Millman's theorem

Tapas Kumar Bhattacharya

Department of Electrical Engineering
I.I.T Kharagpur

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1 Millman's Theorem

Imagine you have number of sources with voltages E_1,$E_2 \cdots E_N$, with internal impedances Z_1,$Z_2 \cdots Z_N$ respectively and assume that these sources are connected in parallel as shown in figure 1. Millman's

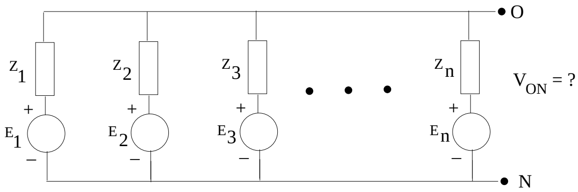

Figure 1:

theorem gives you a handy formula to calculate the the terminal voltage V_{ON}. We write the KCL at point O.

$$
\begin{aligned}
\frac{V_{ON}-E_1}{Z_1}+\frac{V_{ON}-E_2}{Z_2}+\cdots+\frac{V_{ON}-E_n}{Z_n} &= 0\\
\text{or, } \left(\frac{1}{Z_1}+\frac{1}{Z_2}+\cdots+\frac{1}{Z_n}\right)V_{ON} &= \frac{E_1}{Z_1}+\frac{E_2}{Z_2}+\cdots\frac{E_n}{Z_n}\\
\text{or, } V_{ON} &= \frac{\frac{E_1}{Z_1}+\frac{E_2}{Z_2}+\cdots\frac{E_n}{Z_n}}{\frac{1}{Z_2}+\cdots+\frac{1}{Z_n}}\\
\text{we know, Admittance } \; Y &= \frac{1}{Z}\\
\therefore \; V_{ON} &= \frac{E_1Y_1+E_2Y_2+\cdots+E_nY_n}{Y_1+Y_2+\cdots+Y_n}
\end{aligned}
$$

After knowing V_{ON}, we can calculate currents through each branch. For example current through Z_1 from bottom to top is $\frac{E_1-V_{ON}}{Z_1}$.

2 Unbalanced star connected load fed from a balanced 3-ph supply

Look at the circuit 2 where E_{AN}, E_{BN} and E_{CN} represent a balanced three phase voltage, N being the supply neutral. Magnitudes of the voltages are same and they are mutually 120° apart which means that $E_{AN}+E_{BN}+E_{CN}=0$, O being the star point of the load **not connected to the supply neutral N**. We want to calculate the currents I_A, I_B and I_C.

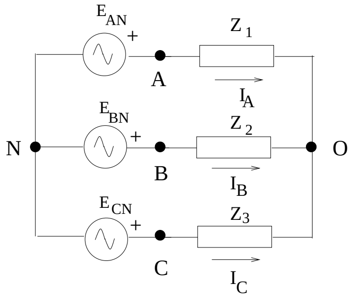

Figure 2:

These currents can be calculated provided we know V_{ON} which can be calculated by using Millman's theorem as follows.

$$V_{ON} = \frac{E_{AN}Y_1 + E_{BN}Y_2 + E_{CN}Y_3}{Y_1 + Y_2 + Y_3}$$

where, $Y_1 = 1/Z_1$, $Y_2 = 1/Z_2$ and $Y_3 = 1/Z_3$. After knowing V_{ON}, the currents are calculated as follows.

$$\begin{aligned} I_A = V_{AO}Y_1 &= (V_{AN} - V_{ON})Y_1 \\ I_B = V_{BO}Y_2 &= (V_{BN} - V_{ON})Y_2 \\ I_C = V_{CO}Y_3 &= (V_{CN} - V_{ON})Y_3 \end{aligned}$$