Probability and Statistics
Assignment No. 6

1. Let (X,Y) have the joint pmf

Y\ X	-1	0	1
-2	1/6	1/12	1/6
1	1/6	1/12	1/6
2	1/12	0	1/12

Find the joint pmf of (U,V) where U = $|X|$ and $V = Y^2$.

2. Projectiles are fired at the origin of an XY – coordinate system. Assume that the point which is hit, say (X,Y), consists of a pair of independent standard normal r.v.'s. For two projectiles fired independently of one another, let (X_1, Y_1) and (X_2, Y_2) represent the points which are hit and Z be the distance between them. What is the distribution of Z^2 ?

3. Let X_1 and X_2 be independent r.v.'s each with negative exponential distribution with pdf $\lambda\exp\{-\lambda x\}$, $x > 0$. Find the joint and marginal distributions of $Y_1 = X_1/X_2$ and $Y_2 = X_1+X_2$.

4. Let X_1, X_2 be i.i.d. N(0,1) and $Y_1 = X_1^2 + X_2^2$, $Y_2 = X_1/X_2$. Are Y_1, Y_2 independent?

5. Let X_1 and X_2 have independent gamma distributions with parameters (n_1,λ) and (n_2,λ). Find the distributions of $Y = X_1/(X_1+X_2)$. Is Y independent of $Z = X_1+X_2$? Is Z independent of $U = X_1/X_2$?

6. Let X_1, X_2, X_3 be independent exponential random variables with the probability density $f(x) = e^{-x}$, $x > 0$. Define random variables Y_1, Y_2 and Y_3 as $Y_1 = X_1 + X_2 + X_3$, $Y_2 = \dfrac{X_1 + X_2}{X_1 + X_2 + X_3}$, $Y_3 = \dfrac{X_1}{X_1 + X_2}$.

Find the joint and marginal densities of Y_1, Y_2 and Y_3. Are they independent?

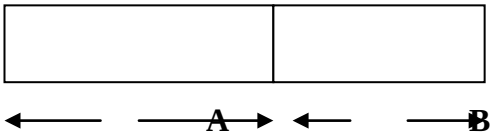
7. Suppose independent random variables Y_1, Y_2, Y_3 are such that $Y_1 = \ln X_1 \sim N(4, 1)$; $Y_2 = \ln X_2 \sim N(3, 1)$; $Y_3 = \ln X_3 \sim N(2, 0.5)$. Find the distribution and the median of $W = e^2 X_1^2 X_2^{1.5} X_3^{1.28}$. Determine L and R such that $P(L \le W \le R) = 0.90$.

8. Let (X, Y) have bivariate normal distribution with density function

$$f(x,y) = \frac{1}{\pi\sqrt{3}} e^{-\frac{2}{3}(x^2 - xy + y^2)}, \quad -\infty < x, y < \infty.$$

Find the correlation coefficient between X and Y, $P(-1 < X < 1|Y=1)$, $V(2X + 3Y)$ and $P(-5 < 2X + 3Y < 8)$.

9. A straight rod consists of two sections **A** and **B**, each of which is manufactured independently on a different machine. The length (in inches) of section **A** is normally distributed with mean **20** and variance **0.03** and the length of section **B** is normally distributed with mean **14** and variance **0.01**. The rod is formed by joining the two sections together as shown below:



Suppose that the rod can be used in the construction of an airplane wing if its total length is between **33.6** to **34.4** inches. What is the probability that the rod can be used in the construction?