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Unsolvable Problems (Unsolvability)

$\{\langle M, w\rangle \mid M$ is a Turing Machine and $w \in \mathcal{L}(M)\}$

let's suppose Alphabet Σ (Finite)

$\Sigma^* \rightarrow$ countable infinite

No. of languages over $\Sigma^* = 2^{\Sigma^*}$ [Uncountable]

$\exists$ language that are not recognized by Turing machine

Proof $\rightarrow$ Show that the possible TMs are countable

We can encode it like DFA

Taking $\Sigma = \{a\}$

let's suppose we map TM with the 2^{Σ^*}

Supposed ~~language~~ & mapping

	ϵ	a^1	a^2	a^3
M_0	0	1		
M_1			1	
M_2				

$L = \{a^i \mid a^i \in \mathcal{L}(M_i)\}$

L will not be in the list as TM are countable

But there will be a language of for L

So this new language will not be recognized by any TM.

Halting Problem

Given a Turing Machine and a string x, show that whether M halts on x or not beforehand?

Proof

→ Suppose you've a TM that takes M and x as i/p

Simulates M on x

→ If M halts and ~~accepts~~ accepts/reject $x \rightarrow$ halt accept

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or it loops on $x \rightarrow$ Outer machine also loops

$\Sigma = \{0, 1\}$, tape alphabet, i/p alphabet

let M_x ~~with~~ be a Turing Machine with encoding x

	ϵ	0	00	01	10	11
M_ϵ	H	L	L	H	L	L
M_0	L	L	L	L	L	L
M_1						
$M_{\dots}$						

Each Turing Machine will have some behaviour for each string

new TM S (Takes input TM and x)

If the problem was decidable then it would have determine the (K, Y) element of the matrix

TM string

$K \rightarrow M_x \# x$

If ~~(K,Y)~~ $(K, Y) = H$ [Original machine accepts]

" " $= L$ [" " rejects]

Build a new TM N such that takes x as i/p

→ Uses x to create description M_x

→ Puts $M_x \# x$ on i/p tape

→ Runs / Simulates K on $M_x \# x$

N goes in a loop ⇐ if K accepts

N halts on x ⇐ " " rejects

[Make N like this]

This will actually result in a turing machine (N here) which has a behaviour different from all those present in a list. This will be a new element in the list.

{<M#w> / M is a TM such that $w \in \mathcal{L}(M)$}

- Membership Problem

Turing Recognizable → $x \in \mathcal{L}(M)$ [it accepts] otherwise we donot kn..

" Decidable → " " " " rejects

Reducing Halting Problem to this problem.

$HP \xrightarrow{reduce} MP$

Take M and modify M.

$(\text{accept}, \text{rejects})_M \longrightarrow (\text{accept})_{M_1}$

Suppose R is a TM that solves ~~MP~~ the given problem

Run R on $\langle M_1, w\rangle$

Now If R accepts → M (accepts, rejects), meaning M halts

" " " rejects/loop → M (loops), meaning M loops

Now this means that halting problem is ~~un~~decidable which is not possible.

$A \xrightarrow{reduce} B$

Undecidable (known) | To prove undecidability

$A \leq_m B$

← mapping

Q {<M> | M is a TM and $L(M) = \phi$}

- Emptiness Problem

Assume this is decidable

=> ∃ a TM R that decides E_{TM}

Create a TM S which solves MP using R

$\langle M, w\rangle \longrightarrow M_1$

- Create M_1 that on input x
- If $x \neq w$, rejects
- If $x = w$, it runs M over w, accepts if M accepts

$$\mathcal{L}(M_1) = \begin{cases} \phi, & \text{if } M \text{ rejects } w \\ \{w\}, & \text{if } M \text{ accepts } w \end{cases}$$

$Regular_{TM} = \{ \langle M \rangle \mid M$ is a TM and $L(M)$ is regular $\}$

Reduce $MP \leq_m Regular_{TM}$

$\langle M, w \rangle \rightarrow M_1$

- Create M_2 that on input x
  - If x is of the form $0^n 1^n$, accepts
  - Otherwise, Run M on w, accept x if M accept w

$$L(M_2) = \begin{cases} \{0,1\}^* & \text{if } M \text{ accept } w \\ 0^n 1^n, & \text{if } M \text{ rejects } w \end{cases}$$

$EQ_{TM} = \{ \langle M_1, M_2 \rangle \mid L(M_1) = L(M_2) \}$

Run R on $\langle M, M_1 \rangle$ $\quad L(M_1) = \phi$

if R accepts $\rightarrow L(M) = \phi \rightarrow$ S accepts

" " rejects $\rightarrow L(M) \neq \phi \rightarrow$ S rejects