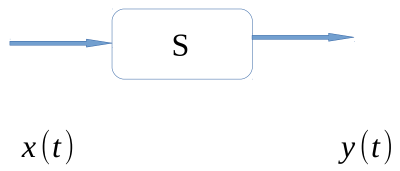
Linear, Time invariant System : how to check?

Linear System:



It is always better you draw the above picture showing the input and output of the system. The relationship between output $y(t)$ and $x(t)$ will be given either in the form of an algebraic relation or in the form of a differential equation.

If you can establish $y(t) \neq 0$ for x(t) = 0 then you can immediately conclude the system is non-linear. So as a precondition y(t) must be equal to 0 for x(t) = 0 (called the Homogenity condition i.e., no output for no input) must be satisfied if the system is to be linear.

Example-1:

$y(t) = -3x(t) + 1$ is a non-linear system, since $y(t) = 1$ when $x(t) = 0$. Does not satisfy the *Homogenity* condition. No further checking is necessary.

Example-2:

$$\frac{d^2 y}{dt^2} + 10\frac{dy}{dt} + 2y = x(t)$$ Is this system linear?

With all initial condition relaxed, check first for homogenity condition. We see that $y(t) = 0$ for $x(t) = 0$. So system satiesfies homogenity condition. We must note that mere satisfaction of homogenity condition, does not make a system linear.

So It is now necessary to check swhether the system satisfy the scaling up and superposition principle.

Suppose $x_1(t) \rightarrow y_1(t)$ and $x_2(t) \rightarrow y_2(t)$.

If we can establish that , $a_1 x_1(t) + a_2 x_2(t) \rightarrow a_1 y_1(t) + a_2 y_2(t)$ then we conclude that the system is linear.

Now $x_1(t) \rightarrow y_1(t)$ means: $$\frac{d^2 y_1}{dt^2} + 10\frac{dy_1}{dt} + 2y_1 = x_1(t)$$

and $x_2(t) \rightarrow y_2(t)$ means: $$\frac{d^2 y_2}{dt^2} + 10\frac{dy_2}{dt} + 2y_2 = x_2(t)$$

Multiply first equation by a_1 and second equation by a_2 and then add these two equations to get:

$$\frac{d^2(a_1 y_1 + a_2 y_2)}{dt^2} + 10\frac{d(a_1 y_1 + a_2 y_2)}{dt} + 2(a_1 y_1 + a_2 y_2) = a_1 x_1(t) + a_2 x_2(t)$$

It indeed means $a_1 x_1(t) + a_2 x_2(t) \rightarrow a_1 y_1(t) + a_2 y_2(t)$. So the system is lnear.

Example-3: Comment on linearity of the following system

$$t\frac{dy}{dt} + 2y = x(t)$$

With initial condition relaxed, Homogenity condition is satisfied.

Now we go for scaling & superposition conditions.

Now $x_1(t) \rightarrow y_1(t)$ means: $t\frac{dy_1}{dt} + 2y_1 = x_1(t)$

Now $x_2(t) \rightarrow y_2(t)$ means: $t\frac{dy_2}{dt} + 2y_2 = x_2(t)$

Multiply first equation by a_1 and second equation by a_2 and then add these two equations to get:

$$t\frac{d(a_1 y_1 + a_2 y_2)}{dt} + 2(a_1 y_1 + a_2 y_2) = a_1 x_1(t) + a_2 x_2(t)$$

It indeed means $a_1 x_1(t) + a_2 x_2(t) \rightarrow a_1 y_1(t) + a_2 y_2(t)$. So the system is lnear.

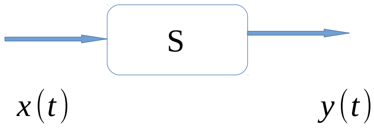
Example-4: Comment on linearity of the following system

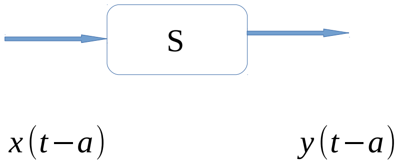
$$y(t) = x^2(t)$$

Here x(t) =0 gives y(t) =0.
We can easily see if x(t) is doubled output increases by 4 times. Non-linear system.

***Time Invariant* system or *non-time invariant* system**

Pictorially it means





In language if input is delayed by some amount, ouput too, will be delayed by the same amount.

Example 5
consider the simple system $y(t)=4x(t)$. Is ithe system time invariant or not?

$x(t)\rightarrow 4x(t)=y_1(t)$ Which means $x(t-a)\rightarrow 4x(t-a)=y_2(t)$.

which means $y_2(t)=y_1(t-a)$ hence output is delayed by same amount: so time invariant system.

Example 6
consider the system $y(t)=t\,x(t)$. Is ithe system time invariant or not?

$x(t)\rightarrow t\,x(t)=y_1(t)$ Which means $x(t-a)\rightarrow t\,x(t-a)=y_2(t)$.

which means $y_2(t)\neq y_1(t-a)=(t-a)x(t-a)$ so not a time invariant system.

**** Example 7**
consider the system $y(t)=x(3t)$. Is ithe system time invariant or not?

$x(t)\rightarrow x(3t)=y_1(t)$

Now let us say that $x_2(t)=x(t-a)$

$x_2(t)\rightarrow x_2(3t)=y_2(t)$

Since $x_2(t)=x(t-a)$ therefore $x_2(3t)=x(3t-a)$

So, $y_2(t)=x(3t-a)$

But $y_1(t)=x(3t)$, therefore $y_1(t-a)=x(3t-3a)\neq y_2(t)$

so **not a time invariant** system.