Module 09: CS31003: Compilers: Fundamentals of Data Flow Analysis

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October 15, 2019

Data-flow analysis

- These are techniques that derive information about the flow of data along program execution paths
- An *execution path* (or *path*) from point p_1 to point p_n is a sequence of points $p_1, p_2, ..., p_n$ such that for each $i = 1, 2, ..., n-1$, either
  1. p_i is the point immediately preceding a statement and p_{i+1} is the point immediately following that same statement, or
  2. p_i is the end of some block and p_{i+1} is the beginning of a successor block
- In general, there is an infinite number of paths through a program and there is no bound on the length of a path
- Program analyses summarize all possible program states that can occur at a point in the program with a finite set of facts
- No analysis is necessarily a perfect representation of the state

Uses of Data-flow Analysis

- Program debugging
  - Which are the definitions (of variables) that *may* reach a program point? These are the *reaching definitions*
  - Can a variable may potentially be used without being initialized?
- Program optimization
  - Constant folding
  - Copy propagation
  - Common sub-expression elimination etc.

Data-Flow Analysis Schema

- A *data-flow value* for a program point represents an abstraction of the set of all possible program states that can be observed for that point
- The set of all possible data-flow values is the *domain* for the application under consideration
  - Example: for the *reaching definitions* problem, the domain of data-flow values is the set of all subsets of definitions in the program
  - A particular data-flow value is a set of definitions
- $IN[s]$ and $OUT[s]$: data-flow values *before* and *after* each statement s
- The *data-flow problem* is to find a solution to a set of constraints on $IN[s]$ and $OUT[s]$, for all statements s

Data-Flow Analysis Schema (2)

- Two kinds of constraints
  - Those based on the semantics of statements (*transfer functions*)
  - Those based on flow of control
- A DFA schema consists of
  - A control-flow graph
  - A direction of data-flow (forward or backward)
  - A set of data-flow values
  - A confluence operator (usually set union or intersection)
  - Transfer functions for each block
- We always compute *safe* estimates of data-flow values
- A decision or estimate is *safe* or *conservative*, if it never leads to a change in what the program computes (after the change)
- These safe values may be either subsets or supersets of actual values, based on the application

Reaching Definitions

Reaching Definitions

- We *kill* a definition of a variable *a*, if between two points along the path, there is an assignment to *a*
- A definition *d* reaches a point *p*, if there is a path from the point immediately following *d* to *p*, such that *d* is not *killed* along that path
- Unambiguous and ambiguous definitions of a variable

  a := b+c

  (unambiguous definition of 'a')

  ...

  *p := d

  (ambiguous definition of 'a', if 'p' may point to variables other than 'a' as well; hence does not kill the above definition of 'a')

  ...

  a := k-m

  (unambiguous definition of 'a'; kills the above definition of 'a')

Reaching Definitions

- We compute super-sets of definitions as *safe* values
- It is safe to assume that a definition reaches a point, even if it does not.
- In the following example, we assume that both a=2 and a=4 reach the point after the complete if-then-else statement, even though the statement a=4 is not reached by control flow

```
if (a==b) a=2; else if (a==b) a=4;
```

Reaching Definitions: How to use them?

- Build use / def Chains
- Constant Propagation: For a use like

  ```
  n: x = ... v ...
  ```

  if all definitions that reach `n` are of the form

  ```
  d: v = c // c is a constant
  ```

  we can replace `v` in `n` by `c`.
- Un-initialized Variables: How to detect?
- Loop-invariant Code Motion: For

  ```
  d1: a = . . .;
  d2: b = . . .;
  for (. . .) {
      . . .
      n: x = a + b;
      . . .
  }
  ```

  if all definitions of variables on RHS of `n` and that reach `n` are outside the loop like `d1` and `d2`, `n` can also be moved outside the loop.

Reaching Definitions Problem: DFA Formulation

- The data-flow equations (constraints)

$$\begin{aligned}
IN[B] &= \bigcup_{P \ is \ a \ predecessor \ of \ B} OUT[P] \\
OUT[B] &= GEN[B] \bigcup (IN[B] - KILL[B]) \\
IN[B] &= \phi, for\ all\ B\ (initialization\ only)
\end{aligned}$$

- If some definitions reach B_1 (entry), then $IN[B_1]$ is initialized to that set
- Forward flow DFA problem (since $OUT[B]$ is expressed in terms of $IN[B]$), confluence operator is $\cup$
    - Direction of flow does not imply traversing the basic blocks in a particular order
    - The final result does not depend on the order of traversal of the basic blocks

Reaching Definitions Problem: DFA Formulation

- $GEN[B]$ = set of all definitions inside B that are "visible" immediately after the block - *downwards exposed* definitions
  - If a variable x has two or more defintions in a basic block, then only the last definition of x is downwards exposed; all others are not visible outside the block
- $KILL[B]$ = union of the definitions in all the basic blocks of the flow graph, that are killed by individual statements in B
  - If a variable x has a definition d_i in a basic block, then d_i kills all the definitions of the variable x in the program, except d_i

Reaching Definitions Analysis: GEN and KILL

In other blocks:

d5: b = a+4
d6: f = e+c
d7: e = b+d
d8: d = a+b
d9: a = c+f
d10: c = e+a

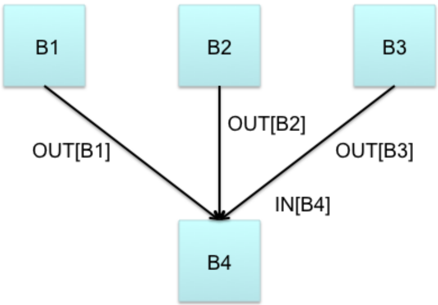
B:

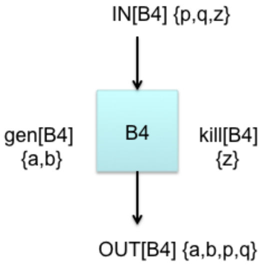
d1: a = f + 1
d2: b = a + 7
d3: c = b + d
d4: a = d + c

Set of all definitions = {d1,d2,d3,d4,d5,d6,d7,d8,d9,10}

GEN[B] = {d2,d3,d4}
KILL[B] = {d4,d9,d5,d10,d1}

Reaching Definitions Analysis: DF Equations





IN[B4] = OUT[B1] U OUT[B2] U OUT[B3]

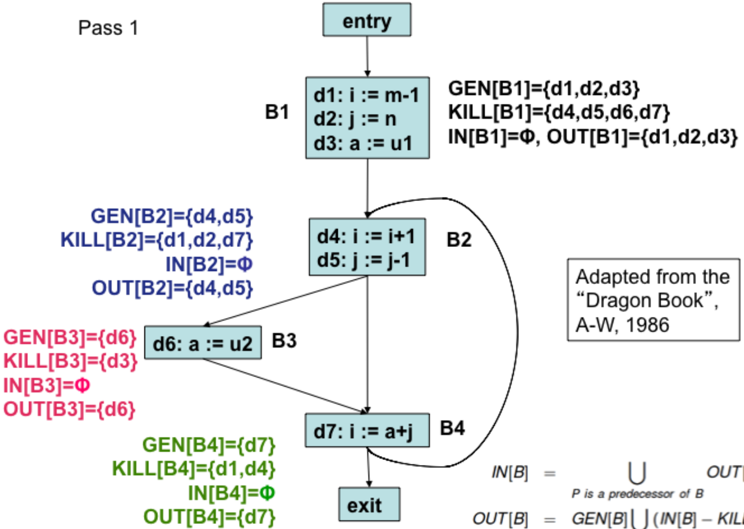
$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

OUT[B4] = gen[B4] U (IN[B4] – kill[B4])

Reaching Definitions Analysis: An Example - Pass 1

Pass 1



Adapted from the "Dragon Book", A-W, 1986

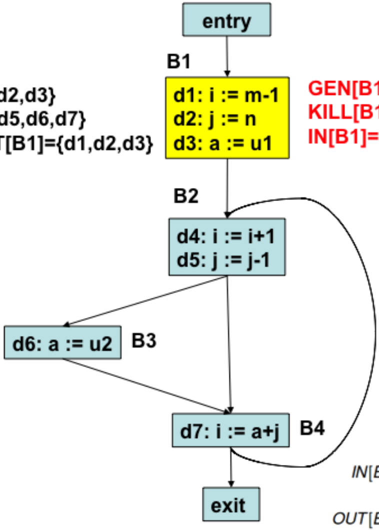
$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

Reaching Definitions Analysis: An Example - Pass 2.1

Pass 2

GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

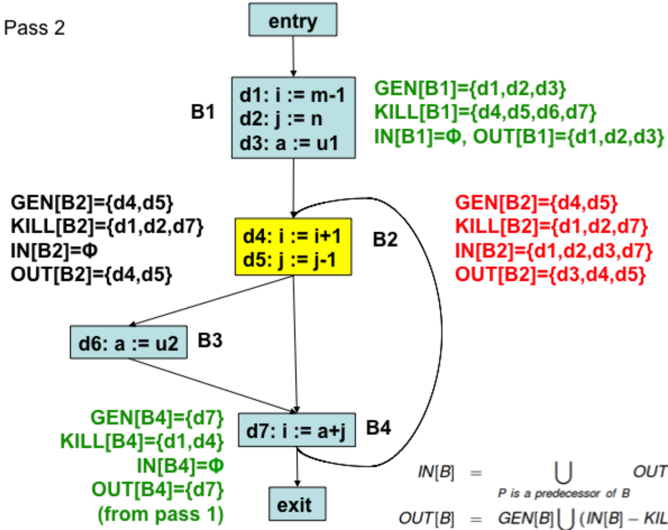


GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

Reaching Definitions Analysis: An Example - Pass 2.2

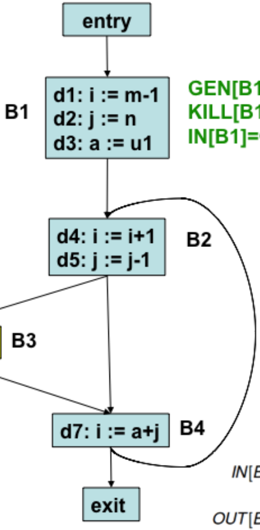


$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

Reaching Definitions Analysis: An Example - Pass 2.3

Pass 2



GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

GEN[B3]={d6}
KILL[B3]={d3}
IN[B3]=Φ
OUT[B3]={d6}

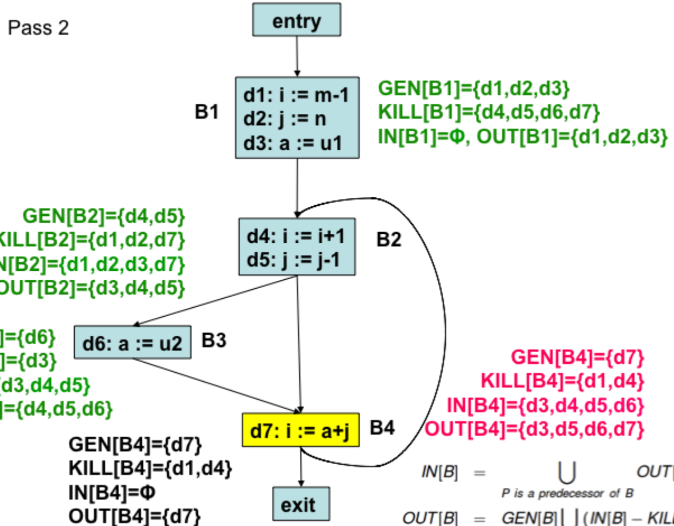
GEN[B2]={d4,d5}
KILL[B2]={d1,d2,d7}
IN[B2]={d1,d2,d3,d7}
OUT[B2]={d3,d4,d5}

GEN[B3]={d6}
KILL[B3]={d3}
IN[B3]={d3,d4,d5}
OUT[B3]={d4,d5,d6}

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

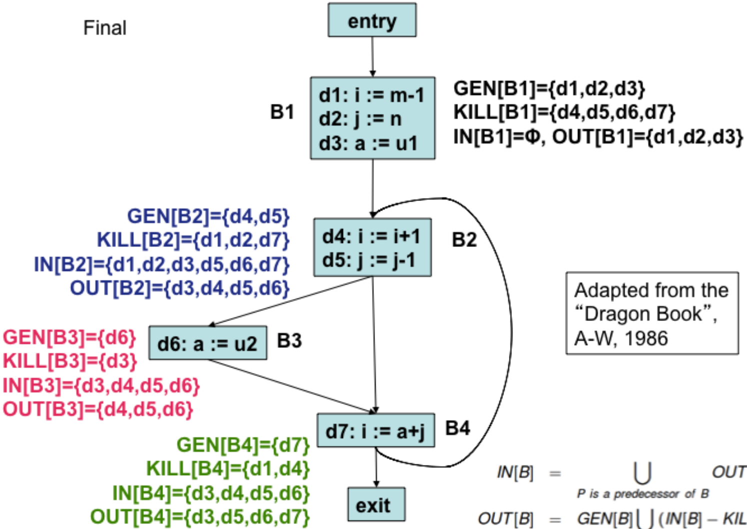
Reaching Definitions Analysis: An Example - Pass 2.4



$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

Reaching Definitions Analysis: An Example - Final



Adapted from the "Dragon Book", A-W, 1986

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

An Iterative Algorithm for Computing Reaching Def.

for each block B do { $IN[B] = \phi$; $OUT[B] = GEN[B]$; }
$change = true$;
while $change$ do { $change = false$;
  for each block B do {

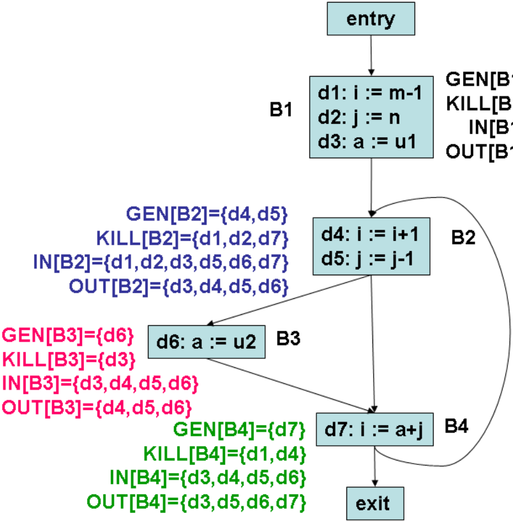
$$
\begin{aligned}
IN[B] &= \bigcup_{P \ a \ predecessor \ of \ B} OUT[P]; \\
oldout &= OUT[B]; \\
OUT[B] &= GEN[B] \bigcup (IN[B] - KILL[B]);
\end{aligned}
$$

    if ($OUT[B] \neq oldout$) $change = true$;
  }
}

- GEN, $KILL$, IN, and OUT are all represented as bit vectors with one bit for each definition in the flow graph

Reaching Definitions: Bit Vector Representation

Final dataflow value sets shown in bit vector format



	d1	d2	d3	d4	d5	d6	d7
GEN[B1]=	1	1	1	0	0	0	0
KILL[B1]=	0	0	0	1	1	1	1
IN[B1]=	0	0	0	0	0	0	0
OUT[B1]=	1	1	1	0	0	0	0

Adapted from the "Dragon Book", A-W, 1986

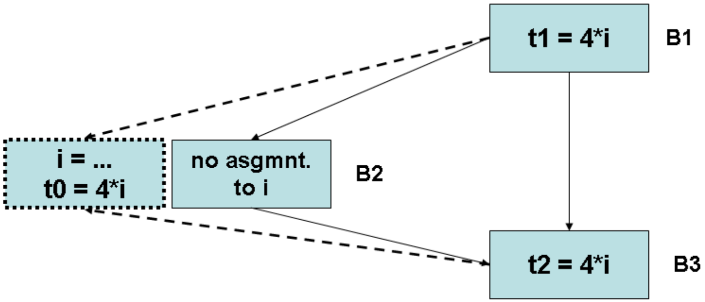
Available Expressions

Available Expression Computation

- Sets of expressions constitute the domain of data-flow values
- Forward flow problem
- Confluence operator is $\cap$
- An expression $x + y$ is *available* at a point p, if every path (not necessarily cycle-free) from the initial node to p evaluates $x + y$, and after the last such evaluation, prior to reaching p, there are no subsequent assignments to x or y
- A block *kills* $x + y$, if it assigns (or may assign) to x or y and does not subsequently recompute $x + y$.
- A block *generates* $x + y$, if it definitely evaluates $x + y$, and does not subsequently redefine x or y

Available Expression Computation(2)

- Useful for global common sub-expression elimination
- $4 * i$ is a CSE in $B3$, if it is available at the entry point of $B3$ *i.e.*, if i is not assigned a new value in $B2$ or $4 * i$ is



Computing e_gen and e_kill

- For statements of the form $x = a$, step 1 below does not apply
- The set of all expressions appearing as the RHS of assignments in the flow graph is assumed to be available and is represented using a hash table and a bit vector

e_gen[q] = A   **q •**
x = y + z
p •

Computing e_gen[p]

1. A = A **U** {y+z}
2. A = A – {all expressions involving x}
3. e_gen[p] = A

e_kill[q] = A   **q •**
x = y + z
p •

Computing e_kill[p]

1. A = A – {y+z}
2. A = A **U** {all expressions involving x}
3. e_kill[p] = A

Available Expression Computation - EGEN and EKILL

In other blocks:

d5: b = a+4
d6: f = e+c
d7: e = b+d
d8: d = a+b
d9: a = c+f
d10: c = e+a

B:

d1: a = f + 1
d2: b = a + 7
d3: c = b + d
d4: a = d + c

Set of all expressions = {f+1,a+7,b+d,d+c,a+4,e+c,a+b,c+f,e+a}

EGEN[B] = {f+1,b+d,d+c}
EKILL[B] = {a+4,a+b,e+a,e+c,c+f,a+7}

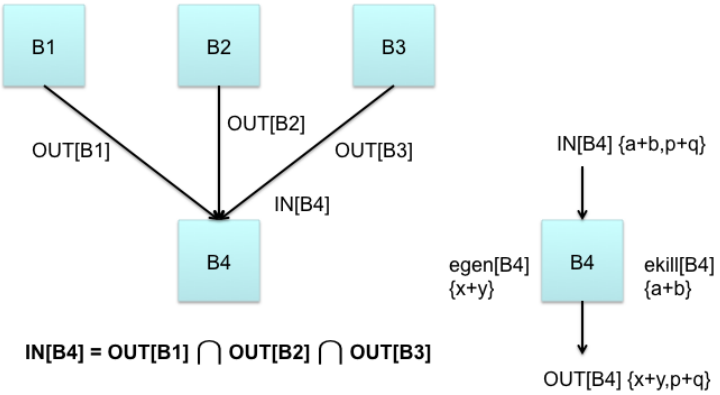
Available Expression Computation - DF Equations (1)

- The data-flow equations

$$
\begin{aligned}
IN[B] &= \bigcap_{P \text{ is a predecessor of } B} OUT[P], \; B \text{ not initial} \\
OUT[B] &= e\_gen[B] \bigcup (IN[B] - e\_kill[B]) \\
IN[B1] &= \phi \\
IN[B] &= U, \; \text{for all } B \neq B1 \; (\text{initialization only})
\end{aligned}
$$

- $B1$ is the intial or entry block and is special because nothing is available when the program begins execution
- $IN[B1]$ is always ϕ
- U is the universal set of all expressions
- Initializing $IN[B]$ to ϕ for all $B \neq B1$, is restrictive

Available Expression Computation - DF Equations (2)



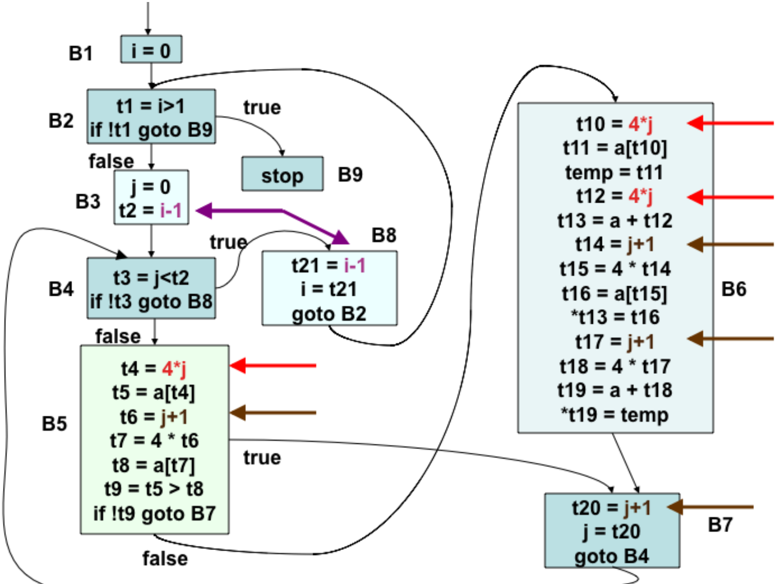
IN[B4] = OUT[B1] $\bigcap$ OUT[B2] $\bigcap$ OUT[B3]

OUT[B4] = egen[B4] $\bigcup$ (IN[B4] – ekill[B4])

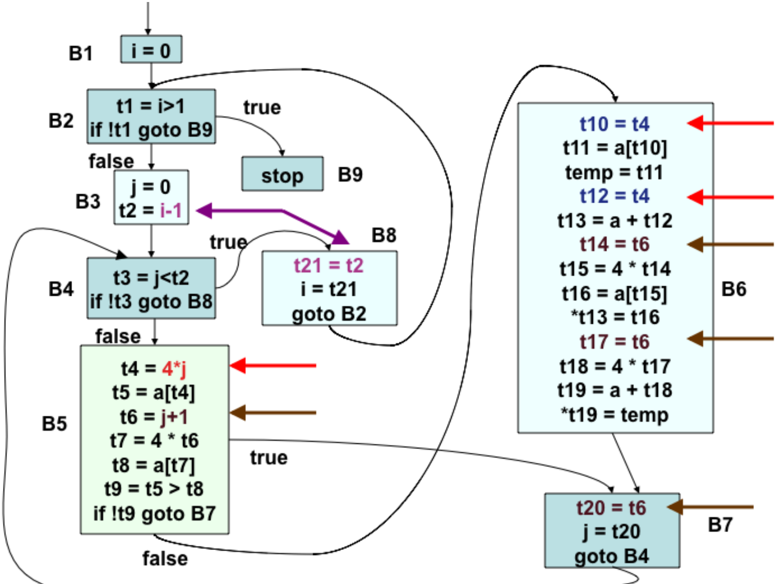
$$IN[B] = \bigcap_{P \text{ is a predecessor of } B} OUT[P], \; B \text{ not initial}$$

$$OUT[B] = e\_gen[B] \bigcup (IN[B] - e\_kill[B])$$

Available Expression Computation - An Example



Available Expression Computation - An Example (2)



An Iterative Algorithm for Computing Available Expressions

for each block $B \neq B1$ do $\{OUT[B] = U - e\_kill[B];\ \}$
/* You could also do $IN[B] = U;$*/
/* In such a case, you must also interchange the order of */
/* $IN[B]$ and $OUT[B]$ equations below */
$change = true;$
while $change$ do $\{\ change = false;$
for each block $B \neq B1$ do $\{$

$$\begin{aligned} IN[B] &= \bigcap_{P\ a\ predecessor\ of\ B} OUT[P]; \\ oldout &= OUT[B]; \\ OUT[B] &= e\_gen[B] \bigcup (IN[B] - e\_kill[B]); \end{aligned}$$

if $(OUT[B] \neq oldout)$ $change = true;$
$\}$
$\}$

Live Variables

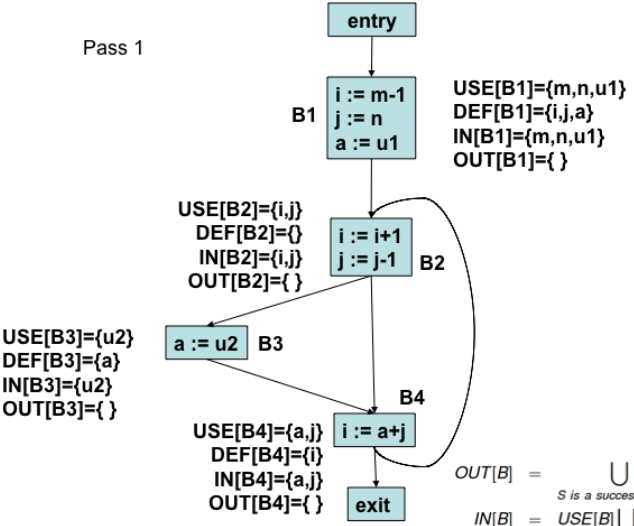
Live Variable Analysis

- The variable x is *live* at the point p, if the value of x at p could be used along some path in the flow graph, starting at p; otherwise, x is *dead* at p
- Sets of variables constitute the domain of data-flow values
- Backward flow problem, with confluence operator $\bigcup$
- $IN[B]$ is the set of variables live at the beginning of B
- $OUT[B]$ is the set of variables live just after B
- $DEF[B]$ is the set of variables definitely assigned values in B, prior to any use of that variable in B
- $USE[B]$ is the set of variables whose values may be used in B prior to any definition of the variable

$$
\begin{aligned}
OUT[B] &= \bigcup_{S \text{ is a successor of } B} IN[S] \\
IN[B] &= USE[B] \bigcup (OUT[B] - DEF[B]) \\
OUT[B] &= \phi, \textit{for all } B \textit{ (initialization only)}
\end{aligned}
$$

Live Variable Analysis: An Example - Pass 1

Pass 1

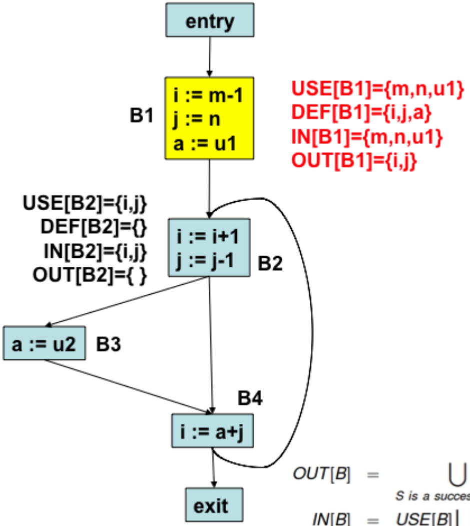


$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

Live Variable Analysis: An Example - Pass 2.1

Pass 2

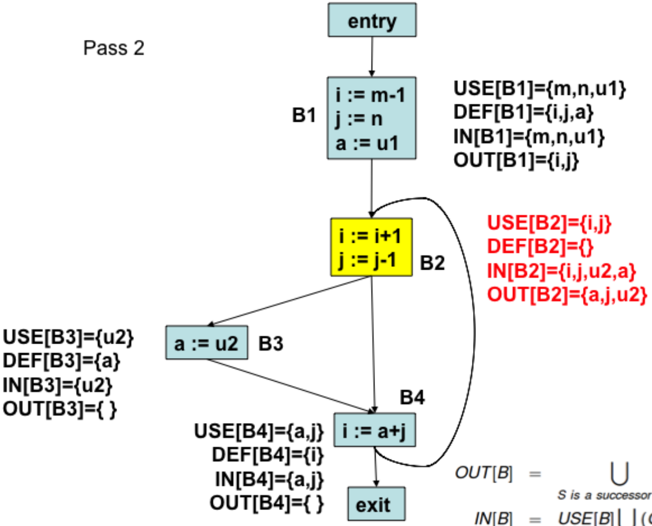


$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

Live Variable Analysis: An Example - Pass 2.2

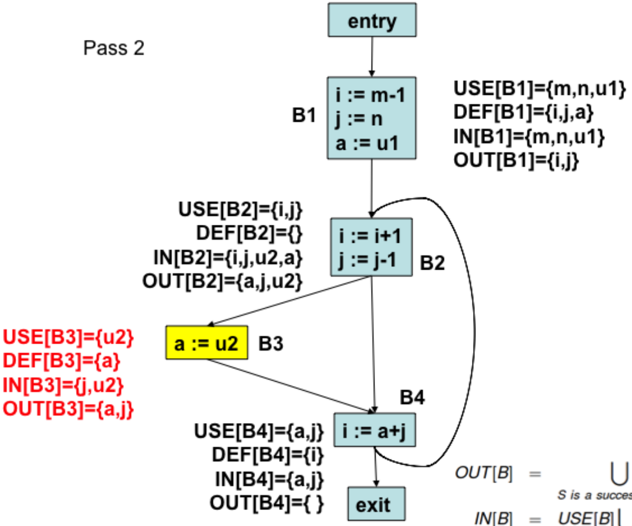
Pass 2



$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

Live Variable Analysis: An Example - Pass 2.3

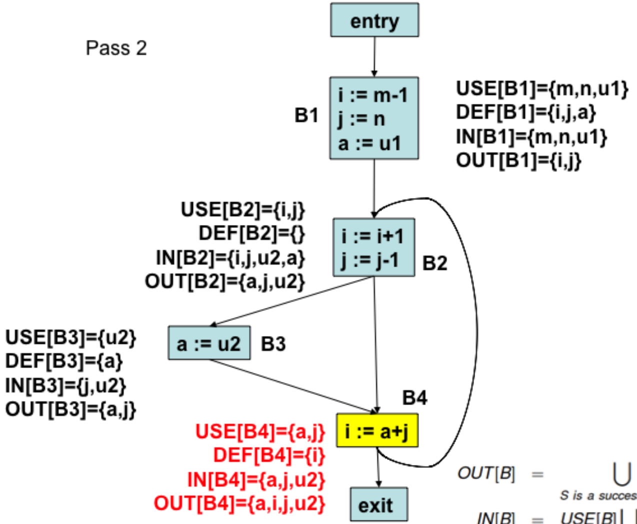


$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

Live Variable Analysis: An Example - Pass 2.4

Pass 2

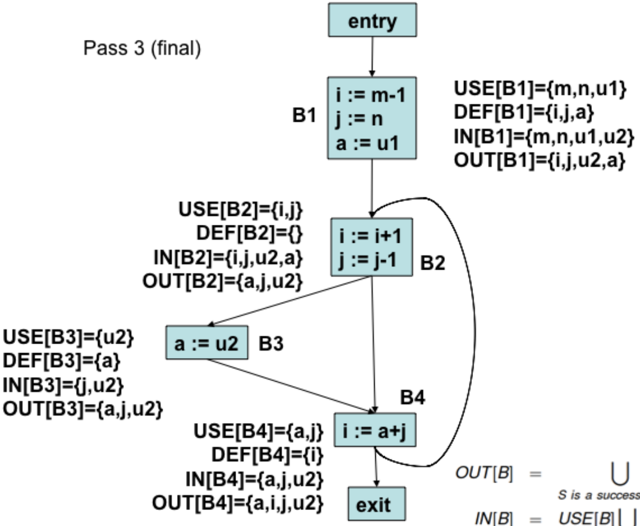


$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

Live Variable Analysis: An Example - Final pass

Pass 3 (final)



$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

Definition-Use Chains

DFA: Definition-Use Chains

- For each definition, we wish to attach the statement numbers of the uses of that definition
- Such information is very useful in implementing register allocation, loop invariant code motion, etc.
- This problem can be transformed to the data-flow analysis problem of computing for a point p, the set of uses of a variable (say x), such that there is a path from p to the use of x, that does not redefine x.
- This information is represented as sets of (x; s) pairs, where x is the variable used in statement s
- In live variable analysis, we need information on whether a variable is used later, but in (x; s) computation, we also need the statement numbers of the uses
- The data-flow equations are similar to that of LV analysis
- Once IN[B] and OUT[B] are computed, d-u chains can be computed using a method similar to that of u-d chains

Data Flow Analysis for (x, s) Pairs

- Sets of pairs (x, s) constitute the domain of data-flow values
- Backward flow problem, with confluence operator $\bigcup$
- $IN[B]$ is the set of pairs (x, s), such that statement s uses variable x and the value of x at $IN[B]$ has not been modified along the path from $IN[B]$ to s
- $OUT[B]$ is the set of pairs (x, s), such that statement s uses variable x and the value of x at $OUT[B]$ has not been modified along the path from $OUT[B]$ to s
- $DEF[B]$ is the set of pairs (x, s), such that s is a statement which uses x, s is not in B, and B contains a definition of x
- $USE[B]$ is the set of pairs (x, s), such that s is a statement in B which uses variable x and such that no prior definition of x occurs in B

Data Flow Analysis for (x, s) Pairs

$$
\begin{aligned}
OUT[B] &= \bigcup_{S \ is \ a \ successor \ of \ B} IN[S] \\
IN[B] &= USE[B] \bigcup (OUT[B] - DEF[B]) \\
OUT[B] &= \phi, for \ all \ B \ (initialization \ only)
\end{aligned}
$$

Copy Propagation

Copy Propagation

- Eliminate copy statements of the form $s: \ x := y$, by substituting y for x in all uses of x reached by this copy
- Conditions to be checked
  1. u-d chain of use u of x must consist of s only. Then, s is the only definition of x reaching u
  2. On every path from s to u, including paths that go through u several times (but do not go through s a second time), there are no assignments to y. This ensures that the copy is valid
- The second condition above is checked by using information obtained by a new data-flow analysis problem
  - $c\_gen[B]$ is the set of all copy statements, $s : x := y$ in B, such that there are no subsequent assignments to either x or y within B, after s
  - $c\_kill[B]$ is the set of all copy statements, $s : x := y$, s not in B, such that either x or y is assigned a value in B
  - Let U be the universal set of all copy statements in the program

Copy Propagation - The Data-flow Equations

- $c\_in[B]$ is the set of all copy statements, $x := y$ reaching the beginning of B along every path such that there are no assignments to either x or y following the last occurrence of $x := y$ on the path
- $c\_out[B]$ is the set of all copy statements, $x := y$ reaching the end of B along every path such that there are no assignments to either x or y following the last occurrence of $x := y$ on the path

$$
\begin{aligned}
c\_in[B] &= \bigcap_{P \text{ is a predecessor of } B} c\_out[P], \; B \textit{ not initial} \\
c\_out[B] &= c\_gen[B] \bigcup (c\_in[B] - c\_kill[B]) \\
c\_in[B1] &= \phi, \; \textit{where } B1 \textit{ is the initial block} \\
c\_out[B] &= U - c\_kill[B], \; \textit{for all } B \neq B1 \; (\textit{initialization only})
\end{aligned}
$$

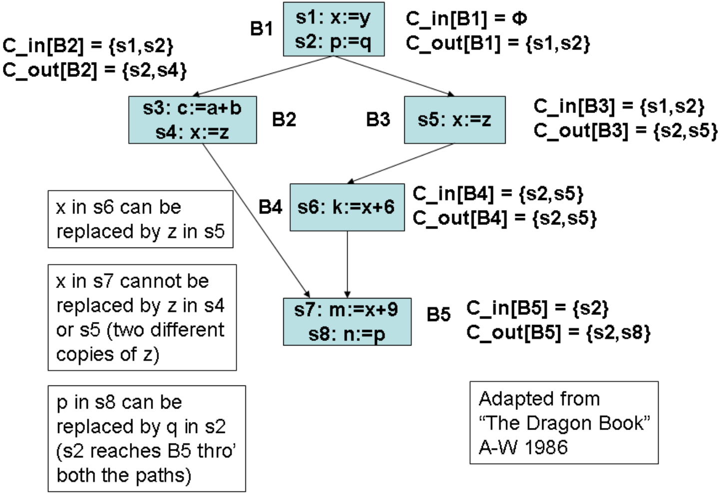
Algorithm for Copy Propagation

For each copy, $s : x := y$, do the following

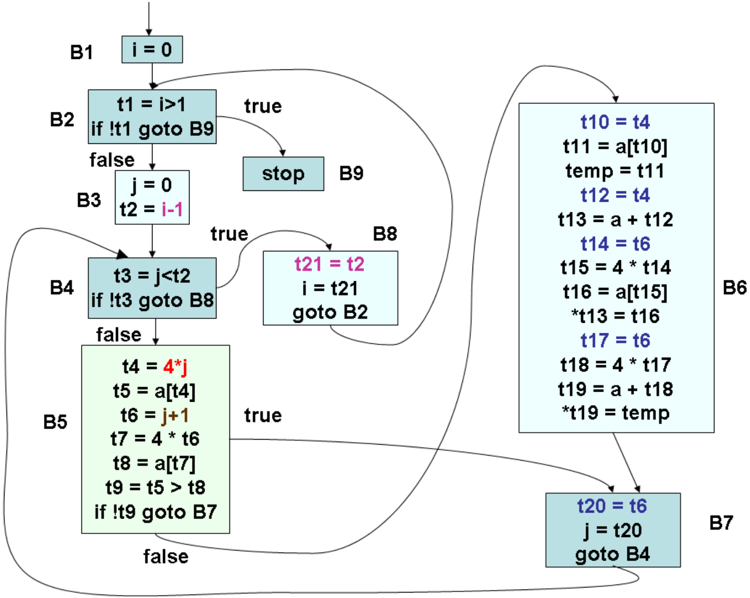
1. Using the $du - chain$, determine those uses of x that are reached by s
2. For each use u of x found in (1) above, check that
   (i) u-d chain of u consists of s only
   (ii) s is in $c\_in[B]$, where B is the block to which u belongs. This ensures that
      - s is the only definition of x that reaches this block
      - No definitions of x or y appear on this path from s to B

   (iii) no definitions x or y occur within B prior to u found in (1) above
3. If s meets the conditions above, then remove s and replace all uses of x found in (1) above by y

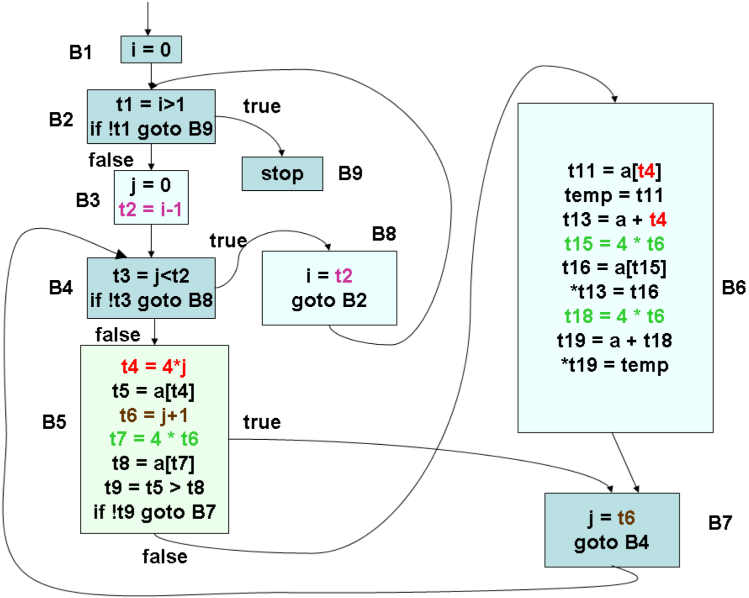
Copy Propagation Example 1



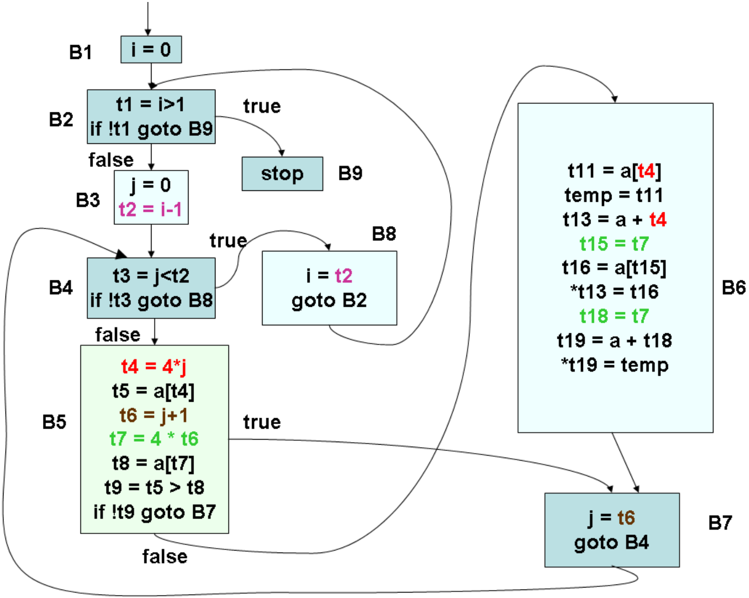
Copy Propagation on Running Example 1.1



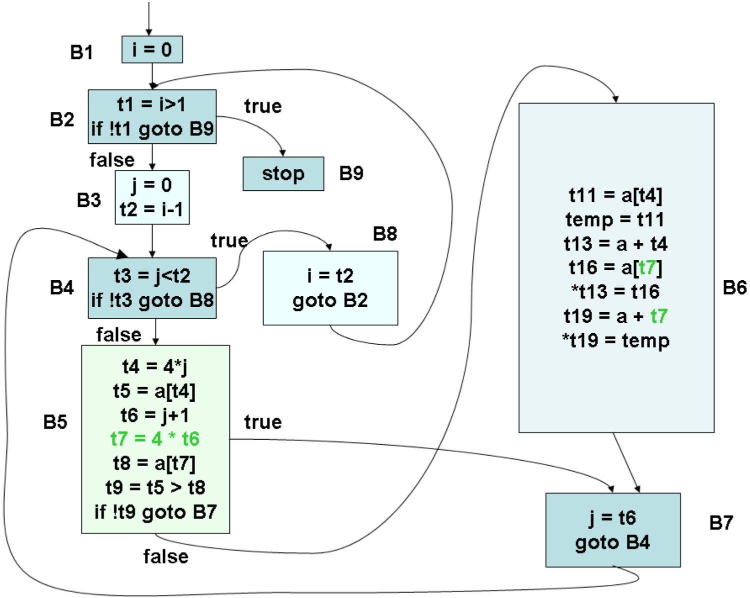
Copy Propagation on Running Example 1.2



GCSE and Copy Propagation on Running Example 1.1



GCSE and Copy Propagation on Running Example 1.2



Optimization by Repeated DFA

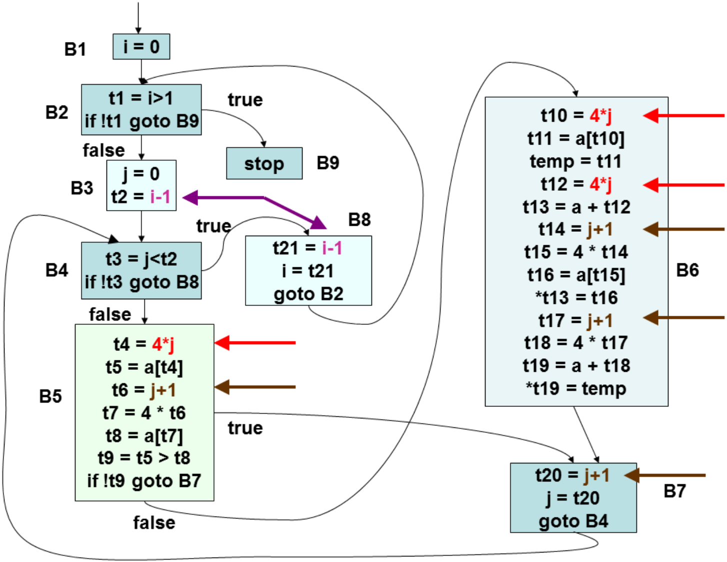
Bubble Sort Source

Bubble Sort

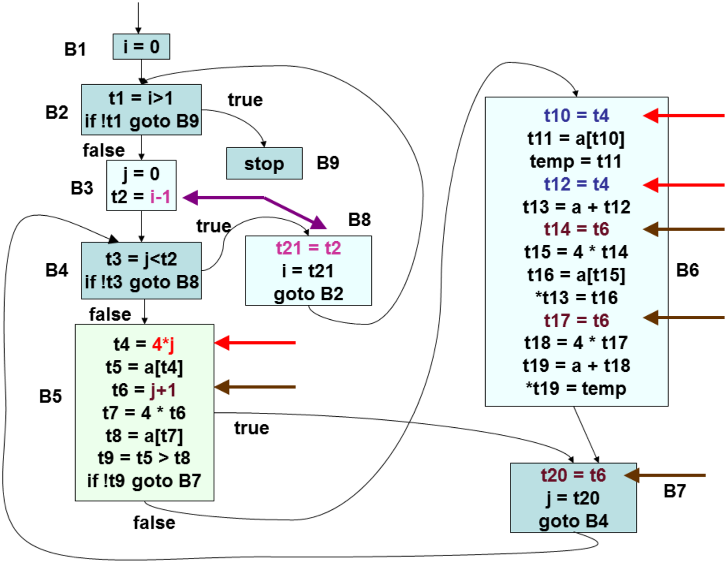
```
for (i=100; i>1; i--) {
    for (j=0; j<i-1; j++) {
        if (a[j] > a[j+1]) {
            temp = a[j];
            a[j] = a[j+1];
            a[j+1] = temp;
        }
    }
}
```

- int a[100]
- array a runs from 0 to 99
- No special jump out if array is already sorted

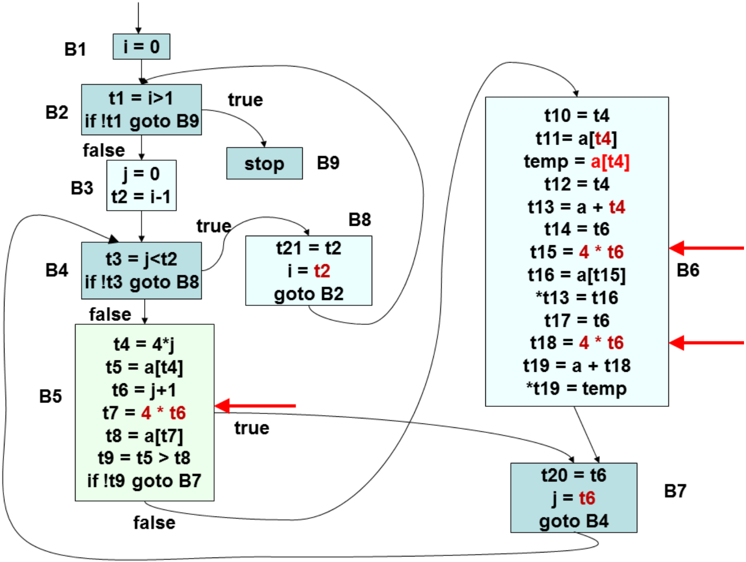
DFA: Common Sub-Expressions



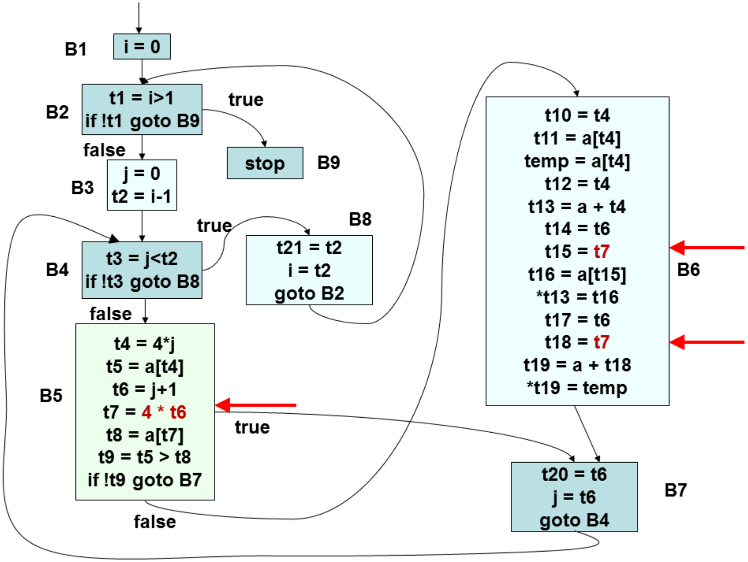
Common Sub-Expressions Eliminated



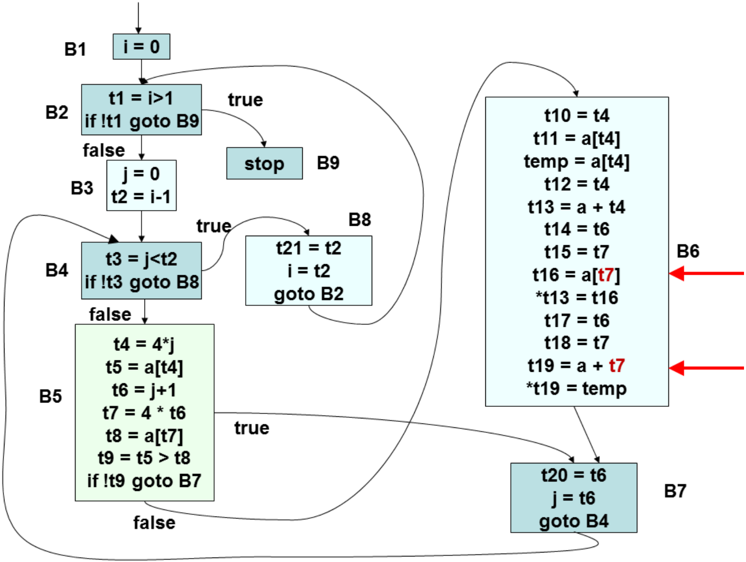
DFA: Reaching Definitions & Copy Propagated



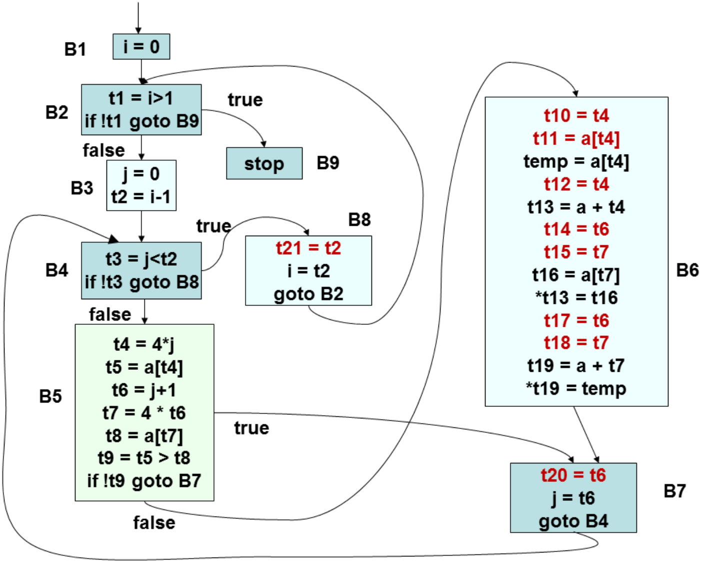
DFA: Common Sub-Expressions & Elimination



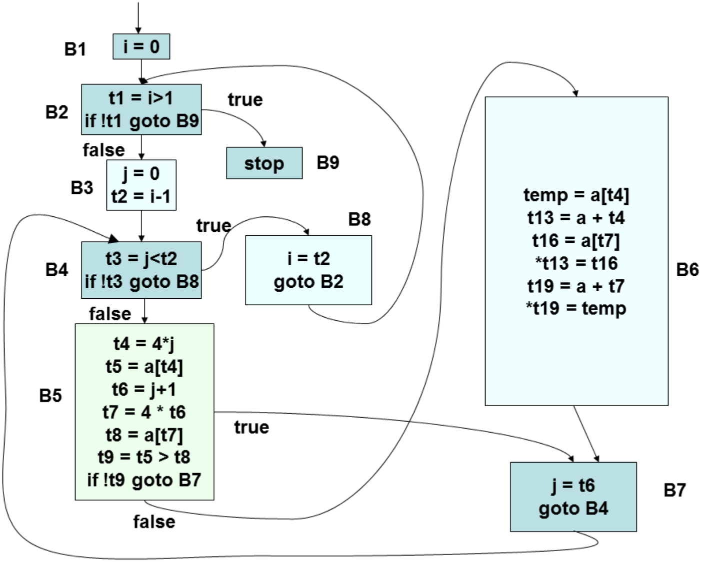
DFA: Reaching Definitions & Copy Propagated



DFA: Live Variable Analysis



DFA: Deadcode Elimination



Optimization Example

Optimizations like Strength Reduction, Induction Variables Analysis and Control Flow Optimization are yet to be done on this code.