Equivalent circuit of transformer from circuit point of view

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1 Transformer equivalent circuit

Transformer is nothing but collection of two mutually coupled coils with self inductances L_1 & L_2 and mutual inductance M as shown in figure 1

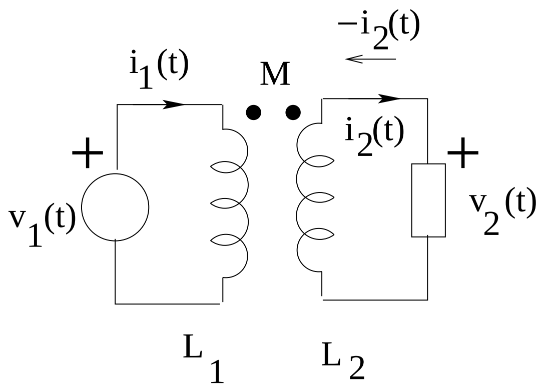

Figure 1: Transformer as coupled coils.

Keeping with the convention of current direction in a transformer, the direction of the secondary current i_2 is shown to be flowing to the load - which means $-i_2$ is flowing towards dot of the second coil.

Our goal here will be to obtain the equivalent circuits referred to either side purely from circuit point of view. Let us assume the turns ratio between the first and the second coil be $a = \frac{N_1}{N_2}$. The instantaneous KVL equations in the two coils will be:

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$$\begin{aligned} v_1 &= r_1 i_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \\ v_2 &= -r_2 i_2 - L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \end{aligned}$$

In the above two equation replace v_2 by $\frac{av_2}{a}$ and i_2 by $a\frac{i_2}{a}$

$$\begin{aligned} v_1 &= r_1 i_1 + L_1 \frac{di_1}{dt} - aM \frac{d(\frac{i_2}{a})}{dt} \\ \frac{av_2}{a} &= -r_2 a \frac{i_2}{a} - aL_2 \frac{d(\frac{i_2}{a})}{dt} + M \frac{di_1}{dt} \\ \text{or, } av_2 &= -a^2 r_2 (\frac{i_2}{a}) - a^2 L_2 \frac{d(\frac{i_2}{a})}{dt} + aM \frac{di_1}{dt} \end{aligned}$$

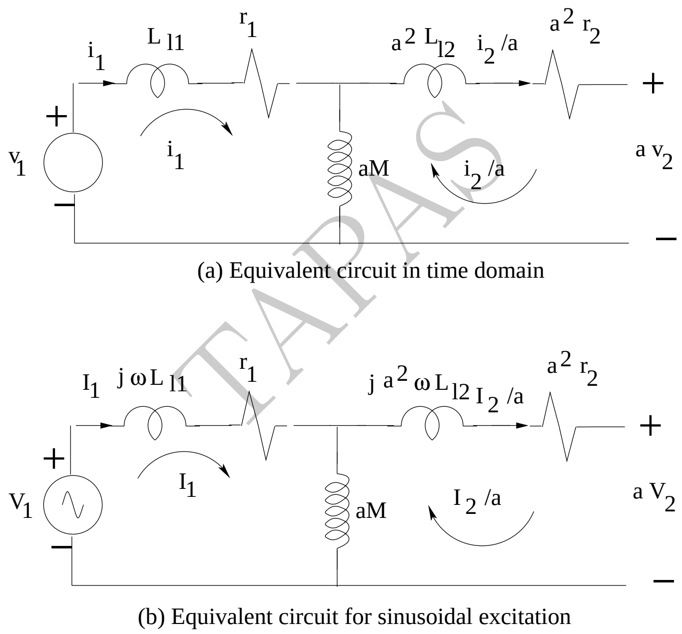
Now we know that,

$$\begin{aligned} L_1 &= (L_{l1} + aM) \\ L_2 &= (L_{l2} + \frac{M}{a}) \end{aligned}$$

Put L_1 and L_2 in the KVL equations to get:

$$
\begin{aligned}
v_1 &= r_1 i_1 + (L_{l1} + aM)\frac{di_1}{dt} - aM\frac{d(\frac{i_2}{a})}{dt} \\
\text{or, } v_1 &= r_1 i_1 + L_{l1}\frac{di_1}{dt} + aM\frac{di_1}{dt} - aM\frac{d(\frac{i_2}{a})}{dt} \\
\text{and, } av_2 &= -a^2 r_2(\frac{i_2}{a}) - a^2(L_{l2} + \frac{M}{a})\frac{d(\frac{i_2}{a})}{dt} + aM\frac{di_1}{dt} \\
\text{or, } av_2 &= -a^2 r_2(\frac{i_2}{a}) - a^2 L_{l2}\frac{d(\frac{i_2}{a})}{dt} - aM\frac{d(\frac{i_2}{a})}{dt} + aM\frac{di_1}{dt}
\end{aligned}
$$

From the expressions of v_1 and av_2, the following equivalent circuit can easily be drawn which is shown in figure 2(a).



(a) Equivalent circuit in time domain

(b) Equivalent circuit for sinusoidal excitation

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Figure 2: Equivalent circuit referred to coil-1.

If the supply voltages are sinusoidal with frequency ω, then steady state KVL equations of the coils in terms of phasors will be obtained by replacing the operator $\frac{d}{dt}$ with $j\omega$ in the above equations.

$$
\begin{aligned}
\bar{V}_1 &= r_1\bar{I}_1 + j\omega L_1\bar{I}_1 - j\omega M\bar{I}_2 \\
\bar{V}_2 &= -r_2\bar{I}_2 - j\omega L_2\bar{I}_2 + j\omega M\bar{I}_1
\end{aligned}
$$

Now let us manipulate the KVL equation for the first coil:

$$
\begin{aligned}
\bar{V}_1 &= r_1\bar{I}_1 + j\omega L_1\bar{I}_1 - j\omega M\bar{I}_2 \\
&= r_1\bar{I}_1 + j\omega(L_{l1} + aM)\bar{I}_1 - j\omega aM\frac{\bar{I}_2}{a} \\
\text{By rearranging, } \bar{V}_1 &= (r_1 + j\omega L_{l1})\bar{I}_1 + j\omega aM(\bar{I}_1 - \frac{\bar{I}_2}{a})
\end{aligned}
$$

We now manipulate the KVL equation of the second coil as follows:

$$\begin{aligned}
\bar{V}_2 &= -r_2\bar{I}_2 - j\omega L_2\bar{I}_2 + j\omega M\bar{I}_1 \\
&= -r_2\bar{I}_2 - j\omega(L_{l2} + \frac{M}{a})\bar{I}_2 + j\omega M\bar{I}_1 \\
&= -(r_2 + j\omega L_{l2})\bar{I}_2 - j\omega\frac{M}{a}\bar{I}_2 + j\omega M\bar{I}_1 \\
\text{or, } \bar{V}_2 &= -(r_2 + j\omega L_{l2})a\frac{\bar{I}_2}{a} - j\omega M\frac{\bar{I}_2}{a} + j\omega M\bar{I}_1
\end{aligned}$$

Now multiplying both sides by a, we get

$$\begin{aligned}
a\bar{V}_2 &= -a^2(r_2 + j\omega L_{l2})\frac{\bar{I}_2}{a} - j\omega aM\frac{\bar{I}_2}{a} + j\omega aM\bar{I}_1 \\
a\bar{V}_2 &= -a^2(r_2 + j\omega L_{l2})\frac{\bar{I}_2}{a} + j\omega aM(\bar{I}_1 - \frac{\bar{I}_2}{a})
\end{aligned}$$

Let us rewrite the manipulated KVL equations of the two coils below:

$$\begin{aligned}
\bar{V}_1 &= (r_1 + j\omega L_{l1})\bar{I}_1 + j\omega aM(\bar{I}_1 - \frac{\bar{I}_2}{a}) \\
a\bar{V}_2 &= -a^2(r_2 + j\omega l2)\frac{\bar{I}_2}{a} + j\omega aM(\bar{I}_1 - \frac{\bar{I}_2}{a})
\end{aligned}$$

From the above two KVL equation it is quite simple to draw the equivalent circuit of the mutually coupled coils referred to the first coil as shown in figure 2(b).

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