SOLVED EXAMPLES

EXAMPLE 3.1: *Find moment of the force about point O for each case as shown in Fig. 3.35.*

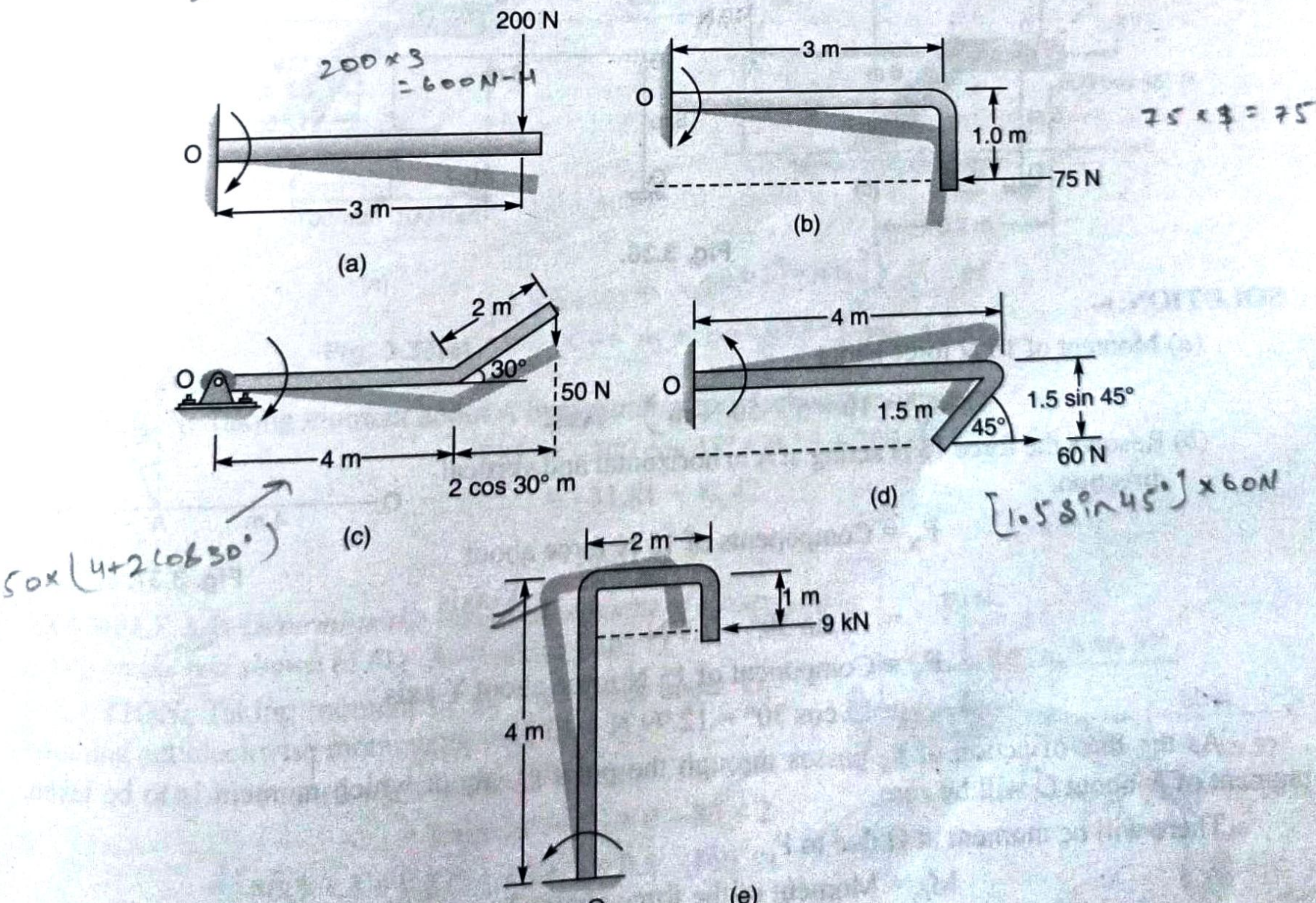


Fig. 3.35.

SOLUTION:

(*a*) Moment of 200 N force about O, $M_O = 200 \times 3 = 600$ N-m **Ans.**

(*b*) Moment of 75 N force about O, $M_O = 75 \times 1 = 75$ N-m **Ans.**

(*c*) Moment of 50 N force about O, $M_O = 50 \times (4 + 2 \cos 30°)$

$= 50 \times 5.73$

$= 286.5$ N-m **Ans.**

(*d*) Moment of 60 N force about O, $M_O = 60 \times 1.5 \sin 45°$

$= 60 \times 1.06$

$= 63.6$ N-m **Ans.**

(*e*) Moment of 9 kN force about O, $M_O = 9 \times (4 - 1) = 9 \times 3$

$= 27$ kN-m **Ans.**

EXAMPLE 3.2: *Find the moment of force shown in Fig. 3.36.*

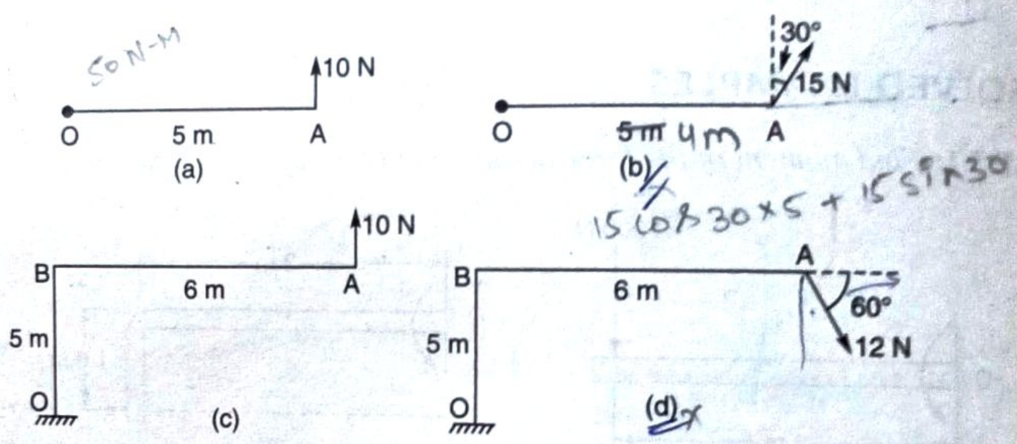


Fig. 3.36.

SOLUTION:

(*a*) Moment of 10 N force about

$O(M_O) = 10 \times 5 = 50$ N-m **Ans.**

(*b*) Resolve the force 15 N acting at A in horizontal and vertical direction.

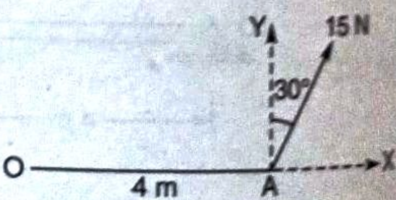


Fig. 3.37.

F_X = Components of 15 N force about X-axis

$= 15 \sin 30° = 7.5$ N

F_Y = Component of 15 N force about Y-axis

$= 15 \cos 30° = 12.99 \text{ N} \approx 13$ N

As the line of action of F_x passes through the point O, about which moment is to be taken, moment of F_x about O will be zero.

There will be moment at O due to F_y.

$\therefore$ M_O = Moment of the force about O

$= F_Y \times 4 = 13 \times 4 = 52$ N-m **Ans.**

: 59.46

. 51.96

(c) The perpendicular distance between the line of action of the force 10 N and the point O about which moment is to be taken, is 6 m.

$\therefore \quad M_O = 10 \times 6$

$= 60$ N-m ↻

(d) Resolve the force 12 N at A in two directions *i.e.*, X and Y-axis.

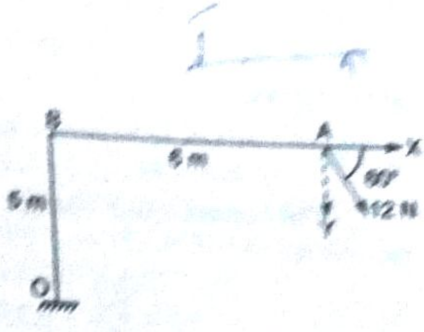


Fig. 3.38.

$\therefore \quad F_X = 12 \cos 60° = 6$ N

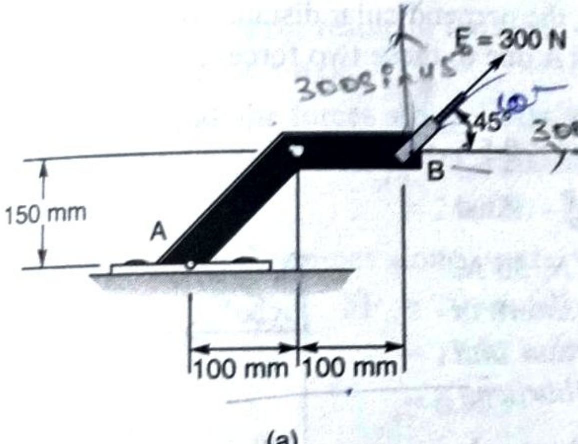
$F_Y = 12 \sin 60° = 10.39$ N

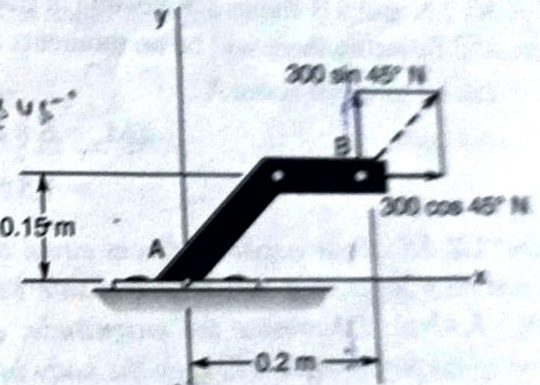
The perpendicular distance of the line of action of the force 10.39 N from O is 6 m and that of 6 N force is 5 m.

$\therefore \quad M_O = F_X \times BO + F_Y \times AB$

$= 6 \times 5 + 10.39 \times 6 = 92.34$ N-m ↻ **Ans.**

EXAMPLE 3.3: *A 300 N force acts on the bracket as shown in Fig. 3.39. Determine the moment of the force about point A.*





−(200 mm × 300 sin 45°)

Fig. 3.39(a) + 150 m + 300 cos 45° Fig. 3.39(b)

SOLUTION: Taking moment about A by assuming anticlockwise moment as +ve.

$$\Sigma M_A = -300 \cos 45° \times 0.15 + 300 \sin 45° \times 0.2$$

$$= -31.81 + 42.42$$

$$= 10.61 \text{ N} \circlearrowleft$$

EXAMPLE 3.4: *Determine the resultant moment of forces acting on the rod shown in Fig. 3.40 about point O.*

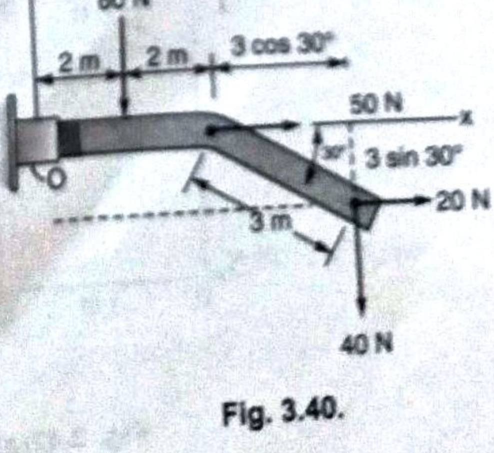


Fig. 3.40.

SOLUTION: Taking moment of all the forces about O, assuming anticlockwise moment as +ve.

$M_O = -40 \times (2 + 2 + 3 \cos 30°) + 20 \times 3 \sin 30° + 50 \times 0 - 80 \times 2$

$= -40(4 + 3 \times 0.87) + 20 \times 3 \times 0.5 - 160$

$= -40 \times 6.61 + 30 - 160$

$= -264.4 + 30 - 160 = -394.4$ N-m

$= 394.4$ N-m ↻ **Ans.**

EXAMPLE 3.5: *Four forces 2, 3, 6 and 5 N act along the sides AB, CB, CD and DA respectively of a square ABCD of side 0.5 m. Find the sum of their moments about*

(i) the centre of the square *(ii) point A.*

SOLUTION:

(*i*) Let ABCD be square in which the four forces are acting as shown in Fig. 3.41. Let O be the centre of the square.

The perpendicular distance of the point O from the sides of the square

$$= \frac{0.5}{2} = 0.25 \text{ m}$$

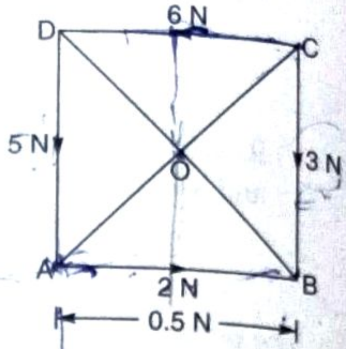


Fig. 3.41.

Let ΣM_O = Sum of moments of all the forces about O

$\Sigma M_O = 2 \times 0.25 - 3 \times 0.25 + 6 \times 0.25 + 5 \times 0.25$ (taking anticlockwise moment as +ve)

$= 2.5$ N-m **Ans.**

(*ii*) Let ΣM_A = Sum of moments of all the forces about A.

As 2 N and 5 N forces are meeting at point A, the perpendicular distance of these forces from A is zero and therefore there will be no moments about A due to these two forces.

Taking moment about A

$$\Sigma M_A = 6 \times 0.5 - 3 \times 0.5 \quad \text{(anticlockwise +ve)}$$

$$= 1.5 \text{ N-m} \quad \textbf{Ans.}$$

EXAMPLE 3.6: *Four coplanar forces equal to 20 N, 30 N, 50 N and 70 N are acting on a square of side 1 m as shown in the Fig. 3.42(a). Determine the magnitude, direction and position of the force which will keep the body in equilibrium.*

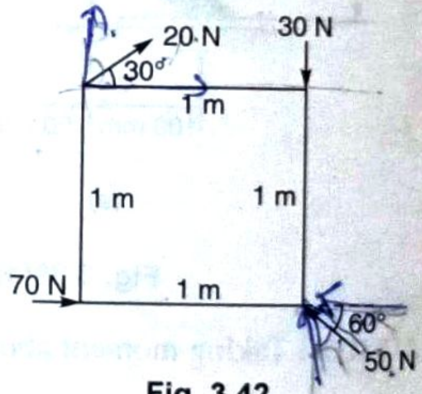


Fig. 3.42.

SOLUTION: Let R be the resultant force θ be the angle between the resultant R and the *x*-axis.

Resolving all the forces along *x*-axis, we get

$$\Sigma F_x = 70 + 20 \cos 30° - 50 \cos 60°$$

$$= 70 + 17.32 - 25 = 62.32 \text{ N}$$

$$\Sigma F_y = 50 \sin 60° + 20 \sin 30° - 30$$

$$= 43.30 + 10 - 30 = 23.30 \text{ N}$$

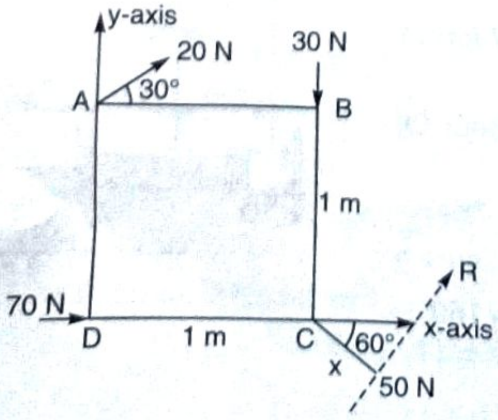


Fig. 3.42(*a*)

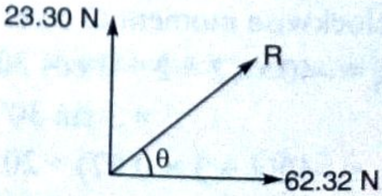


Fig. 3.42(*b*)

Now

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(62.32)^2 + (23.30)^2} = 66.53 \text{ N}$$

and

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{23.30}{62.32}$$

$\therefore$

$$\theta = 20.50° = 20°30'$$

Let the resultant force R acts at a distance x from point C as shown in the Fig. 3.42(a).

Moment of the resultant about C = Algebraic sum of moments of all the forces about C.

$$Rx = 20 \cos 30° \times 1 + 20 \sin 30° \times 1$$

$$66.53x = 17.32 + 10$$

$\therefore$

$$x = 0.41 \text{ m}$$

$\therefore$ The resultant R acts at distance of 0.41 m from point C.

EXAMPLE 3.7: *Three forces 24 N, 20 N and 12 N are acting on a equilateral triangle of side 100 mm as shown in the Fig. 3.43. Determine the magnitude direction and position of the resultant force.*

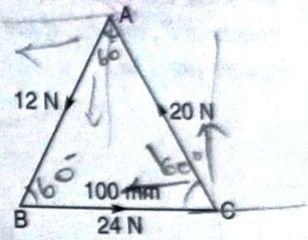


Fig. 3.43.

SOLUTION: Let R is the resultant force and θ be the angle between the resultant R and the x-axis.

Resolving all the forces along x-axis, we get

$$\Sigma F_x = 24 - 20 \cos 60° - 12 \cos 60°$$

$$= 24 - 10 - 6 = 8 \text{ N}$$

Resolving all the forces along y-axis, we get

$$\Sigma F_y = 20 \sin 60° - 12 \sin 60°$$

$$= 17.32 - 10.39$$

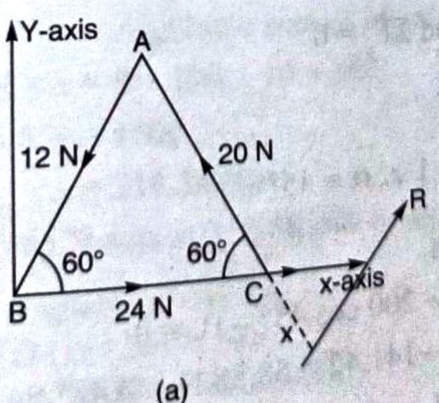
$$= 6.93 \text{ N}$$

↑ (+ve) →

(−ve) ↓

Σx 24 − 20cos60 − 12sin60

Σy = 20sin60° − 20cos60°



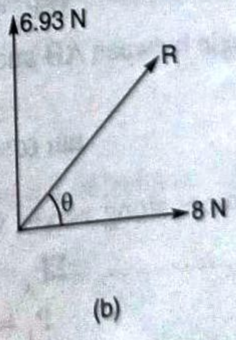


Fig. 3.43(b)

Fig. 3.43(a)

Now,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(8)^2 + (6.93)^2}$$

$$= \sqrt{64 + 48.02} = 10.58 \text{ N}$$

and $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{6.93}{8.0}$

$\therefore$ $\theta = 40.9° = 40°54'$

Let the resultant R acts at a distance x from point C as shown in Fig. 3.43(a).

Moment of the resultant about C

= Algebraic sum of moment of all the forces about C

$$Rx = 12 \sin 60° \times 100$$

or $$10.58x = 1039.23$$

$\therefore$ $$x = 98.22 \text{ mm}$$

$\therefore$ The resultant force of 10.58 N acts at a distance of 98.22 mm from point C.

EXAMPLE 3.8: *A square ABCD has forces acting along its sides as shown in the Fig. 3.44. Find the values of P_1 and P_2, if the system reduces to couple. Also find magnitude of the couple, if the sides of the square are 1 m.*

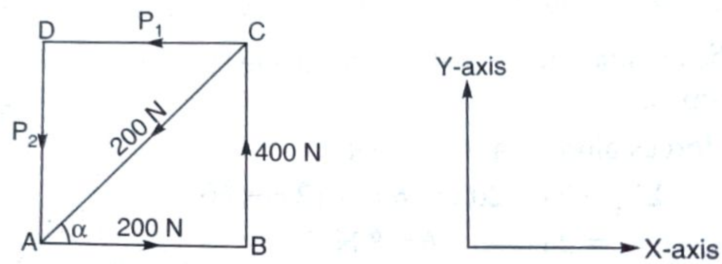


Fig. 3.44.

SOLUTION:

The force system acting on the square reduces to a couple if the resultant force of the system is zero *i.e.*, if

$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$

Let α be the angle between AB and AC

$$\tan\alpha = \frac{1}{1} = 1 \quad \therefore \alpha = 45°$$

magnitude ~ $P_1 \times P_2$ or $F_1 \times F_2$

Resolving the forces along x-axis, we get

$$\Sigma F_x = 200 - 200\cos 45° - P_1 = 0$$

$\therefore$ $$P_1 = 200 - 141.42 = 58.58 \text{ N} \quad \textbf{Ans.}$$

Resolving the forces along y-axis, we get

$$\Sigma F_y = 400 - P_2 - 200\sin 45° = 0$$

$\therefore$ $$P_2 = 400 - 141.42 = 258.58 \text{ N} \quad \textbf{Ans.}$$

Taking moment of all the forces about A, we get

$$\Sigma M_A = 400 \times 1 + 58.58 \times 1$$
$$= 458.58 \text{ N-m}$$

$\therefore$ The magnitude of the couple is 458.58 N-m **Ans.**

EXAMPLE 3.9: *Forces of 6 N, 8 N, 10 N and 12 N respectively act along the sides of a square taken in order. Find the magnitude, direction and position of line of action of their resultant if each side of a square is 100 mm long.*

fn=6-10=-4.

fy=8-12 -4.

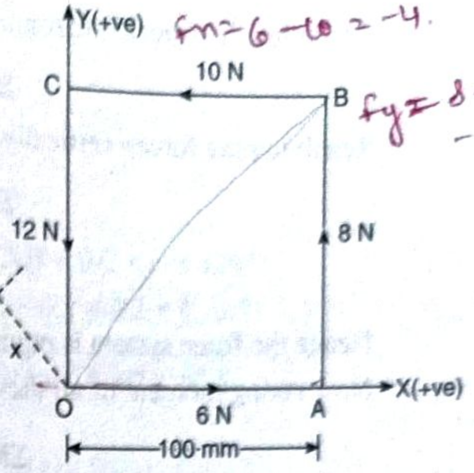


Fig. 3.45.

SOLUTION: Let OABC be a square in which four forces 6 N, 8N, 10 N and 12 N acting along OA, AB, BC and CO respectively.

Given OA = AB = BC = CO = 100 mm

Resolving all the forces horizontally, we get

$$\Sigma F_X = 6 - 10 = -4 \text{ N}$$

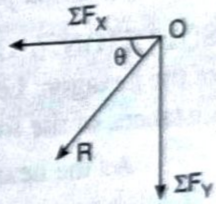
Resolving all the forces vertically, we get

$$\Sigma F_Y = 8 - 12 = -4 \text{ N}$$

Resultant, $R = \sqrt{(\Sigma F_X)^2 + (\Sigma F_Y)^2} = \sqrt{(-4)^2 + (-4)^2}$

$\therefore$ $R = 5.65$ N **Ans.**

Let θ be the angle which the resultant R makes with horizontal.



$\therefore$ $\tan\theta = \dfrac{\Sigma F_Y}{\Sigma F_X}$

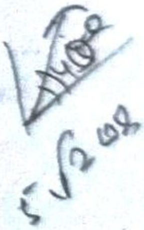
$= \dfrac{4}{4} = 1$

$\therefore$ $\theta = 45°$ **Ans.**

Let R be acting at distance x from point O, then

Moment of the resultant about O

= Algebraic sum of moment of all the forces about O.



$R \times x = 8 \times 100 + 10 \times 100$

$5.65x = 1800$

$\therefore$ $x = 318.58$ mm

The resultant force 5.65 N acts at 318.58 mm away on the left of point o. **Ans.**

EXAMPLE 3.10: *ABCD is a square whose side is 2 m. Along AB, BC, CD and DA act forces equal to 1, 2, 8 and 5 N, along AC and DB act force equal to $5\sqrt{2}$ and $2\sqrt{2}$ N. Show that they are equivalent to a couple whose moments is equal to 16 N-m.*

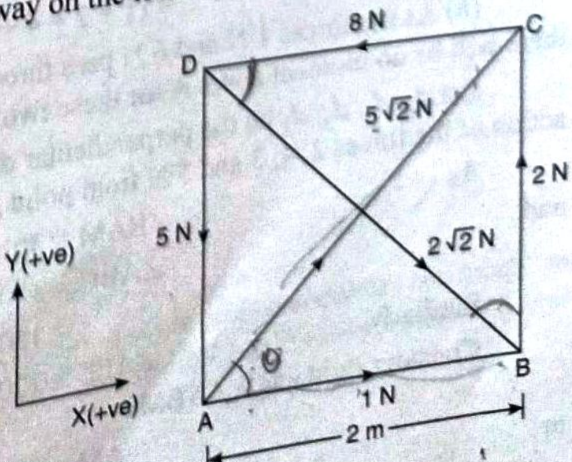


Fig. 3.46.

SOLUTION: The forces acting on the square ABCD are shown in the Fig. 3.46.

This force system is equivalent to a couple if the resultant of the force system is zero.

Resolving the forces horizontally, we get

$$\Sigma F_X = 1 - 8 + 5\sqrt{2}\ \cos 45° + 2\sqrt{2}\ \cos 45° = 0$$

Resolving the forces vertically, we get

$$\Sigma F_X = 2 - 5 + 5\sqrt{2}\ \sin 45° - 2\sqrt{2}\ \sin 45° = 0$$

$$\therefore \qquad R = \sqrt{(\Sigma F_X)^2 + (\Sigma F_Y)^2} = 0$$

Hence the force system is equivalent to a couple.

Now taking moment of all the forces about A

$$\Sigma M_A = 2 \times 2 + 8 \times 2 - 2\sqrt{2}\ \sin 45° \times 2$$

-ve +ve

$$= 16 \text{ N-m}$$

$\therefore$ Moment of the couple is 16 N-m. **Ans.**

EXAMPLE 3.11: *Forces 1, 2, 4, 3, 5 and 6 N act along the sides of a regular hexagon AB, BC, CD, DE, EF and FA respectively. If each side of the hexagon is 0.2 m. Find the sum of the moments of the forces.*

(a) about the centre of the forces

(b) about point A.

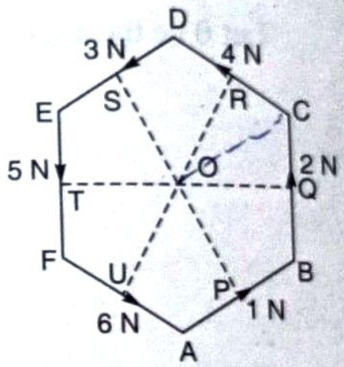


Fig. 3.47.

SOLUTION: (*a*) Let ABCDEF be a regular hexagon and O be the centre of the hexagon. Let OP, OQ, OR, OS, OT and OU be the perpendicular distances of the forces 1, 2, 4, 3, 5 and 6 N from O respectively.

As the hexagon is regular,

$$\therefore \qquad OP = OQ = OR = OS = OT = OU$$

Consider right angle ΔOCQ

$$OC^2 = OQ^2 + QC^2$$

or $$(0.2)^2 = (OQ)^2 + (0.1)^2$$

$$\therefore \qquad OQ = \sqrt{0.03} = 0.1732 \text{ m}$$

Now taking moment about O

$$\Sigma M_O = 1 \times 0.1732 + 2 \times 0.1732 + 4 \times 0.1732 + 3 \times 0.1732 + 5 \times 0.1732 + 6 \times 0.1732$$

$$= (1 + 2 + 4 + 3 + 5 + 6) \times 0.1732 = 3.64 \text{ N-m} \quad \textbf{Ans.}$$

(*b*) As the forces 1 N and 6 N pass through the point A, about which moment is to be taken, so there will be no moment about A for these two forces.

Let d_1, d_2, d_3, d_4 be the perpendicular distance of the line of action of the forces 2, 4, 3 and 5 N from point A.

As $\qquad \angle BAM = 30°, \therefore \angle ABM = 60°$

and $\qquad \angle ABC = 180° - \angle ABM$

$$= 180° - 60° = 120°$$

Similarly, $\qquad \angle AFE = 120°$

Consider right angle ΔABM

$$AM = AB \cos 30°$$

or $$d_1 = 0.2 \times 0.866 = 0.1732 \text{ m}$$

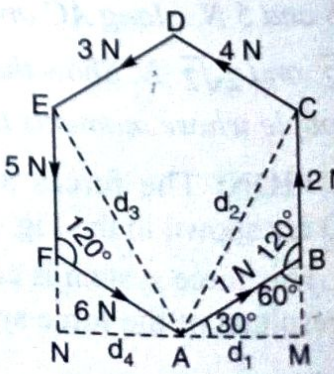


Fig. 3.48.

Similarly from ΔAFN

$$AN = AF \cos 30^\circ$$

or $$d_4 = 0.2 \times 0.866$$

$\therefore$ $$d_4 = 0.1732 \text{ m}$$

In ΔABC using consine rule, we get

$$AC^2 = AB^2 + BC^2 - 2 \times AB \times BC \times \cos 120^\circ$$

or $$(d_2)^2 = (0.2)^2 + (0.2)^2 - 2 \times 0.2 \times 0.2 \times (-0.5)$$

or $$d_2 = \sqrt{(0.2)^2 + (0.2)^2 + 2 \times 0.2 \times 0.2 \times 0.5}$$

$\therefore$ $$d_2 = 0.346 \text{ m}$$

Similarly from ΔAFE

$$AE = d_3 = 0.346 \text{ m}$$

Now taking moment of all the forces about A.

$$\Sigma M_A = 2 \times d_1 + 4 \times d_2 + 3 \times d_3 + 5 \times d_4$$
$$= 2 \times 0.1732 + 4 \times 0.346 + 3 \times 0.346 + 5 \times 0.1732$$
$$= 3.634 \text{ N-m}$$ **Ans.**

EXAMPLE 3.12: *A painter's scaffold 10 m long and weighing 0.75 kN is supported in a horizontal position by vertical ropes attached at equal distances from the ends of the scaffold. Find the greatest distance from the ends that the ropes may be attached to permit a 1 kN painter to stand at one end of the scaffold.*

SOLUTION: The F.B.D. of the scaffold is shown in Fig. 3.49.

Let T_1 and T_2 be the tensions in the ropes. Let the ropes attached at a distance of x m from both ends.

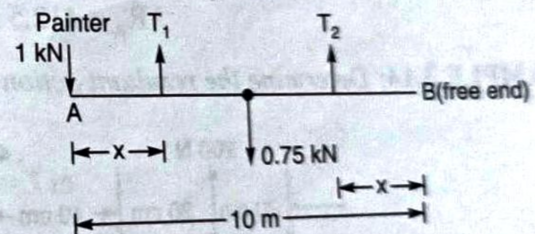


Fig. 3.49.

Resolving the forces vertically, we get

$$T_1 + T_2 = 1 + 0.75$$
$$= 1.75 \text{ kN} \quad ...(i)$$

Taking moment of all the forces about A, we get

$$T_1 x + T_2(10 - x) = 0.75 \times 5 \quad ...(ii)$$

For x to be greatest, $T_2 = 0$

Putting the value of T_2 in (*i*), we get

$$T_1 = 1.75 \text{ kN}$$

Putting the value of T_1 in (*ii*), we get

$$1.75x = 0.75 \times 5$$

$\therefore$ $$x = 2.14 \text{ m}$$ **Ans.**

EXAMPLE 3.13: *A horizontal beam 12 m long is supported at the ends and carries two loads, one of 250 N, 3 m from the left end and another of 600 N, 7.50 m from the left end. Neglecting the weight of the beam, calculate the reactions at the supports.*

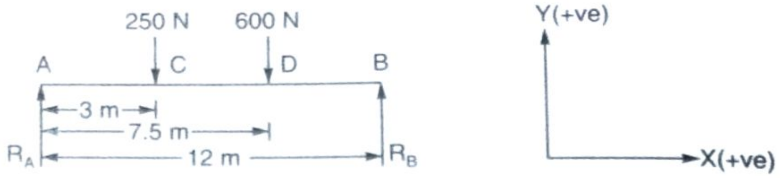


Fig. 3.50.

SOLUTION: Let R_A and R_B be the reactions at A and B respectively.

Since the beam is in equilibrium under the actions of the force

$$\therefore \qquad \Sigma F_y = 0$$

$$R_A + R_B - 250 - 600 = 0$$

$$\therefore \qquad R_A + R_B = 850 \text{ N} \qquad \ldots(i)$$

Now, $$\Sigma M_A = 0$$

i.e., Algebraic sum of moments of all the forces about A is zero.

$$R_B \times 12 = 600 \times 7.5 + 250 \times 3 = 5250$$

$$\therefore \qquad R_B = 437.5 \text{ N} \quad \textbf{Ans.}$$

Putting the value of R_B in (*i*), we get

$$R_A + 437.5 = 850$$

$$\therefore \qquad R_A = 412.5 \text{ N} \quad \textbf{Ans.}$$

EXAMPLE 3.14: *Determine the resultant action of a coplanar parallel force system in the Fig. 3.51.*

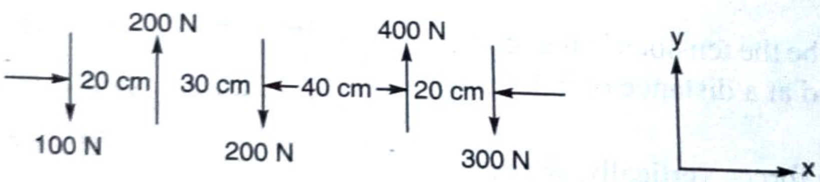


Fig. 3.51.

SOLUTION: $$\Sigma F_y = -100 + 200 - 200 + 400 - 300 = 0$$

$\therefore$ The resultant force of the force system is zero.

As the resultant force is zero, the resultant of the coplanar force system may be a couple moment.

Taking moment of all the forces about 100 N force, we get

$$\Sigma M = 200 \times 20 - 200 \times 50 + 400 \times 90 - 300 \times 110$$

(By taking anticlockwise moment as +ve)

$$= -3000 \text{ N-cm} = -30 \text{ N-m}.$$

$\therefore$ The force system can be replaced by a couple moment of –30 N-m. **Ans.**

EXAMPLE 3.15: *Determine the smallest force F that must be applied to the rope in order to cause the pole to break at its base O. This requires a moment of M = 900 N-m to be developed at O.*

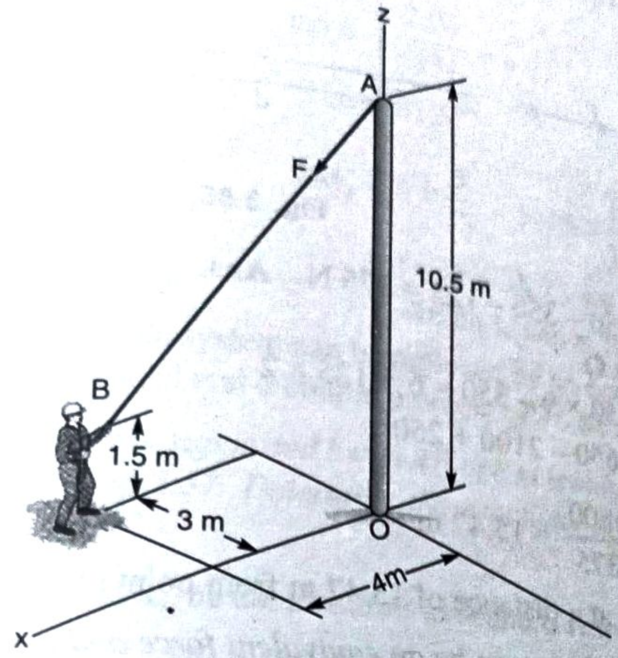


Fig. 3.52.

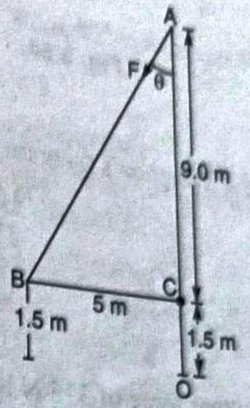


Fig. 3.53.

From Fig. 3.52.

$$BC^2 = 3^2 + 4^2$$

$\therefore$ $$BC = \sqrt{9+16} = 5 \text{ m}$$

$$\tan\theta = \frac{5}{9}$$

$\therefore$ $$\theta = 29.05^\circ$$

$$\Sigma M_O = F \sin\theta \times 10.5 = 900$$

$$F \sin 29.05^\circ \times 10.5 = 900$$

$$F \times 0.48 \times 10.5 = 900$$

$\therefore$ $$F = \frac{900}{0.48 \times 10.5} = 178.57 \text{ N.}$$ **Ans.**

EXAMPLE 3.16: *Replace the force system by a single force resultant and specify its point of application, measured along the x-axis from point O.*

SOLUTION: Let R be the resultant of the given force system which is acting at a distance of '*d*' from O.