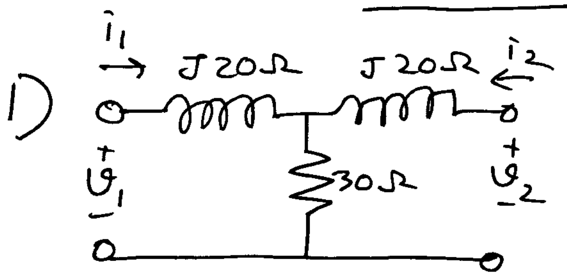
Tutorial 9 Solution

1) 

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$z_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0} = (30 + J20)\ \Omega$$

$$z_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0} = 30\ \Omega$$

$$z_{21} = \left.\frac{v_2}{i_1}\right|_{i_2=0} = 30\ \Omega$$

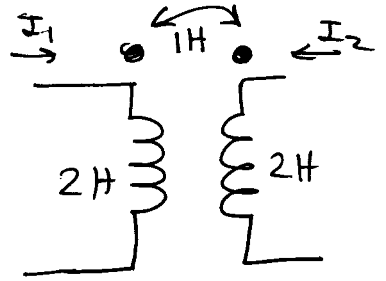
$$z_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0} = (30 + J20)\ \Omega.$$

$$\therefore\ Z = \begin{bmatrix} 30+20J & 30 \\ 30 & 30+20J \end{bmatrix} \Omega$$

$$y = Z^{-1} = \frac{\begin{bmatrix} 30+20J & -30 \\ -30 & 30+20J \end{bmatrix} \mho}{(30+20J)^2 - 30^2}$$

= Do it yourselves.

The network is <u>reciprocal</u> because $z_{21} = z_{12}$. Also the network is completely <u>symmetric</u> (and hence also reciprocal) (by observation only and need not verify the mathematical conditions additionally). Also the condition $z_{11} = z_{22}$ is satisfied

2) 

$$V_1 = 2J\omega I_1 + J\omega I_2$$
$$V_2 = 2J\omega I_2 + J\omega I_1$$

ω is in rad/sec

$$\therefore \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2J\omega & J\omega \\ J\omega & 2J\omega \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \Omega$$

$$\therefore Z = \begin{bmatrix} 2J\omega & J\omega \\ J\omega & 2J\omega \end{bmatrix} \Omega$$

$$Y = Z^{-1} = \frac{\begin{bmatrix} 2J\omega & -J\omega \\ -J\omega & 2J\omega \end{bmatrix} \mho}{(4-1)J^2\omega^2} = \frac{\begin{bmatrix} 2J\omega & -J\omega \\ -J\omega & 2J\omega \end{bmatrix} \mho}{-3\omega^2}$$

Again by observation, the network is symmetric and reciprocal. Also $Z_{21} = Z_{12}$ (reciprocal) and $Z_{11} = Z_{22}$ (symmetric)

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3) $$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

(i) with o/p shorted $(V_2 = 0)$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{25V}{1A} = 25\Omega$$

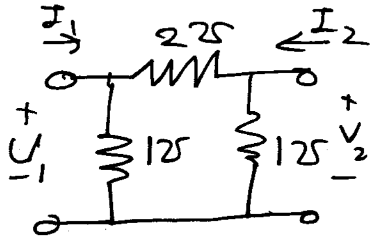
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{2A}{1A} = 2$$

(ii) with i/p open $(I_1 = 0)$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{10V}{50V} = \frac{1}{5}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{2A}{50V} = \frac{1}{25} \mho$$

4)
(a)



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{V_1\,(1\mho + 2\mho)}{V_1} = 3\mho$$

$$y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} = \frac{-V_2 \times 2\mho}{V_2} = -2\mho$$

From visual/observed symmetry

$Y_{22} = Y_{11} = 3\mho$ and $Y_{21} = Y_{12} = -2\mho$

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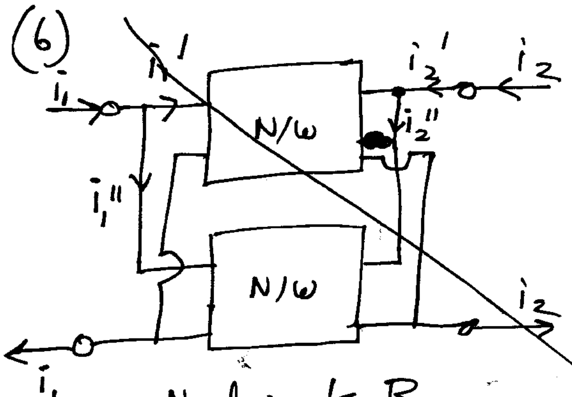
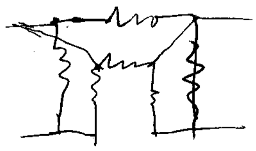
(6)



Network B



$$i_1 = i_1' + i_1''$$

$$i_2 = i_2' + i_2''$$

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} i_1' \\ i_2' \end{bmatrix} + \begin{bmatrix} i_1'' \\ i_2'' \end{bmatrix}$$

$$= \begin{bmatrix} Y \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} Y \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

[$\because$ the two networks are in parallel same voltage is applied across them]

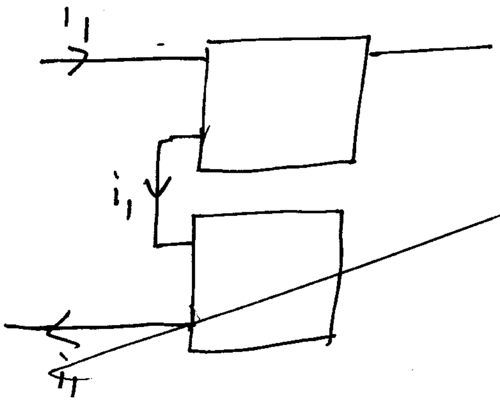
$$= 2 \times \begin{bmatrix} Y \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2Y \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$\therefore$ effective Y-parameters of N/w B

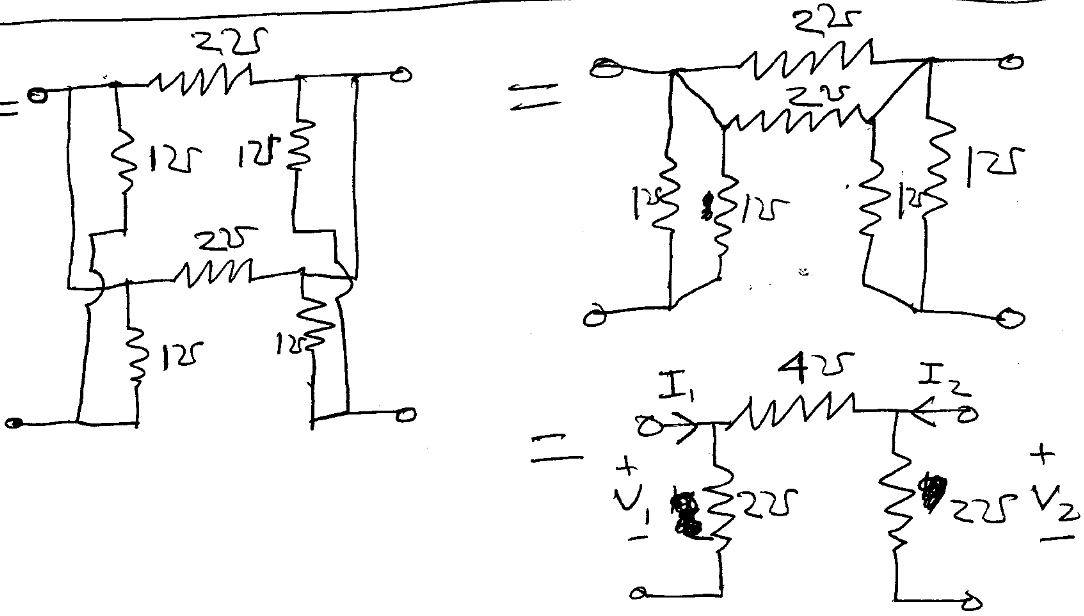
$$= 2 \times \begin{bmatrix} \text{Y param of N/W A} \end{bmatrix}$$

$$= 2 \times \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 6 & -4 \\ -4 & 6 \end{bmatrix}$$

~~5) N/W A & B are in series.~~
~~i.e. same i_1 & i_2 flows through them,~~
~~But the voltages are added in series~~



4) b) N/W B =



Now
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{V_1\,(2\mho + 4\mho)}{V_1} = 6\mho$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-V_2\,4\mho}{V_2} = -4\mho$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{V_1\,4\mho}{V_1} = -4\mho$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{V_2\,(4\mho + 2\mho)}{V_2} = 6\mho$$

5) We have not discussed this theory in our class ~~and~~ ~~it~~ ~~is~~ ~~not~~ but it is very interesting and easy. You may read D. Roy. Choudhury. This will not come in the exam.

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6) $$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$ (Given)

$\Rightarrow V_1 = 2I_1 + I_2$ —— (i)

$V_2 = 3I_1 + 0I_2$ —— (ii)

for h-parameters

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

or $V_1 = h_{11} I_1 + h_{12} V_2$ —— (iii)

$I_2 = h_{21} I_1 + h_{22} V_2$ —— (iv)

from (ii) $I_2 = \frac{V_2}{0} - \frac{3}{0} I_1$

putting this in (i) $V_1 = 2I_1 - \frac{3}{0} I_1 + \frac{V_2}{0}$

comparing this with (iii) $h_{11} = 2 - \frac{3}{0}$ = undefined

and $h_{12} = \frac{1}{0}$ = undefined.

Now from (ii)

$I_2 = \frac{V_2}{0} - \frac{3}{0} I_1 = -\frac{3}{0} I_1 + \frac{V_2}{0}$

comparing this with (iv) $h_{21} = \frac{-3}{0}$ = undefined

$h_{22} = \frac{1}{0}$ = undefined.

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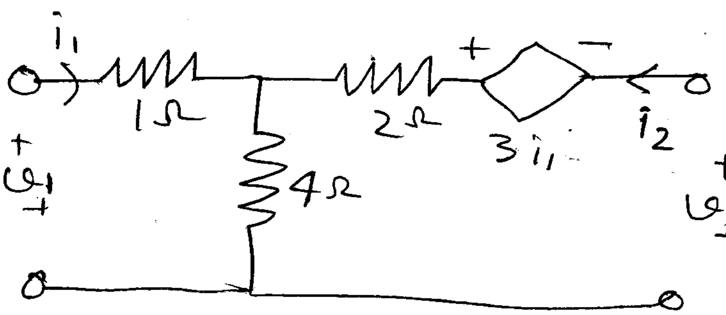
7)



$$\begin{bmatrix} v_2 \\ i_2 \end{bmatrix} = \begin{bmatrix} E & F \\ G & H \end{bmatrix} \begin{bmatrix} v_1 \\ -i_1 \end{bmatrix}$$

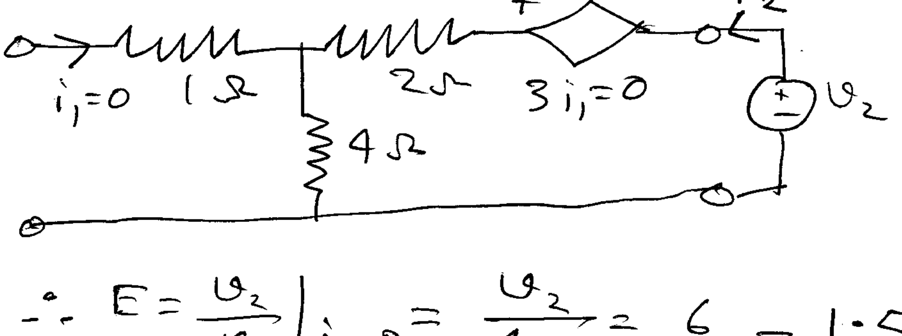
~~$E = \frac{v_2}{v_1}\Big|_{i_1=0}$ (secondary open) $= \frac{v_1 \times \frac{4}{5} - 3i_1}{v_1} = \frac{\frac{4}{5}v_1 - \frac{3}{5}v_1}{v_1} = \frac{1}{5}$~~

~~$G = \frac{i_2}{v_1}\Big|_{i_1=0}$ (secondary open) $=$~~

~~$E = \frac{v_2}{v_1}\Big|_{i_1=0}$ (~~secondary~~ primary open and v_2 source connected to secondary) $= \frac{v_2}{\;}$~~

$E = \frac{v_2}{v_1}\Big|_{i_1=0}$ ~~and $G = \frac{i_2}{v_1}\Big|_{v_2=0}$~~

To get E open circuit primary & excite secondary with v_2 and measure v_1



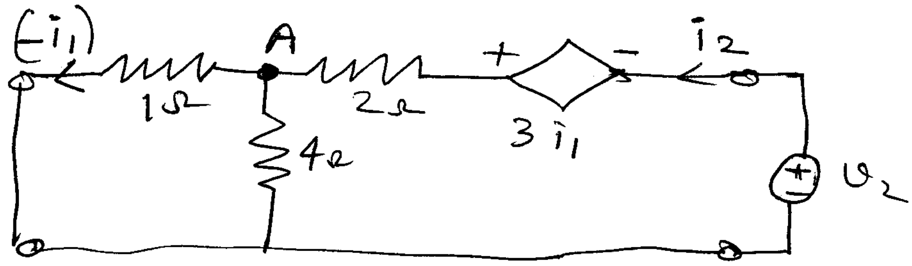
$i_2 = \frac{v_2}{6}$

$v_1 = 4i_2 = \frac{4v_2}{6}$

$\therefore E = \frac{v_2}{v_1}\Big|_{i_1=0} = \frac{v_2}{\frac{4}{6}v_2} = \frac{6}{4} = 1\cdot5$

$$F = \left.\frac{v_2}{-i_1}\right|_{v_1=0}$$

To get F, short circuit primary and excite secondary with v_2 and measure $-i_1$



KCL at $A \Rightarrow i_2 = (-i_1) + (-i_1) \times \frac{1\Omega}{4\Omega} = \frac{5}{4}(-i_1)$

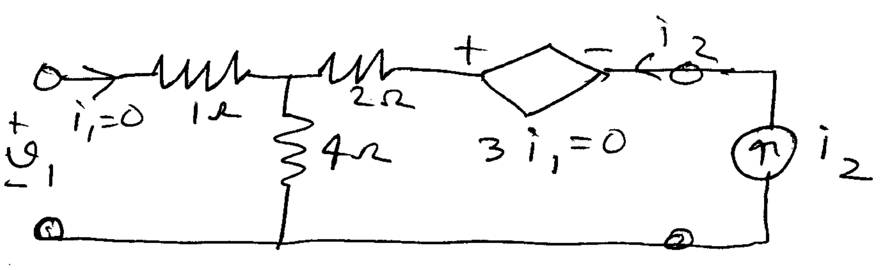
KVL in the outer loop ⇒

$$v_2 = -3i_1 + 2i_2 + 1(-i_1)$$
$$= (3+1)(-i_1) + 2 \times \frac{5}{4}(-i_1)$$
$$= 6.5\,(-i_1)$$

$$\therefore\; F = \left.\frac{v_2}{(-i_1)}\right|_{v_1=0} = 6.5$$

$$G = \left.\frac{i_2}{v_1}\right|_{i_1=0}$$

To get G open circuit primary, excite secondary with i_2 and measure v_1



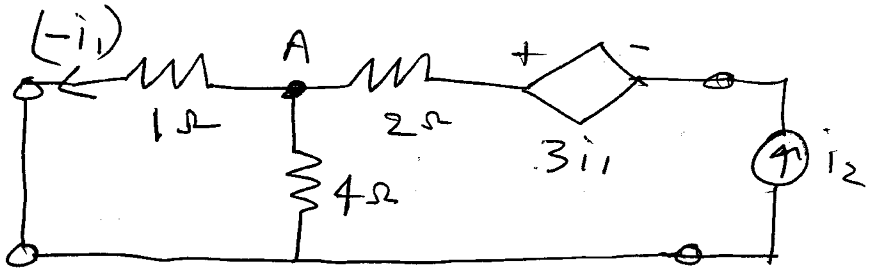
$v_1 = 4i_2$

$\Rightarrow \frac{i_2}{v_1} = \frac{1}{4} = 0.25$

$$\therefore\; G = \left.\frac{i_2}{v_1}\right|_{i_1=0} = 0.25$$

$$H = \frac{i_2}{(-i_1)}\bigg|_{v_1=0}$$

To get H, short circuit primary and excite secondary with i_2 and measure $(-i_1)$.



KCL at $A \Rightarrow i_2 = (-i_1) + (-i_1) \times \frac{1\Omega}{4\Omega} = \frac{5}{4}(-i_1)$

$$\therefore H = \frac{i_2}{-i_1}\bigg|_{v_1=0} = \frac{5}{4} = 1.25$$

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10) Since the network is ~~resistive and~~ linear
$V = aI_1 + bI_2$ for some constant values of a & b

~~Given~~ ~~that~~ from given information

$80 = a8 + b12$
and $0 = a(-8) + b(4)$

$(+)$ $80 = 16b$
$\Rightarrow b = 5$
$\therefore a = \frac{4b}{8} = 2.5$

$\therefore$ when $I_1 = I_2 = 20$
$V = 2.5 \times 20 + 5 \times 20 = 150$
$\therefore$ $V = 150V$

11) Given $V_1 = 1 I_1 + 2 I_2$ --- (i)

$V_2 = 3 I_1 + 4 I_2$ --- (ii)

and $V_2 = -5 I_2$ --- (iii)

substituting (iii) in (ii)

$$-5 I_2 = 3 I_1 + 4 I_2$$

$$\Rightarrow I_2 = -\frac{1}{3} I_1$$

putting this in (i) $V_1 = I_1 + 2\left(-\frac{1}{3}\right) I_1$

$$= \frac{1}{3} I_1$$

$\therefore$ Driving point impedance $= \frac{V_1}{I_1} = \frac{1}{3}\,\Omega$

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13) This can have multiple solution. Obviously given a circuit its h-parameters are fixed. But given the h-parameters which one can find many circuits which will have same h-parameter. Here is one solution [which is motivated from the differential equivalent circuit of a the BJT (bipolar Junction transistor)]

$$V_1 = h_{11} I_1 + h_{12} V_2$$

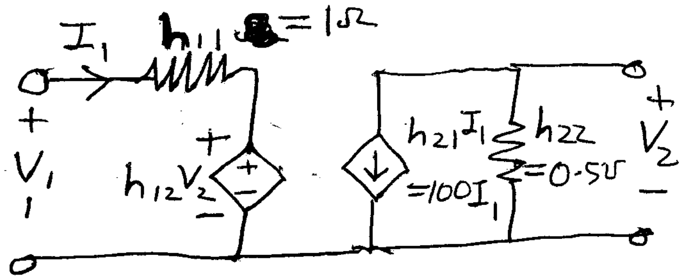
$$I_2 = h_{21} I_1 + h_{22} V_2$$

where $h_{11} = 1\,\Omega$

$h_{12} = 0.1$

$h_{21} = 100$

$h_{22} = 0.5\,℧$



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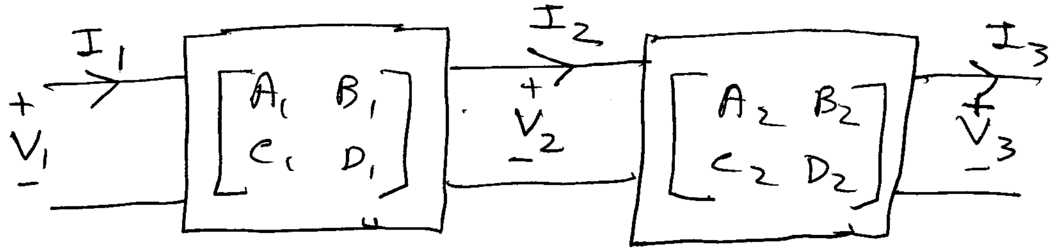
12) <u>Theory</u> : If ~~the~~ two networks with known T-parameters are connected in cascade as follows



then $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$ --- (i)

and $\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$ --- (ii)

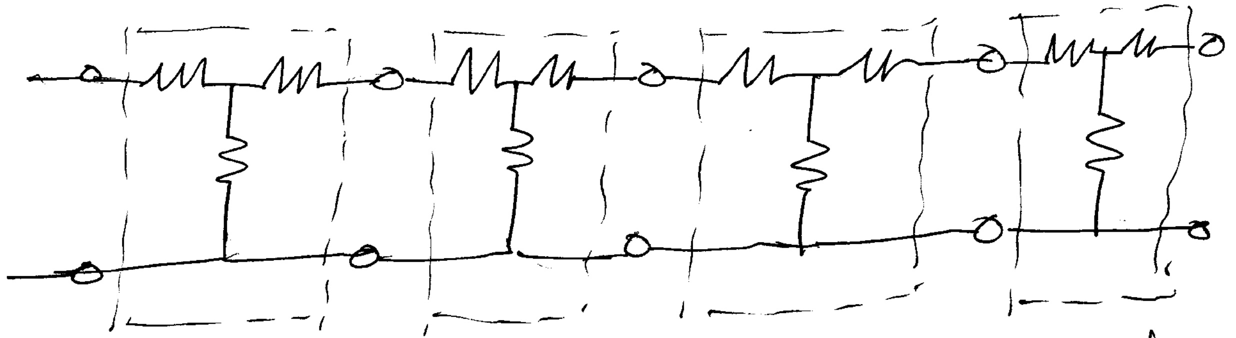
combining (i) & (ii) we get

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \left(\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \right) \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

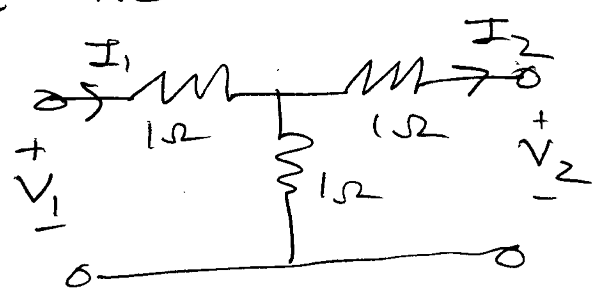
$\therefore$ T-parameters of the two cascaded network is

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \times \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

12) The given network can be thought to be a cascade of 4 smaller networks



Let us find the T- parameters of an individual network



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{V_1}{\frac{V_1}{2}} = 2$$

(secondary open)

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{V_1}{\frac{I_1}{2}} = \frac{V_1}{\left(\frac{1}{2} \times \frac{V_1}{1+\frac{1}{2}}\right)}$$

(secondary short)

$$= \frac{1}{\frac{1}{2} \times \frac{1}{1\cdot 5}} = 3$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_1}{I_1 \times 1} = 1$$

(secondary open)

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{I_1}{\frac{I_1}{2}} = 2$$

(secondary short)

$$\therefore \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$\therefore$ Overall T parameters

$$= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}^4 = \left(\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \right)^2$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}^2 = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 97 & 168 \\ 56 & 97 \end{bmatrix}$$

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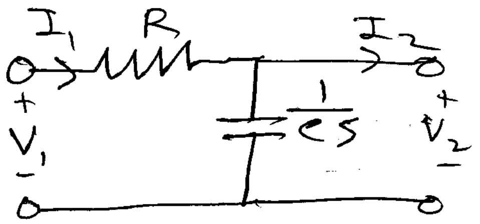
14) Lets find the ABCD parameters of an individual network first



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{V_1}{V_1 \times \frac{\frac{1}{es}}{R+\frac{1}{es}}} = \frac{1}{\frac{1}{1+Res}} = (1+Res)$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{V_1}{\frac{V_1}{R}} = R$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_1}{\frac{I_1}{cs}} = cs$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = 1$$

$$\therefore \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+Rcs & R \\ cs & 1 \end{bmatrix}$$

$\therefore$ ABCD Parameters of the overall / cascaded network

$$= \begin{bmatrix} 1+Rcs & R \\ cs & 1 \end{bmatrix}^4 = \left(\begin{bmatrix} 1+Rcs & R \\ cs & 1 \end{bmatrix} \begin{bmatrix} 1+Rcs & R \\ cs & 1 \end{bmatrix} \right)^2$$

$$= \left[\begin{array}{c|c} ((1+Rcs)^2 + Rcs) & ((1+Rcs)R + R) \\ \hline ((1+Rcs)cs + cs) & (Rcs+1) \end{array} \right]^2$$

$$= \left[\begin{array}{c|c} ((1+Rcs)^2 + Rcs)^2 + ((1+Rcs)R+R)((1+Rcs)cs + cs) & \text{something}_B \\ \hline \text{something}_C & \text{something}_D \end{array} \right]$$

$$= \left[\begin{array}{c|c} (1+3Rcs+(Rcs)^2)^2 + (2R+R^2cs)(2cs+Rc^2s^2) & \text{some } B \\ \hline \text{some } C & \text{some } D \end{array} \right]$$

$$= \left[\begin{array}{c|c} \text{some } A & \text{some } B \\ \hline \text{some } C & \text{some } D \end{array} \right]$$

where $(\text{some } A) = \left(1 + 3Rcs + (Rcs)^2\right)^2 +$

$$(2R + R^2cs)(2cs + Rc^2s^2)$$

~~Vin~~ If we call

V_{in} = input to overall network

V_{out} = output of " "

then $V_{in} = (\text{Some } A)\, V_{out} + (\text{Some } B)\, I_{out}$

where I_{out} = output current ~~when~~

Now under no load condition

(as usual in this type of analysis), $I_{out} = 0$

$\therefore V_{out} = \dfrac{V_{in}}{(\text{some } A)}$ where some A is

given above.

$$\therefore H(s) = \frac{V_{out}}{V_{in}} = \frac{1}{\text{some } A}$$

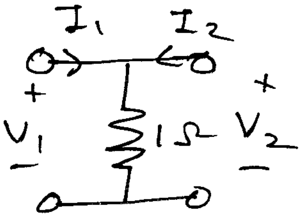
$$= \frac{1}{\left(1 + 3Rcs + (Rcs)^2\right)^2 + (2R + R^2cs)(2cs + Rc^2s^2)}$$

$$\therefore H(j\omega) = \frac{1}{\left(1 + 3Rc\omega j - R^2c^2\omega^2\right)^2 + \left(2R + R^2cj\omega\right)\left(2cj\omega - Rc^2\omega^2\right)}$$

Now if input is $V_1 = \cos(\omega t)$

output will be $|H(j\omega)| \cos\left(\omega t + \angle H(j\omega)\right)$

9) a)



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = 1 \qquad z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = 1$$

$$z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} = 1 \qquad z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = 1$$

$$\therefore\ Z = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

so $Y = Z^{-1}$ = undefined
$[\because \det[Z] = 0]$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} = 0$$

$$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0} = 1$$

$$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0} = -1$$

$$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0} = 1$$

$$\therefore\ h = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$$

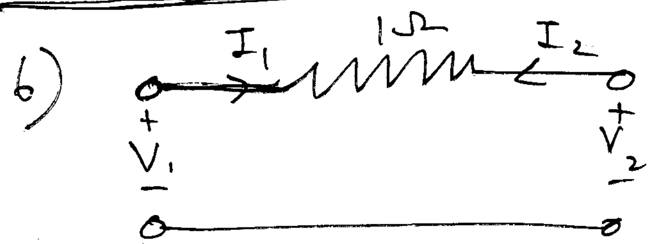
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$A = \frac{V_1}{V_2}\Big|_{I_2=0} = 1 \qquad C = \frac{I_1}{V_2}\Big|_{I_2=0} = 1$$

$$B = \frac{V_1}{-I_2}\Big|_{V_2=0} = 0 \qquad D = \frac{I_1}{-I_2}\Big|_{V_2=0} = 1$$

$$\therefore \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

b)



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = 1 \qquad y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} = -1$$

$$y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} = -1 \qquad y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0} = 1$$

$$\therefore y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$\therefore Z = y^{-1}$ = undefined

$[\because \det[Y] = 0]$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = 1$$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} = 1$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} = -1$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} = 0$$

$$\therefore h = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = 1 \qquad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1$$

$$\therefore \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$