ASSIGNMENT RM2: Corporate Bond Portfolio: Market Implied vs Real World Default Probabilities

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1 Q1: Default Intensities for the Two Bonds

1.1 Results

- **1-Year Bond Intensity:** 2.43896%
- **2-Year Bond Intensity:** 2.43377%

1.2 Analysis

The computed intensity values suggest a slightly higher risk perception for the two-year bond compared to the one-year bond. Since both bonds are issued by the same firm and have a recovery rate of 30%, we might have expected the intensities to be identical under a constant hazard rate assumption. However, the slight difference arises due to market pricing differences and investor expectations about credit risk beyond the first year. The increasing intensity for the longer bond suggests a mild expectation of worsening creditworthiness over time.

2 Q2: Default Probabilities Based on Intensity Values

2.1 Results

- **1-Year Default Probability:** 2.40439%
- **2-Year Default Probability:** 4.76367%

2.2 Analysis

These default probabilities are derived by computing survival probabilities using the Q1 solution. In particular, we applied the two-year bond intensity as an approximation that also reflects what occurs between one and two years. In our specific setting—where the second maturity is effectively double the first—one might consider using the average of the intensities. However, we chose to use the two-year intensity to better capture the overall credit risk.

Notably, the two-year default probability is roughly twice the one-year default probability, indicating a consistent application of the intensity function and aligning with expectations under a constant intensity assumption. Moreover, it is evident that the probability of default over two years is greater than the probability of default in one year, as the former necessarily includes the latter as a subset event.

3 Q3: Bond Z-Spreads

3.1 Results

- **Z-Spread (1-Year Bond):** 1.72144%
- **Z-Spread (2-Year Bond):** 1.72730%

3.2 Analysis

The Z-spread represents the additional yield required over the risk-free rate to compensate for credit risk.

3.2.1 Question: Why do practitioners tend to use the Z-spread instead of the intensity?

Practitioners prefer using the **Z-spread** over **intensity** because it is directly observable from bond market prices and is easier to interpret in relative terms. The **Z-spread represents the additional yield required over the risk-free rate to account for credit risk**, making it a practical measure for pricing and comparing bonds. In contrast, intensity (default rate) requires modeling assumptions and is not directly traded in the market. The Z-spread is also more intuitive for portfolio managers and traders, as it aligns with spread-based valuation approaches commonly used in fixed-income markets.

3.2.2 Question: If the two spreads (Z-spread and intensity) are constant and calibrated from the same bond price, which is the lowest?

The intensity is lower than the Z-spread because the Z-spread includes default risk and premium due to low liquidity, while intensity reflects only default risk. By looking at the dirty–price formulas for a bond, we observe that the intensity–based valuation includes an additional term with respect to the Z–spread formula. Specifically, that extra piece is:

$$\pi \sum_{n=1}^{N} B\big(t, t_n\big) \Big[P\big(t, t_{n-1}\big) \; - \; P\big(t, t_n\big)\Big].$$

This quantity is always greater than or equal to zero, so the remaining part of the sum (coupon plus redemption) must be smaller when using intensity. Because all other variables remain the same in both approaches, that portion depends solely on the intensity in the intensity-based framework, whereas it depends solely on the Z-spread in the Z-spread framework.

Since both intensity λ and Z–spread z enter discounting via negative exponents, the Z–spread calibrated to match the same bond price must be strictly lower than the intensity. Moreover, they coincide if and only if the recovery rate π is zero.

Finally, a practical rule of thumb is that the Z–spread is approximately

$$z \approx \lambda\big(1 - \pi\big).$$

4 Q4: Default Probability Under Piece-Wise Constant Intensity

4.1 Results

- **First Intensity:** 2.43896%
- **Second Intensity:** 2.42823%
- **1-Year Default Probability:** 2.40946%
- **2-Year Default Probability:** 4.76332%

4.2 Analysis

In general, a piecewise-constant intensity function offers a more flexible and realistic modeling approach than a constant hazard rate. The results do show slight deviations from the probabilities in Q2, demonstrating how time-varying intensities can affect cumulative default probabilities. A more refined bootstrap method could further improve the capture of market-implied credit risk. However, in our specific scenario—where we expect the intensity to remain constant from the first period to the second—the default probabilities at one and two years end up virtually identical. This would not be the case under market shocks, where time-varying intensities would lead to more pronounced differences.

5 Q5: Historical Default Probability from Rating Transition Matrix

5.1 Results

- **1-Year Real World Default Probability:** 2.00%
- **2-Year Real World Default Probability:** 4.71%

5.2 Analysis

These probabilities are derived from historical transition matrices rather than market-implied intensities. The lower probabilities compared to Q2 and Q4 indicate that historical defaults for firms with an IG rating have been less frequent than what the market currently prices in. This could reflect market risk aversion, economic conditions, or a divergence between historical and forward-looking risk assessments.

6 Q6: Scenario 1: Drop in Two-Year Bond Price

6.1 Results

- **1-Year Default Probability:** 2.40946%
- **2-Year Default Probability:** 11.92589%

6.2 Analysis

A significant increase in the two-year default probability reflects the impact of the price drop to €97. This suggests that the market perceives an increased likelihood of distress over the second year, as the higher probability indicates deteriorating creditworthiness expectations. The first-year probability remains unchanged, which is consistent with the assumption that the short-term outlook is unaffected.

6.2.1 Question: Given a movement of the price a corporate bond (e.g. 2% loss in 1 day for a 10y bond) is it possible to state something about the riskiness of the corporation?

A 2% loss in a 10y corporate bond in one day indicates increased risk, possibly due to concerns about the corporation's creditworthiness or market-risk factors. However, to assess the riskiness of the corporation, other factors like credit ratings, bond yield spreads, economic conditions, and company-specific events must also be considered.

7 Q7: Scenario 2: Acquisition by a High-Quality Company

7.1 Results

- **1-Year Default Probability:** 1.00%
- **2-Year Default Probability:** 3.43%

7.2 Analysis

Following the price increase of both bonds, default probabilities decline significantly, indicating improved market confidence. This aligns with expectations, as an acquisition by a high-quality firm typically strengthens financial stability and reduces perceived credit risk.

8 Q8: Consistency of Default Probabilities (under scenario 1) with Historical Data?

Under Scenario 1, Beta maintains the same credit risk level for the first year. This is evident because the price of the one-year coupon bond remains unchanged, and we know that Beta initially holds an Investment Grade (IG) rating. However, the decline in the price of the two-year coupon bond suggests an increase in the company's credit risk, but only during the second year. This indicates that the probability of default in the first year is lower than in the second year.

From this reasoning, we deduce that Beta has transitioned from IG to High Yield (HY), as these are the only two rating categories present in the Markov transition matrix mentioned in point 5 of historical probabilities.

The probability of default in the first year is therefore 2% (corresponding to the [1,3] element of the Markov transition matrix), while in the second year, it increases to 5% (element [2,3] of the matrix). Since we assume with certainty that Beta transitions to HY after the first year, we do not need to consider the sum of all possible paths leading from IG to default within two years.

We computed the conditional probability using the exponential formula linking intensity to survival probability. The result, 9.75139%, is considerably higher than the historical value.

9 Additional Questions

9.1 Beyond IRS, CDS and ASW, other swaps are:

1. **Currency Swaps**: Exchange of cash flows in different currencies.
2. **Commodity Swaps**: Based on commodity price fluctuations.
3. **Equity Swaps**: Based on stock or equity index performance.
4. **Total Return Swaps (TRS)**: Exchange of total return on an asset for fixed/floating payments.
5. **Inflation Swaps**: Based on inflation rate changes.
6. **Variance Swaps**: Based on the variance (volatility squared) of an asset's price.
7. **Forward Rate Agreements (FRAs)**: Agreement on future interest rate differences.

9.2 Do you expect the assumption of independence between "risk-free" rate and default to be reasonable?

No, the assumption that the risk-free rate and default risk are independent is not reasonable, as both are influenced by similar economic factors. For instance, during periods of economic downturns, such as a recession, government bond yields (the risk-free rate) typically decrease, while default risk tends to increase. This occurs because, in challenging economic conditions, central banks often reduce interest rates to stimulate growth, while companies face greater financial difficulties, increasing the probability of default. Therefore, in practice, the risk-free rate and default risk are often correlated and influence one another.

Errors Found in the Code

1. The expiry date of the second bond was set to 3 years instead of 2 years and has been corrected.
2. The function `bond_cash_flows` has been appropriately modified.