Random Numbers

Anshul Sangrame
CS21BTECH11004

1 UNIFORM RANDOM NUMBERS

Let U be a uniform random variable between 0 and 1.

1.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution:
Download the following files,

```
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/solution/1/
    coeffs.h
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/solution
    /1/1_1.c
```

Execute the above files using code,

```
gcc 1_1.c −lm
./a.out
```

1.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \le x) \qquad (1.1)$$

Solution:
Download the following files

```
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/solution
    /1/1_2.py
```

Execute the code using command

```
python3 1_2.py
```
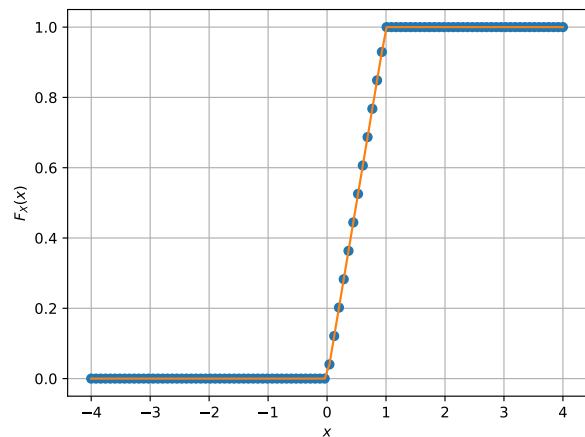
Plot 1.1 is obtained,

Fig. 1.1

1.3 Find a theoretical expression for $F_U(x)$.
Solution:
Pdf of Uniform distribution between [0,1] is given by,

$$f_U(x) = \begin{cases} 1, & x \in [0,1] \\ 0, & \text{otherwise} \end{cases} \qquad (1.2)$$

$$F_U(x) = \int_{-\infty}^{x} f_U(x)dx \qquad (1.3)$$

Case-1: x < 0,

$$F_U(x) = \int_{-\infty}^{x} 0dx \qquad (1.4)$$

$$= 0 \qquad (1.5)$$

Case-2: x ∈ [0,1],

$$F_U(x) = \int_{-\infty}^{0} 0\,dx + \int_{0}^{x} 1\,dx \qquad (1.6)$$

$$= x \qquad (1.7)$$

Case-3: x > 1,

$$F_U(x) = \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} 1\,dx + \int_{1}^{x} 0\,dx \qquad (1.8)$$

$$= 1 \qquad (1.9)$$

Hence,

$$F_U(x) = \begin{cases} 0, & x < 0 \\ x, & x \in [0,1] \\ 1, & x > 1 \end{cases} \qquad (1.10)$$

1.4 The mean of U is defined as

$$E[U] = \frac{1}{N} \sum_{i=1}^{N} U_i \qquad (1.11)$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \qquad (1.12)$$

Write a C program to find the mean and variance of U.
Solution:
Download the following files,

wget https://github.com/Anshul−Sangrame/
AI1110/blob/main/Assignment/solution/1/
coeffs.h
wget https://github.com/Anshul−Sangrame/
AI1110/blob/main/Assignment/solution
/1/1_4.c

Execute the above files using code,

gcc 1_4.c −lm
./a.out

The following result is obtained

$$E[U] = 0.500007$$
$$\text{var}[U] = 0.083301$$

1.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x)\,dx \qquad (1.13)$$

Solution:

$F_U(x)$ for uniform distribution,

$$F_U(x) = \begin{cases} 0, & x < 0 \\ x, & x \in [0,1] \\ 1, & x > 1 \end{cases} \qquad (1.14)$$

$$E[U] = \int_{-\infty}^{\infty} x dF_U(x) \qquad (1.15)$$

$$= \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} x\,dx + \int_{1}^{\infty} 0\,dx \qquad (1.16)$$

$$= \frac{1}{2} \qquad (1.17)$$

$$E[U^2] = \int_{-\infty}^{\infty} x^2 dF_U(x) \qquad (1.18)$$

$$= \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} x^2\,dx + \int_{1}^{\infty} 0\,dx \qquad (1.19)$$

$$= \frac{1}{3} \qquad (1.20)$$

$$E[U - E[U]]^2 = E[U^2] - [E[U]]^2 \qquad (1.21)$$

From Equation (1.17) and (1.20),

$$E[U - E[U]]^2 = \frac{1}{3} - \left(\frac{1}{2}\right)^2 \qquad (1.22)$$

$$= \frac{1}{3} - \frac{1}{4} \qquad (1.23)$$

$$= \frac{1}{12} \approx 0.083 \qquad (1.24)$$

2 Central Limit Theorem

2.1 Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \qquad (2.1)$$

using a C program, where $U_i, i = 1, 2, \ldots, 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat
Solution:
Download the following files,

wget https://github.com/Anshul−Sangrame/
AI1110/blob/main/Assignment/solution/1/
coeffs.h
wget https://github.com/Anshul−Sangrame/
AI1110/blob/main/Assignment/solution
/2/2_1.c

Execute the above files using code,

```
gcc 2_1.c −lm
./a.out
```

2.2 Load gau.dat in python and plot the empirical CDF of X using the samples in gau.dat. What properties does a CDF have?
Solution:
Download the following files,

```
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/solution
    /2/2_2.py
```

Execute the above file using code,

```
python3 2_2.py
```

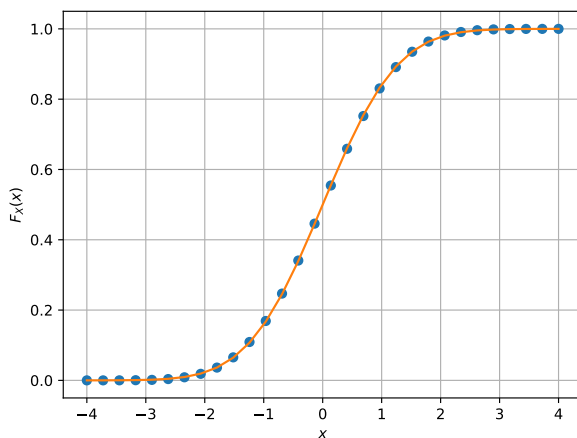Plot 2.1 obtained is symmetric about (0,0.5)

Fig. 2.1

2.3 Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as

$$p_X(x) = \frac{d}{dx}F_X(x) \qquad (2.2)$$

What properties does the PDF have?
Solution:
Download the following files,

```
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/solution
    /2/2_3.py
```

Execute the above file using code,
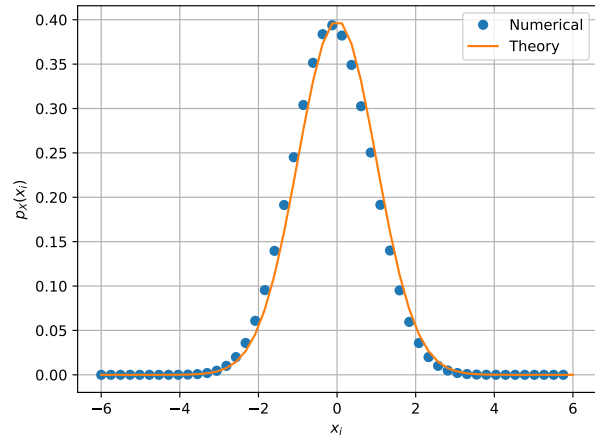
```
python3 2_3.py
```


Fig. 2.2

Plot 2.2 obtained is symmetric about y-axis

2.4 Find the mean and variance of X by writing a C program.
Solution:
Download the following files,

```
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/soltion/1/
    coeffs.h
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/solution
    /2/2_4.c
```

Execute the above files using code,

```
gcc 2_4.c −lm
./a.out
```

The result is,

$$E[U] = 0.000294 \qquad (2.3)$$
$$\text{var}[U] = 0.999561 \qquad (2.4)$$

2.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \quad (2.5)$$

repeat the above exercise theoretically.
Solution:

$$E[U] = \int_{-\infty}^{\infty} u p_X(u) du \qquad (2.6)$$

$$(2.7)$$

$up_X(u)$ is a odd function.

$$E[U] = 0 \tag{2.8}$$

$$\tag{2.9}$$

$$E[U^2] = \int_{-\infty}^{\infty} \frac{u^2}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du \tag{2.10}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} u\left(u \exp\left(-\frac{u^2}{2}\right)\right) du \tag{2.11}$$

Using integration by parts,

$$E[U^2] = -u\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right)\Big|_{-\infty}^{\infty} +$$

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) \tag{2.12}$$

$$= 0 + \frac{1}{\sqrt{2\pi}} \sqrt{2\pi} \tag{2.13}$$

$$= 1 \tag{2.14}$$

Hence,

$$\text{var}[U] = E[U^2] - [E[U]]^2 \tag{2.15}$$

$$= 1 \tag{2.16}$$

3 FROM UNIFORM TO OTHER

3.1 Generate samples of

$$V = -2\ln(1 - U) \tag{3.1}$$

and plot its CDF.
Solution:
Download the following files,

wget https://github.com/Anshul−Sangrame/
  AI1110/blob/main/Assignment/solution/
  coeffs.h
wget https://github.com/Anshul−Sangrame/
  AI1110/blob/main/Assignment/solution
  /3/3_1.c
wget https://github.com/Anshul−Sangrame/
  AI1110/blob/main/Assignment/solution
  /3/3_1.py

Execute the above files using code,

```
gcc 3_1.c −lm
./a.out
python3 3_1.py
```

Plot 3.1 is obtained
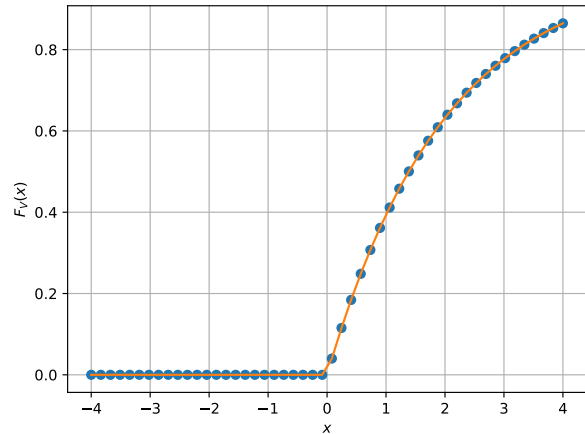### 3.2 Find a theoretical expression for $F_V(x)$.

Fig. 3.1

Solution:

$$F_V(x) = Pr(V \leq x) \tag{3.2}$$

$$= Pr(-2\ln(1 - U) \leq x) \tag{3.3}$$

$$= Pr\left(U \leq 1 - e^{-\frac{x}{2}}\right) \tag{3.4}$$

$$F_V(x) = F_U\left(1 - e^{-\frac{x}{2}}\right) \tag{3.5}$$

$$= \begin{cases} 0, & 1 - e^{-\frac{x}{2}} < 0 \\ 1 - e^{-\frac{x}{2}}, & 1 - e^{-\frac{x}{2}} \in [0, 1] \\ 1, & 1 - e^{-\frac{x}{2}} > 1 \end{cases} \tag{3.6}$$

Now,

$$1 - e^{-\frac{x}{2}} < 0 \tag{3.7}$$

$$\implies x < 0 \tag{3.8}$$

$$0 \leq 1 - e^{-\frac{x}{2}} \leq 1 \tag{3.9}$$

$$\implies x \geq 0 \tag{3.10}$$

$$1 < 1 - e^{-\frac{x}{2}} \tag{3.11}$$

$$\implies x \in \phi \tag{3.12}$$

Hence,

$$F_V(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-\frac{x}{2}}, & x \geq 0 \end{cases} \tag{3.13}$$

4 TRIANGULAR DISTRIBUTION

4.1 Generate

$$T = U_1 + U_2 \tag{4.1}$$

5

Solution:

Download the following files,

> wget https://github.com/Anshul−Sangrame/
> AI1110/blob/main/Assignment/solution/
> coeffs.h
> wget https://github.com/Anshul−Sangrame/
> AI1110/blob/main/Assignment/solution
> /4/4_1.c

Execute the above files using code,

> gcc 4_1.c −lm
> ./a.out

4.2 Find the CDF of T.
Solution:
Download the following files,

> wget https://github.com/Anshul−Sangrame/
> AI1110/blob/main/Assignment/solution
> /4/4_2.py

Execute the above files using code,
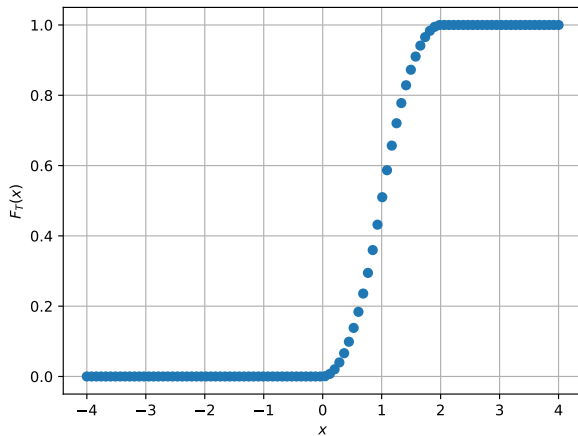
> python3 4_2.py

Plot 4.1 is obtained

Fig. 4.1

4.3 Find the PDF of T.
Solution:
Download the following files,

> wget https://github.com/Anshul−Sangrame/
> AI1110/blob/main/Assignment/solution
> /4/4_3.py

Execute the above files using code,

> python3 4_3.py
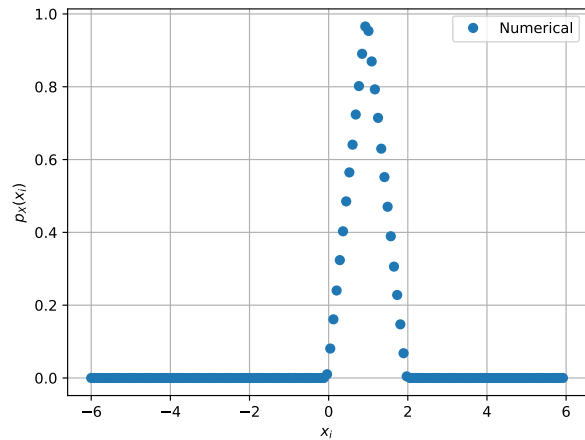
Plot 4.2 is obtained

Fig. 4.2

4.4 Find the theoretical PDF and CDF of T.
Solution:
The CDF of T is given by

$$F_T(t) = \Pr(T \le t) = \Pr(U_1 + U_2 \le t) \quad (4.2)$$

Since $U_1, U_2 \in [0,1] \implies U_1 + U_2 \in [0,2]$
Therefore, if $t \ge 2$, then $U_1 + U_2 \le t$ is always true and if $t < 0$, then $U_1 + U_2 \le t$ is always false.
Now, fix the value of U_1 to be some x

$$x + U_2 \le t \implies U_2 \le t - x \quad (4.3)$$

If $0 \le t \le 1$, then x can take all values in $[0, t]$

$$F_T(t) = \int_0^t \Pr(U_2 \le t - x)\, p_{U_1}(x)\mathrm{d}x \quad (4.4)$$

$$= \int_0^t F_{U_2}(t - x)p_{U_1}(x)\mathrm{d}x \quad (4.5)$$

$$0 \le x \le t \implies 0 \le t - x \le t \le 1 \quad (4.6)$$
$$\implies F_{U_2}(t - x) = t - x \quad (4.7)$$

$$F_T(t) = \int_0^t (t - x) \cdot 1 \cdot \mathrm{d}x \quad (4.8)$$

$$= tx - \frac{x^2}{2}\Big|_0^t \quad (4.9)$$

$$= \frac{t^2}{2} \quad (4.10)$$

If $1 < t < 2$, x can only take values in $[0, 1]$ as $U_1 \leq 1$

$$F_T(t) = \int_0^1 F_{U_2}(t - x) \cdot 1 \cdot dx \qquad (4.11)$$

$$0 \leq x \leq t - 1 \implies 1 \leq t - x \leq t \qquad (4.12)$$
$$t - 1 \leq x \leq 1 \implies 0 < t - 1 \leq t - x \leq 1 \qquad (4.13)$$

$$F_T(t) = \int_0^{t-1} 1\,dx + \int_{t-1}^1 (t - x)\,dx \qquad (4.14)$$

$$= t - 1 + t(1 - (t - 1)) - \frac{1}{2} + \frac{(t - 1)^2}{2} \qquad (4.15)$$

$$= t - 1 + 2t - t^2 - \frac{1}{2} + \frac{t^2}{2} + \frac{1}{2} - t \qquad (4.16)$$

$$= -\frac{t^2}{2} + 2t - 1 \qquad (4.17)$$

Therefore,

$$F_T(t) = \begin{cases} 0 & t < 0 \\ \dfrac{t^2}{2} & 0 \leq t \leq 1 \\ 2t - \dfrac{t^2}{2} - 1 & 1 < t < 2 \\ 1 & t \geq 2 \end{cases} \qquad (4.18)$$

The PDF of T is given by

$$p_T(t) = \frac{d}{dt} F_T(t) \qquad (4.19)$$

$$\therefore p_T(t) = \begin{cases} 0 & t < 0 \\ t & 0 \leq t \leq 1 \\ 2 - t & 1 < t < 2 \\ 0 & t \geq 2 \end{cases} \qquad (4.20)$$

4.5 Verify your results through a plot.
   Solution:
   Download the following files,

   wget https://github.com/Anshul−Sangrame/
       AI1110/blob/main/Assignment/solution
       /4/4_5_1.py
   wget https://github.com/Anshul−Sangrame/
       AI1110/blob/main/Assignment/solution
       /4/4_5_2.py

   Execute the above files using code,

   python3 4_5_1.py

python3 4_5_2.py

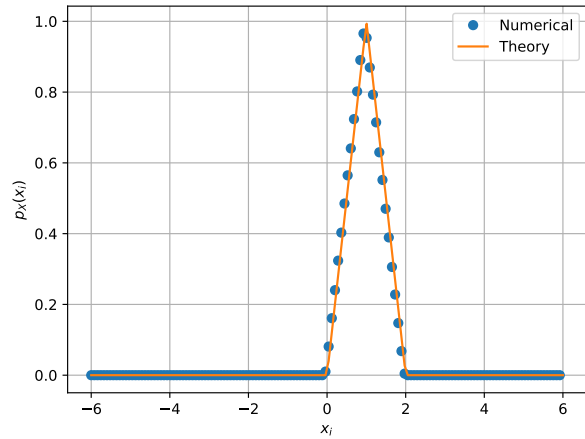Plot 4.3 is obtained is pdf Plot 4.4 is obtained
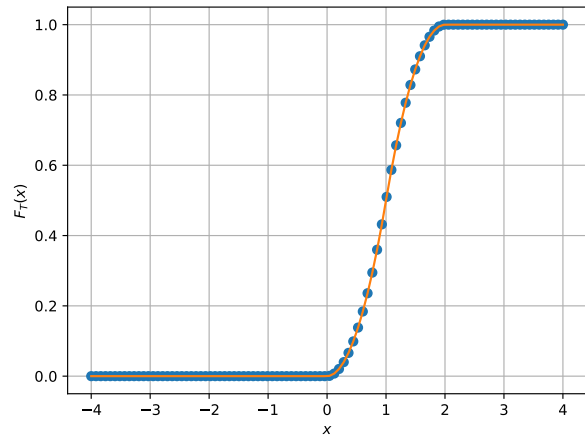

Fig. 4.3

is cdf

Fig. 4.4

5 MAXIMUM LIKELIHOOD

5.1 Generate equiprobable $X \in \{1, -1\}$.
   Solution:
   Download the following files,

   wget https://github.com/Anshul−Sangrame/
       AI1110/blob/main/Assignment/solution
       /5/5.1.c
   wget https://github.com/Anshul−Sangrame/
       AI1110/blob/main/Assignment/solution/
       coeffs.h

Execute the above files using code,

```
gcc 5.1.c −lm
./a.out
```

5.2 Generate

$$Y = AX + N \qquad (5.1)$$

where $A = 5$ dB, $X \in \{1, -1\}$ is Bernoulli and $N \sim \mathcal{N}(0, 1)$.
Solution:
Download the following files,

```
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/solution
    /5/5.2.c
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/solution/
    coeffs.h
```

Execute the above files using code,

```
gcc 5.2.c −lm
./a.out
```

5.3 Plot Y using a scatter plot.
Solution:
Download the following files,

```
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/solution
    /5/5.3.py
```

Execute the above files using code,

```
python3 5.3.py
```
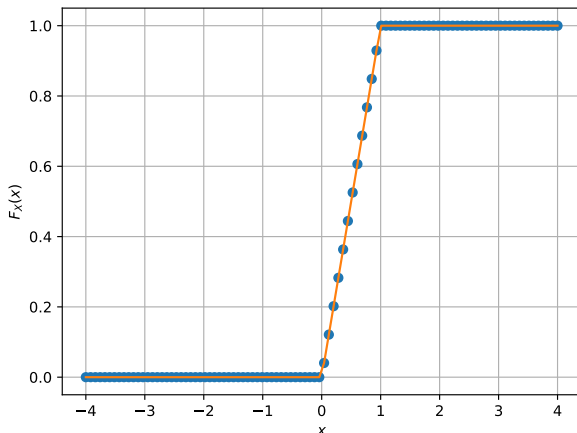
Plot 5.1 is obtained

Fig. 5.1

5.4 Guess how to estimate X from Y.
Solution:
From the plot of Y, we see that the estimate model can be written as

$$\hat{X} = \begin{cases} 1 & Y > 0 \\ 0 & Y < 0 \end{cases} \qquad (5.2)$$

5.5 Find

$$P_{e|0} = \Pr\left(\hat{X} = -1 | X = 1\right) \qquad (5.3)$$

and

$$P_{e|1} = \Pr\left(\hat{X} = 1 | X = -1\right) \qquad (5.4)$$

Solution:
Letting $X = 1$ and $X = -1$ respectively, we see the number of mismatched data points to compute the error probabilities. The simulation is coded in

```
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/solution
    /5/5.5.py
```

and can be run by typing

```
python3 5.5.py
```

The results are

$$P_{e|0} = 5.02 \times 10^{-4} \qquad (5.5)$$
$$P_{e|1} = 5.24 \times 10^{-4} \qquad (5.6)$$

5.6 Find P_e assuming that X has equiprobable symbols.
Solution:
Here, $\Pr(X = 1) = \Pr(X = -1) = 0.5$. Thus,

$$P_e = \Pr(X = 1) P_{e|1} + \Pr(X = -1) P_{e|0} \qquad (5.7)$$
$$= \frac{1}{2}\left(P_{e|0} + P_{e|1}\right) = 5.13 \times 10^{-4} \qquad (5.8)$$

5.7 Verify by plotting the theoretical P_e wrt A from 0 dB to 10 dB.
Solution:

$$P_{e|0} = \Pr\left(\hat{X} = 1 | X = -1\right) \qquad (5.9)$$
$$= \Pr(Y > 0 | X = -1) \qquad (5.10)$$
$$= \Pr(AX + N > 0 | X = -1) \qquad (5.11)$$
$$= \Pr(N > A) = Q(A) \qquad (5.12)$$

since X and N are independent. Writing a

similar expression for $P_{e|1}$ and noting that

$$\Pr(N < -A) = \Pr(N > A) = Q(A) \quad (5.13)$$

it follows that $P_e = Q(A)$. This is the idea used to plot the theoretical P_e. The plot is coded both in the rectangular axes and the semilog-y axes. Download the relevant codes using

```
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/solution
    /5/5.7.py
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/solution
    /5/5.7_semilog.py
```

Execute the above files using code,

```
python3 5.7.py
python3 5.7_semilog.py
```
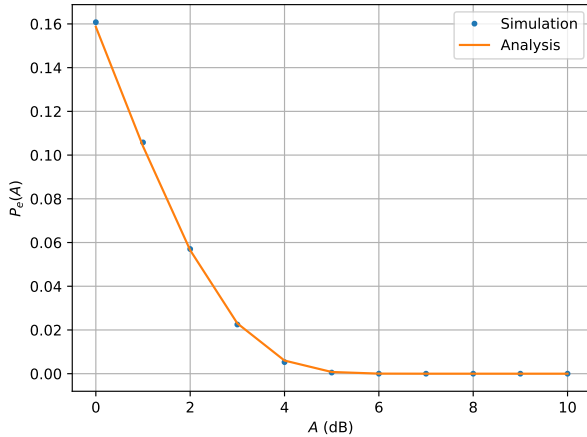
Plot 5.2 obtained in rectangular axis

Fig. 5.2

Plot 5.3 obtained in semilog-y axis

5.8 Now, consider a threshold δ while estimating X from Y. Find the value of δ that maximizes the theoretical P_e.

Solution:

To estimate X from Y, we now consider the following:

$$X = \begin{cases} 1, & Y > \delta \\ -1, & Y < \delta \end{cases} \quad (5.14)$$
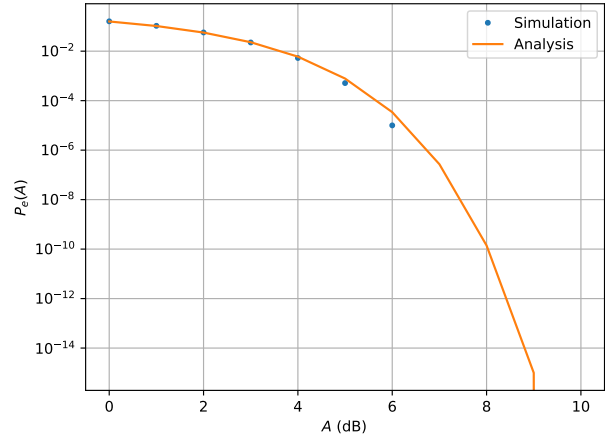

Fig. 5.3

Therefore,

$$P_e = \Pr(X = -1)Q(A + \delta)$$
$$+ \Pr(X = 1)Q(A - \delta) \quad (5.15)$$
$$= \frac{1}{2}(Q(A + \delta) + Q(A - \delta)) \quad (5.16)$$

To minimise P_e, we differentiate the above equation wrt δ:

$$0 = \frac{d}{d\delta}\left(\frac{1}{2}(Q_N(A - \delta) + Q_N(A + \delta))\right) \quad (5.17)$$

$$= \frac{1}{2}\left(\frac{1}{\sqrt{2\pi}}e^{-\frac{(\delta - A)^2}{2}} - \frac{1}{\sqrt{2\pi}}e^{-\frac{(A + \delta)^2}{2}}\right) \quad (5.18)$$

Therefore,

$$(\delta - A)^2 = (\delta + A)^2 \quad (5.19)$$
$$\implies \delta = 0 \quad (5.20)$$

5.9 Repeat the above exercise when

$$p_x(0) = p$$

Solution:

$$P_e = P_{e|0}p + P_{e|1}(1 - p) \quad (5.21)$$
$$= pQ_N(A - \delta) + (1 - p)Q_N(A + \delta) \quad (5.22)$$

Differentiating as before, we get:

$$0 = p \frac{1}{\sqrt{2\pi}} e^{-\frac{(\delta-A)^2}{2}} - (1-p) \frac{1}{\sqrt{2\pi}} e^{-\frac{(A+\delta)^2}{2}}$$
(5.23)

Taking ln on both sides we have:

$$\ln p - \frac{(\delta-A)^2}{2} = \ln 1 - p - \frac{(\delta+A)^2}{2} \quad (5.24)$$

$$\implies 2\delta A = \ln \frac{1-p}{p} \quad (5.25)$$

$$\implies \delta = \frac{1}{2A} \ln \frac{1-p}{p} \quad (5.26)$$

5.10 Repeat the above exercise using the MAP criterion.
Solution:
Using theorem of conditional probabilities,

$$p_{X|Y}(x|y) = \frac{p_{Y|X}(y|x) \times p_X(x)}{p_Y(y)} \quad (5.27)$$

$$p_{X|Y}(x|y) = \frac{p_X(x)Pr(Ax+N=y)}{p_Y(y)} \quad (5.28)$$

$$p_{X|Y}(x|y) = \frac{p_X(x)Pr(N=y-Ax)}{p_Y(y)} \quad (5.29)$$

$$p_{X|Y}(x|y) = \frac{p_X(x)p_N(y-Ax)}{p_Y(y)} \quad (5.30)$$

$$p_{X|Y}(x|y) = \frac{\frac{1}{\sqrt{2\pi}} p_X(x) e^{-\frac{(y-Ax)^2}{2}}}{p_Y(y)} \quad (5.31)$$

$$(5.32)$$

When $X = 1$, we have:

$$p_{X|Y}(1|y) = \frac{p_{Y|X}(y|1) \times p_X(1)}{p_Y(y)} \quad (5.33)$$

$$= \frac{(1-p) \frac{e^{-\frac{(y-A)^2}{2}}}{\sqrt{2\pi}}}{p \frac{e^{-\frac{(y+A)^2}{2}}}{\sqrt{2\pi}} + (1-p) \frac{e^{-\frac{(y-A)^2}{2}}}{\sqrt{2\pi}}} \quad (5.34)$$

$$= \frac{(1-p) e^{2yA}}{p + (1-p) e^{2yA}} \quad (5.35)$$

Similarly when $X = -1$,

$$p_{X|Y}(-1|y) = \frac{p}{p + (1-p)e^{2yA}} \quad (5.36)$$

Therefore, when $p_{X|Y}(1|y) > p_{X|Y}(-1|y)$, we have:

$$\frac{(1-p) e^{2yA}}{p + (1-p) e^{2yA}} > \frac{p}{p + (1-p) e^{2yA}} \quad (5.37)$$

$$e^{2yA} > \frac{p}{(1-p)} \quad (5.38)$$

$$y > \frac{1}{2A} \ln \frac{p}{(1-p)} \quad (5.39)$$

Therefore, when Eq. (5.39), we can assert that $X = 1$, and $X = -1$ otherwise.

6 Gaussian to Other

6.1 Let $X_1 \sim \mathcal{N}(0,1)$ and $X_2 \sim \mathcal{N}(0,1)$. Plot the CDF and PDF of

$$V = X_1^2 + X_2^2 \quad (6.1)$$

Solution:
Applying following transformation on X_1 and X_2,

$$X_1 = R \cos \Theta \quad (6.2)$$
$$X_2 = R \sin \Theta \quad (6.3)$$

Where $R \in [0, \infty), \Theta \in [0, 2\pi)$. The Jacobian Matrix for this transformation is given by

$$\mathbf{J} = \begin{pmatrix} \frac{\partial X_1}{\partial R} & \frac{\partial X_2}{\partial R} \\ \frac{\partial X_1}{\partial \Theta} & \frac{\partial X_2}{\partial \Theta} \end{pmatrix} \quad (6.4)$$

$$= \begin{pmatrix} \cos \Theta & \sin \Theta \\ -R \sin \Theta & R \cos \Theta \end{pmatrix} \quad (6.5)$$

$$\implies |\mathbf{J}| = R \quad (6.6)$$

We also know that

$$|\mathbf{J}| p_{X_1,X_2}(x_1, x_2) = p_{R,\Theta}(r, \theta) \quad (6.7)$$

$$\implies p_{R,\Theta}(r, \theta) = R p_{X_1}(x_1) p_{X_2}(x_2) \quad (6.8)$$

$$= \frac{R}{2\pi} \exp\left(-\frac{X_1^2 + X_2^2}{2}\right) \quad (6.9)$$

$$= \frac{R}{2\pi} \exp\left(-\frac{R^2}{2}\right) \quad (6.10)$$

Where (6.8) follows as X_1, X_2 are iid random variables. Thus,

$$p_R(r) = \int_0^{2\pi} p_{R,\Theta}(r, \theta) d\theta \quad (6.11)$$

$$= R \exp\left(-\frac{R^2}{2}\right) \quad (6.12)$$

However, $V = X_1^2 + X_2^2 = R^2 \geq 0$, thus $F_V(x) = 0$. For $x < 0$,

$$F_V(x) = F_R(\sqrt{x}) \qquad (6.13)$$

$$= \int_0^{\sqrt{x}} r \exp\left(-\frac{r^2}{2}\right) dr \qquad (6.14)$$

$$= \int_0^{\frac{x}{2}} e^{-t} dt = 1 - e^{-\frac{x}{2}} \qquad (6.15)$$

Where $t = \frac{r^2}{2}$ for $x \geq 0$, thus

$$F_V(x) = \begin{cases} 1 - e^{-\frac{x}{2}} & x \geq 0 \\ 0 & x < 0 \end{cases} \qquad (6.16)$$

$$p_V(x) = \frac{dF_V(x)}{dx} \qquad (6.17)$$

$$= \begin{cases} 0, & x < 0 \\ \frac{1}{2}e^{-\frac{x}{2}}, & x \geq 0 \end{cases} \qquad (6.18)$$

Download the following files to verify the plot,

```
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/solution
    /6/6.1.c
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/solution/
    coeffs.h
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/solution
    /6/6.1.cdf.py
wget https://github.com/Anshul−Sangrame/
    AI1110/blob/main/Assignment/solution
    /6/6.1.pdf.py
```

Execute the programs using the code in terminal,

```
gcc 6.1.c −lm
./a.out
python3 6.1.cdf.py
python3 6.1.pdf.py
```
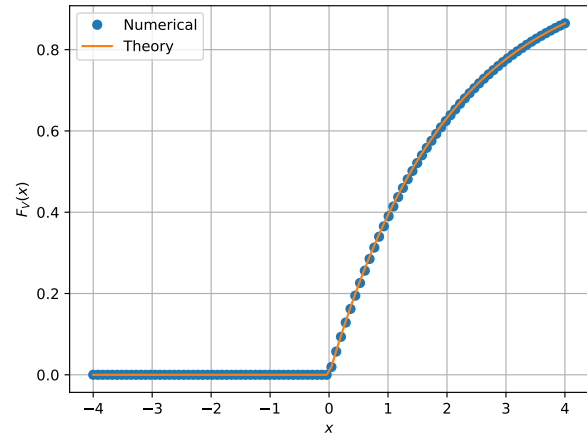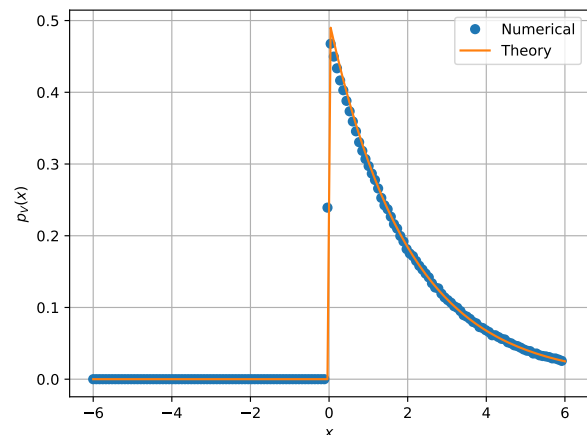
Plot 6.1 is CDF
Plot 6.2 is PDF

Fig. 6.1

Fig. 6.2