

Assignment-2

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Abstract—This document contains solution for ICSE
2019 class 12 maths Q.16(b)

Problem 16(b): If \vec{a} and \vec{b} are non-collinear vectors, find the value of x such that vectors $\vec{\alpha} = (x - 2) \vec{a} + \vec{b}$ and $\vec{\beta} = (3 + 2x) \vec{a} - 2 \vec{b}$ are collinear.

Solution: α and β can be written as,

$$\alpha = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} x-2 \\ 1 \end{pmatrix}, \beta = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} 3+2x \\ -2 \end{pmatrix} \quad (1)$$

Since α and β are collinear,

$$\alpha = \lambda \beta \quad (2)$$

$$\Rightarrow \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \left(\begin{pmatrix} x-2 \\ 1 \end{pmatrix} - \lambda \begin{pmatrix} 3+2x \\ -2 \end{pmatrix} \right) = 0 \quad (3)$$

$$\Rightarrow \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} (1-2\lambda)x - (2+3\lambda) \\ 1+2\lambda \end{pmatrix} = 0 \quad (4)$$

$$\Rightarrow \begin{pmatrix} (1-2\lambda)x - (2+3\lambda) \\ 1+2\lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (5)$$

Comparing every element of matrix in equation (5), we get,

$$1 + 2\lambda = 0 \quad (6)$$

$$\Rightarrow \lambda = \frac{-1}{2} \quad (7)$$

$$(1 - 2\lambda)x - (2 + 3\lambda) = 0 \quad (8)$$

Using equation (7) in (8),

$$2x - \frac{1}{2} = 0 \quad (9)$$

$$\Rightarrow x = \frac{1}{4} \quad (10)$$