# CS201

Mathematics For Computer Science

Indian Institute of Technology, Kanpur

# Assignment 4

Due by: Dec 14, 2020

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## Instructions.

- Solutions should be mandatorily LaTeXed using the template shared and submitted through GradeScope before time. Mention Group Numbers and member names in solutions (refer template instructions).
- Clearly express solutions avoiding unnecessary details. Everything discussed in class is not required to be proved again. And anything non-trivial must be proved.
- Write the solutions on your own. Acknowledge the source wherever required. Keep in my mind department's Anti-Cheating Policy.

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1. Let $S$ be a finite set and $F$ be set of all bijections from $S$ to $S$. Show that $F$ along with the composition operation is a group.
2. Let $G$ be a non-commutative group and $e \in G$ be the identity element. The **order** of an element $g \in G$ denoted as $ord(g)$ is the smallest natural number $s$ such that $g^s = e$ where
$$g^i = \underbrace{g.g.g \dots g}_{\text{number of } g \text{ is } i}$$
Let $a$ and $b$ be elements of $G$ such that $ord(a) = 7$ and $a^3 b = ba^3$. Prove that $ab = ba$.
3. Let $\mathbb{Q}[\alpha, \beta]$ denote the smallest subring of $\mathbb{C}$ containing rational numbers $\mathbb{Q}$ and the element $\alpha = \sqrt{2}$ and $\beta = \sqrt{3}$. Let $\gamma = \alpha + \beta$. Is $\mathbb{Q}[\alpha, \beta] = \mathbb{Q}[\gamma]$?
4. An element $n$ of a ring $R$ is called **nilpotent** if there exists $j \in \mathbb{N}$ such that $n^j = 0$. An element $u$ of a ring $R$ is called a **unit** if there exists $v \in R$ such that $uv = 1$. Prove that if $r \in R$ is nilpotent, then $1 - r$ is a unit.

5. Let $I$ and $J$ be ideals of a ring $R$ such that $I + J = R$. Prove that $IJ = I \cap J$ where $IJ = \{xy | x \in I, y \in J\}$.