ASSIGNMENT 1: Option pricing in Matlab

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1 Point (a): European Call Option Pricing

Three different methods have been used to price a European call option: the `blkprice` Matlab function, the Cox-Ross-Rubinstein (CRR) tree method, and the Monte Carlo (MC) approach.

- **`blkprice` function**: `blkprice` uses a closed-form formula to compute the price, which results in **0.02887443**.
- **CRR tree method**: The result depends on the number of iterations. With M = 100 iterations, the call option price is **0.02880454**.
- **Monte Carlo approach**: Since the result is both iteration-dependent and stochastic, the price is not unique with M = 100 iterations. However, by fixing the seed, we obtain a price of **0.04331091**.

As we can observe, with 100 iterations, the CRR method is already sufficiently close to the actual value i.e. the `blkprice` result (we will analyze this in more detail later), whereas the Monte Carlo method requires a larger number of iterations to converge. Additionally, we noticed that for large M, the CRR method slows down significantly more compared to the Monte Carlo approach.

Remark: All the given prices are in euros per contract. To obtain the price for 10^6 contracts, each price simply needs to be multiplied by this number.

2 Point (b) & (c): Selection of M and Numerical Errors

In order to select an appropriate M for the CRR method we considered the error as the absolute value of the difference between the exact price and the price computed with the tree approach. For the MC approach we considered as error the unbiased standard deviation of the price. Considering a bid/ask spread of $1bp$ we look for a value of M such that the corresponding error becomes lower than $1bp$. By plotting on a loglog-scale the errors with different values of M we can see that a good value of M for the CRR tree approach would be around 100 while for the MC it is around 500000. The graphic also shows that the numerical error rescales with M as $1/M$ for the CRR and as $1/\sqrt(M)$ for the MC.

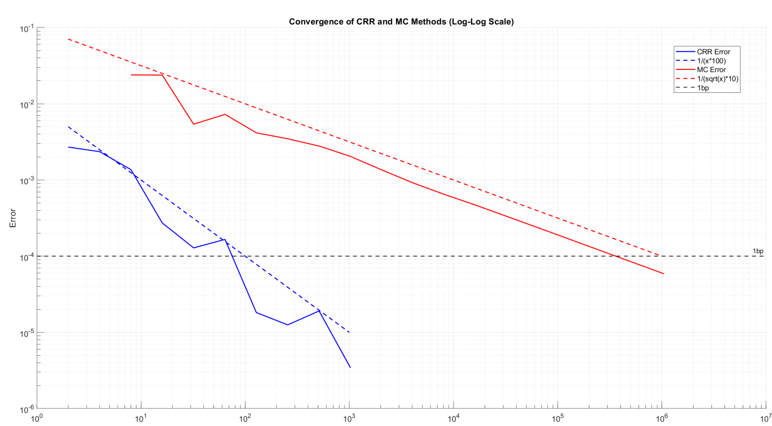

Figure 1: Error comparison: CRR V/s MC

3 Point (d): European Call Option with European Barrier Pricing

Note that a European call option with a European knock-out (KO) barrier has the same payoff as the following portfolio: a long position in a European call with strike K, a short position in a European call with strike KO ($KO > K$, with $KO = 1.4$ in our case), and a short position in $(KO - K)$ digital options with strike KO.

Remark: We recall that the payoff of a digital option is given by:

$$\text{Payoff}_{\text{digital}} = \begin{cases} 1, & \text{if } S_T \geq K \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where S_T is the underlying asset price at maturity and K is the strike price.

Due to this equivalence (and the absence of arbitrage), we have that also their prices are equivalent, hence the existence of the European barrier price is guaranteed and we can compute it as:

$$V_{\text{KO-Call}} = C_K - C_{KO} - (KO - K)D \tag{2}$$

where $D = B(t_0, t)\mathcal{N}(d_2)$. The resulting value is **0.02792323**.

As requested, the price was also computed using the CRR and Monte Carlo methods. To ensure convergence, we used $M = 100$ for CRR and $M = 10^4$ for the Monte Carlo approach. The prices obtained with these two numerical methods are **0.02772767** and **0.02730513**, respectively.
According to our expectations, the price of the KO Barrier Call Option is less then the price of the original European Call Option since the payoff of the first one is always less or equal to the second one.

4 Point (e): Vega of KO European Call Option

We recall that Vega is defined as:

$$\mathcal{V} = \frac{\partial C}{\partial \sigma} \Delta\sigma \tag{3}$$

where in our case $\Delta\sigma = 0.01$. From this definition, we know that Vega is linear. Therefore, based on what was said in the previous point, we can compute the Vega of a European call with a knock-out (KO) as:

$$\mathcal{V}_{KO-Call} = \mathcal{V}_K - \mathcal{V}_{KO} - (KO - K)\mathcal{V}_D \tag{4}$$

where the Vega of a standard Call option is:

$$\mathcal{V} = B(t_0, t)F_0\sqrt{t - t_0}\phi(d_1)\Delta\sigma \tag{5}$$

while the Vega of a digital option is:

$$\mathcal{V}_D = B(t_0, t)\phi(d2)\frac{d1}{\sigma} \tag{6}$$

where ϕ is the standard normal density. This ensures the existence of a closed-form solution.

We have also implemented a numerical approximation of Vega using the CRR and MC methods and applying the finite difference central scheme. By varying the initial price F_0 between the required values, we obtained the following graphs.

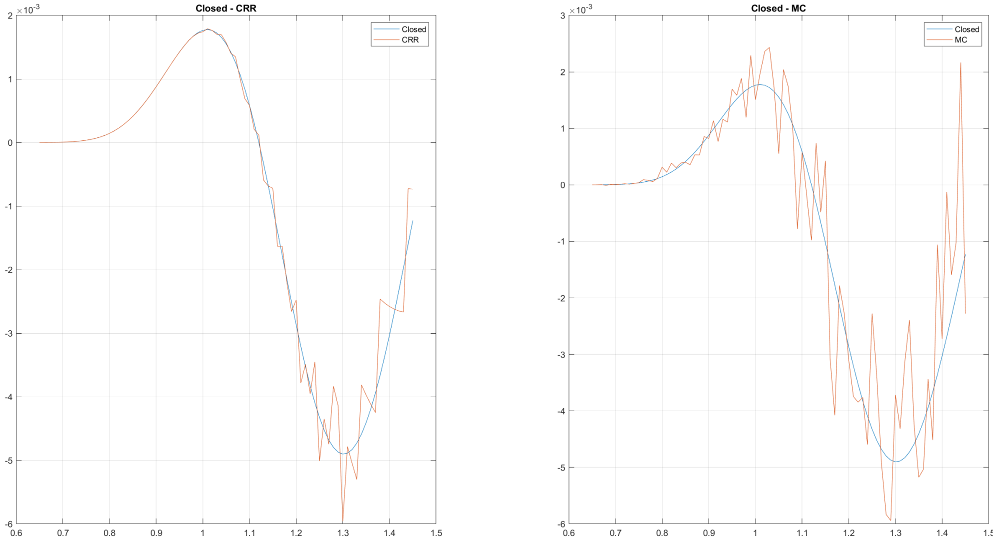

Figure 2: Vega comparison

5 Point (f): Antithetic variable technique

The antithetic variable technique for the MC approach consists, instead of taking a sample of dimension M, in taking a sample of dimension $M/2$ and considering the same sample with a negative sign. this allows to have two samples that are negatively correlated, so the total variance will be lower than the one obtained with a traditional MC approach; as it can be seen in the graphic for same values of M we get a lower error in the MC made with antithetic variables technique; this has an important computational advantage since we can get a certain error with a significantly lower number of simulations.

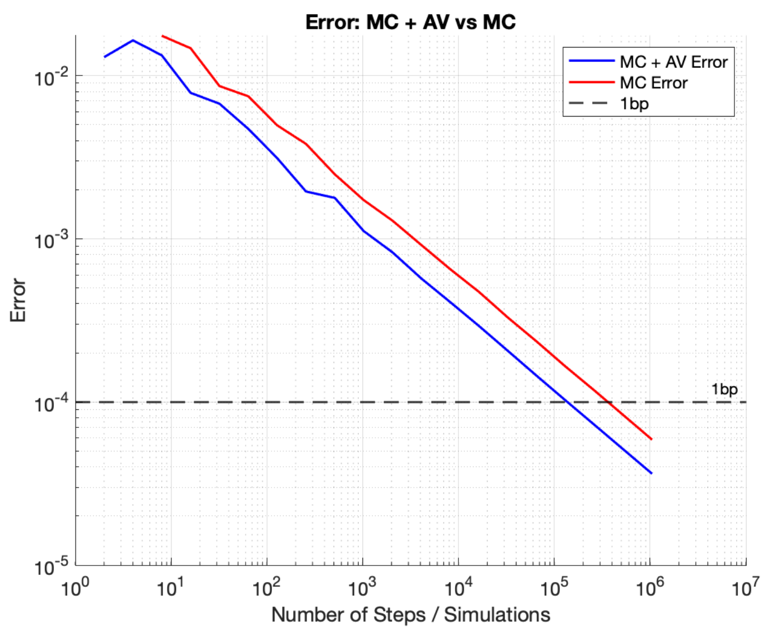

Figure 3: Antithetic Variable Technique For Error Reduction

6 Point (g): Pricing a Bermudan Option Using the Binomial Tree

6.1 What is a Bermudan option?

A Bermudan option is a derivative that allows the holder to exercise the option at a finite set of predetermined dates before expiration. This differs from a European call option, which can only be exercised at expiration, and an American option, which can be exercised at any time before maturity.

6.2 Methodology

Pricing of a Bermudan option is performed using the binomial tree model by checking at each allowed exercise date whether early exercise is optimal. This involves comparing the immediate exercise value with the continuation value at each node. In our case, the holder has the right to exercise at the end of every month, acquiring the stock at the strike price $K = \mathbf{1.05}$ **€**.

6.3 Result & Difference

- Bermudan Option Price: **0.0288740€**
  - Total value of the contract: **28874.0€**
- European Option Price (CRR): **0.0288737€**
  - Total value of the contract: **28873.7€**

The Bermudan option is priced slightly (not significantly) higher than the European option because it offers more flexibility, allowing the holder to exercise the option at multiple points in time, rather than just at maturity. This added flexibility increases the value of the Bermudan option. The small price difference suggests that, in this case, the additional value of this flexibility is relatively low, since the time-to-maturity is short (4 months), which limits the potential for significant exercise opportunities that a Bermudan option offers, making both option prices almost identical.

7 Point (h): Bermudan Option with Varying Dividend Yields

7.1 Objective

Now, we observe the effect of varying dividend yields on the pricing of the Bermudan option. The dividend yield will range from **0% - 5%**, and the corresponding prices will be compared to those of an European option. Dividends typically reduce the value of options, as they cause a decrease in the underlying stock price on the ex-dividend date. Thus, dividends may affect the optimal exercise strategy for Bermudan options compared to European options.

7.2 Methodology

The binomial tree model will be used to price the Bermudan option with different dividend yields, and the results will be contrasted with the European call option price.

Dividend Yield Discretization

The dividend yield interval $[0, 0.05]$ is discretized into d_{step} steps (**1000** steps in our case) using the `linspace` function in MATLAB. The resulting array, di, contains d_{step} evenly spaced dividend yield values, starting from **0** and ending at **0.05**.

$$di = \texttt{linspace}(0, 0.05, d_{\text{step}})$$

Each value in di represents the dividend yield for a specific step, which we use in the option's pricing. Then, the dividend yield at each step is updated during each iteration of the CRR, and for each step, we use the corresponding value from di.

7.3 Result & Comparison

As the dividend yield increases, the prices of both options decrease, with the European option being slightly more sensitive to the change. This is because higher dividends lowers the value of the call option. However, the difference between option prices remains small due to the short time-to-maturity and moderate volatility (21% p.a). The impact of dividends is more noticeable with higher yields, but given the short time horizon, the added value of flexibility in the Bermudan option does not result in a significant price difference.
Finally, we notice that if $d = 0$ the Bermudan and the European call options have the same price.

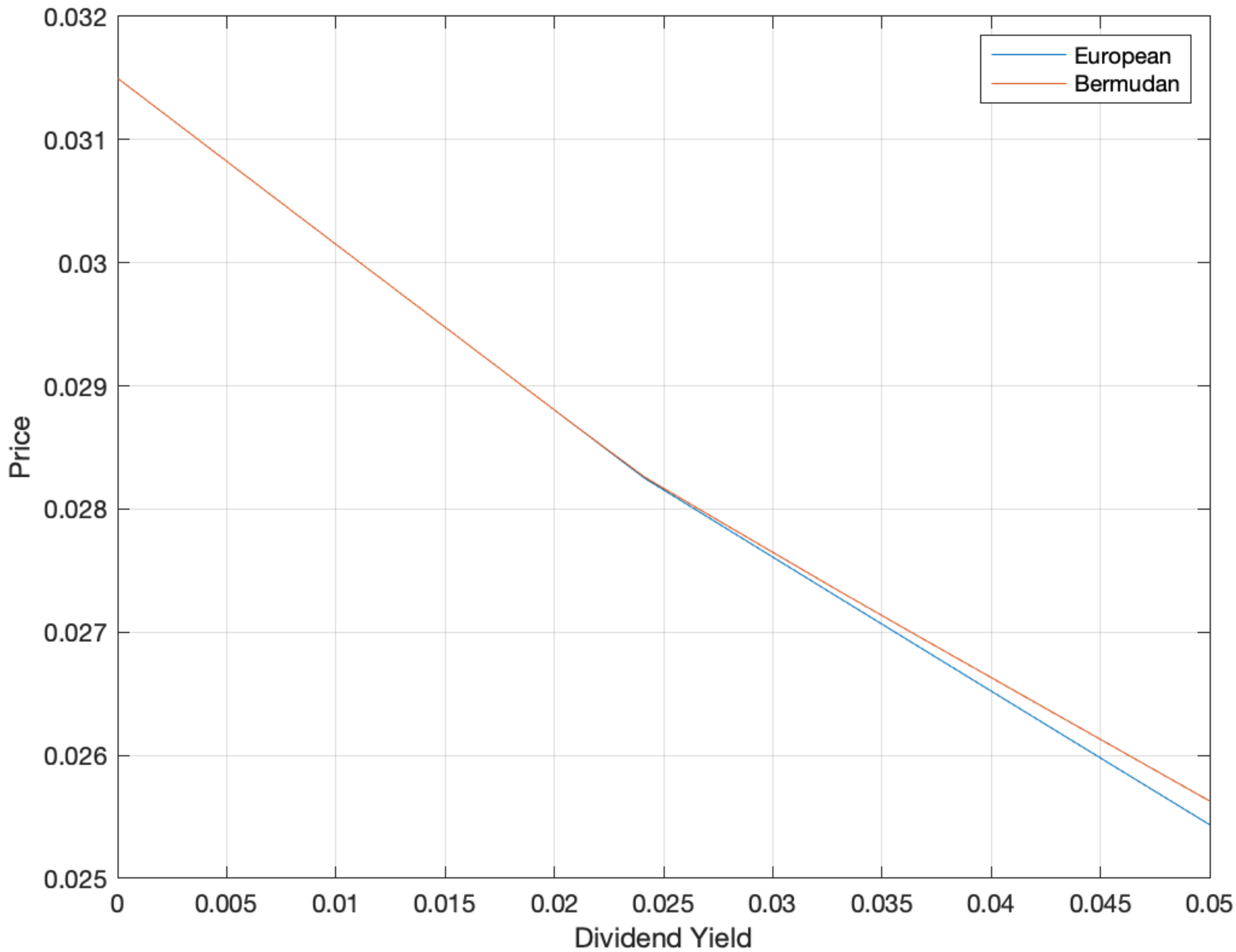

Figure 4: Option Price Comparison With Dividend Changes