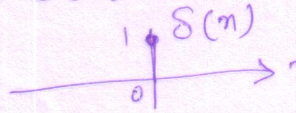
- Solution of 2^{nd} order linear constant co-efficients Difference equation — Classical Approach.

Obtain total solution of the following linear system when the input is unit impulse i.e., $\delta(n)$



$$\boxed{y(n) - y(n-1) - 2y(n-2) = \delta(n)}$$

$\because$ Causal $y(n) = 0$ for $n < 0$ (no output with no input; input is at $n = 0$).

$\therefore \quad y(-1) = 0, \; y(-2) = 0$

$\because$ a 2^{nd} order equ^n, two B.C needed namely $y(0)$ and $y(1)$

$n = 0 \Rightarrow \quad y(0) - y(-1) - 2y(-2) = \delta(0) = 1$

$\therefore \boxed{y(0) = 1}$

$n = 1 \Rightarrow \quad y(1) - y(0) - 2y(-1) = \delta(1) = 0$

$\therefore \; y(1) = y(0) = 1 \qquad \therefore \boxed{y(1) = 1}$

After knowing this, the problem boils down to solve

$y(n) - y(n-1) - 2y(n-2) = 0$

with initial conditions: $\boxed{y(0) = y(1) = 1}$

$n \to n+2: \quad y(n+2) - y(n+1) - 2y(n) = \cancel{\delta(n+2)} \; 0$

or $\quad (E^2 - E - 2)\, y(n) = \cancel{E^2\{\delta(n)\}} \; 0$

ch. equ^n $\quad m^2 - m - 2 \quad$ roots $m_{1,2} = 2, -1$

$\therefore \quad y(n) = c_1 (2)^n + c_2 (-1)^n$

$y(0) = 1 \Rightarrow \quad c_1 + c_2 = 1$

$y(1) = 1 \Rightarrow \quad 2c_1 - c_2 = 1$

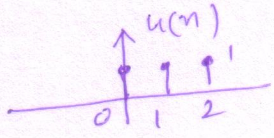
$3c_1 = 2 \qquad \therefore c_1 = 2/3$

$c_2 = 1 - \frac{2}{3} = \frac{1}{3}$

$$\therefore \boxed{y(n) = \frac{2}{3}(2)^n u(n) + \frac{1}{3}(-1)^n u(n)}$$

Ans.

solving
Same problem when $x(n) = u(n)$



$\therefore$ Eq.^{n} $\quad y(n) - y(n-1) - 2y(n-2) = u(n)$

$\because$ Causal $\quad y(n) = 0$ for $n < 0$

$\therefore \quad y(-2) = y(-1) = 0$

We need two B.C $\; y(0)$ & $y(1)$

$n = 0$: $\quad y(0) - y(-1) - 2y(-2) = u(0) = 1$

$\therefore \; \boxed{y(0) = 1}$

$n = 1$ $\quad y(1) - y(0) - y(-1) = u(1) = 1$

$\therefore \; \boxed{y(1) = 1 + y(0) = 2}$

$n \to n+2 \Rightarrow \quad y(n+2) - y(n+1) - 2y(n) = u(n+2)$

$E^2\{u(n)\} = E^2\{1^n u(n)\}$.

$\therefore \; (E^2 - E - 2)\, y(n) =$

Ch. equ.^{n} $\quad m^2 - m - 2 = 0 \quad \therefore m_{1,2} = 2, -1$

Total solution is

$$y(n) = c_1 (2)^n + c_2 (-1)^n + \left.\frac{E^2}{E^2 - E - 2}\right|_{E=1} 1^n u(n)$$

$$y(n) = c_1 \underset{\times u(n)}{(2)^n} + c_2 \underset{\times u(n)}{(-1)^n} - \frac{1}{2} u(n)$$

B.C

$y(0) = 1$: $\quad c_1 + c_2 - \frac{1}{2} = 1 \quad$ or $c_1 + c_2 = \frac{3}{2}$

$y(1) = 2$: $\quad 2c_1 - c_2 - \frac{1}{2} = 2 \quad$ or $2c_1 - c_2 = \frac{5}{2}$

$c_1 = \frac{4}{3}$

$c_2 = \frac{1}{6}$

$\therefore$ total sol.^{n} is :-

$$\boxed{y(n) = \frac{4}{3} (2)^n u(n) + \frac{1}{6} (-1)^n u(n) - \frac{1}{2} u(n)}$$

Ans.