Probability and Statistics
Assignment No. 1

1. If 5 balls are placed at random into 5 cells, find the probability that exactly one cell remains empty.

2. Let event E be independent of events F, $F \cup G$ and $F \cap G$. Show that E is independent of G.

3. A pair of dice is rolled until a sum of 5 or an even number appears. Find the probability that a 5 appears first.

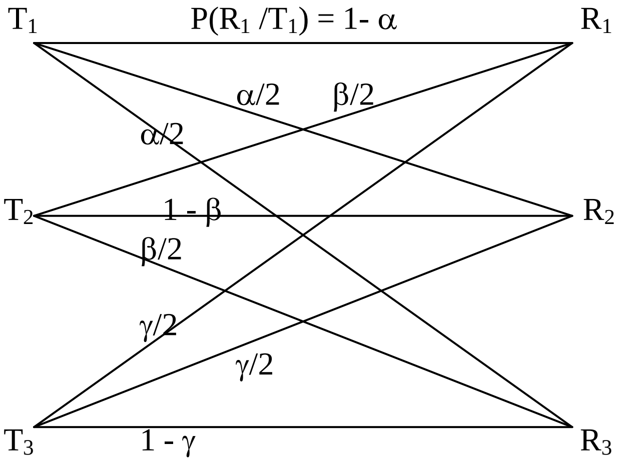
4. In a certain colony, 60% of the families own a car, 30% own a house and 20% own both a car and a house. If a family is randomly chosen, what is the probability that this family owns a car or a house but not both?

5. A survey of people in given region showed that 20% were smokers. The probability of death due to lung cancer, given that a person smoked, was 10 times the probability of death due to lung cancer, given that a person did not smoke. If the probability of death due to lung cancer in the region is 0.006, what is the probability of death due to lung cancer given that a person is a smoker?

6. Show that if $P(A \mid B) = 1$, then $P(B^C \mid A^C) = 1$.

7. If $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^c) = 0.5$, find $P(B / A \cup B^c)$.

8. Consider a trinary communication channel whose channel diagram is shown below:



For i = 1,2,3, let T_i denote the event "Digit i is transmitted" and let R_i denote the event "Digit i is received". Assume that a 3 is transmitted three times more frequently than a 1, and a 2 is transmitted twice as often as 1. (i) If a one has been received, what is the probability that a 1 was sent? (ii) Find the probability of a transmission error. (iii) Find the probability that digit i is received for i = 1,2,3.

9. In any given year a male automobile policyholder will make a claim with probability p_m and a female automobile policyholder will make a claim with probability p_f, where $p_m \neq p_f$. The fraction of policyholders that are male is α, $0 < \alpha < 1$. A policyholder is randomly chosen and A_i denotes the probability that this policyholder will make a claim in the year i, i = 1, 2,…. Find $P(A_1)$ and $P(A_2|A_1)$ and show that $P(A_2|A_1) > P(A_1)$.

10. Which of the following statements is true?
    (i) $P(A) = 0.3, P(B) = 0.7, P(A \cup B) = 0.5, P(A \cap B) = 0.5$.
    (ii) $P(A) = 0.5, P(A \cup B) = 0.7$, A and B are independent, $P(B) = 0.4$.
    (iii) $P(A) = 0.2, P(A \cup B) = 0.9$, A and B are disjoint, $P(B) = 0.6$.
    (iv) none of these.

11. Four players A, B, C and D are distributed thirteen cards each at random from a complete deck of 52 cards. What is the probability that player C has all four kings?

12. Each coefficient in the equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die. Find the probability that the equation will have real roots.

13. An Integrated M.Sc. student has to take 5 courses a semester for 10 semesters. In each course he/she has a probability 0.5 of getting an 'Ex' grade. Assuming the grades to be independent in each course, what is the probability that he/she will have all 'Ex' grades in at least one semester.

14. If $P(A) > 0$, show that $P(A \cap B \mid A) \geq P(A \cap B \mid A \cup B)$.

15. Let $S = \{1, 2, \ldots, n\}$ and suppose that A and B are, independently, equally likely to be any of the 2^n subsets (including) the null set and S itself) of S. Let X denote the number of elements of B. Find $P(X = i)$, $P(A \subset B \mid X = i)$, $P(A \subset B)$ and deduce that $P(A \cap B = \phi) = \left(\frac{3}{4}\right)^n$.

16. A question paper consists of six True-False and four multiple choice (A, B, C, D) questions. Each question carries one mark for the correct answer and zero for wrong answer. Assume that an unprepared student answers all questions independently with guess. What is the probability that he/she will score at least 8 marks?

17. Four computer firms A, B, C, D are bidding for a certain contract. A survey of past bidding success of these firms on similar contracts shows the following probabilities of winning: $P(A) = 0.35$, $P(B) = 0.15$, $P(C) = 0.3$, $P(D) = 0.2$. Before the decision is made to award the contract, firm B withdraws its bid. Find the new probabilities of winning the bid for A, C and D.

18. Boys and girls are equally likely to qualify an examination. If 2n students qualify, what is the probability that more girls qualify than boys?

19. Suppose n men take part in a get-together in a hall. When they enter into the hall they put off their hats and keep on a table. While leaving the hall they pick up a hat at random. What is the probability that no one will get back his own hat? What is the limiting probability when n is large?

20. Let there are R boxes numbered 1, 2,…, R. At random n balls are places in R boxes. What is the probability that exactly k balls will be placed in first r (< R) boxes?

21. In a deck of cards there are 52 cards. Out of these 26 cards are red and the rest are black in colour. If you choose 13 cards at random without replacement, what is the probability that you will get 3 red cards?

22. In a deck of cards there are 52 cards with four suits namely club, heart, diamond and spade of equal sizes. If you choose 13 cards at random without replacement then what is the probability that you will get exactly 3 clubs, 4 diamonds, 4 hearts and 2 spades?

23. In a deck of 52 cards, each of the four suits has 13 denominations (Ace, 2,3,4,5,6,7,8,10, Jack, Queen, King). If you choose 4 cards at random with replacement, what will be the probability that you will draw (a) four distinct kings ? (b) a queen each time ?

24. Let n balls be distributed in n numbered boxes so that all n^n arrangements are equally likely (of equal probability). What is the probability that only the 1st box will remain empty ?

25. In village of (n+1) people someone originates a rumour. He chooses a person at random and tells it. Second person finds another at random and repeats it and so on … for r (< n) times. (a)What is the probability that the rumour will not come back to the originator? (b) What is the probability that the rumour will not come back to a person who already knew it ?