Q.7

Prove that for any natural number n, $2+2^2+\ldots+2^n=2^{n+1}-2$

Proof: By induction, if $n=1$, the statement is true.

Assume $2+2^2+\ldots+2^n=2^{n+1}-2$.

By the inductive step, $2+2^2+\ldots+2^n+2^{n+1}=2^{n+2}-2$.

By Algebra, $2+2^2+\ldots+2^n+2^{n+1}=2^{n+1}-2+2^{n+1}=2^{n+2}-2$.

By more Algebra, $2(2^{n+1})=2^{n+2}$.

Thus, the statement is proven.

QED