What is hedging?

Definition: it refers to buying or selling futures opposite to the position held in cash/spot market so as to reduce the price risk.

1. A short hedge is a hedge that involves a short position in futures contract.
2. A hedger who is **long in cash/spot market** (already holding the underlying commodity or intending to buy in future) takes short position in futures.
3. It involves the short position in futures contract.
4. A short hedge is appropriate when the hedger already owns an asset and expects to sell it at some time in the future.
    1. An Exporter wishes to minimize the risk of appreciation of currency.
    2. Farmer with certain product at hand wants to hedge the risk of price fall

What is hedging?

Definition: it refers to buying or selling futures opposite to the position held in cash/spot market so as to reduce the price risk.

1. A hedger with **short cash position** (already sold the underlying commodity, but yet to deliver or intending to sell in future) takes long position in futures.
2. Hedges that involve taking a long position in a futures contract are known as ***long hedges***.
3. A long hedge is appropriate when a company knows it will have to purchase a certain asset in the future and wants to lock in a price now.

Appropriate when you are sure that the price of futures will be lower than the spot market. Today it is selling at Rs. 500 and the futures on the same is being sold at 400 and after maturity it may rise max up to Rs. 450. Still by buying (long) on futures is yield you the spread of Rs. 50 (500- 450).

Short hedge: Example

Suppose a company is planning to hedge the exchange rate risk on a cash flow of $100,000 after 3 months. The current exchange rate is Rs. 65 = 1USD

- It means that the company will take a short position for appreciation of the currency. What happens when the Indian rupee appreciates?
- INR today is Rs. 65 and it may go up further to Rs. 50. If the price after 6 months is Rs. 70, company is going to get benefit by taking long position on Rs. (70*100000 – 65*100000) = 70,00000 – 65,00000 = Rs. 500,000.
- What happens if the rupee appreciates from Rs. 65 to Rs. 50. In this case, the company will take short position and hedge the risk.
- Rs. 65,00000 – Rs. 50,00000 = Rs. 1500000
- By this way, the company minimizes the risk of Rs. Appreciation.

A. Overview of Hedging Strategies

3. Under what circumstances can risk be eliminated?

Can attain a perfect hedge ($\sigma = 0$):

a. If the ***asset you are hedging*** is identical
to the ***asset underlying the futures*** contract; and

b. If you wish to hedge risk until futures expiration;

c. Then have perfect correlation ($\rho = 1$).

4. Why hedge anyway? (+ = for hedging; - = against)

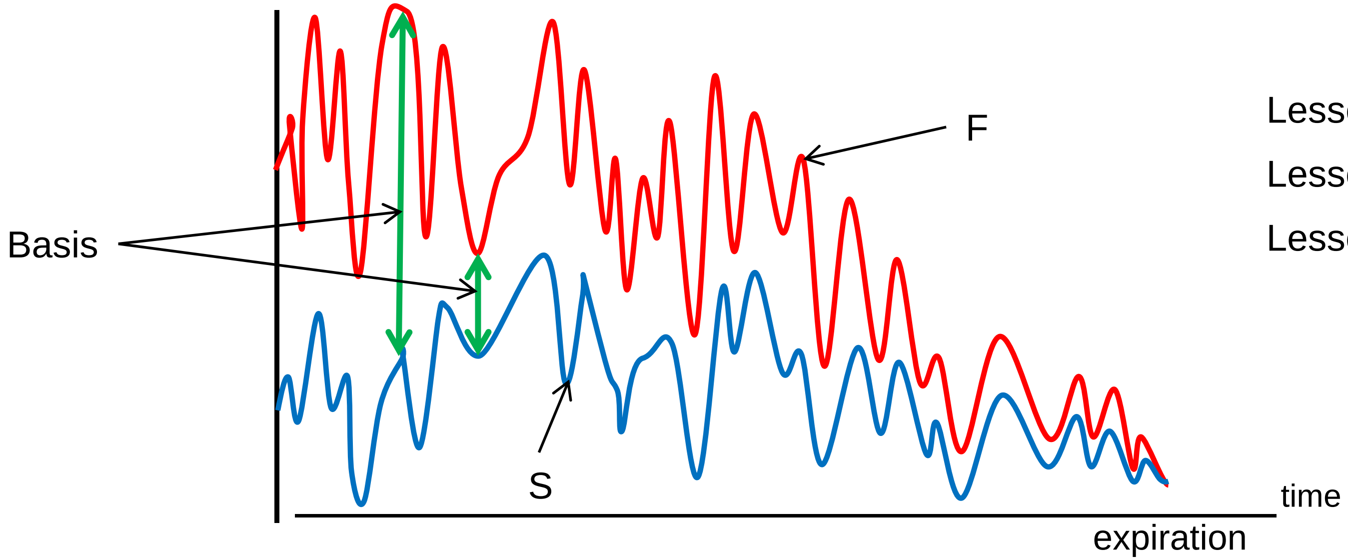
+ Risks (uncertainty) outside your control could ruin firm.
- But shareholders can manage risk themselves.
+ But firm can hedge more efficiently (cheaply, effectively).
- But shareholders can at least *diversify* cheaply, effectively.
+ Firm has a duty to take prudent risks (legal issues, …).

5. Other questions to be answered in this lecture:

- If perfect hedge is not possible, how much can risk be reduced?
- Which futures contract should be used to hedge?
- How many futures contracts should be bought or sold?

B. Basis Risk

1. Definition: **Basis = S - F**.



Lesson 1: $\sigma_F > \sigma_S$

Lesson 2: $\rho < 1$

Lesson 3: Basis varies

2. ***Complications*** for hedger that lead to **Basis Risk**:

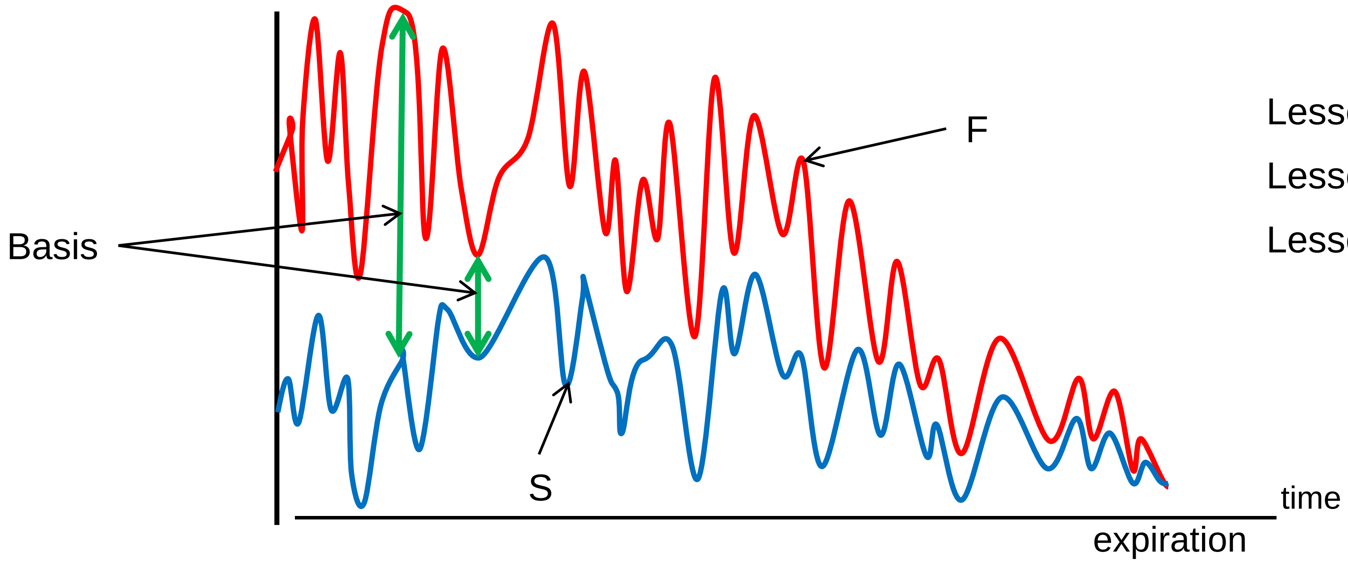
   a. ***Asset being hedged*** may not be the **same** as ***Asset underlying futures*** contract.

   b. Hedger may be **uncertain about exact date** when asset must be bought / sold.

   c. Hedge may require futures to be **closed out before** its **expiration date**.

B. Basis Risk

1. Definition: **Basis = S - F**.



Lesson 1: $\sigma_F > \sigma_S$

Lesson 2: $\rho < 1$

Lesson 3: Basis varies

- If the asset to be hedged and the asset underlying the futures contract are the same, the basis should be zero at the expiration of the futures contract.
- An increase in the basis is referred to as a strengthening of the basis; a decrease in the basis is referred to as a weakening of the basis.

B. Basis Risk

1. A Basis is negative for Contango market while it is positive for backwardation market.

2. As the delivery approaches, basis reduces. Basis goes on reducing as the delivery date approaches and becomes zero on the delivery date as the spot and futures price converges on the delivery date.

B. Basis Risk in Futures

1. Changes in spot and futures prices leads to changes in basis.
2. Change in basis affects the commodity futures pay-off.
3. The impact of change in basis is explained below with a simple example involving commodity hedging.

B. Long Cash (Short Hedge) Market Position

EXAMPLE:

On 10th September 2018, the CEO of a copper mining firm decides to hedge its copper production using futures contract as he feels that copper price will fall during December 2018. The copper mining firm intends to extract and process 2000 tons of copper and sell copper by 9th December 2018.

So on 10th September 2018, the firm enters into short futures position maturing in December 2018. On the contract initiation date, the spot market price is USD 8, 324 per and December 2018 copper futures available at a commodity exchange at a price of USD 8095. Contract expires on 24th December 2018.

As copper firm has to sell copper on 9th December 2018, the firm squares of its position on 9th December 2018, even though the contract matures on 24th December 2018.

Suppose we assume two scenarios:

Scenario 1: Spot and futures price decreased to **USD 8120** and **USD 7673**, respectively.

Scenario 2: Spot and futures increased to **USD 8723** and **USD 8880**, respectively.

B. Long Cash (Short Hedge) Market Position

EXAMPLE:

It shows the different possible spot and futures price scenarios and the pay-off to the copper producer on 9th December 2018.

Contract date (10th September 2018) spot price: USD 8324, Futures price: USD 8095			
Scenario 1			
Squaring of date (9th December 2018): Spot Price: USD 8120, Futures Price: USD 7673			
Sell underlying in the cash market on 9th December 2018		Cash Receipt	USD 8120
Long futures position to offset the short futures position		Net cash receipt (Cash receipt for short futures – cash payment for long futures)	USD 422 (8095 – 7643)
	Total receipt		USD 8542 (8120+422)
Scenario 2 Squaring of date (9th December 2018): Spot Price: USD 8723, Futures Price: USD 8880			
Sell underlying in the cash market on 9th December 2018		Cash receipt	USD 8723
Long futures position to offset the short futures position			- 785 (8095 – 8880)
	Total Receipt		USD 7938

B. Long Cash (Short Hedge) Market Position

EXAMPLE:

It shows the different possible spot and futures price scenarios and the pay-off to the copper producer on 9th December 2018.

Final Conclusion:

Without futures contract, the firm would have earned either USD 8120 or USD 8723. With futures contract, the firm earns USD 8542 or USD 7938. These are different than the futures price of USD 8095 prevailing on the contract date.

In other words, even with a futures contract, the copper mining firm is realizing a price which is different than the futures price prevailing on the contract start date.

B. Short Cash (Long Futures) Market Position

EXAMPLE:

As a part of a long-term supply arrangement, wheat wholesaler has agreed to deliver 100 MT of wheat to a bread manufacturer at a negotiated price of Rs. 13, 500 per MT. Spot date is 10 August 2018. Next due for delivering what is 15th March 2019. The wholesaler can buy wheat in the open market and deliver to the bread manufacturer on 15th March 2019. But this exposes the wholesaler to price risk. So the wheat wholesaler buys futures contract maturing in March 2019.

So, on 10th August 2018 (spot date), the wholesaler enters into long futures contracts maturing in March 2018 with contract maturity date of 15th March 2019. On 10th August 2018, spot price is Rs. 13, 735 per contract and march 2019 futures price is Rs. 14, 476. The market is in contango i.e. futures price is higher than spot price. The spot price and futures price difference is negative – Rs. 841.

Suppose we assume two scenarios:

Scenario 1: Spot and futures price decreased to **Rs. 12, 908**and **Rs. 14, 239** respectively.

Scenario 2: Spot and futures increased to **Rs. 14, 412** and **Rs. 14, 230**, respectively.

B. Long Cash (Short Hedge) Market Position

EXAMPLE:

It shows the different possible spot and futures price scenarios and the pay-off to the copper producer on 15th March 2019.

Contract date (10th August 2018) spot price: Rs. 13, 735, Futures price: Rs. 14, 576		
Scenario 1		
Squaring of date (15th March 2019): Spot Price: Rs. 13, 735, Futures Price: Rs. 14, 576		
Buying underlying in the cash market	Cash payment	Rs. 12, 908 per contract
Short futures position to offset the long futures position	Net cash payment (Cash payment for long futures – cash receipt for short futures)	Rs. 337 per contract Rs (14, 576 – 14, 239)
	Total Cash Payment	Rs. 13, 245
Scenario 2 Squaring of date (15th March 2019): Spot Price: Rs. 14,412, Futures Price: Rs. 14, 230		
Buying underlying in the cash market	Cash payment	Rs. 14, 412 per contract
Short futures position to offset the long futures position	Net cash payment (Cash payment for long futures – cash receipt for short futures)	Rs. 346 (14, 576 – 14, 230)
	Total Cash Payment	Rs. 14, 758

B. Long Cash (Short Hedge) Market Position

EXAMPLE:

Final Conclusion:

- Without futures contract, the wholesaler would have purchased wheat from open market at cash price of Rs. 12, 908 or Rs. 14, 412. With futures contract, his total purchase price is either Rs. 13, 245 or Rs. 14, 758.

- In any case, the futures prices in both scenarios are different from the futures price of Rs. 14, 576 prevailing on contract start date.

B. Basis Risk

5. **Short Hedge for Sale of an Asset:**

(wheat farmer)

Define

F_1 : Futures price at time hedge is set up (sell at F_1)

F_2 : Futures price at time asset is sold (buy at F_2)

S_2 : Asset price at time of sale (sell for S_2)

b_2 : Basis at time of sale (S_2 - F_2)

Price received for asset = S_2

Gain on Futures = $F_1 - F_2$ (assuming F↓)

Net amount received = $S_2 + (F_1 - F_2) = F_1 + (S_2 - F_2)$

$= F_1 + b_2$

Lock into receiving F_1, but don't know what basis (b_2) will be.

B. Basis Risk

Problem

Does a short hedger's position improve or worsen
when the basis strengthens (increases) unexpectedly?

A short hedger (wheat farmer) is long the asset (has wheat) and short futures.

If S↑, the long position makes money – get more for wheat (improves).

If F↓, the short position makes money – gain on futures (improves).

Basis = S - F. The basis increases when S↑ or F↓.

Therefore, a short hedger's position improves when the basis strengthens (↑).

HOWEVER! F & S typically move in the same direction.

That's why this is an effective hedge!

The question is whether the DIFFERENCE (S - F) will widen or narrow.

That's what matters to the hedger.

B. Basis Risk

Example: Suppose S = $3.00 / bu and F = $3.05 / bu;
Basis = -$.05; The farmer has wheat and shorts futures.
Then suppose **S↓ by $1** to $2.00 / bu. What happens to F? To Basis?

1. Suppose F↓ also by $1.00, to $2.05;
   Here Basis remains at -$.05, before & after price changes.
   The $1.00 lost on wheat is all gained back on futures. Basis has not changed.
   The short hedger's position has not improved or worsened – hedged effectively.
   All they lost on the spot is gained back on futures.

2. Suppose F↓ by less than $1.00, to $2.07; (F ↓ by $.98)
   Here the Basis↓ (becomes a bigger negative number – weakens) to -$.07;
   The $1.00 lost on wheat is not all gained back on futures ($.98).
   Basis has decreased, the short hedger's ***position has worsened*** slightly.

3. Suppose F↓ by more than $1.00, to $2.03; (F ↓ by $1.02)
   Here the Basis↑ (becomes a smaller negative number – strengthens) to $-.03;
   The $1.00 lost on wheat is more than gained back on futures ($1.02).
   Basis has increased, the short hedger's ***position has improved*** slightly.

B. Basis Risk

6. If asset being hedged is ***not same*** as underlying asset.

(For example, hedging spot price of jet fuel with heating oil futures.)

a. **Cross-Hedge**: Basis may not = 0 at expiration.

b. Basis risk is usually greater.

There is another component of basis risk. (No futures for jet fuel.)

Before, **Basis = S - F**. (e.g., S for jet fuel - F for jet fuel)

Now, Let S = spot price of asset being hedged (jet fuel);

S* = spot price of asset underlying F (heating oil);

Now there is clear relation between **S*** & F (both heating oil)

rather than between S & F (jet fuel & oil).

Add and subtract **S***, and rearrange terms:

Basis = (S* - F) + (S - S*); has another component.

B. Basis Risk

7. For Cross-Hedge, choice of contract has 2 components:

   a. Choice of ***asset underlying*** the futures contract.

      i. If asset being hedged is same as that for futures, choice is easy.

      ii. Otherwise, pick F (and ***S****) most closely correlated with S.

   b. Choice of ***delivery month***.

      i. May choose delivery month same as hedge expiration, or may choose later delivery month.

**   ii. Usually choose next delivery month *after* hedge expiration:
   - F is often erratic during expiration month (more Basis risk).
   - long hedger runs risk of having to take delivery.
   - Basis risk increases as time between hedge expiration and delivery month increases.

      iii. This choice depends on *liquidity* of contracts with different maturities.
   - In practice, nearby contract is most liquid (low TC).
   - Hedger may choose short maturity futures, and roll them forward.

C. Minimum Variance Hedge Ratio

Consider **Cross-Hedge** - if the assets are different.

Deriving the **Minimum Variance Hedge Ratio (h*)**.

1. Given that Basis may not = 0 at expiration,

   want to know its possible outcomes.

   a. What is mean and variance of Basis?

      Now "hedging" means managing the tails of this distribution.

2. Definition: **Hedge Ratio (h)**.

   Let N_A = # of units of asset held (the risk exposure);

   N_F = # of units of (similar) asset hedged with futures.

   a. $h = N_F / N_A$ = (size of futures position) ÷ (size of exposure).

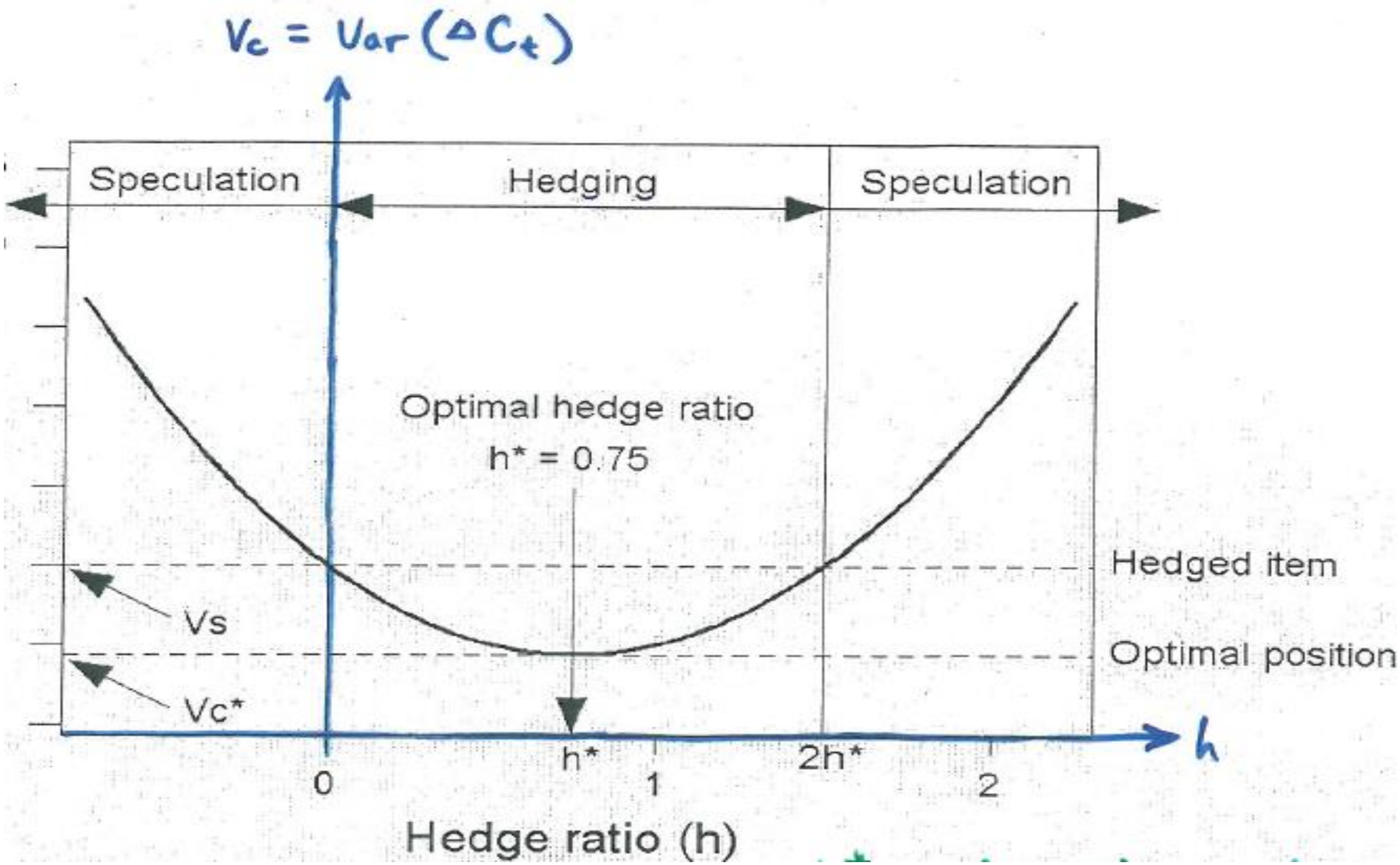
   b. Up to now, have assumed h = 1 [amt hedged = amt exposed].

C. Minimum Variance Hedge Ratio

3. As we vary h, the variance of hedger's position changes.

   a. **h*** = value of h that minimizes the variance of hedger's position.

   b. If objective is to minimize risk, **h*** may not = 1!



C. Minimum Variance Hedge Ratio

4. **The Framework.** Consider a short hedge (long S_t, short h x F_t).

Focus on ***combined hedged position*** in asset and futures.

Derive minimum variance hedge ratio (***h****).

a. Notation:

S_t = Spot price of asset at time t (t = 1, 2).

F_t = Futures price at time t.

ΔS = Change in S during period of hedge (S_2 - S_1).

ΔF = Change in F " " " " (F_2 - F_1).

σ_S = Standard deviation of ΔS.

σ_F = Standard deviation of ΔF.

ρ = Correlation between ΔS and ΔF.

b. Definitions: $C_t = S_t - h\,F_t$ = ***combined hedged position***;

[Long the asset (S_t) and short h futures (F_t), where h = hedge ratio.]

Note: $\Delta C_t = \Delta S_t - h\,\Delta F_t$,

and: **h^* =** $\rho(\sigma_S / \sigma_F)$ = optimal hedge ratio $\rightarrow$ min Var(ΔC_t).

C. Minimum Variance Hedge Ratio

c. Derivation of formula for **h***:

Want to minimize uncertainty about **combined hedged position**;
Want to minimize $V_C = Var(\Delta C_t) = Var[\Delta S - h\,\Delta F]$.

//　　　　　　　　　　　　　　　　　　　　　　　　　　\\

Digression: $Var[aX + bY] = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab(\rho\,\sigma_X\sigma_Y)$.

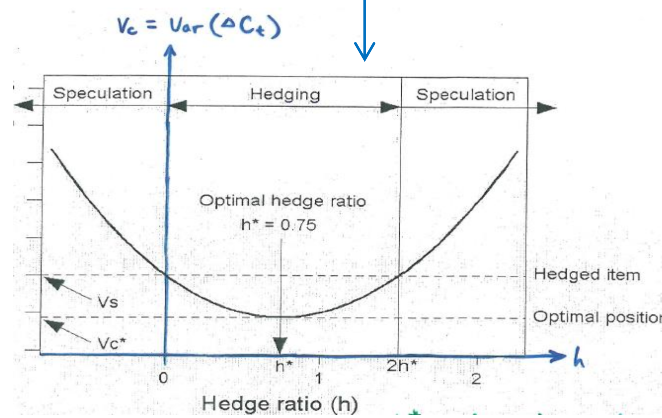
\\　　　　　　　　　　　　　　　　　　　　　　　　　　//

$Var[\Delta S - h\,\Delta F] = \sigma_S^2 + h^2\sigma_F^2 - 2h(\rho\,\sigma_S\sigma_F)$ ---(parabola in h)

$\partial Var[\Delta S - h\,\Delta F] / \partial h = 2(\mathbf{h^*})\sigma_F^2 - 2(\rho\,\sigma_S\sigma_F) = 0$

Solving for **h***: $\mathbf{h^*} = (\rho\,\sigma_S\sigma_F)/\sigma_F^2$

or: $\boxed{h^* = \rho(\sigma_S/\sigma_F)}$



C. Minimum Variance Hedge Ratio

5. Intuition: $\boxed{h^* = \rho(\sigma_S / \sigma_F) = \text{optimal } (N_F / N_A)}$

Or, Optimal N_F = $\mathbf{N_F^*}$ = ($\mathbf{h^*}$) N_A .

Note: If F changes 1%, expect S to change **h***%;

If S changes 1%, expect F to change (1 / **h***)%.

a. If $\rho = 1$ and $\sigma_F = \sigma_S$, then $\mathbf{h^*} = 1$;

F and S mirror each other perfectly.
If S changes 1%, expect F to change 1%;
Perfect hedge; $\mathbf{N_F^*} = (1)\, N_A$ --- hold same amount of F as S.

b. If $\rho = 1$ and $\sigma_F = 2\sigma_S$, then $\mathbf{h^*} = 1/2$; (Recall lesson 1: $\sigma_F > \sigma_S$)

F always changes by twice as much as S.
If S changes 1%, expect F to change 2%;
Perfect hedge; $\mathbf{N_F^*} = (1/2)\, N_A$ --- hold half as much of F as S.

C. Minimum Variance Hedge Ratio

$$\boxed{\mathbf{h^*} = \rho(\sigma_S / \sigma_F)}$$

c. If $\rho < 1$, then **h*** depends on magnitude of ρ and ratio, σ_S / σ_F.

Intuition still holds true.

If S changes 1%, expect F to change (1 / **h***)%;

Imperfect hedge; $\mathbf{N_F^* = h^*}N_A$ --- hold **h*** as much F as S.

d. Example: If **h*** = .786, futures position should have

78.6% of face value of asset being hedged.

If S changes 1%, expect F to change more (1 / .786)%;

Imperfect hedge; if you hold $\mathbf{N_F^*}$ = (.786) N_A,

the value of your futures position is

expected to just offset the change in S.

C. Minimum Variance Hedge Ratio

$$\boxed{h^* = \rho(\sigma_S / \sigma_F)}$$

e. To minimize risk, should the hedger hold more or less futures, compared to the amount held of the underlying risk exposure?

Is $h^* > 1$ or < 1?

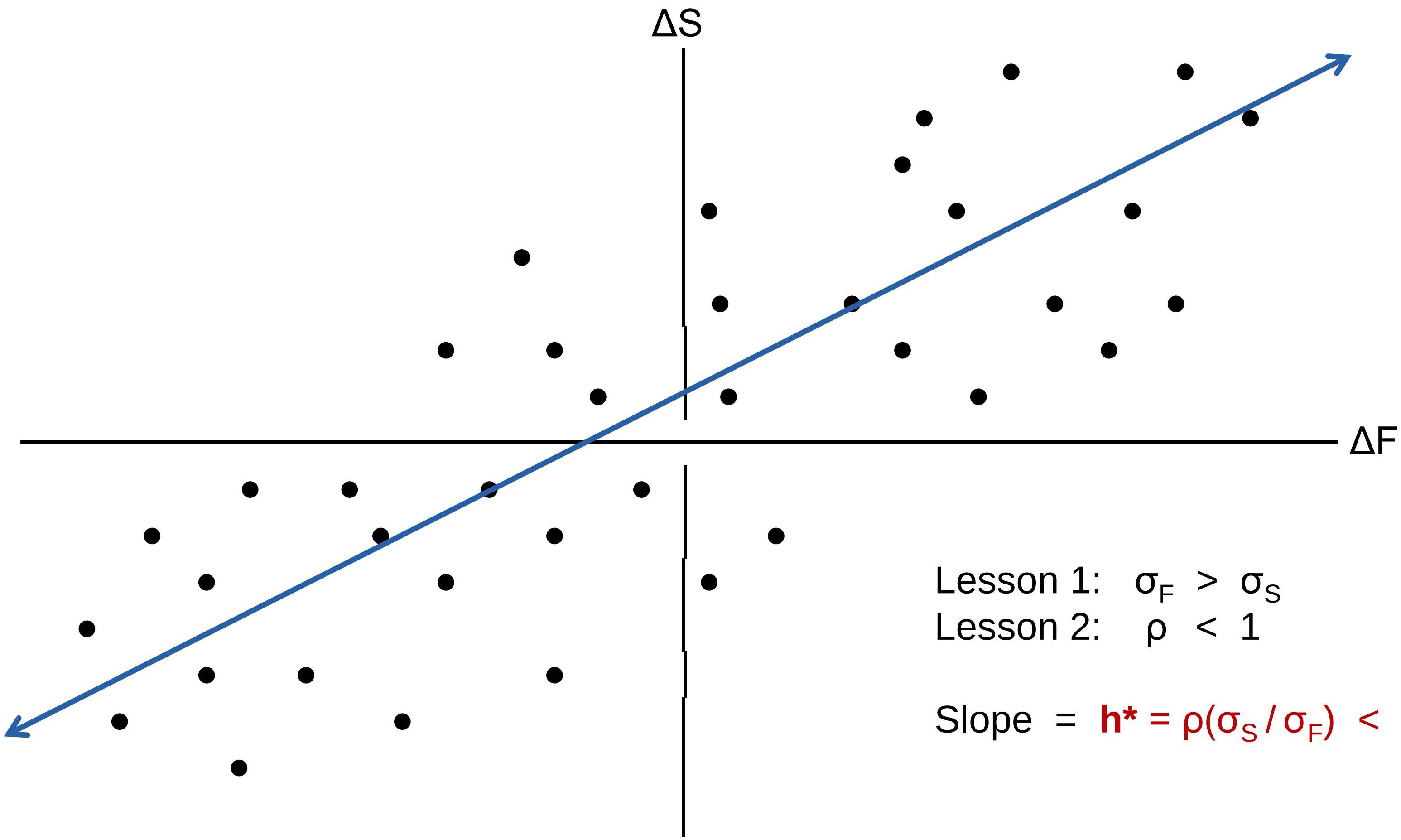
Lesson 1: $\sigma_F > \sigma_S$

Lesson 2: $\rho < 1$

This means: $h^* = \rho(\sigma_S / \sigma_F) < 1$ for most situations.

C. Minimum Variance Hedge Ratio

f. **h*** is slope of best fit line when ΔS is regressed on ΔF.
In practice, get data on ΔS & ΔF, measured over hedge horizon.
Regression coefficient is hedge ratio, **h***.



Lesson 1: $\sigma_F > \sigma_S$
Lesson 2: $\rho < 1$

Slope = $\mathbf{h^*} = \rho(\sigma_S / \sigma_F) < 1$

C. Minimum Variance Hedge Ratio

6. **Hedge effectiveness**: ρ^2 or $h^{*2} [\, \sigma_F / \sigma_S \,]^2$.

   a. The R^2 in this simple regression.

   b. Proportion of Var [ΔS] explained by movemts in ΔF:

   c. Proportion of Var [ΔS - hΔF] eliminated by hedging.

FAS 133 requires hedgers to show they expect hedge to be "highly effective." R^2 of regression! See papers on FAS133;

Kawaller & Koch, *Journal of Derivatives*, 2000;

Charnes, Berkman & Koch, *Journal of Applied Corporate Finance*, 2003;

Juhl, Kawaller, & Koch, *Journal of Futures Markets*, 2012;

Kawaller & Koch, *Journal of Derivatives*, 2013;

Jiang, Kawaller, & Koch, *Journal of Financial Research, 2016*.

C. Minimum Variance Hedge Ratio

7. **Optimal number of futures contracts (N*).**

a. Define: N_A = size of position being hedged (units);

Q_F = size of one futures contract (units).

Recall, $h^* = N_F^* / N_A$; or, $N_F^* = (h^*) N_A$.

The futures hedge should have a face value of $(h^*) N_A$.

Thus, get **N*** by simply dividing this face value (N_F^*) by Q_F:

$$\boxed{N^* = [(h^*) N_A] / Q_F = [(\rho\sigma_S / \sigma_F) N_A] / Q_F}$$

b. In practice, # of futures contracts must be an integer;

Thus, hedger can only approximate optimal hedge.

C. Minimum Variance Hedge Ratio

8. Tailing the Hedge, for short hedger:

a. When **futures** used for hedging, small adjustment can be made to account for the **impact of daily settlement** (tailing the hedge).

b. Old equation: $\mathbf{N^*} = [\ (h^*)\ \mathbf{N}_A\ ] \ /\ \mathbf{Q}_F$

c. New Equation: $\mathbf{N^{**}} = [\ (h^*)\ \mathbf{V}_A\ ] \ /\ \mathbf{V}_F = [\ (h^*)\ \mathbf{N}_A\ ] \ /\ \mathbf{Q}_F * (S\ /\ F)$

where V_A = \$ value of position being hedged ($V_A = (S)\ N_A$);
V_F = \$ value of one futures contract ($V_F = (F)\ Q_F$).

d. Effect of tailing the hedge is to multiply the hedge ratio (h*) by the ratio of spot price to futures price, or (S / F).

e. If contango (F > S), then (S / F) < 1, & tailing the hedge ↓ N**.
If backward. (F < S), then (S / F) > 1, & tailing the hedge ↑ N**.

f. If **forwards** are used, no daily settlement, usually don’t tail hedge.

C. Minimum Variance Hedge Ratio

8. **Tailing the Hedge** (continued):

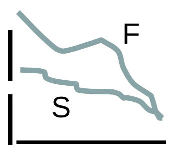
g. **For Short hedger**, Long the asset (+S), Short futures (-F).

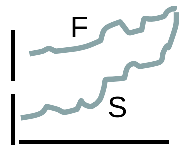
If contango (F > S), convergence means:

if F & S both ↓, F will ↓ more than S ↓ (hedge gain),

or if F & S both ↑, F will ↑ less than S ↑ (hedge gain).

Thus, over time, Expect cash **in**flows;
should hold less futures (N** < N*).



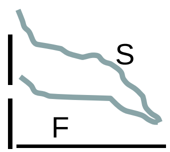


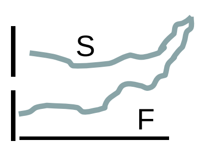
h. **If backwardation** (F < S), opposite. Then convergence means:

if F & S both ↓, F will ↓ less than S ↓ (hedge loss),

or if F & S both ↑, F will ↑ more than S ↑ (hedge loss).

Thus, over time, Expect cash **out**flows;
should hold more futures (N** > N*).





C. Minimum Variance Hedge Ratio

9. **Example:** Cross-Hedge; (no daily settlemt; no tailing)

- Airline uses Heating Oil **forwards** to hedge Jet Fuel costs.
- Airline will purchase 2 million gallons of Jet Fuel in 1 month, and hedges by buying xxxx gallons of Heating Oil **forward**.
- From historical data $\sigma_F = 0.0313$, $\sigma_S = 0.0263$, & $\rho = 0.928$

$$h^* = 0.928 \times \frac{0.0263}{0.0313} = 0.7777$$

Airline should buy forward (.7777) x (2,000,000 gal) = 1,555,400 gal (fwd 1 mo).

- Or, if they use Heating Oil futures: 1 contract = 42,000 gal.

$N^* = [(h^*) N_A] / Q_F = [(.7777) \times (2{,}000{,}000 \text{ gal})] / (42{,}000 \text{ gal})$

$= 37.03$ contracts.

The Airline would buy 37 futures contracts (if no tailing).

C. Minimum Variance Hedge Ratio

10. **Example:** Cross-Hedge; (daily settlemt; tailing hedge)

- If Airline uses Heating Oil **futures** to hedge Jet Fuel costs.
- From historical data $\sigma_F = 0.0313$, $\sigma_S = 0.0263$, & $\rho = 0.928$

$$h^* = 0.928 \times \frac{0.0263}{0.0313} = 0.7777$$

- The size of one Heating Oil contract is 42,000 gal.
- Spot price = S = \$1.94 / gal; Futures price = F = \$1.99 / gal.
- Optimal number of contracts after tailing the hedge:

$N^{**} = [\,(h^*)\ V_A\,] / V_F \quad = [\,(h^*)\ \mathbf{N}_A\,] / \mathbf{Q}_F \times (S / F)$

$= [\,(.7777) \times (\$1.94 \times 2{,}000{,}000 \text{ gal})\,] / (\$1.99 \times 42{,}000 \text{ gal})$

$=$ 36.10 contracts. (Note: $S < F$, $(S / F) < 1$; $N^{**} \downarrow$)

The Airline should buy 36 contracts.

D. Hedging a Stock Portfolio

Hedging stock portfolio with Stock Index Futures.

1. Define: V_A = Current value of stock portfolio;
   V_F = Current value of one futures contract.

2. **If** stock portfolio perfectly matches the index ($\beta = 1$),
   Optimal Number of Futures Contracts = **N*** = V_A / V_F.

   a. Example: V_A = \$1,000,000;
   Index Value = 1,000;
   V_F = \$250 x Index = \$250,000;

Short **N*** = V_A / V_F = \$1,000,000 / \$250,000 = 4 contracts.

(Value of 4 contracts matches Value of stock portfolio.)

D. Hedging a Stock Portfolio

3. **If** stock portfolio does not perfectly match index ($\beta \neq 1$), Need to consider Beta of portfolio, β_{pm}.

   a. Appeal to CAPM: $k_p = r + [\, E(R_m) - r \,]\, \beta_{pm}$.

   Now Optimal Number of Contracts = $\mathbf{N^*} = \beta_{pm} (V_A / V_F)$.

   b. Steps for hedging:

   i. Estimate β_{pm} .

   ii. Compute optimal number of futures contracts:

$$\boxed{\mathbf{N^*} = \beta_{pm} (V_A / V_F)}$$

   iii. Short **N*** futures contracts.

   c. Example: $V_A = \$1{,}000{,}000$; $\beta_{pm} = .75$;

   Index Value = 1,000; $V_F = \$250 \times \text{Index} = \$250{,}000$;

   Short $\mathbf{N^*} = \beta_{pm} (V_A / V_F) = .75\,(4) = 3$ contracts.

D. Hedging a Stock Portfolio

d. This optimal hedge (N*) reduces market beta to zero
[for the hedged portfolio (stock portfolio + futures)].
In the example, if market ↓ 10.0%,
Expect portfolio to ↓ 7.5% = β_{pm} (10%);
Short position in 3 contracts will increase 7.5%.
Thus, the total value of hedged position
should be roughly independent of index value;
Expect to earn riskfree rate over life of hedge.

e. This hedging strategy assumes:
  i. Dividend yield on index is predictable;
  ii. Riskfree rate is constant during life of hedge;
  iii. Portfolio has a stable β_{pm} during life of hedge.
  iv. Portfolio is diversified (only systematic risk).
If assumptions not true, hedge less effective.

D. Hedging a Stock Portfolio

4. Two reasons for hedging with stock index futures.
(Why hedge so you can expect to earn riskfree rate?)

a. α Fund:

If you can consistently pick undervalued stocks (that beat mkt),
this hedge removes the market risk;
Hedged portfolio should perform > r,
to the extent that your picks outperform the market.

b. May like stock portfolio long term,
but worried about short term.
Need short term protection; Use futures for timing.

[If you sold entire portfolio, & later bought back, high TC!]

D. Hedging a Stock Portfolio

5. Can change beta of portfolio to anything desired.

   a. $\mathbf{N^*} = \beta_{pm} (V_A / V_F)$ contracts reduce β from β_{pm} to zero.

   b. $\mathbf{N^*} / 2$ contracts reduce β from β_{pm} to $\beta_{pm} / 2$.

   c. In general, can change β from β_{pm} to any β^* desired:

      i. If $\beta_{pm} > \beta^*$, short $(\beta_{pm} - \beta^*) (V_A / V_F)$ contracts;

      ii. If $\beta_{pm} < \beta^*$, long $(\beta^* - \beta_{pm}) (V_A / V_F)$ contracts.

6. Can use stock index futures to hedge individual stock.

   a. Special case of 5. above, where $\beta_{pm} = \beta_{im}$.
      $\mathbf{N^*} = \beta_{pm} (V_A / V_F)$ contracts reduce β of *stock* from β_{im} to 0.

   b. Note: **N*** is calculated same way, but performance worse!
      This hedge only protects against systematic (market) risk,
      not unsystematic risk in individual stock. (See Prob. 3.18)

E. Stack and Roll

1. Expiration date of the hedge may be later than delivery dates of all usable (liquid) futures contracts.

   User chooses short term (nearby) futures contract to hedge until it expires;

   Then rolls hedge forward; as futures expire, close one & open another.

   Procedure: company expects to receive price in future at time Tn;

T0:	Short futures contract #1 expiring at T1;
T1:	Close out futures contract #1;
	Short futures contract #2 expiring at T2;
T2:	Close out futures contract #2;
	Short futures contract #3 expiring at T3; . . .
Tn-1:	Close out futures contract #n-1;
	Short futures contract #n expiring at Tn;
Tn:	Close out futures contract #n.

   Will not be perfect hedge. Outcome depends on futures prices (& Basis!) you'll get at times T1, T2, …, Tn. If market moves against, margin calls!

E. Stack and Roll

2. Metallgesellschaft Case (MG).

MG sold **huge volume** of 10-year forward heating oil contracts.

Customers paid 6-7 cents above market price;
Locked into heating oil prices for 5-10 years.

Then MG hedged risk that oil prices would rise over next 10 years,
by buying ***short term*** futures; and rolling them over each month.

Problem: Prices dropped; huge margin calls on long futures.

No Problem?
Short term outflows should be offset by long term inflows (in 10 yr!).
Problem? Short term outflows were huge!
MG decided to bail out – unwound hedges; Lost $1.3 Billion.