Indian Institute of Technology, Kharagpur
Computer Science and Engineering Department

B.Tech. End-Semester Examination, Autumn 2018-19
Algorithms II (CS31005)

Full marks: 100 **Time: 3 hours**

Answer Questions 1 and 2, and any two from the rest.

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1. Write only the answer/answers (a/b/c/d) in the **1st page** of your answer-script in a tabular form as follows. $(2 \times 10 = 20)$

Question	i	ii	iii	iv	v	vi	vii	viii	ix	x
Answer										

i) For an input of size n, the time complexity of an algorithm is given by $T(1) = 1, T(n) = 3T(n-1)+1$ for $n \geq 2$. Then $T(n)$ solves to
(a) $\Theta(n)$. (b) $\Theta(n^2)$. (c) $\Theta(n^3)$. (d) $\Theta(3^n)$.

ii) Time complexity of Edmonds-Karp algorithm for max-flow computation is
(a) $O(VE)$. (b) $O(V^2E)$. (c) $O(VE^2)$. (d) $O(V^2E^2)$.

iii) Space complexity of doubly connected edge list in Voronoi diagram algorithm with n sites as input, is
(a) $O(n)$. (b) $O(n \log n)$. (c) $O(n^2)$. (d) $O(n^{3/2})$.

iv) $G(V,E)$ is a flow network with s and t as the source and the sink, f is a flow in G, and (S,T) is a cut on V. Then $f(S,T) = |f|$ is true for
(a) exactly one cut (S,T) with $s \in S, t \in T$. (b) possibly for more than one cut (S,T) with $s \in S, t \in T$. (c) for any cut (S,T) with $s \in S, t \in T$. (d) cannot be said.

v) Time complexity to compute all intersection points among n line segments by the plane-sweep algorithm, when each line segment intersects at most k line segments, is
(a) $O(kn^2)$. (b) $O(kn \log n)$. (c) $O((k+n)\log n)$. (d) $O(k \log n)$.

vi) An unfair dice has probability $1/12$ of showing any of its even faces and $1/4$ of showing any of its odd faces. The expected value on throwing this dice is
(a) 3.20. (b) 3.25. (c) 3.50. (d) 3.75.

vii) Time complexity for nondeterministic sorting algorithm is
(a) $O(n \log n)$. (b) $O(n^{4/3})$. (c) $O(n^{3/2})$. (d) none of these.

viii) There are n input boolean variables and g gates in an instance of SAT (circuit satisfiability problem). Its equivalent 3-SAT instance can be constructed in time:
(a) $O(n)$. (b) $O(g)$. (c) $O(n+g)$. (d) $O(ng)$.

ix) Number of clauses in the 3-CNF of $(x_1 \vee \neg x_2) \wedge (\neg x_1 \vee x_2)$ is
(a) 2. (b) 3. (c) 4. (d) none of these.

x) Which of the following is/are not an $\mathcal{NP}$-complete problem?
(a) Max cut. (b) Min cut. (c) Max flow. (d) Minimum set cover.

2. Answer all questions. $(5 \times 10 = 50)$

    i) Write an algorithm to multiply two n-bit numbers, which will run in less than $O(n^2)$ time.

    ii) Consider the distance function $d(p_i, p_j) = \max\{|x_i - x_j|, |y_i - y_j|\}$ between any two points $p_i(x_i, y_i)$ and $p_j(x_j, y_j)$. Using this distance function (not the usual Euclidean distance function), construct the Voronoi diagram of the following sites: $(2,2), (2,6), (6,2), (6,6)$.

    iii) Given a list of n points on the xy-plane and (a, b, c) for a straight line $L : ax + by + c = 0$, we have to decide whether L intersects the convex hull of P. Suggest a deterministic approach and write its best- and worst-case time complexities.

    iv) Derive the 3-CNF of $y \leftrightarrow (x_1 \vee \neg x_2)$.

    v) Give an example to illustrate how an instance of circuit satisfiability problem can be reduced to an equivalent instance of formula satisfiability problem.

    vi) Derive the maximum number of edge-contracts that can be done in any undirected graph such that the probability of survival of a specific min-cut is at least $\frac{1}{2}$.

    vii) Given a rectangle R containing n points (no two points have same x- or y-coordinate), suggest an expected-linear-time Las Vegas algorithm that can partition R into four rectangles such that each rectangle contains at least $\lfloor n/4 \rfloor$ points.

    viii) Prove or disprove: "Constant-factor approximation algorithm cannot be designed for TSP."

    ix) Give an example to show that a greedy strategy for minimum vertex cover does not yield a constant approximation ratio.

    x) Suggest a derandomization technique for computation of max cut in an undirected graph.

3. A paint manufacturing company makes a survey of n different colors with m dealers. Each dealer puts YES or NO against each color. The company now attempts to find the smallest set of colors such that it contains at least three colors liked by each dealer. Can it be done in a time polynomial in m and n? If so, suggest an algorithm and explain its time complexity. Otherwise, explain why not. (15 marks)

4. An auction house has $3n$ items and $4n$ bidders. Ever bidder bids against 3 items and every item is bidden by 4 bidders. Once all biddings are over, the auction house wants to sell all its items to the bidders, whatsoever be the profit margin. Can it be done? Provide arguments.
   Also suggest an algorithm to determine which item has to be sold to whom. Explain its time complexity. $(5 + 5 + 5 = 15$ marks$)$

5. There are n identical houses in a land. The land can be assumed as a planar surface and each house can be assumed as a circle of radius ρ. The positions of the houses are known by (x, y) coordinates of their centers. A circular boundary wall of minimum diameter has to be constructed to accommodate all the houses inside the boundary wall. Suggest a randomized algorithm to compute the diameter of the wall and derive its expected time complexity. $(10 + 5$ marks$)$

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