ASSIGNMENT 2: Discount Factor Bootstraping in Matlab

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1 Point (1): Bootstrap Of Discount Factor

1.1 Methodology:

To create the bootstrap curve we first need a complete set of dates; to do this we had to complete the dates of the swaps to get one for each year, being careful that the ones we add are actually business dates; so we used a modified following convention. To get the corresponding rates we used a spline interpolation on the mid rates that we already had. For our curve we used the first four deposits, then the first seven futures, which are the most liquid contracts, and finally the swaps from the 2y one to the 50y one. After getting the discounts for the deposits we interpolated between the third and the fourth one to get the discount at the settlement of the first futures; for all the futures we got the discount at expiry as the product between the discount at settlement and the forward discount between settlement and expiry and to get the settlement ones we interpolated (or extrapolated depending on the situation) on the previous discounts. We then interpolated on the futures to get the discount for the first swap and from that we got all the remaining discounts. Finally combining all the discounts of the different instruments we got our bootstrap curve.

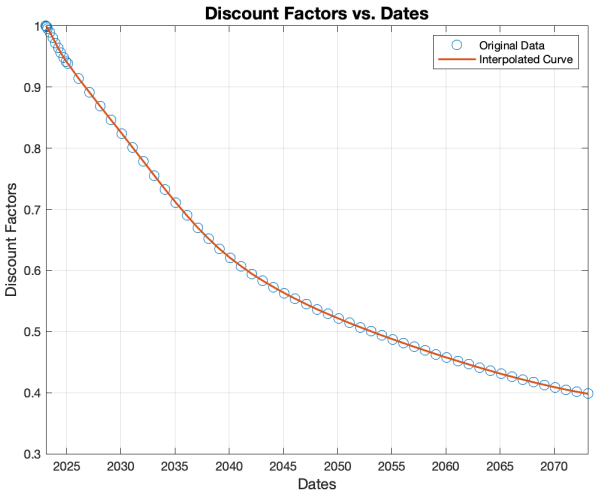

Figure 1: Bootstraped Discount Factors

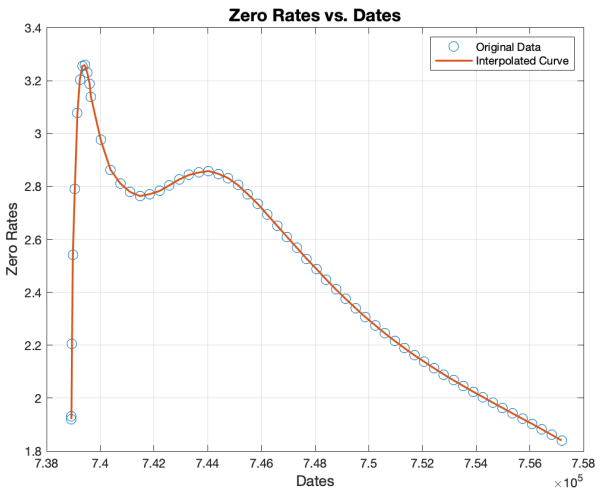

Figure 2: Zero Rate Curve

Question: Bootstrap is not the only technique to obtain Discount Factors from quoted rates. Why is it so relevant the bootstrap of DFs in finance?

Answer: Bootstrapping discount factors is relevant in finance as it ensures the construction of a smooth, arbitrage-free, and market-consistent yield curve. And, by sequentially deriving discount factors from observable market rates such as deposits, FRAs and swaps, it facilitates accurate pricing, hedging and risk management of fixed-income instruments. The resulting swap curve serves as a reliable benchmark, particularly as the supply of government bonds declines. This methodology ensures precise mark-to-market valuation and improves interest rate risk management, making it a fundamental tool in financial markets.

2 Point (2): Pricing An Inter-Bank floater coupon bond

2.1 Methodology

Using the discount factors obtained from point (1), we can compute the price of the I.B. floater at expiry of 7 years with the fixed mid-rate for the 7-year swap, with the coupon payments as the given swap expiries till the year 2030 using the given formula, yielding a value close to the face value of €100 million.

2.2 Price

- Notional Value: €100 million
- Coupon Price: €100012894.62

2.2.1 Remark:

The coupon price should generally be equal to the notional value. However, the slight difference observed in our result may be due to day-count conventions or small valuation adjustments.

3 Point (3): Sensitivities For A 7y plain vanilla IR swap

3.1 Results

- Notional Value: €100 million
    1. DV01-parallel shift: 0.000625100968976194
    2. DV01$^{(z)}$: 0.000646127855658429
    3. BPV of the 7y IRS: 0.00062570260449797
    4. Macaulay Duration of the "I.B. coupon bond": 6.45758168469037

The results from the 7-year plain vanilla IRS with a fixed rate of 2.8175% vs. Euribor 3-month reflect a moderate sensitivity to interest rate movements.

3.2 Analysis

The respective DV01(parallel-shift) and DV01$^{(z)}$indicate that the swap price is relatively insensitive to small shifts in interest rates, with the slight difference between the two values suggesting a marginally higher sensitivity under specific curve movements. The BPV implies that for a $1bp$ change in interest rates, the swap's value changes by approximately €625. This value confirms that the swap has a limited interest rate exposure. The Macaulay duration of 6.46 years indicates a moderate level of interest rate sensitivity. A duration greater than 6 years means the swap is somewhat sensitive to rate changes, with cash flows weighted towards the latter part of the swap's life.

3.2.1 Remark:

The DV01$^{(z)}$ should be approximately equal to the Macaulay Duration, differing only by a factor of 10^n, and also, similar to the given BPpar IRS. The observed values confirm this relationship, reinforcing the consistency of our calculations.

3.3 Conclusion

The values indicate that the IRS is relatively stable with respect to small interest rate movements, with moderate exposure to longer-term interest rate risks due to the 7-year maturity and the cash flow structure.

4 Point (4): NPV Of Monthly Cashflows

4.1 Methodology

To get the net present value we first computed all the cash flows, taking into account also the AAGR. In order to actualize them, we multiplied each flow with the corresponding discount factor, which we got by interpolation using the datas that we obtained from the bootstrap curve of point 1. Summing all the discounted flows we got the NPV.

4.2 Results

- NPV with initial flow of €$1.5K$ = €397148.09
- NPV with initial flow of €$6K$ = €1588592.35