Let X_1, X_2 be independent RVs each having geometric distribution $q^k p$, $k = 0, 1, 2 \dots$. Show that the conditional distribution of X_1 given $X_1 + X_2$ is uniform.

$$P(X_1 = r / X_1 + X_2 = n) = \frac{P\{(X_1 = r) \cap (X_1 + X_2 = n)\}}{P(X_1 + X_2 = n)}$$

$$= \frac{P\{(X_1 = r) \cap (X_2 = n - r)\}}{\sum P\{(X_1 = s) \cap (X_2 = n - s)\}} = \frac{P(X_1 = r)\, P(X_2 = n - r)}{\sum P(X_1 = s)\, P(X_2 = n - s)}$$

$$= \frac{q^r p \cdot q^{n-r} p}{\sum_{s=0}^{n} q^s p \cdot q^{n-s} p} = \frac{p^2 q^n}{\sum_{s=0}^{n} q^n p^2} = \frac{p^2 q^n}{(n+1) p^2 q^n} = \frac{1}{n+1}$$

If X_1 and X_2 are independent Poisson RVs with respective means λ_1 and λ_2, find the conditional pmf of X_1 given $X_1 + X_2 = n$ and conditional expected value of X_1 given $X_1 + X_2 = n$.

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$$P(X_1 = r / X_1 + X_2 = n) = \frac{P(X_1 = r)\, P(X_2 = n-r)}{P(X_1 + X_2 = n)}$$

$$= \frac{e^{-\lambda_1} \cdot \frac{\lambda_1^{r}}{r!} \cdot e^{-\lambda_2} \cdot \frac{\lambda_2^{n-r}}{(n-r)!}}{e^{-(\lambda_1+\lambda_2)} \cdot \frac{(\lambda_1+\lambda_2)^n}{n!}}$$

$$= {}^nC_r \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^r \left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-r}$$

$$\sim \text{Bin}\left(n, \frac{\lambda_1}{\lambda_1+\lambda_2}\right)$$

$\Rightarrow$ Expected value is $= \dfrac{n\lambda_1}{\lambda_1+\lambda_2}$.

If $X_1, X_2 \cdots X_K$ are k independent Poisson variates with parameters $\lambda_1, \lambda_2 \cdots \lambda_K$ respectively, Prove that conditional distribution $P(X_1 \cap X_2 \cdots \cap X_K / X)$ where $X = X_1 + X_2 + \cdots + X_K$ is multinomial.

In a game of billiards, a player continues to play until she misses a shot. If a particular player misses any of her shots with probability $p = \frac{1}{4}$. What is the probability that this player's turn will last (a) exactly 6 shots (b) atmost 5 shots (c) atleast 4 shots?

(a) $P(X = 6) = \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^5$

(b) $P(X \leq 5) = \sum_{s=1}^{5} \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^{s-1}$

(c) $P(X \geq 4) = 1 - \sum_{s=1}^{3} \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^{s-1}$

Draw 6 cards from a deck without replacement.
What is the probability of getting 2 hearts.

HyprGeo $\quad N = 52$ Total

$M = 13$ number of hearts

$$P(X = 2\text{hearts})$$

$$= \frac{({}^{13}C_2) \times ({}^{39}C_4)}{({}^{52}C_6)} = .31513.$$

Draw 5 cards from a deck without replacement.
What is the probability of getting at most 2 diamonds.

$$P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{{}^{39}C_5}{{}^{52}C_5} + \frac{{}^{13}C_1 \; {}^{39}C_4}{{}^{52}C_5} + \frac{{}^{13}C_2 \; {}^{39}C_3}{{}^{52}C_5}$$

$$= .907.$$

In a book of 520 pages, 390 typo-graphical erros occur. What is the probability that one page, selected randomly will be free from errors?

Here, $\lambda = \frac{390}{520} = .75$ errors per page.

$P(X=0) = P(\text{no errors}) = e^{-\lambda} = .4724.$

→ What is the probability that 5 pages contain no error

$$[P(X=0)]^5 = (.4724)^5 =$$

→ Find the mgf of this distribution.

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-.75}.(.75)^x}{x!}$$

$$mgf = e^{\lambda(e^t-1)} = e^{-.75(e^t-1)}$$

An insurance company insures 4000 people against loss of both eyes in a car accident. Based on previous data the rates were computed on the assumption that on the average 10 persons in 1,00,000 will have car accident each year that result in this type of injury. What is the probability that more than 3 of the insured will use their policy in a given year?

Poisson distn.

$$p = \frac{10}{1,00,000} = \cdot 0001$$

p small & n large.

$$\lambda = np = 4000 \times \cdot 0001 = \cdot 4$$

$$P(X > 3) = 1 - \sum_{x=0}^{3} P(X = x)$$

$$= \cdot 0008$$