

Q.5

Prove that for any integer  $n$ , at least one of the integers  $n$ ,  $n+2$ , or  $n+4$  is divisible by 3.

Proof: Every integer is of the forms  $3k$ ,  $3k+1$ ,  $3k+2$ .

If  $n=3k$ ,  $n$  is divisible by 3.

If  $n=3k+1$ ,  $n+2=3k+3$ ,  $n+2$  is divisible by 3.

If  $n=3k+2$ ,  $n+2=3k+4$ ,  $n+4=3k+6$ ,  $n+4$  is divisible by 3.

In all cases, one of the integers  $n$ ,  $n+2$ , or  $n+4$ , was divisible by 3.

Thus, the statement is proven.

QED 