ASSIGNMENT - 2

NAME : AMRITANSHU KESHARI
BRANCH : COMPUTER SCIENCE ENGG.
ROLL NUMBER : 19402060006
SUBJECT : ENGG. MATHEMATICS - III
SESSION : 2019 - 22
SEMESTER : 3rd SEMESTER
COLLEGE : GOVT. POLYTECHNIC COLLEGE, ADITYAPUR.

Page No. :
Date :

Q1. Bisection method a root lies between 2 and 3 of equation $x^3 - 5n + 1 = 0$.

Solution : Given,

$f(x) = x^3 - 5n + 1$

There is a root between (2, 3)

$f(n) = x^3 - 5n + 1$

$f(2) = 8 - 10 + 1 = -1$

$f(3) = 27 - 15 + 1 = 14$

$a = 2$, $f(2) = -1$

$b = 3$, $f(3) = 14$

1st iteration :

$$m = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$$

$$f(m) = (2.5)^3 - 5 \times (2.3) + 1$$
$$= 4.125$$

Because $f(m) > 0$, we replace two to with m

$a = 2$, $f(2) = -1$

$b = 2.5$, $f(2.5) = 4.125$

2nd iteration:

$$m = \frac{a+b}{2} = \frac{2+2.5}{2} = 2.25$$

$$f(2.25) = (2.25)^3 - 5 \times (2.25) + 1$$
$$= 1.141$$

Because $f(m) > 0$, we replace b with m

$a = 2$, $f(2) = -1$ 29
$b = 2.25$, $f(2.25) = 1.141$ 10

3rd iteration: 41

$$m = \frac{a+b}{2} = \frac{2+2.25}{2} = 2.125$$

$$f(2.125) = (2.125)^3 - 5(2.125) + 1$$
$$= -0.029$$

Because $f(m) < 0$, we replace 'a' with m

$a = 2.125$, $f(2.125) = -0.029$
$b = 2.25$, $f(2.25) = 1.141$

4th iteration:

$$m = \frac{a+b}{2} = \frac{2.125 + 2.25}{2}$$
$$= 2.1875$$

$$f(m) = (2.1875)^3 - 5(2.1875) + 1$$
$$= 0.53$$

Hence, Approximate root of the equation $x^3 - 5n + 1$ using bisection method is 2.1875.

Q2. Regular falsi method (0, 1)
$$x^3 + x - 1 = 0$$

Solution : Given, $x^3 + x - 1 = 0$
There is a root lie between (0, 1).
By using Regular falsi method

$$f(n) = x^3 + x - 1$$
$$f(0) = 0 + 0 - 1 = -1$$
$$f(1) = 1 + 1 - 1 = 1$$

$$x_0 = 0, \quad f(0) = -1$$
$$x_1 = 1, \quad f(1) = 1$$

1st iteration :

ANNU

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \times f(x_0)$$

$$= 0 - \frac{1-0}{1+0}(-1)$$

$$= 0.5$$

$$f(x_2) = (0.5)^3 + (0.5) - 1$$
$$= -0.375 < 0$$

Because, $f(x_2) < 0$,
we replace x_0 with x_2

$x_0 = 0.5$, $f(x_0) = -0.375$
$x_1 = 1$, $f(x_1) = 1$

2nd iteration:

$$x_3 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \times f(x_0)$$

$$= 0.5 - \frac{1 - 0.5}{1 + 0.375}(-0.375)$$

$$= 0.5 + \frac{0.5 \times 0.375}{1.375}$$

$$= 0.5 + 0.14$$
$$= 0.64$$

$$f(x_3) = (0.64)^3 + (0.64) - 1$$
$$= -0.098 < 0$$

Because $f(x_3) < 0$, we replace x_0 with x_3

Now, $x_0 = 0.64$, $f(x_0) = -0.098$
$x_1 = 2$, $f(x_1) = 1$

3rd iteration:

$$x_4 = 0.64 - \frac{1 - 0.64}{1 + 0.098}(-0.098)$$
$$= 0.64 + \frac{0.36 \times 0.098}{1.098}$$
$$= 0.64 + 0.03$$
$$= 0.67$$

$$f(x_4) = (0.67)^3 + (0.67) - 1$$
$$= -0.006 < 0$$

Because $f(x_4) < 0$, we replace with x_4

Now, $x_0 = 0.67$, $f(x_0) = -0.006$
$x_1 = 1$ $f(x_1) = 1$

4th iteration:

$$x_5 = 0.68 + \frac{0.32 \times 0.006}{1.006}$$

$= 0.68 + 0.002$
$= 0.682$

$f(x_8) = -0.00078$

Hence, the approximate root of the equation $x^3 + x - 1 = 0$ using regular falsi method is 0.682.

Q3. Newton's Raphson Method
$x^3 - 5x^2 + 4x - 3 = 0$

Solution: Given, $x^3 - 5x^2 + 4x - 3 = 0$
By newton's raphson method.

$f(x) = x^3 - 5x^2 + 4x - 3$
$f'(x) = 3x^2 - 10x + 4$

x	0	1	2	3	4	5
$f(x)$	-3	-3	-7	-9	-3	17

Here $f(4) = -3 < 0$ and
$f(5) = 17 > 0$

$\therefore$ Root lies between 4 & 5
Let $x_0 = \frac{4+5}{2} = 4.5$

1st Iteration :

$f(x_0) = (4.5)^3 - 5(4.5)^2 + 4 \times (4.5) - 3$

$= 4.875$

$f'(x_0) = 3 \times (4.5)^2 - (4.5) \times 10 + 4$

$= 19.75$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 4.5 - \frac{4.875}{19.75}$

$= 4.253$

2nd iteration :

$f(x_1) = (4.253)^3 - 5 \times (4.253)^2 - 4(4.253) - 3$

$= 0.5003$

$f'(x_1) = 3 \times (4.253)^2 - 10 \times (4.253) + 4$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 4.253 - 0.0318$

$x_2 = 4.2212$

3rd iteration :

$f(x_2) = (4.2212)^3 - 5(4.2212)^2 + 4 \times (4.2212) - 3 = 0.0078$

$f'(x_2) = 3 \times (4.2212)^2 - 10 \times (4.2212) + 4$

$= 15.2436$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 4.2212 - \frac{0.0078}{15.2436}$

$= 4.2212 - 0.00051$

$= 4.22$

Hence, the approximate root of the equation $x^3 - 5x^2 + 4x - 3 = 0$ using newton raphson method is 4.22.

Q4. $\sqrt[3]{41}$ by newton raphson method

Solution: $f(x) = \sqrt[3]{41}$

$x^3 = 41$

$x^3 - 41 = 0$

By newton raphson method :

$f(x) = x^3 - 41$

$f'(x) = 3x^2$

Here,

x	0	1	2	3	4
$f(x)$	-41	-40	-33	-14	23

Here root lies between 3 and 4.

$x_0 = 4$

1st iteration:

$f(x_0) = (4)^3 - 41 = 23$

$f'(x_0) = 3 \times (4)^2 = 48$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 4 - \frac{23}{48}$

$= 4 - 0.48$

$x_1 = 3.52$

2nd iteration :

$f(x) = (3.52)^3 - 41 = 2.614$
$f'(x) = 3(3.52)^2 = 37.171$
$x_2 = x_1 - \frac{f(x_1)}{f'(x_2)} = 3.52 - \frac{2.614}{37.171}$
$= 3.52 - 0.0703$
$x_2 = 3.4497$

3rd iteration:
$f(x_2) = (3.4497)^3 - 41 = 0.0529$
$f'(x_2) = 3 \times (3.4497)^2 = 35.7012$
$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 3.4497 - \frac{0.0529}{35.70}$
$x_3 = 3.4497 - 0.0015$
$x_3 = 3.4482$
Hence, the approximate value of $\sqrt[3]{41}$ lies using newton raphson method is 3.4482

Q5. Using Gauss elimination process solved the equation.
$3x - y - z = 4$
$x - 4y - z = -5$
$x + y - 6z = -12$

Solution: Given, $3x - y - z = 4$ —— (I)
$x - 4y - z = -5$ —— (II)
$x + y - 6z = -12$ —— (III)
By using gauss elimination process.

CLASSMATE
Date:
Page: 10

Step 1 :- Multiply (II) by 3 and substract from (I)

$$3x - y - z = 4$$
$$\cancel{3x} + 12y - 3z = -15$$
$$- \quad - \quad + \qquad +$$

$$-13y + 2z = 19$$

Step 2: Multiply (III) by 3 and substract from (I)

$$3x - y - z = 4$$
$$3x - 3y - 18z = -36$$
$$- \quad + \quad + \qquad +$$

$$-4y + 17z = 40$$

Now, required equation,

$$3x - y - z = 4$$
$$-13y + 2z = 19 \quad \text{--- (IV)}$$
$$-4y + 17z = 40 \quad \text{--- (V)}$$

Step 3: Multiply (V) by 13/4 and substract from (IV)

$$-13y + 2z = 19$$
$$-4 \times \frac{13}{4} y + 17 \times \frac{3}{4} z = 40 \times \frac{13}{4}$$

$$2z - \frac{17 \times 13}{4} z = 19 - \frac{40 \times 13}{4}$$

$$\Rightarrow \frac{(8 - 221)}{4} z = \frac{76 - 520}{4}$$

$\Rightarrow \frac{-213}{4} z = \frac{-444}{4}$

$\therefore z = \frac{444}{213} = \frac{148}{71}$

Put the value of z in eq^n (V)

$-4y + 17 \times \frac{148}{71} = 40$

$-4y = 40 - \frac{17 \times 148}{71}$

$y = -10 + \frac{17 \times 37}{71}$

$y = \frac{-710 + 629}{71}$

$\therefore y = \frac{-81}{21}$