Assignment 5 Structured Products

1. Case Study: Structured bond

On the 31st of January 2023 at 10:45 C.E.T., the Bank XX issues a structured bond, whose hedging termsheet is described in the annex. Consider the Structured bond issue in a single-curve interest rate modeling setting and neglecting the counterparty risk. Market parameters for (flat) Cap Volatilities are:

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EUR Caps/Floors - Implied Volatilities
   Please call +44 (0)20 7532 3080 for further details
    1.50 1.75 2.00 2.25
                         2.5
                              3.0
                                    3.5
                                         4.0
                                              5.0
                                                   6.0
                                                         7.0
                                                              8.0 10.0
 14.0 | 13.0 | 12.9 | 12.1 | 13.3 | 13.8 | 14.4 | 15.0 | 17.2 | 19.1 | 20.2 | 21.6 | 23.9 |
 2Y|22.4|19.7|17.5|18.0|19.2|20.4|21.0|21.4|22.3|23.6|24.9|26.1|28.1|
 3Y|23.8|21.7|20.0|19.8|20.3|20.5|20.8|21.4|22.9|24.3|25.6|26.7|28.2|
 4Y|24.2|22.4|20.9|20.4|20.4|20.2|20.2|20.5|21.7|22.9|24.0|25.0|26.6|
 54 24.3 22.6 21.2 20.6 20.4 19.8 19.5 19.6 20.5 21.5 22.6 23.5 25.0
 64/24.3/22.7/21.4/20.7/20.2/19.4/18.9/18.8/19.3/20.2/21.2/22.0/23.5/
 7Y|24.1|22.6|21.4|20.7|20.1|19.1|18.4|18.1|18.4|19.1|20.0|20.8|22.2|
 84 23.9 22.5 21.4 20.6 20.0 18.8 18.0 17.6 17.6 18.2 19.0 19.8 21.1
 9Y|23.7|22.4|21.3|20.5|19.8|18.5|17.6|17.1|17.0|17.6|18.3|19.0|20.3|
10Y|23.5|22.2|21.2|20.4|19.6|18.3|17.3|16.8|16.5|16.9|17.6|18.3|19.5|
12Y|23.0|21.7|20.8|20.0|19.3|17.9|16.9|16.2|15.8|16.0|16.5|17.1|18.1|
15Y|22.3|21.2|20.3|19.5|18.7|17.3|16.3|15.5|15.0|15.1|15.5|16.0|16.9|
20Y|21.6|20.4|19.5|18.8|18.0|16.6|15.5|14.7|14.1|14.1|14.5|15.0|15.9|
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It is required to

- a. Compute the LMM spot vols on the same grid of Flat Vols. [Calibration]
- b. Determine the upfront X% [Pricing].

Hint: Use (if possible) the LMM spot vols on the same strike of derivative's payoff (spline interpolation on strike).

- c. Compute Delta-bucket sensitivities for all buckets [Risk measurement].
- d. Compute total Vega. [Risk measurement]
- e. Consider the course-grained buckets (0-2y; 2y-6y; 6y-10y). Completely hedge with swaps the Delta risk. [Portfolio risk management]

Hint: Select 3 swap notionals (2y, 6y, 10y) s.t. the corresponding coarse-grained bucket deltas are zero in the hedged portfolio.

Hint: Start with the longest swap. Why?

- f. Hedge the Vega. Consider the buckets for the vega (0-6y and 6y-10y) hedge the bucketed Vega with a 6y ATM Cap (strike = ATM 6y Swap rate same conventions for the two legs) and 10y ATM Cap. [Portfolio risk management]
 - Hint: Start hedging the longest cap. Why?
- g. [Facultative] Is there any digital risk? If yes, can you correct the price? [Pricing]

2. Case Study: Exotic cap [Calibration & Pricing]

On the same date, price with a BMM the option, whose quarterly payoff is at reset date T_{i+1}

$$\delta(T_i, T_{i+1})[L(T_i, T_{i+1}) - L(T_{i-1}, T_i) - 5 bps]^+$$

First payment quarter: 6m after the Start date.

Maturity Date: 4y (last payment date).

Vol: BMM spot vol for the i^{th} term corresponding to the strike $K_i =$

 $L(t_0; T_i, T_{i+1})$

Correlation: $\rho_{ij} = e^{-\lambda \delta(T_i, T_{i+1})}$ with $\lambda = 0.1$ and δ with an Act/365 yearfrac.

Exercise Annex:

Indicative Terms and Conditions as of 31st of January 2023

Swap Termsheet

Principal Amount (N): 50 MIO EUR

Party A: Bank XX

Party B: I.B.

Trade date: today

Start Date: 2 Feb 2023

Maturity Date (t): 10 years after the Start Date, subject to the Following Business Day

Convention.

Party A pays: Euribor 3m + 2.00%

Party A payment dates: Quarterly, subject to Modified Business Convention

Daycount: Act/360

Party B pays @ Start Date: X% of the Principal Amount

Party B pays @ payment dates: Coupon

Party B payment dates: Quarterly, subject to Modified Business Convention

First Quarter Coupon: 3%

Next Quarter Coupons: [Up to (and including) the 3rd year] uptil the 3rd year euribor

€ 3m+1.10% if € 3m ≤ 4.30%

else 4.50%

[After 3y and up to (and including) the 6y]

€ 3m+ 1.10% if € 3m ≤ 4.60% from 3rd year - 6th year

else 4.80%

[After 6y] € 3m+ 1.10% capped at 5.10% € 3m+ 1.10% if € 3m \le 5.20%

else 5.40%

from 6th to the 10th year & the cap rate is 5.10%(strike rate)