DFA and NFA

21 Jan 2019

Instructions : For the problems with (To submit), please write the answers neatly in loose sheets with your name and roll number. Submit to the TA at the end of the class.

1. Construct DFAs for the following languages.
   (a) $L_1 = \{\omega | \omega$ contains an equal number of occurrences of 01 and 10$\}$
   (b) Ternary Strings (base3), (i.e. $\Sigma = \{0, 1, 2\}$) whose integer equivalent is divisible by 7. (To submit)

2. Construct NFAs for the following languages.
   (a) $L_2 = \{\omega | \omega$ is a string in which at least one a_i occurs even number of times (not necessarily consecutively), where $1 \leq i \leq 3$ over $\Sigma = \{a_1, a_2, a_3\}\}$.
   (b) $L_3 = \{\omega | \omega$ contains two 0s separated by a substring whose length is a multiple of 3 $\}$, $\Sigma = \{0, 1\}$. (To submit)

3. Prove the following properties.
   (a) For languages A and B, the shuffle of A and B is the language $L = \{\omega | \omega = a_1 b_1 \cdots a_k b_k\}$, where $a_1 \cancel{\circ} \cdots \cancel{\circ} a_k \in A$ and $b_1, \cdots, b_k \in B$, $\forall a_i, b_i \in \Sigma^*$. Prove that the class of regular languages is closed under Shuffle operation.
   (b) Let B and C be languages over $\Sigma = \{0, 1\}$. We have defined a language $L = B \leftarrow C$ as $L = \{\omega \in B |$ for some $y \in C$, strings ω and y contain equal numbers of 1's. $\}$. Show that the class of regular languages is closed under the $\leftarrow$ operation. (To submit)
   (c) A homomorphism is a mapping h with domain Σ^* for some alphabet Σ which preserves concatenation: $h(v \cdot w) = h(v) \cdot h(w)$. Prove that the class of regular languages is closed under Homomorphism operation. (Home)

4. Consider $\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$. A string $\sigma \in \Sigma^*$ can be interpreted as two binary numbers, for example

$$\sigma = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 101100 \\ 010011 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

where $x, y \in \{0,1\}^*$. Design a DFA which accepts strings in Σ^* such that $2x - y \leq 2$. Note that, for such a DFA transitions will be labeled with elements from $\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$. (Home)