

Q.1

Prove $(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m+5n=12)$

Proof: Assume $3m+5n=12$ and $\exists m, n \in \mathbb{N}$.

Then, $3m+5n=3 \times 4$.

By rearranging, $5n=3(4-m)$.

Since n is natural, $5n$ is also natural. Thus, $3(4-m)$ is natural.

In order for $3(4-m)$ to be natural, ~~m~~ $m < 4$ and $m \in \mathbb{N}$.

By Addition, $5n = 3 \times 4 - 3m$
 $+ 4 > m$

$$5n + 4 > 12 - 2m$$

By Algebra, $5n + 2m > 12 + 4$.

By ^{Subtraction} ~~Addition~~, $5n + 2m > 12 + 4$
 $- (5n + 3m = 12)$

$$-m > 4$$

By Addition, ~~m~~ $> m$
 $+ -m > 4$
 $\hline 4 - m > 4 + m$

→ Subtracting both sides by 4,

$$-m > m$$

Divide both sides by m .

$$-1 > 1$$

↑
 m is natural

Thus, the statement is not true.

QED \square