ESC201T : Introduction to Electronics

L11: Transient Analysis of RLC Circuits

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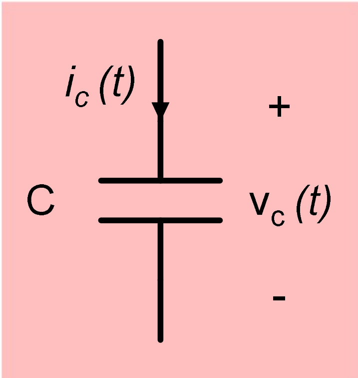
Two important concepts

Voltage across a capacitor cannot change instantaneously

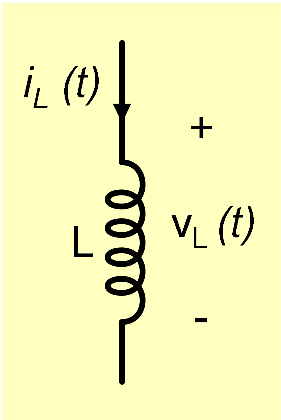


$$i_c = C\,\frac{dv_c}{dt}$$

$$v_C = \frac{1}{C}\int i_C(t)\,dt$$

Instantaneous change implies infinite current!

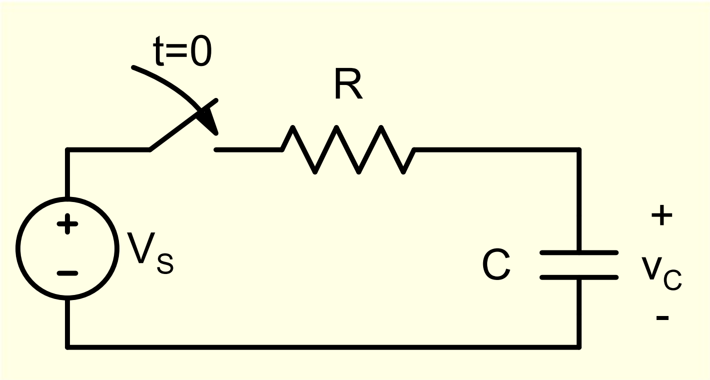
Current through an inductor cannot change instantaneously



$$v_L = L\,\frac{di_L}{dt}$$

$$i_L = \frac{1}{L}\int v_L(t)\,dt$$

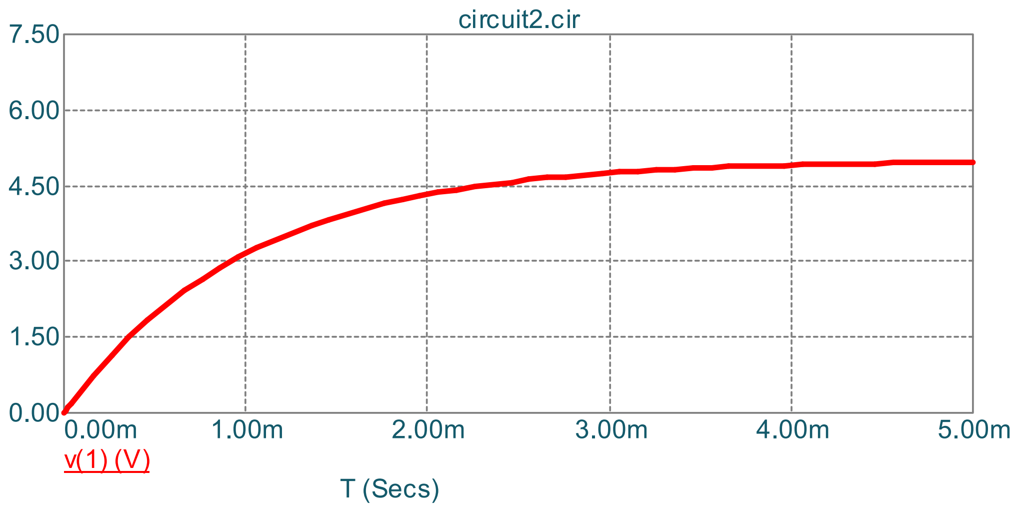
Instant change in voltage implies infinite voltage!



$$\frac{dx}{dt} = -a_1 x + a_2$$

$$x(t) = x(\infty) + \{x(0) - x(\infty)\}\, e^{-a_1 t}$$

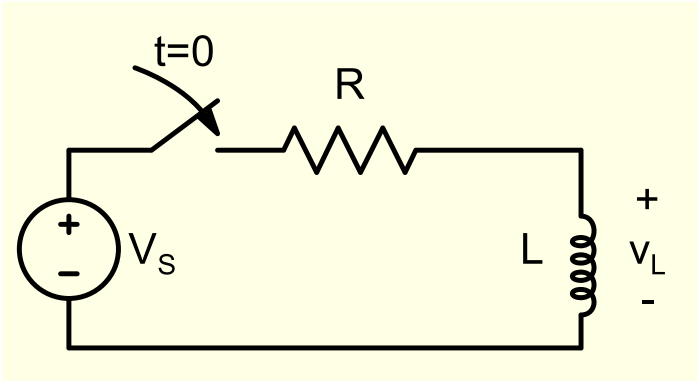
$$v_C(t) = V_S(1 - e^{-\frac{t}{RC}})$$

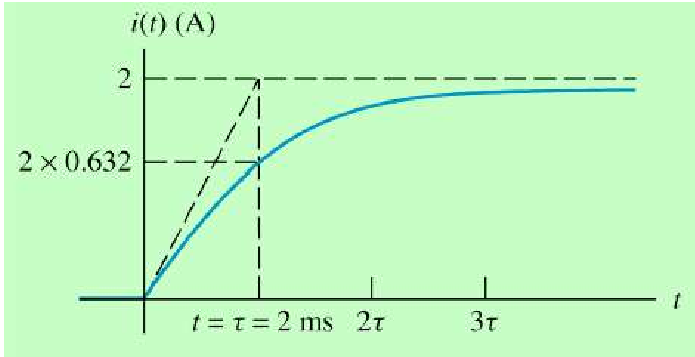


Time	τ	2τ	3τ	4τ	5τ
$V_C(t)/V_i$	0.632	0.865	.95	0.982	0.993

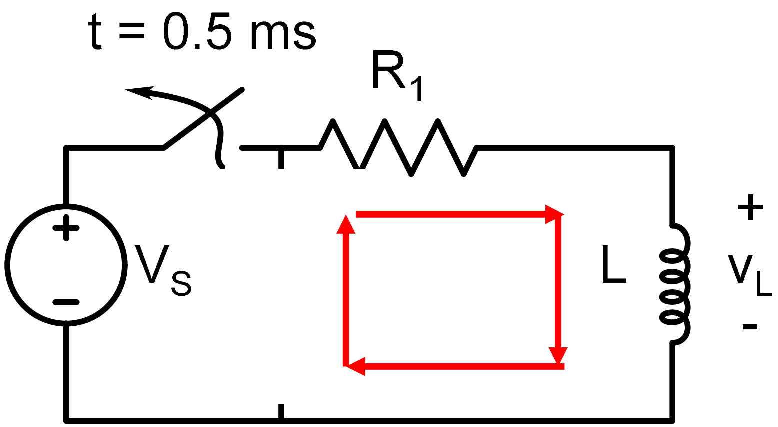
R-L Circuits For High Voltage Generation

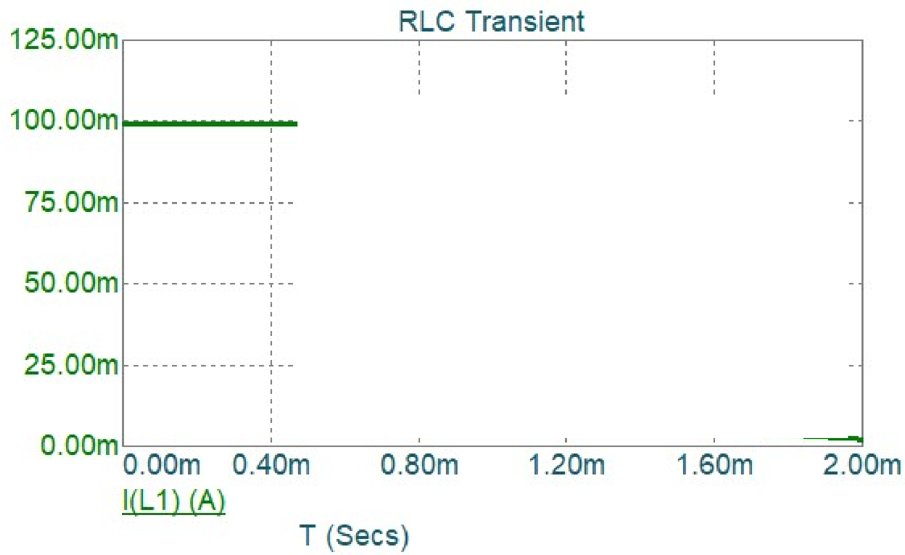
$$i(t) = \frac{V_S}{R} \times (1 - e^{-\frac{t}{\tau}})$$

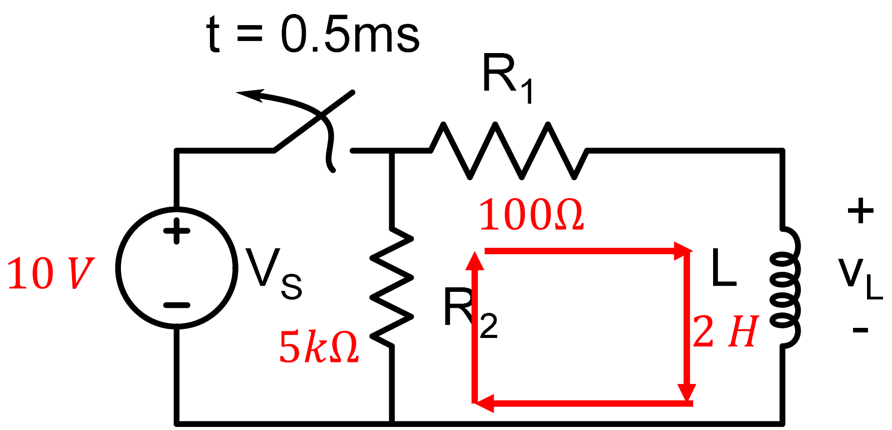


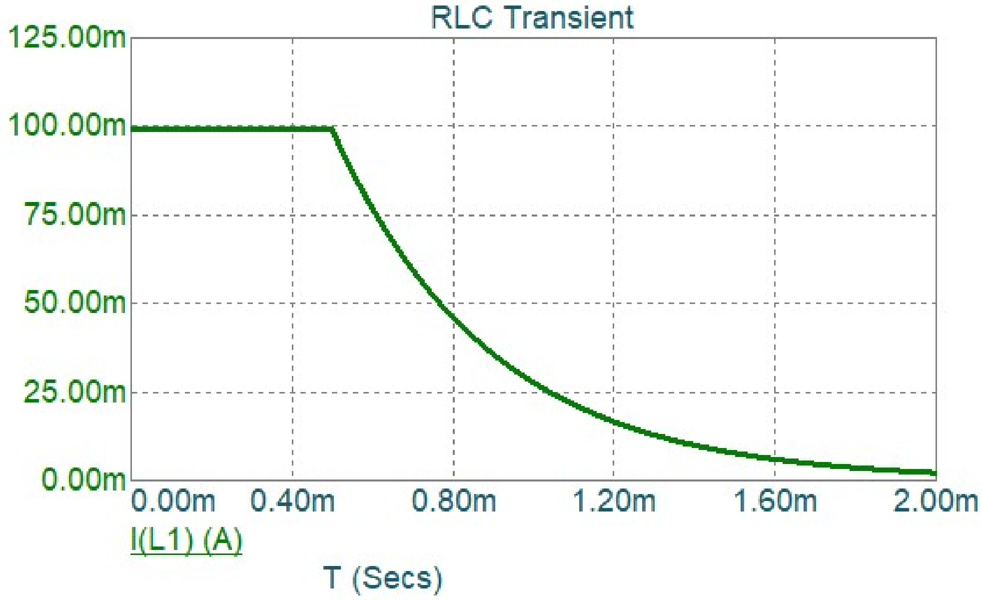


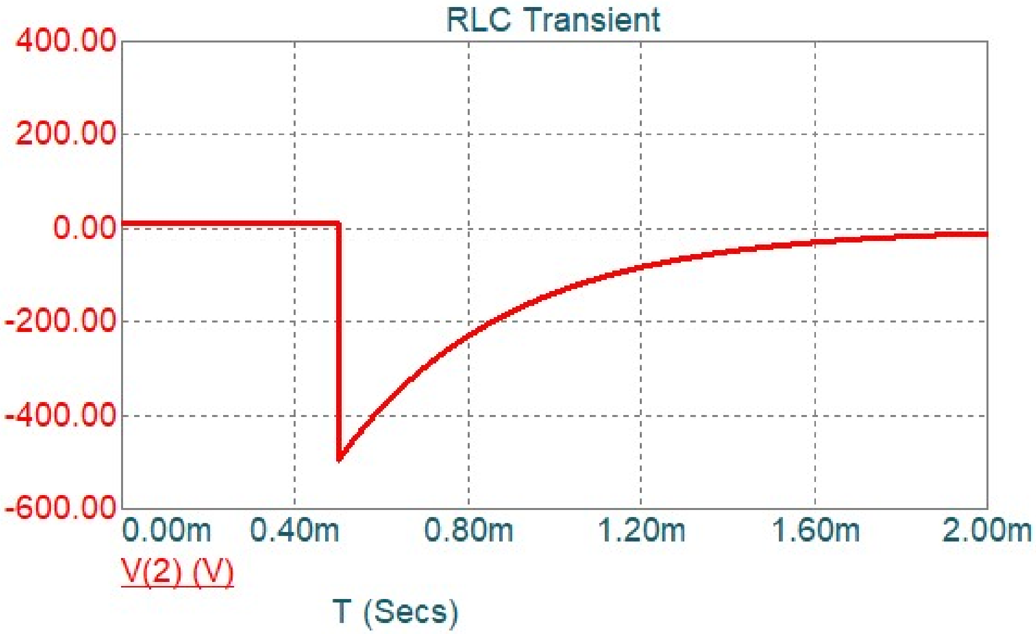
$$I_L(t) = \frac{V_S}{R_1} \times \exp(-\frac{t}{\tau})$$



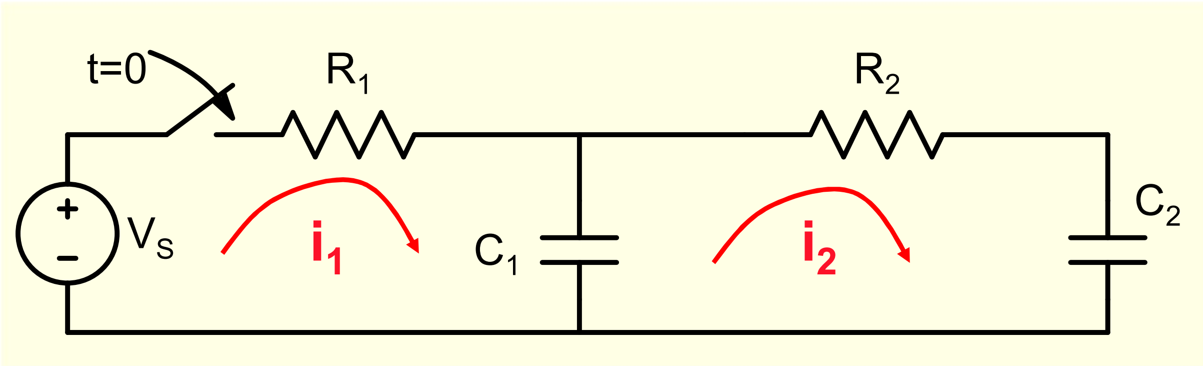



t = 0.5ms
R1
100Ω
10 V
VS
5kΩ
R2
L
2 H
+
vL
-


RLC Transient
125.00m
100.00m
75.00m
50.00m
25.00m
0.00m
0.00m
0.40m
0.80m
1.20m
1.60m
2.00m
I(L1) (A)
T (Secs)


RLC Transient
400.00
200.00
0.00
-200.00
-400.00
-600.00
0.00m
0.40m
0.80m
1.20m
1.60m
2.00m
V(2) (V)
T (Secs)

Second Order System



$$V_S = i_1 R_1 + v_{C1} \quad (1)$$

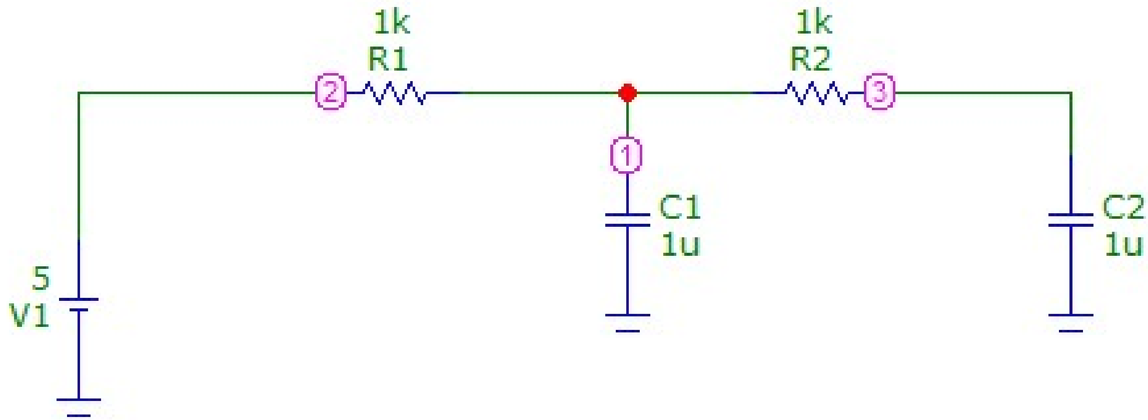
$$v_{C1} = i_2 R_2 + v_{c2} \quad (2)$$

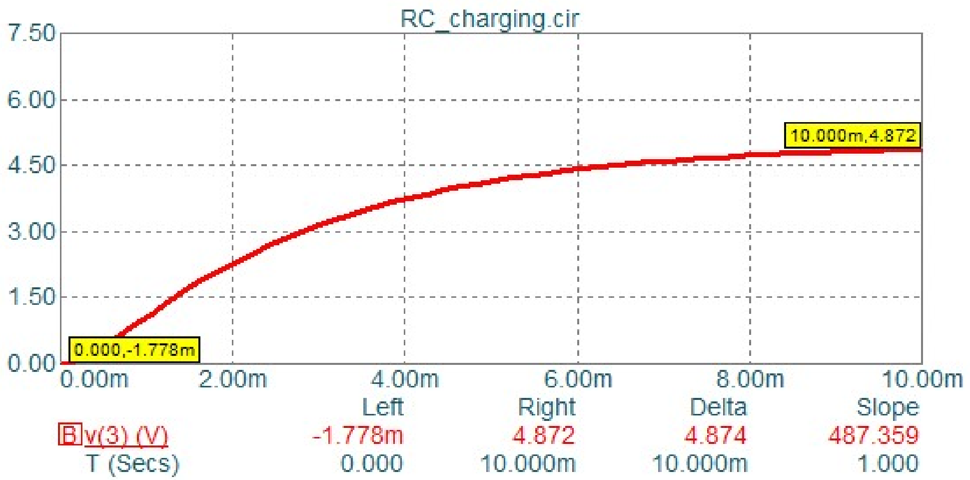
$$i_1 - i_2 = C_1 \frac{dv_1}{dt} (3)$$

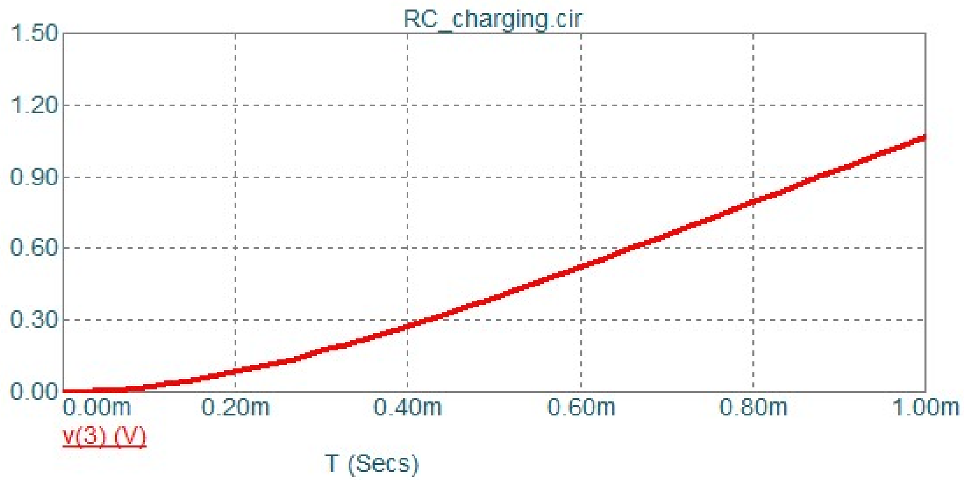
$$i_2 = C_2 \frac{dv_2}{dt} (4)$$

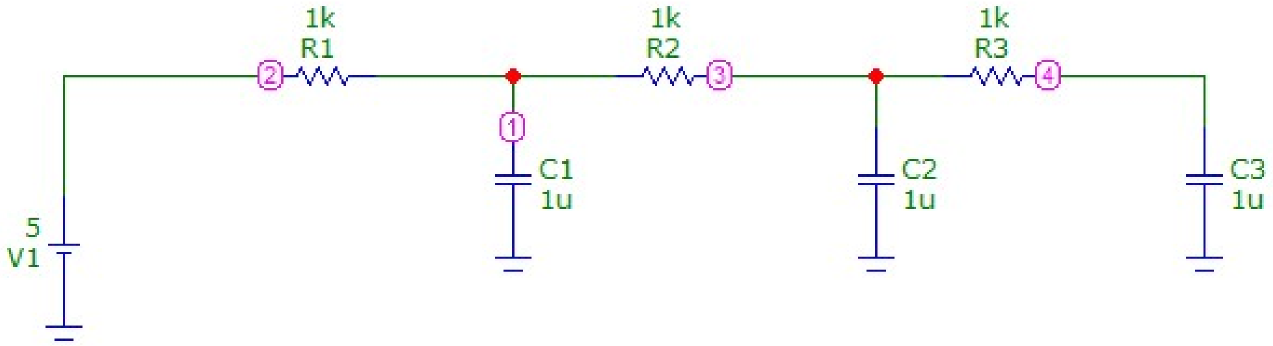
$$R_1 R_2 C_1 C_2 \frac{d^2 v_{c2}}{dt^2} + (R_1 C_1 + R_1 C_2 + R_2 C_2) \frac{dv_{c2}}{dt} + v_{c2} = V_S$$

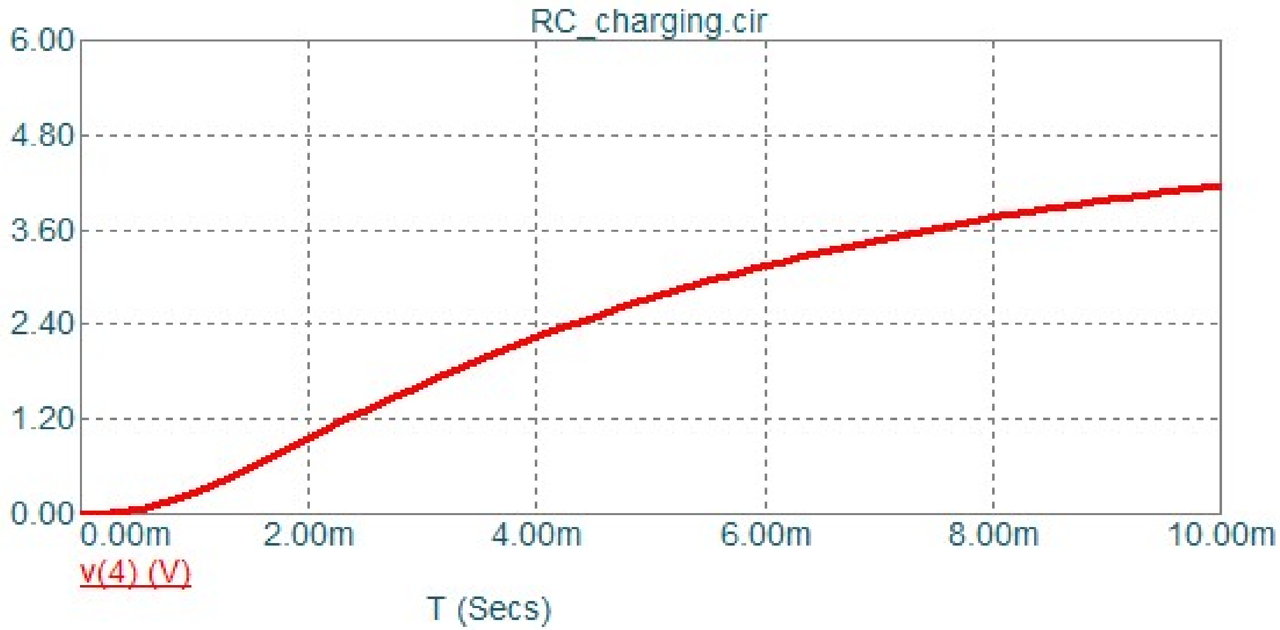
$$v_{c2}(t) = K_0 + K_1 e^{s_1 t} + K_2 e^{s_2 t}$$


1k
R1
1k
R2
2
1
3
C1
1u
C2
1u
5
V1

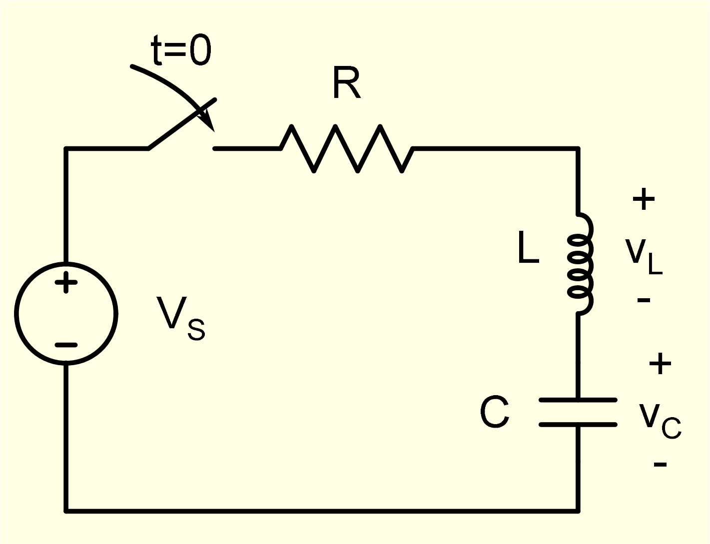

RC_charging.cir
7.50
6.00
4.50
3.00
1.50
0.00
0.00m
2.00m
4.00m
6.00m
8.00m
10.00m
10.000m,4.872
0.000,-1.778m
Left
Right
Delta
Slope
B v(3) (V)
-1.778m
4.872
4.874
487.359
T (Secs)
0.000
10.000m
10.000m
1.000


RC_charging.cir
1.50
1.20
0.90
0.60
0.30
0.00
0.00m
0.20m
0.40m
0.60m
0.80m
1.00m
v(3) (V)
T (Secs)


1k
R1
1k
R2
1k
R3
2
1
3
4
C1
1u
C2
1u
C3
1u
5
V1


RC_charging.cir
6.00
4.80
3.60
2.40
1.20
0.00
0.00m
2.00m
4.00m
6.00m
8.00m
10.00m
v(4) (V)
T (Secs)

Series RLC Circuit



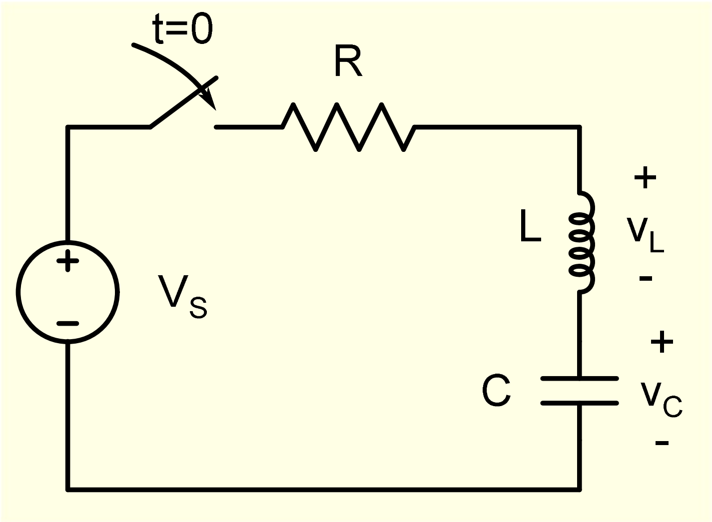
$$V_S = I \times R + L\frac{dI}{dt} + V_C \qquad I = C\frac{dV_C}{dt}$$

$$V_S = C\frac{dV_C}{dt} \times R + LC\frac{d^2V_C}{dt} + V_C$$

$$\frac{d^2V_C}{dt^2} + \frac{R}{L} \times \frac{dV_C}{dt} + \frac{V_C}{LC} = \frac{V_S}{LC} \qquad V_C(t) = A \times e^{st} \qquad As^2e^{st} + \frac{R}{L}Ase^{st} + \frac{A}{LC}e^{st} = 0$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \qquad s = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

$$\omega_O = \frac{1}{\sqrt{L \times C}} \qquad Q = \frac{\omega_O L}{R} \qquad \frac{s}{\omega_O} = -\frac{1}{2Q} \pm \sqrt{\frac{1}{4Q^2} - 1}$$

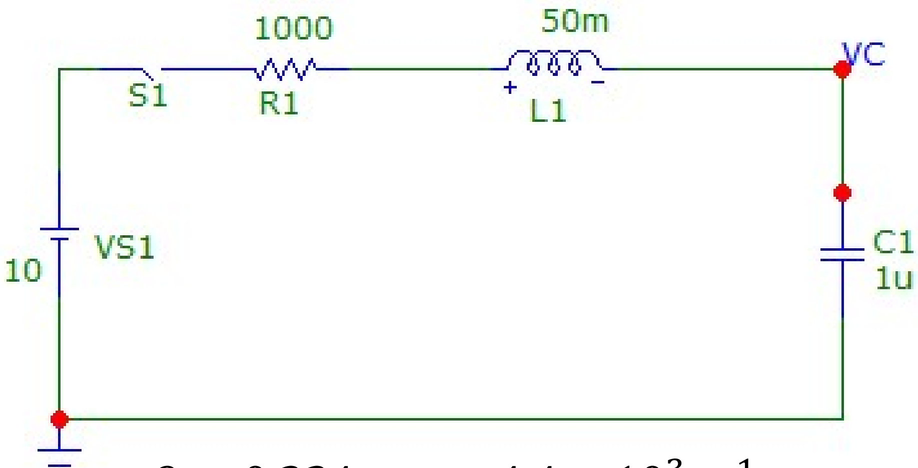


$$V_C(t) = V_S + A \times e^{s_1 t} + B \times e^{s_2 t}$$

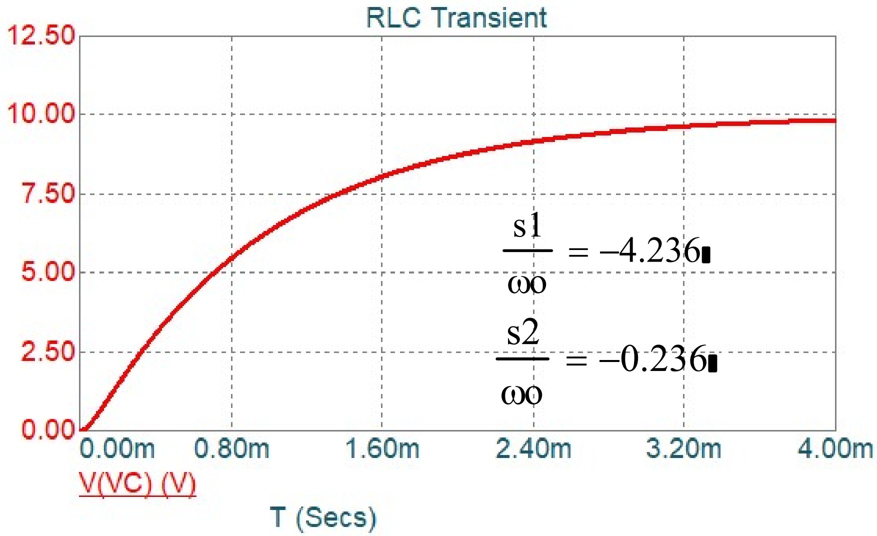
$$\omega_O = \frac{1}{\sqrt{L \times C}} \qquad Q = \frac{\omega_O L}{R} \qquad \frac{s}{\omega_O} = -\frac{1}{2Q} \pm \sqrt{\frac{1}{4Q^2} - 1}$$

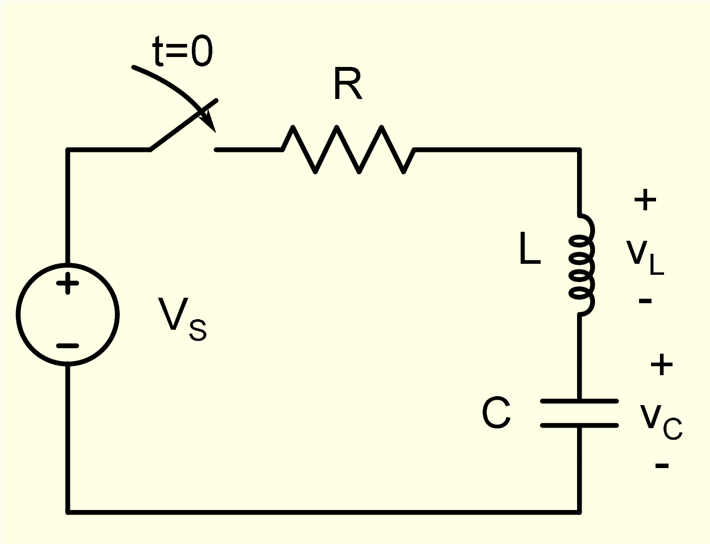
$$\frac{d^2 V_C}{dt^2} + \frac{R}{L} \times \frac{dV_C}{dt} + \frac{V_C}{LC} = \frac{V_S}{LC}$$

Case-1 $Q < 0.5 \Rightarrow s_{1,2}\ are\ real\ and\ negative$



$$Q = 0.224; \omega_O = 4.4 \times 10^3 s^{-1}$$





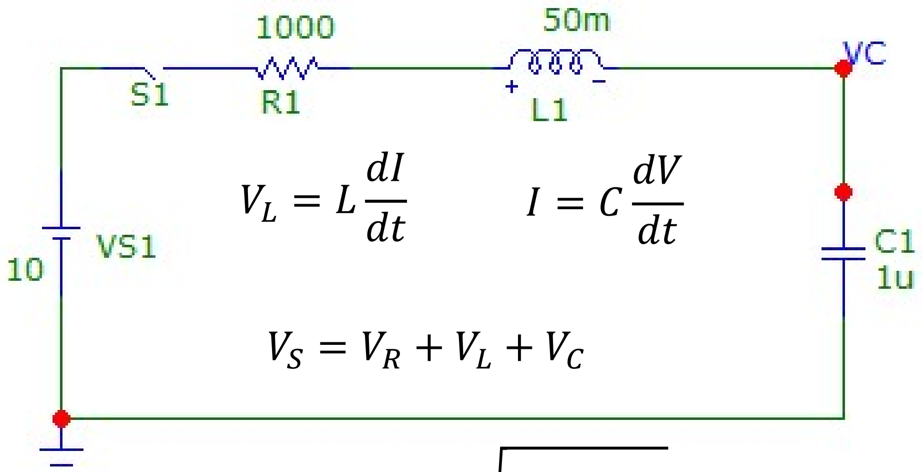
$$V_C(t) = V_S + A \times e^{s_1 t} + B \times e^{s_2 t}$$

$$\omega_O = \frac{1}{\sqrt{L \times C}} \qquad Q = \frac{\omega_O L}{R} \qquad \frac{s}{\omega_O} = -\frac{1}{2Q} \pm \sqrt{\frac{1}{4Q^2} - 1}$$

$$V_C(0) = V_S + A + B = 0 \qquad (1)$$

$$I_C(t) = C\frac{dV_C(t)}{dt} = C\{V_S + As_1 \times e^{s_1 t} + Bs_2 \times e^{s_2 t}\}$$

$$I_C(0) = 0 = C\{V_S + As_1 + Bs_2\} \qquad (2)$$

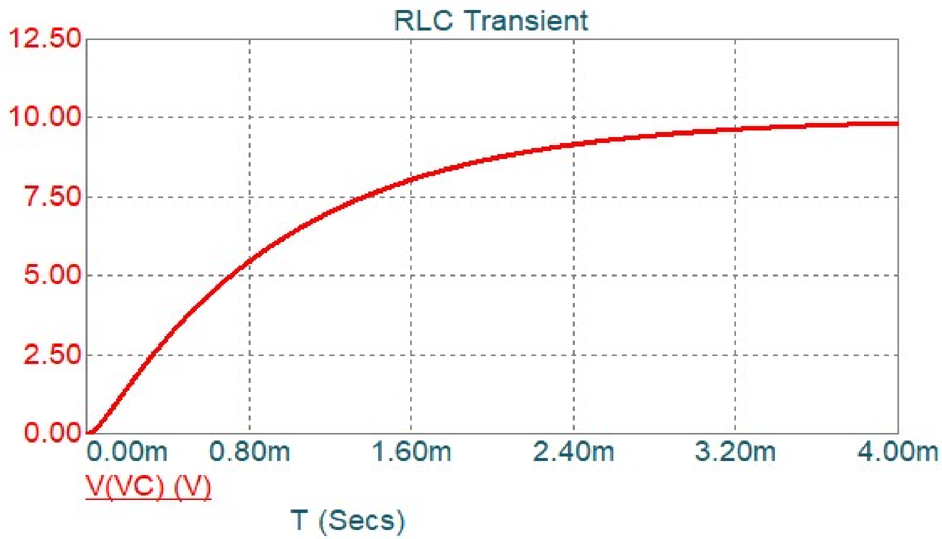


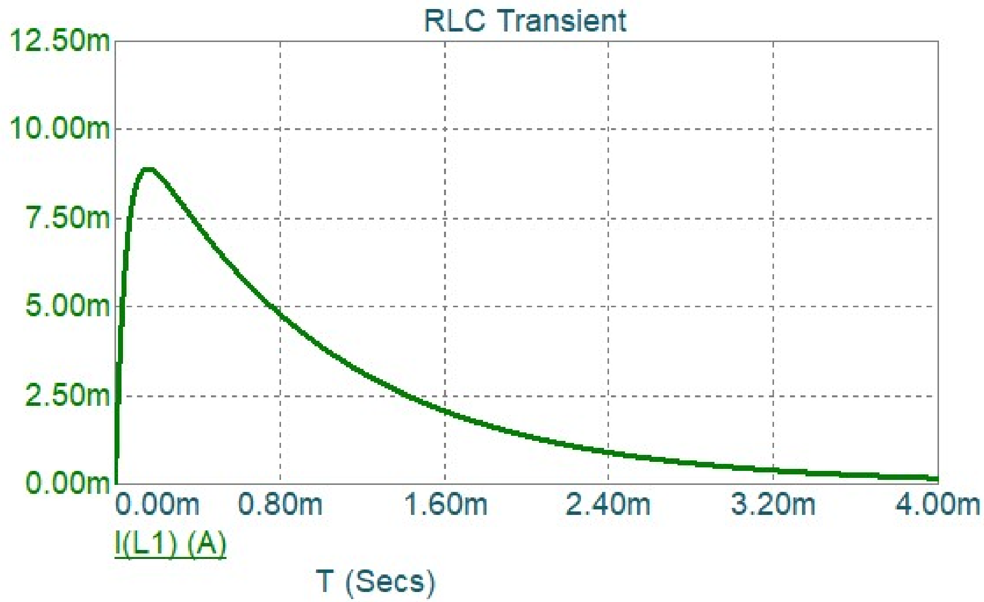
$$V_L = L\frac{dI}{dt} \qquad I = C\frac{dV}{dt}$$

$$V_S = V_R + V_L + V_C$$

$$\frac{s}{\omega_O} = -\frac{1}{2Q} \pm \sqrt{\frac{1}{4Q^2} - 1}$$

$$V_C(t) = 10 + 0.59 \times e^{-1.9\times10^4\, t} - 10.59 \times e^{-10^3 t}$$

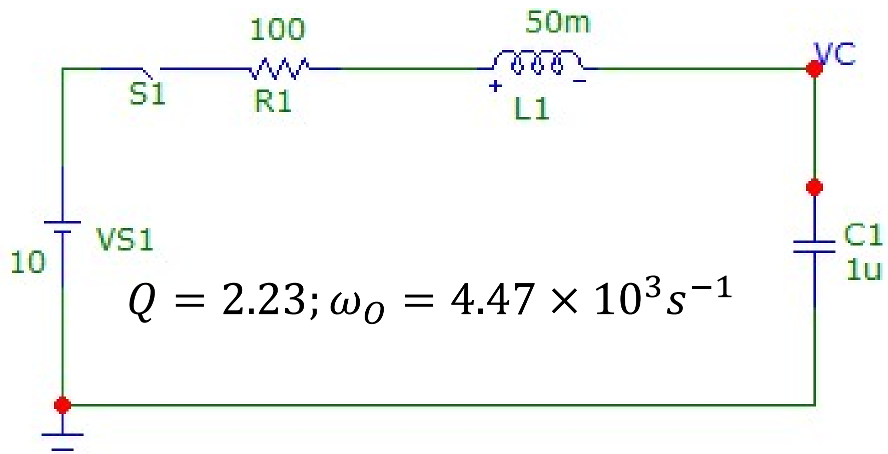




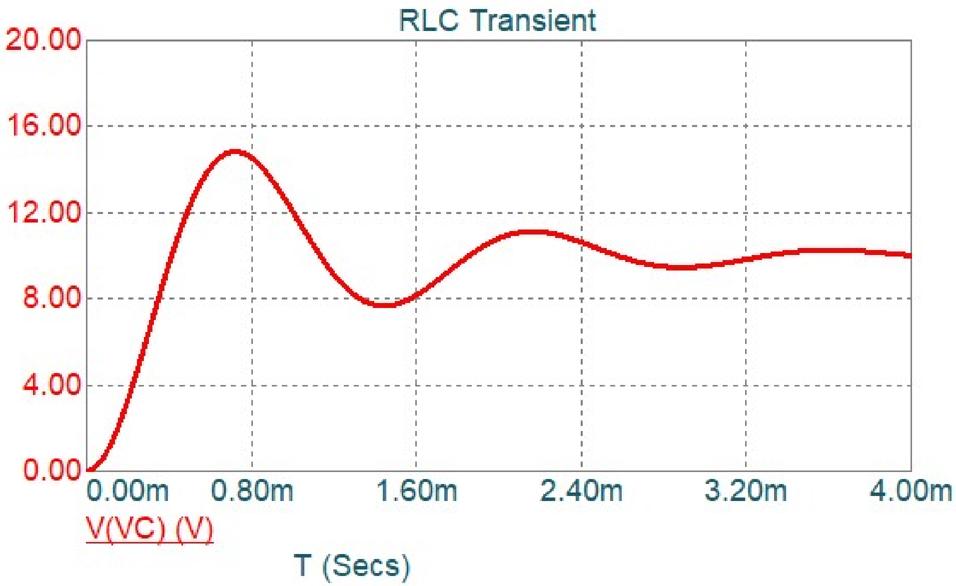
Q< 0.5 Overdamped Case

Case-2 : Q= 0.5 critically damped Case

Case-3 underdamped case : Q > 0.5



$$Q = 2.23; \omega_O = 4.47 \times 10^3 s^{-1}$$



$$\frac{s}{\omega_O} = -\frac{1}{2Q} \pm \sqrt{\frac{1}{4Q^2} - 1} \qquad \omega_O = \frac{1}{\sqrt{L \times C}} \qquad Q = \frac{\omega_O L}{R}$$

$$s = -\frac{\omega_o}{2Q} \pm j\omega_O \sqrt{1 - \frac{1}{4Q^2}}$$

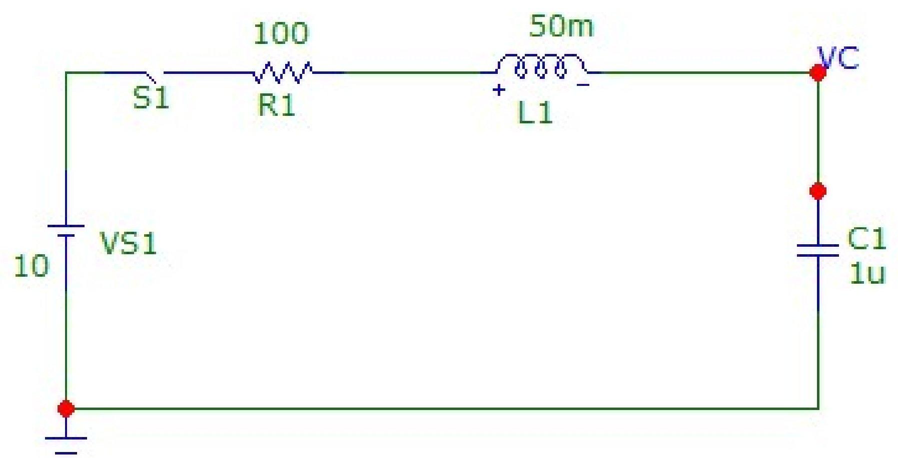
$\Rightarrow s_{1,2}$ *have real and imaginary* components

$$s1 = -1 \times 10^3 - 4.359i \times 10^3$$

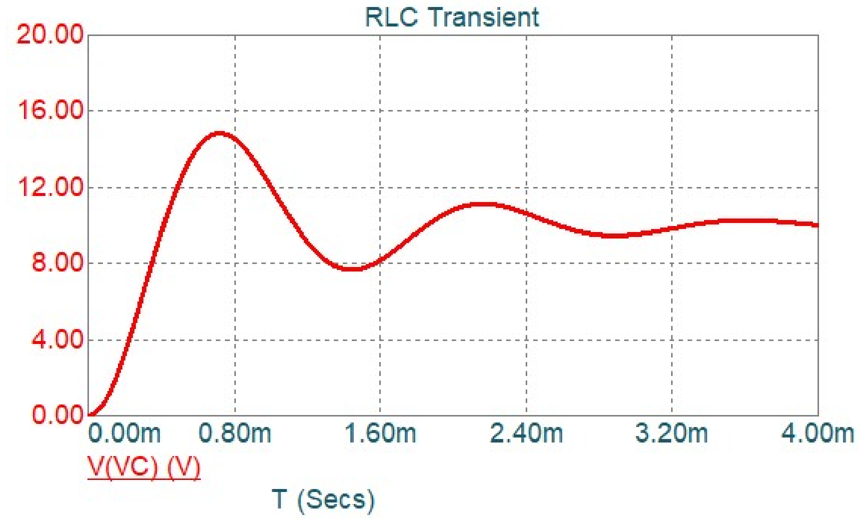
$$s2 = -1 \times 10^3 + 4.359i \times 10^3$$

$$V_C(t) = 10 - e^{-10^3 t} \times (10 \times Cos(\omega_1 t) + 2.3 \times Sin(\omega_1 t))$$

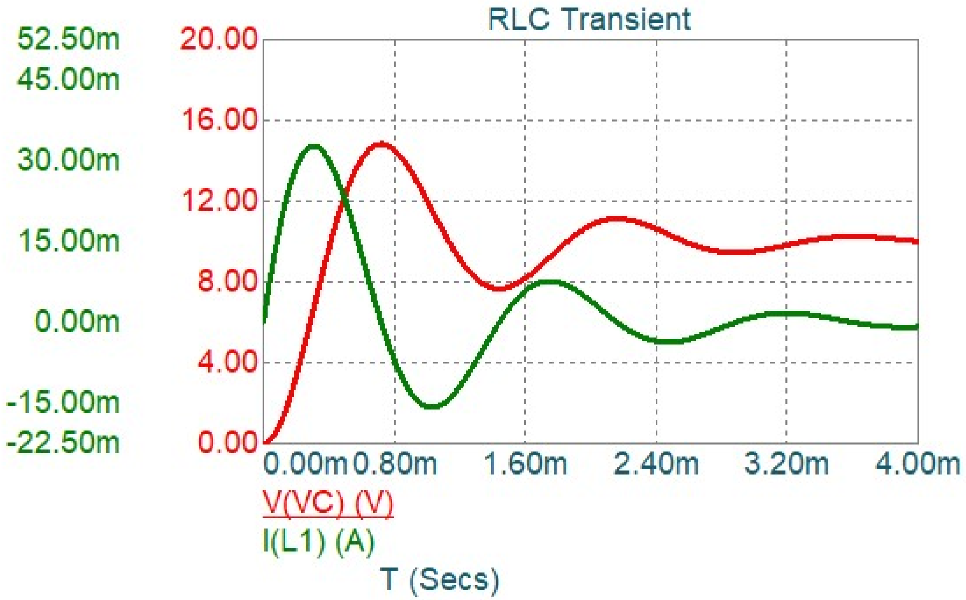
$$\omega_1 = 4.36 \times 10^3 s^{-1}$$



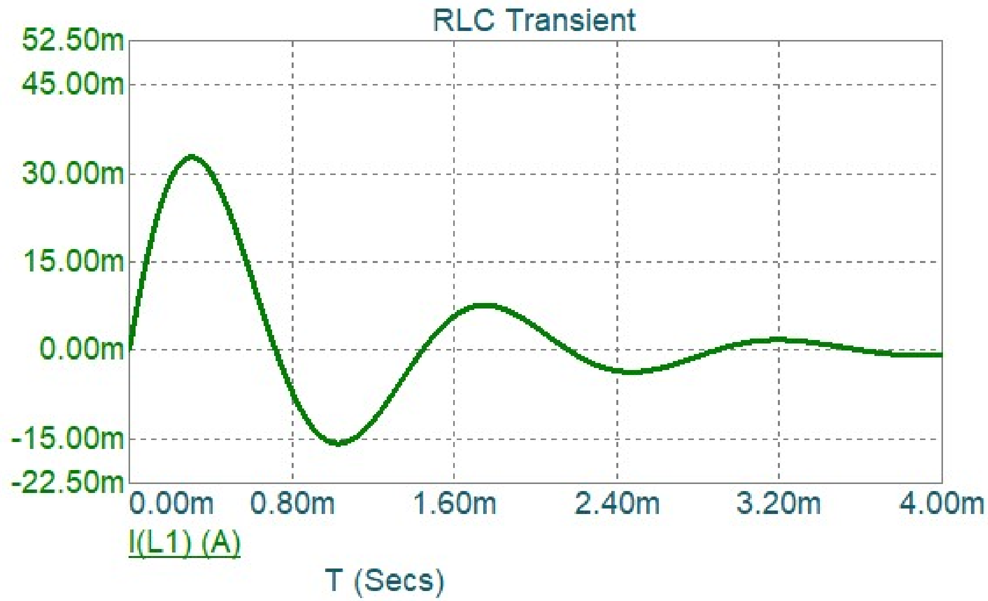
100
50m
VC
S1
R1
L1
VS1
10
C1
1u



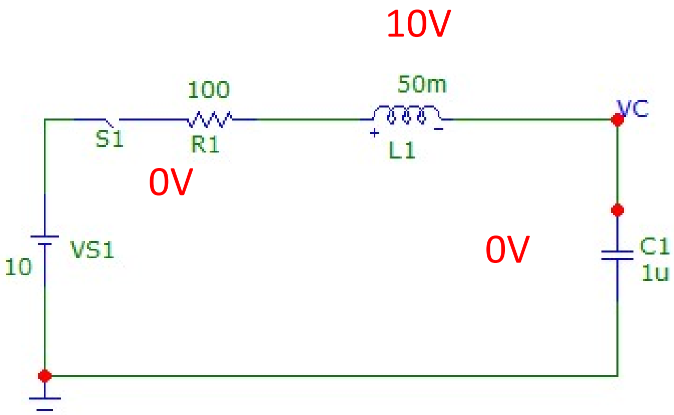
RLC Transient
20.00
16.00
12.00
8.00
4.00
0.00
0.00m
0.80m
1.60m
2.40m
3.20m
4.00m
V(VC) (V)
T (Secs)

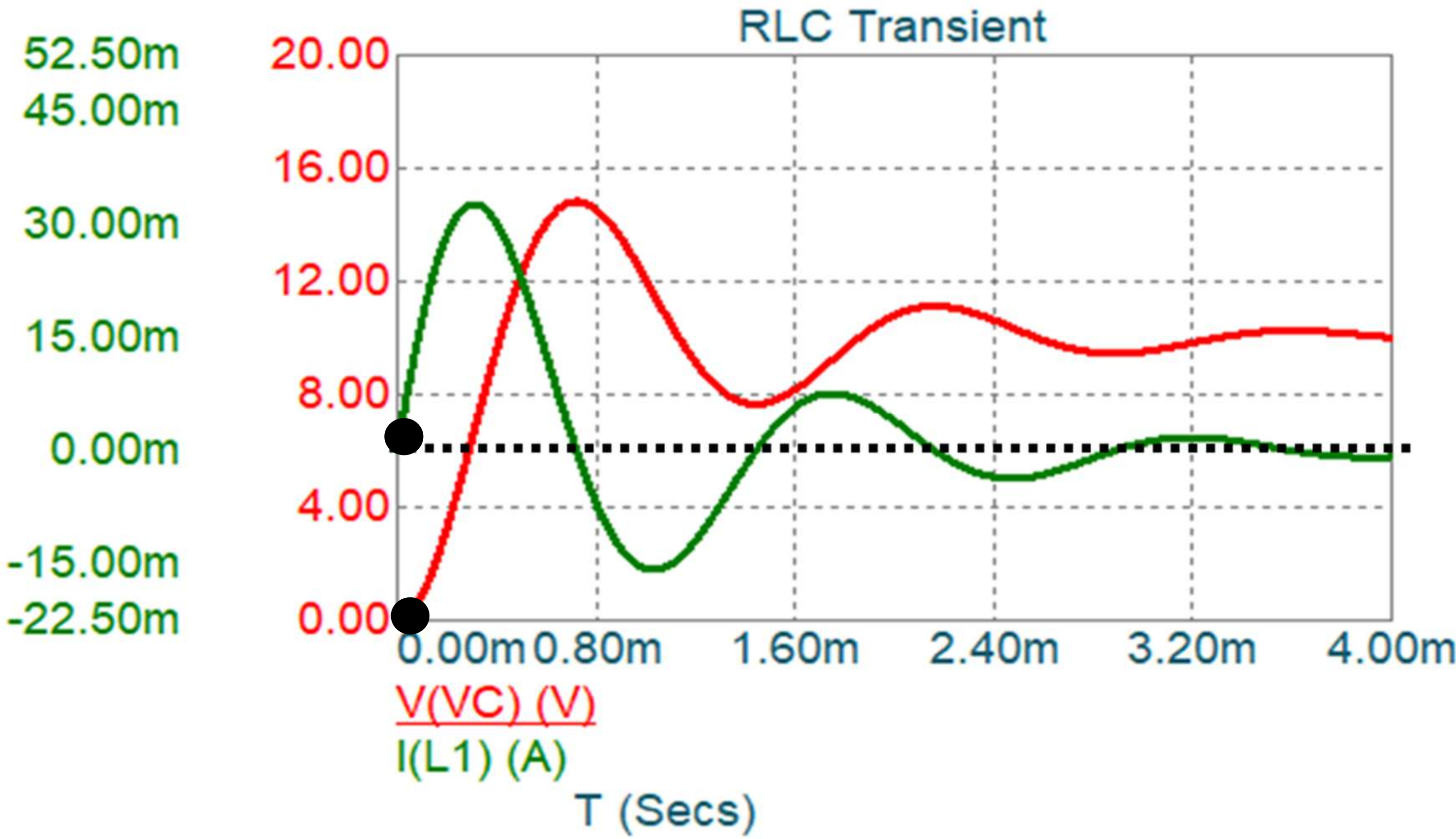


RLC Transient
52.50m
45.00m
30.00m
15.00m
0.00m
-15.00m
-22.50m
20.00
16.00
12.00
8.00
4.00
0.00
0.00m
0.80m
1.60m
2.40m
3.20m
4.00m
V(VC) (V)
I(L1) (A)
T (Secs)



RLC Transient
52.50m
45.00m
30.00m
15.00m
0.00m
-15.00m
-22.50m
0.00m
0.80m
1.60m
2.40m
3.20m
4.00m
I(L1) (A)
T (Secs)

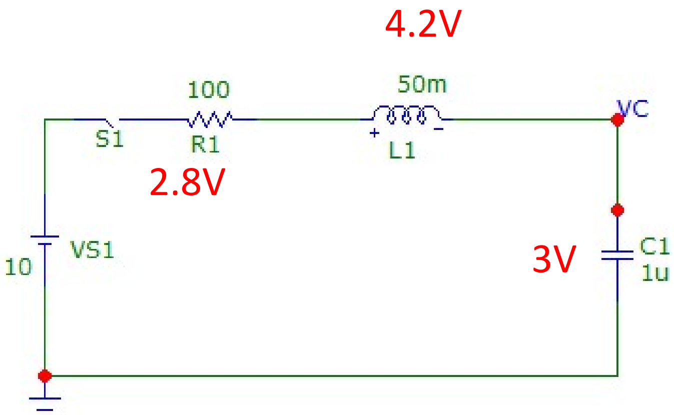


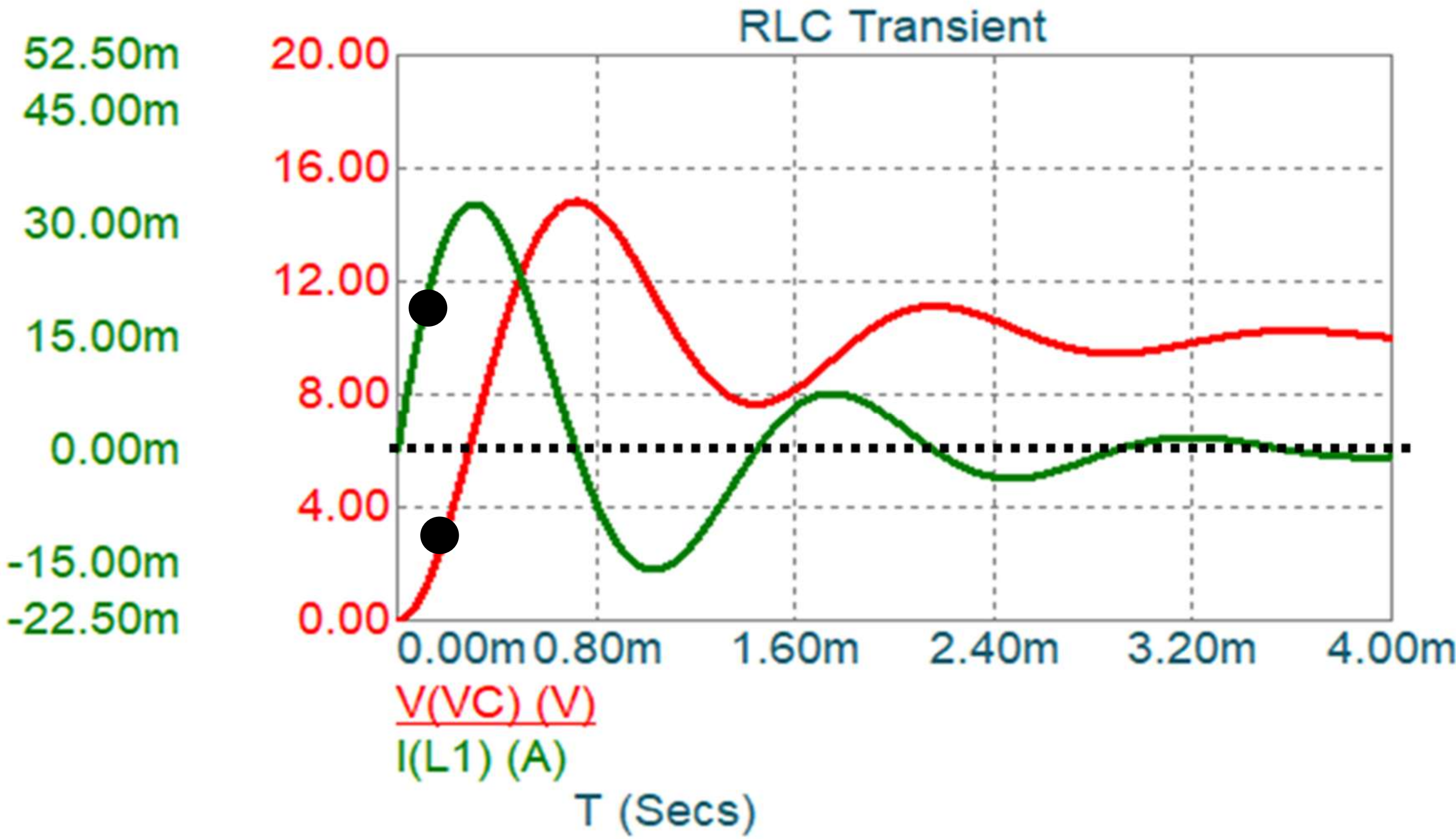


$$\boxed{V_L = L\frac{dI}{dt}}$$

$$I = C\frac{dV}{dt}$$

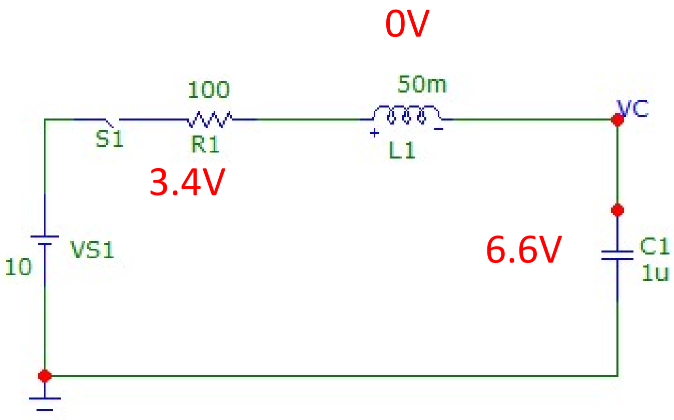
$$V_S = V_R + V_L + V_C$$

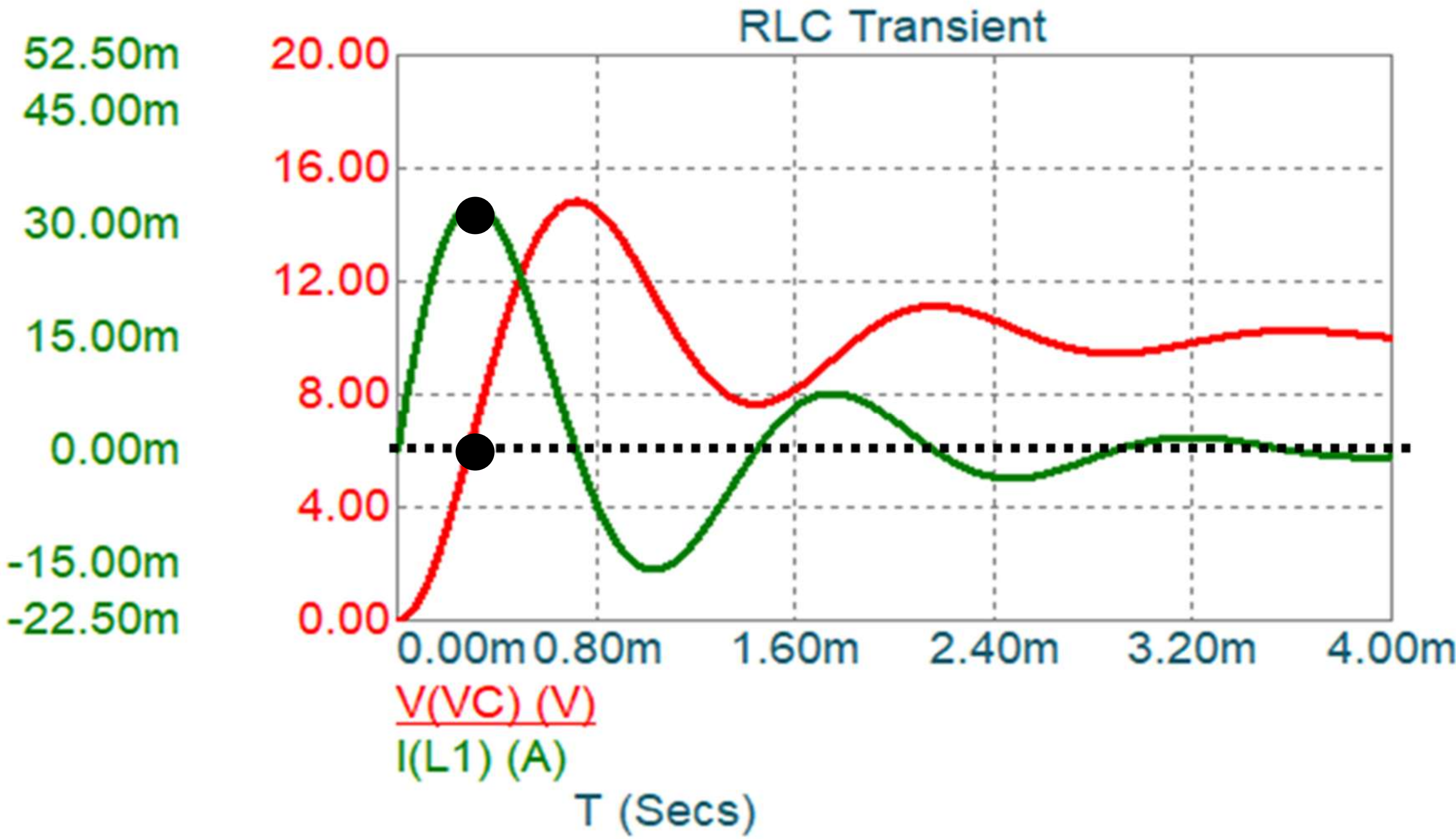




$$V_L = L\frac{dI}{dt} \qquad I = C\frac{dV}{dt}$$

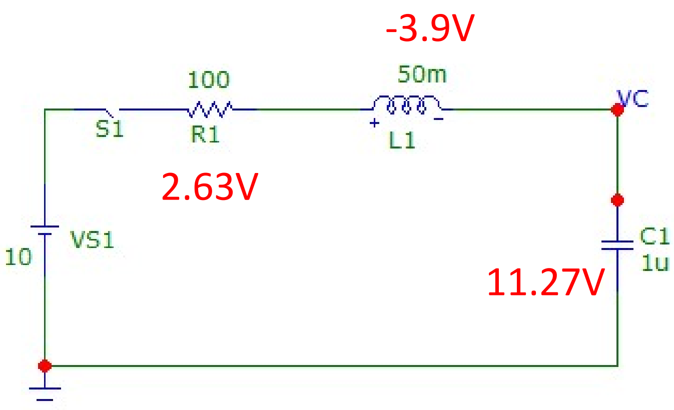
$$V_S = V_R + V_L + V_C$$

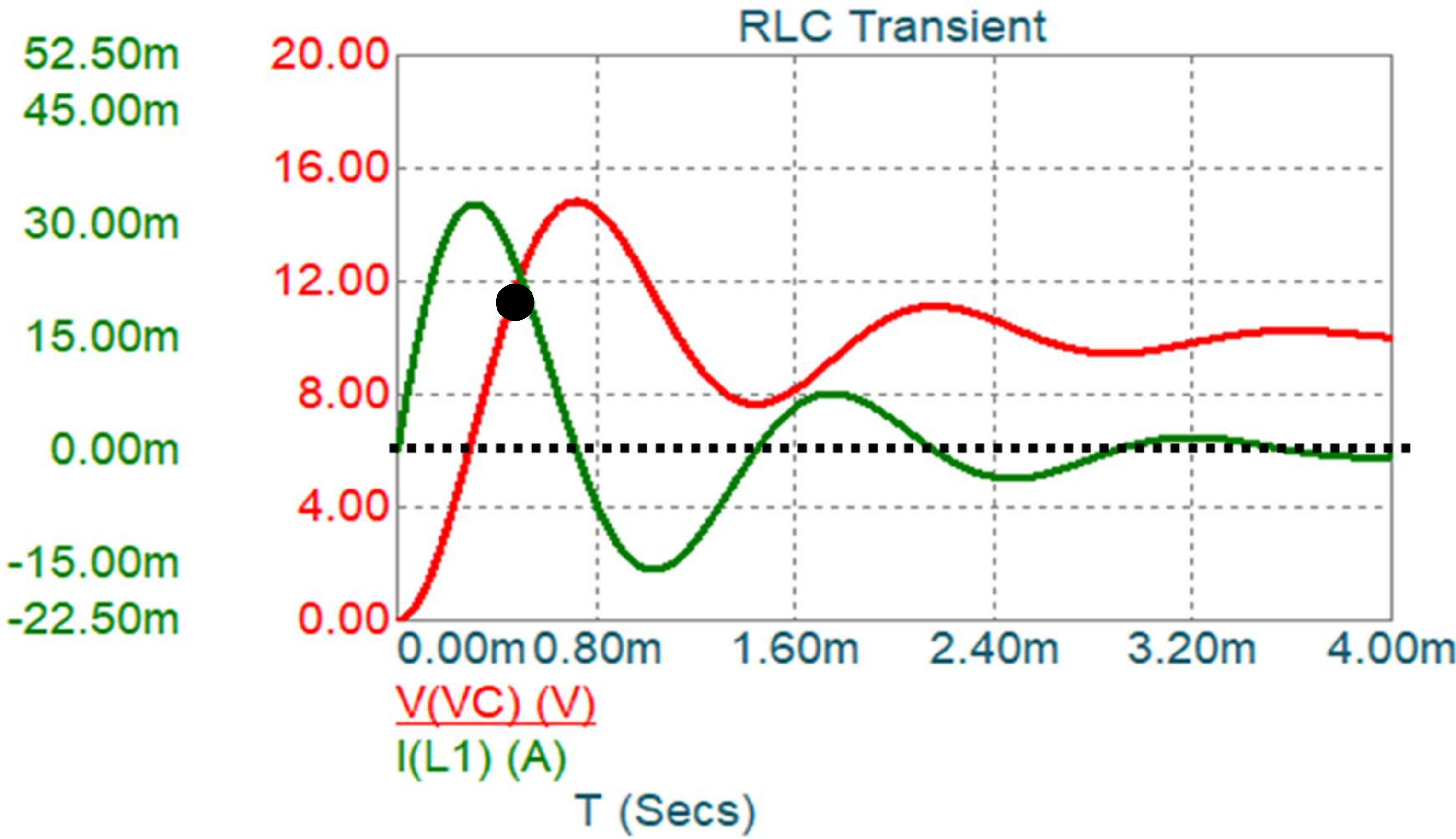




$$V_L = L\frac{dI}{dt} \qquad I = C\frac{dV}{dt}$$

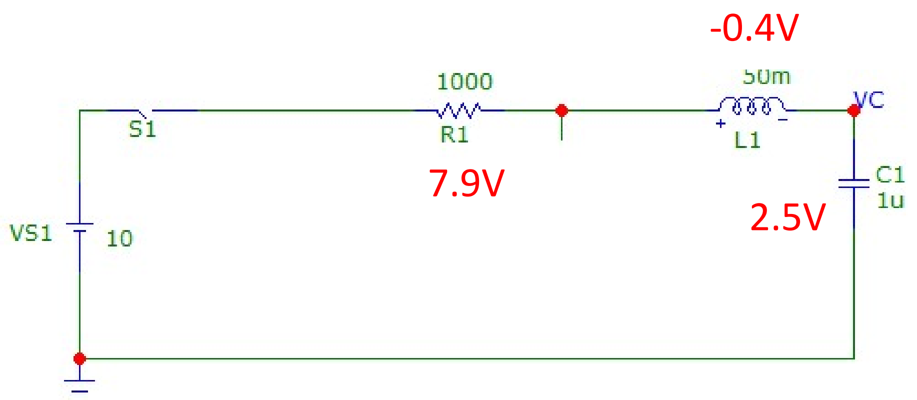
$$V_S = V_R + V_L + V_C$$

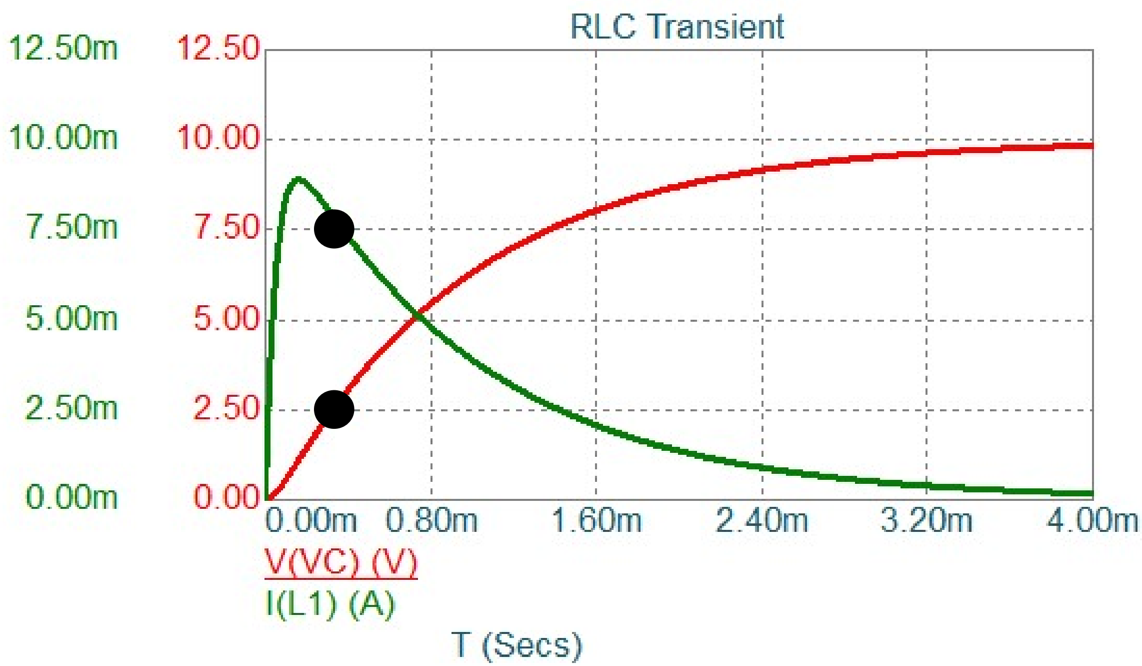




$$V_L = L\frac{dI}{dt} \qquad I = C\frac{dV}{dt}$$

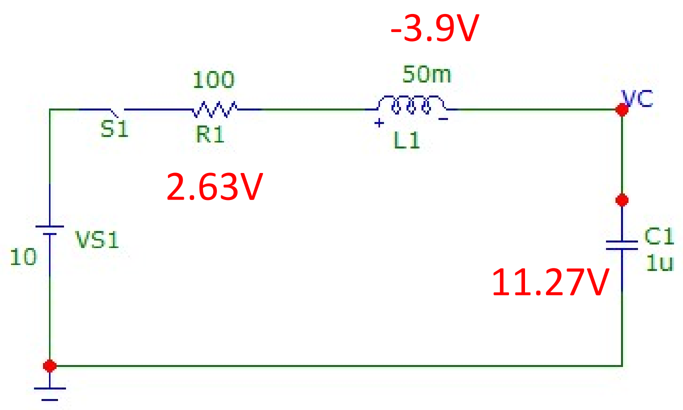
$$V_S = V_R + V_L + V_C$$





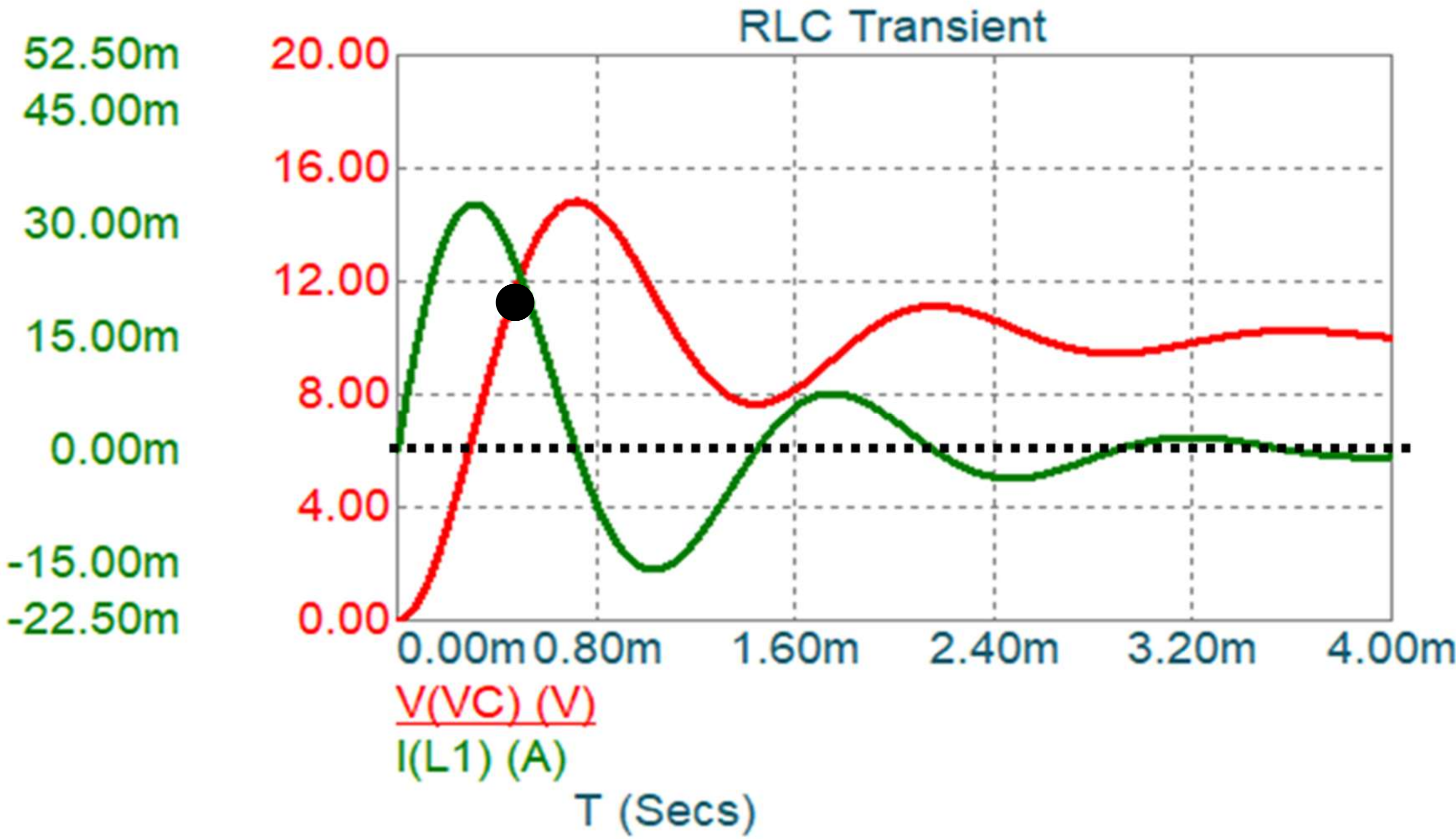
$$V_L = L\frac{dI}{dt} \qquad I = C\frac{dV}{dt}$$

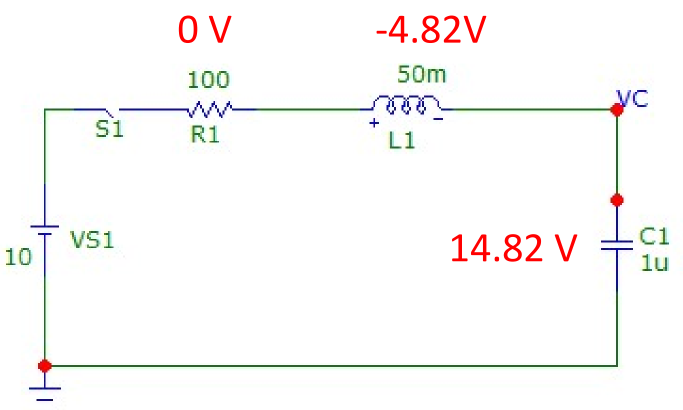
$$V_S = V_R + V_L + V_C$$



$$V_L = L\frac{dI}{dt} \qquad I = C\frac{dV}{dt}$$

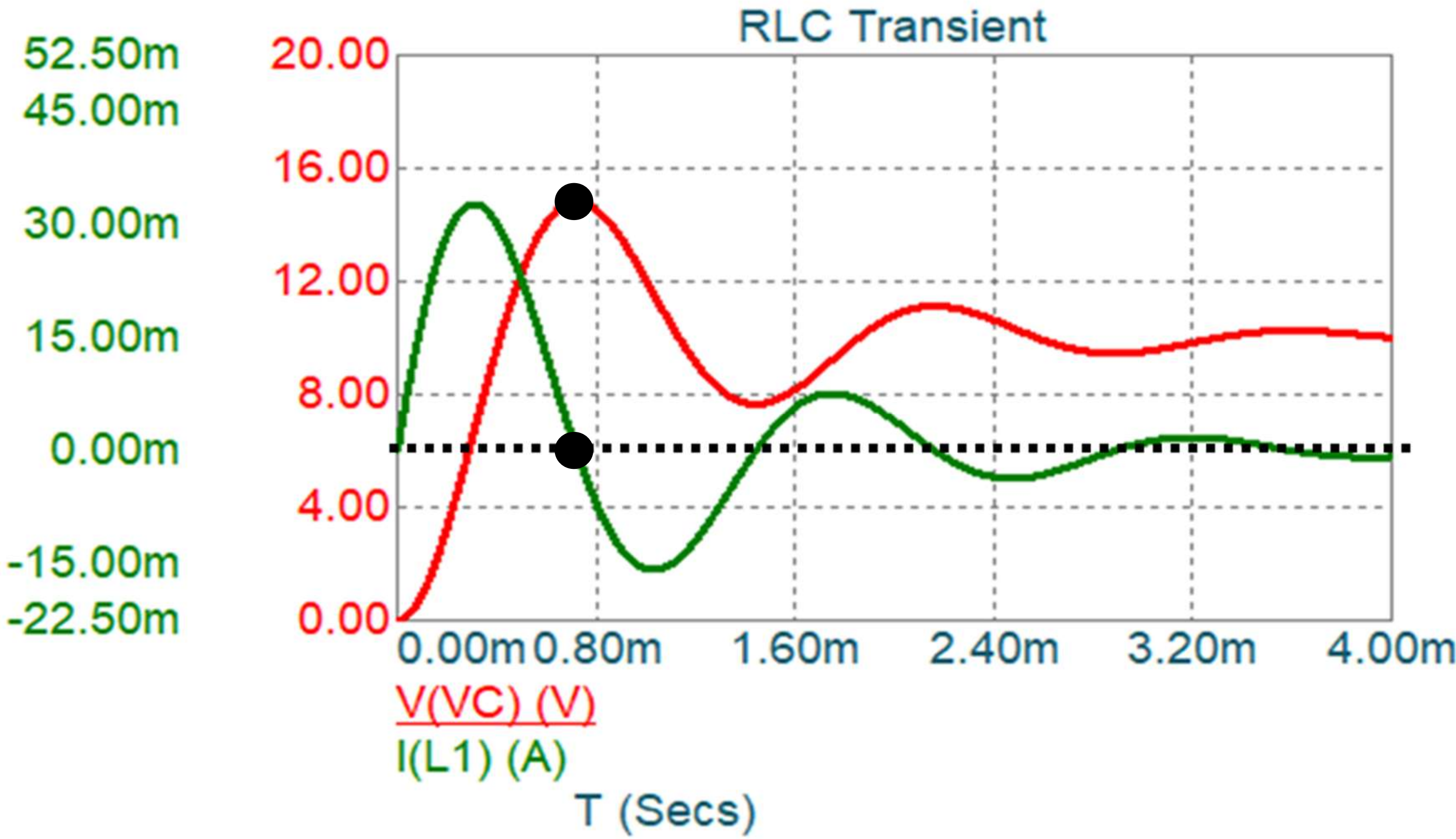
$$V_S = V_R + V_L + V_C$$

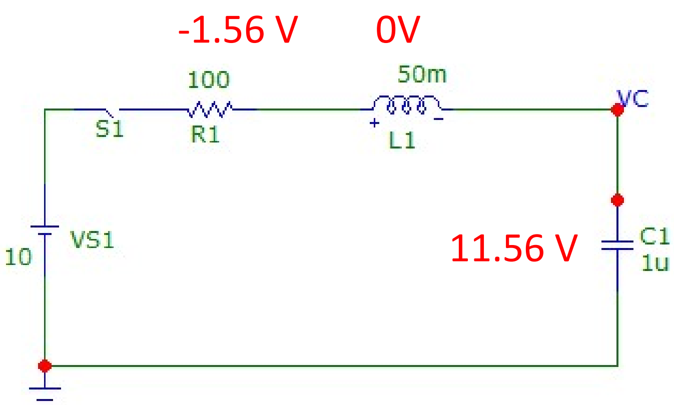


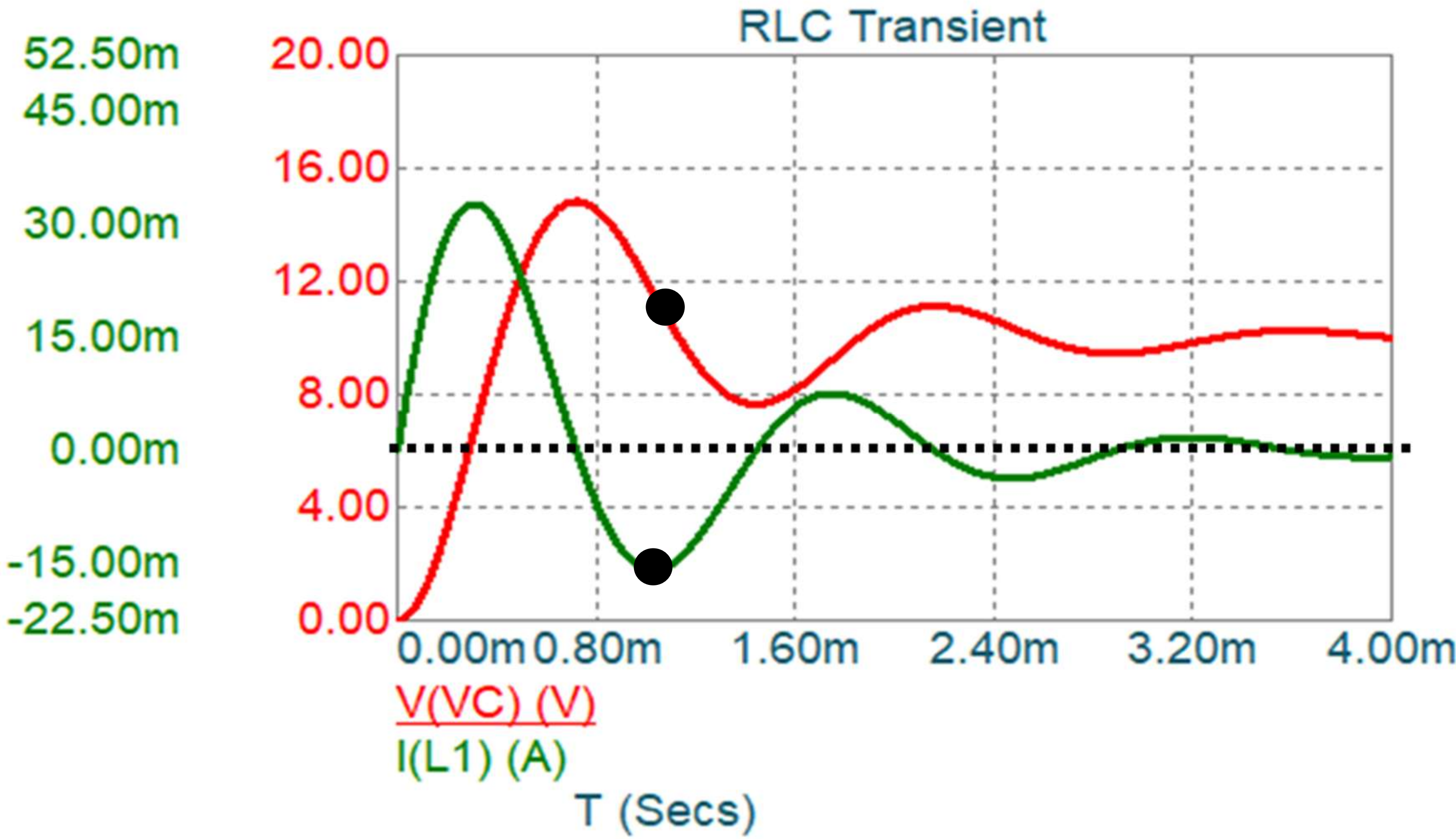


$$V_L = L\frac{dI}{dt} \qquad I = C\frac{dV}{dt}$$

$$V_S = V_R + V_L + V_C$$

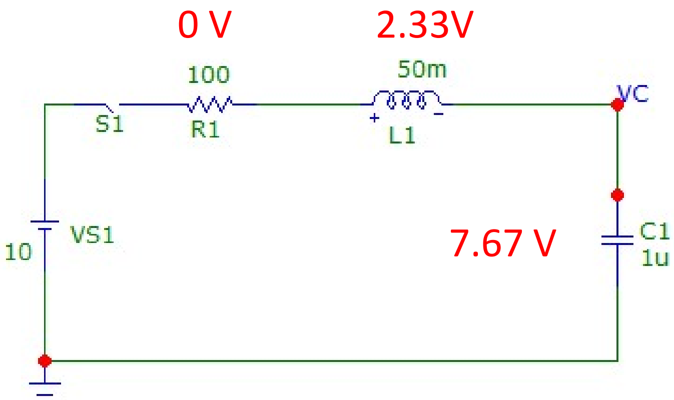


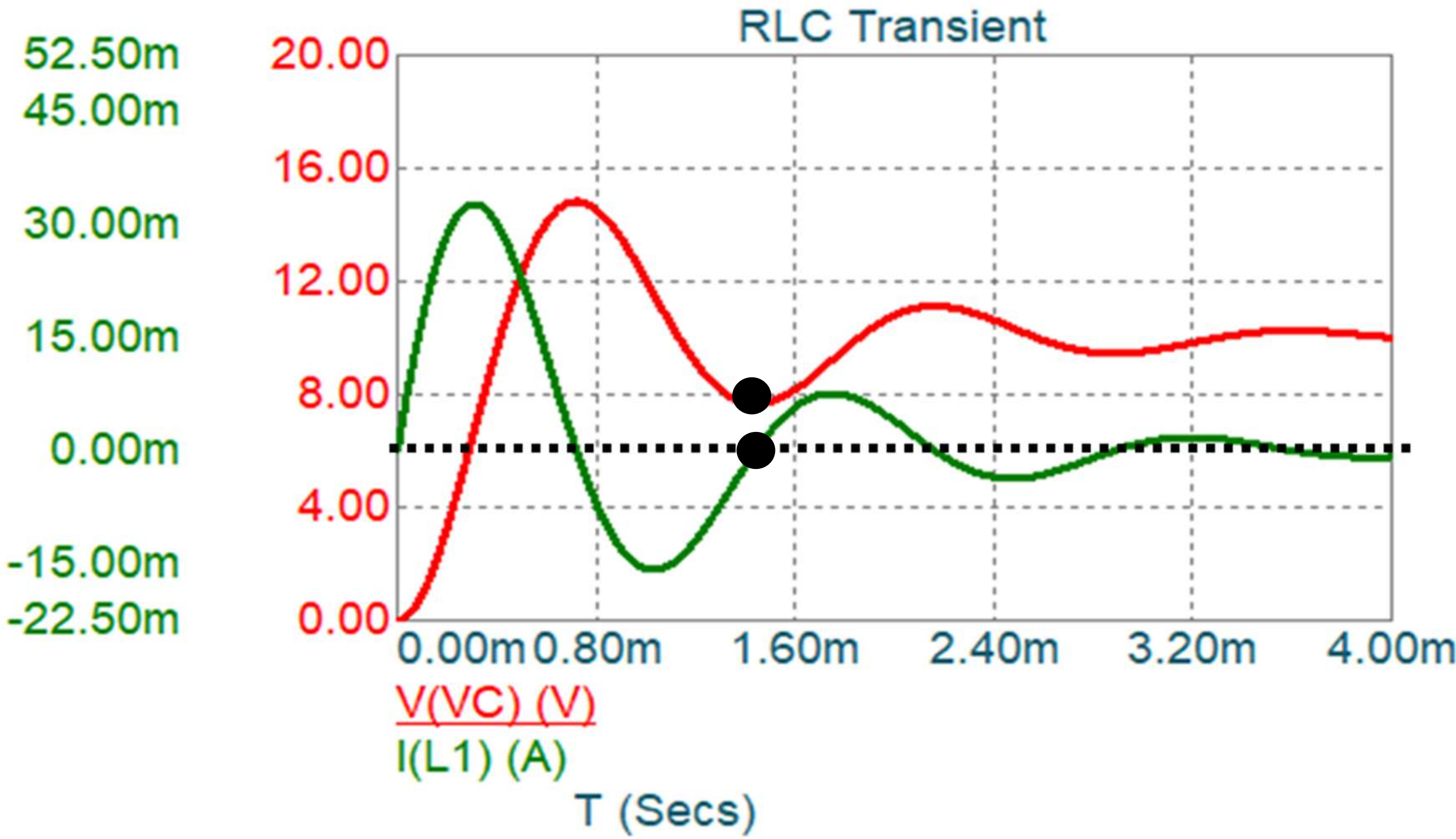




$$V_L = L\frac{dI}{dt} \qquad I = C\frac{dV}{dt}$$

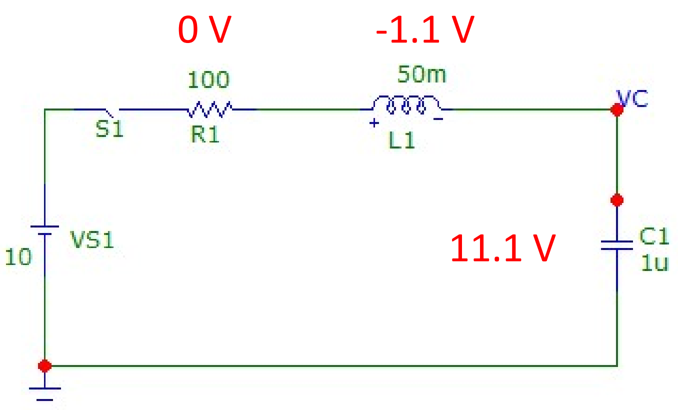
$$V_S = V_R + V_L + V_C$$

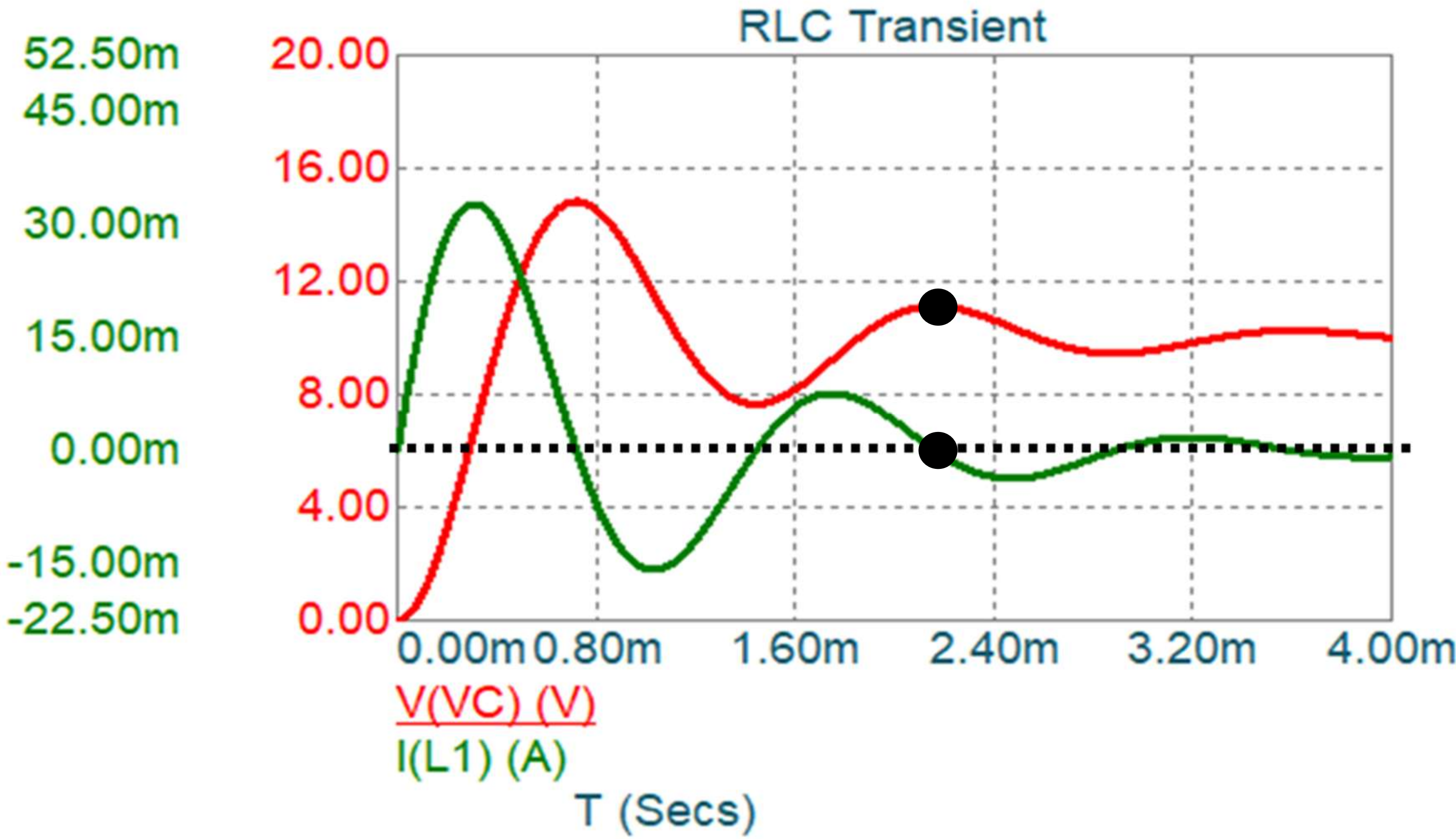




$$V_L = L\frac{dI}{dt} \qquad I = C\frac{dV}{dt}$$

$$V_S = V_R + V_L + V_C$$



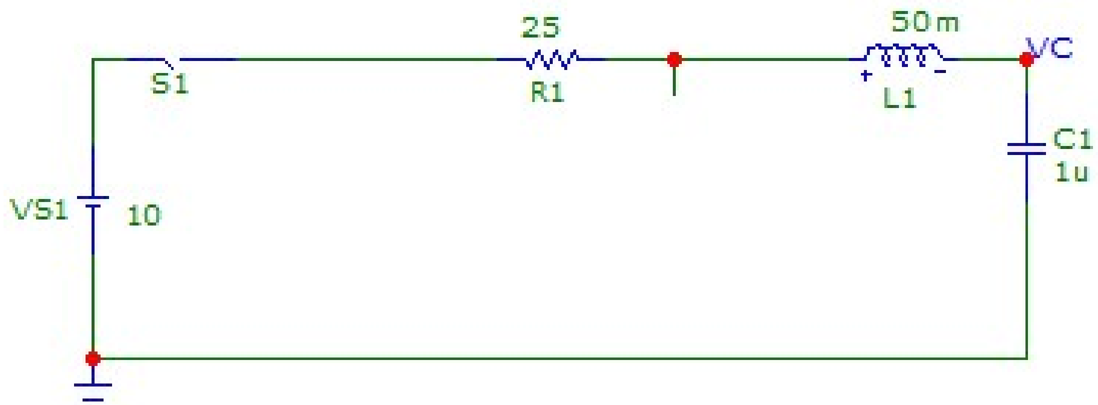


$$V_L = L\frac{dI}{dt} \qquad I = C\frac{dV}{dt}$$

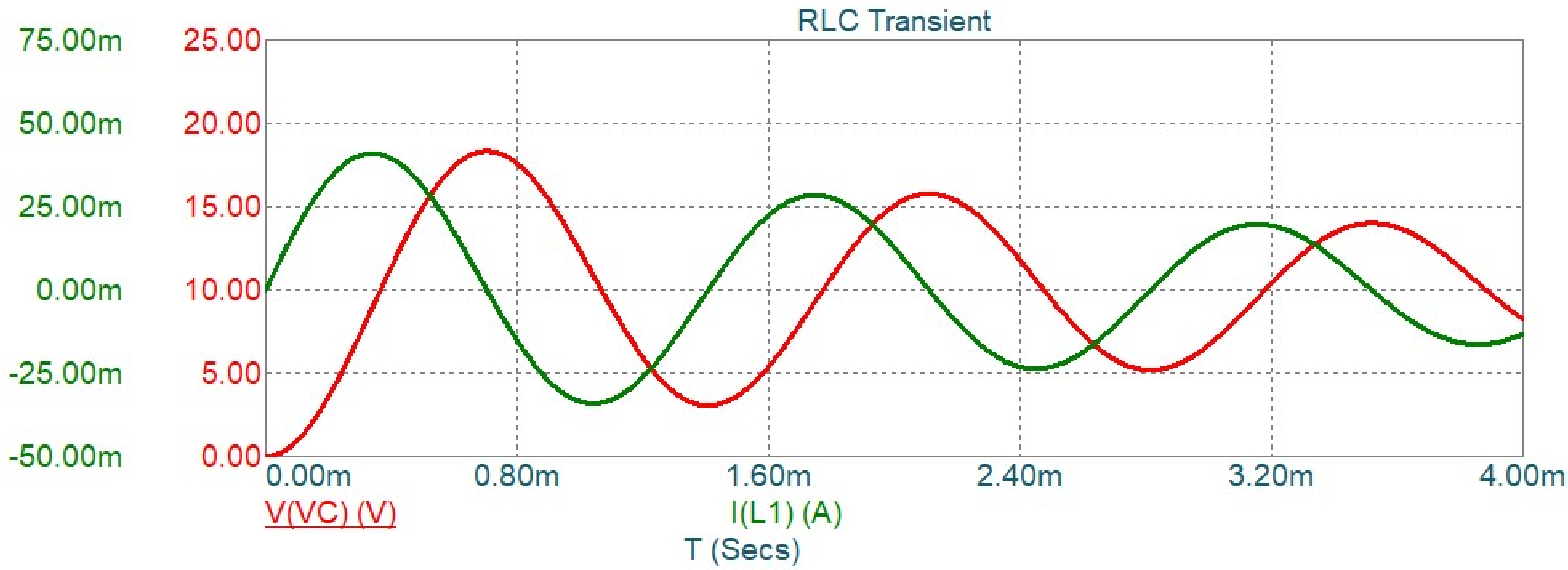
$$V_S = V_R + V_L + V_C$$

$$V_C(t) = 10 - e^{-10^3 t} \times (10 \times Cos(\omega_1 t) + 2.3 \times Sin(\omega_1 t))$$

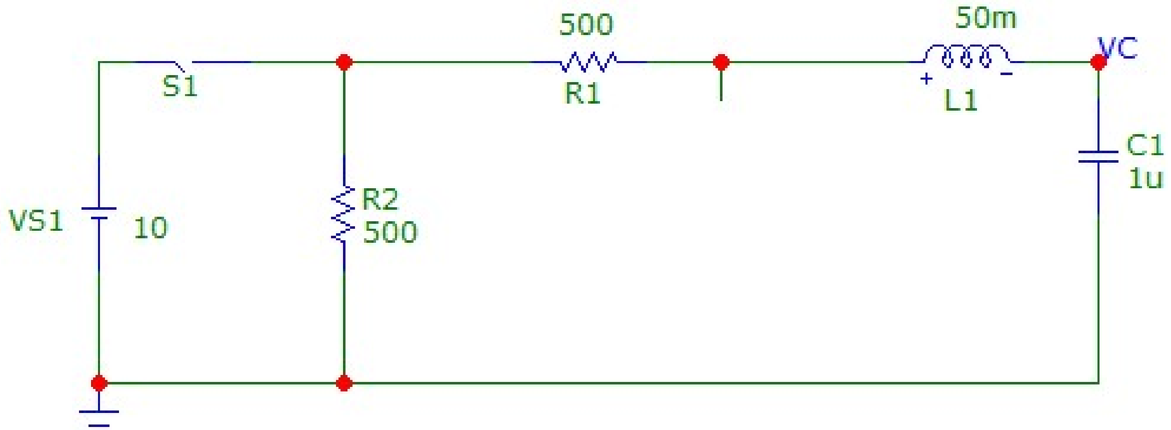
$$s = -\frac{\omega_o}{2Q} \pm j\omega_o\sqrt{1 - \frac{1}{4Q^2}} \qquad Q = \frac{\omega_O L}{R} \qquad \sim e^{-\frac{R}{2L}t}$$


25
R1
50m
L1
VC
S1
VS1
10
C1
1u

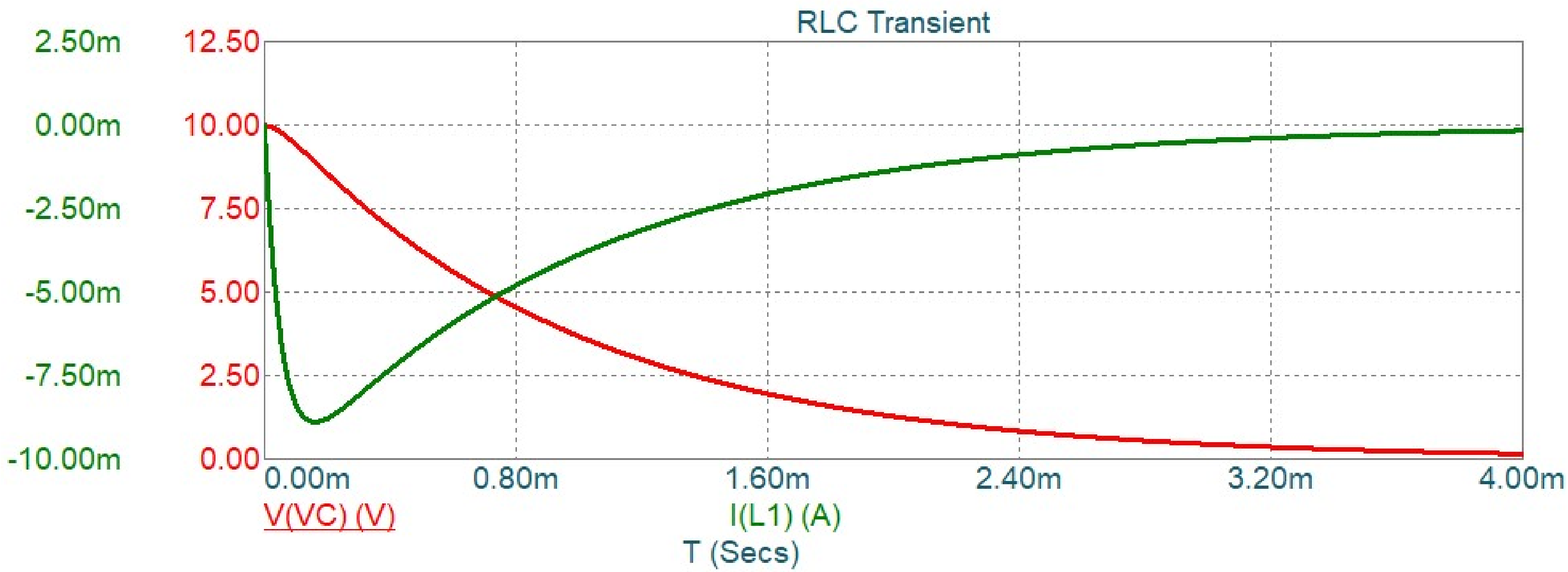
Q = 8.92

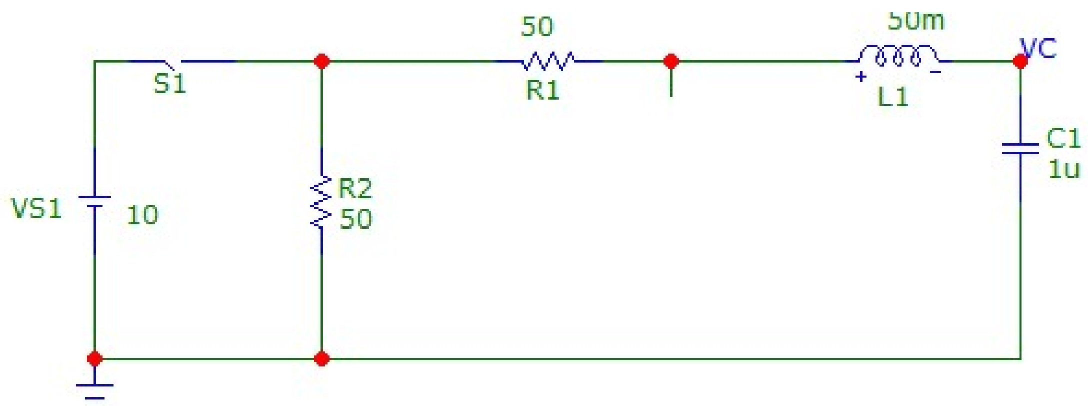

RLC Transient
75.00m
50.00m
25.00m
0.00m
-25.00m
-50.00m
25.00
20.00
15.00
10.00
5.00
0.00
0.00m
0.80m
1.60m
2.40m
3.20m
4.00m
V(VC) (V)
I(L1) (A)
T (Secs)

RLC Discharge

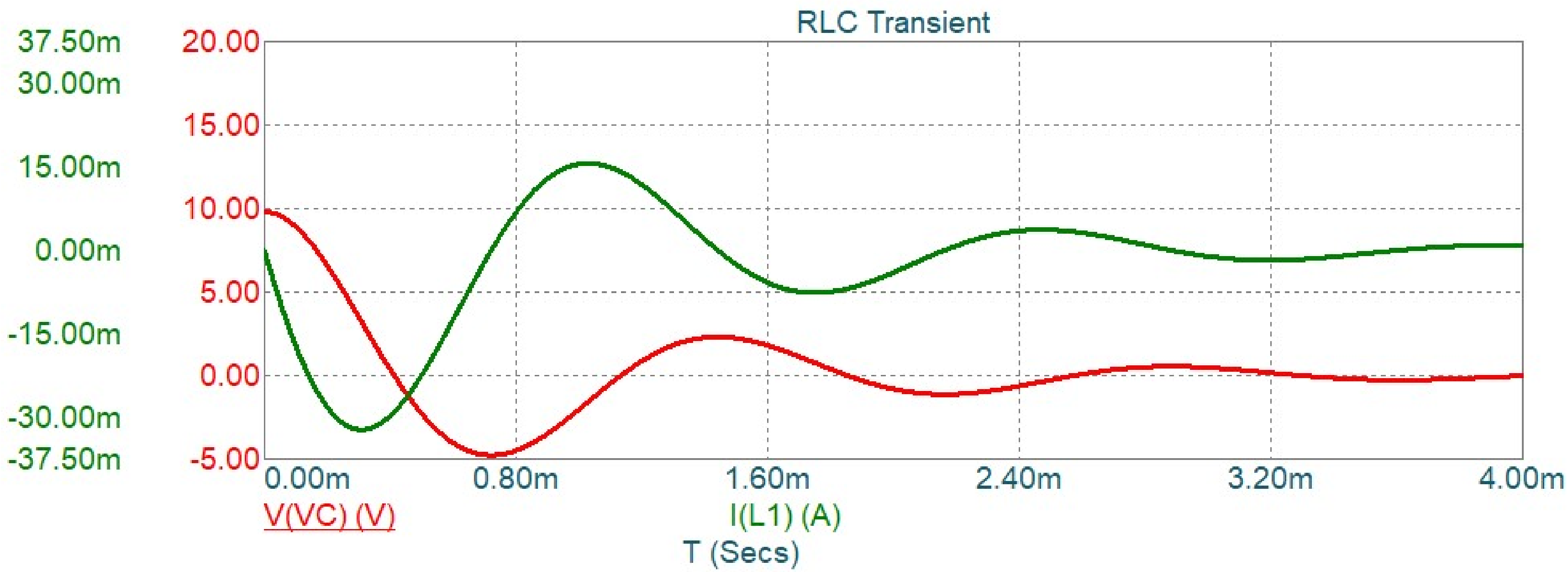


Over-damped Case

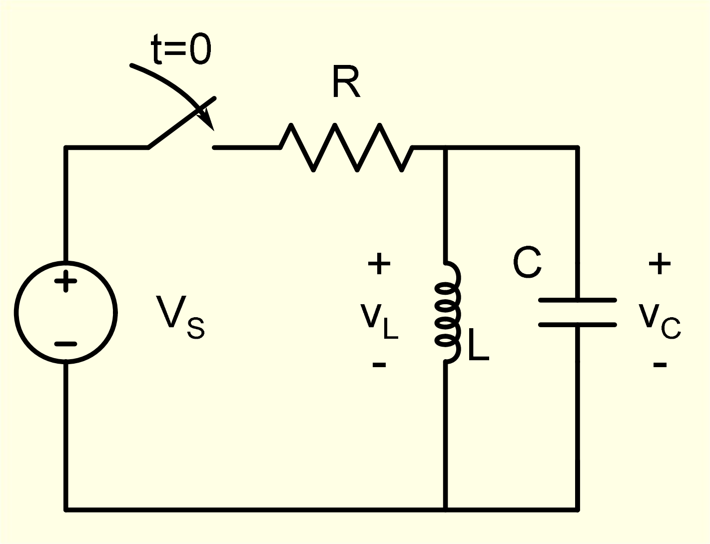




Under-damped Case



Parallel RLC circuit



$$V_S = I \times R + V_L \qquad I = C\frac{dV_C}{dt} + I_L \qquad V_L = L\frac{dI_L}{dt}$$

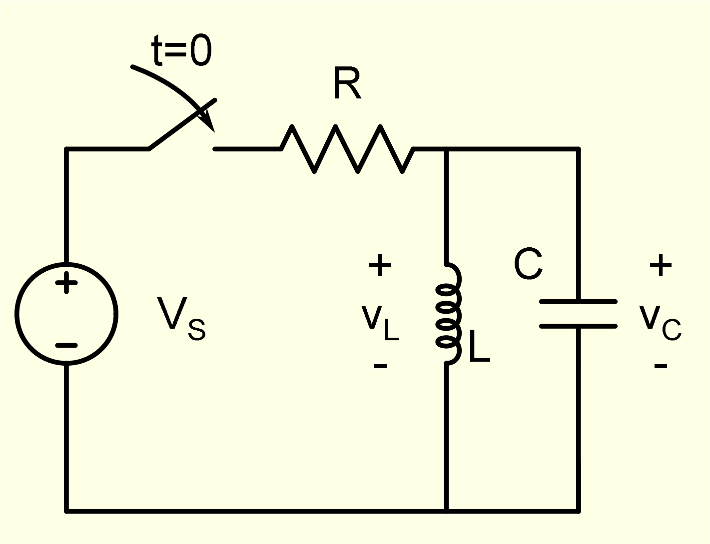
$$\frac{V_S}{R} = LC\frac{d^2 I_L}{dt^2} + \frac{L}{R} \times \frac{dV_L}{dt} + I_L$$

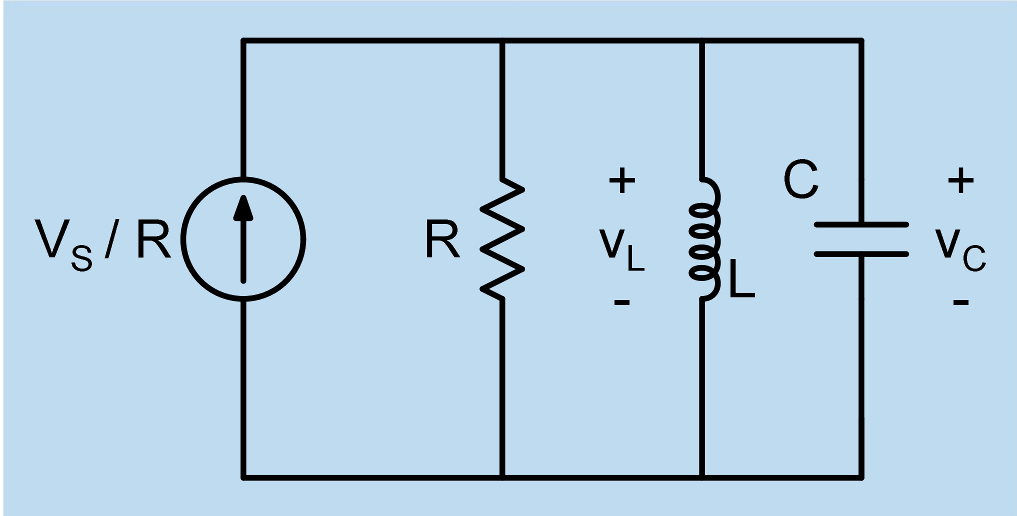
$$\frac{d^2 I_L}{dt^2} + \frac{1}{RC} \times \frac{dI_L}{dt} + \frac{I_L}{LC} = \frac{V_S}{RLC}$$

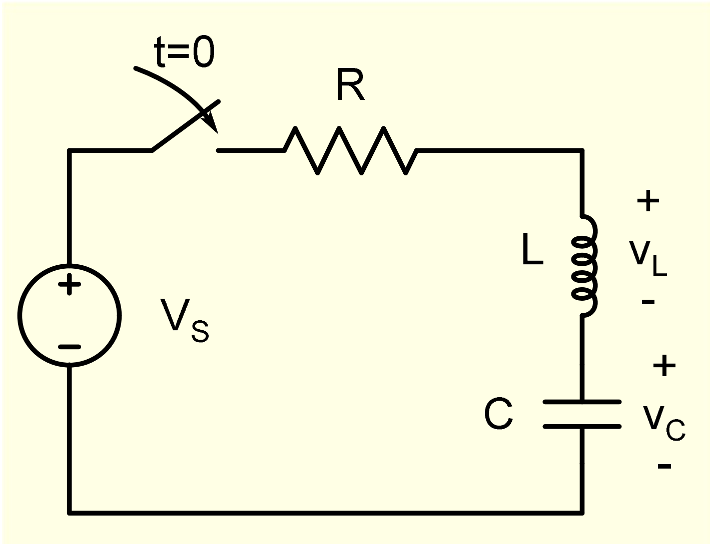
$$I_L(t) = A \times e^{st} \qquad As^2 e^{st} + \frac{1}{RC}Ase^{st} + \frac{A}{LC}e^{st} = 0$$

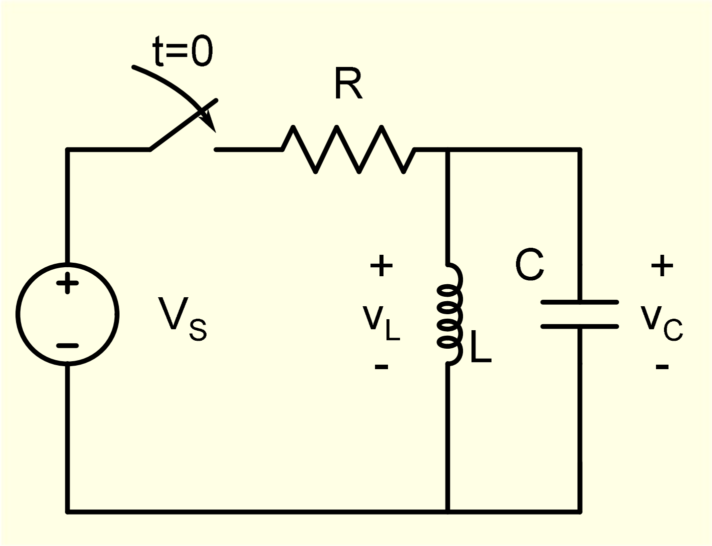
$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \qquad s = -\frac{1}{2RC} \pm \sqrt{\frac{1}{4R^2C^2} - \frac{1}{LC}}$$

$$\omega_O = \frac{1}{\sqrt{L \times C}} \qquad Q = \frac{R}{\omega_O \times L} \qquad \frac{s}{\omega_O} = -\frac{1}{2Q} \pm \sqrt{\frac{1}{4Q^2} - 1}$$


t=0
R
+
-
V_S
+
v_L
-
L
C
+
v_C
-


V_S / R
R
+
v_L
-
L
C
+
v_C
-


t=0
R
+
-
V_S
L
+
v_L
-
C
+
v_C
-

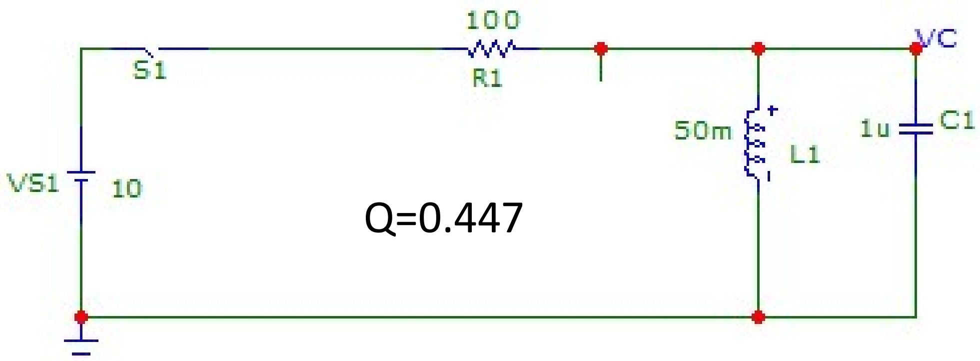


$$\frac{d^2 I_L}{dt^2} + \frac{1}{RC} \times \frac{dI_L}{dt} + \frac{I_L}{LC} = \frac{V_S}{RLC}$$

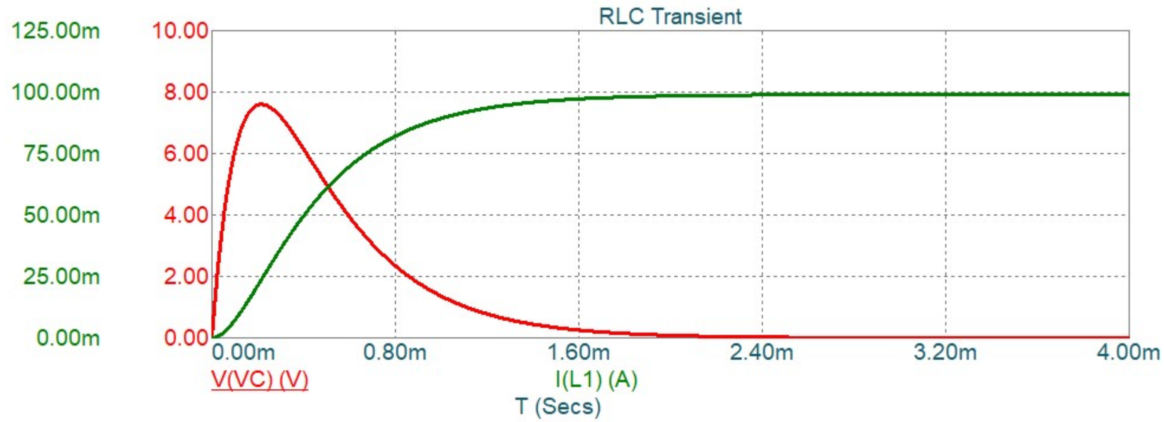
$$I_L(t) = \frac{V_S}{R} + A \times e^{s_1 t} + B \times e^{s_2 t}$$

$$\omega_O = \frac{1}{\sqrt{L \times C}} \qquad Q = \frac{R}{\omega_O \times L} \qquad \frac{s}{\omega_O} = -\frac{1}{2Q} \pm \sqrt{\frac{1}{4Q^2} - 1}$$

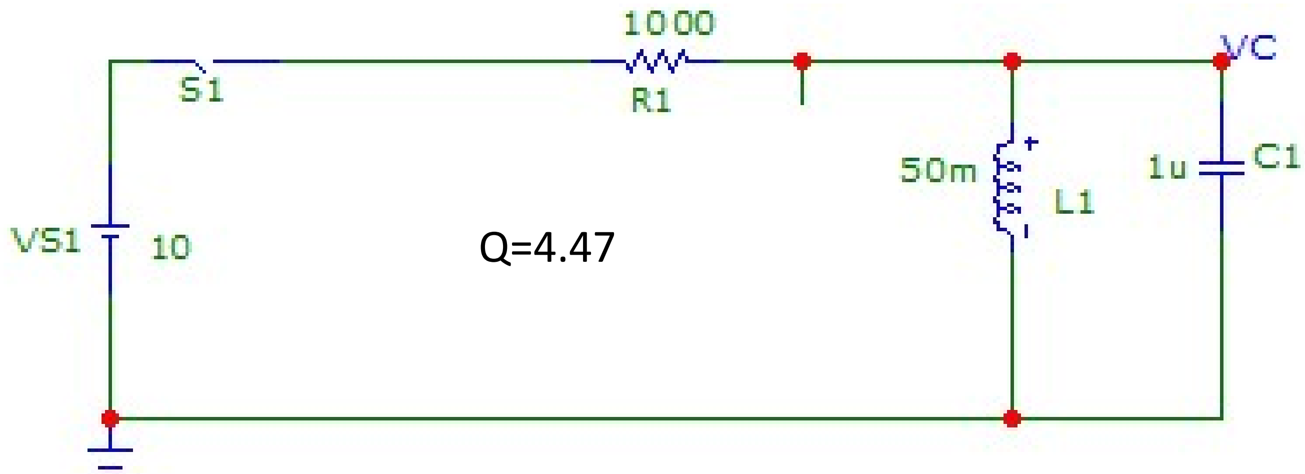
Case-1: Overdamped : Q< 0.5



Q=0.447

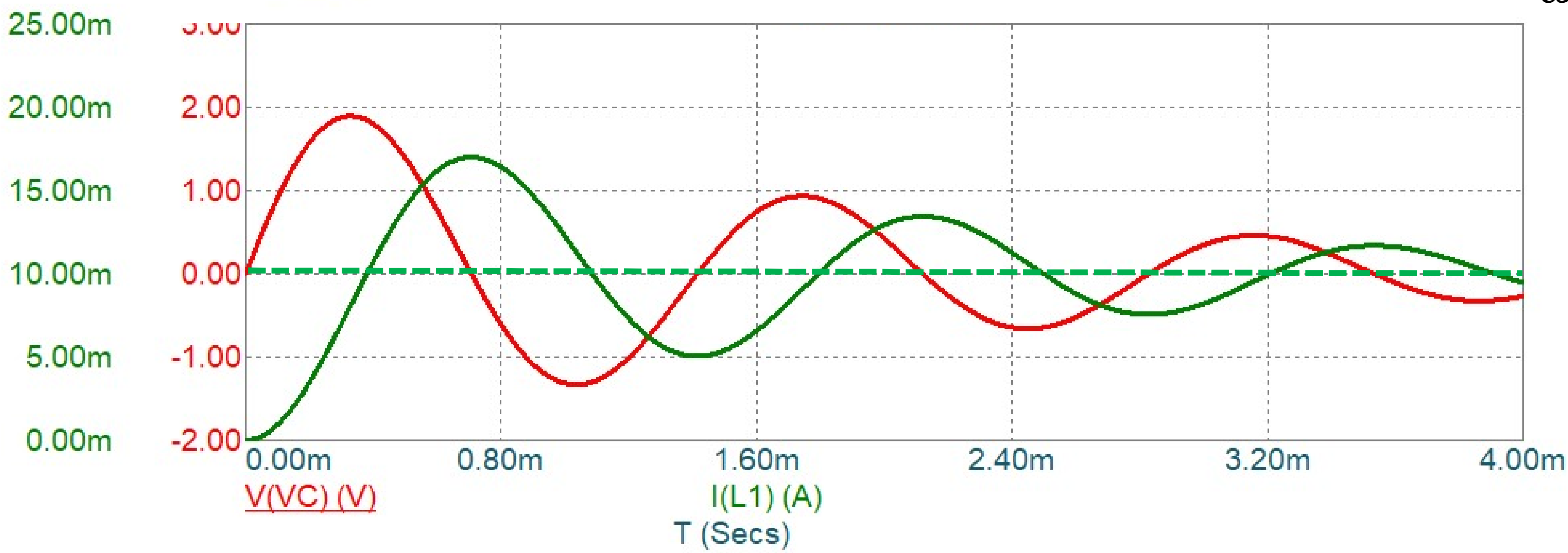


Underdamped Ca



Q=4.47

$$Q = \frac{R}{\omega_O \times L}$$



LC Oscillators exploit this feature