Random Walker

Imagine a "random walker" on the real line, starting at origin $z = 0$ at time $t = 0$.
Within each timeframe Δt , the walker takes either a
Δz step right with probability p, **OR** a Δz step left with probability $q := 1 - p$

Let total number of steps taken $= n$

Consider an event where walker took 'x' steps right; '$n - x$' steps left.

- Random walker is located at $z = \Delta z(2x - n)$ (How ?)
- For $(x + f)$ steps right, where $0 < f < 1$, location can be linearly interpolated as $z + \Delta z. 2. f$
- Probability of this event is $P(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x}$ (figure this out)
- We want to make the walker's location a continuous (real-valued) random variable. So, we want to make $n,\ x,\ (n - x)$ (infinitely) large, and Δt and Δz (infinitesimally) small.
- Factorial values can be approximated for large 'n' using Stirling's formula
  $n! = n^n e^{-n} \sqrt{2\pi n}\left[1 + O(\frac{1}{n})\right]$ and

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

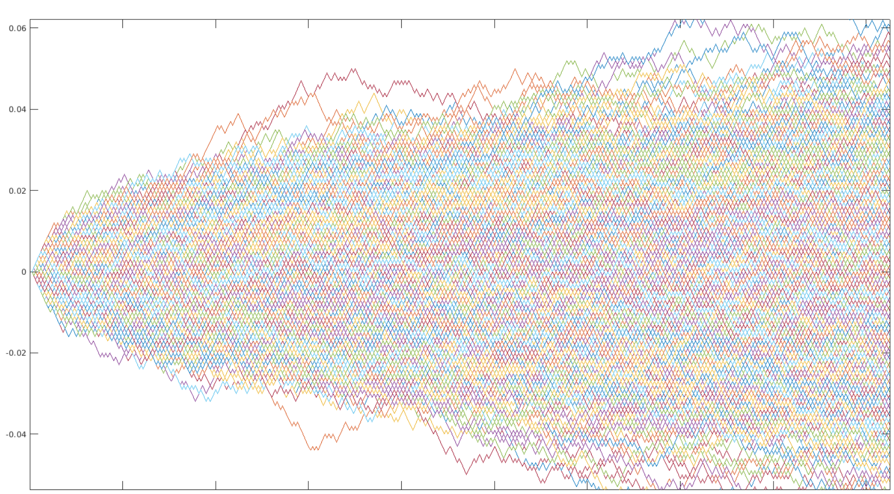
- On solving and taking limits $P(x) = \frac{1}{\sqrt{2\pi npq}} e^{-(x-np)^2/2npq}$

(Try to get this if you want to do the math. But it is not really required)

Lets see how will the distribution will be in different cases

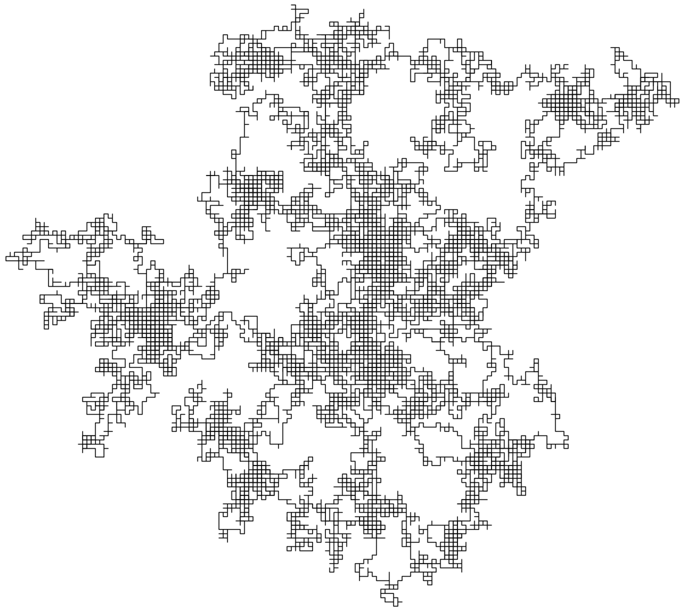
Random walk on 1D real line (for 1000 persons)

- Horizontal axis: time
- Vertical axis: location



Random walk in 2D

- Small steps, numbering 25,000 (for 1 person)



Random walk in 2D

- Tiny steps, numbering 2 million (for 1 person)

Brownian motion in 2D

Real Life simulation
[Brownian motion]

Galton Board (also called bean machine) (Simulation of Random Walk)

- Gaussian (Normal) Distribution
- Device invented by Sir Francis Galton
- Shows that, binomial distribution can be visualized as a Gaussian

[Galton Board in Slow Motion]

[Galton Board]