CS21003 - Tutorial 6

September 12th, 2018

1. Consider the following variation of interval scheduling problem – we have n requests labeled $1, \ldots, n$ with each request a_i having a start time s_i and a finish time f_i. Each request a_i also comes with a value, or weight v_i. The goal is to select a subset $S \subseteq \{1, \ldots, n\}$ of mutually compatible intervals, so as to maximize the sum of the values of the selected intervals $\sum_{i \in S} v_i$. [**Hint:** The complexity is much better than $O(n^3)$]

2. Given a sequence of n real numbers $a_1, \ldots, a_n$, determine a contiguous subsequence $a_i, \ldots, s_j$ for which the sum of elements in the subsequence is maximized. E.g., for input $\{3, -4, 9, -8, 8, 7\}$, the output will be $\{9, -8, 8, 7\}$ with sum = 16. Propose a dynamic programming algorithm for this problem and show the working of your algorithm using the example provided.

3. You are traveling by a canoe down a river and there are n trading posts along the way. Before starting your journey, you are given for each $1 \leq i < j \leq n$, the fee $f_{i,j}$ for renting a canoe from post i to post j. These fees are arbitrary. For example it is possible that $f_{1,3} = 10$ and $f_{1,4} = 5$. You begin at trading post 1 and must end at trading post n (using rented canoes). Your goal is to minimize the rental cost. Give the most efficient algorithm you can for this problem.

4. You are given k coin values $c_0 < c_1 < \ldots < c_{k-1}$ (where $c_0 = 1$), and a value v, find a way to give the value v in change using as few coins as possible.

5. You are given a collection C of n positive integers $a_0, a_1, \ldots, a_n$ with sum $S = \sum_0^{n-1} a_i$. Your task is to partition C into two subcollections A and B ($C = A \cup B$) such that the sum of the members of A is as close as possible to the sum of the members of B. In other words, if $S_1 = \sum_{a \in A} a$ and $S_2 = \sum_{b \in B} b$, your objective is to minimize $|S_1 - S_2|$. Design an efficient dynamic programming algorithm to solve this problem. What is the complexity of your algorithm? Take an example set $C = \{1, 2, 4\}$ and show the working of your algorithm.

6. Design a dynamic programming algorithm for the following problem: Given a sequence $a_1 \circ_1 a_2 \circ_2 \ldots a_{n-1} \circ_{n-1} a_n$, in which each a_i is a postive integer and each $\circ_i$ is '+' or '-', compute a parenthesization of the expression such that the resulting value is the maximum possible. It suffices to compute the resulting value instead of the parenthesization. Estimate the time complexity of the algorithm.

   For example, if the given sequence is 3-4-5, ((3-4)-5) results in -6 while (3-(4-5)) results in 4. The second parenthesization results in the maximum possible value, and the output is 4. Show the working of your algorithm on this example.

7. A sequence is called a good sequence if $a_1 < a_2 > a_3 < a_4 \ldots, a_k$, i.e., $a_i < a_{i+1}$ if i is odd and $a_i > a_{i+1}$ if i is even for all $i < k$. You are given a sequence A containing n integers. You need to find the length of longest good subsequence of A using dynamic programming algorithm. For example, for the input sequence $\{1, 2, 6, 5, 3, 4\}$, the largest such sequence is $\{2, 6, 3, 4\}$. Show the working of your algorithm using this example.