Assignment

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CS21BTECH11004

1.5: Verify your result theoretically given Z that,

$$E[U^k] = \int_{-\infty}^{\infty} x^k dF_U(x)$$

Solution:

$F_U(x)$ for uniform distribution,

$$F_U(x) = \begin{cases} 0, & x < 0 \\ x, & x \in [0,1] \\ 1, & x > 1 \end{cases} \tag{1}$$

$$E[U] = \int_{-\infty}^{\infty} x dF_U(x) \tag{2}$$
$$= \int_{-\infty}^{0} 0dx + \int_{0}^{1} xdx + \int_{1}^{\infty} 0dx \tag{3}$$
$$= \frac{1}{2} \tag{4}$$

$$E[U^2] = \int_{-\infty}^{\infty} x^2 dF_U(x) \tag{5}$$
$$= \int_{-\infty}^{0} 0dx + \int_{0}^{1} x^2 dx + \int_{1}^{\infty} 0dx \tag{6}$$
$$= \frac{1}{3} \tag{7}$$

$$E[U - E[U]]^2 = E[U^2] - [E[U]]^2 \tag{8}$$

From Equation (4) and (7),

$$E[U - E[U]]^2 = \frac{1}{3} - \left(\frac{1}{2}\right)^2 \tag{9}$$
$$= \frac{1}{3} - \frac{1}{4} \tag{10}$$
$$= \frac{1}{12} \approx 0.083 \tag{11}$$