ASSIGNMENT RM1: Risk Management in Python

Group 5:
Catina Andrea, Crivellaro Federico,
Kharbanda Ansh

1 Question 1: Mark-to-Market (MtM) the Portfolio

1.1 Methodology

To compute the **Mark-to-Market** (MtM) of the portfolio, we first determine the **ATM swaption strike**, which is set equal to the forward swap rate. This is done by generating the underlying swap's fixed-leg schedule and computing the par swap rate using discount factors. Next, we price the swaption using the Black model, calculating its MtM based on the swaption notional. For the IRS component, we determine its fixed-leg payment dates, compute its par swap rate, and then calculate its MtM using standard IRS valuation. Since, the IRS is entered on the settlement date, its MtM is initially zero, making the total portfolio MtM equal to the swaption's value.

1.2 Results and Conclusion

- **Forward-Swap Rate**: 2.97979%
- **Swaption price:** €57,062,717.42
- **Swaption delta:** −0.3561
- **IRS rate:** 2.85050%
- **IRS MtM:** €−0.00
- **Portfolio MtM:** €57,062,717.42

An Interest Rate Swap (IRS) and an Interbank (IB) fixed coupon bond, with a coupon set equal to the swap rate $S(t_0; t_0, t_N)$ (the par swap rate at value date), share structural similarities. However, an IRS has no initial cost (NPV = 0 at value date), while an IB bond has a present value based on its discount factors. The bondholder also bears credit risk, whereas an IRS is collateralized through margin agreements.

2 Question 2: DV01-Parallel

2.1 Methodology

To evaluate the portfolio's $DV01$ (parallel), we shock the market rates by increasing each rate by $1bp$ (0.01%). The new rates are bootstrapped to obtain updated discount factors, and the forward swap rate is recalculated. The swaption price and IRS MtM are then recalculated using the updated discount factors. The $DV01$ of the swaption and IRS is computed by comparing their prices (or MtM) before and after the shock. Finally, the portfolio's $DV01$ is determined as the difference between the shocked and unshocked portfolio MtM, indicating its sensitivity to parallel shifts in interest rates.

2.2 Results and Conclusion

- **Swaption $DV01$-parallel:** €−97,265.47
- **IRS $DV01$-parallel:** €514,969.19
- **Portfolio $DV01$-parallel:** €417,703.71

3 Question 3: DV01 Approximation

3.1 Methodology

To estimate the portfolio $DV01$ analytically, we calculate the IRS duration and the swaption $DV01$ using its delta, both multiplied by their respective notional and a $1bp$ shift. The portfolio's proxy $DV01$ is the sum of the swaption and IRS $DV01$ approximations. The difference from the previous method is that this approach provides a quicker estimation, while the shocked rates method captures the full nonlinear impact of the rate shift, offering a more precise result.

3.2 Results and Conclusion

- **Approximated Swaption $DV01$:** €$-24,924.50$
- **Approximated IRS DV01:** €$530,630.66$
- **Approximated Portfolio proxy DV01:** €$505,706.16$

A discrepancy is observed between the portfolio's DV01 computed in Question 2 and the approximated DV01. This difference arises because the **formula** used to compute the swaption Δ is actually **incorrect**. The error occurs due to the fact that when swap rates change, the discount factors also change, leading to potential mispricing in volatile markets, which was not accounted for in the initial formula:

$$\Delta(t_0) = \frac{\partial SP_{\alpha,\omega}(t_0)}{\partial S_{\alpha,\omega}(t_0)} = B(t_0, t_\alpha) \cdot BPV_{\alpha,\omega}(t_0) \cdot N(d_1) \tag{1}$$

3.2.1 Why do we need also this $DV01^z$?

DV01-parallel differs from $DV01^z$. The zero-rate curve version isolates the impact of rate changes without being affected by the interpolation method used in bootstrapping. This helps in better sensitivity analysis.

3.3 Why $DV01^z = -DU$ for an IB Par Fixed Coupon Bond ?

For an IB bond valued at par, duration (DU) measures the weighted average time of cash flows. Since, $DV01^z$ measures the bond's price change for a $1bp$ shift, it is numerically equal to $-DU$ due to the inverse relationship between price and yield:

$$DV01(z) = -DU \times \frac{\text{Notional}}{10,000} \tag{2}$$

4 Question 4: Delta Hedging with a 10Y IRS

4.1 Methodology

To hedge the portfolio's interest rate sensitivity, we compute the required **10-year IRS notional** such that the portfolio DV01 is neutralized. The DV01 (Dollar Value of 1 Basis Point) measures how the portfolio value changes with a 1 bp move in rates.

- **Exact Hedge Calculation:**
  1. Compute the change in IRS mark-to-market (MtM) for a 1 bp rate shift.
  2. Determine the required IRS notional using:

  $$\text{IRS Notional} = -\frac{\text{Portfolio DV01}}{\Delta\text{IRS MtM}}$$

  3. Round down to the nearest €1Mln (since IRS contracts are traded in €1Mln lots)
- **Approximated Hedge Using Duration:**
  1. Using **approximate DV01** formula where the IRS DV01 is estimated using duration:

  $$\text{IRS Notional} \approx -\frac{\text{Portfolio Proxy DV01}}{\text{IRS Duration} \times 10^{-4}}$$

  2. Again, round to the nearest €1Mln
- **Verification:**

1. Compute the new hedged portfolio DV01 to confirm the hedge effectiveness.
2. Comparing the **exact** and **approximate** approaches to evaluate hedge precision.

By applying this method, we derive the **optimal IRS notional** to offset the swaption's DV01 exposure, ensuring a neutral position.

4.2 Results:

- With €$-487,000,000$ swap notional the DV01 is: €-280
  - **Net IRS Notional:** €$113,000,000$
- With €$-572,000,000$ swap notional the $DV01$ is: €$-73,234$
  - **Net IRS Notional:** €$28,000,000$

5 Question 5: Coarse-grained bucket DV01

5.1 Methodology

To evaluate the portfolio coarse-grained bucket $DV01$, we first shift the market rates according to the 10y and 15y bucket maturities. For each bucket, we perform a bootstrap on the adjusted rates, compute the swaption and IRS prices, and calculate their respective $DV01$s. After determining the portfolio $DV01$ for each bucket, we sum them up to obtain the total portfolio $DV01$. This method provides a simplified view of the portfolio's sensitivity to interest rate changes, focusing on larger maturity bands for the shock analysis.

5.2 Results & Comparison:

- **Portfolio $DV01$-parallel bucket 1(10y):** €$559,114.34$
- **Portfolio $DV01$-parallel bucket 2(15y):** €$-140,977.46$
- **Portfolio Total $DV01$-parallel:** €$418,136.88$

6 Question 6: Delta-hedging with two liquid IRS

6.1 Methodology

First, the portfolio is delta-hedged using two IRS contracts: a 10-year IRS for the IRS exposure and a 15-year IRS for the swaption exposure. The 10-year IRS is used to hedge the IRS exposure, while the 15-year IRS matches the swaption's sensitivity to rate changes. The notional amounts for each IRS are calculated based on the respective $DV01$s, and the portfolio's bucket $DV01$ is recalculated with the hedging positions for both 10y and 15y IRS contracts. The results show the hedging effect on the portfolio's total $DV01$ across the different rate buckets.
Key Steps used:

1. Compute notional for 10y and 15y IRS.
2. Recalculate portfolio $DV01$ with hedged positions.

6.2 Result & Question:

- **Notional IRS 10y for hedging the IRS:** €$-600,000,000.00$
- **Notional IRS 15y for hedging the Swaption:** €$81,000,000.00$
- **Portfolio $DV01$-parallel bucket 1(10y):** €$-18,173.83$
- **Portfolio $DV01$-parallel bucket 2(15y):** €$43,433.43$
- **Portfolio Total $DV01$-parallel:** €$25,259.60$

6.2.1 Ineffectiveness of a DV01-Parallel Hedge

A parallel shift in rates does not capture realistic market movements. If hedging a swaption only with a 5Y IRS:

1. A 1bp parallel shift affects both the swaption and the IRS but does not account for curve twists.
2. A steepening movement (15Y rate up 1bp, 5Y rate down proportionally) causes hedging errors, as the IRS does not offset the swaption's true sensitivity across maturities.

7 Question 7: Curve steepening scenario

7.1 Part (a) (P&L hedging with a 10y IRS)

result for part a: €-694660.77

7.2 Part (b) (P&L hedging with two IRS)

result for part b: €-81841.63

7.3 Comparing Hedging Strategies: IRS vs. Bucket-DV01s

1. **P&L with Only a 10-Year IRS:**
   - A 1bp decrease in the 10y IRS rate reduces the IRS value.
   - The P&L is calculated by multiplying the change in IRS value by the notional.
2. **P&L with Two IRS (10y and 15y):**
   - Curve steepening (1bp decrease in 10y, 1bp increase in 15y) affects both IRS positions.
   - The P&L is the combined change in value of the 10y IRS and 15y IRS, reflecting the portfolio's total exposure.
3. **Best Hedging Strategy:**
   - The two-IRS strategy (10y and 15y) is generally more effective because it addresses both IRS and swaption risks.
   - This strategy offers better overall hedge and less volatility compared to using only the 10y IRS.
4. **More Accurate Hedging with Bucket-DV01s:**
   - Bucket-DV01s break the curve into smaller segments (e.g., 2y, 5y, 10y, 15y) for more precise hedging.
   - This improves the hedge's accuracy by targeting specific curve risks.
5. **Pros and Cons of Bucket-DV01s:**
   - **Pros:** More precise hedging, better risk alignment.
   - **Cons:** Increased complexity and cost.
6. **Comparison of Hedging Strategies:**
   - The two-IRS strategy provides a more balanced hedge than using only the 10y IRS.
   - Bucket-DV01s can refine this hedge further, though it can increase time complexity.

7.4 Question: Hedging with a 5Y and 15Y IRS (Coarse-Grained Sensitivity Analysis)

Using two swaps (5Y and 15Y), we solve:

$$w_1 \cdot DV01(\text{IRS}_1) + w_2 \cdot DV01(\text{IRS}_2) = -\text{Portfolio DV01} \tag{3}$$

where w_1 and w_2 are the weights assigned to each IRS. A steepening scenario (10Y $-1bp$, 15Y $+1bp$) is tested, and the hedge effectiveness is verified using recalculated NPVs. This method better captures non-parallel shifts compared to a single IRS hedge.

8 Main Errors Encountered

During the implementation, the following key errors were identified and corrected:

1. **Error in the `swap_par_rate` function:** The return value was incorrectly divided by `discount_factor_t0`
2. **Error in the `swap_mtm` function:** The multiplier was incorrectly applied in the expression:

   $$\text{multiplier} \times (\text{float\_leg} - \text{fixed\_leg})$$

   leading to incorrect mark-to-market (MTM) calculations. **Correction:** The terms should be inverted to:

   $$\text{multiplier} \times (\text{fixed\_leg} - \text{float\_leg})$$

   to properly reflect the valuation logic.
3. **Error in the `swap_price_calculator` function:** The function incorrectly passed `fwd_swap` as input. **Correction:** `fwd_swap_up` should be used to correctly compute the impact of a rate shift.

These corrections ensure that the swap pricing and delta-hedging calculations are accurately performed.