CS21004 - Tutorial 2

January 14th, 2019

Instructions: For the problems with (To submit), please write the answers neatly in loose sheets and submit to the TA before the end of the tutorial.

1. Consider the following two languages over the alphabet $\Sigma = \{a, b\}$

$$L_1 = \{a^n : n \geq 1\}$$
$$L_2 = \{b^n : n \geq 1\}$$

Describe the following languages as per the set notations (e.g., as above) as well as the precise definitions in English (e.g., L_1 can be defined as the set of all strings that have one or more a's but no b's).

- $L_3 = \overline{L_1}$
- $L_4 = (L_1 L_2)^+$ (To submit)

2. Let $\Sigma = \{0, 1\}$. Give DFA's accepting the following strings

(a) The set of all strings containing 1101 as substring

(b) $\{0^n | n \geq 0, n \neq 3\}$

(c) The set of all strings beginning with 101 (To submit)

(d) The set of all strings, which are divisible by 5. (To submit)

(e) $\{01^4 x 1^3 | x \in \{0, 1\}^*\}$ (To submit)

3. For any language L over Σ, the *prefix closure* of L is defined as

$$Pre(L) = \{x \in \Sigma^* \mid \exists y \in \Sigma^* \text{ such that } xy \in L\}$$

Prove that if L is regular then so is $Pre(L)$. (To submit)

4. Prove that $\forall L_1, L_2$, $(L_1 L_2)^R = L_2^R L_1^R$. (Home)

5. Construct a DFA for the set of all strings over the alphabet $\{0, 1\}$ that, when interpreted in reverse as a binary integer are divisible by 5. Example of strings in this language are 0, 10011, 1001100, 101, while strings like 111 are not in the language. Note, $10011 = 25$, when interpreted in reverse as a binary integer. (Home)