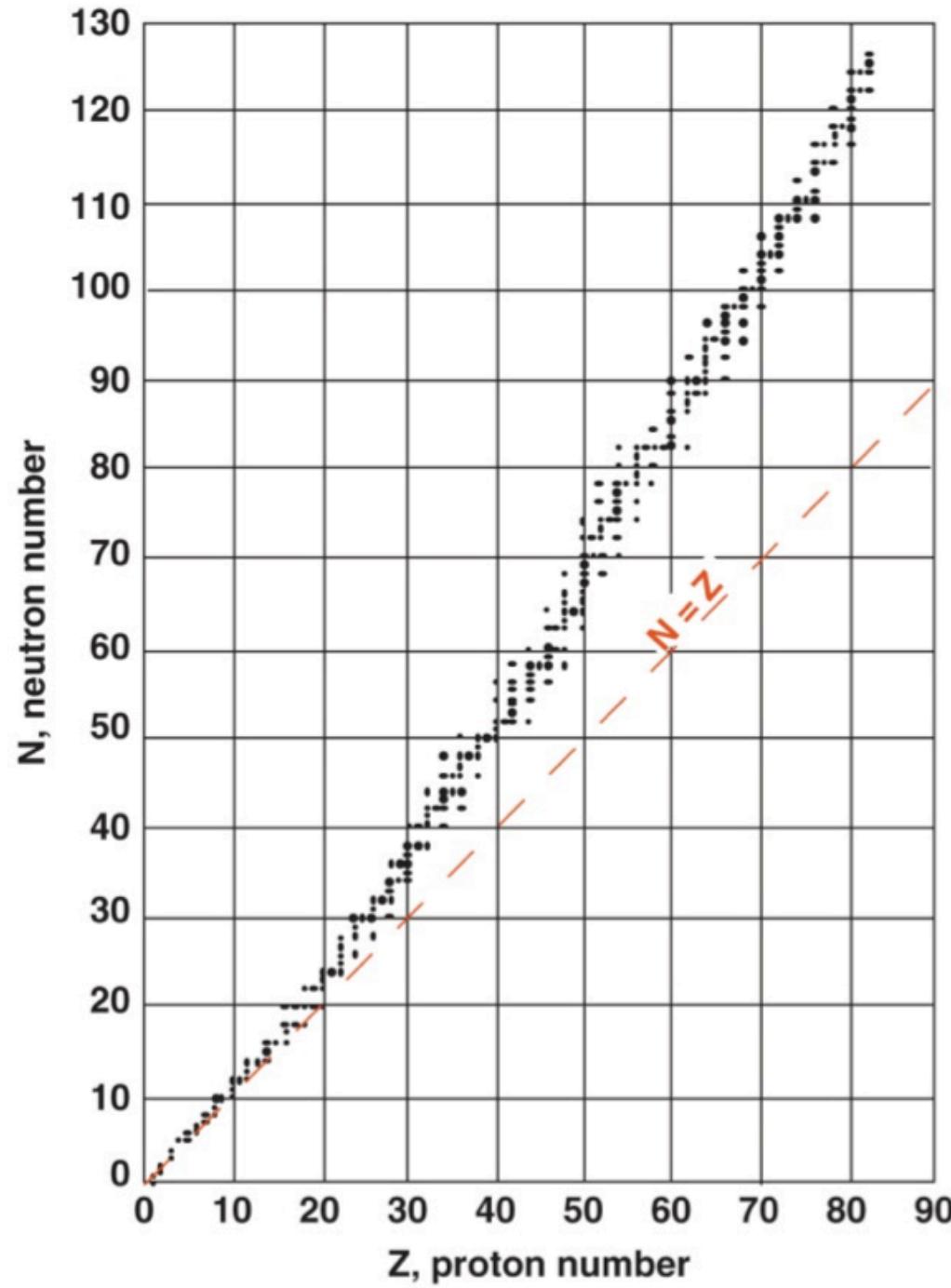


Isotope Geochemistry





I-b Properties of nuclides

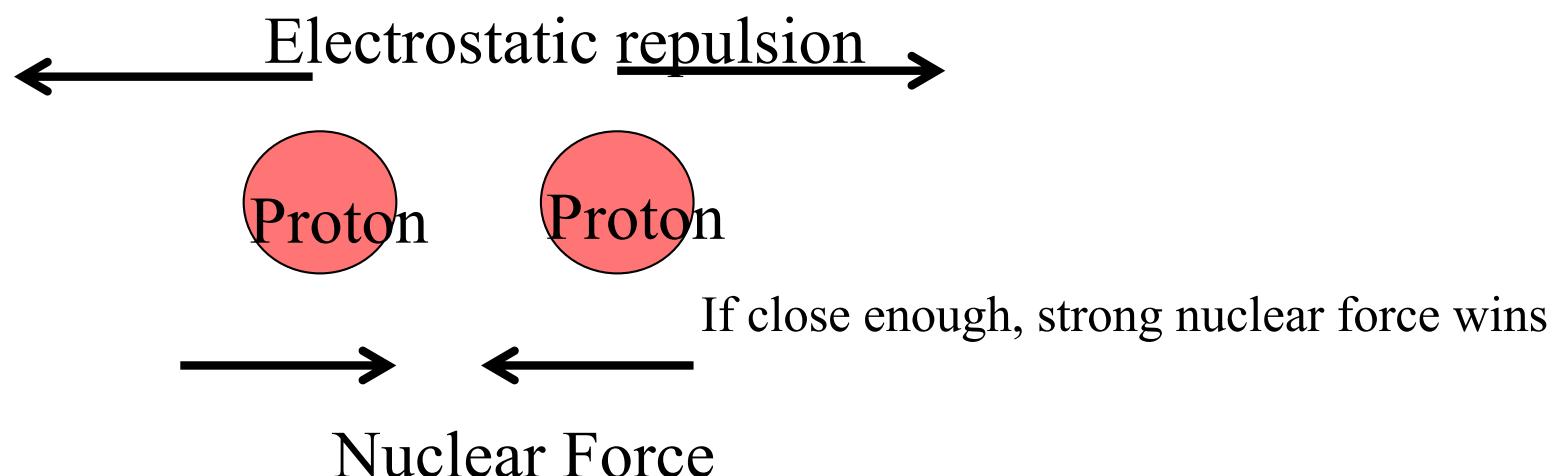
- Mass:

The mass of a nuclide is given in amu (atomic mass unit)

Definition: $M(^{12}\text{C})=12 \text{ amu}$

$m_{\text{proton}} \approx m_{\text{neutron}} \approx 1 \text{ amu}$

- The mass of one atom of carbon 12 (^{12}C) is determined by mass spectrometry and is equal to $1.9922 \times 10^{-23} \text{ g}$
- Electrostatic repulsion vs. nuclear attraction



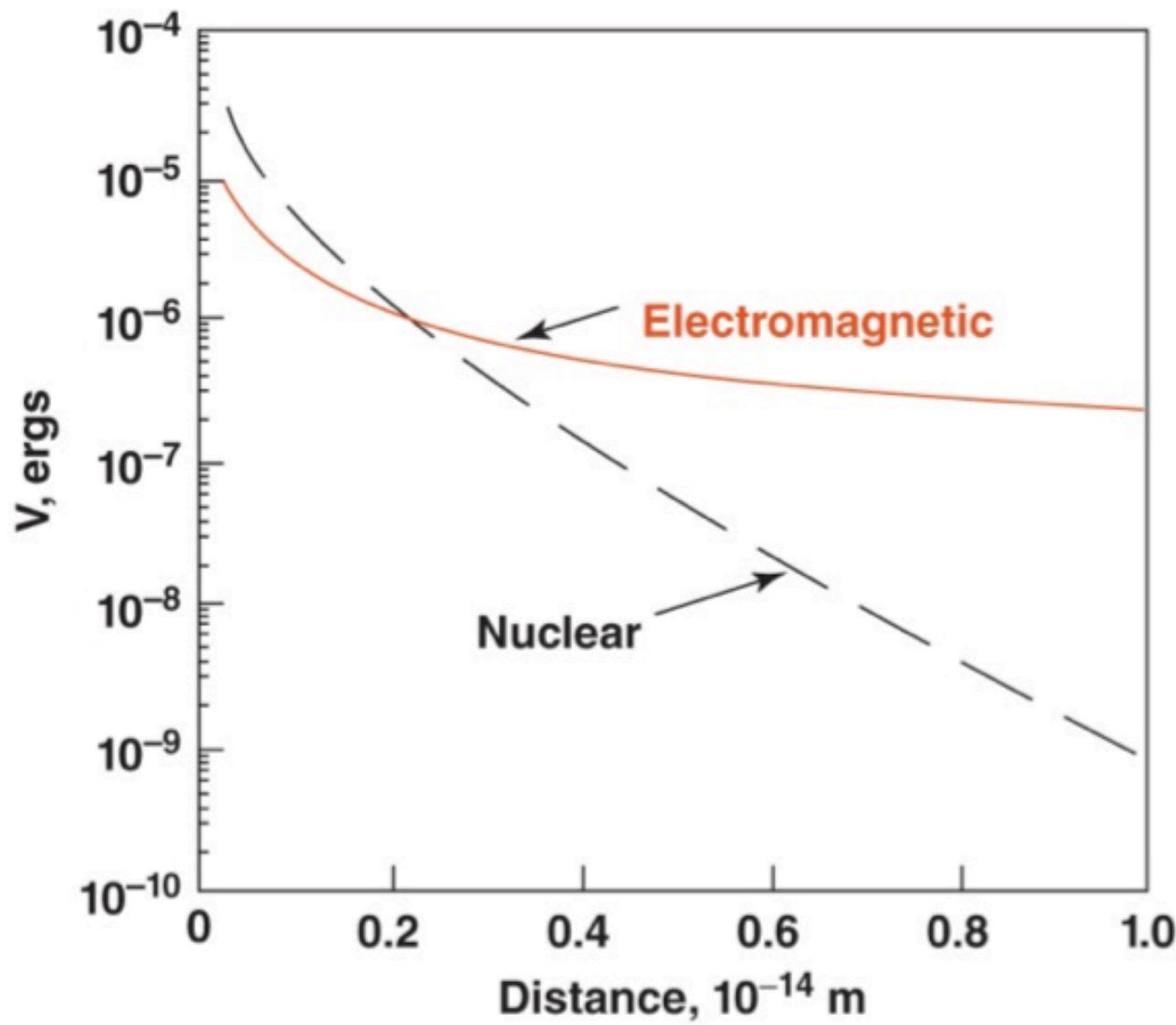
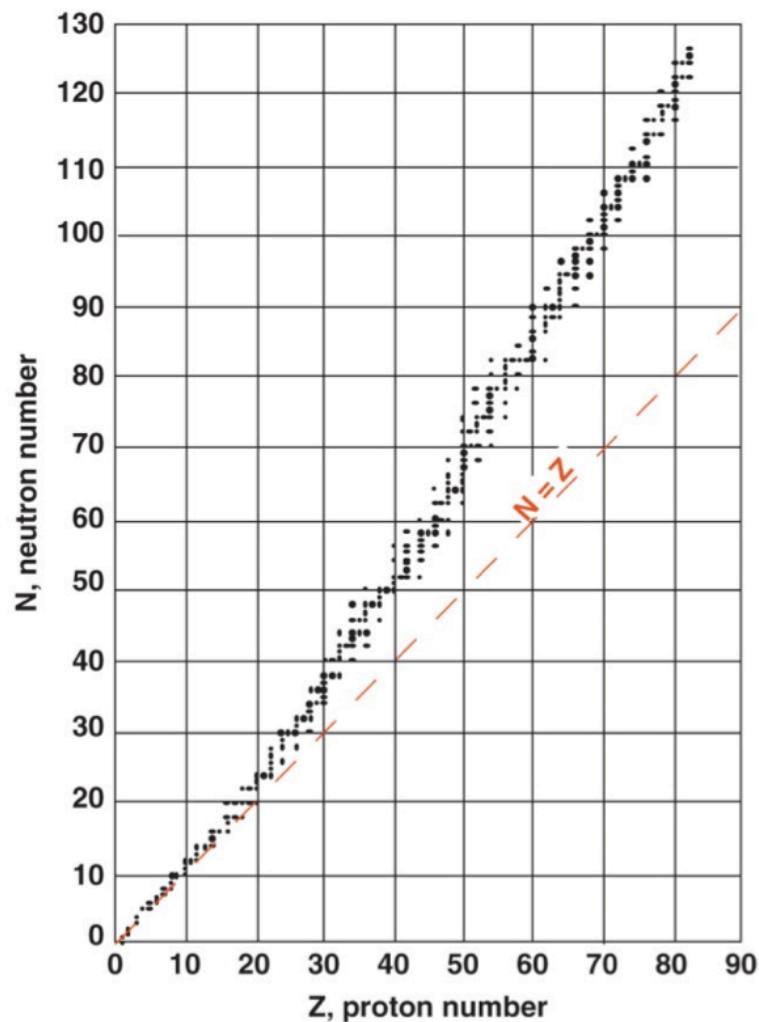


Figure 1.2 The nuclear and electromagnetic potential of a proton as a function of distance from the proton.



The elements deviate from the 1:1 line.

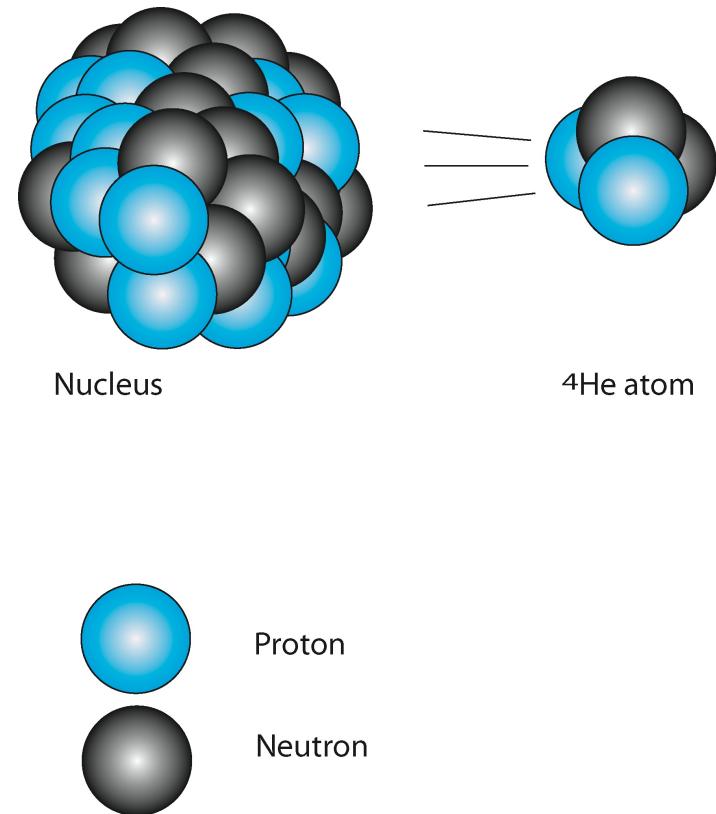
As the number of protons increases in a nucleus, the electromagnetic force increases faster than the strong nuclear force. Therefore larger atoms needs more and more neutrons to remain stable.

II. Natural Radioactivity

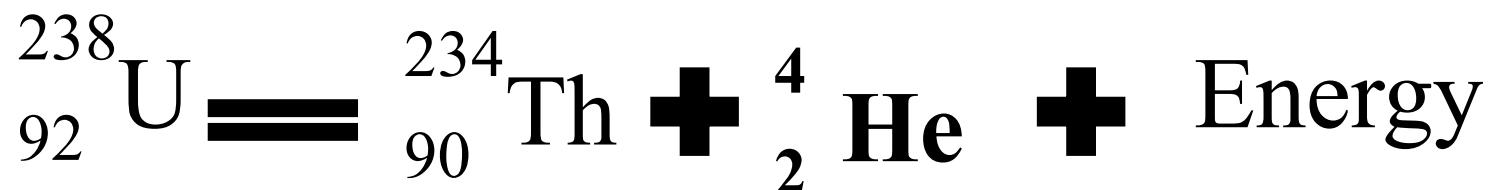
- Radioactivity is the emission of particles from unstable nuclei. Radioactivity allows a nucleus to evolve from an unstable state to a more stable configuration.
- The emitted particles by radioactivity are of three types: α , β , γ .
 $\alpha = {}^4\text{He}$ nucleus;
 $\beta^- = \text{electron}$ or $\beta^+ = \text{positron}$;
 $\gamma = \text{photons}$

α - decay

- Occurs for unstable nuclides with $A \geq 56$ (except ${}^5\text{He}$, ${}^5\text{Li}$ and ${}^6\text{Be}$)
- Consists of 2 proton and 2 neutron



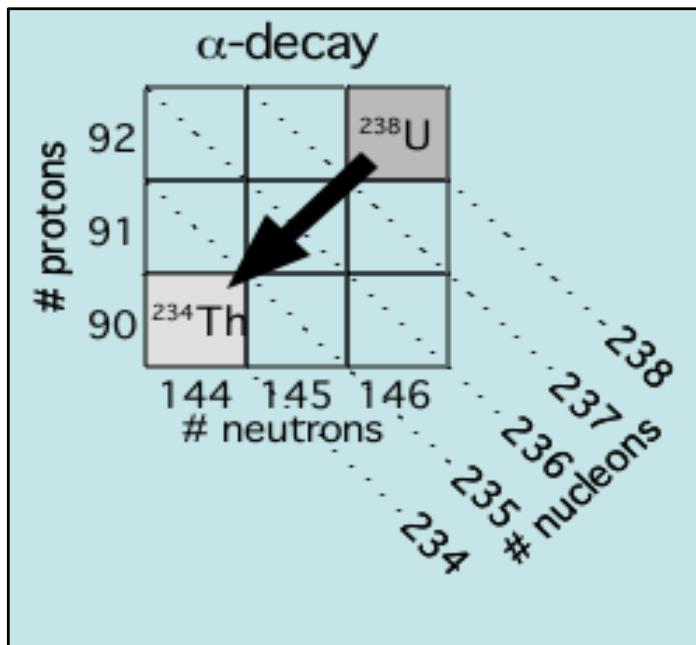
	<i>Atomic No.</i>	<i>Neutron No.</i>	<i>Mass No.</i>
<i>Parent</i>	Z	N	Z+N = A
<i>Daughter</i>	Z-2	N-2	Z-2+N-2 = A-4



α - decay

Emission of an α particle =⁴He nucleus (2 neutrons, 2 protons)

The number of nucleons A decreases by 4 and its Z by 2.



Example: $^{238}\text{U} \rightarrow ^{234}\text{Th} + ^4\text{He}$

Mass balance:

^{238}U	238.0508 amu
^{234}Th	-234.0436 amu
^4He	-4.00260 amu

Mass defect 0.0046 amu

$$= 6.86 \times 10^{-10} \text{ J/decay}$$

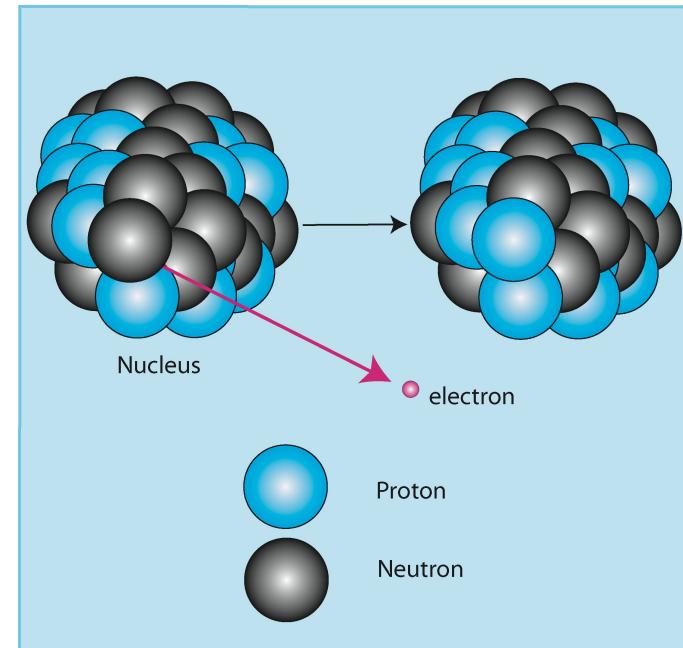
$$= 1.74 \times 10^{12} \text{ J/g } ^{238}\text{U}$$

$$=\sim 480,000 \text{ KW-H (1.6 Billion BTUs)}$$

This is the principal mode of decay of heavy isotopes such as U, Th, ²⁰⁹Bi

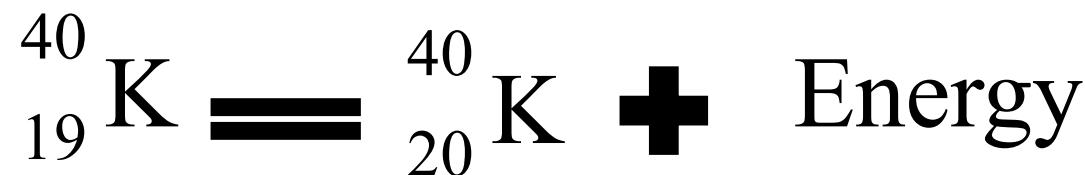
β^- decay

Decay by emitting a negatively charged beta particle



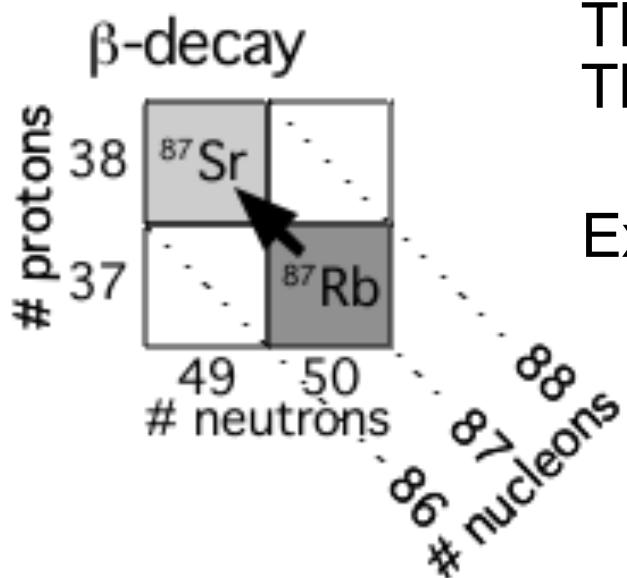
Neutron = Electron and Proton

	<i>Atomic No.</i>	<i>Neutron No.</i>	<i>Mass No.</i>
<i>Parent</i>	Z	N	Z+N =A
<i>Daughter</i>	Z+1	N-1	Z+1+N-1 = A

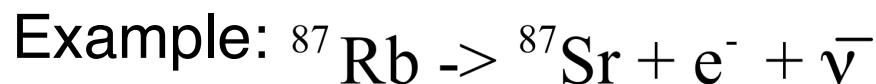


β^- decay

Emission of an electron (and an antineutrino) during the conversion of a neutron to a proton



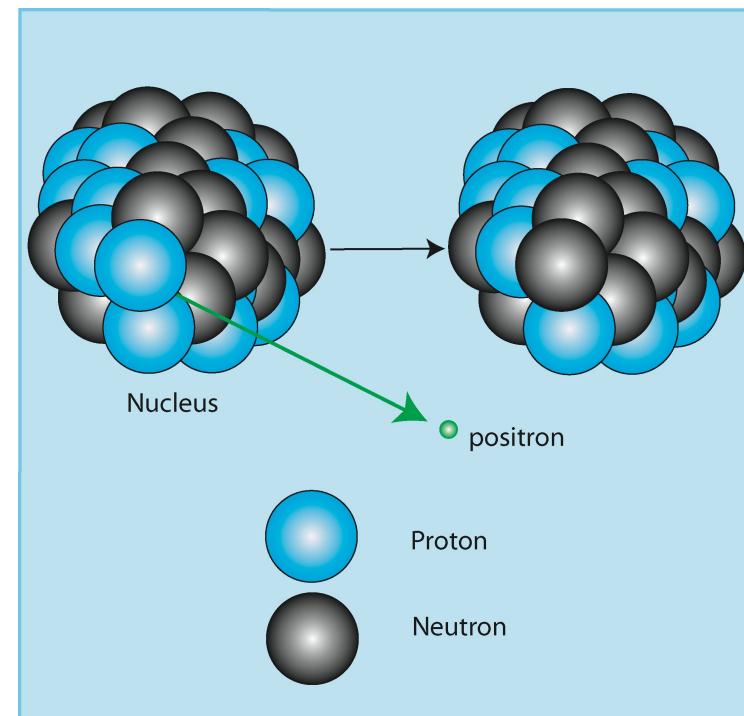
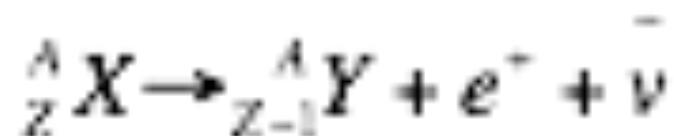
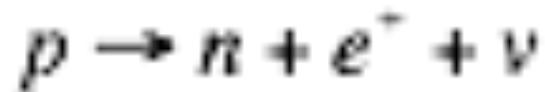
The number of nucleons does not change.
The atomic number increase by one:



This is the preferred mode of decay for nuclei with an excess number of protons relative to the stability valley.

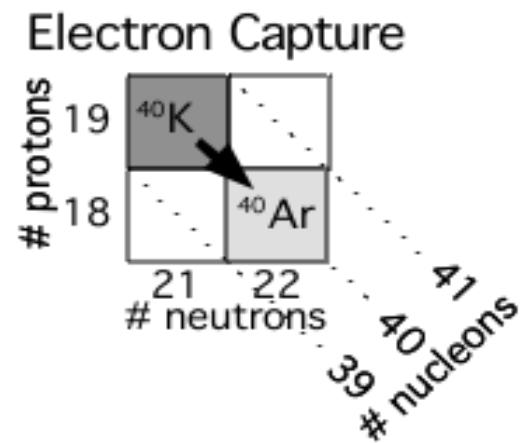
β^+ decay and electron capture

- A proton transforms into a neutron with emission of a positron and a neutrino:



β^+ decay and electron capture

Emission of a positron (and a neutrino) *or* capture of an electron from an internal layer. This is the conversion of a proton into a neutron



The number of nucleons does not change. The atomic number decreases by one :

Examples: ${}^{40}_{19}\text{K} \rightarrow {}^{40}_{18}\text{Ar} + e^- + \nu$

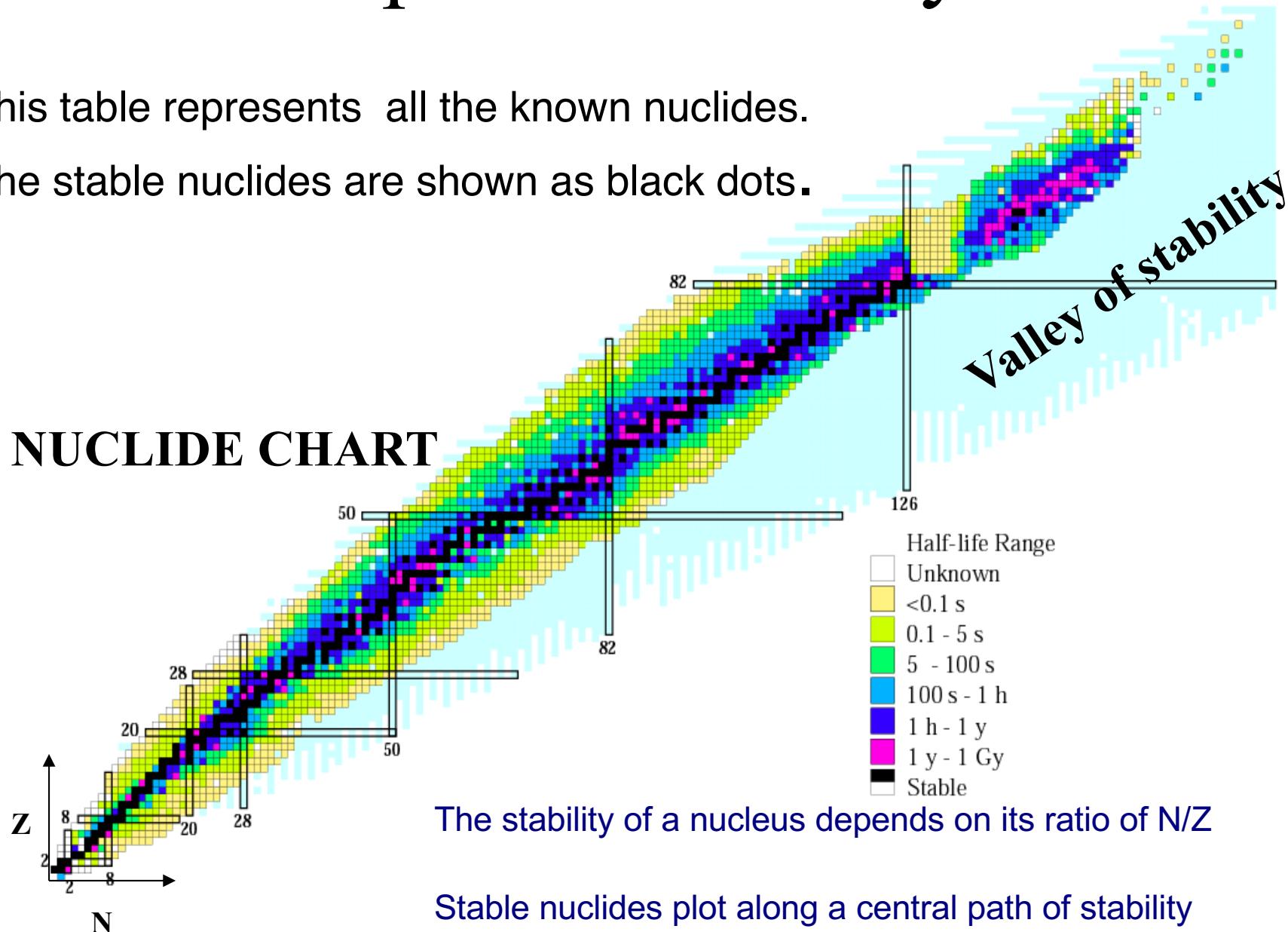
These are the decay mode for nuclei with an excess of protons relative to the stability valley.

Central path of stability

This table represents all the known nuclides.

The stable nuclides are shown as black dots.

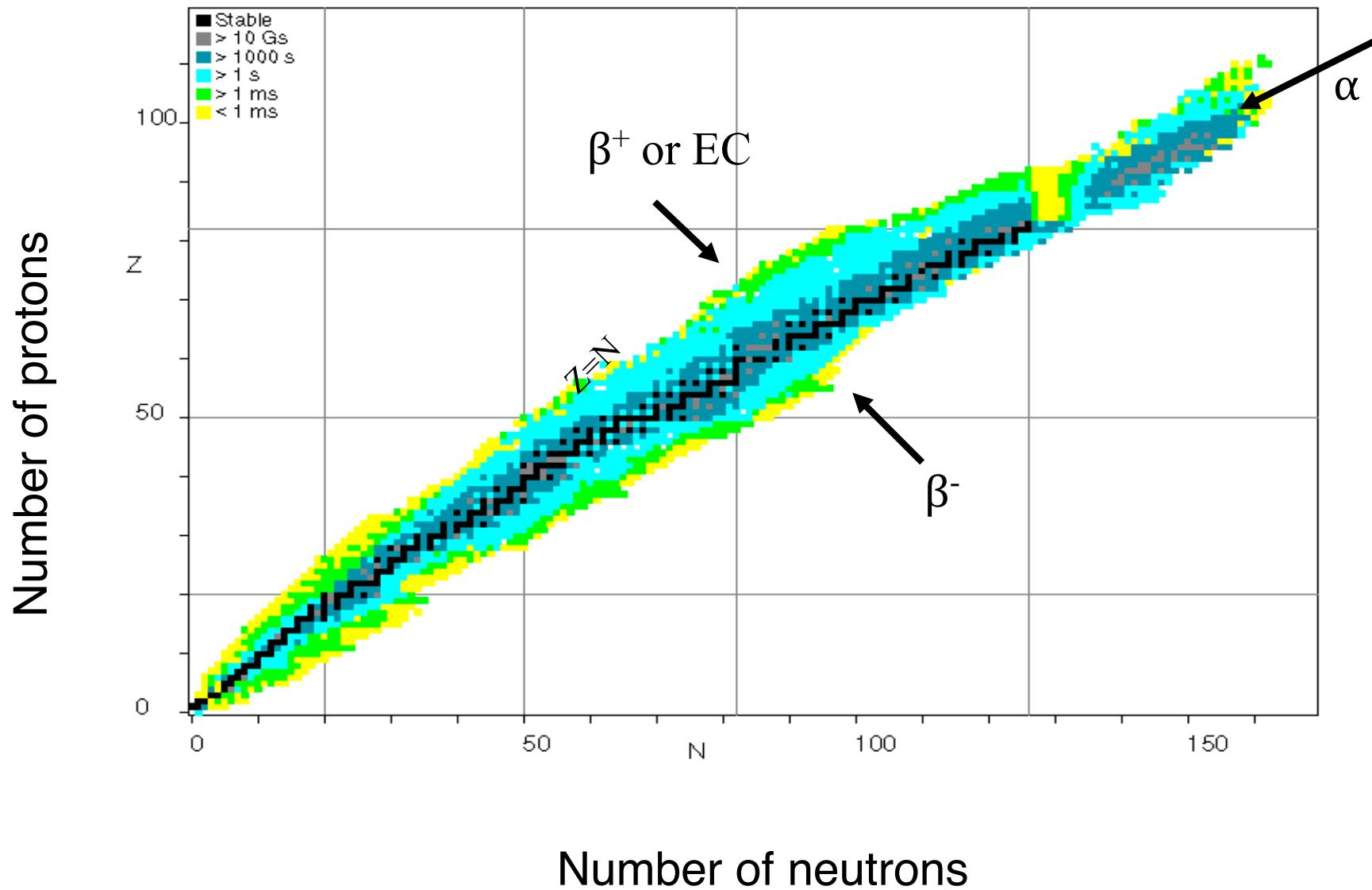
NUCLIDE CHART



The stability of a nucleus depends on its ratio of N/Z

Stable nuclides plot along a central path of stability
At low masses, $N/Z \approx 1$ ($Z=N$)
At high masses, $N/Z \approx 3$

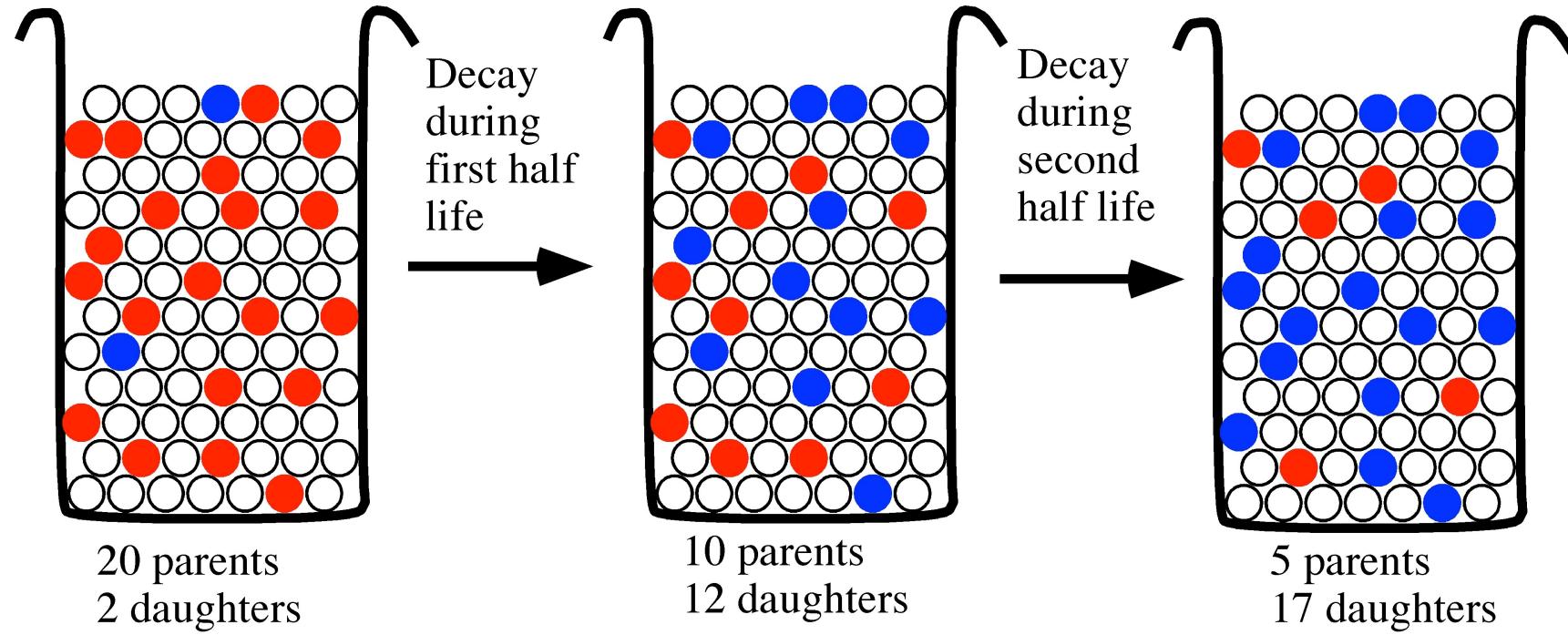
NUCLIDE CHART



Isotopes



Dating with Radioactivity



*The actual number of atoms that decay (**radioactive parent**) continually decreases and the number of stable daughter atoms (**radiogenic daughter**) increases.*

Simple Radioactive Decay

Radioactive decay is a stochastic process linked to the stability of nuclei. The rate of change in the number of radioactive nuclei is a function of the total number of nuclei present and the decay constant λ .

$$-\frac{dN}{dt} = \lambda N$$

The sign on the left hand is negative because the number of nuclei is decreasing. Rearranging this equation yields

$$-\frac{dN}{N} = \lambda dt$$

and integrating yields

$$-\ln N = \lambda t + C$$

C is the integration constant. We solve for C by setting $N = N_0$ and $t = t_0$. Then

$$C = -\ln N_0$$

Substituting for C gives

$$-\ln N = \lambda t - \ln N_0$$

We rearrange

$$\ln N - \ln N_0 = -\lambda t$$

Rearrange again

$$\ln N/N_0 = -\lambda t$$

Eliminate the natural log

$$N/N_0 = e^{-\lambda t}$$

And rearrange

$$N = N_0 e^{-\lambda t}$$

...continue...

Unfortunately, we don't know N_0 a priori, but decayed N have produced radiogenic daughters D^* .

Therefore

$$D^* = N_0 - N$$

Replacing N_0 with $N e^{\lambda t}$ yields

$$D^* = N e^{\lambda t} - N$$

Rearranged

$$D^* = N (e^{\lambda t} - 1) \quad \text{or, for small } \lambda t, \quad D^* = N \lambda t,$$

The number of daughter isotopes is the sum of those initially present plus those radiogenically produced.

$$D = D_0 + D^*$$

Therefore,

$$D = D_0 + N (e^{\lambda t} - 1) \quad \text{or, for small } \lambda t, \quad D = D_0 + N \lambda t,$$

This is the basic radioactive decay equation used for determining ages of rocks, minerals and the isotopes themselves. D and N can be measured and λ has been experimentally determined for nearly all known unstable nuclides. The value D_0 can be either assumed or determined by the **isochron method**.

How to obtain a “half-life”?

The rate of decay of a radioactive parent nuclide to a stable daughter product is proportional to the number of nuclides present at any time (Rutherford and Soddy, 1902)

$$\frac{dN}{dt} = -\lambda N$$

(1) whereby N is number of nuclides , t is time and λ is the decay constant

Integration of 1

$$\int_{N_0}^N \frac{dN}{dt} = -\lambda \int_0^t dt$$

results in

$$\ln \frac{N}{N_0} = -\lambda t$$

(2) which can be rewritten as

$$N = N_0 e^{-\lambda t}$$

(3)

Half life: $N=N_0/2$ and $t=t_{1/2}$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Points to Remember

$$-dN/dt = \lambda N$$

N = # atoms of the radioactive nuclide, t = time,
 λ = decay constant = probability that an atom will decay in a unit time

$$N = N_0 e^{-\lambda t}$$

$$N_0 = N \text{ at } t=0$$

$$t_{1/2} = (\ln 2) / \lambda = .693/\lambda$$

Half-life = time it takes for half a sample to decay (t when $N = \frac{1}{2}N_0$)

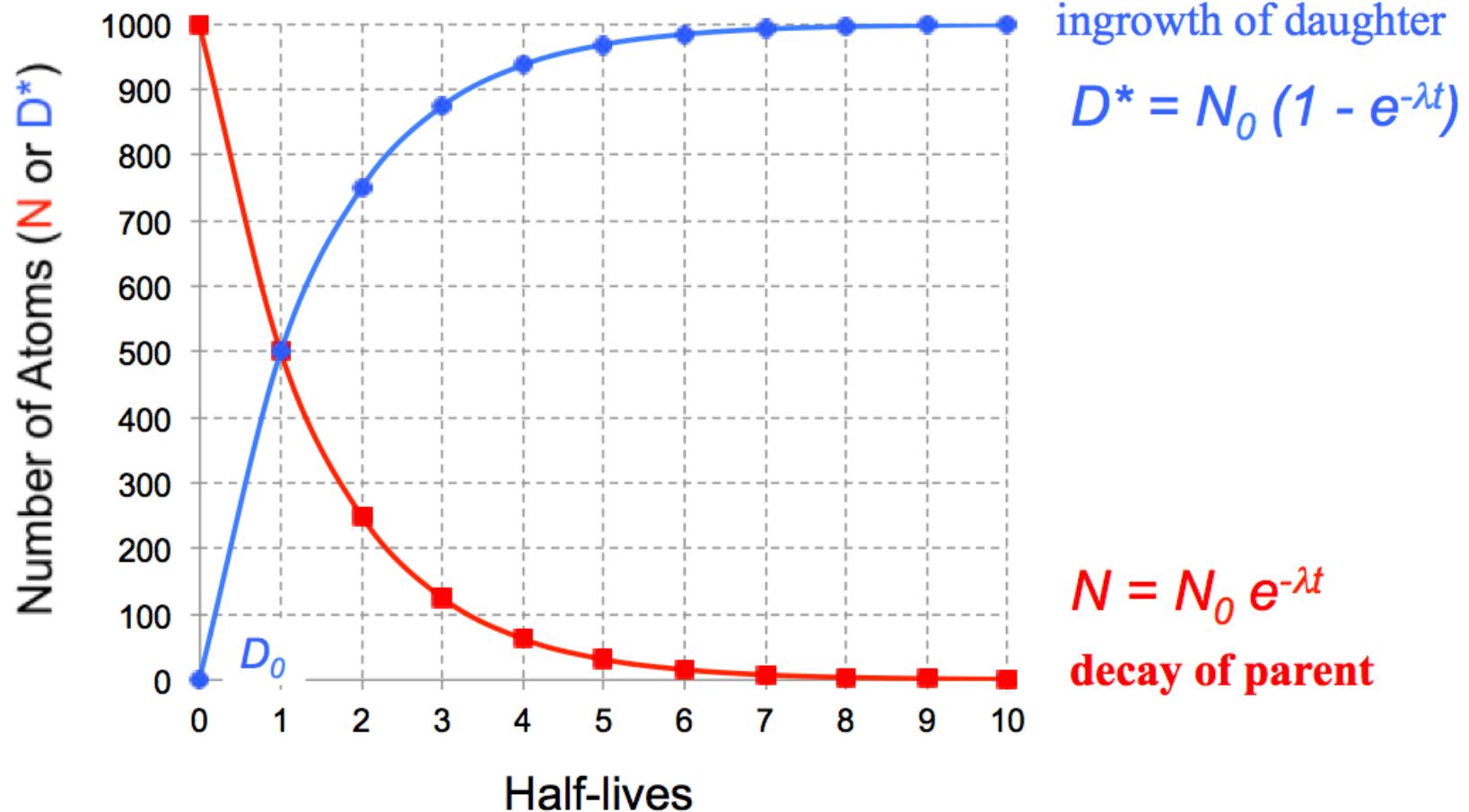
$$D = D_0 + N(e^{\lambda t} - 1)$$

D^* = radiogenic daughter = # daughter atoms produced by radioactive decay of a parent, P ($D = D_0 + D^*$)

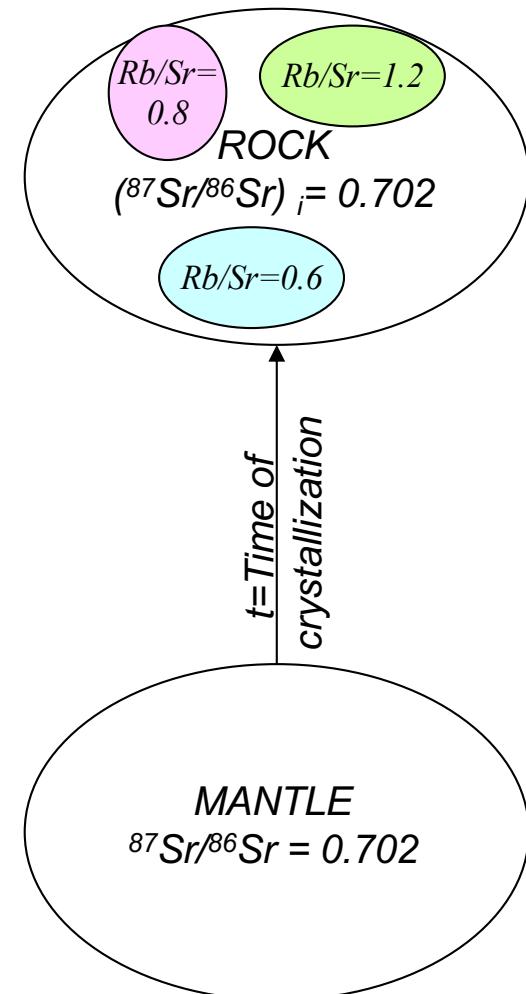
Divide by a stable, non-radiogenic isotope of the daughter element to get ratios e.g. for $^{87}\text{Rb} \rightarrow ^{87}\text{Sr} + \beta^-$:

$$^{87}\text{Sr}/^{86}\text{Sr} = (^{87}\text{Sr}/^{86}\text{Sr})_0 + ^{87}\text{Rb}/^{86}\text{Sr} (e^{\lambda t} - 1) \text{ [Example]}$$

Simple Decay: Radioactive Parent \Rightarrow Stable Daughter



^{87}Rb - ^{87}Sr decay equation (widely used in river geochemistry)



Uses of Radiogenic Isotopes

- Age Determination
- Source Identification
- Mixing Calculation

Dating

We can use the radioactive decay equation to calculate the age of a sample. We can measure the present day ratios and λ , but we still have 2 unknowns: R_0 and t . What can we do?

- 1) **Assume zero initial daughter.** This approach can be valid if you know something about mineralogy. For example, zircons are often used for U-Pb dating because the mineral incorporates U, but not lead. Therefore any Pb measured is radiogenic. Similarly, micas accept Rb but not Sr. Another example is K-Ar dating of volcanic rocks. Ar, the daughter, is lost upon eruption, and the only Ar present is radiogenic Ar.
- 2) **Use 2 different isotope systems.** For example, if you know what the age should be from a system where you think you know the initial daughter ratio (e.g. U-Pb), you can calculate the initial daughter ratio.
- 3) **Assume one and calculate the other.** If we assume the initial ratio and calculate an age, it is called a “Model Age”
- 4) **Use an isochron diagram**

The Isochron

The radioactive decay equation is in the form of a line:

$$D = D_0 + P(e^{\lambda t} - 1) \dots y = b + xm$$

Plot D ratio vs. P/D for several comagmatic or cogenetic samples and draw a best fit line through the data

y-intercept = initial D ratio, slope is related to t

This line is called an “**Isochron**”

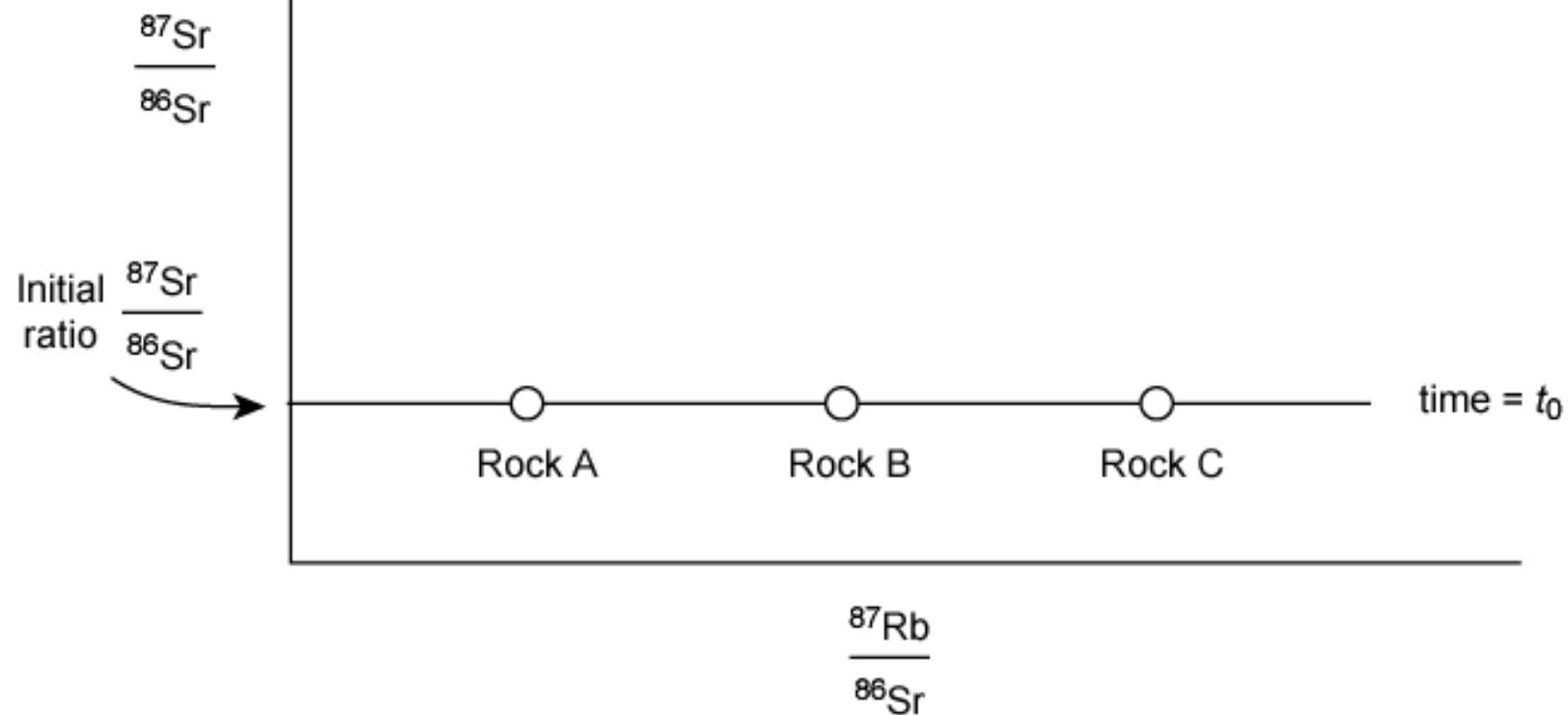
Represents true age if:

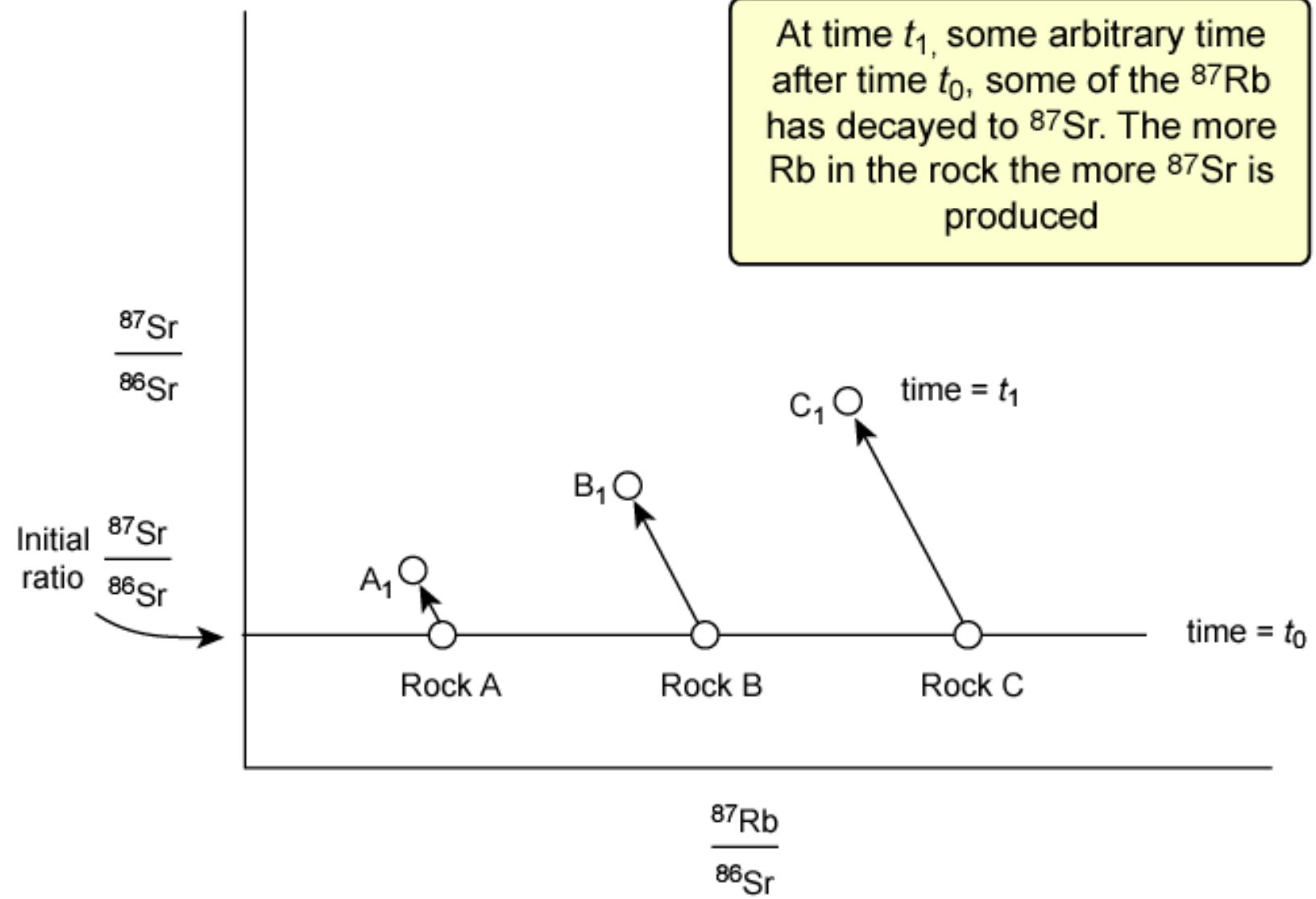
- (1) The system was at isotopic equilibrium at time $t = 0$.
i.e. all the samples formed with the same initial daughter isotope ratio
- (2) Closed system since formation

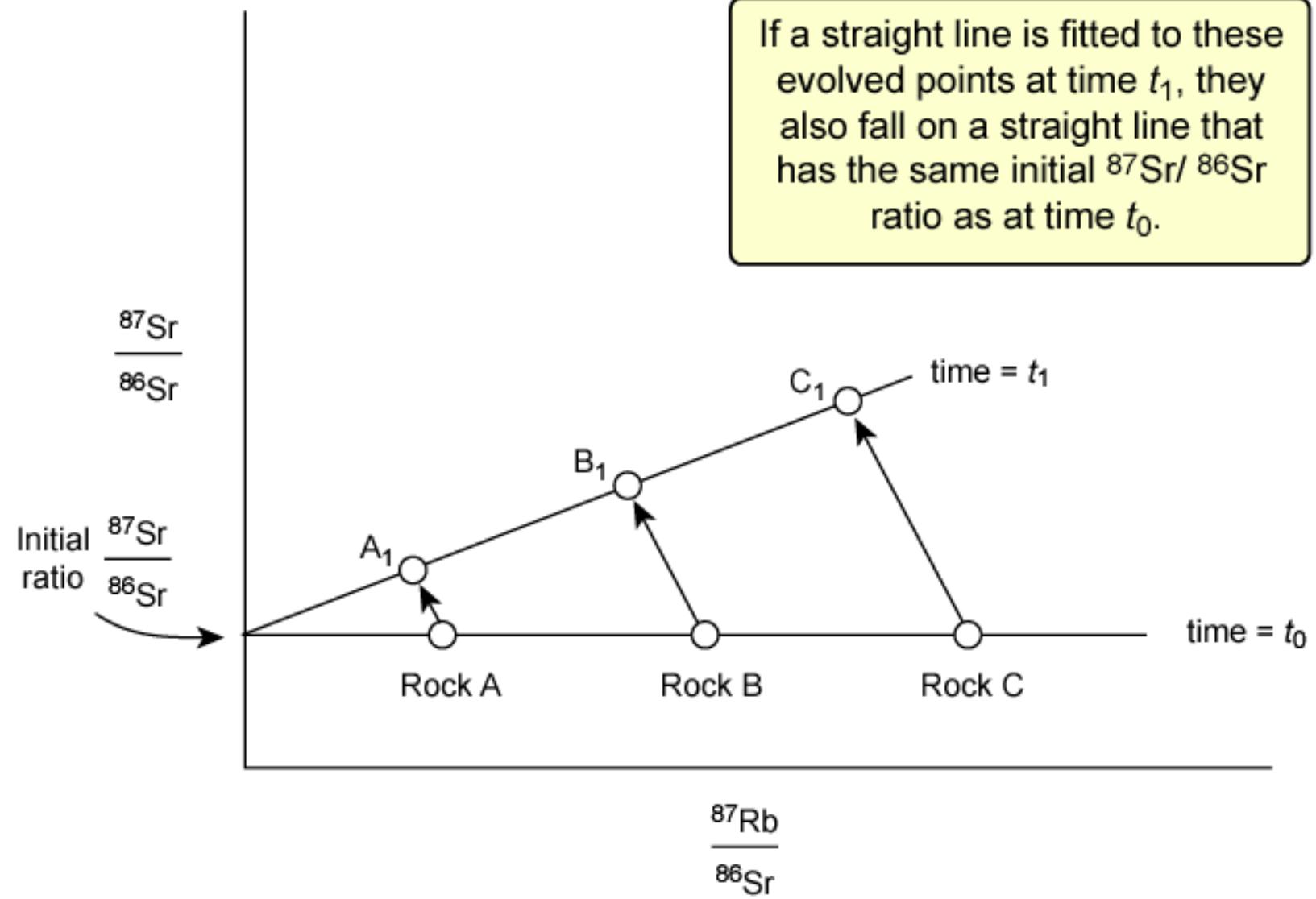
Whole-rock isochron represents age of formation

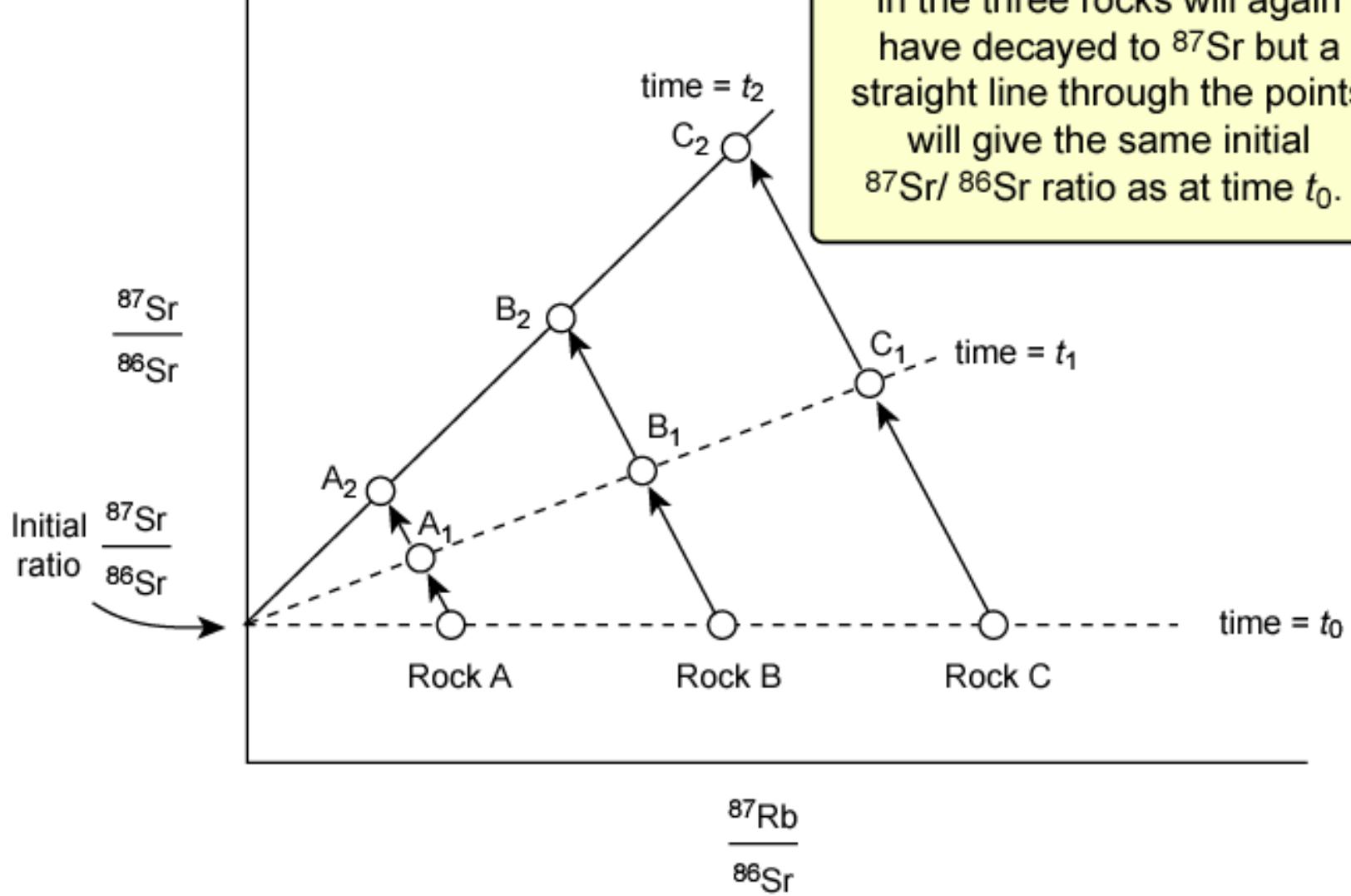
Mineral isochron represents age of last metamorphosis

Start by plotting three rocks with different Rb contents at time t_0



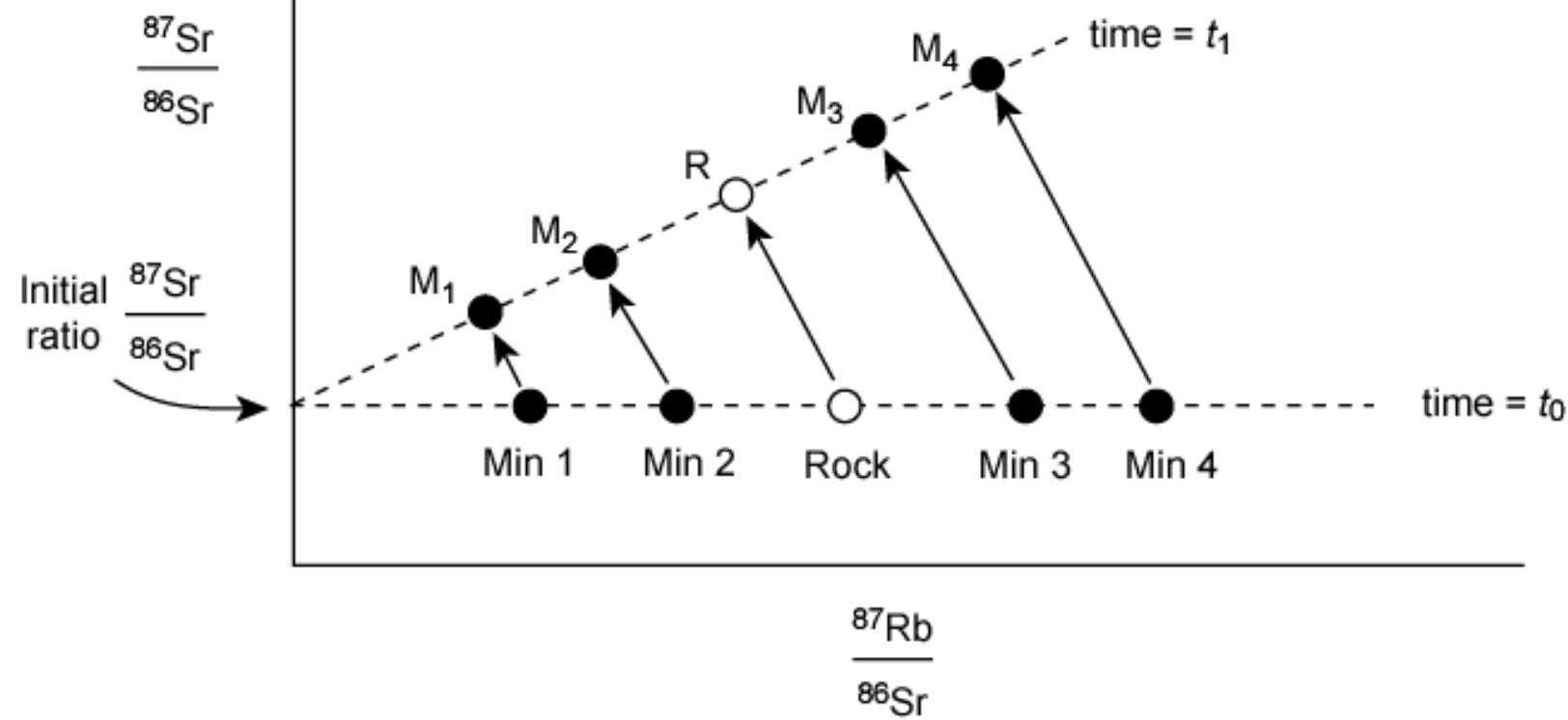






Similarly after time t_2 , the ^{87}Rb in the three rocks will again have decayed to ^{87}Sr but a straight line through the points will give the same initial $^{87}\text{Sr}/^{86}\text{Sr}$ ratio as at time t_0 .

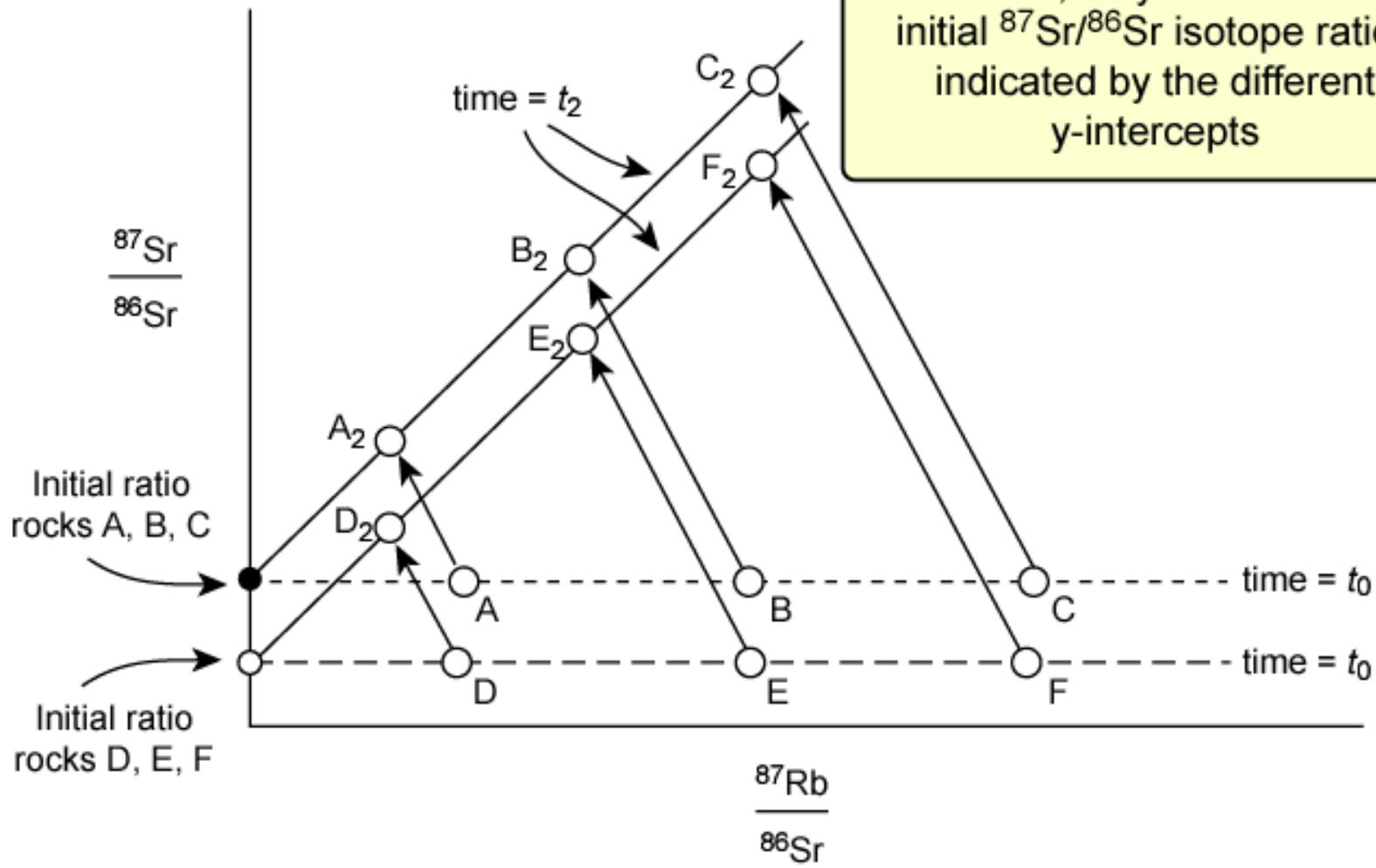
One rock with four different minerals each with different Rb/Sr ratios. Also evolve so that they lie along a steeper but still linear line going through the same initial $^{87}\text{Sr}/^{86}\text{Sr}$ ratio

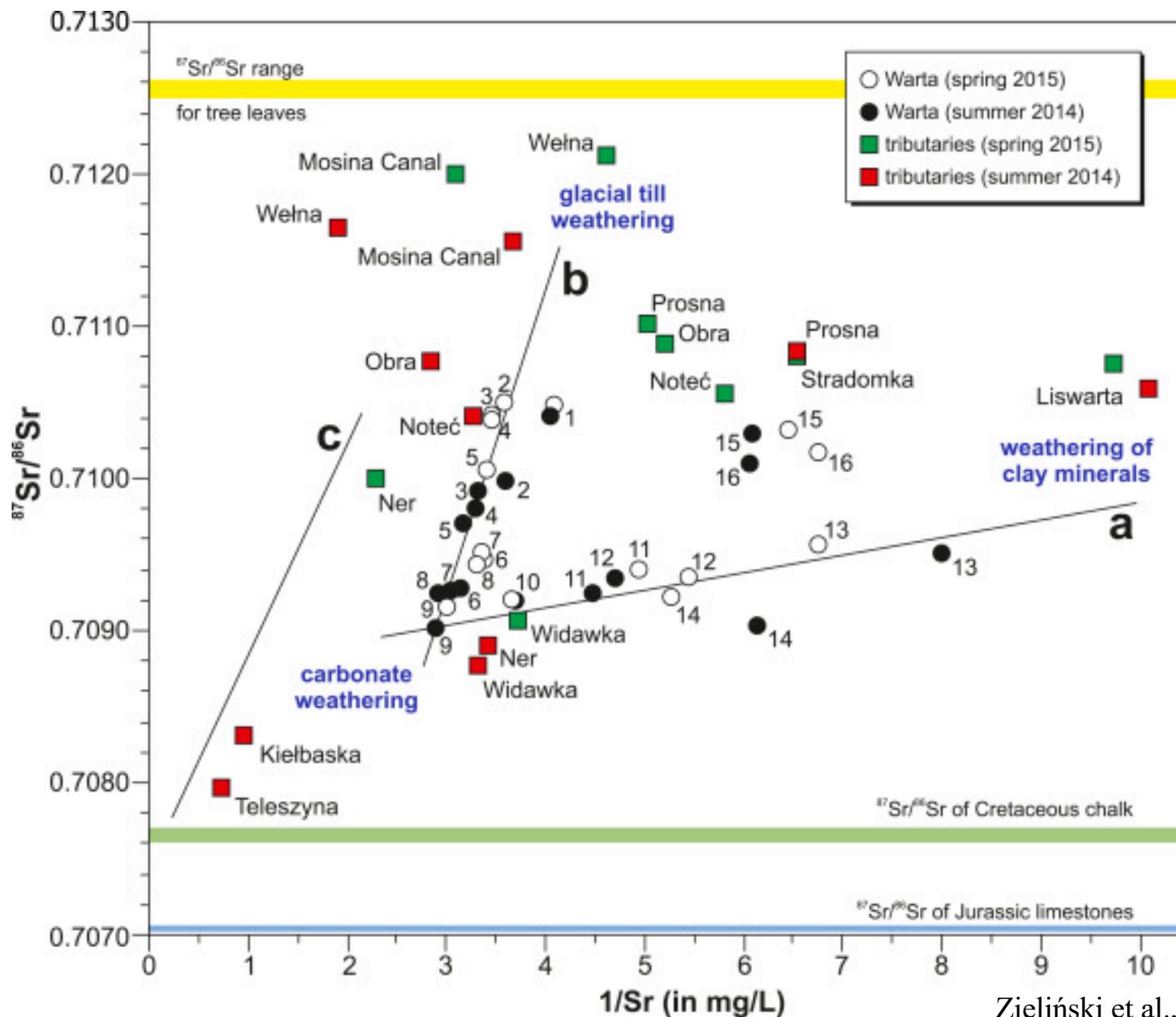


The initial ratio

- How do we know if a series of sediments are co-genetic?
- For sediments to be co-genetic, implies that they are derived from the same parent material.
- This parent material would have had a single $^{87}\text{Sr}/^{86}\text{Sr}$ isotope value, ie the initial isotope ratio
- Therefore, all samples derived from the same parent magma should all have the same $^{87}\text{Sr}/^{86}\text{Sr}$ isotope ratio
- If they don't, it implies that they are derived from a different parent source.

Two suites of samples that are not co-genetic





Mixing Theory

- Geochemistry often tries to model variations in measured composition as the result of mixing of a small number of *components* [N.B. a different use of component from the thermodynamic usage] or *end members*
 - This reduces highly multivariate data to a few manageable dimensions
 - It allows identification of the end members with particular source or fluxes, hence a meaningful interpretation of data
 - Many geochemical processes are easily understood in terms of mixing or unmixing:
 - river water + ocean water = mixing
 - primary liquid – fractionated crystals = unmixing
- We will work out the mixing relations for several spaces:
 - Element-element plots
 - Element-ratio plots (including elemental and isotope ratios)
 - Ratio-ratio plots

Mixing Theory

- Mixing is simplest to see and understand when there are only two end members: *Binary mixing*
- For concreteness, instead of a bunch of general symbols, let's do all this with two end members, a and basalt b , with the following compositions:

	a	b
[Sr]	100 ppm	400 ppm
[Nd]	2 ppm	20 ppm
$^{87}\text{Sr}/^{86}\text{Sr}$	0.712	0.704
$^{143}\text{Nd}/^{144}\text{Nd}$	0.511	0.513

- The same relationships will apply for mixing of major elements as for trace elements.
- The same relationships will apply for ratios of major elements, ratios of trace element concentrations, and ratios of isotopes.

Binary Mixing I: element-element

- This is the simplest case. Binary mixing in concentration-concentration space always generates *lines*.
- Let mixtures be generated with mass fraction f_a of end member a and f_b of end member b , such that $f_a + f_b = 1$.
- Then for two species, say Sr and Nd, we have conservation of atoms and mass in the form

$$[\text{Sr}]_{\text{mix}} = f_b[\text{Sr}]_b + (1 - f_b)[\text{Sr}]_a$$

$$[\text{Nd}]_{\text{mix}} = f_b[\text{Nd}]_b + (1 - f_b)[\text{Nd}]_a$$

- This can be written

$$[\text{Sr}]_{\text{mix}} - [\text{Sr}]_a = f_b([\text{Sr}]_b - [\text{Sr}]_a)$$

$$[\text{Nd}]_{\text{mix}} - [\text{Nd}]_a = f_b([\text{Nd}]_b - [\text{Nd}]_a)$$

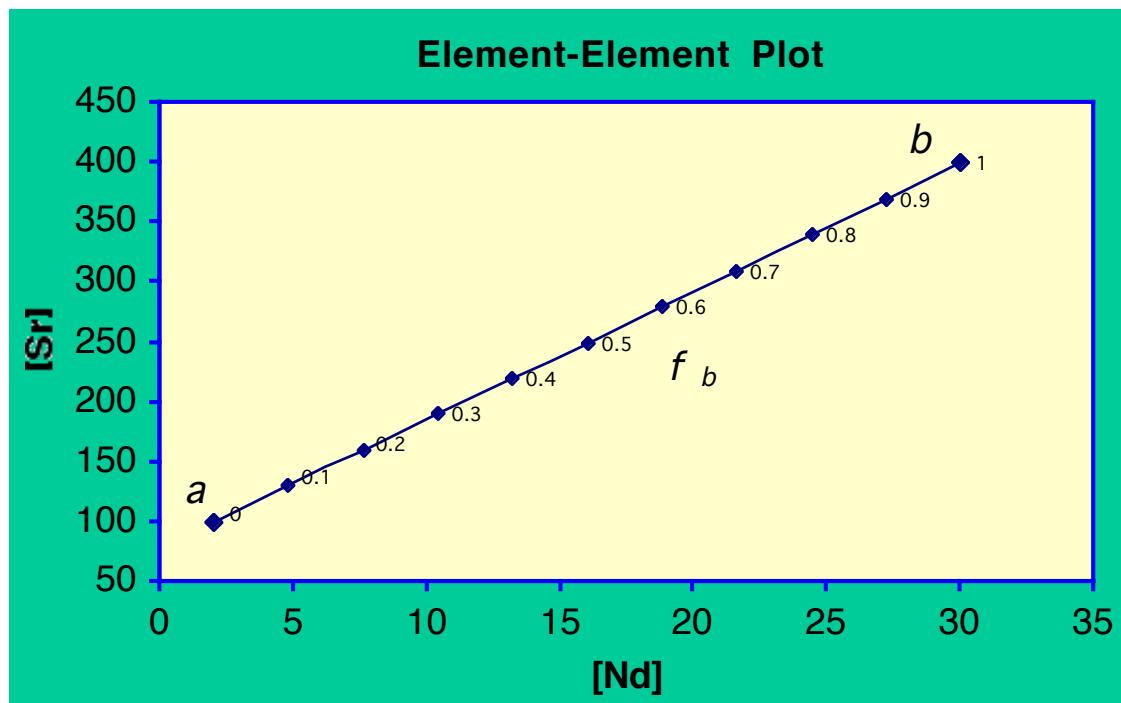
- Dividing these two equations gives the equation of the mixing relationship in $([\text{Sr}], [\text{Nd}])$ space:

$$[\text{Sr}]_{\text{mix}} - [\text{Sr}]_a = \frac{[\text{Sr}]_b - [\text{Sr}]_a}{[\text{Nd}]_b - [\text{Nd}]_a} ([\text{Nd}]_{\text{mix}} - [\text{Nd}]_a)$$

Binary Mixing I: element-element

$$[Sr]_{mix} - [Sr]_a = \frac{[Sr]_b - [Sr]_a}{[Nd]_b - [Nd]_a} ([Nd]_{mix} - [Nd]_a)$$

- This is the equation of a *line* with slope $\frac{[Sr]_b - [Sr]_a}{[Nd]_b - [Nd]_a}$
- Passing through points $([Nd]_a, [Sr]_a)$ and $([Nd]_b, [Sr]_b)$



Binary Mixing I: element-element

- If you *know* the compositions of the end members, you can solve for f_b using the lever rule:

$$f_b = \frac{[Sr]_{mix} - [Sr]_a}{[Sr]_b - [Sr]_a} = \frac{[Nd]_{mix} - [Nd]_a}{[Nd]_b - [Nd]_a}$$

- If you *don't know* the end members, but only the data, what can you learn from a graph showing a linear correlation?
 - You can infer that if generated by mixing there are only two end members, otherwise the data would fill a triangle
 - You can infer that both end members lie on the mixing line, outside the extreme range of the data on both ends if they must have positive amounts (*additive mixing*)

Binary Mixing II: Ratio

- In geochemistry we very frequently work with ratios, either isotope ratios or ratios of concentrations.
 - Sometimes a ratio is all you can measure accurately
 - Sometimes ratios have significance where concentrations are more or less arbitrary (example: during fractionation of olivine from a basalt, [Sr] will change because it is incompatible and the amount of liquid is decreasing, but [Sr]/[Nd] will not)
- You might think that the $^{87}\text{Sr}/^{86}\text{Sr}$ ratio of mixtures could be obtained in the same way as for [Sr]

$$(^{87}\text{Sr}/^{86}\text{Sr})_{\text{mix}} = f_b (^{87}\text{Sr}/^{86}\text{Sr})_b + (1 - f_b) (^{87}\text{Sr}/^{86}\text{Sr})_a$$

- You would be wrong!
- The isotope ratio of the mixture is going to be weighted by the concentration of Sr in each end member
 - More generally, the weighting of ratios in the mixture is controlled by the denominator of the ratio, in this case ^{86}Sr .

Binary Mixing II: Ratio

- Let's do it right: we have

$$\begin{aligned}[{}^{87}\text{Sr}]_{mix} &= f_b [{}^{87}\text{Sr}]_b + (1 - f_b) [{}^{87}\text{Sr}]_a \\ [{}^{86}\text{Sr}]_{mix} &= f_b [{}^{86}\text{Sr}]_b + (1 - f_b) [{}^{86}\text{Sr}]_a\end{aligned}$$

- Taking the ratio of these, we have

$$\left(\frac{{}^{87}\text{Sr}}{{}^{86}\text{Sr}}\right)_{mix} = \frac{f_b \left[{}^{87}\text{Sr}\right]_b + (1 - f_b) \left[{}^{87}\text{Sr}\right]_a}{f_b \left[{}^{86}\text{Sr}\right]_b + (1 - f_b) \left[{}^{86}\text{Sr}\right]_a}$$

- And substituting $[{}^{87}\text{Sr}] = ({}^{87}\text{Sr}/{}^{86}\text{Sr})[{}^{86}\text{Sr}]$ for a and b :

$$\left(\frac{{}^{87}\text{Sr}}{{}^{86}\text{Sr}}\right)_{mix} = \frac{f_b \left[{}^{86}\text{Sr}\right]_b \left(\frac{{}^{87}\text{Sr}}{{}^{86}\text{Sr}}\right)_b + (1 - f_b) \left[{}^{86}\text{Sr}\right]_a \left(\frac{{}^{87}\text{Sr}}{{}^{86}\text{Sr}}\right)_a}{f_b \left[{}^{86}\text{Sr}\right]_b + (1 - f_b) \left[{}^{86}\text{Sr}\right]_a}$$

- We can approximate this using $[\text{Sr}]$ instead of $[{}^{86}\text{Sr}]$ as the weighting factors.

Binary Mixing II: Ratio

- Now let's consider plotting the isotope ratio of the mixture against an elemental concentration, perhaps $[Nd]$.
- If we eliminate f_b between the mixing equation for $(^{87}\text{Sr}/^{86}\text{Sr})$ and $[Nd]$, we obtain the following equation (using the approximation $\text{Sr} \sim ^{86}\text{Sr}$):

$$A\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_{mix} + B[Nd]_{mix}\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_{mix} + C[Nd]_{mix} + D = 0$$

$$A = [Nd]_b[\text{Sr}]_a - [Nd]_a[\text{Sr}]_b$$

$$B = [\text{Sr}]_b - [\text{Sr}]_a$$

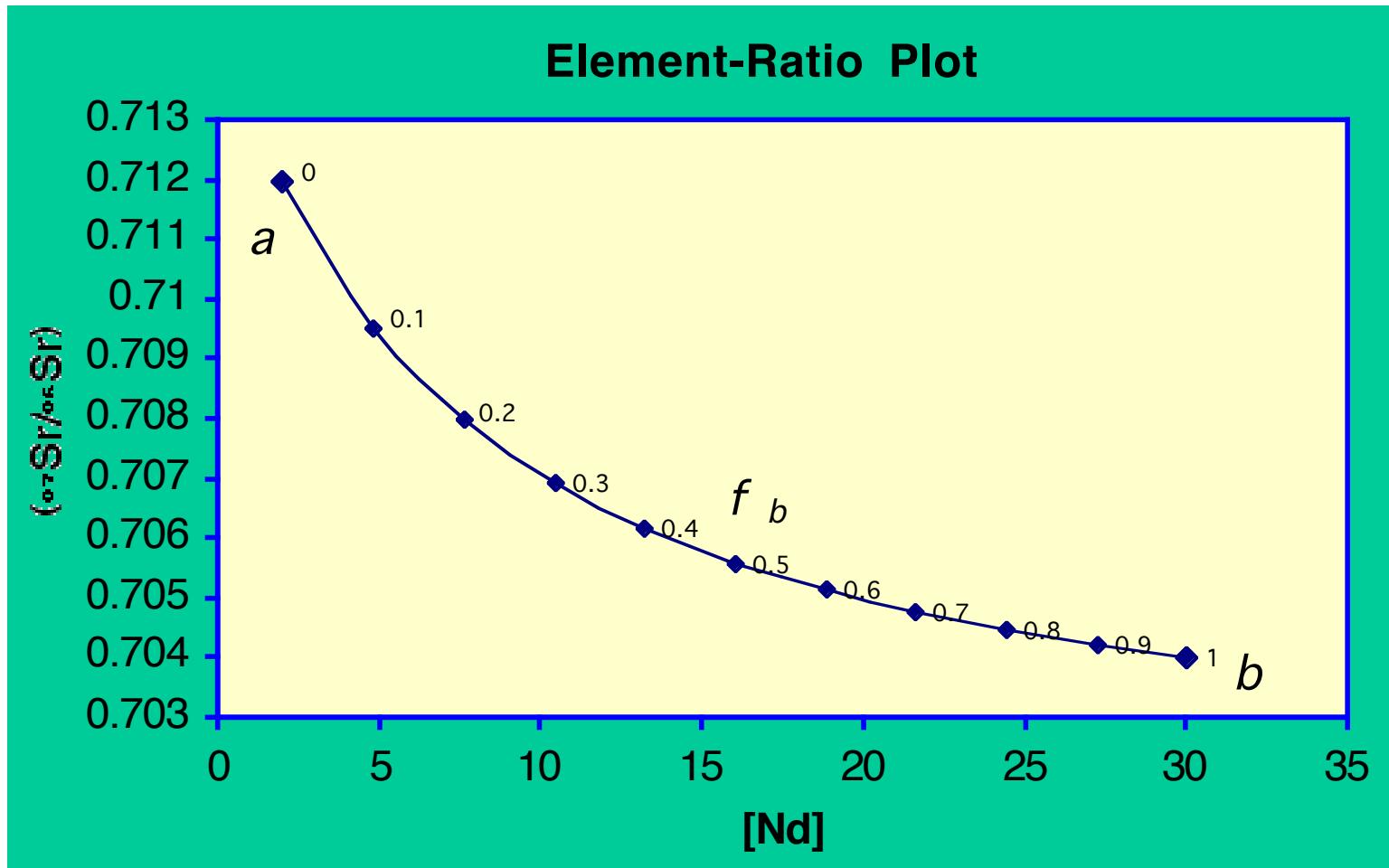
$$C = [\text{Sr}]_a\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_a - [\text{Sr}]_b\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_b$$

$$D = [Nd]_a[\text{Sr}]_b\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_b - [Nd]_b[\text{Sr}]_a\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_a$$

- Equation $Ax + Bxy + Cy + D = 0$?

Binary Mixing II: element-ratio

- In general, element-ratio mixing generates a *hyperbola*.
 - The only case in which it is linear is $B = [\text{Sr}]_b - [\text{Sr}]_a = 0$



- The index of curvature $r = [\text{Sr}]_b / [\text{Sr}]_a$ tells you “how hyperbolic” the hyperbola is going to be.

Binary Mixing III: Inverse Element-Ratio

- Although there is nothing special about the element-ratio case A/B vs. B (thus $(^{87}\text{Sr}/^{86}\text{Sr})$ vs. [Sr] is still hyperbolic), there is an especially useful test for mixing if you plot A/B vs. 1/B (in this case, $(^{87}\text{Sr}/^{86}\text{Sr})$ vs. 1/[Sr]).
 – Going back to our hyperbolic equation, if we replace $[\text{Nd}]_{mix}$, $[\text{Nd}]_a$ and $[\text{Nd}]_b$ with $[\text{Sr}]_{mix}$, $[\text{Sr}]_a$ and $[\text{Sr}]_b$ we have:

$$A\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_{mix} + B[\text{Sr}]_{mix}\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_{mix} + C[\text{Sr}]_{mix} + D = 0$$

$$A = [\text{Sr}]_b[\text{Sr}]_a - [\text{Sr}]_a[\text{Sr}]_b = 0, B = [\text{Sr}]_b - [\text{Sr}]_a$$

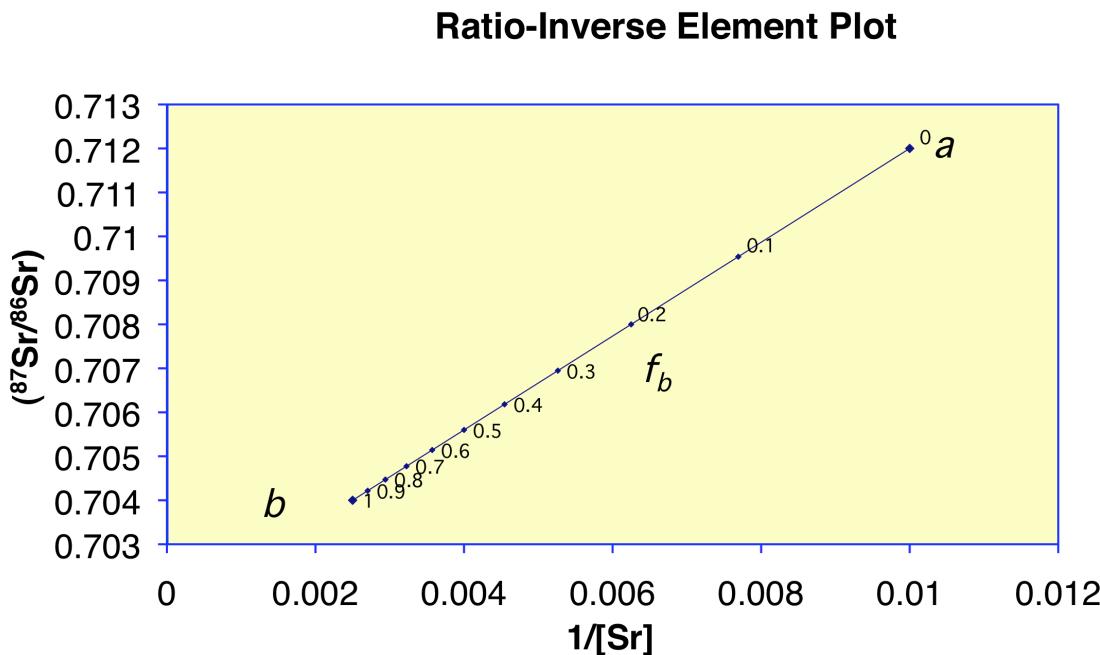
$$C = [\text{Sr}]_a\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_a - [\text{Sr}]_b\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_b$$

$$D = [\text{Sr}]_a[\text{Sr}]_b\left(\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_b - \left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_a\right)$$

- Or, $B\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_{mix} + C + D\frac{1}{[\text{Sr}]_{mix}} = 0$, which is a *line*.

Binary Mixing III: inverse element-ratio

- Mixing in A/B vs. 1/B space always generates a line.



- The value of $r = [\text{Sr}]_b/[\text{Sr}]_a$ now controls how hyperbolic the *spacing* of equal increments of mixing fraction are along the line. Since linear correlation is easy to calculate, it is much easier to test whether data are consistent with binary mixing in this space than in ratio-element space.

Binary Mixing IV: Ratio-Ratio

- Our final case is plots of ratios against ratios, whether isotope ratios, trace element ratios, or major element ratios.
- For example, let's do $(^{87}\text{Sr}/^{86}\text{Sr})$ vs. $(^{143}\text{Nd}/^{144}\text{Nd})$.
- We now have two equations of the same form:

$$\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_{mix} = \frac{f_b[\text{Sr}]_b \left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_b + (1-f_b)[\text{Sr}]_a \left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_a}{f_b[\text{Sr}]_b + (1-f_b)[\text{Sr}]_a}$$

$$\left(\frac{^{143}\text{Nd}}{^{144}\text{Nd}}\right)_{mix} = \frac{f_b[\text{Nd}]_b \left(\frac{^{143}\text{Nd}}{^{144}\text{Nd}}\right)_b + (1-f_b)[\text{Nd}]_a \left(\frac{^{143}\text{Nd}}{^{144}\text{Nd}}\right)_a}{f_b[\text{Nd}]_b + (1-f_b)[\text{Nd}]_a}$$

Where once again for the particular case of small variations in isotope ratios I have weighted by concentration rather than by the stable denominator isotope.

Binary Mixing IV: ratio-ratio

- This time eliminating f_b between the mixing equations gives

$$A\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_{mix} + B\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_{mix}\left(\frac{^{143}\text{Nd}}{^{144}\text{Nd}}\right)_{mix} + C\left(\frac{^{143}\text{Nd}}{^{144}\text{Nd}}\right)_{mix} + D = 0$$

$$A = [\text{Nd}]_b[\text{Sr}]_a\left(\frac{^{143}\text{Nd}}{^{144}\text{Nd}}\right)_b - [\text{Nd}]_a[\text{Sr}]_b\left(\frac{^{143}\text{Nd}}{^{144}\text{Nd}}\right)_a$$

$$B = [\text{Nd}]_a[\text{Sr}]_b - [\text{Nd}]_b[\text{Sr}]_a$$

$$C = [\text{Nd}]_b[\text{Sr}]_a\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_a - [\text{Nd}]_a[\text{Sr}]_b\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_b$$

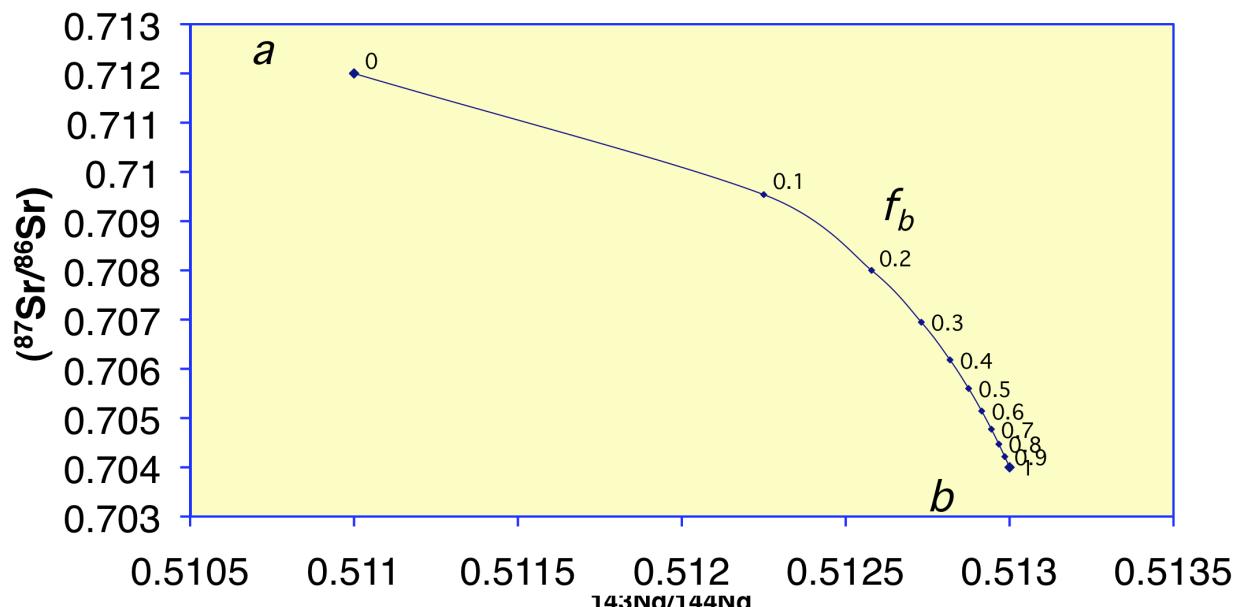
$$D = [\text{Nd}]_a[\text{Sr}]_b\left(\frac{^{143}\text{Nd}}{^{144}\text{Nd}}\right)_a\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_b - [\text{Nd}]_b[\text{Sr}]_a\left(\frac{^{143}\text{Nd}}{^{144}\text{Nd}}\right)_b\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}}\right)_a$$

- Still a hyperbola. Now term B gives the curvature index
- $r = ([\text{Sr}]_a/[\text{Sr}]_b)/([\text{Nd}]_a/[\text{Nd}]_b)$

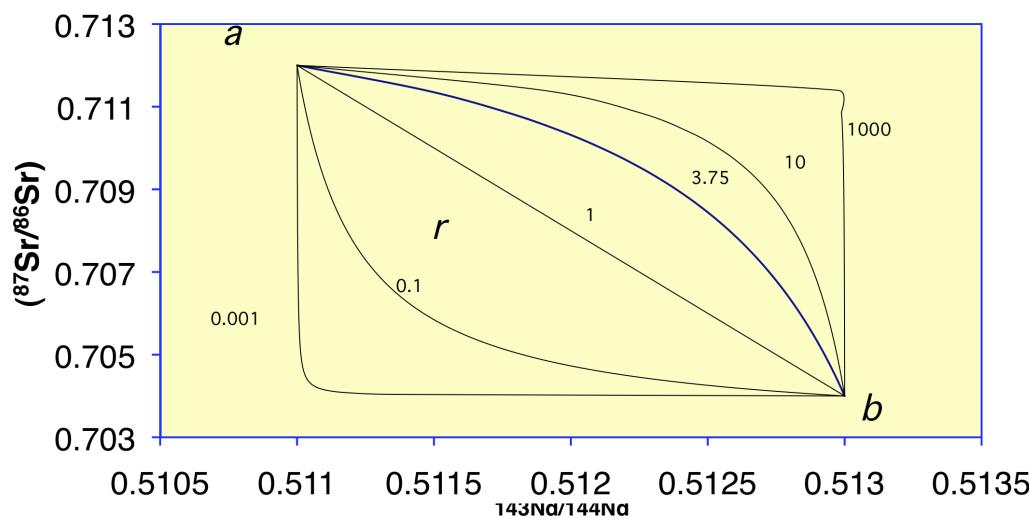
Binary Mixing IV: ratio-ratio

This looks different from the element-ratio hyperbola because now the *spacing* of equal increments of mixing fraction is no longer regular.

Ratio-Ratio Plot



Ratio-Ratio Plot



Given an array of ratio-ratio data, you can constrain the curvature parameter as well as the isotope ratios of the end members.

Some mixing plots

