Mesh & nodal analysis

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1 Introduction

You are already familiar with solving a network problem either by mesh analysis or by nodal analysis. Briefly we review these methods first. The emphasis is on hoe to write down the KVL equations in meshes or KCL equations at nodes **just by inspection**.

However there may be situations where writing down those equations by inspection may be difficult. One such situation is when a ideal voltage source exist between two nodes or a current source exists in a branch shared by two adjacent meshes. By suitably transferring the sources else where in the original network, it is possible to write down the equations by inspection in the so called transformed circuit.

2 Solving circuit by mesh analysis

Following steps are to be followed to solve network problem by *Mesh* analysis.

1. First identify the number of meshes in the network. Remember that any closed path (loop) may not be a mesh. Mesh are those closed paths within which no other mesh exist.
2. Assign unknown mesh currents I_1, I_2, $I_3 \cdots$, I_n where n is the number of meshes.
3. Mesh currents are solved by forming n number of independent KVL equations in those meshes.
4. If current sources are present in the outer meshes, those mesh currents are already known and KVL equations are not written in those meshes.
5. Solve the mesh currents. All the branch currents then can be calculated.
6. It is interesting to note that the KVL equations in each mesh can be written just by inspection.

Let us write down the KVL equations in mesh of the circuit shown in Figure 1

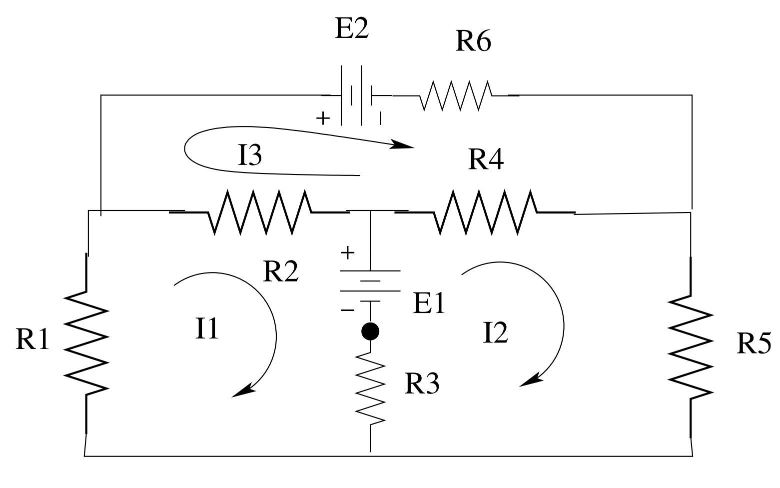

Figure 1:

1. There are 3 meshes in the circuit and we assume three mesh currents I_1, I_2 and I_3, all in the clockwise direction.
2. While writing KVL in mesh-1, coefficient of I_1 will be sum of all the resistances in mesh-1; coefficient of I_2 will be negative value of the common resistance existing between mesh-1 and mesh-2; coefficient of I_3 will be negative value of the common resistance existing between mesh-1 and mesh-3;

3. While writing KVL in mesh-2, coefficient of I_2 will be sum of all the resistances in mesh-2; coefficient of I_1 will be negative value of the common resistance existing between mesh-1 and mesh-2; coefficient of I_3 will be negative value of the common resistance existing between mesh-2 and mesh-3;

4. While writing KVL in mesh-3, coefficient of I_3 will be sum of all the resistances in mesh-3; coefficient of I_1 will be negative value of the common resistance existing between mesh-1 and mesh-3; coefficient of I_2 will be negative value of the common resistance existing between mesh-2 and mesh-3;

The right hand side of each of the above equation will contain sum of all the emfs acting in that particular mesh. If the +ve polarity of the emf source is in agreement with the mesh current direction, assign +ve sign to that emf. On the other hand, if the +ve polarity of the emf source is in opposition to the mesh current direction, assign -ve sign to that emf.

Following the above steps, three KVL equations for the circuit shown in Figure 1 are written below by inspection.

$$\begin{aligned}(R_1 + R_2 + R_3)I_1 - R_3 I_2 - R_2 I_3 &= -E_1 \\ -R_3 I_1 - (R_3 + R_4 + R_5)I_2 - R_4 I_3 &= E_1 \\ -R_2 I_1 - R_4 I_2 + (R_2 + R_4 + R_6)I_3 &= -E_2\end{aligned}$$

2.1 Problem solving with mesh analysis

Find out all the branch currents in the following circuit shown in Figure 2. Also calculate the voltage across the current source.

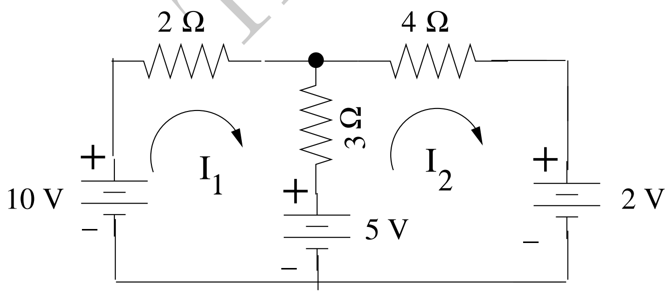

Figure 2:

Solution:

If you recall this problem was solved by applying KVL & KCL methods earlier. There are two meshes and we assume the mesh currents as I_1 and I_2 as shown in Figure 2. Then we write down the two KVL equations for mesh-1 and for mesh-2 by inspection which are shown below.

$$\begin{aligned}5I_1 - 3I_2 &= 10 - 5 = 5 \\ -3I_1 + 7I_2 &= 5 - 2 = 3\end{aligned}$$

After solving the above equations we get the mesh currents as:

$$I_1 = 1.692 \text{ A and } I_2 = 1.154 \text{ A}$$

Note current through the $3\,\Omega$ resistance is $I_1 - I_2 = 0.638$ A and its direction is from top to bottom. The result obtained remain same. We redraw the circuit showing all the branch currents along with their directions in the following Figure 3.

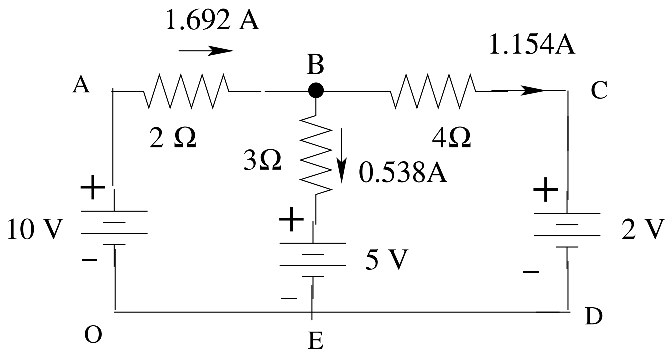

Figure 3:

2.2 Another Example using mesh analysis

Let us take another previously solved problem involving a current source, the circuit is shown in Figure 4 and we have to solve it by mesh analysis.

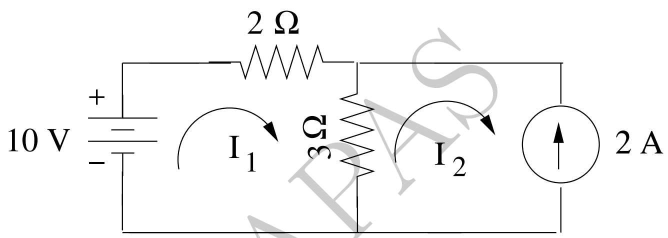

Figure 4:

Solution:

There are two meshes in the circuit and we assume the mesh currents as I_1 and I_2. But look at the second mesh where a current source of 2 A is present. Therefore the second mesh current is already known and $I_2 = -2$ A. Since I_2 is known we have to write only one KVL equation for mesh-1 by inspection.

$$\begin{aligned}
5I_1 - 3I_2 &= 10 \\
\text{or } 5I_1 - 3 \times (-2) &= 10 \text{ putting the value of } I_2 \\
\text{or } 5I_1 &= 4 \\
\therefore\ I_1 &= 0.8 \text{ A} \\
\text{Current through } 3\ \Omega &= I_1 - I_2 = 0.8 - (-2) = 2.8 \text{ A}
\end{aligned}$$

3 Solving circuit by Node analysis

Following steps are to be followed for solving a circuit problem by Node analysis.

1. First find out the number of nodes (N) present in the circuit. Recall that a node is a junction where ends of more than two elements meet.
2. Choose any one of those N nodes as reference node and label it as O. Other (N-1) nodes can be suitably named as A,B,C etc.
3. In node analysis unknown are the (N-1) node voltages namely V_{AO}, V_{BO},V_{CO},V_{DO} etc.

4. To find out unknown node voltages, KCL equations are written at each node A, B, C, D etc. Number of KCL equations will be in general (N-1). The KCL equations can be written just by inspection.

5. It may be noted while writing KCL at node A, coefficient of V_{AO} is sum of reciprocal of all the resistances connected to node A; coefficient of V_{BO} should be -ve of the reciprocal of the resistance connected between A and B; coefficient of V_{CO} should be -ve of the reciprocal of the resistance connected between A and C; and so on.

6. Write hand side of each of the KCL equations will have source terms connected to that particular node. EMF sources will appear as $\pm E/R$ where E is the emf connected to that node and R is the resistance in series with E. If +ve terminal of E is connected to that node then +ve sign is to be used. If -ve terminal of E is connected to that node then -ve sign is to be used before E/R. In addition, If a current source I, is connected to a particular node, $\pm I$ will also appear on the right hand side of each KCL equation. +ve sign to be used when direction of the current is towards the node and -ve sign is to be used when direction of I is away from the node.

7. After getting the values of the node voltages, all the branch currents can be calculated.

Let us write down the KCL equations at different nodes of the following circuit.

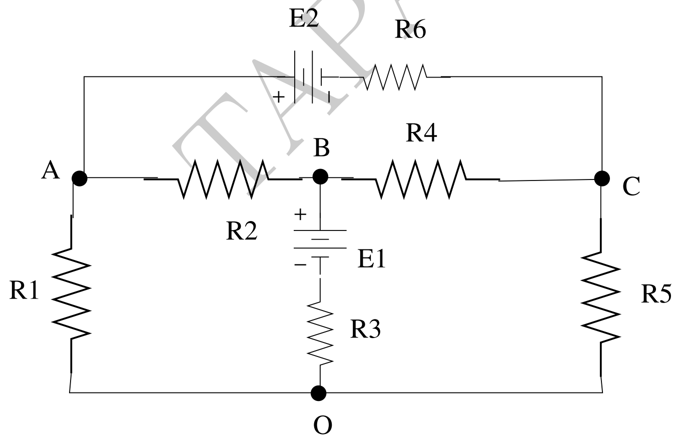

Figure 5:

In this circuit there are 4 nodes and we have named them as A,B,C and O. Terminal O is chosen to be reference node. So unknowns are V_{AO}, V_{BO}, V_{CO}.

$$\begin{aligned}
\text{KCL at node A gives: } \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_6}\right)V_{AO}-\frac{1}{R_2}V_{BO}-\frac{1}{R_6}V_{CO} &= \frac{E_2}{R_6}\\
\text{KCL at node B gives: } -\frac{1}{R_2}V_{AO}+\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)V_{BO}-\frac{1}{R_4}V_{CO} &= \frac{E_1}{R_3}\\
\text{KCL at node C gives: } -\frac{1}{R_6}V_{AO}-\frac{1}{R_4}V_{BO}+\left(\frac{1}{R_4}+\frac{1}{R_5}+\frac{1}{R_6}\right)V_{CO} &= -\frac{E_2}{R_6}
\end{aligned}$$

Thus V_{AO}, V_{BO}, V_{CO} can be calculated by solving the above three equations. Then various branch currents can be calculated as follows:

$$\begin{aligned}
I_{AB} &= (V_{AO}-V_{BO})/R_2\\
I_{AO} &= V_{AO}/R_2
\end{aligned}$$

$$
\begin{aligned}
I_{BO} &= (V_{BO} - E_1)/R_3 \\
I_{BC} &= (V_{BO} - V_{CO})/R_4 \\
I_{CO} &= V_{CO}/R_5
\end{aligned}
$$

4 Ideal voltage source between two nodes

Look a the circuit (figure 6(a)) where an ideal voltage source E_3 is present between nodes A and B. Since resistance in the branch (AB), is zero, KCL at nodes A or B can not be written by inspection as one of the terms will blow up to ∞.

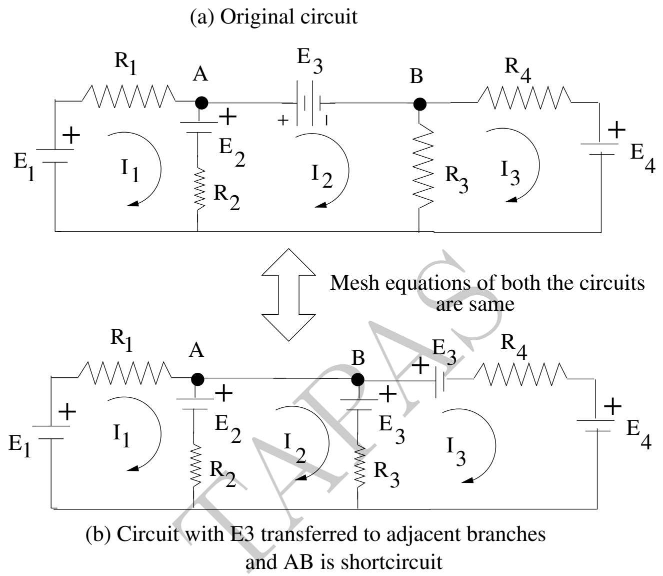

Figure 6:

From the original circuit, a circuit can be manufactured (shown in figure 6(b)) by replacing E_3 by a short circuit and transferring E_3 in the adjacent branches in such a way so that the mesh KVL equations are same as mesh equations of the original circuit. This will ensure the branch currents of both the circuits to be same. So, we shall now solve the transformed circuit and calculate the branch currents, These branch currents indeed will be the same as that of the original circuit.

Example-1

For the network shown in figure 7, solve for the branch currents by nodal method using the concept of transferring the voltage source.

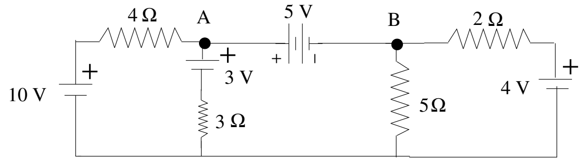

Figure 7:

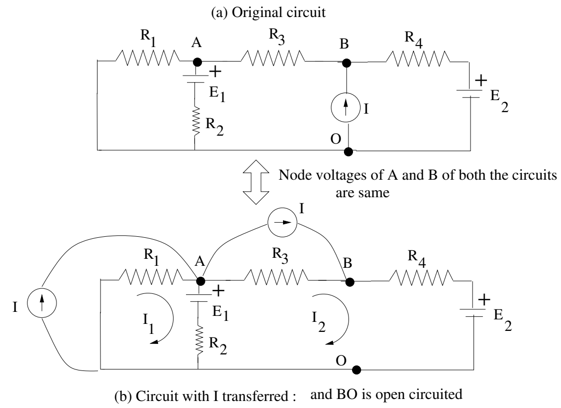

Figure 8:

5 Ideal current source between adjacent meshes

Look a the circuit (figure 8(a)) where an ideal current source I is present in the branch BO. If we want to write down mesh KVL equations by inspection, difficulty will be faced as voltage across the current source is not known. We transfer the current source I (making BO open circuited)suitably so that node voltages of A and B remains same as that of the original circuit.

Example-2

For the network shown in figure 9, solve for the branch currents by mesh analysis using the concept of transferring the current source.

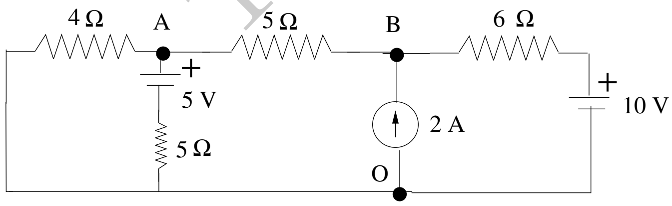

Figure 9: