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# Simple Harmonic Motion

Periodic Motion ⇒ A motion which repeat itself over and again after a regular interval of time is called a periodic motion and the fixed interval of time after which the motion repeated is called period of ~~m~~ the motion

Examples ⇒

(i) Revolution of Earth around the Sun (period one year)

(ii) Rotation of Earth about its polar axis (period one day)

(iii) Motion of hour's hand of a clock (period 12-hour)

## Oscillatory or Vibratory motion

Oscillatory or Vibratory motion is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point in a definite interval of time. In such a motion, the body is confined with in well-defined limits on either side of mean position.

Oscillatory motion is also called as harmonic motion.

Example

(i) The motion of the pendulum of a wall clock

(ii) The motion of a load attached to a spring, when it is pulled and then released.

(iii) The motion of liquid contained in U-tube when it is compressed once in one limb and left to itself.



## Harmonic ~~osc~~ and Non-harmonic Oscillation

Harmonic oscillation is that oscillation which can be expressed in terms of single harmonic function (i.e sine or cosine ~~formula~~ function)

$y = a\sin\omega t$ or ~~$y = a$~~ $y = a\cos\omega t$.

$\boxed{y = a\sin\omega t}$ $\boxed{y = a\cos\omega t}$

$y = a\cos\omega t$
$y = a\sin\omega t$



Non harmonic oscillation is that oscillation which can not be expressed in terms of single harmonic function. It is a combination of two or more than two harmonic oscillations.

Example: $y = a\sin\omega t + b\sin 2\omega t$..

Time period ⇒ It is the least interval of time after which the periodic motion of a body ~~to~~ repeats itself

S.I unit of time period is second.

frequency ⇒ It is defined as the number of periodic motions executed by body per second. S.I unit of frequency is hertz (Hz)

Angular frequency ⇒ Angular frequency of a body executing periodic ... of the body with fact...

In terms of Single harmonic function ( ... ) function
$y = a\sin\omega t$ or ~~y = a~~ $y = a\cos\omega t$.
$\boxed{y = a\sin\omega t}$ $\boxed{y = a\cos\omega t}$ $y = a\cos\omega t$
$y = a\sin\omega t$

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Non harmonic Oscillation is that oscillation which can not be Expressed in terms of Single harmonic fuction. It is a Combination of two or more than two Harmonic oscillations.

Example: $y = a\sin\omega t + b\sin 2\omega t$

Time period => It is the least interval of time after which the periodic motion of a body repeats itself.

S.I unit of time period is Second.

frequency => It is defined as the number of periodic motions Executed by body per Second. S.I unit of frequency is hertz (Hz)

Angular frequency => Angular frequency of a body Executing periodic motion is Equal to product of frequency of the body with factor of $2\pi$. Angular frequency $\omega = 2\pi n$.

S.I units of $\omega$ is Hz [S.I] $\omega$ also represents angular Velocity. In that case unit will be rad/sec.

Displacement => In general, the name displacement is given to a physical quantity which undergoes a change with time in a periodic motion.

Example ->

(1) In an oscillation of a loaded spring, displacement variable is its deviation from the mean position.

(2) During the propagation of Sound wave in air, the displacement Variables ~~are electric and magnetic fields which vary periodic~~ is the local change in pressure.

(3) During the propagation of Sound wave in air, the displacement Variables are Electric and magnetic fields, which vary Periodically.



(5) Phase => Phase of a ~~Vibrato~~ Vibrating particle at any instant is a physical quantity which completely express the position and direction of motion, of the particle at that instant with respect to its mean position.

In oscillatory motion the phase of a Vibrating particle is ... of Sine or Cosine fuction involved to resp...

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2) During the propagation of sound wave in air, the displacement variables are Electric and magnetic fields, which vary periodically



(5) Phase ⇒ phase of a ~~vibrato~~ vibrating particle at any instant is a physical quantity which completely express the position and direction of motion, of the particle at that instant with respect to its mean position.

In oscillatory motion the phase of a vibrating particle is the argument of sine or cosine fuction involved to ~~resp~~ represent the generalised equation of motion of the vibrating particle.

$$y = a\sin\theta = a\sin(\omega t+\phi_0) \text{ here } \theta = \omega t+\phi_0$$
= phase of vibrating particle

(i) Initial phase or epoch: It is the phase of a vibrating particle at $t=0$

In $\theta = \omega t+\phi_0$ where $t=0$ $\theta=\phi_0$ here $\phi_0$ is the angle of epoch.

(ii) Same phase: Two vibrating particle are said to be in same phase if the phase difference between them is an even multiple of $\pi$ or path difference is an even multiple of $(\lambda/2)$ or time interval is an even multiple of $(T/2)$ because time period is ~~Equal~~ Equivalent to $2\pi$ radon 1 wave length $(\lambda)$.

(iii) Opposite phase: ⇒ When the two vibrating particle cross their respective mean positions at the same time moving in opposite directions, then the phase difference between the two vibrating particles is 180°

Opposite phase means the phase difference between the particle is an odd multiple of $\pi$ (say $\pi, 3\pi, 5\pi, 7\pi \ldots$) or the path difference is an odd multiple of $\lambda$ (say $\frac{\lambda}{2}, \frac{3\lambda}{2} \ldots$) or the time interval is an odd multiple of $(T/2)$



Phase difference ⇒ If two particles performs S.H.M and their equations are

$$y_1 = a\sin(\omega t+\phi_1) \text{ and } y_2 = a\sin(\omega t+\phi_2)$$

then phase difference $\Delta\phi = (\omega t+\phi_2)-(\omega t+\phi_1) = \cancel{\phi_1-\phi_2}$ $\phi_2-\phi_1$

...le Harmonic Motion ... type of periodic difference is an odd multiple of $\pi$ (say $\frac{\pi}{2}, \frac{3\pi}{2}$ ...)

time interval is an odd multiple of $(T/2)$



Phase difference ⇒ If two particles perform S.H.M and their Equations are

$y_1 = a\sin(\omega t + \phi_1)$ and $y_2 = a\sin(\omega t + \phi_2)$

then phase difference $\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1) = \phi_2 - \phi_1$

$\phi_2 - \phi_1$

## Simple Harmonic Motion

Simple Harmonic motion is a ~~stood spec~~ special type of periodic motion, in which a particle moves to and from repeatedly about a mean position under a restoring force which is always directed towards the mean position and whose magnitude at any instant is directly proportional to the displacement of the particle from ~~the~~ the mean position at that instant.

Restoring force $\propto$ Displacement of the particle from mean Position

$F \propto -x$

$F = -kx$

Where k is known as force constant, ~~its~~ its S.I unit ~~is~~ is Newton/meter and ~~its~~ dimension is $[MT^{-2}]$

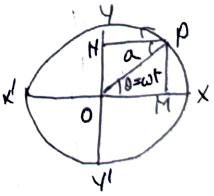


## Displacement in S.H.M

The displacement of a particle executing S.H.M at an instant is defined of particle from the mean position at that instant.

As we know that simple harmonic motion is defined as the projection of uniform circular motion on any diameter of circle of reference. If the projection is taken on y-axis.

then from the figure $y = a\sin\omega t$




$y = a\sin\frac{2\pi}{T}t$

$y = a\sin 2\pi n t$

$y = a\sin(\omega t \pm \phi)$

Where $a$ = Amplitude

$\omega$ = Angular frequency $t$ = Instantaneous time,

$T$ = Time period $n$ = frequency and $\phi$ = Initial phase of particle.



If the projection of P is taken on X-axis then Equations of SHM can be given as

$x = a\cos(\omega t \pm \phi)$

$x = a\cos\left(\frac{2\pi}{T}t \pm \phi\right)$

$y = a \sin \frac{2\pi}{T} t$

$y = a \sin 2\pi n t$

$y = a \sin(\omega t \pm \phi)$

where $a$ = Amplitude
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$x = a \cos\left(\frac{2\pi}{T} t \pm \phi\right)$

$x = a \cos(2\pi n t \pm \phi)$



(1) $y = a \sin \omega t$ when the time is noted from the instant when the vibrating particle is at mean position.

(2) $y = a \cos \omega t$ when the time is noted from the instant when the vibrating particle is at Extreme position.

(3) $y = a(\sin(\omega t \pm \phi))$ when the vibrating particle is $\phi$ phase leading or lagging from the mean position.

(4) Direction of displacement is always away from the Equilibrium position. Particle either is moving away from or is coming towards the Equilibrium position.

(5) If $t$ is given or phase ($\theta$) is given, we can calculate the displacement of the particle.

if $t = \frac{T}{4}$ (or $\theta = \frac{\pi}{2}$) then from Equation $y = a \sin \frac{2\pi}{T} t$

we get $y = a \sin \frac{2\pi}{T} \cdot \frac{T}{4} = a \sin\left(\frac{\pi}{2}\right) = a$

Similarly if $t = \frac{T}{2}$ (or $\theta = \pi$) then we get $y = 0$



## Velocity of S.H.M

Velocity of the particle executing S.H.M at any instant, is defined as the time rate of change of its displacement at that instant.



In case of S.H.M when motion is considered from the equilibrium position

$y = a \sin \omega t$

$v = \frac{dy}{dt} = a\omega \cos \omega t$

Velocity of the particle executing ... defined as the time rate of change of its displacement at that instant



In case of S.H.M when motion is considered from the equilibrium position

$y = a \sin\omega t$

$v = \frac{dy}{dt} = a\omega \cos\omega t$

$v = a\omega \cos\omega t$ —(i)

or $v = a\omega \sqrt{1-\sin^2\omega t}$ As $\sin\omega t = \frac{y}{a}$

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$v = \omega\sqrt{a^2-y^2}$ —(ii)



(1) In SHM velocity is maximum at equilibrium position

From Equation(i) $V_{max} = a\omega$ when $|\cos\omega t| = 1$ i.e $\theta = \omega t = 0$

From Equation(ii) $V_{max} = a\omega$ when $y = 0$

(2) In SHM velocity is ~~max~~ minimum at extreme position

From Equation(i) $V_{min} = 0$ when $|\cos\omega t| = 0$ $\theta = \omega t = \frac{\pi}{2}$

From Equation(ii) $V_{min} = 0$ when $y = a$

(3) Direction of velocity is either towards or away from mean position depending on the position of particle.

Acceleration in S.H.M

The acceleration of the particle executing S.H.M at any instant defined as the rate of change of its velocity at that instant

So acceleration $A = \frac{dv}{dt} = \frac{d}{dt}(a\omega \cos\omega t)$

$A = -\omega^2 a \sin\omega t$ —(i)

$A = -\omega^2 y$ —(ii) As $(y = a\sin\omega t)$



(1) In SHM as $|\text{Acceleration}| = \omega^2 y$ is not constant, so equations of translatory motion ~~can~~ can not be applied

(2) In SHM acceleration is maximum at extreme position

From Equation(i) $|A_{max}| = \omega^2 a$ when $|\sin\omega t|$ - maximum - 1

i.e at $t = \frac{T}{4}$ or $\omega t = \frac{\pi}{2}$

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i.e at $t = \frac{T}{4}$ or $\omega t = \frac{\pi}{2}$

From Equation(ii) $|A_{max}| = \omega^2 a$ when $y = a$

(iii) from Equation In SHM acceleration is minimum at Mean Position

From Equation(i) $A_{min} = 0$ when $\sin\omega t = 0$ i.e at $t = 0$ or $t = \frac{T}{2}$ or $\omega t = \pi$

From Equation(ii) $A_{min} = 0$ when $y = 0$

(4) Acceleration is always directed towards the mean position and so is always opposite to ~~dis~~ displacement

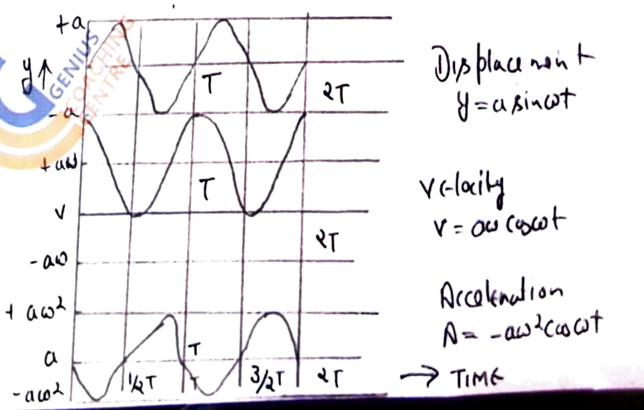
i.e $\boxed{A \propto -y}$

Comparative Study of displacement, velocity and Acceleration.

Displacement $y = a\sin\omega t$

velocity $v = a\omega\cos\omega t = a\omega \sin(\omega t + \frac{\pi}{2})$

Acceleration $A = -a\omega^2 \sin\omega t = a\omega^2 \sin(\omega t + \pi)$



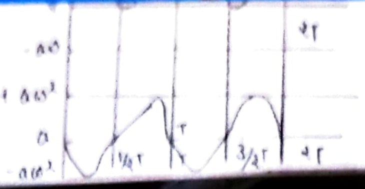



All three quantities displacement, velocity and acceleration show harmonic variation with time having same period.

The velocity amplitude is $\omega$ times the displacement amplitude

The acceleration amplitude is $\omega^2$ times the displacement amplitude

... displacement by a phase angle




$v = a\omega\cos\omega t$

Acceleration
$A = -a\omega^2\cos\omega t$

→ Time



(1) All three quantities displacement, velocity and acceleration show harmonic variation with time having same period.
(2) The velocity amplitude is $\omega$ times the displacement amplitude
(3) The acceleration amplitude is $\omega^2$ times the displacement amplitude.
(4) In SHM the velocity is ahead of displacement by a phase angle $\pi/2$
(5) In SHM the acceleration is ahead of displacement by a phase angle of $\pi$
(6) The acceleration is ahead of velocity by a phase angle $\pi/2$
(7) Various physical quantities in SHM at different position.



| quantities | Equilibrium quantities ($y=0$) | Extreme position ($y=\pm a$) |
|---|---|---|
| Displacement $y = a\sin\omega t$ | Minimum (zero) | Maximum ($a$) |
| velocity $V = \omega\sqrt{a^2 - y^2}$ | Maximum ($a\omega$) | Minimum (zero) |
| Acceleration $\lvert A\rvert = \omega^2 y$ | Minimum (zero) | Maximum ($a\omega^2$) |





Energy in SHM
A particle executing SHM possesses two types of Energy : Potential Energy and Kinetic Energy
(1) Potential Energy This is an ~~on~~ account of the displac

Energy in SHM

A particle executing SHM possesses two types of Energy : Potential Energy and Kinetic Energy

(1) Potential Energy  This is an account of the displacement of the particle from its mean position.

The restoring force $F = -ky$ against which work has to be done

$$U = -\int dw = -\int_0^x F\,dx = \int_0^y ky\,dy = \frac{1}{2}ky^2$$

Potential Energy $U = \frac{1}{2}m\omega^2 y^2$ $\left[As\ \omega^2 = K/m\right]$

$$U = \frac{1}{2}m\omega^2 a^2 \sin^2\omega t \quad \left[As\ y = a\sin\omega t\right]$$

(i) potential Energy is maximum and Equal to total Energy at Extreme position

$$U_{max} = \frac{1}{2}Ka^2 = \frac{1}{2}m\omega^2 a^2$$

when $y = \pm a$; $\omega t = \frac{\pi}{2}$ $\therefore t = T/4$



Potential Energy is minimum at mean position

$U_{min} = 0$  when $y = 0$  $\omega t = 0$; $t = 0$



(2) Kinetic Energy ⇒ This is because of the velocity of the particle

Kinetic Energy = $K = \frac{1}{2}mv^2$

$$K = \frac{1}{2}ma^2\omega^2\cos^2\omega t \quad \left[As\ v = a\omega\cos\omega t\right]$$

$$K = \frac{1}{2}m\omega^2(a^2 - y^2) \quad \left[As\ v = \omega\sqrt{a^2 - y^2}\right]$$



Kinetic Energy is maximum at mean position and Equal to total Energy at mean position

$K_{max} = \frac{1}{2}m\omega^2 a^2$  when $y = 0$  $t = 0$  $\omega t = 0$

Kinetic Energy is minimum at Extreme position

when $y = a$; $t = T/4$  $\omega t = \pi/2$

$$K = \frac{1}{2} m\omega^2 (a^2 - y^2) \qquad [\text{As } v = \omega\sqrt{a^2-y^2}]$$



(i) Kinetic Energy is maximum at mean position and equal to total Energy at mean position

$$K_{max} = \frac{1}{2} m\omega^2 a^2 \quad \text{when } y=0 \quad t=0 \quad \omega t = 0$$

(ii) Kinetic Energy is minimum at Extreme position

$$K_{min} = 0 \quad \text{when } y = a : \; t = T/4 \quad \omega t = \pi/2$$

Total Energy ⇒ Total mechanical Energy = Kinetic Energy + potential Energy

$$E = \frac{1}{2} m\omega^2 (a^2 - y^2) + \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2 a^2$$

Total Energy is not a position function

it always remains Constant.

Time period and frequency of SHM

For SHM restoring force is proportional to the displacement

$F \propto y$ or $F = -Ky$ —(i) (where K is a force constant)

For S.H.M acceleration of the body $A = -\omega^2 y$ —(ii)

∴ Restoring force on the body $F = mA = -m\omega^2 y$ —(iii)

Eq. (i) and (iii) $Ky = m\omega^2 y \Rightarrow \omega = \sqrt{\frac{K}{m}}$

$$\text{Time period} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$$

$$\text{Frequency } (n) = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

In different types of SHM the quantities m and K will go on taking different forms and names.



In general m is called inertia factor and K is called spring factor

$$\text{Thus} = T = 2\pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$$

$$n = \frac{1}{\ldots}\sqrt{\frac{\text{spring factor}}{\ldots}}$$

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Thus $T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$

$$n = \frac{1}{2\pi}\sqrt{\frac{\text{Spring factor}}{\text{Inertia factor}}}$$

In linear SHM the spring factor stands for force per unit displacement and inertia factor for mass of the body executing SHM and in angular SHM K stands for restoring torque per unit angular displacement and inertial factor for moment of inertia of body executing SHM

For linear SHM $T = 2\pi\sqrt{\frac{m}{K}} = \sqrt{\frac{m}{\text{Force/displacement}}} = 2\pi\sqrt{\frac{m \times \text{displacement}}{m \times \text{Acceleration}}}$

$$= 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{y}{A}}$$

Or $n = \frac{1}{2\pi}\sqrt{\frac{\text{Acceleration}}{\text{Displacement}}} = \frac{1}{2\pi}\sqrt{A/y}$

## Differential Equation of SHM

For SHM (linear) Acceleration $\propto -(\text{Displacement})$

$$A \propto -y$$

or

$$A = -\omega^2 y$$

or

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

$$m\frac{d^2y}{dt^2} + Ky = 0 \qquad \boxed{\text{As } \omega = \sqrt{\frac{K}{m}}}$$

For angular SHM $\tau = -c\theta$ and $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$ I = MOI Moment of inertia

$\omega^2 = \frac{c}{I}$ [As c = Restoring torque constant and



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## Simple Pendulum

An ideal simple pendulum consists of a heavy point mass body suspended by a weightless, inextensible and perfectly flexible string from a rigid body support about which it is free to oscillate.

... string exist, some

In angular SHM  $m\frac{d^2y}{dt^2}+ky=0$  $\left[As\ \omega=\sqrt{\frac{k}{m}}\right]$

$\tau = -c\theta$ and $\frac{d^2\theta}{dt^2}+\omega^2\theta=0$

$\omega^2 = \frac{c}{I}$ As c = Restoring torque constant and I = ...



## Simple Pendulum

An ideal simple pendulum consists of a heavy point mass body suspended by a weightless, inextensible and perfectly flexible string from a rigid body support about which it is free to oscillate.

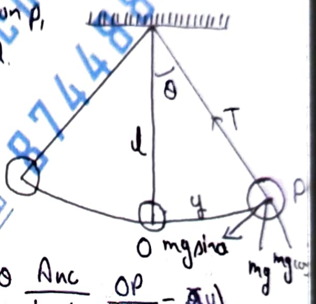
But in reality neither point mass nor weightless string exist, so we can never construct a simple pendulum strictly according to the defination.

let mass of the bob = m

Length of simple pendulum = $l$

Displacement of mass from mean position (OP) = x

When the bob is displaced to position P, through a small angle θ from the vertical. Restoring force acting on the bob




$F = -mg\sin\theta$

$F = -mg\theta$ (when θ is small $\sin\theta \simeq \theta$)   $\frac{\text{Arc}}{\text{length}}$   $\frac{OP}{l} = \frac{y}{l}$

$F = -mg\frac{y}{l}$

$\frac{F}{y} = -\frac{mg}{l}$ K (spring factor)

So time period $T = 2\pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}} = 2\pi\sqrt{\frac{m}{mg/l}}$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$





Important points

(i) The period of simple pendulum is independent of amplitude as long as its motion is simple harmonic. But if θ is not small $\sin\theta \neq \theta$ then motion will not remain simple harmonic but will become oscillatory. In this situation ... the amplitude of motion

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Important points

(1) The period of simple pendulum is independent of amplitude as long as its motion is simple harmonic. But if $\theta$ is not small $\sin\theta \neq \theta$ then motion will not remain simple Harmonic but will become oscillatory. In this situation if $\theta_0$ is the amplitude of motion

$$T = 2\pi\sqrt{\frac{l}{g}}\left[1+\frac{1}{2^2}\sin^2\left(\frac{\theta_0}{2}\right)+\cdots\right] \simeq T_0\left[1+\frac{\theta_0^2}{16}\right]$$

(2) Time period of simple pendulum is also independent of mass of the bob. This is why.

(a) If the solid bob is replaced by a ~~hole~~ hollow sphere of same radius but different mass time period remains unchanged.

(b) If a girl is swinging in a swing and another sits with her the time period remains unchanged.

(3) Time period $T \propto \sqrt{l}$ where $l$ is distance between point of suspension and centre of mass of bob and also called effective length.

(a) When a sitting girl on a swinging swings stand up, then centre of mass will go up and so $l$ and hence $T$ will decrease

(b) If a hole is made at the bottom of hollow sphere full water and water comes out slowly through the hole and time period is recorded till sphere is empty. initially and finally the centre of mass will be at the centre of the sphere. However, as water drains off the sphere, the center of mass of the system will the first move down and then will come up. Due to this $l$ and hence $T$ first increase, reaches a ~~min~~ max and decrease till it becomes equal to its initial value.



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(4) If the length of the pendulum is comparable to the radius of earth then $T = 2\pi\sqrt{\frac{1}{g\left[\frac{1}{l}+\frac{1}{R}\right]}}$

(a) if $l << R$ then $\frac{1}{l} >> \frac{1}{R}$ so $T = 2\pi\sqrt{\frac{l}{g}}$

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However, as [illegible] and then will [illegible] of the system will the first move down and [illegible] Due to this and hence T first increase, reaches a maximum and decrease till it becomes equal to its initial value.



(4) If the length of the pendulum is comparable to the radius of Earth then $T = 2\pi\sqrt{\dfrac{1}{g\left[\frac{1}{l}+\frac{1}{R}\right]}}$

(a) if $l \ll R$ then $\frac{1}{l} \gg \frac{1}{R}$ so $T = 2\pi\sqrt{\frac{l}{g}}$

(b) if $l \gg R$ ($\to\infty$) $1/l < 1/R$ so $T = 2\pi\sqrt{\frac{R}{g}}$

$$= 2\pi\sqrt{\frac{6.4\times10^6}{10}} = 84.6 \text{ minutes}$$

and it is maxi timeperiod which an oscillating simple pendulum can have

(c) If $l = R$ so $T = 2\pi\sqrt{\frac{R}{2g}} \simeq 1$ hour.

(5) If the bob of simple pendulum is suspended by a wire the effective length of pendulum will increase with the rise of temperature due to which the time period will increase.

$l = l_0(1+\alpha\Delta\theta)$ (if $\Delta\theta$ is the rise in temperature
$l_0$ = initial length of wire
$l$ = final length of wire)

$$\frac{T}{T_0} = \sqrt{\frac{l}{l_0}} = (1+\alpha\Delta\theta)^{1/2} \simeq 1+\frac{1}{2}\alpha\Delta\theta$$

$$\frac{T}{T_0} - 1 = \frac{1}{2}\alpha\Delta\theta \quad \text{i.e.} \quad \frac{\Delta T}{T} \simeq \frac{1}{2}\alpha\Delta\theta$$



In absence of resistive force of the [illegible] pendulum in one complete oscillation is zero
Work done in giving an angular displacement θ to the pendulum from its mean position

$W = U = mgl(1-\cos\theta)$

($l_0$ initial length of wire
$l$ final length of wire)

$$\frac{T}{T_0} = (1+\alpha\Delta\theta)^{1/2} \approx 1 + \frac{1}{2}\alpha\Delta\theta$$

$$\frac{T}{T_0} - 1 = \frac{1}{2}\alpha\Delta\theta \quad \text{i.e.} \quad \frac{\Delta T}{T} \approx \frac{1}{2}\alpha\Delta\theta$$



(6) In absence of resistive force of simple pendulum in one complete oscillation is zero

(7) Work done in giving an angular displacement θ to the pendulum from its mean position

$$W = U = mgl(1-\cos\theta)$$

(8) Kinetic Energy of the bob at mean position = work done on potential Energy at extreme

$$KE_{mean} = mgl(1-\cos\theta)$$

(9) Various graph of simple pendulum