Circuit Elements : a deeper look

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1 Introduction

An electrical circuit or network will consist of sources and three circuit elements R, L and C. Inductor and capacitor are energy storing elements while a resistor is an energy dissipative element. In this note $v - i$ characteristics of these three circuit elements are discussed. The present value of the current in a resistor depends on the present value of the voltage only. In contrast present value of the current in an inductor is not only decided by the present value of the voltage across it but also depends upon the previous voltage applied earlier across it. Similarly present value of the voltage in a capacitor is not only decided by the present value of the current flowing through it but also depends upon the previous value of the current, the capacitor carried earlier. For this reason, inductor and capacitor are called elements with memory and a resistor is called a memory less element. It may be noted that to build up current in an inductor we have to apply voltage across it while we have to pump current in to capacitor to build up voltage across it.

If the excitation (voltage or current) are reasonably good (!) functions of time, then current through an inductor and voltage across a capacitor will be continuous in time. This conditions will fail or will be different if we allow excitation to be an impulse ($\delta(t)$). How to handle situations with $\delta(t)$ functions is also discussed.

2 v vs i relations of R, L and C

The current value of the current in a resistor depends solely upon the current value of the voltage. Hence it is called a memory less element. In a circuit if energy storing elements (inductor & capacitor) are present along with resistance and the D.C sources, response of the current due to switching of the circuit causes time varying current to flow before a steady state is reached. The voltage-current relationship of a resistor, inductor and a capacitor are shown in Figure 1.

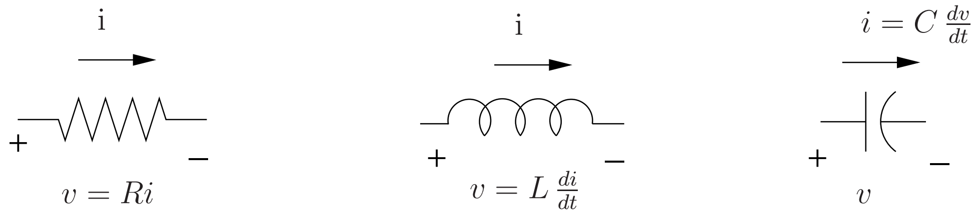

Figure 1:

$$\begin{aligned} \text{For resistance: } v &= Ri \\ \text{For inductance: } v &= L\frac{di}{dt} \\ \text{For capacitance: } i &= C\frac{dv}{dt} \end{aligned}$$

One should carefully note the direction of the current through the element and the polarity of the voltage across the element. The following important boundary conditions will be satisfied for the three elements when a switching is executed say at $t = 0$.

1. In a resistance since $v = Ri$, current can change in steps i.e., can change instantaneously. A resistor is an energy dissipating elements. In fact the current value of the current in a resistor depends solely upon the current value of the voltage. Hence it is called a memory less element.

2. In an inductor since, $v = L\,\frac{di}{dt}$, current can not change instantaneously as such a situation will call for voltage to be infinitely large ($\because\ \frac{di}{dt} \to \infty$). This essentially means that $i(0_-) = i(0_+)$. 0_- is the time immediately before switching is done and 0_+ is the time immediately after switching is done. The current value of the current in an inductor depends not only upon the current value of the voltage but also upon the previous voltage applied across it.. Hence it is called a element with memory.

3. When an inductor carries current, it stores energy which is equal to $\frac{1}{2}\,Li^2$

4. In a capacitor, since, $i = C\,\frac{dv}{dt}$, voltage can not change instantaneously as such a situation will call for current to be infinitely large ($\because\ \frac{dv}{dt} \to \infty$). This essentially means that $v(0_-) = v(0_+)$. 0_- is the time immediately before switching is done and 0_+ is the time immediately after switching is done. The current value of the voltage across a capacitor depends not only upon the present value of the current but also upon the currents passed through it previously. Hence it is called an element with memory.

5. When there is a voltage across a capacitor, it stores energy which is equal to $\frac{1}{2}\,Cv^2$

2.1 Deeper look at inductor & capacitor

About inductor

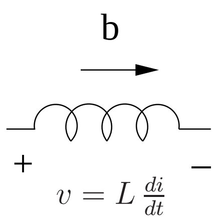

Figure 2:

Let us try to write current i in an inductor in terms of voltage:

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(t)\,dt$$

Thus the present value of the inductor current depends on the past history of the applied voltage. Suppose in a circuit at $t = 0$ some switching is executed, then the RHS of the above equation can be broken up into various pieces as below. Note $t = 0^-$ represent time immediately before $t = 0$ and $t = 0^+$ represents time immediately after $t = 0$.

$$i(t) = \frac{1}{L}\int_{-\infty}^{0^-} v(t)\,dt + \frac{1}{L}\int_{0^-}^{0^+} v(t)\,dt + \frac{1}{L}\int_{0^+}^{t} v(t)\,dt$$

$$\begin{aligned}
\text{Now, } \frac{1}{L}\int_{0^-}^{0^+} v(t)\,dt &= 0 \text{ provided } v(t) \text{ is not an impulse} \\
\therefore\ i(t) &= \frac{1}{L}\int_{-\infty}^{0^-} v(t)\,dt + \frac{1}{L}\int_{0^+}^{t} v(t)\,dt \\
\therefore\ i(t) &= i(0^-) + \frac{1}{L}\int_{0^+}^{t} v(t)\,dt \\
\text{Now putting } t = 0^+,\ i(0^+) &= i(0^-) + \frac{1}{L}\int_{0^+}^{0^+} v(t)\,dt \\
\text{Thus } i(0^+) &= i(0^-) \\
\text{and } i(t) &= i(0^-) + \frac{1}{L}\int_{0^+}^{t} v(t)\,dt = i(0^-) + \frac{1}{L}\int_{0}^{t} v(t)\,dt
\end{aligned}$$

Conclusion is that "in a circuit if some switching is carried out (at $t = 0$, current through an inductor can not change instantaneously".

Recall:

$$i(t) = i(0^-) + \frac{1}{L}\int_{0}^{t} v(t)\,dt$$

This equation suggests that an inductor with initial current $i(0^-)$ is equivalent to an inductor with no initial current and in parallel with a current source of strength $i(0^-)$ as depicted in Figure 3.

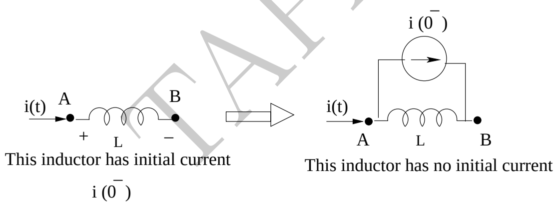

Figure 3:

If somebody loves to see inductor without any initial current $i_(0_-)$ and a capacitor without any initial voltage $v_(0_-)$ then he has to introduce additional current source of constant magnitude $i_(0_-)$ across the inductor and additional voltage source of constant magnitude $v_(0_-)$ in series the capacitor and pretend that initial conditions of the inductor and capacitor to be relaxed.

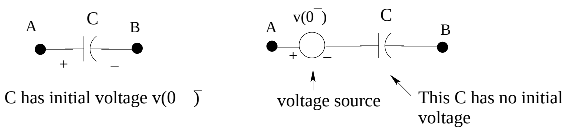

Figure 4:

About capacitor

Let us try to write down voltage $v(t)$ across a capacitor in terms of current:

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$$

Thus the present value of the capacitor voltage depends on the past history of the injected current. Suppose in a circuit at $t = 0$ some switching is executed, then the RHS of the above

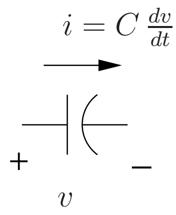

Figure 5:

equation can be broken up into various pieces as below. Note $t = 0^-$ represent time immediately before $t = 0$ and $t = 0^+$ represents time immediately after $t = 0$.

$$
\begin{aligned}
v(t) &= \frac{1}{C}\int_{-\infty}^{0^-} i(t)\,dt + \frac{1}{C}\int_{0^-}^{0^+} i(t)\,dt + \frac{1}{C}\int_{0^+}^{t} i(t)\,dt \\
\text{Now, } \frac{1}{C}\int_{0^-}^{0^+} i(t)\,dt &= 0, \text{ provided } i(t) \text{ is not an impulse} \\
v(t) &= \frac{1}{C}\int_{-\infty}^{0^-} i(t)\,dt + \frac{1}{C}\int_{0^+}^{t} i(t)\,dt \\
\therefore\; v(t) &= v(0^-) + \frac{1}{C}\int_{0^+}^{t} i(t)\,dt \\
\text{Now putting } t = 0^+,\; v(0^+) &= v(0^-) + \frac{1}{C}\int_{0^+}^{0^+} i(t)\,dt \\
\text{Thus, } v(0^+) &= v(0^-) \\
\text{and at any time } t,\; v(t) &= v(0^-) + \frac{1}{C}\int_{0^+}^{t} i(t)\,dt = \frac{1}{C}\int_{0}^{t} i(t)\,dt
\end{aligned}
$$

Conclusion is that "in a circuit if some switching is carried out (at $t = 0$, voltage across the capacitor can not change instantaneously".

Recall:

$$v(t) = v(0^-) + \frac{1}{C}\int_{0}^{t} i(t)\,dt$$

This equation suggests that a capacitor with initial voltage $v(0^-)$ is equivalent to an capacitor with no initial voltage and in series with a voltage source of strength $v(0^-)$ as depicted in Figure 6.

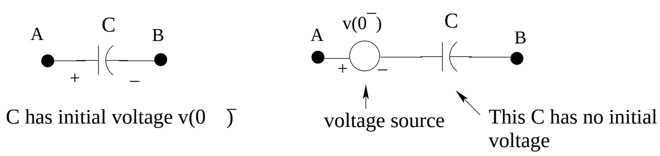

Figure 6: