

Q.3

Prove that for any integer n , n^2+n+1 is odd.

Proof: By induction, if $n=1$, n^2+n+1 is odd.

Assume, n^2+n+1 is odd.

By the inductive step, $(n+1)^2+n+1+1=2k+1$.

By Algebra, $n^2+2n+1+n+1+1=n^2+3n+3$.

By Algebra, $n^2+3n+3=n^2+n+1+2n+2=2k+1$.

By more Algebra, $n^2+n+1+2=2(k-n)-1$

$2(k-n)-1$ is odd. We assumed that n^2+n+1 is odd.

Thus, the statement is proven.

QED \square