INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date 22.04.10 ~~FN~~ / AN Time: ~~2~~/3 Hrs. Full Marks 75 No. of Students 80
~~Autumn~~ / Spring Semester, 2009-2010 Deptt. CSE Sub No. CS21004
2ND Yr. B. Tech.(Hons.) / ~~B. Arch.~~ / ~~M. Sc.~~ Sub. Name FORMAL LANGUAGE AND AUTOMATA THEORY

Instruction : **Answer Q1 and any Four(4) from the Remaining Questions.**

1. Answer with a short justification whether the following **claims** are **true** or **false**. No credit will be given for writing only *true* or *false*. Assume the alphabet $\Sigma = \{0,1\}$ unless specified otherwise. $< A >$ means an encoding of the object A. CFL (CFG): context-free language (grammar), PDA: push-down automaton, r.e.: recursively enumerable, RE_Σ: the collection of all r.e. languages over Σ.

   (a) **Claim**: A deterministic finite state transition system over Σ with n states (the start state is fixed) can accept 2^n different languages for different choices of the set of final states, and these languages form a Boolean algebra.

   (b) **Claim**: There are only finite number of unambiguous CFGs for the language $L = \{0^n 1^n : n \geq 1\}$.

   (c) Let $\mathcal{C} = \{L \subseteq \Sigma^* : L \text{ is } \textit{co-finite}\}$.
   Claim: Each element of $\mathcal{C}$ is a CFL and intersections of any two of them is also a CFL.

   (d) L is a CFL, $x \in L$, and a *proper prefix* of x is also in L.
   Claim: L cannot be accepted by a *deterministic push-down automaton* (DPDA) in *empty stack*.

   (e) **Claim**: If $L = \{1^p : p \text{ is a prime}\}$, then there is no *context-sensitive language* L' so that LL' is a *regular* language.

   (f) **Claim**: If L is a CFL and $x \in L$ is of length greater than or equal to the *pumping constant*, the number of strings of L is infinite.

   (g) **Claim**: The collection of *decidable* or *recursive* languages over Σ is a Boolean algebra with countably infinite number of elements.

   (h) **Claim**: The length of encoding (using $0, 1$) of a *deterministic Turing machine* over $\{0,1\}$, with the tape alphabet $\{0, 1, \not{b}\}$, and the number of states n, is $O(n)$.

   (i) **Claim**: Every r.e. language over $\{0,1\}$ is not reducible to $L_{HALT} = \{< M, w >: M \text{ is a Turing machine that halts on input } x\}$.

   [9×3]

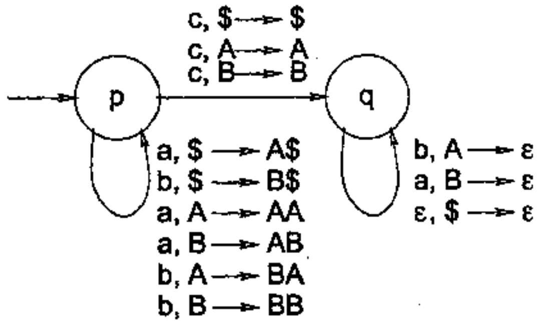
2. (a) Use pumping theorem to prove that $L = \{0^p 1^q : p + q \text{ is } \underline{not} \text{ a perfect square}\}$ is not a regular language.

   (b) Use Myhill-Nerode theorem and other closure properties of regular languages to show that $L = \{0^m 1^n : \text{hcf}(m, n) > 1\}$ is not regular. [$6 + 6$]

3. (a) Design a PDA (state transition diagram) that recognises the language $L = \{x \in \{0,1\}^* : x \neq ww\}$. \$ is the bottom marker of the stack.

   (b) Use pumping theorem to prove that $L = \{x \in \{0,1\}^* : x = ww\}$ is not a context-free language. [$7 + 5$]

4. (a) Consider the *push-down automaton* $P = (\{p, q\}, \{a, b, c\}, \{\$, A, B\}, \delta, p, \$, \phi)$ and formally construct an equivalent *context-free grammar*. The acceptance is by *empty-stack*. Clearly explain the non-terminals and the production rules.



(b) Let L be a prefix closed infinite context-free language. Prove that there is an infinite regular language $L' \subseteq L$. [8 + 4]

5. (a) Prove that $L_d = \{x_i :$ the Turing machine M_i does not accept $x_i\}$ is not Turing recognisable.

(b) Prove that $L_\emptyset = \{< M >: M$ is a Turing machine and $L(M) = \emptyset\}$ *mapping reducible* to $L_= = \{< M_1, M_2 >: M_1$ and M_2 are equivalent Turing machines$\}$ as well $L_{\neq} = \{< M_1, M_2 >: M_1$ and M_2 are not equivalent Turing machines$\}$.
What is your conclusion from this result?

(c) Prove that the universal language $L_u = \{< M, x >:$ the Turing machine M accepts $x\}$ is not *mapping reducible* to $L_\emptyset$. [3 + 6 + 3]

6. Give proper justification for the following statements.

(a) Context-free languages are closed under inverse-homomorphism.

(b) If L_1 and L_2 are recognised by deterministic Turing machines (DTMs) M_1 and M_2, then there is a DTM that recognises $L_1 L_2$.

(c) Any *context-free language* over a one-letter alphabet is a regular language. [3 × 4]