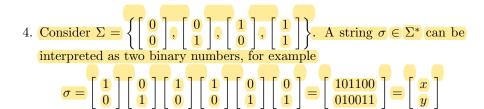
## DFA and NFA

## 21 Jan 2019

**Instructions**: For the problems with (To submit), please write the answers neatly in loose sheets with your name and roll number. Submit to the TA at the end of the class.

- 1. Construct DFAs for the following languages.
  - (a)  $L_1 = \{\omega | \omega \text{ contains an equal number of occurrences of } 01 \text{ and } 10\}$
  - (b) Ternary Strings (base 3), (i.e.  $\Sigma=\{0,1,2\})$  whose integer equivalent is divisible by 7. (To submit)
- 2. Construct NFAs for the following languages.
  - (a)  $L_2 = \{\omega | \omega \text{ is a string in which at least one } a_i \text{ occurs even number of times (not necessarily consecutively), where } 1 \leq i \leq 3 \text{ over } \Sigma = \{a_1, a_2, a_3\}\}.$
  - (b)  $L_3 = \{\omega | \omega \text{ contains two 0s separated by a substring whose length is a multiple of 3 }, \Sigma = \{0, 1\}.$  (To submit)
- 3. Prove the following properties.
  - (a) For languages A and B, the shuffle of A and B is the language  $L = \{\omega | \omega = a_1b_1 \cdots a_kb_k\}$ , where  $a_1 \circ \cdots \circ a_k \in A$  and  $b_1, \cdots, b_k \in B$ ,  $\forall a_i, b_i \in \Sigma^*$ . Prove that the class of regular languages is closed under Shuffle operation.
  - (b) Let B and C be languages over  $\Sigma = \{0,1\}$ . We have defined a language  $L = B \leftarrow C$  as  $L = \{\omega \in B | \text{ for some } y \in C, \text{ strings } \omega \text{ and } y \text{ contain equal numbers of 1's. }$ . Show that the class of regular languages is closed under the  $\leftarrow$  operation. (To submit)
  - (c) A homomorphism is a mapping h with domain  $\Sigma^*$  for some alphabet  $\Sigma$  which preserves concatenation:  $h(v \cdot w) = h(v) \cdot h(w)$ . Prove that the class of regular languages is closed under Homomorphism operation. (Home)



where  $x, y \in \{0, 1\}^*$ . Design a DFA which accepts strings in  $\Sigma^*$  such that  $2x - y \le 2$ . Note that, for such a DFA transitions will be labeled with elements from  $\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ . (Home)