ASSIGNMENT 3: Credit Pricing, Bootstraping & Simulation in Matlab

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1 Point (1): Pricing An Asset Swap

1.1 Methodology

To compute the Asset Swap Spread over Euribor 3M for a bond issued by issuer YY, with a maturity date of 31st March 2028, we followed these steps:

1. **Accrual Price**: We calculated the year fraction from the issue date (31st March 2022) to the settlement date (31st January 2023) and computed the accrual by multiplying it for the coupon. The dirty price was then obtained by adding the accrual to the clean price.

2. **Fixed Leg Present Value**: Using the discounting curve, we discounted the bond's annual coupon payments, with coupon dates determined by the backward date convention.

3. **Floating Leg Present Value**: Similarly, the floating leg, based on Euribor 3M, was discounted using the same curve over the relevant quarterly periods.

4. **Asset Swap Spread ($\mathbf{s^{asw}}$)**: Finally, the Asset Swap Spread was calculated by determining the difference between the present values of the fixed and floating legs, divided by the BPV (Basis Point Value) for the floating leg.

1.2 Result & Conclusion

By following the above methodology, the Asset Swap Spread is $\mathrm{s}^{\mathrm{asw}} = 0.017565952315215$. This result was obtained ignoring the accrual term for the computation of the dirty price since it should cancel out with the calculations of the interbank bond.

2 Point (2): CDS(Credit Default Swap) Bootstrapping

2.1 Methodology

- **Part a (No Accrual):** To construct the piecewise constant intensity $\lambda(t)$ for ENI, neglecting the accrual term, we first computed the survival probability for (t_0, t_1) using the given CDS spreads and a 40% recovery rate. Then, using recursion, we derived survival probabilities for later periods (t_{i-1}, t_i) and inverted them to obtain the corresponding intensity.

- **Part b (With Accrual):** To construct the piecewise constant intensity $\lambda(t)$ for ENI, **with** the accrual term, the same procedure was followed as **part a**, but considering in our formulas another term considering the accrual.

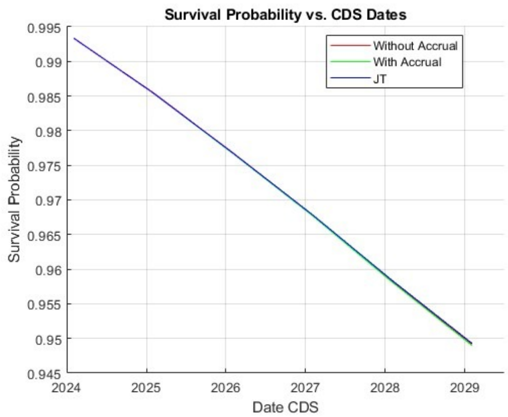
- **Part c (Jarrow-Turnbull):** To construct the constant intensity $\lambda(t)$ for ENI, using the **J.T** approximation method, using the given formula, since the CDS spreads for all the years and the recovery value are given.

Results Bootstrap CDS

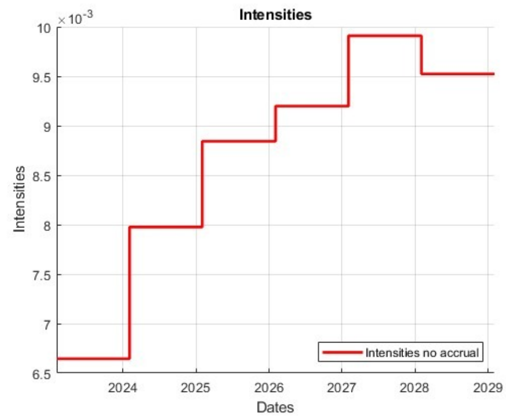
bootstrapCDS_approx	
survProbs	intensities
0.99337748	0.00664454
0.98550772	0.00797562
0.97678478	0.00884216
0.96784124	0.00919828
0.95829776	0.00990952
0.94918991	0.00952355

bootstrapCDS_exact	
survProbs	intensities
0.99335548	0.00666669
0.98545503	0.00800702
0.97669456	0.00888085
0.96771137	0.00924010
0.95812299	0.00995771
0.94897369	0.00956885

bootstrapCDS_JT	
survProbs	intensities
0.99335550	0.00666666
0.98546016	0.00733333
0.97675301	0.00783333
0.96783947	0.00816666
0.95836814	0.00850000
0.94928378	0.00866666



Survival Probabilities



Estimated Intensities

2.2 Conclusion

From the comparison of survival probabilities with and without the accrual, it is evident that the two probabilities are very close to each other for every year. This suggests that the impact of the accrual term on the overall survival probability is negligible, and the results remain almost identical whether or not the accrual term is included.

3 Point (3): Credit Simulation

3.1 Methodology

We simulate the default time τ using $M = 10^5$ uniform random variables, where each value represents a probability from the cumulative distribution function. The simulation follows a piecewise constant hazard rate model with given parameters. The empirical survival probability is then estimated by computing the proportion of simulations where default has not occurred by each time step. This is compared to the theoretical survival probability for validation. Next, we fit a non-linear least squares model to estimate λ_1 and λ_2, obtaining a 95% confidence interval for both parameters. Finally, we visualize the results by plotting the simulated and theoretical survival probabilities on a linear scale to assess the fit. This ensures estimation of default values and validates the theoretical survival probability model.

3.2 Results

Estimated Parameters:

- $\hat{\lambda}_1 = 0.0005044$
- $\hat{\lambda}_2 = 0.00090352$

Confidence Intervals:

- CI for $\lambda_1 = [0.00048005, 0.00052876]$
- CI for $\lambda_2 = [0.00089694, 0.00091009]$

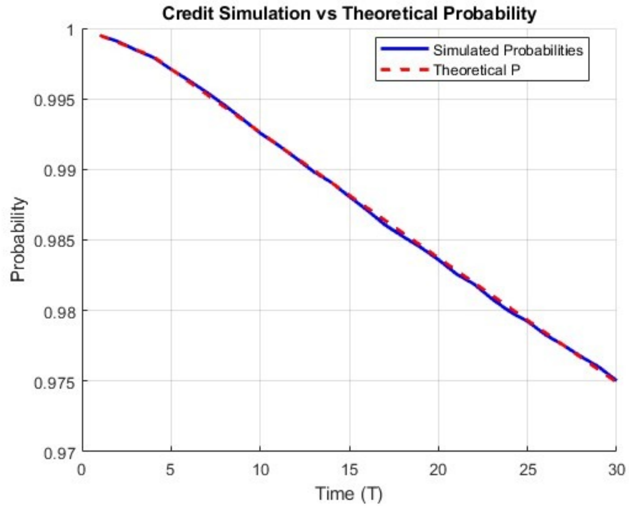


Figure 1: Survival Probability Comparison

4 Point (4): Asset-Backed Security Pricing (MBS)

4.1 Part (a) (Mezzanine Tranche)

We employ the Vasicek model to price the mezzanine tranche while varying the number of obligors in the reference portfolio of mortgages held by a bank.
To price the mezzanine tranche, we start with the **Homogeneous Portfolio (HP) method**, which calculates tranche losses by summing over possible defaults. This approach is accurate but, becomes impractical when the portfolio size **I** is large. To handle this, we use the **Large Homogeneous Portfolio (LHP)** approximation, which simplifies the calculation by treating defaults as continuous variable. However, for moderate values of **I**, we cannot use the sterling approximation anymore for the binomial coefficient. To solve this issue, we apply the **Large Deviation (LD)** approach, which refines the probability of defaults using a correction based on **Kullback-Leibler (KL) entropy**. This method provides a better estimate of tranche losses by capturing deviations from the **LHP** assumption. This approach ensures reliable pricing across different portfolio sizes.

4.2 Result(part a)

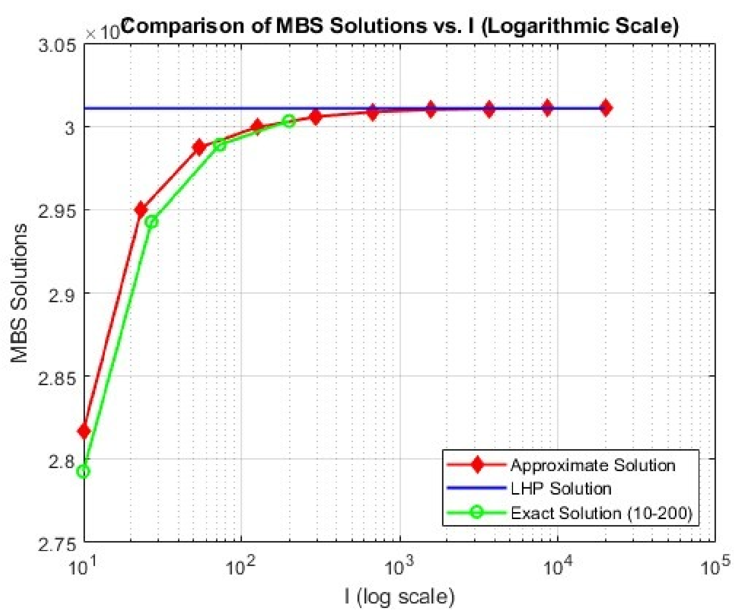


Figure 2: Mezzanine Tranche Prices

4.3 Part (b) (Equity Tranche)

We extend the pricing analysis to the **equity tranche** using the same methods. However, the KL approximation does not perform well in this case due to its high sensitivity to small probabilities and its inability to properly account for frequent, high-impact losses. A possible alternative approach could be to determine the equity tranche price as the difference between the **expected total portfolio loss** and a **mezzanine tranche price**.

4.4 Result(part b)

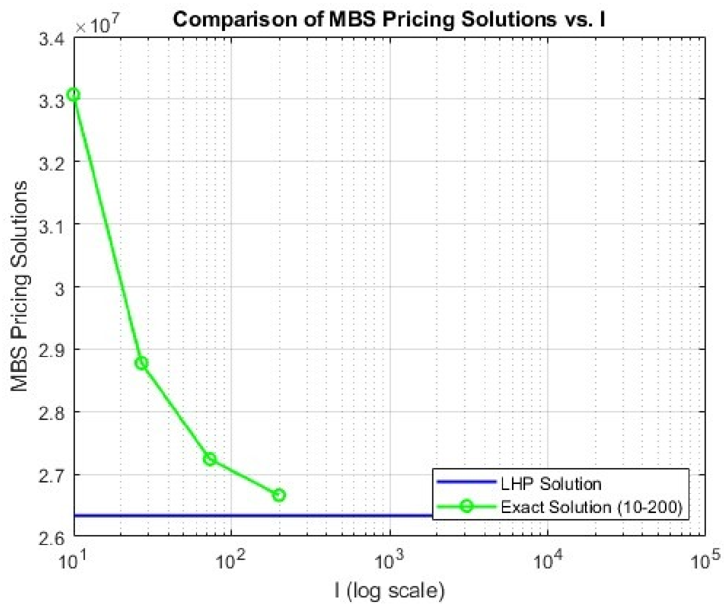


Figure 3: Equity Tranche Prices