

A Project Dissertation Presentation
on
Shape Alignment and their Procrustes
Analysis

Presented By:

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Under The Guidance

Of

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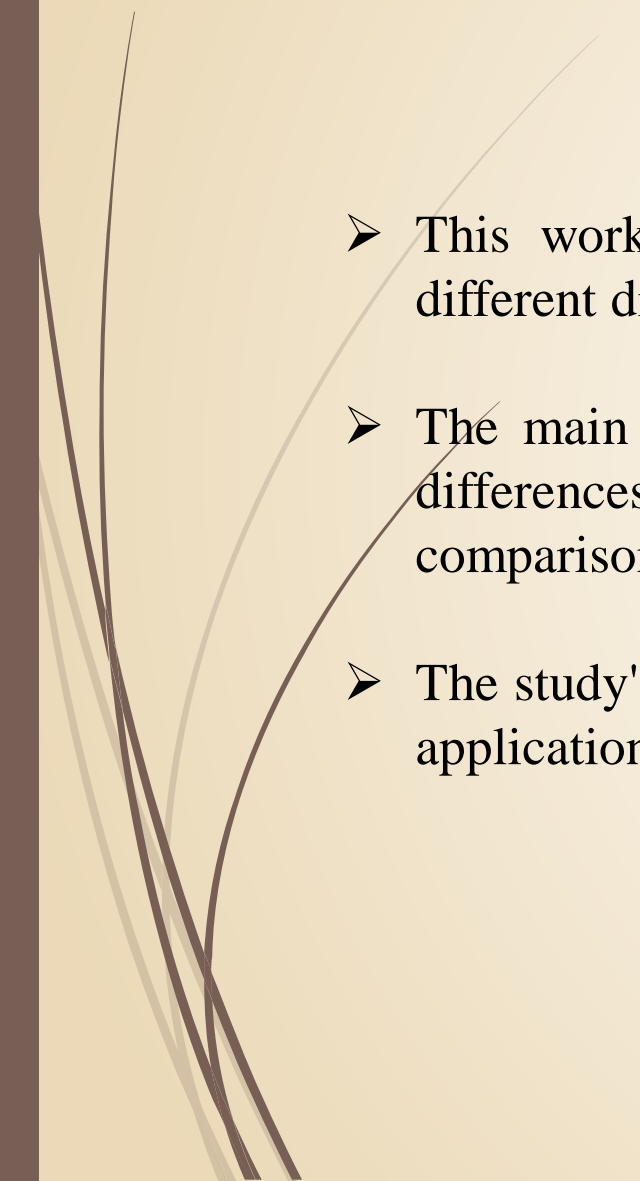
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Objective

- This work aims to explore and analyse shape alignment using Procrustes Analysis with different distance metrics.
 - The main objectives of Procrustes analysis are to remove translation, rotation, and scaling differences among shapes, resulting in aligned shapes for meaningful statistical analysis, comparison, and visualization.
 - The study's findings will contribute valuable insights to shape alignment techniques, benefiting applications in science, technology, and medicine.
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Motivation of the Work

- ▶ The motivation behind shape alignment and Procrustes analysis is to gain insights, make discoveries, improve technology, and help us make better decisions across different fields.
 - ▶ By studying these relationships, we gain insights into topics like how living organisms evolve, how objects are recognized by computer, and how different cultures create unique artifacts.
 - ▶ Shape alignment even helps with the tasks like reconstructing missing parts of shapes or combining shape information with other types of data.
- ▶ By conducting work on shape alignment and Procrustes analysis, we aim to advance our understanding of shape relationships, quantify shape variability, facilitate comparative shape analysis, enable shape reconstruction and registration.




Work Completed

- Successfully implemented the Euclidean distance metric in MATLAB to calculate distances between corresponding points for shape alignment.
- Implemented the Manhattan distance metric, allowing measurement of shape similarity based on the sum of absolute differences between landmarks.
- Studied and implemented shape translation, rotation and scaling techniques to align shapes, ensuring a common reference frame.
- Gained a comprehensive understanding of the Mean Squared Error metric and its significance in evaluating alignment accuracy.
- Acquired valuable experience and expertise in shape analysis and alignment for future contributions in the field.



Shape alignment

- **Why Shape alignment?**
 - Shape alignment is the process of making different shapes match and fit together in a meaningful way. It involves finding the best way to transform and adjust the shapes so that they line up correctly. It finds applications in diverse fields such as biology, anthropology, pattern recognition, computer graphics, and medical imaging.
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Generalized Procrustes Analysis (GPA)

- Any shape having 'K' number of landmark points in the 2D space can be defined using:

$$S_i = [x_1, x_2, \dots, x_k, y_1, y_2, \dots, y_k]^T$$

- If there are L number of shapes, then we may write:

$$S = [S_1, S_2, S_3, \dots, S_L]$$

- GPA align the shapes by minimizing the squared difference between the Mean shape and the shapes to be aligned using the formula:

$$D = |S_i - \bar{S}|^2$$

Where, \bar{S} is the mean shape

GENERALISED PROCRUSTES ANALYSIS (GPA) ALGORITHM

- 1) Select any of the example shape (preferably the 1st shape) from the training set as the approximate mean shape or reference shape \bar{S} .
- 2) Align all other shapes with the current approximated mean shape \bar{S} .
 - a) Calculate the centroid of each shape
 - b) Translate all the shapes S_i towards the origin
 - c) Scale each shape to have unit size i.e. $1/S_i$.
 - d) Rotate and align each individual shape S_i towards the current approximated mean shape \bar{S} .
- 3) Calculate the new approximated mean shape \bar{S} from the roughly aligned shapes
- 4) Return to step-2 until the shapes converge



Shape Alignment using GPA

- ▶ Shapes can not be aligned until the all the three vibrational parameters are removed:
 - ▶ Translation
 - ▶ Rotation
 - ▶ Scaling

GPA (Cont...)

► TRANSLATION

Translation in mathematics refers to the operation of moving a geometric object without rotating or changing its size. For 2-dimensional space, translation involves shifting the object horizontally (x-axis) and vertically (y-axis) by specified amounts.

► If T_x & T_y are the Translational parameter, then the Shape Translation can be defined as:

$$S_{TX} = S_x - T_x$$

$$\text{And } S_{TY} = S_y - T_y$$

GPA (Cont...)

➡ ROTATION

Rotation in mathematics refers to the operation of turning a geometric object around a fixed point, called the center of rotation, by a specified angle.

In 2-dimensional space, a point (x, y) can be rotated by an angle θ around the origin (0, 0) using the following formulas:

$$\theta = \tan^{-1} \left(\frac{\sum_{i=1}^L Y_{Ri} X_{Ti} - X_{Ri} Y_{Ti}}{\sum_{i=1}^L X_{Ri} X_{Ti} + Y_{Ri} Y_{Ti}} \right)$$

GPA (Cont...)

➤ SCALING

Scaling refers to the operation of resizing a geometric object while maintaining its shape. In 2-dimensional space, scaling involves multiplying the coordinates of each point by a constant factor to change its size uniformly in all dimensions.

The scaling operation for a point (x, y) with scaling factors s_x and s_y is given by the following function:

$$\rho = \frac{\mathbf{S}_i}{\|\mathbf{S}_i\|_N}$$

Euclidean Distance Matrice

Euclidean distance is a widely used distance metric. It works on the principle of the Pythagoras theorem and signifies the shortest distance between two points.

Mathematically, for an n-dimensional space and (p_i, q_i) as data points, the perfect distance metric is calculated by:

$$D_e = \left(\sum_{i=1}^n (p_i - q_i)^2 \right)^{1/2}$$

Manhattan Distance Matrice


The Manhattan distance is used to calculate the distance between two coordinates in a grid-like path. It is a more effective metric in cases when discrete or binary attributes are present in the dataset as the path derived from the given values of attributes can be perceived in reality.

The generalized formula to calculate the Manhattan distance in an n-dimensional space is:

$$D_m = \sum_{i=1}^n |p_i - q_i|$$

Simulation Result

- To study the alignment process, I have taken JSRT Data set images from the Japanese Radiological Society which contain 247 chest X-ray images.
- The images are manually annotated in the care of Radiologist.
 - The left Lung contains: 17 Landmark points
 - The right lung contains: 16 Landmark Points

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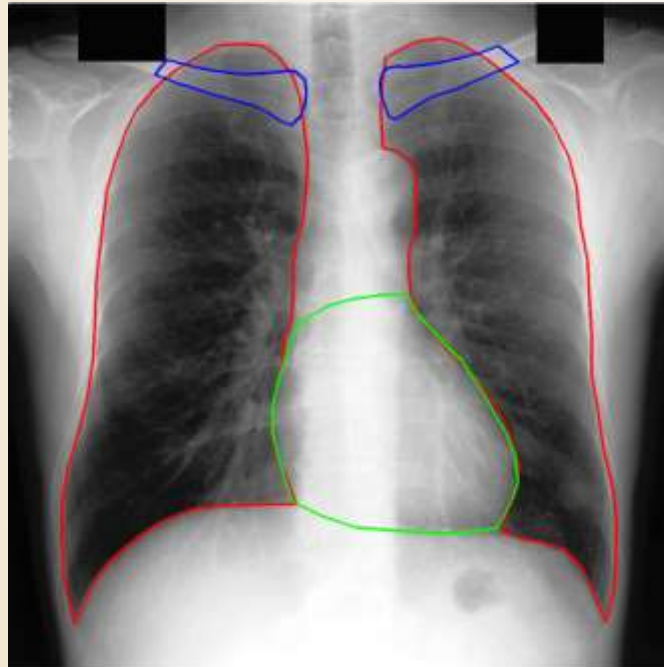


Figure -1: Different anatomical regions in chest X-ray images

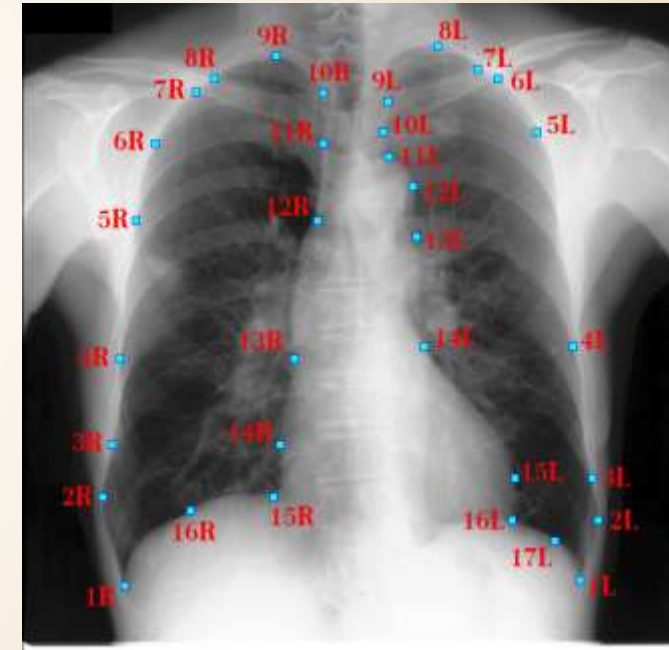


Figure-2: Established Landmark Points

Simulation Result(Cont...)

➤ Lung Field Contours

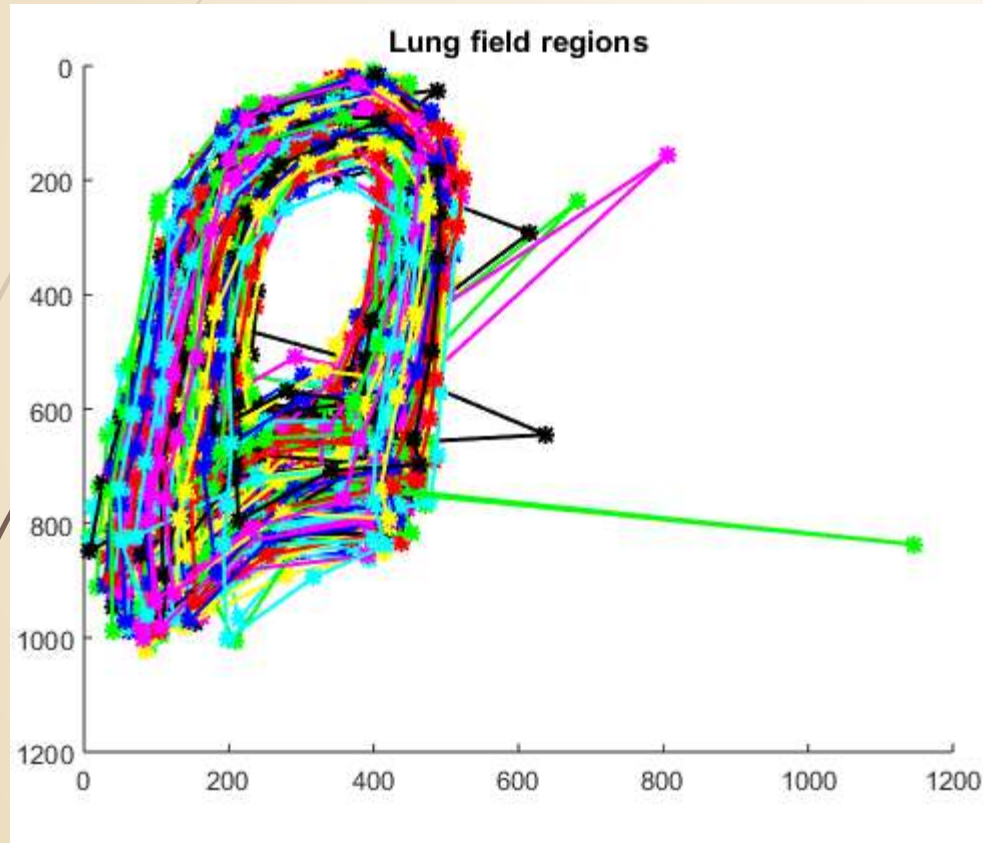


Figure- : Right lung field regions

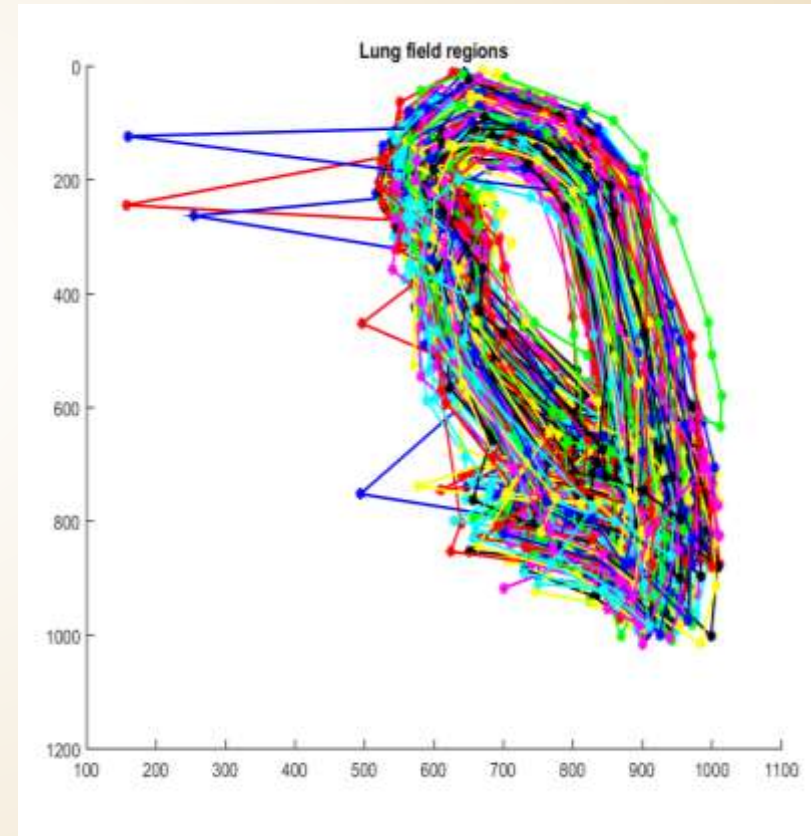


Figure- : Left lung field regions

Simulation Result

- ❖ Shape Alignment: Aligned shapes of Left lung field region

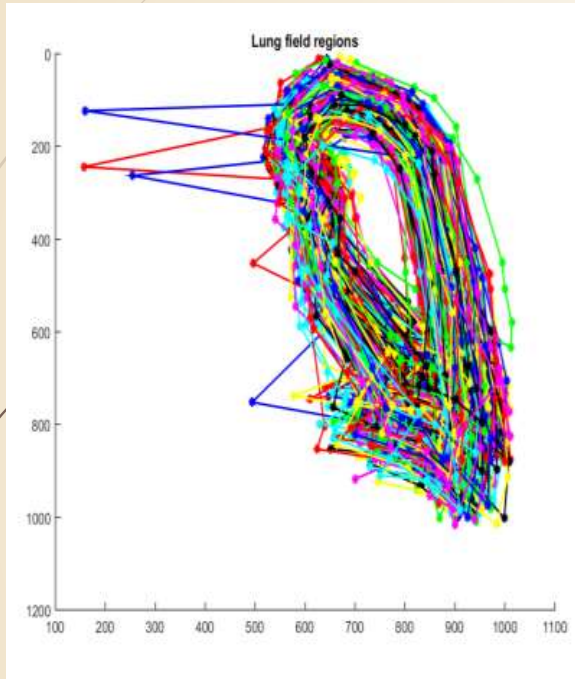


Figure- : Left lung field regions (unaligned)

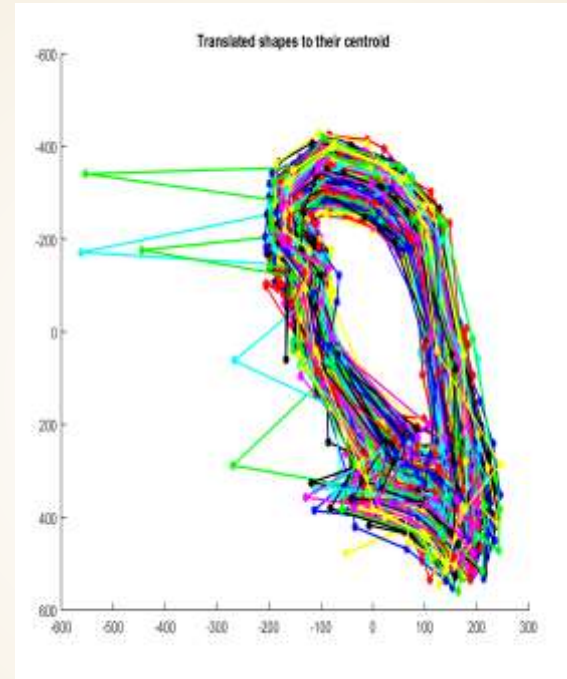


Figure-: Translated shapes to their centroid positions

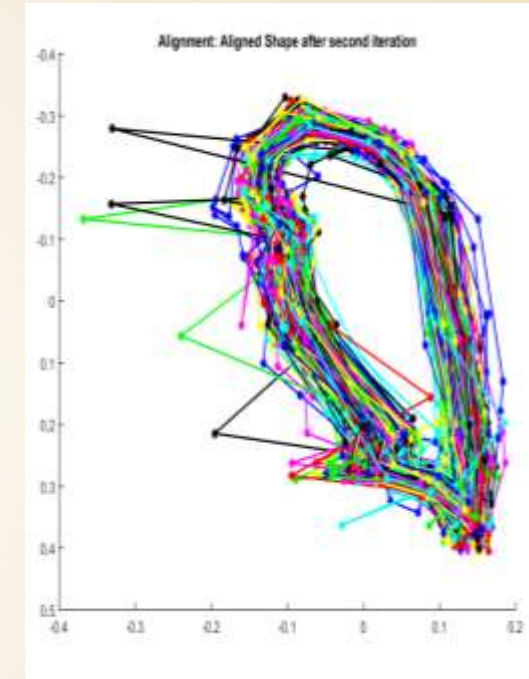


Figure-: Aligned shape after 2nd iteration using GPA

Convergence Parameters:

Scaling convergence=0.001

Theta convergence= $0.001 \cdot \pi / 180$

Translation convergence=0.001

Simulation Result

❖ Shape Alignment: Aligned shapes of Right field region

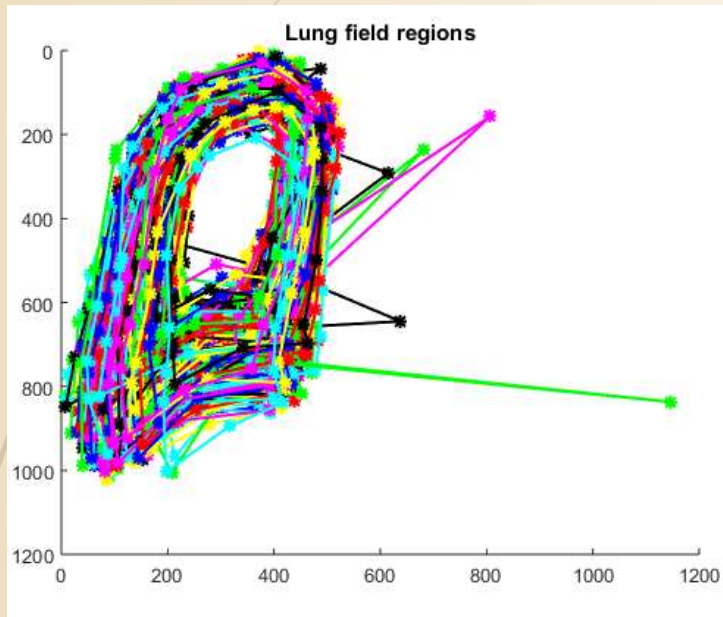


Figure- : Right lung field regions (unaligned)

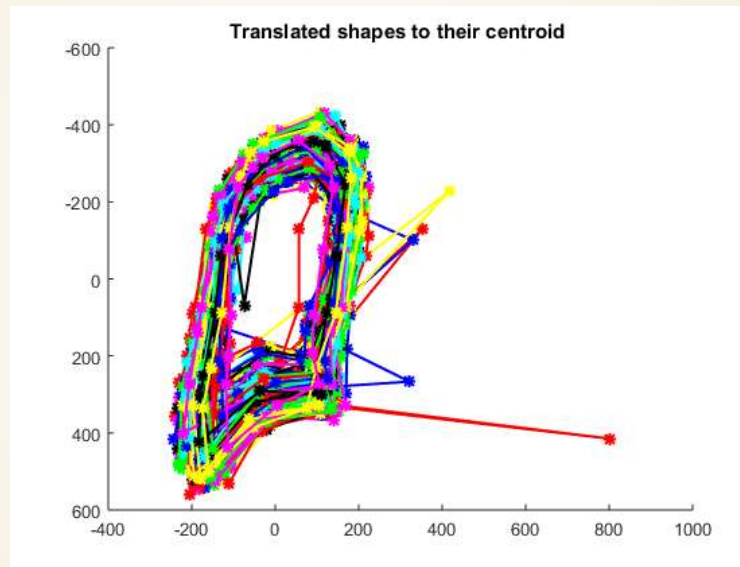


Figure-: Translated shapes to their centroid positions

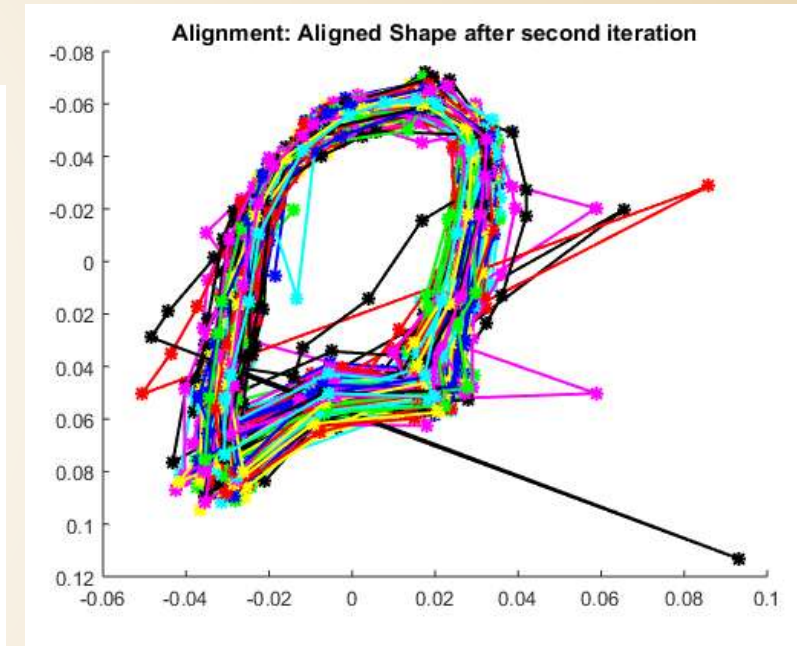


Figure-: Aligned shape after 2nd iteration using GPA

Convergence Parameters:

Scaling convergence=0.01

Theta convergence= $0.01 \cdot \pi / 180$

Translation convergence=0.01

Simulation Result (Cont...)

❖ Alignment Process of Left lung

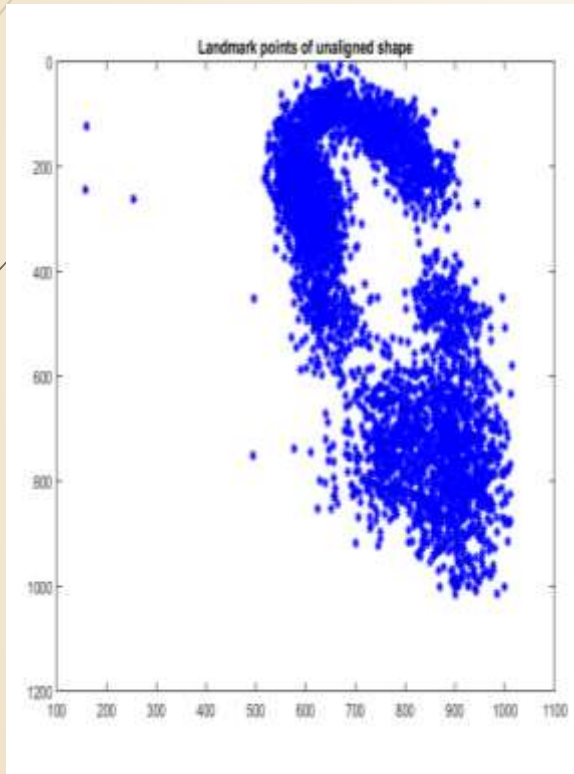


Figure-: Unaligned landmark Points (scatter plot)

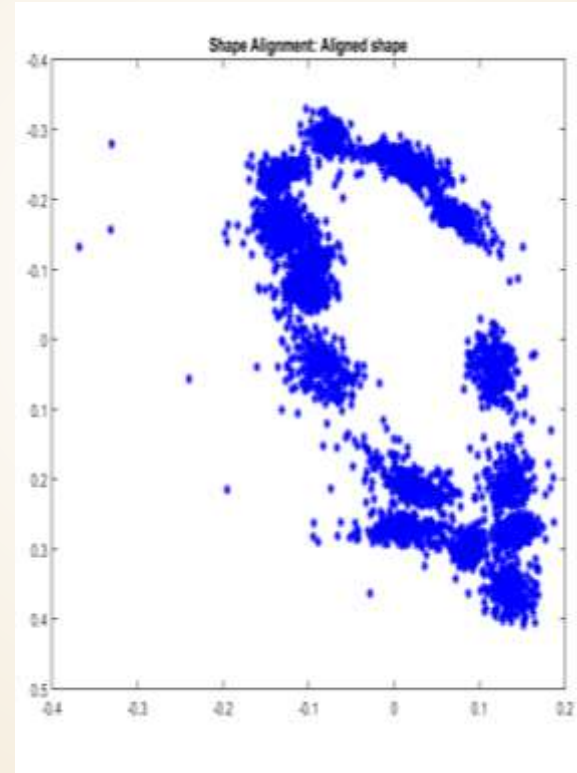


Figure-: Aligned landmark points after 2nd iteration

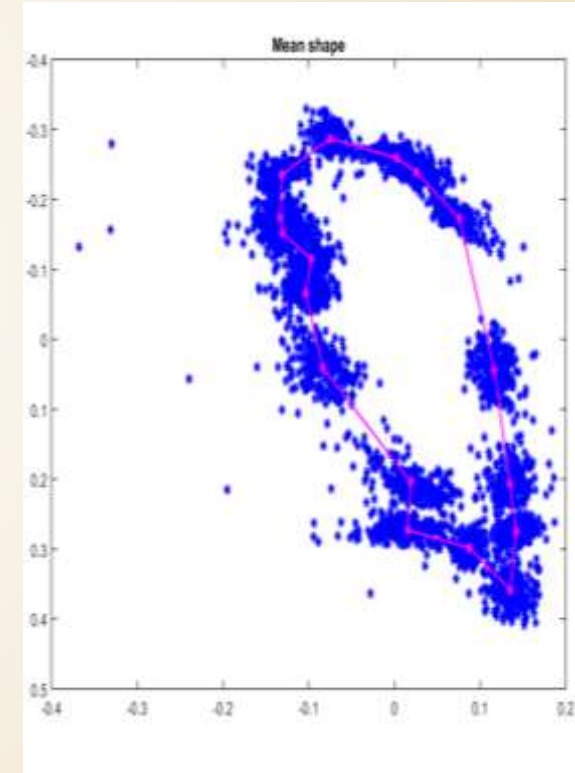


Figure-: Constructed Mean shape

Simulation Result (Cont...)

❖ Alignment Process of Right lung

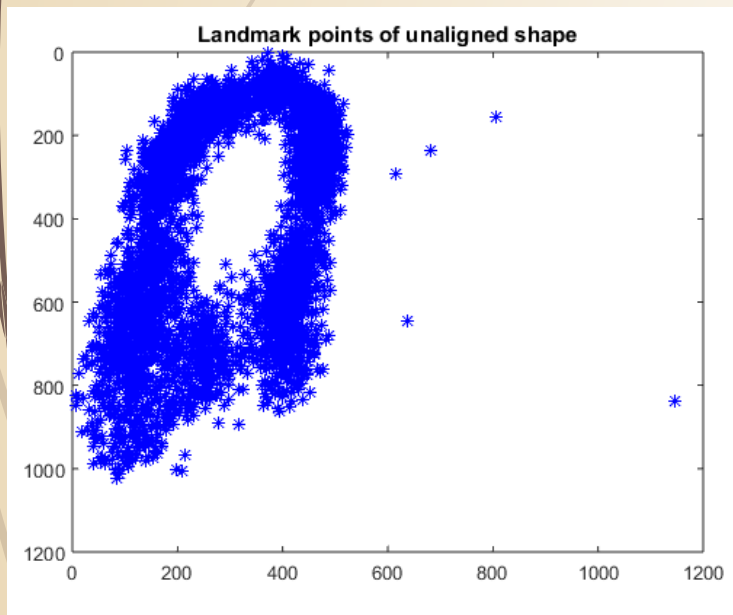


Figure-: Unaligned landmark Points (scatter plot)

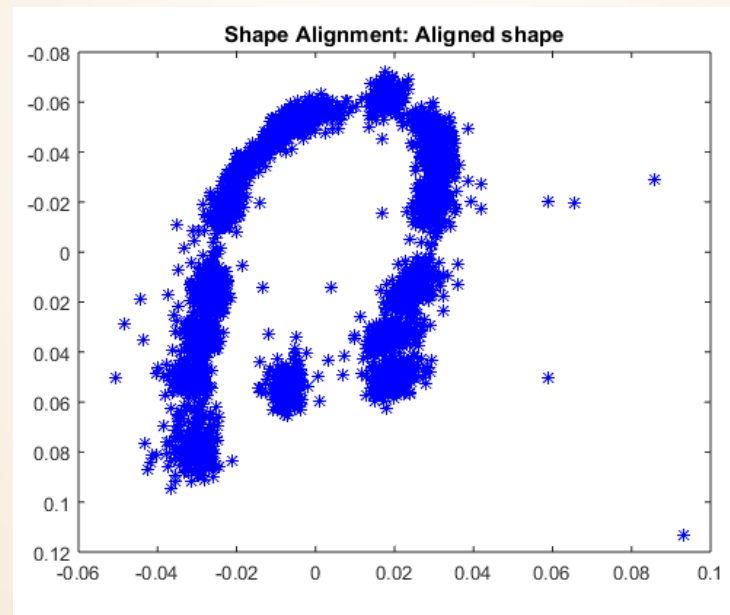


Figure-: Aligned landmark points after 2nd iteration

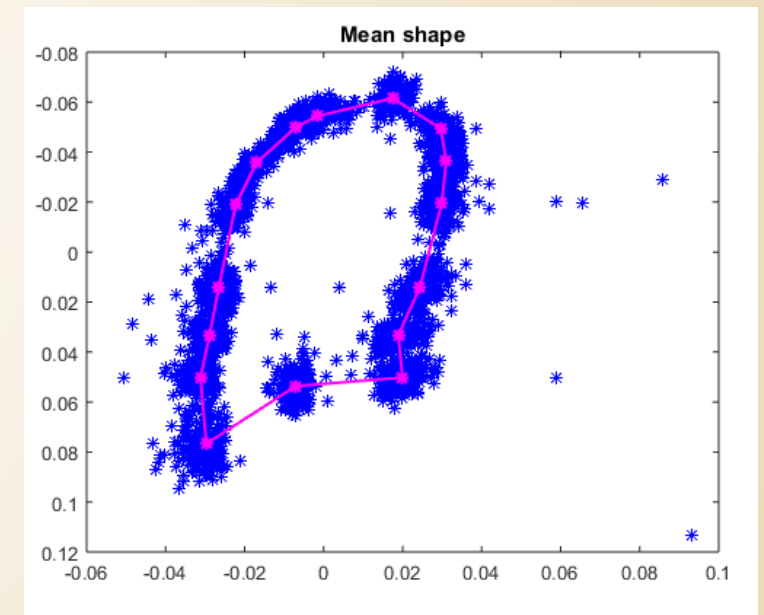
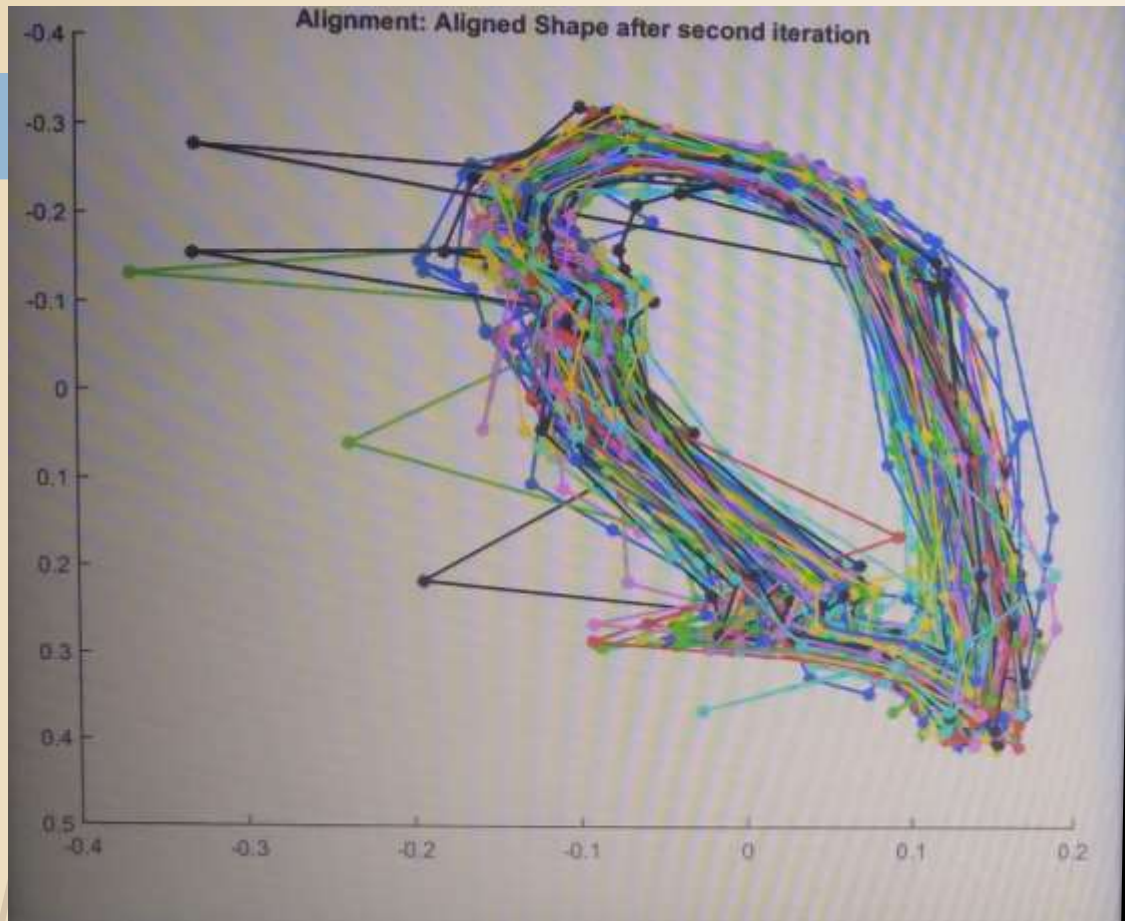
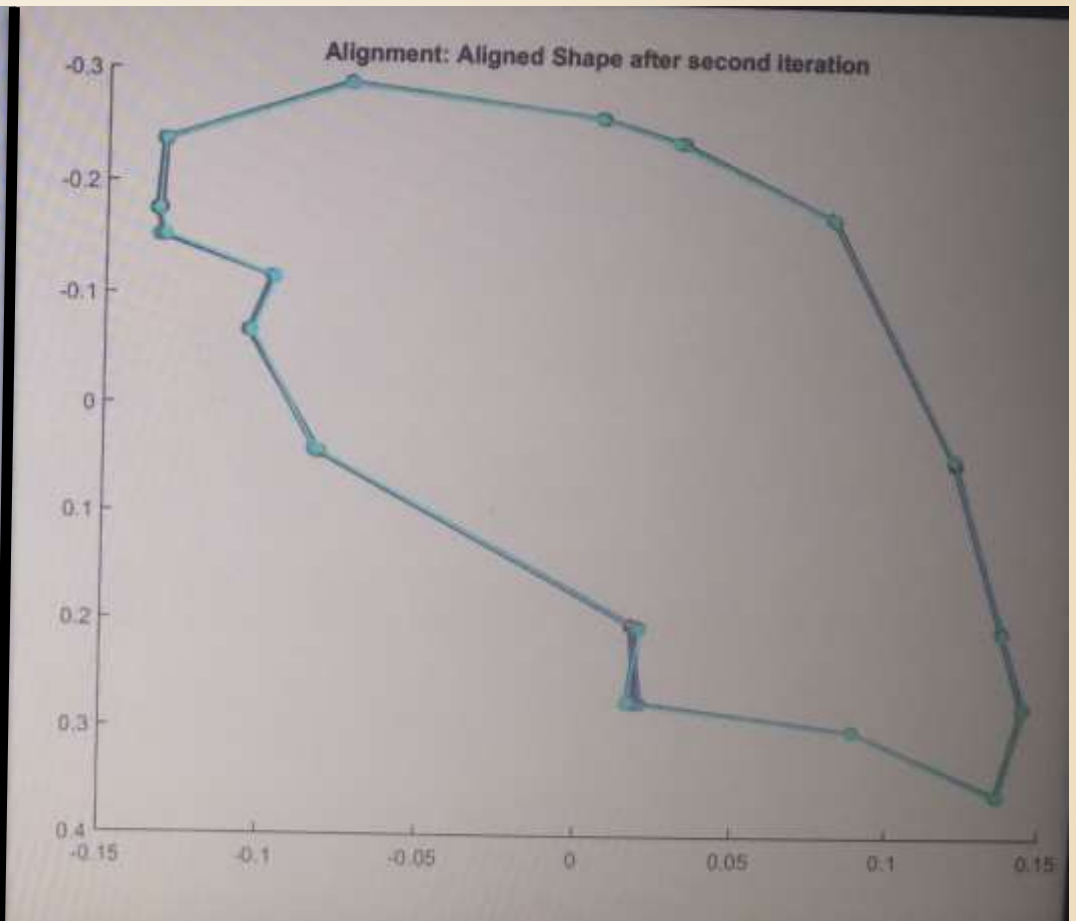


Figure-: Constructed Mean shape



Rotational
convergence= $0.01 \cdot \pi / 180$



Rotational
convergence= $0.0001 \cdot \pi / 180$

Performance Analysis

Table:1 Shape Alignment					
Sl. No.	Convergence Parameter			No. of Iteration taken to converge the shapes	
	Translation	Scaling	Rotation	Manhattan Distance	Euclidean Distance
1	0.01	0.01	$0.001 \cdot \pi / 180$	19	19
2			$0.002 \cdot \pi / 180$	3	3
3			$0.003 \cdot \pi / 180$	3	3
4			$0.004 \cdot \pi / 180$	3	2
5			$0.005 \cdot \pi / 180$	2	2
6			$0.006 \cdot \pi / 180$	2	2
7			$0.007 \cdot \pi / 180$	2	2
8			$0.008 \cdot \pi / 180$	2	2
9			$0.009 \cdot \pi / 180$	2	2
10			$0.01 \cdot \pi / 180$	2	2
11			$0.0001 \cdot \pi / 180$	248	248

Performance Analysis

Sl. No.	Convergence Parameter			No. of Iteration taken to converge the shapes	
	Translation	Rotation	Scaling	Manhattan Distance	Euclidean Distance
1	0.01	$0.001 \cdot \pi / 180$	0.01	19	19
2			0.02	19	19
3			0.03	19	19
4			0.04	19	19
5			0.05	19	19
6			0.09	19	19
7			0.001	19	19
8			0.005	19	19
9			0.009	19	19
10			0.1	19	19

Performance Analysis

Sl. No.	Convergence Parameter			No. of Iteration taken to converge the shapes	
	Scaling	Rotation	Translation	Manhattan Distance	Euclidean Distance
1	0.01	$0.001 \cdot \pi / 180$	0.01	19	19
2			0.05	19	19
3			0.09	19	19
4			0.001	19	19
5			0.005	19	19
6			0.009	19	19
7			0.1	19	19
8			0.5	19	19
9			0.9	19	19




Result & Discussion

- Under the required convergence condition, the GPA takes only 2 iterations to align all the shapes irrespective of the distance metric applied (Euclidean & Manhattan)
- Euclidean & Manhattan distance metric takes equal no. of iterations of shapes, however their Procrustes distance may differ from one another and needs to be analysed.
- The mean shapes represent the average shapes within the dataset and can be seen as a central or prototype shape that captures the common characteristics of the shapes in the dataset.
- By applying translation, rotation and scaling we achieved a meaningful representation of shapes in a consistent reference frame.
- By applying normalization of point sizes we have ensure accurate alignment and meaningful analysis.



Work to be done

- ▶ Conduct a comprehensive performance analysis of Procrustes Analysis on aligned shapes to assess accuracy, robustness, and identify areas for improvement in the alignment techniques.
 - ▶ Application to Different Shape Types to extend alignment algorithms to various shape classes.
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References

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- [5] C. Goodall, "Procrustes methods in the statistical analysis of shape", *J. Roy. Stat. Soc. Series B (Methodological)* 53 (2), pp. 285–339, 1991.
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Thank you