

Q. 8

Given $\lim_{n \rightarrow \infty} a_n = L$ and $M > 0$, Prove $\lim_{n \rightarrow \infty} M a_n = M L$

Proof: $\lim_{n \rightarrow \infty} a_n = L$ is the same as $(\forall m \in \mathbb{N})(\exists \epsilon)(\forall n \geq m) (|a_n - L| < \epsilon)$.

$\lim_{n \rightarrow \infty} M a_n = M L$ is the same as $|M a_n - M L| < M \epsilon$.

By Algebra, $\lim_{n \rightarrow \infty} M a_n = M L$ is the same as $M |a_n - L| < M \epsilon$.

This still conveys that beyond a number m , $M a_n$ is in some range of L .

Thus, the statement is proven.

QED 