SHM

Oscillatory motion

↳ if a body moves back & forth repeatedly about the mean position.

Ex: swinging pendulum, vibratory motion of a mass attached to a spring.

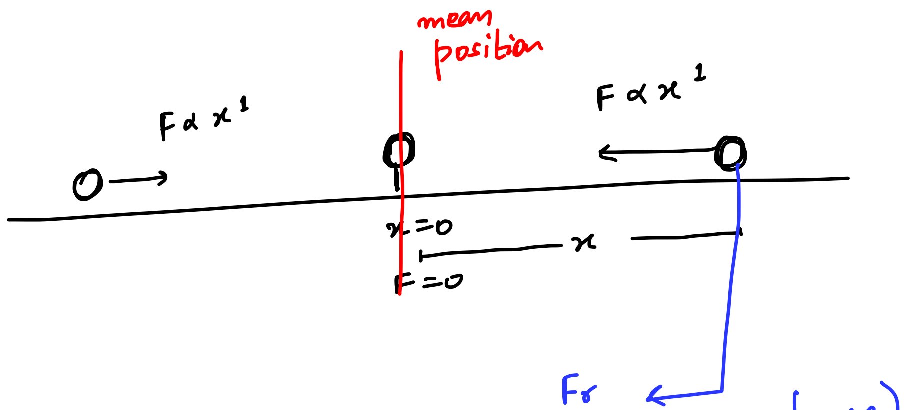
Periodic Motion: A motion that repeats after equal intervals of time is a periodic motion.

Each oscillatory motion is a periodic motion.

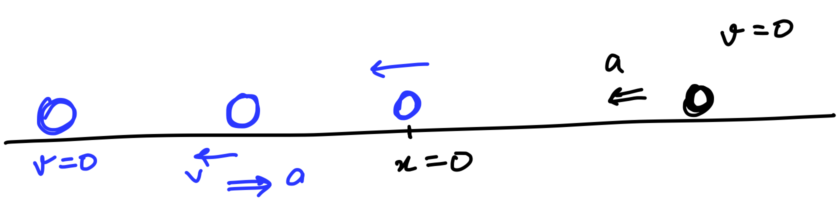
A body moving in a Circle is also an example of periodic motion, but it is not oscillatory.

SHM: - motion of simple pendulum
- vibrating tuning fork

It is the simplest form of Oscillatory motion.
↳ where magnitude of force acting on it is directly proportional to the magnitude of its displac.



$F_r = kx$
↳ proportionality constant



Note: We cannot apply 3 equations of motion

Kinematics of SHM

$$F = -kx$$

$$ma = -kx$$

$$a = -\frac{kx}{m}$$

$$\frac{d^2x}{dt^2} = -\frac{kx}{m} \quad -(i)$$

This is called the differential equation of SHM.

$$a(x) \longrightarrow v(x) \longrightarrow x(t)$$

$$x(t) = A\sin(\omega t + \phi)$$

(ii) General expression for $x(t)$ satisfying the differential equation.

Substituting (ii) in (i), we get

$$\omega = \sqrt{k/m}$$

Note : $\frac{d^2x}{dt^2} = -\omega^2 x$

$$x(t) = C_1 \sin(\omega t + C_2)$$

$$x(t) = A\sin(\omega t + \phi_0)$$

Ex : $\frac{d^2y}{dx^2} = -25y$

$$y = C_1 \sin(5x + C_2)$$

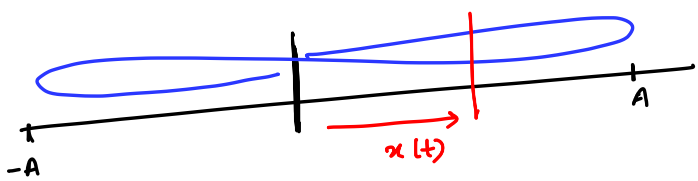
dx/dt

$$v(t) = A\omega\cos(\omega t + \phi_0)$$

$\frac{dv}{dt}$

$$a(t) = -\omega^2 A\sin(\omega t + \phi_0)$$

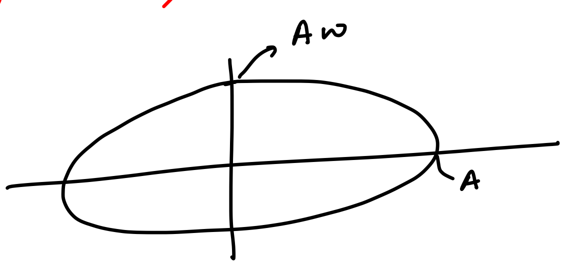
$$\boxed{a = -\omega^2 x}$$



Note $v_{max} = A\omega$

Eliminating t from $x(t)$ & $v(t)$

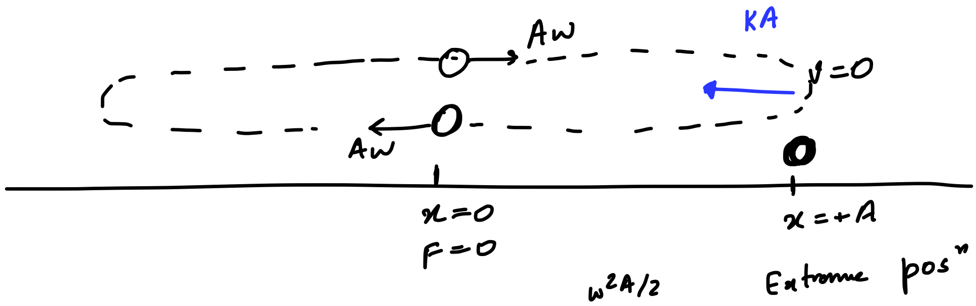
$$\left(\frac{x}{A}\right)^2 + \left(\frac{v}{A\omega}\right)^2 = 1$$

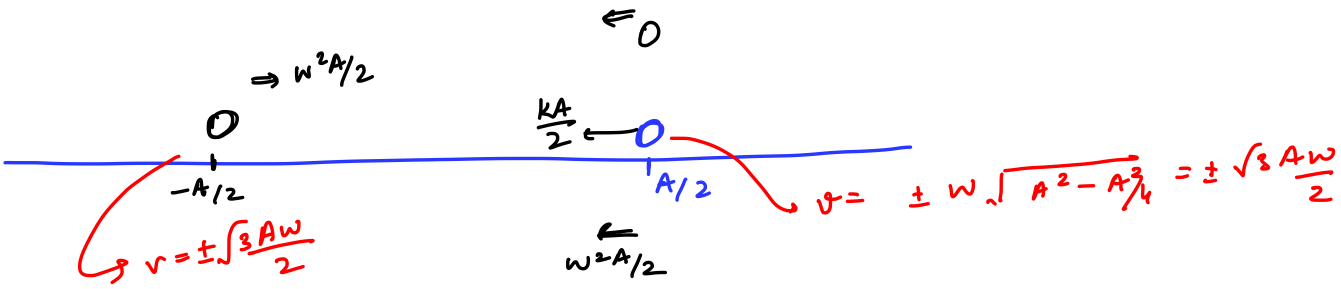


$$v(x) = \pm\omega\sqrt{A^2 - x^2}$$

Eliminating t from $v(t)$ & $a(t)$

$$\left(\frac{v}{A\omega}\right)^2 + \left(\frac{a}{-\omega^2 A}\right)^2 = 1$$





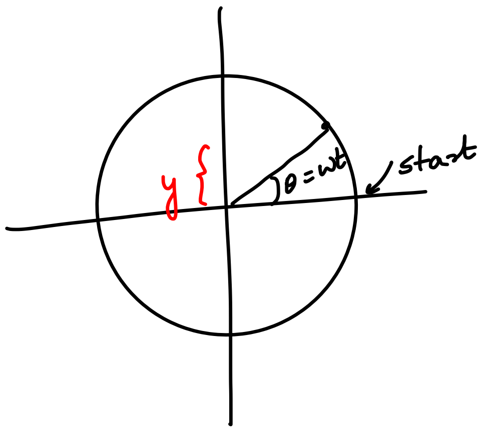
\# $\sin px \rightarrow \frac{2\pi}{p}$ [Time Period]

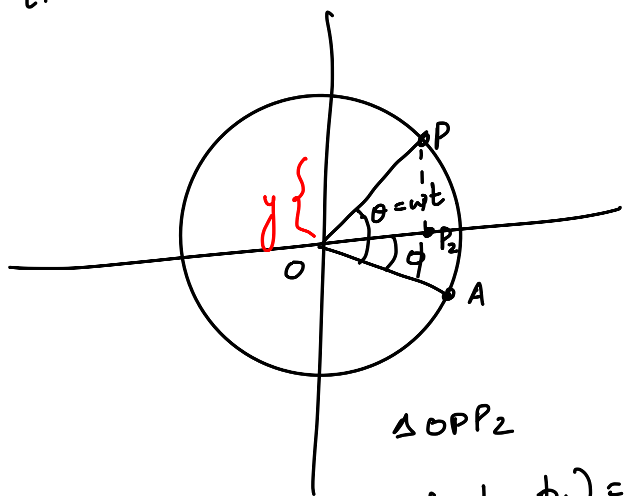
$\sin(\omega t + \phi_0) \rightarrow \frac{2\pi}{\omega}$ [Time period]

lies b/w $[0, 2\pi]$

\# $\phi_0 \longrightarrow$ Initial phase

$\omega t + \phi_0 \longrightarrow$ phase at time t





ω

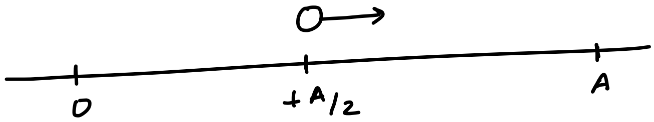
$y = a \sin \omega t$

ΔOPP_2

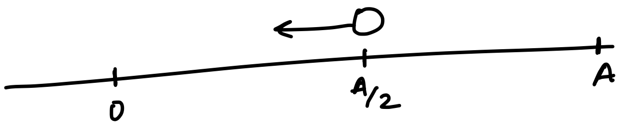
$\sin(\omega t - \phi_0) = \frac{y}{a}$

$y = a\sin(\omega t - \phi_0)$

Ques

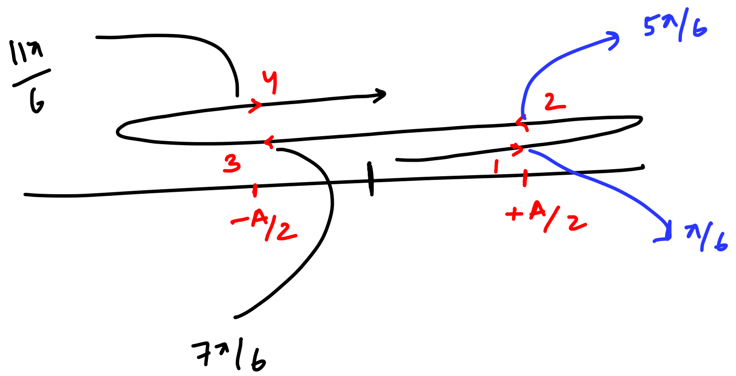


$$x = A\sin\theta$$
$$v = A\omega\cos\theta$$
$$\frac{A}{2} = A\sin\theta$$
$$\theta = n\pi + (-1)^n \frac{\pi}{6}$$
$$= \pi/6,\ 5\pi/6$$



$\because \cos\pi/6 \rightarrow +ve$

$\cos 5\pi/6 \rightarrow -ve$



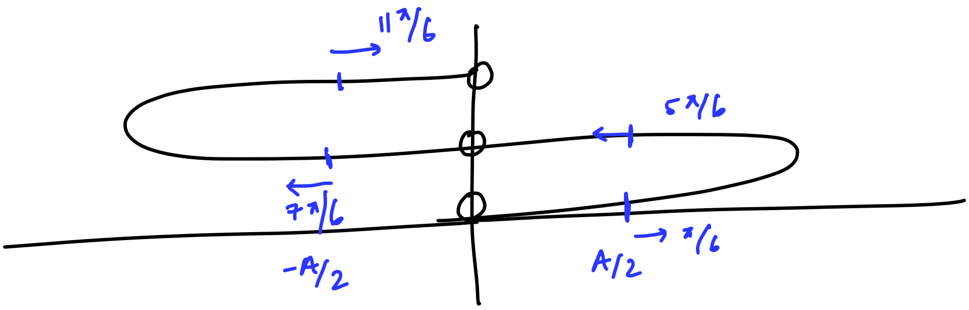
$$-\frac{A}{2} = A\sin\theta$$
$$\theta = n\pi + (-1)^n(-\pi/6)$$
$$-\frac{\pi}{6},\ \frac{7\pi}{6},\ \frac{11\pi}{6}$$

Note :

$$\sin\theta = \sin\alpha$$
$$\theta = n\pi + (-1)^n\alpha$$

$$\cos\theta = \cos\alpha$$
$$\theta = 2n\pi \pm \alpha$$

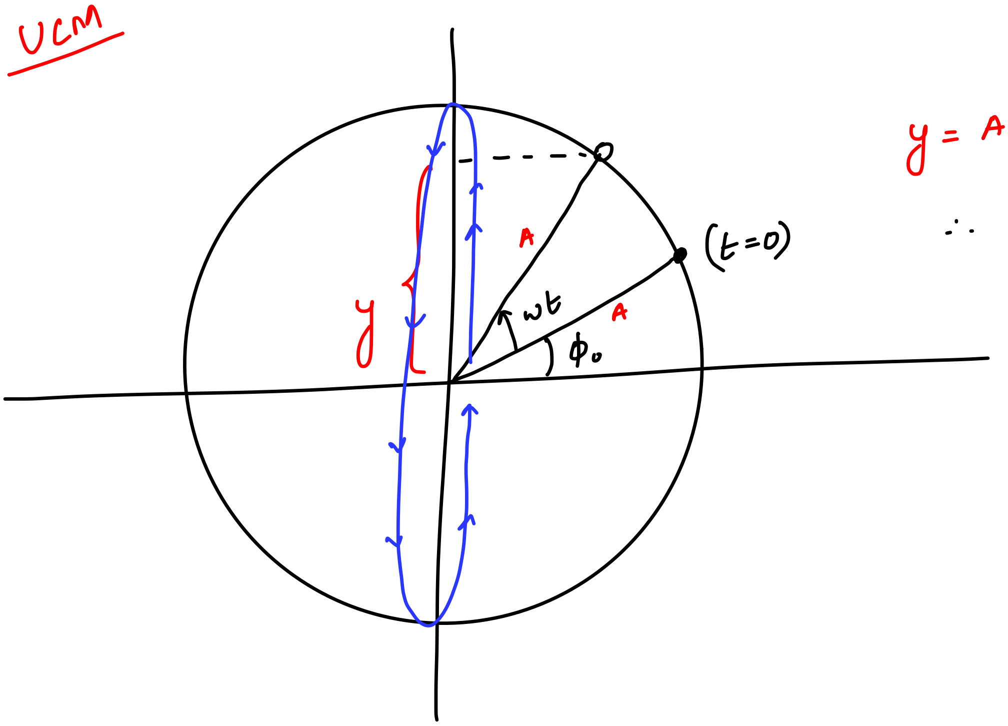
$$\cos^2\theta = \cos^2\alpha$$
$$\theta = n\pi \pm \alpha$$ (used in Interferance)



If I say phase = 7π/6
↓
we know everything

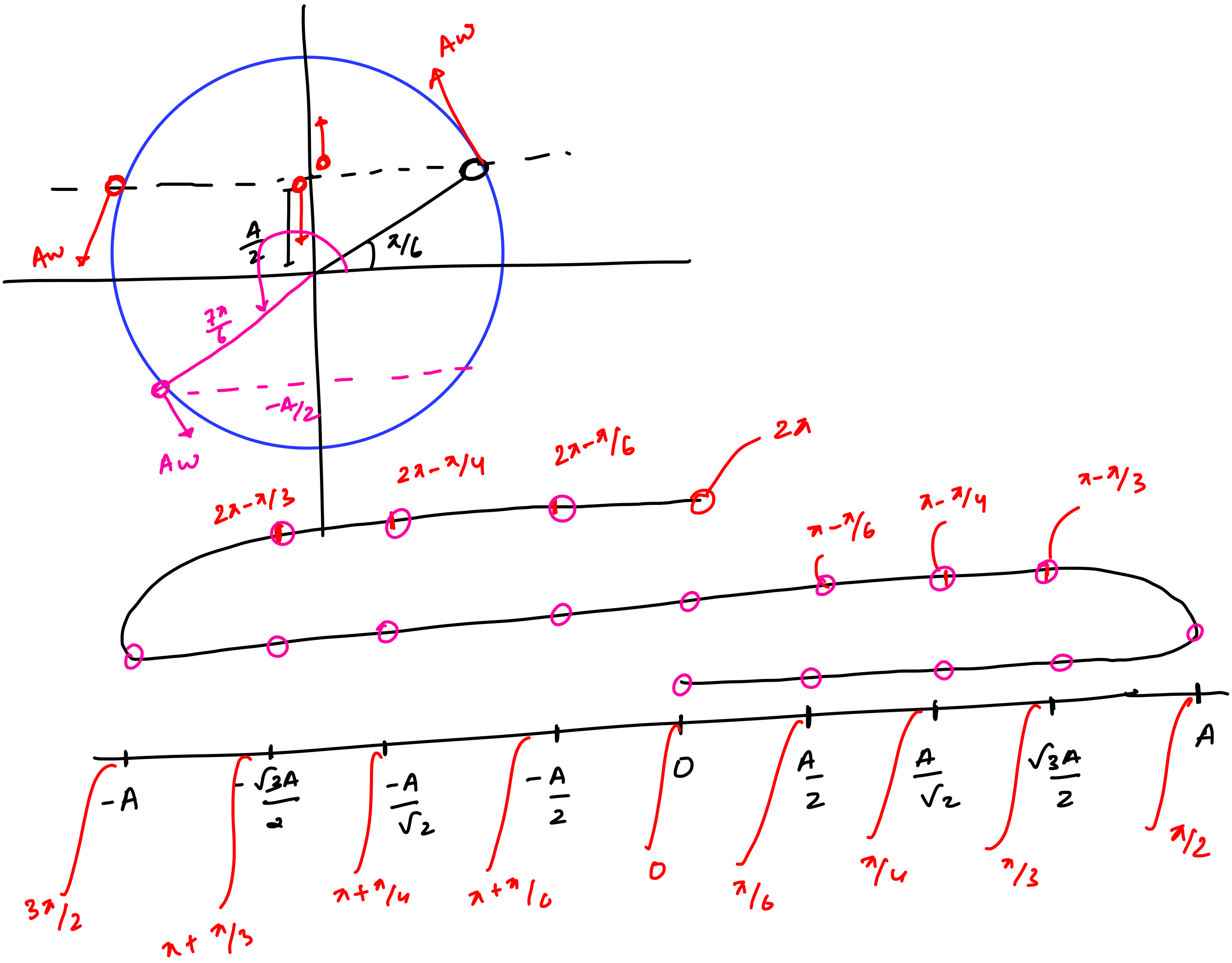
-A/2 | -ve velocity

UCM



$y = A\sin(\omega t + \phi_0)$

$\therefore$ y-coordinate of UCM is doing SHM.



Ques

$F = -50x$

$m = 2\,kg$

at $t=0$ $\quad x = +A/2$ $\quad v = -20\,m/s$

Equation of SHM

Sol^n

$x = A\sin(\omega t + \phi_0)$

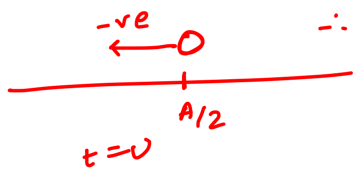
Setup par depend karta h

↳ ki forces kese lag rahi hei

$$\left[\begin{array}{l} a = -25x \\ \omega = 5 \end{array}\right.$$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{5}$

$\text{frequency} = \frac{1}{T}$



$\therefore\ \phi_0 = 5\pi/6$

Also,

$-20 = \pm\,\omega\sqrt{A^2 - A^2/4}$

$A = \frac{4\times 2}{\sqrt{3}}$

$$x(t) = \frac{8}{\sqrt{3}}\sin\left(5t + \frac{5\pi}{6}\right)$$

if $v = 20\,m/s$

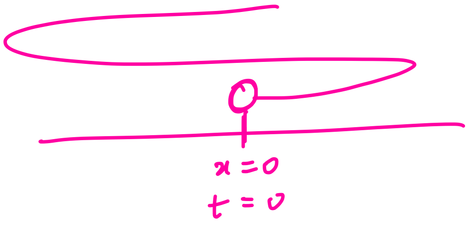
$$x(t) = \frac{8}{\sqrt{3}}\sin\left(5t + \frac{\pi}{6}\right)$$

$\phi_0 \rightarrow$ initial location & direction of velocity

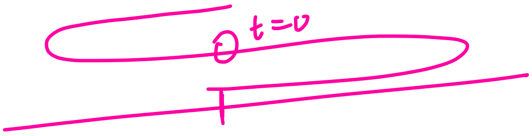
if $v = 30\,m/s$

$$x(t) = \frac{12}{\sqrt{2}}\sin\left(5t + \pi/6\right)$$

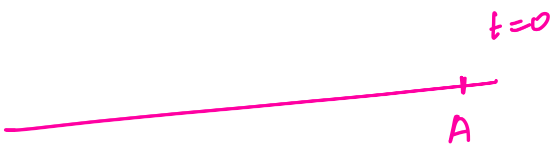
$A \rightarrow$ initial location & direction of velocity



$x(t) = A\sin\omega t$

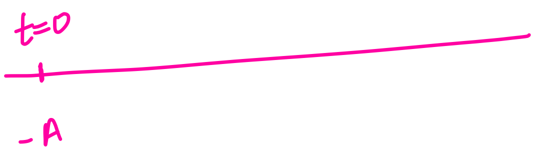


$x(t) = A\sin(\omega t + \pi)$
$= -A\sin\omega t$



$x(t) = A\sin(\omega t + \pi/2)$
$= A\cos\omega t$

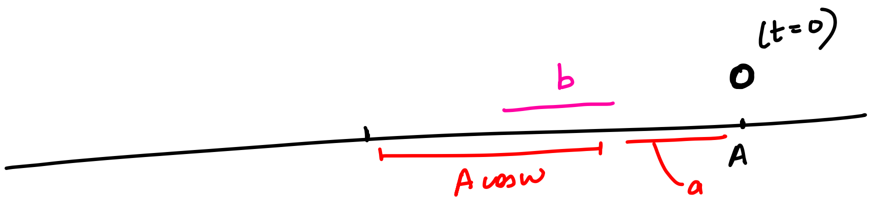
$x(t) = -A\cos\omega t$



Ques Starting from rest $1^{st} \longrightarrow a$

$2^{nd} \longrightarrow b$

Ans



$x(t) = A\cos\omega t$

Put $t=0 \Rightarrow \boxed{x = A}$

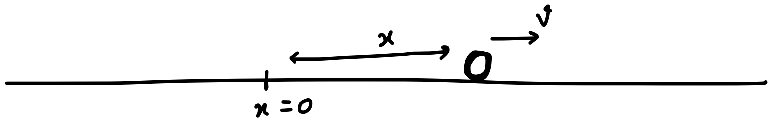
note that x is from mean position.

$x(1) = A\cos\omega$
$= A - a$ —①

$A\cos 2\omega = A - a - b$ —②

$\frac{1+\cos 2\omega}{2} = \cos^2\omega$

Energy

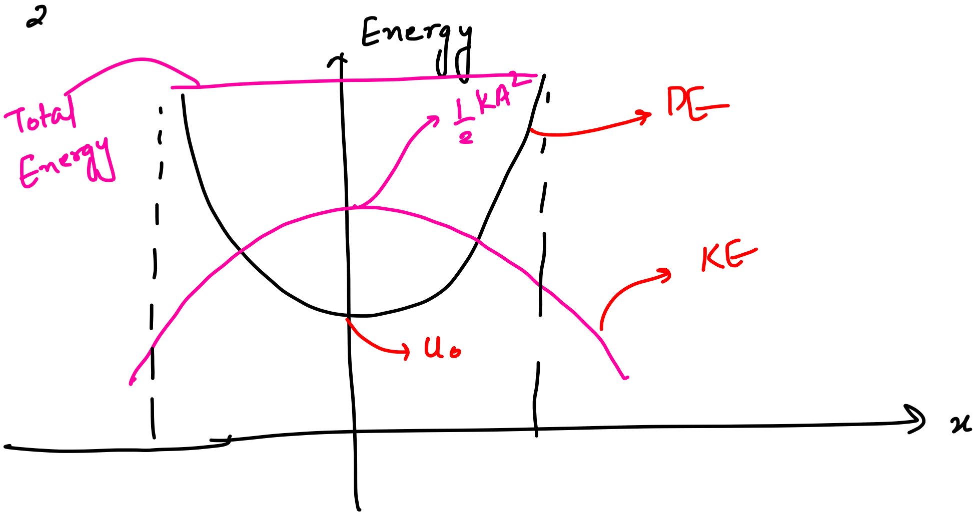


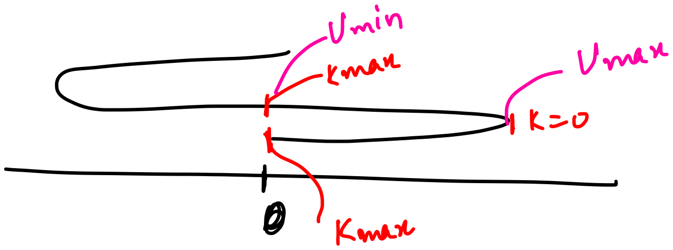
$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$= \frac{1}{2}K\,(A^2 - x^2)$$

$$P.E = \frac{1}{2}Kx^2 + U_0$$ (P.E at mean position)

Total Energy : $\frac{1}{2}KA^2 + U_0$





$$\text{Time KE} = \frac{T_{SHM}}{2}$$

$$f = 2f_{SHM}$$

$$K = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m\,\omega^2A^2\cos^2(\omega t + \phi_0)$$

$$= \frac{1}{2}KA^2\cos^2(\omega t + \phi_0)$$

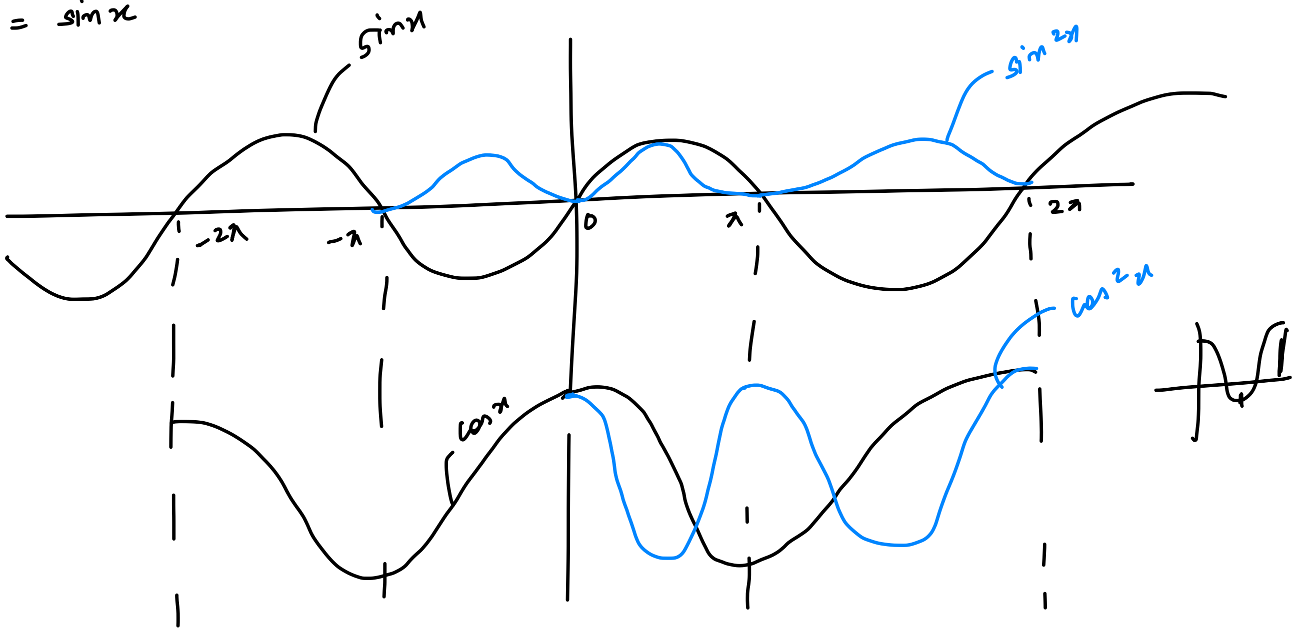
$$= \frac{1 + \cos(2\omega t + 2\phi_0)}{2}$$

This is a periodic function of jiska

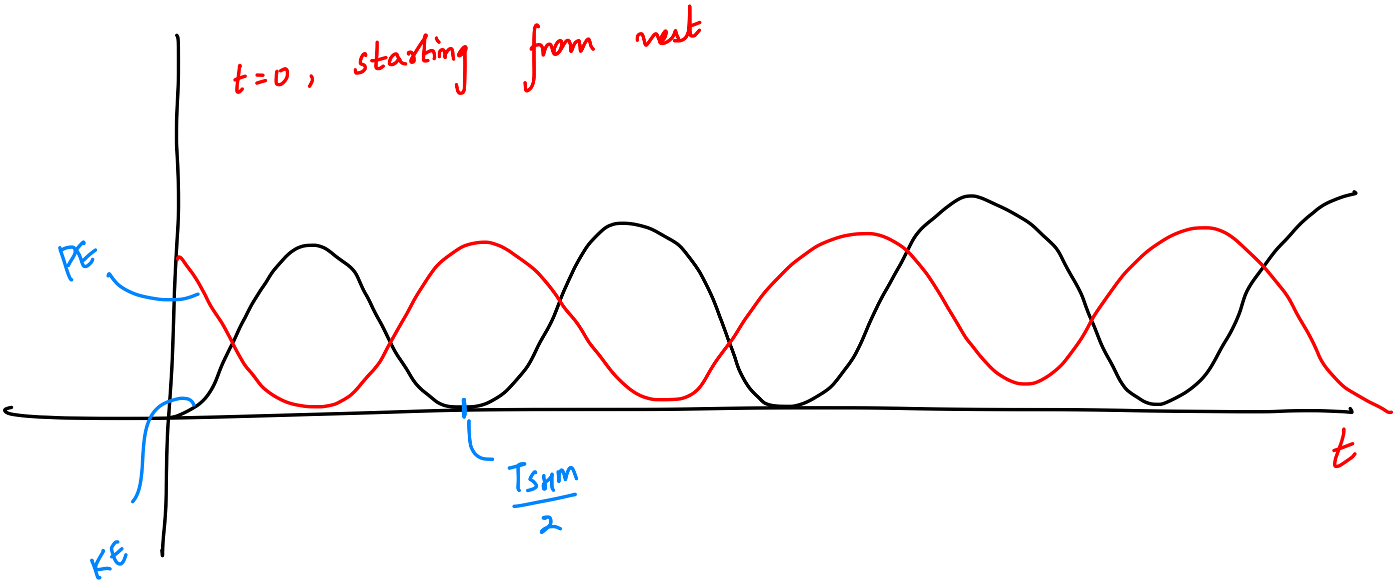
$$T = \frac{2\pi}{2\omega} = \frac{T_{SHM}}{2}$$

$$U = \frac{1}{2} kA^2 \underbrace{\sin^2(\omega t + \phi_0)}_{\frac{1 - \cos 2(\omega t + \phi_0)}{2}} + U_0$$

$y = \sin x$

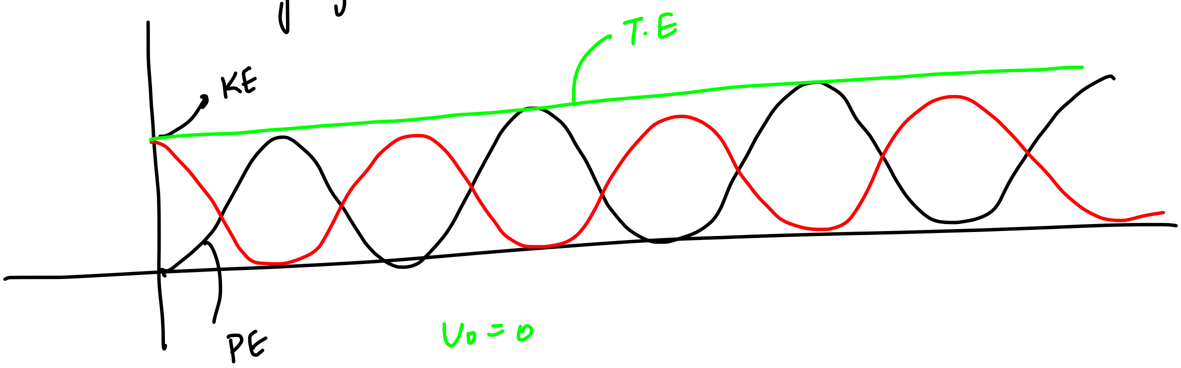


$t = 0$, starting from rest



Also, $U_0 = 0$

starting from mean position



$U_0 = 0$

Dynamics of SHM

Steps

1) Find mean position (FBD) ($F_{net} = 0$)

2) Displace from mean position by x

3) FBD again

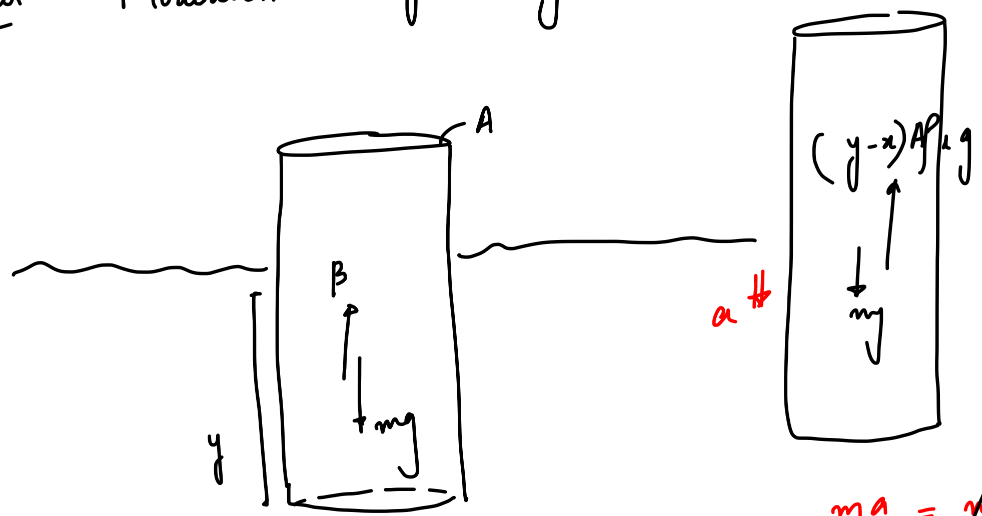
4) Calculate restoring force // always in dirxn of restoring i.e kx not $-kx$

5) $F_r = kx$

$$ma = kx$$

$$\omega = \sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{K}}$$

Ques Floatation of Cylinder



$$ma = \cancel{mg} - (\cancel{y}-x)A\rho_l g$$

$mg = yA\rho_l g$ // mean position

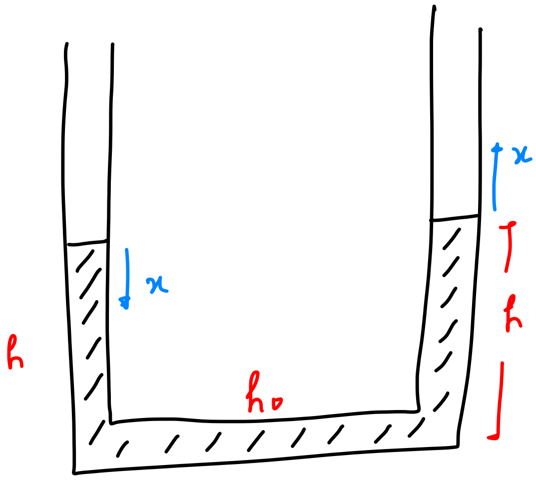
$$a = \left(\frac{A\rho_l g}{m}\right)x$$

by comparing

$$\omega = \sqrt{\frac{A\rho_l g}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{A\rho_l g}}$$

Ques



$$\Delta P = 2x\rho g$$

$$F_x = (2x\rho g)A$$

$$ma = (2x\rho g)A$$

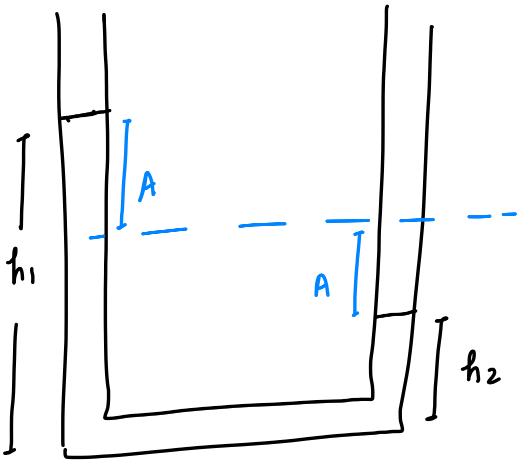
$$a = \left(\frac{2x\rho g A}{m_{total}}\right)$$

$$\omega = \sqrt{\frac{2\rho g A}{m}} = \sqrt{\frac{2\rho g A}{(2h+h_0)A\rho}}$$

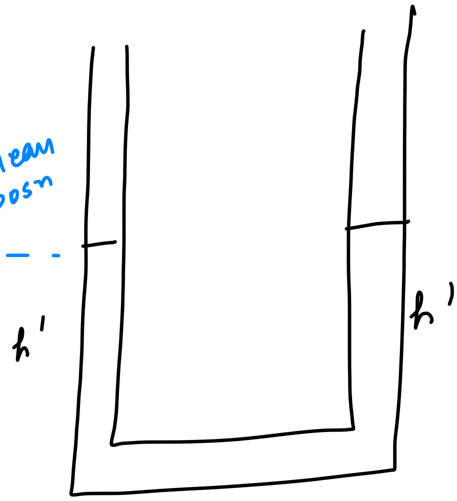
$$T = \frac{2\pi}{\omega}$$ ✓

Ques Rest release

Extreme Posn



Mean posn



$$A = h_1 - h'$$
$$= h' - h_2$$

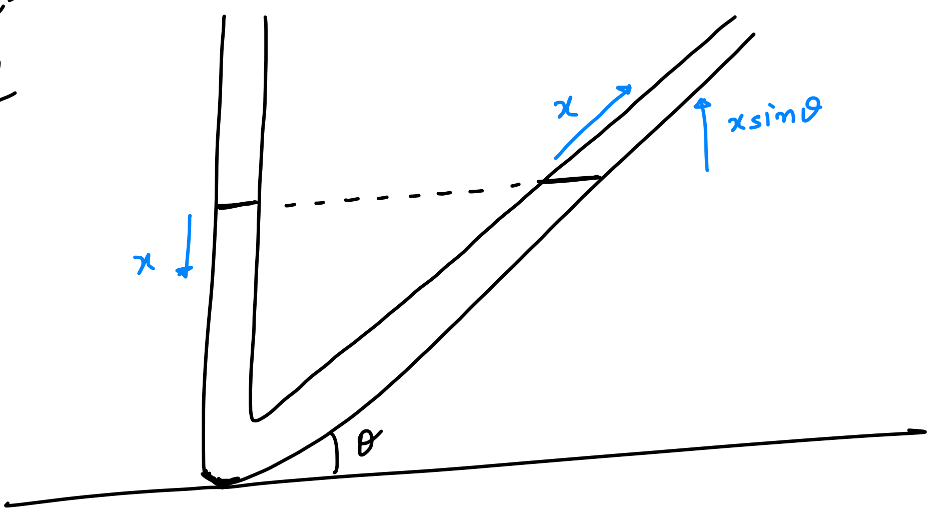
or $$A = \frac{h_1 - h_2}{2}$$

at mean posn : $$v = A\omega = \left(\frac{h_1 - h_2}{2}\right)\left(\frac{2\rho g A}{m_{total}}\right) = \left(\frac{h_1 - h_2}{2}\right)\sqrt{\frac{2\rho g A}{(h_1+h_2+h)A\rho}}$$

or

$$a = \frac{\Delta P \cdot A}{m} = \frac{(2x\rho g)A}{m} \rightarrow \sqrt{\omega}$$

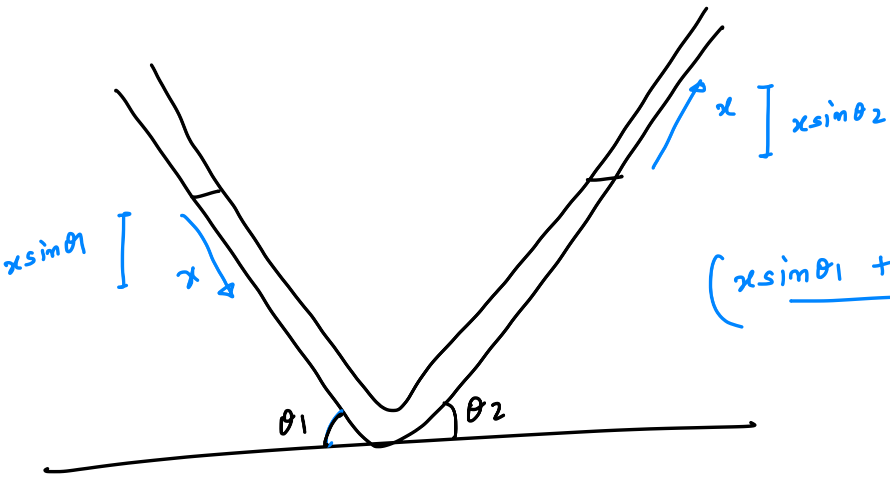
INEB Q6



$$ma = (x + x\sin\theta)\rho g A$$

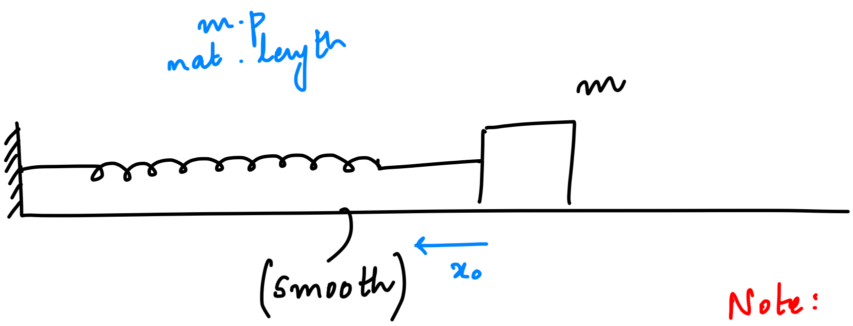
$$\omega = \sqrt{\frac{\rho g A(1+\sin\theta)}{m}}$$

Ques



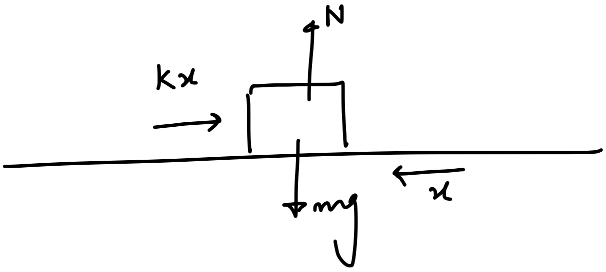
$$\frac{(x\sin\theta_1 + x\sin\theta_2)A\rho g}{m} = a$$

Ques



Note: Now, this x_0 will act as amplitude.

$$x(t) = -x_0 \cos\sqrt{k/m}\,t$$



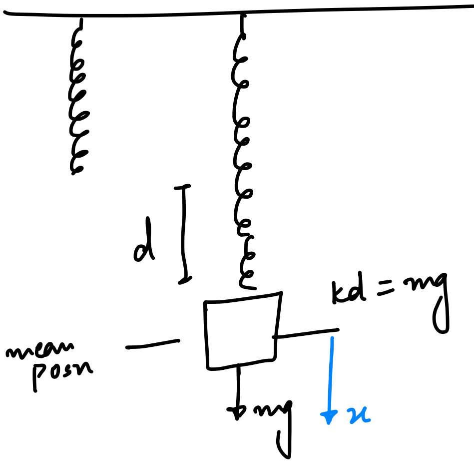
$kx = ma$

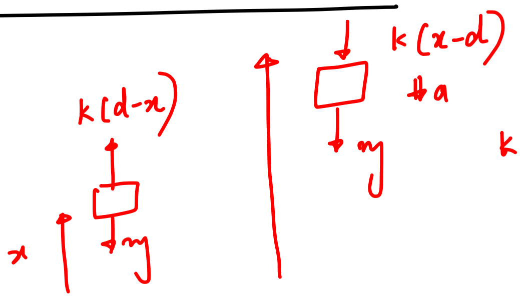
$a = kx/m$

$$\omega = \sqrt{k/m}$$

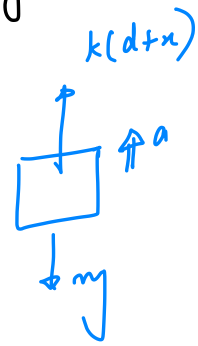
$$T = 2\pi\sqrt{\frac{m}{k_{sp}}}$$

Que





$kx = ma$

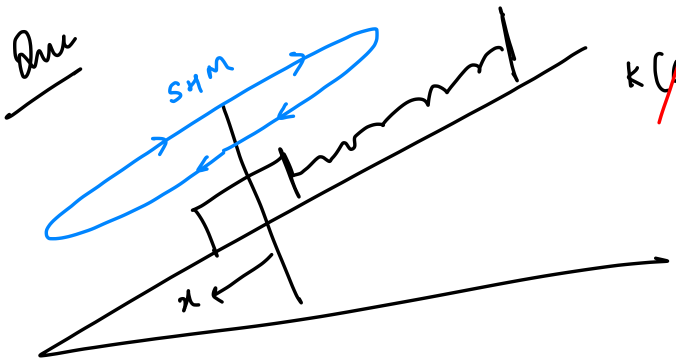


$ma = k(d+x) - mg$

$ma = kx$

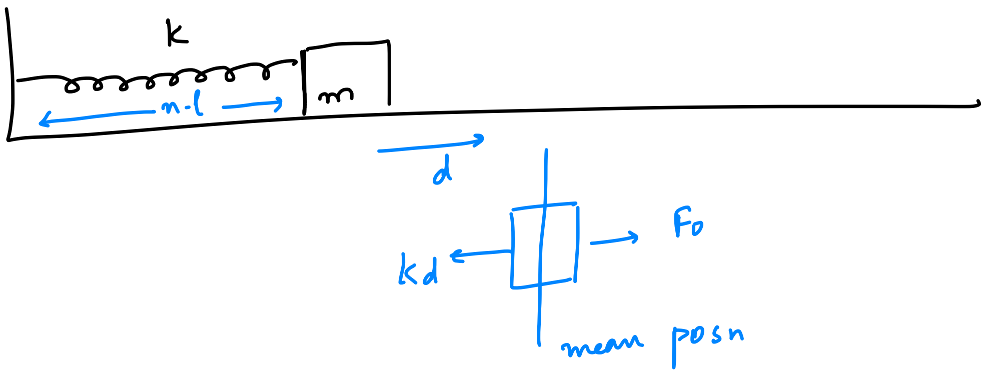
$\omega = \sqrt{k/m}$ $\quad T = 2\pi\sqrt{\frac{k}{m}}$

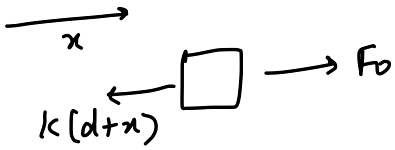
Que



$k(\cancel{d}+x) - \cancel{mg\sin\theta} = ma$

Constant force





$$mA = k(d+x) - F_0$$

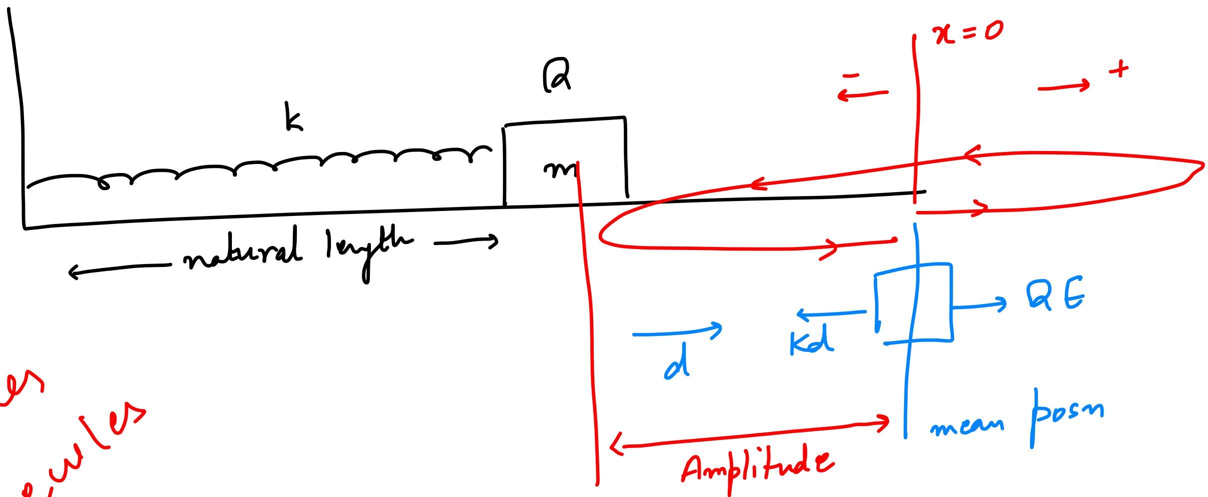
$$ma = kx$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Fo ka koi role nahi h, ab bas mean posn n.l par nahi h.

then → Electric field switched ON

Qn

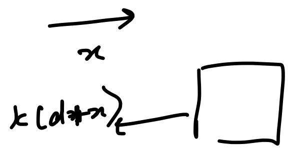


Em waves
Biomolecules

$$k(d+x) - QE = ma$$

$$kx = ma$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

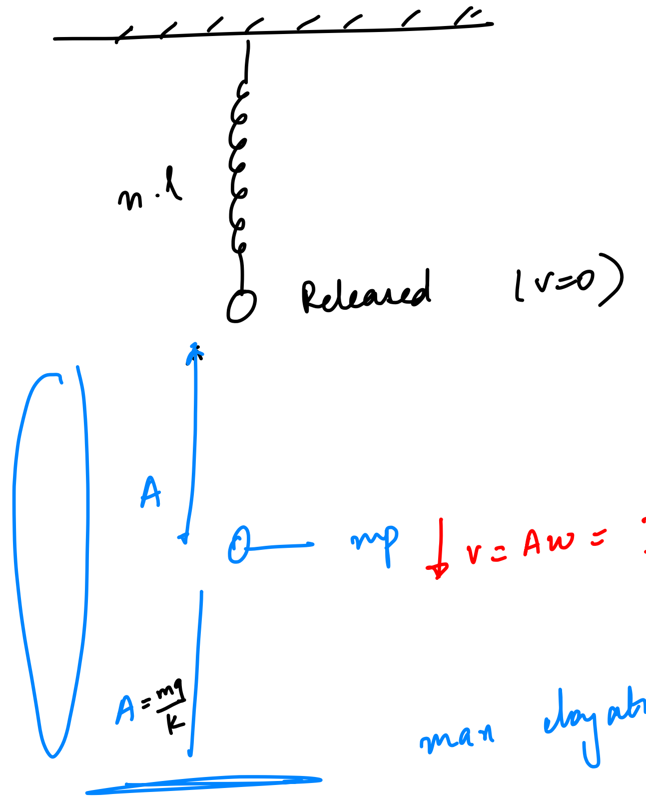


$$x(t) = -A\cos(\omega t)$$

$$= -\frac{QE}{K}\cos\left(\sqrt{\frac{k}{m}}\,t\right)$$

$$v_{max} = A\omega = \left(\frac{QE}{K}\right)\sqrt{\frac{K}{m}}$$

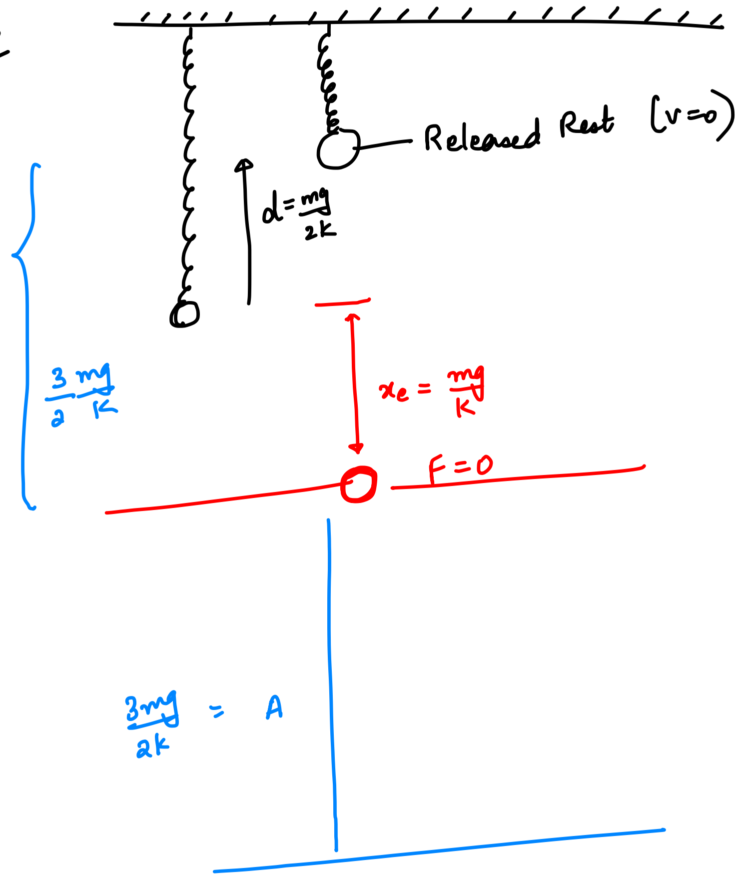
Qun



$$v = A\omega = \frac{mg}{K}\sqrt{\frac{K}{m}}$$

$$\text{max elongation} = 2A = \frac{2mg}{K}$$

Ques



$$x_e = \frac{mg}{K}$$

$$\frac{3mg}{2k} = A$$

i) Max elongation from natural length

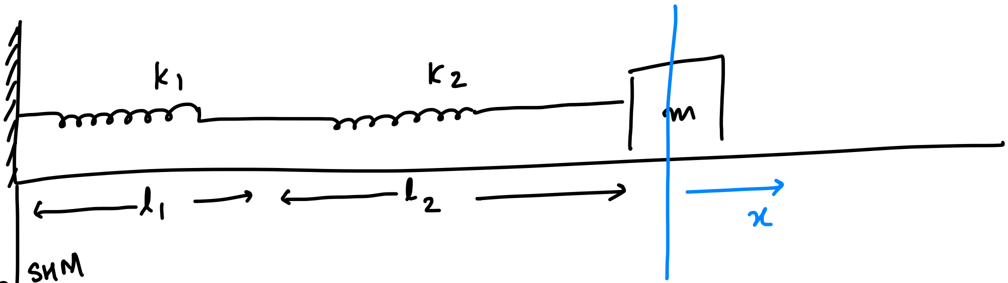
$$= \frac{5mg}{2k} \quad \left(\because \frac{3mg}{2k} + \frac{mg}{k} \right)$$

ii) Velocity at Equilibrium $= A\omega = \frac{3mg}{2k}\sqrt{\frac{k}{m}}$

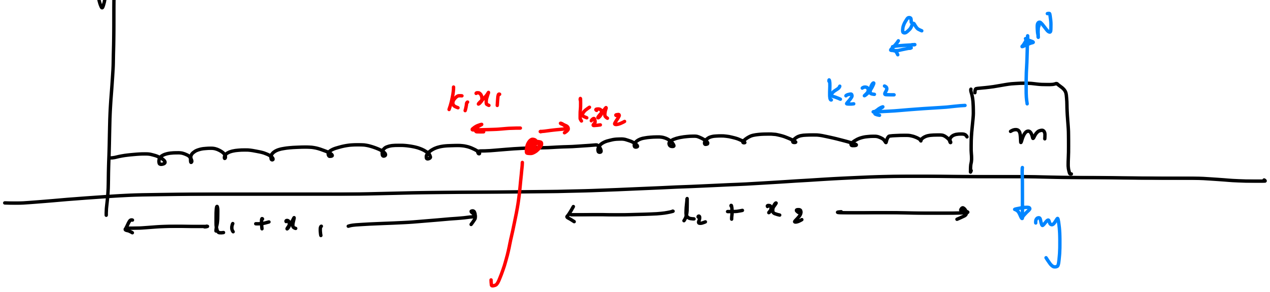
iii) Velocity at natural length $= \omega\sqrt{A^2 - x^2}$

$$= \sqrt{\frac{k}{m}} \sqrt{\left(\frac{3mg}{2k}\right)^2 - \left(\frac{mg}{k}\right)^2}$$

Ques



T of SHM



$F_{net} = ma$

$m = 0$

$\therefore k_1 x_1 = k_2 x_2$

Now, $x_1 + x_2 = x$

$$\frac{k_2 x_2}{k_1} + x_2 = x$$

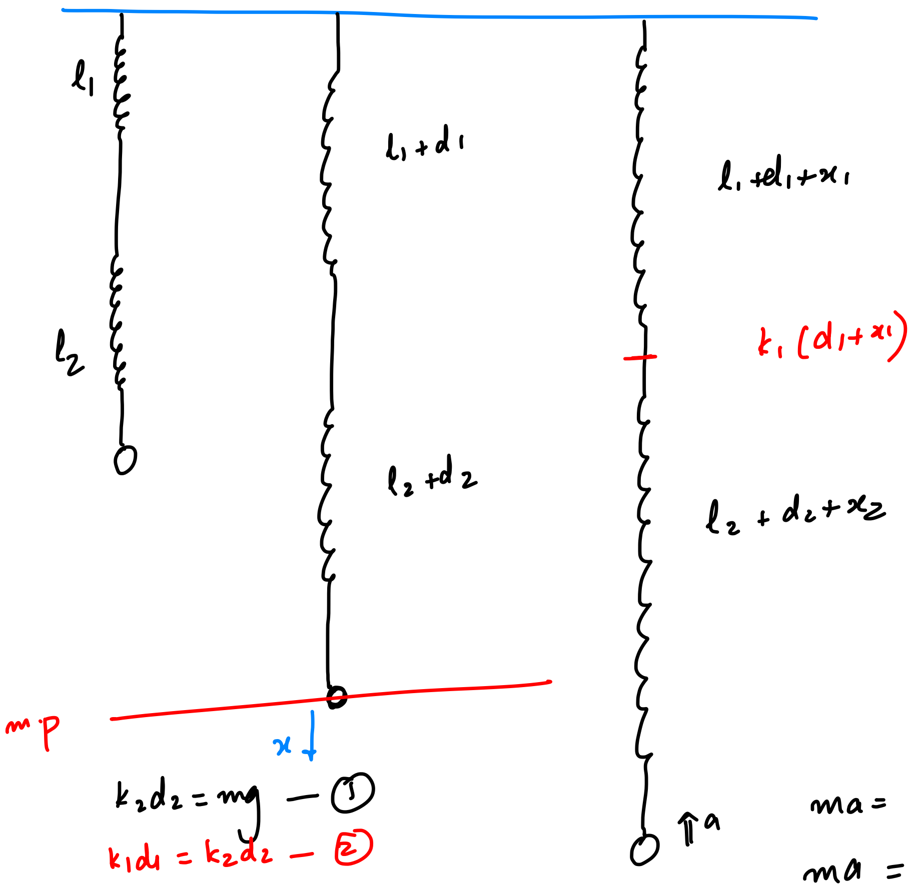
$k_2 x_2 = ma$

$$\frac{k_2 k_1}{k_1 + k_2} x = ma$$

k_{eq}

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k_{eq}}}}$$

Qus



$k_1(d_1+x_1) = k_2(d_2+x_2)$

& $x_1+x_2 = x$

$k_2 d_2 = mg$ — (1)

$k_1 d_1 = k_2 d_2$ — (2)

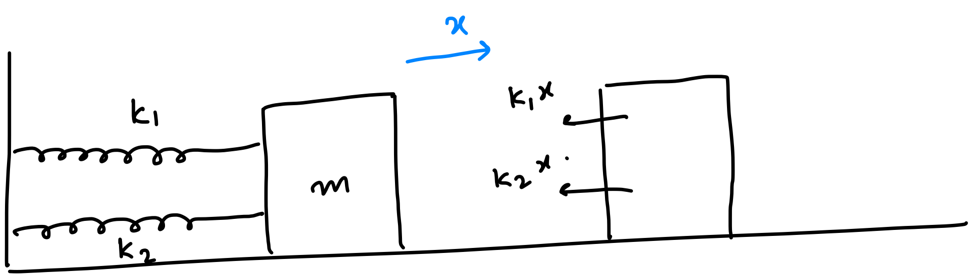
$ma = k_2(\cancel{d}+x_2) - \cancel{mg}$

$ma = k_2 x_2$

$ma = \dfrac{k_1 k_2 x}{k_1+k_2}$

$w = \sqrt{\dfrac{k_1 k_2}{k_1+k_2}}$

Ques

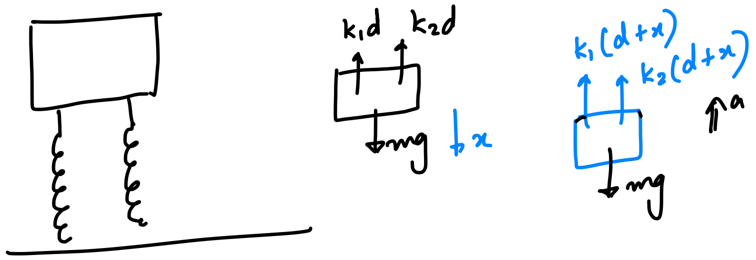


$k_1 x + k_2 x = ma$

$\therefore k_{eq} = k_1 + k_2$

$w = \sqrt{\dfrac{k_{eq}}{m}}$

Ques

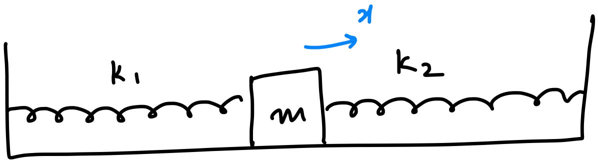


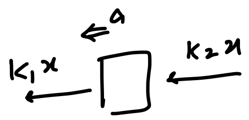
$ma = k_1(d+x) + k_2(d+x)$

$ma = (k_1+k_2)\,x$

$k_{eq} = k_1+k_2$

Ques

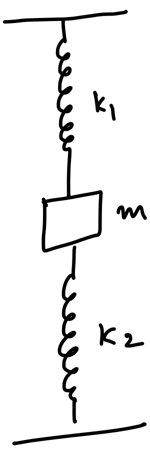




$ma = k_1 x + k_2 x$

$k_{eq} = k_1 + k_2$

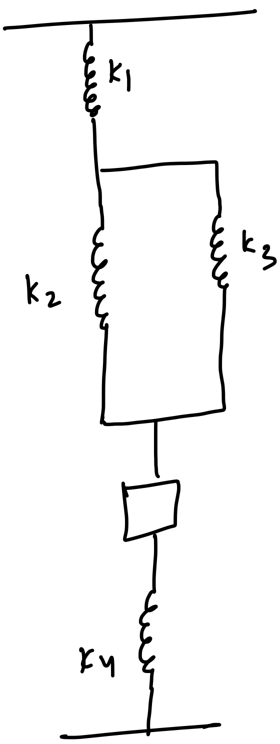
Ques



$T = 2\pi\sqrt{\dfrac{m}{k_1+k_2}}$

Ques

k_2 & k_3 parallel

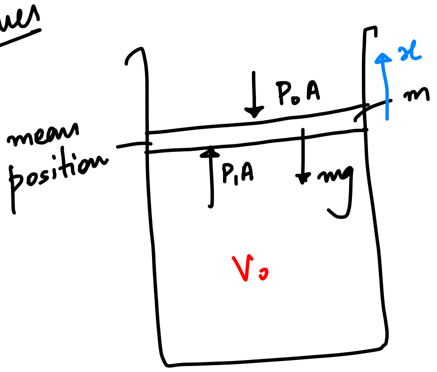


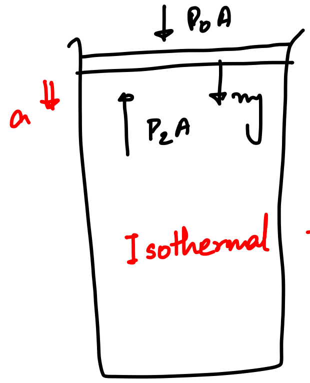
$k_{eq} = \dfrac{k_1\,(k_2+k_3)}{k_1+(k_2+k_3)} + k_4$

k_1 & (k_2+k_3) in series

$(k_1\ k_2\ k_3)$ & k_4 are parallel

Ques





$$P_0 A + mg - P_2 A = ma$$

$$P_1 A - P_2 A = ma$$

$$\longrightarrow \quad P_1 V_0 = P_2 (V_0 + Ax)$$

$$P_0 A + mg = P_1 A$$

$$P_1 A - \frac{P_1 V_0}{(V_0 + Ax)} A = ma$$

$$\frac{P_1 A [Ax]}{(V_0 + Ax)} = ma$$

Here not proportional to x

$\therefore$ Not an SHM

but, if we assume x to be very small

$$\frac{P_1 A^2 x}{V_0 m} = a$$

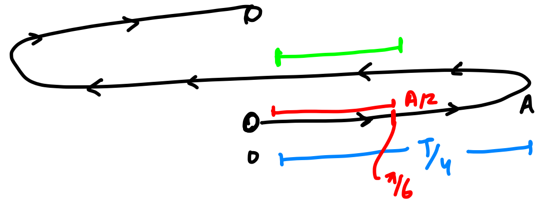
H.W Solve for Adiabatic

$$P_1 A \left[1 - \frac{1}{\left(1 + \frac{Ax}{V_0}\right)^{\gamma}} \right] = ma$$

$$\left[1 - \left[1 - \frac{\gamma A x}{V_0} \right] \right]$$

$$\frac{P_1 A^2 \gamma x}{V_0 m} = a$$

Ques

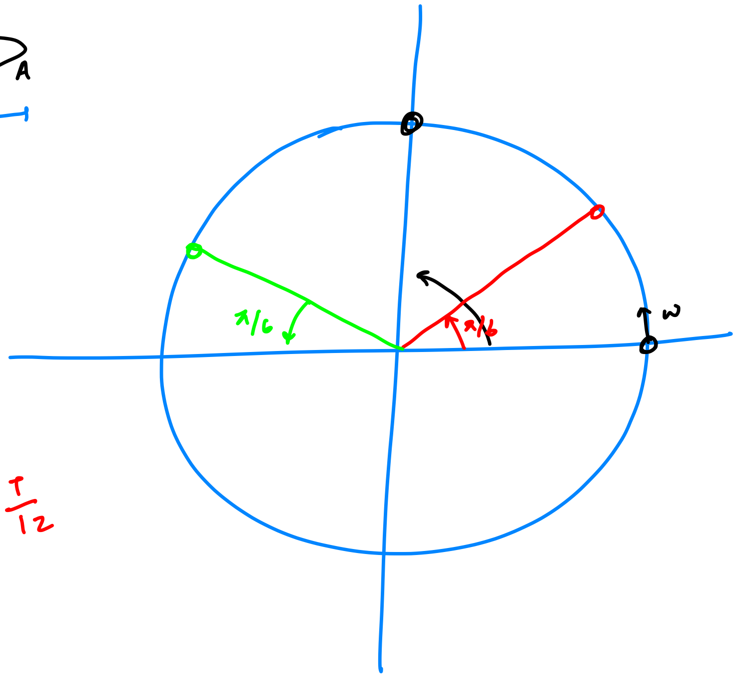


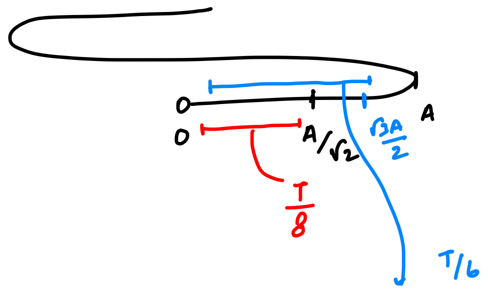
$\frac{\pi}{2} = \omega t$

$t = \frac{\pi}{2\omega} = T/4$

$\frac{\pi}{6} = \omega t$

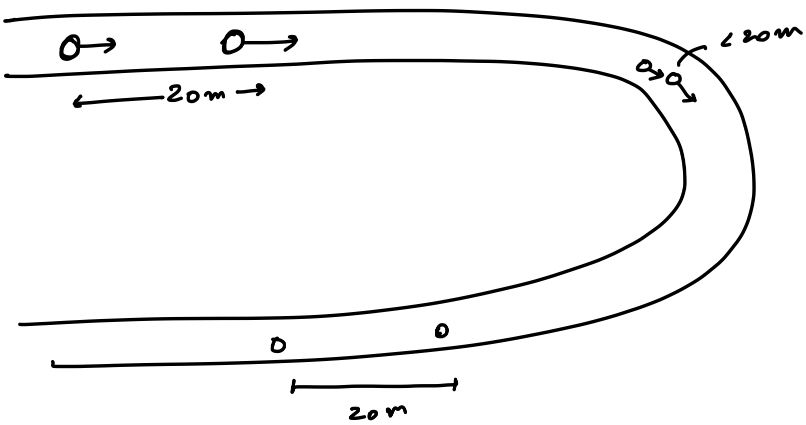
$t = \frac{\pi}{6\omega} = \frac{T}{12}$

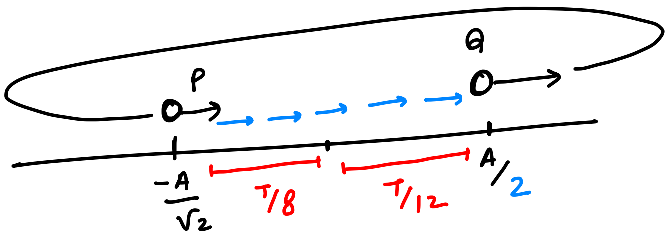




$\left(\because\ t = \frac{\pi}{8\omega}\right)$

Time Lag



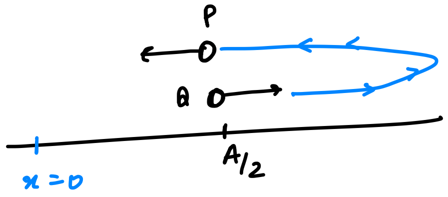


↳ Q is ahead of P by $\frac{T}{8} + \frac{T}{12}$

↳ P is ahead of Q by $T - \left(T/8 + T/12\right)$

OR $\left(\frac{T}{4} - \frac{T}{12}\right) + \frac{T}{2} + \left(\frac{T}{4} - T/8\right)$

Ques



P is ahead of Q by ?

$$= \left(\frac{T}{4} - \frac{T}{12}\right) \times 2$$

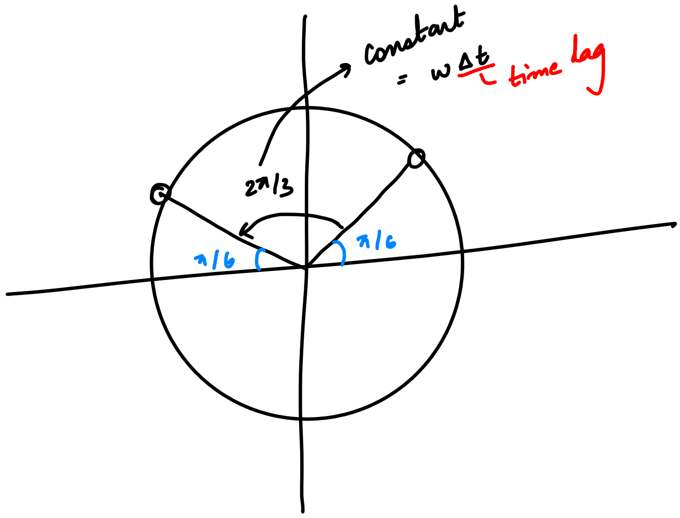
$$= \frac{2T}{12} \times 2 = T/3$$

We can also say phase difference is constant

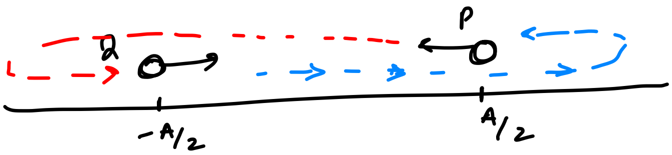
$$\Delta\theta = \omega \Delta t = \omega(T/3)$$

$$= \frac{2\pi}{T}(T/3)$$

$$= \boxed{\frac{2\pi}{3}}$$



Ques



P is ahead of Q by $= T/12 + T/4 + (T/4 - T/8)$

$$= \left(\frac{2+6+6-3}{24}\right)T$$

$$= \frac{11T}{24}$$

Q is ahead of P by $= (T/8 + T/4 + T/4 - T/12)$

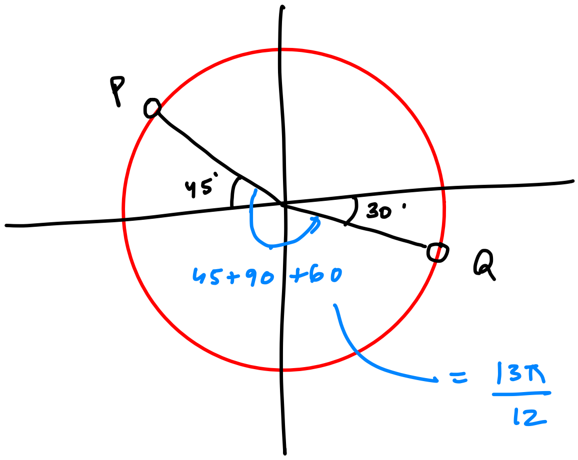
$$= 13T/24$$

phase difference

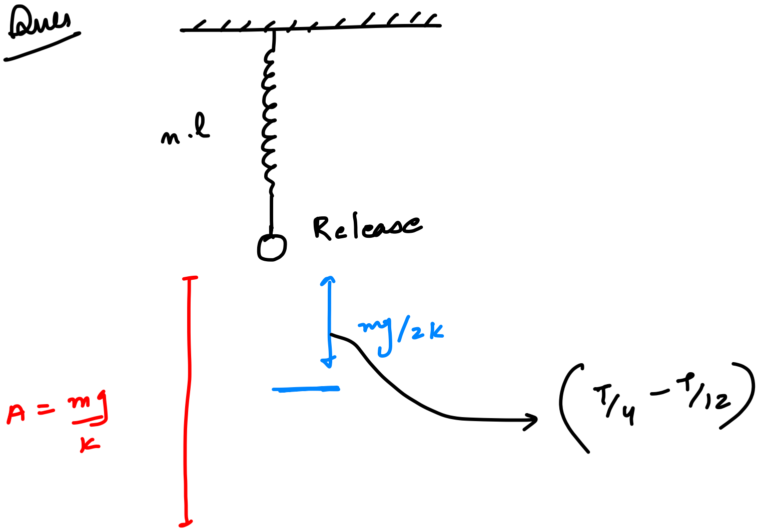
$$\Delta\theta = \omega\Delta t$$

$$= \frac{2\pi}{T}\left(\frac{13T}{24}\right)$$

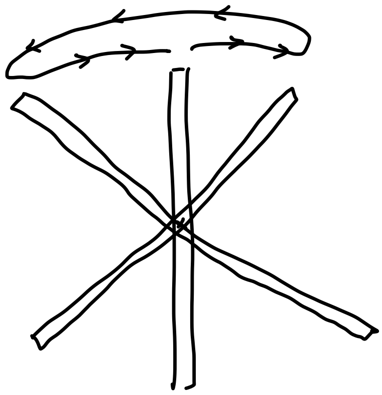
$$= \frac{13\pi}{12}$$



Ques



Ques



Angular SHM

1) m.p $\quad \tau = 0$

2) Rotate by θ from m.p

3) $(\tau_{\text{restoring}})_P = C\theta$

P → Hinge
P → C.M
P → JAOR

$$I_P \, \alpha = C\theta$$

$$\omega = \sqrt{\frac{C}{I_P}} \qquad T = 2\pi\sqrt{\frac{I_P}{C}}$$

$$\theta(t) = \theta_{max} \sin(\omega t + \phi_0)$$

$$\Omega(t) = \theta_{max}\,\omega \cos(\omega t + \phi_0)$$

$$\alpha(t) = -\omega^2 \theta_{max} \sin(\omega t + \phi_0)$$

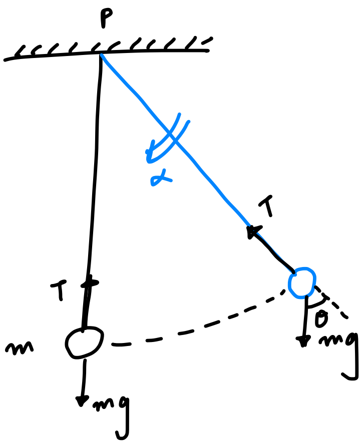
$$\Omega = \pm \omega \sqrt{\theta_{max}^2 - \theta^2}$$

$$KE = \frac{1}{2} C \left(\theta_{max}^2 - \theta^2\right)$$

$$PE = \frac{1}{2} C \theta^2 + U_0$$

$$TE = \frac{1}{2} C \theta_{max}^2 + U_0$$

Ques



$\tau_P = 0$

$\therefore$ P → mean position

$$(mg\sin\theta)l = I_P \alpha$$
$$= (ml^2)\alpha$$

$$\therefore \frac{g\sin\theta}{l} = \alpha \qquad \text{(this is not SHM)}$$

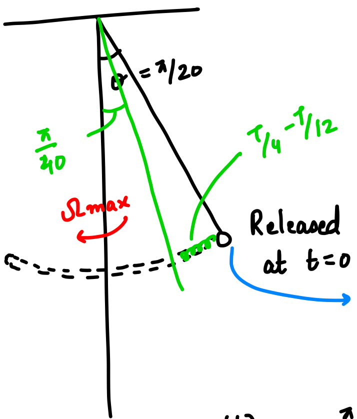
$$\sin\theta = \theta$$

$$\frac{g\theta}{l} = \alpha \rightarrow \omega^2$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$T = 2\pi\sqrt{l/g}$$

Ques



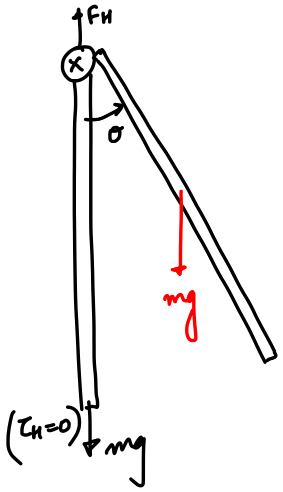
$$\phi_0 = \pi/2$$

$$T = 2\pi\sqrt{l/g}$$

$$\theta(t) = \frac{\pi}{20} \cos\left(\sqrt{\frac{g}{l}}\, t\right)$$

$$\Omega_{max} = \theta_{max}\,\omega$$
$$= \left(\frac{\pi}{20}\right)\sqrt{g/l}$$

Ques



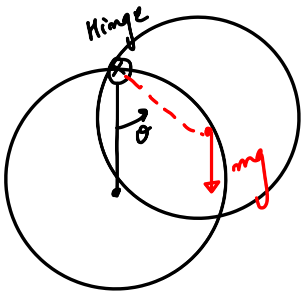
$$\tau_H = mg\,\frac{l}{2}\sin\theta = \left(\frac{ml^2}{3}\right)\alpha$$

$$\alpha = \frac{3g\sin\theta}{2l}$$

$$\alpha = \frac{3g\theta}{2L}$$

$$\omega = \sqrt{\frac{3g}{2l}}$$

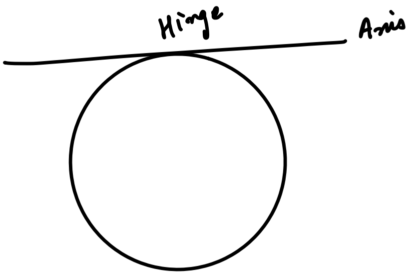
Ques

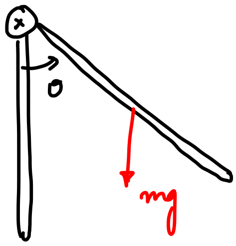


$$(mg\sin\theta)R = \left(\frac{3}{2}mR^2\right)\alpha$$

$$\omega = \sqrt{\frac{2g}{3R}}$$

Ques

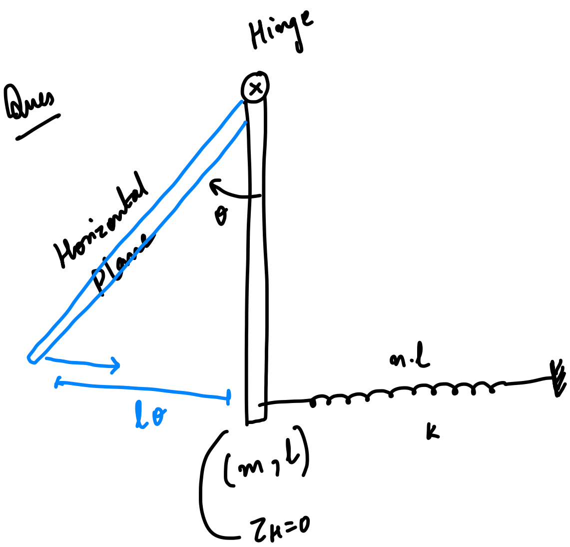




$$mg\sin\theta R = \left(\frac{5}{4}mR^2\right)\alpha$$

$$\omega = \sqrt{\frac{4g}{5R}}$$

Ques



$$(\tau_s)_{hinge} = (kl\theta)\,l = \left(\frac{ml^2}{3}\right)\alpha$$

$$\omega = \sqrt{\frac{3k}{m}}$$

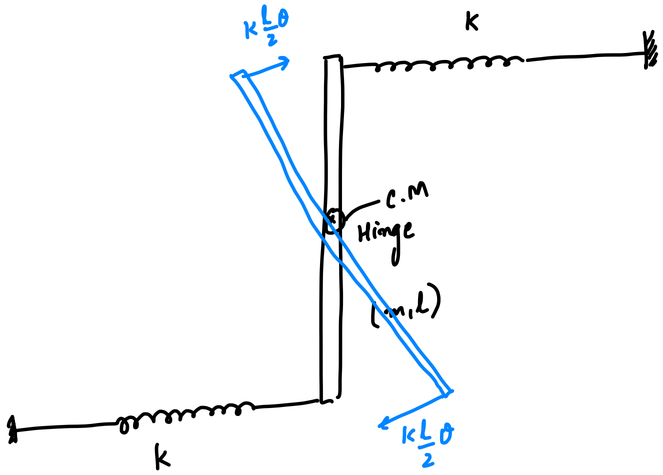
$$T = 2\pi\sqrt{\frac{m}{3k}}$$

ii) Not horizontal plane

$$\circlearrowright \left(mg\frac{l}{2}\sin\theta\right)R + (kl\theta)l = \left(\frac{ml^2}{3}\right)\alpha \circlearrowright$$

$$\omega = \sqrt{\frac{mg\,l/2 + kl^2}{\frac{ml^2}{3}}}$$

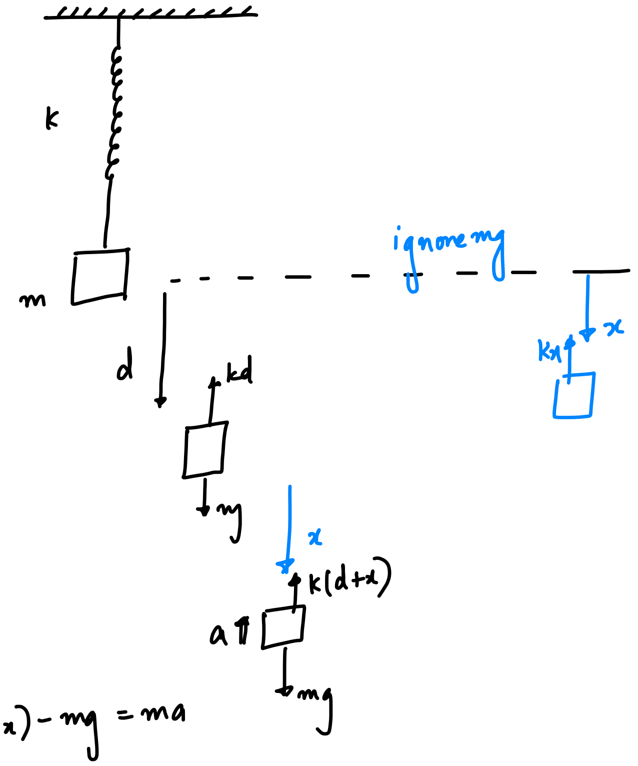
Ques



$$\tau_H = \left[\left(k\frac{l}{2}\theta\right)\frac{l}{2}\right]\times 2 = \left(\frac{ml^2}{12}\right)\alpha$$

$$T = 2\pi\sqrt{\frac{m}{6K}}$$

Ques



$$kx = ma$$

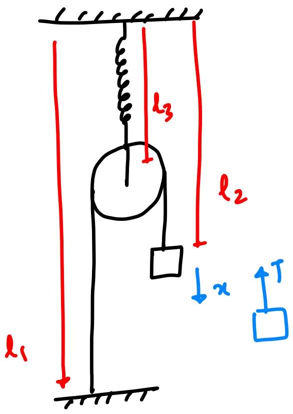
$$\omega = \sqrt{K/m}$$

$\because$ Constant force only changes mean position.

$$k(d+x) - mg = ma$$

$$kx = ma$$

Ques



Ignore mg & kd

$T \propto x$ (show karna h)

($T = ma$)

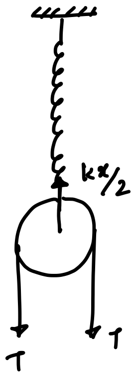
$l_1 + l_2 - 2l_3$ = length of string

$\cancel{\Delta l_1}^{\,0} + \Delta l_2 - 2\Delta l_3 = \Delta l\ \cancel{\text{string}}^{\,0}$ ($\because$ inextensible string)

$\Delta l_2 \rightarrow x$

$x - 2\Delta l_3 = 0$

$\left(\Delta l_3 = x/2\right)$ // Elongation in spring



$\frac{kx}{2} = 2T$

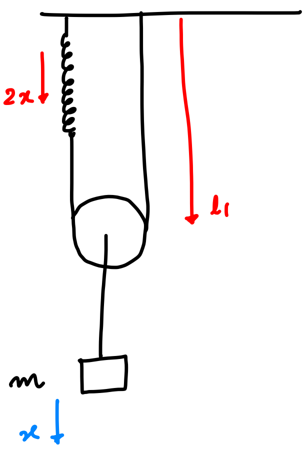
$T = \frac{kx}{4}$ $\longrightarrow$ $T = ma$

$\frac{kx}{4} = ma$

$\omega = \sqrt{\frac{K}{4m}}$

Ques

Find T_{SHM}

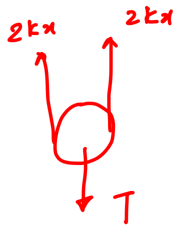


$2l_1$ = length of string + spring

$2\Delta l_1 = \cancel{\Delta l\,\text{string}}^{\,0} + \Delta l\,\text{spring}$

$2\Delta l_1 = \Delta l\,\text{spring}$ ($\because$ here $\Delta l_1 = x$)

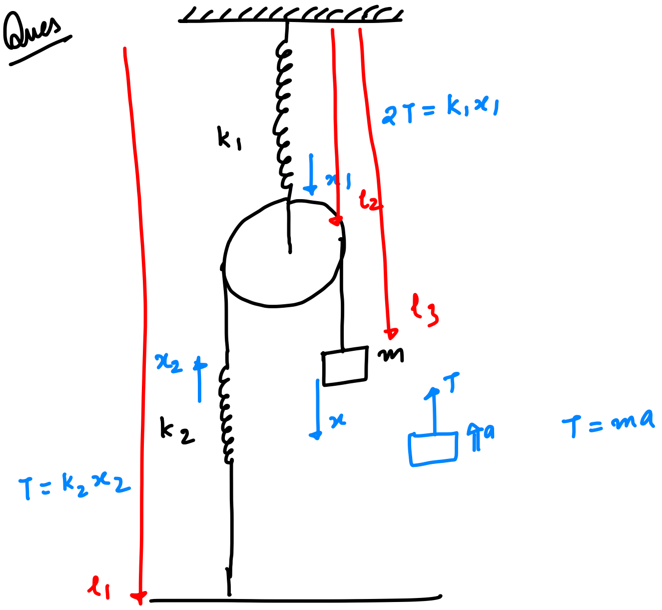
$2x$ = ''



$4kx = ma$

$\omega = \sqrt{\frac{4k}{m}}$

Ques



$T = ma$

$$l_1 + l_3 - 2l_2 = l_{string} + l_{spring2}$$

$$\cancel{\Delta l_1}^{\,0} + \Delta l_3 - 2\Delta l_2 = \Delta l_{spring2}$$

$$x - 2\Delta l_2 = \Delta l_{spring2}$$

$$x_1$$

$$x - 2x_1 = x_2$$

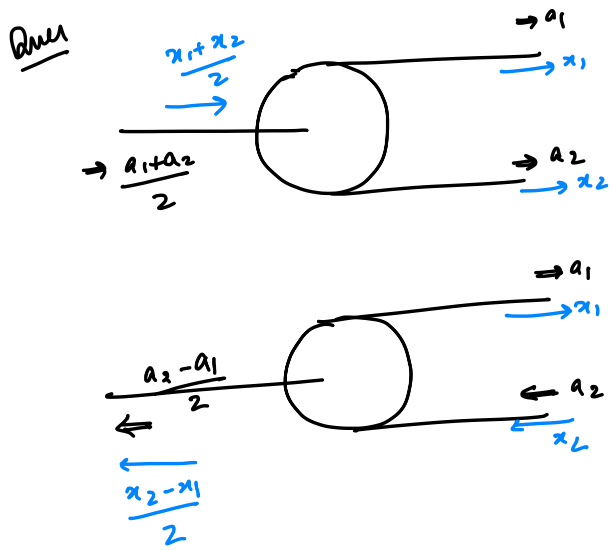
$$x = 2x_1 + x_2$$

$$x = 2\left(\frac{2T}{k_1}\right) + \frac{T}{k_2}$$

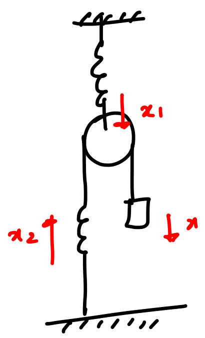
$$\left(\frac{k_1k_2}{4k_2+k_1}\right)\frac{x}{m} = a$$

$$\omega = \sqrt{\frac{k_1k_2}{4k_2+k_1}}$$

Ques



Power Qs

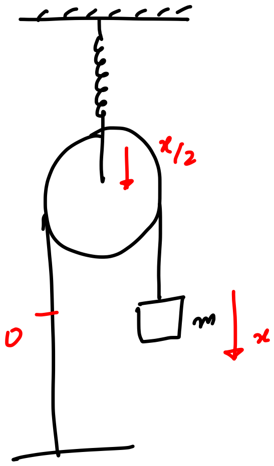


$$\frac{x-x_2}{2} = x_1$$

$$x = 2x_1 + x_2$$

$$x = \frac{4T}{k_1} + \frac{T}{k_2}$$

Qn

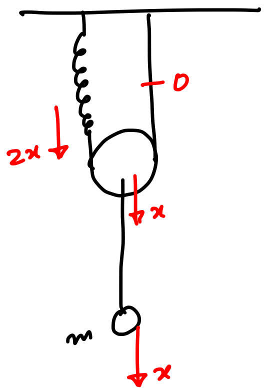


$$\therefore\ 2T = \frac{kx}{2}$$

$$2ma = \frac{kx}{2}$$

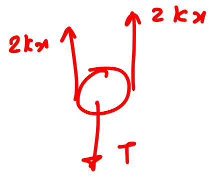
$$a = \frac{kx}{4m}$$ ✓

Qn

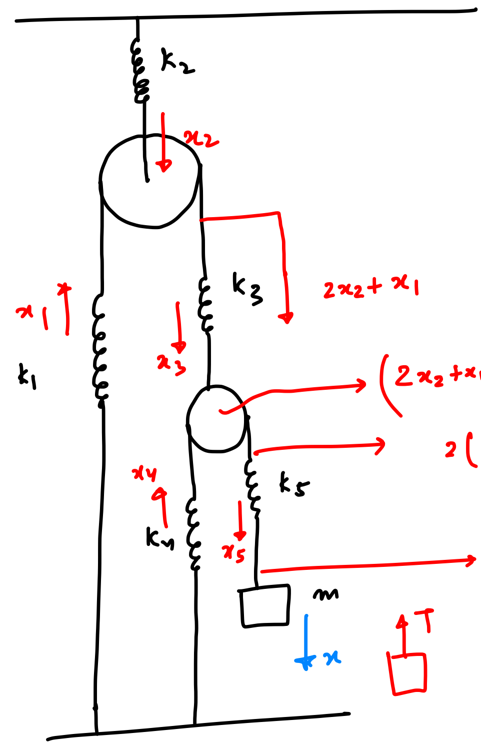


$$2kx + 2kx = T$$
$$= ma$$

$$a = \frac{4kx}{m}$$



Qn



$$(2x_2+x_1+x_3)$$

$$2(2x_2+x_1+x_3)+x_4$$

$$2(2x_1+x_1+x_3)+x_4+x_5 = x$$

$$T = k_5x_5 = k_4x_4$$

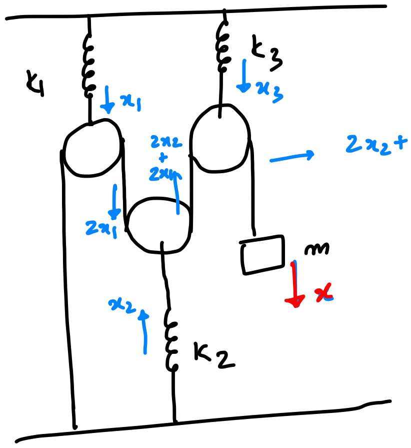
$$2T = k_3x_3 = k_1x_1$$

$$4T = k_2x_2$$

$$T = ma$$

$$2\left(2\left(\frac{4T}{k_2}\right)+\frac{2T}{k_1}+\frac{2T}{k_3}\right)+\frac{T}{k_4}+\frac{T}{k_5} = x$$

Que



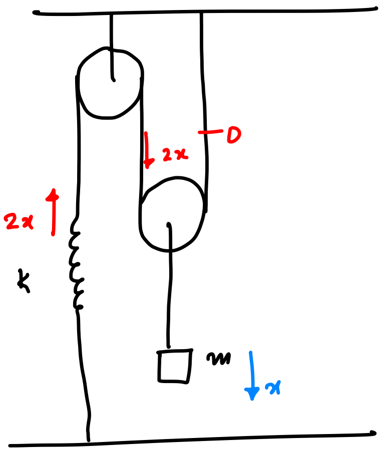
$2x_2 + 2x_1 + 2x_3 = x$

$2T = k_3 x_3 = k_2 x_2 = k_1 x_1$

$T = ma$

Note : mg और ~~देना~~ ~~लेना~~
Ignore initial elongation

H.W



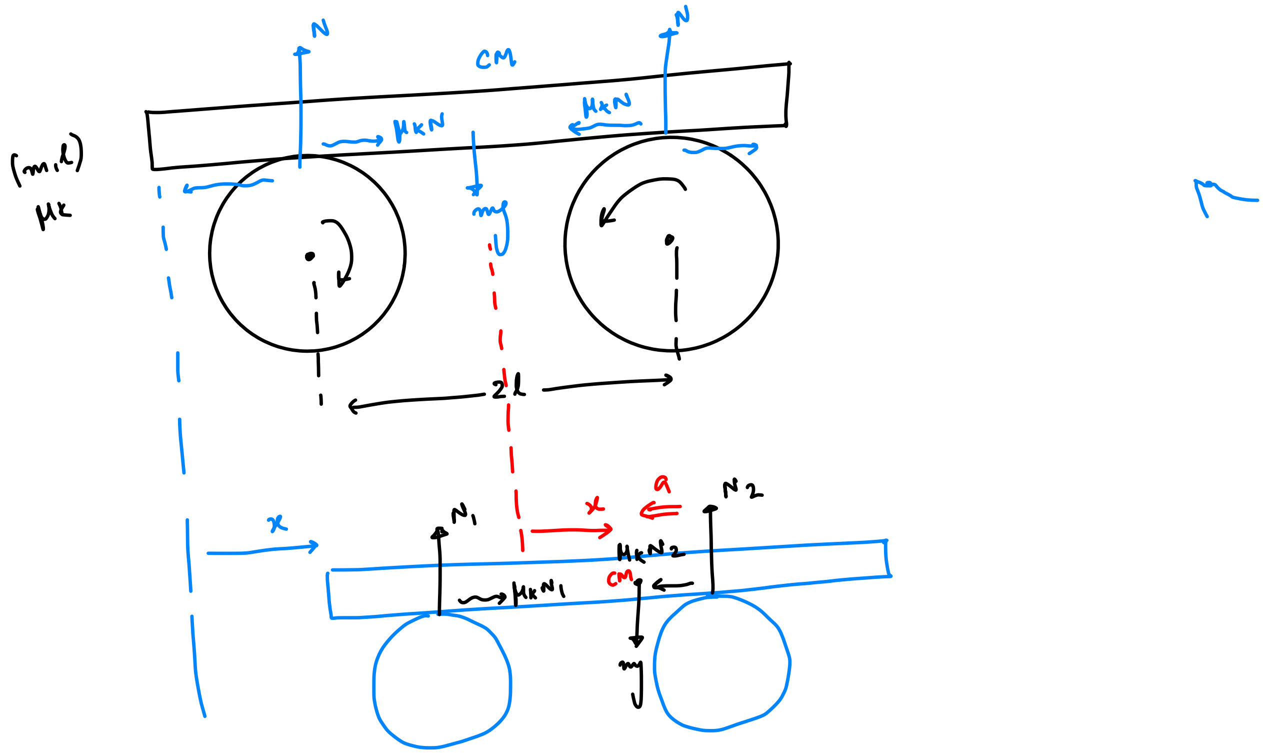
$2T = ma$

and $T = 2kx$

$4kx = ma$

$a = \frac{4kx}{m}$

Ques
Irodov



$F_r = \mu_k N_2 - \mu_k N_1 = ma$

$N_2 + N_1 = mg$

$\because (\tau_{cm} = 0) \therefore N_1(l+x) = N_2(l-x)$

$$\frac{N_1}{N_2} = \frac{l-x}{l+x}$$

Componendo dividendo

applying C & D

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

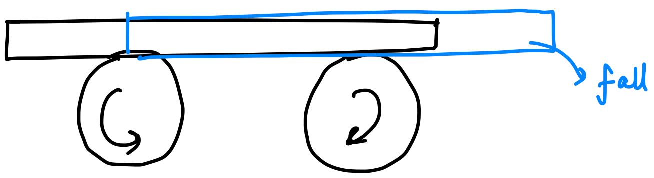
$$\frac{N_2 - N_1}{N_2 + N_1} = \frac{2x}{2l} \Rightarrow N_2 - N_1 = mg\left(\frac{x}{l}\right)$$

$$F_r = \left(mg\frac{x}{l}\right)\mu_k = ma$$

$$\omega = \sqrt{\frac{\mu_k g}{l}}$$

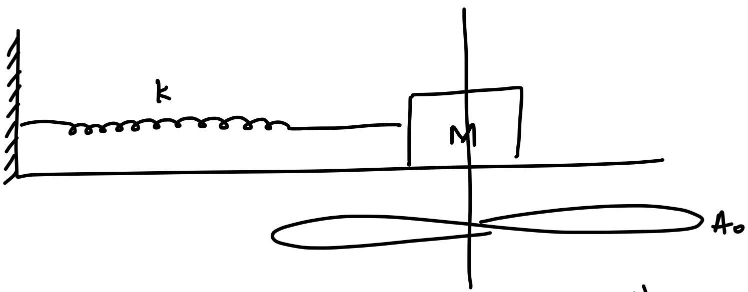
Note : $x < l$

Ques



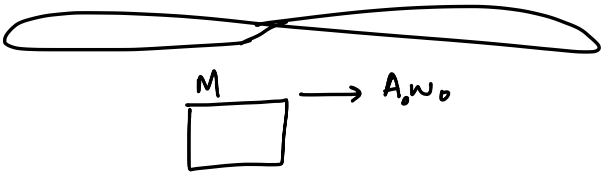
// Unstable Equilibrium.

Ques

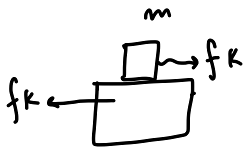


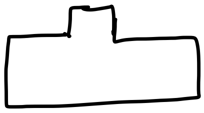
when it passes mean position. m is placed on it.
Assume friction is so large that friction ceases almost instantly. → friction ko impulsive मानने को कह रहा है

Ques



$\longrightarrow A_0\omega_0$ where $\omega_0 = \sqrt{K/M}$





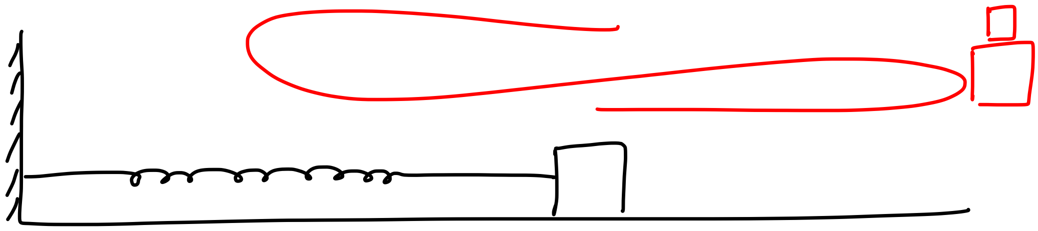
$\longrightarrow v_1 = A_1\omega_1 = A_1\sqrt{\dfrac{k_{sp}}{m+M}}$

Apply COLM ($\because$ I_f internal, $I_{spring} \approx 0$)

$$M A_0 \omega_0 = (M+m) A_f \omega_f$$

$$A_f = A_0\sqrt{\frac{M}{M+m}}$$

$$\text{Heat} = \frac{1}{2}kA_0^2 - \frac{1}{2}kA_0^2\left(\frac{M}{m+M}\right)$$



Agar extreme posn par rakhdei

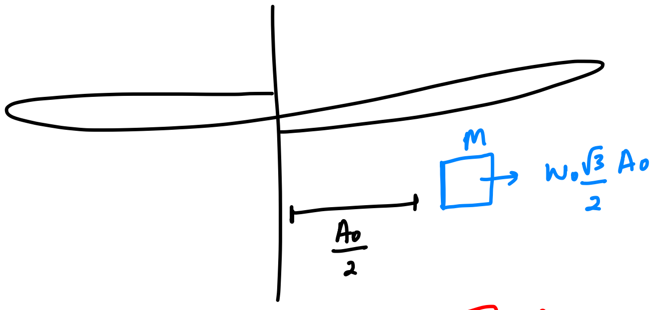
$\because v = 0$

A – same

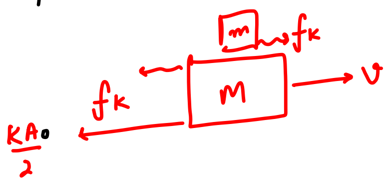
Heat – 0

No slipping

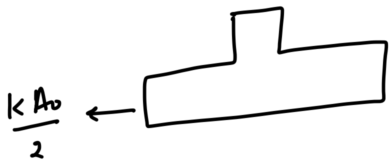
Que



$\left(\because v = \pm w\sqrt{A^2 - A^2/4}\right)$



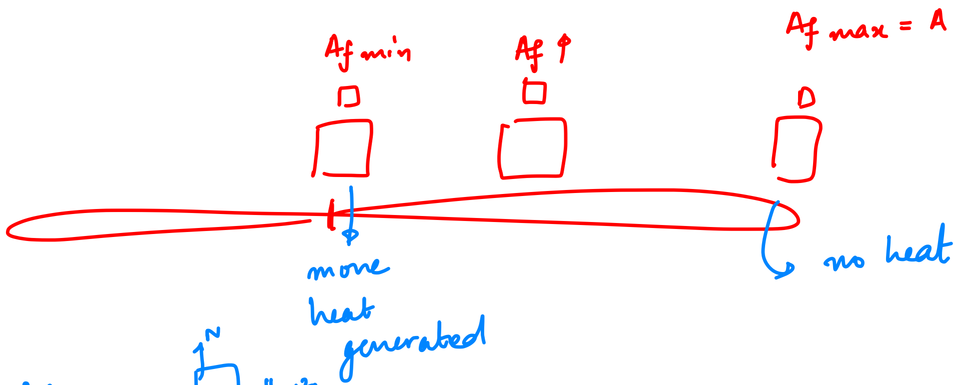
Note : Impulse due to $\frac{KA_0}{2}$ is negligible as $\Delta t \to 0$



COLM

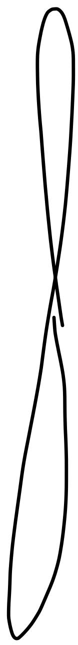
$$M w_0 \frac{\sqrt{3}}{2} A_0 = (M+m) v_f$$
$$= (M+m)\, w_f \sqrt{A_f^2 - \left(\frac{A_0}{2}\right)^2}$$

Note

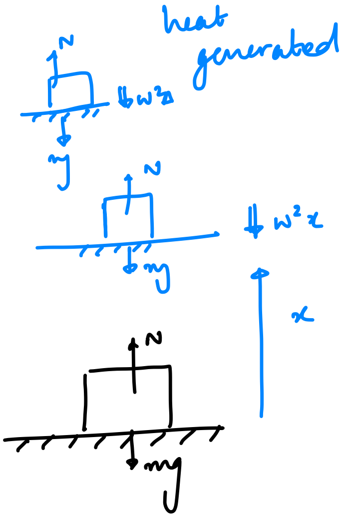


Ques

$mg - N = m w^2 A$



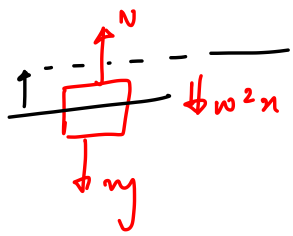
w, A



$mg - N = m w^2 x$

Note : # Normal is min at highest problem

N = 0 , when

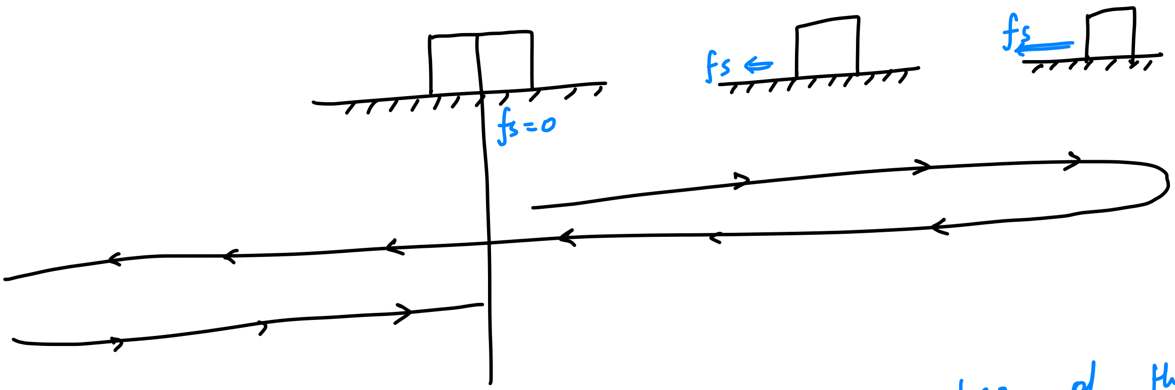


$\omega^2 x > g$

$\omega^2 x = g$

at this moment the platform will do SHM and the particle will free fall.

Que



Note, here dabba is doing SHM bcz of the friction.

at m.p $a = 0$ $\therefore f_s = 0$

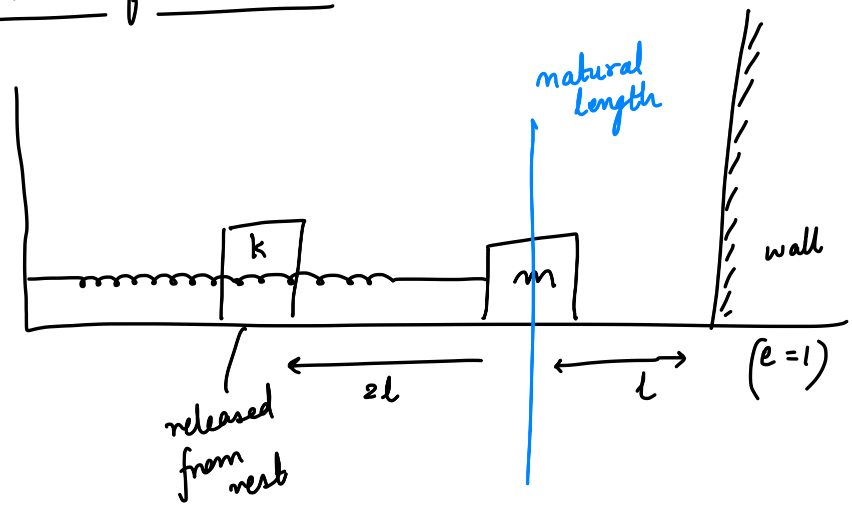
extreme posn a max $\therefore f_s$ max

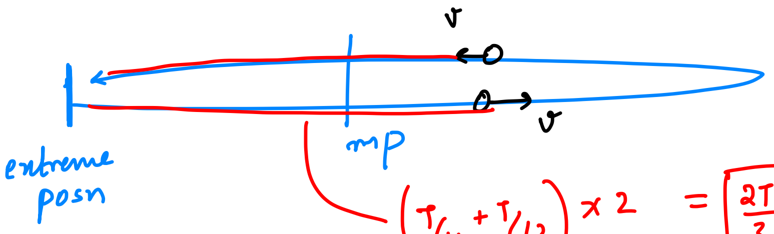
$\therefore$ Frictional force is providing the restoring force.

Max tendency of slipping is at extreme position.

friction is always towards the mean position

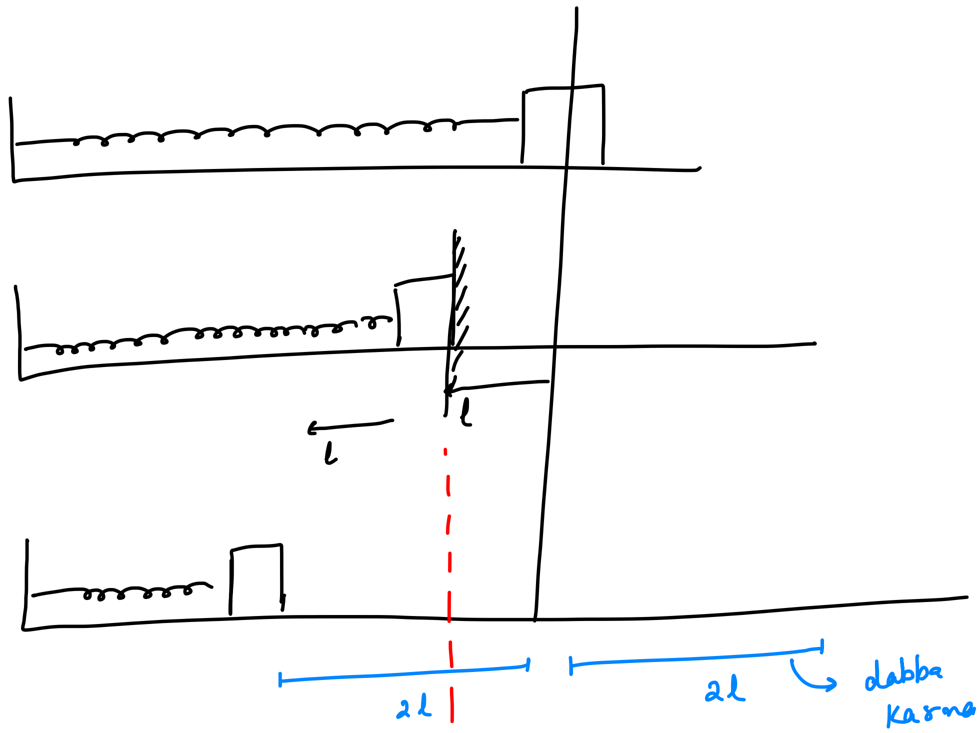
Part of SHM





$\left(\frac{T}{4} + \frac{T}{12}\right) \times 2 = \boxed{\frac{2T}{3}}$ effective time

Ques

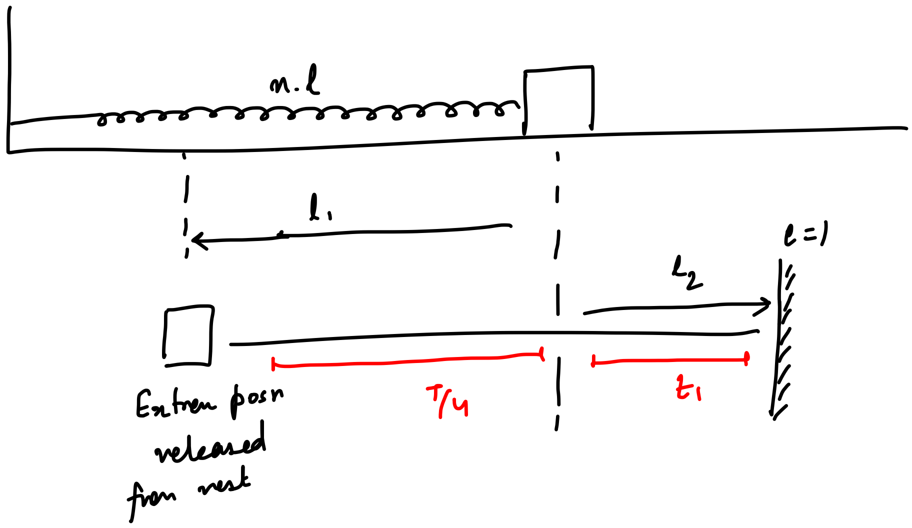


dabba aisa SHM karna chahta h.

$T_{effective} = \left(\frac{T}{4} - \frac{T}{12}\right) \times 2$

Note : Time period is independent of Amplitude.

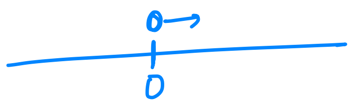
Ques



$l_2 < l_1$

$A = l_1$

$x(t) = A\sin\omega t$

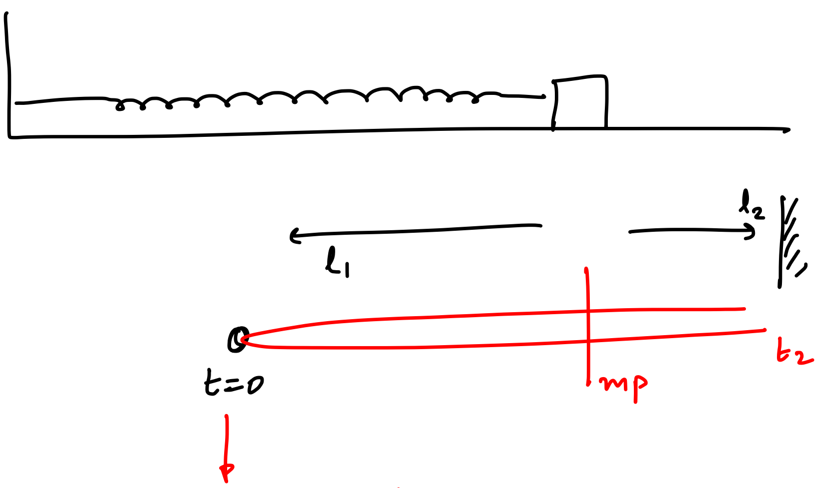


$l_2 = x(t) = A\sin\omega t_1$

$T_{eff} = \left(T/4 + t_1\right)\times 2$

$= \frac{T}{2} + 2t_1 = \left(\frac{2\pi}{\omega}\right)\frac{1}{2} + 2\,\frac{1}{\omega}\sin^{-1}\left(\frac{l_2}{l_1}\right)$

#



$x(t) = -A\cos\omega t$

$x(t) = -l_1\cos\omega t$

$l_2 = x(t) = -l_1\cos\omega t_2$

$t_{eff} = 2t_2 = 2\,\frac{1}{\omega}\cos^{-1}\left(\frac{-l_2}{l_1}\right)$

$$\frac{2}{w}\left[\frac{\pi}{2}+\sin^{-1}\theta\right]$$

Same.

$$\frac{2}{w}\cos^{-1}(-\theta) = \frac{2}{w}\left[\pi - \cos^{-1}\theta\right]$$

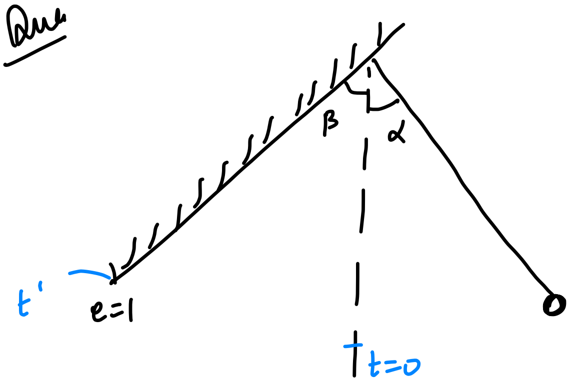
$$= \left\{\pi - \left[\pi/2 - \sin^{-1}\theta\right]\right\} 2/w$$

$$= \left\{\frac{\pi}{2} + \sin^{-1}\theta\right\} 2/w$$

Note :

$$\boxed{\begin{aligned}\cos^{-1}(-x) &= \pi - \cos^{-1}x \\ \sin^{-1}x + \cos^{-1}x &= \pi/2\end{aligned}}$$

Qu

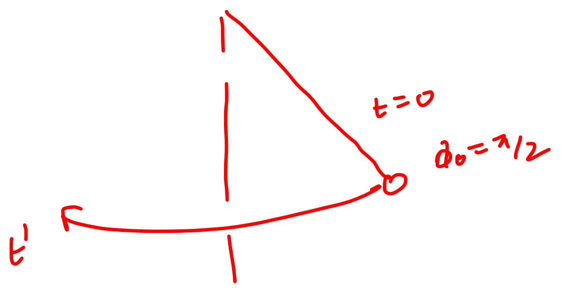


$\beta = \alpha \sin wt'$

$$\longrightarrow \quad T_{eff} = 2\times\left[\frac{T}{4} + \frac{1}{w}\sin^{-1}\left(\frac{\beta}{\alpha}\right)\right]$$

M2

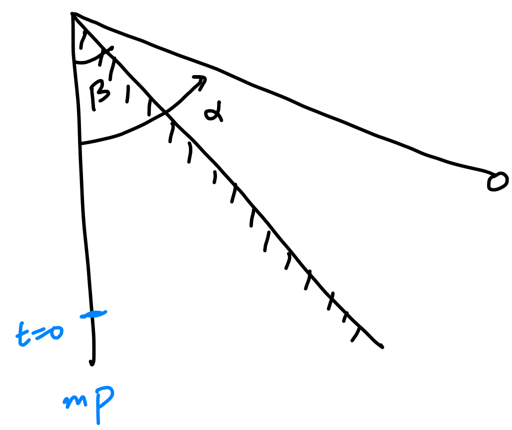
$$\theta(t) = \alpha\cos wt$$



$$-\beta = \theta(t) = \alpha\cos wt'$$

$$T_{eff} = \frac{2}{w}\cos^{-1}\left(\frac{-\beta}{\alpha}\right)$$

Qu



M1

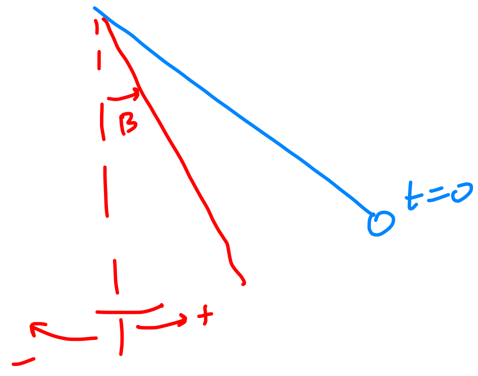
$$2\left(T/4 - t_1\right)$$

$$\downarrow$$

$$\frac{1}{w}\sin^{-1}\left(\frac{\beta}{\alpha}\right)$$

$\because \beta = \alpha \sin wt_1$

M2



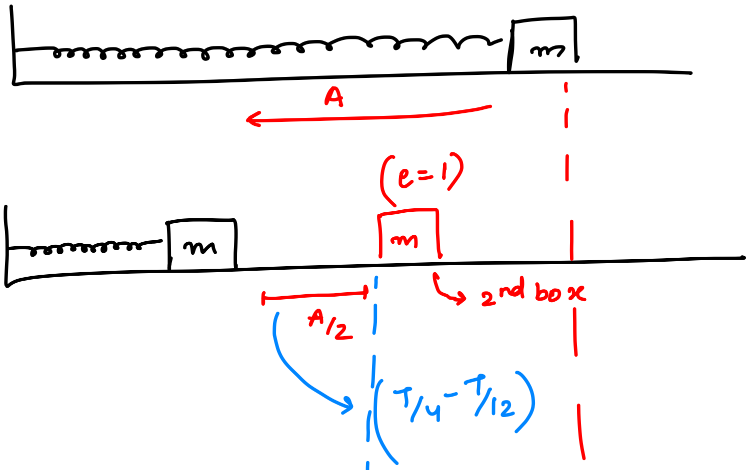
$\theta(t) = \alpha \cos \omega t$

$\downarrow$

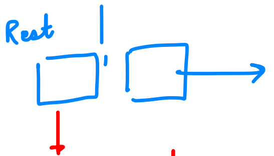
$\beta = \alpha \cos \omega t'$

$t_{eff} = \frac{2}{\omega} \cos^{-1}\left(\frac{\beta}{\alpha}\right)$

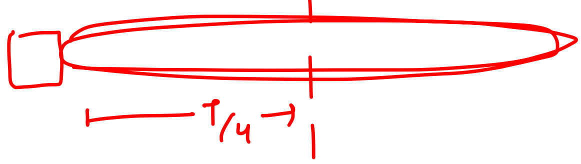
Ques



masses same
e = 1
∴ velocity interchange

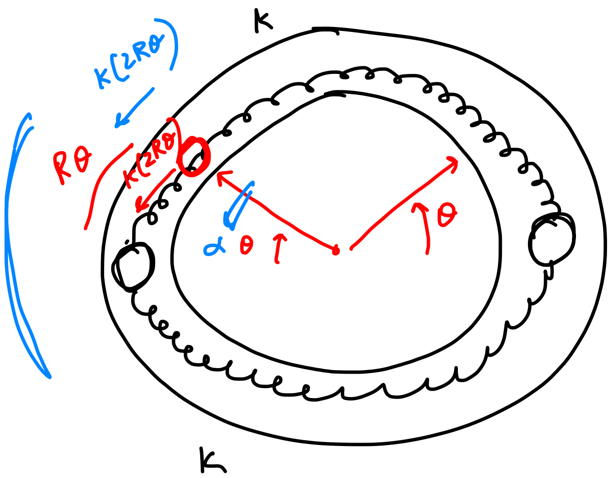


SHM reset
↓
new extreme posn



Time to come at mean $= \left(T/4 - T/12\right) + T/4$

Que



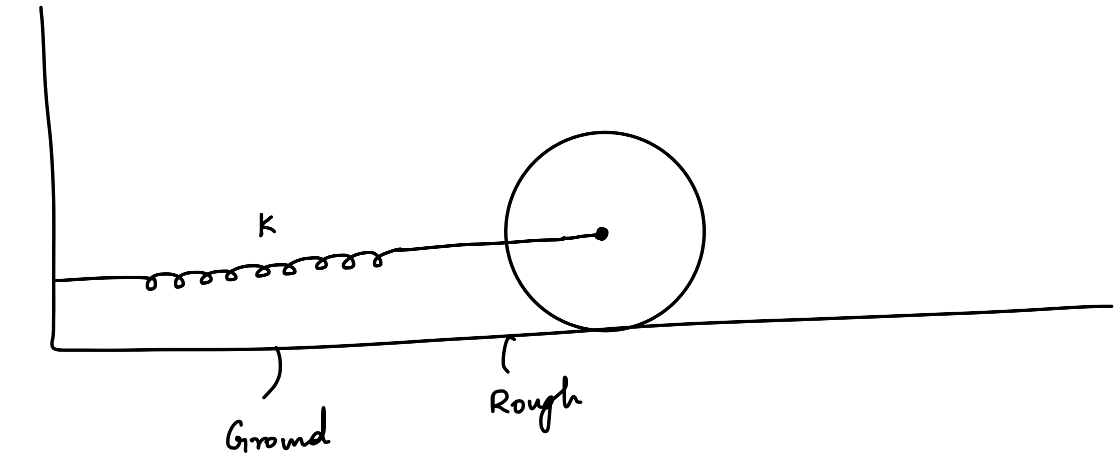
$\circlearrowleft \tau = (4KR\theta)R = (mR^2)\alpha$

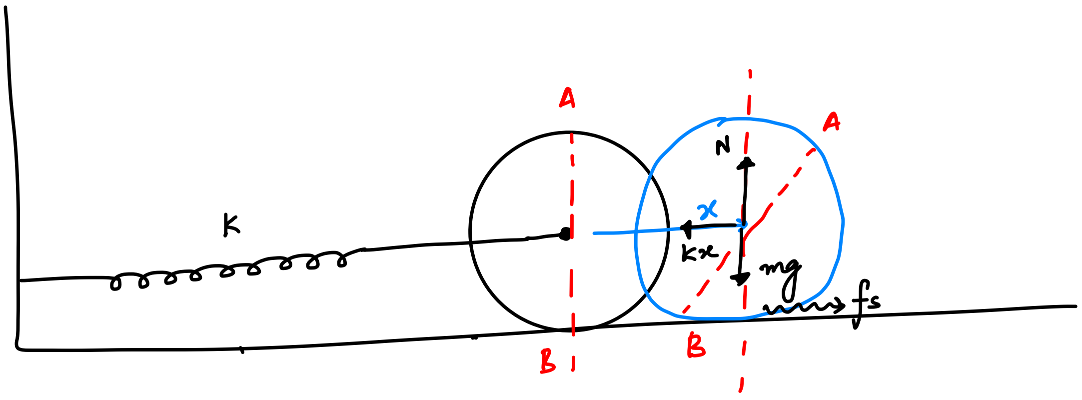
$\left(\frac{4K}{m}\right)\theta = \alpha$

$\therefore \omega = \sqrt{\frac{4K}{m}}$

Ques

Find T_{SHM} ?





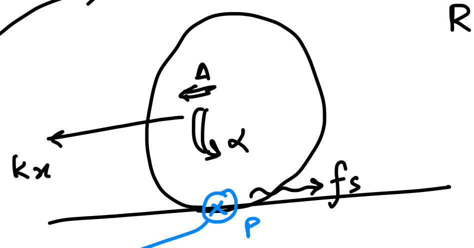
CM → linear SHM

& CM par koi beth jae → angular SHM

Both will have same T

$$\begin{bmatrix} kx - f_s = mA \\ f_s R = I_{cm}\alpha \\ A - R\alpha = 0 \end{bmatrix}$$

$$kx = mA + \frac{\overbrace{I_{cm}}^{mk^2}\overbrace{\alpha}^{A/R}}{R}$$



$$\left(\frac{kx}{m + \frac{mk^2}{R^2}}\right) = A$$

w^2

Also, note

this is acting as

IAOR ∵ no slip condition

$\therefore \tau_P = I_P \alpha$

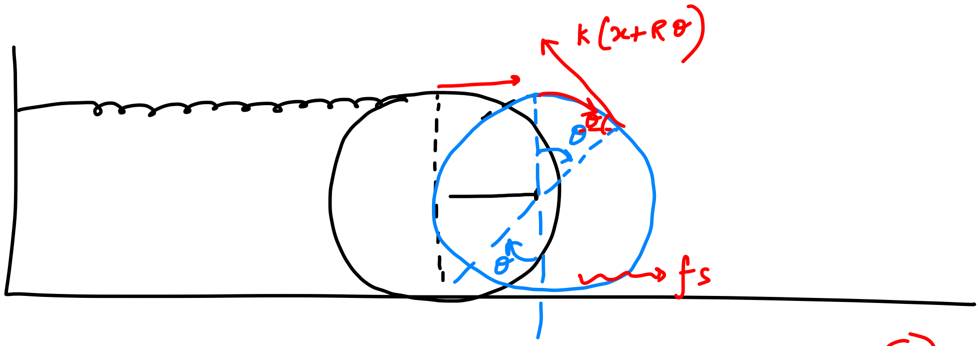
$kxR = (I_{cm} + mR^2)\alpha$

$x \to R\theta$

$kR^2\theta = (I_{cm} + mR^2)\alpha$

$$\Rightarrow \boxed{\alpha = \left(\frac{kR^2}{I_{cm} + mR^2}\right)\theta}$$

Ques



$$k(x+R\theta)\cos\theta - f_s = mA \quad -(1)$$

$$\tau = k(x+R\theta)R + f_s R = I_{cm}\alpha \quad -(2)$$

$$A - R\alpha = 0 \quad -(3)$$

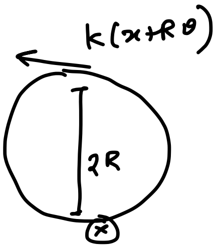
$$k(\underbrace{x+R\theta}_{2R\theta})\underbrace{\cos\theta}_{1} + k(x+R\theta) = mA + \frac{I_{cm}A}{R^2}$$

$$4kR\theta = mA + \frac{I_{cm}A}{R^2}$$

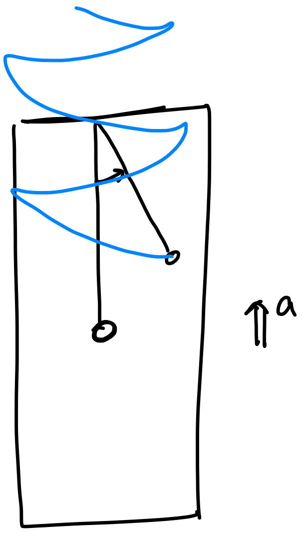
M2 about IAOR

$$\tau = F.R = (I_{cm} + mR^2)\alpha$$

$$k(x+R\theta)\,2R$$

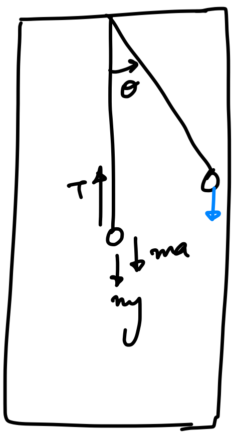


Ques



not an SHM
wrt ground.

∴ First time Psuedo
Let the lift be at rest



$$\tau = I\alpha$$

$$m(g+a)\,l\sin\theta = ml^2\alpha$$

$$\frac{(g+a)\theta}{l} = \alpha$$

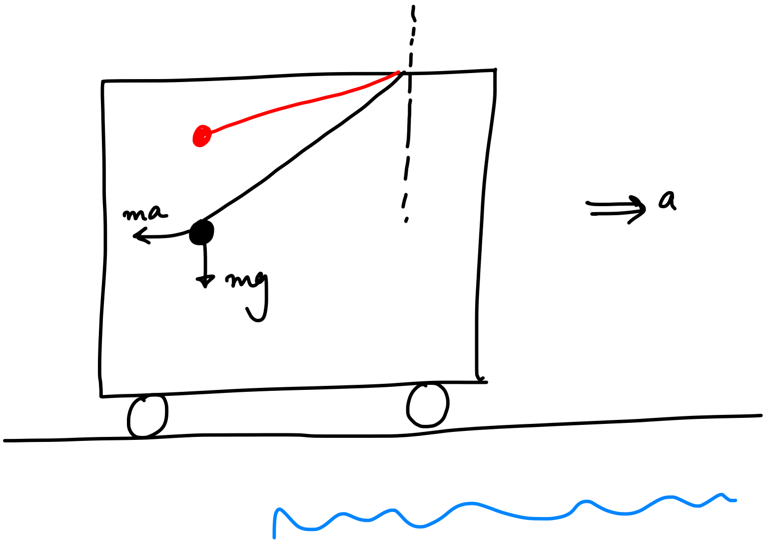
$$\omega = \sqrt{\frac{g+a}{l}}$$

$$T = 2\pi\sqrt{\frac{l}{g+a}}$$

Note : In freefall

$$T = \infty$$

Ques



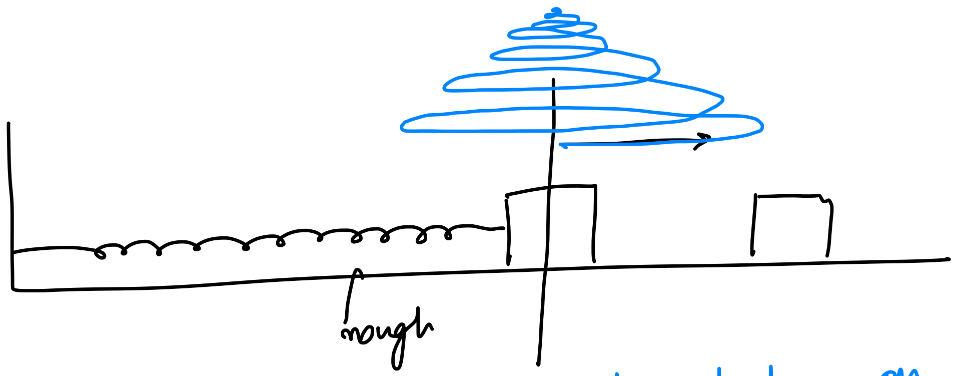
(not SHM wrt ground)

30 mins

Damped SHM

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$x(t) = A\sin(\omega t + \phi_0) \qquad \omega = \sqrt{k/m}$$



Note : Now A is dependent on time
A ↓ with time.

$$\boxed{ma = -kx - bv}$$

b → some proportionality constant

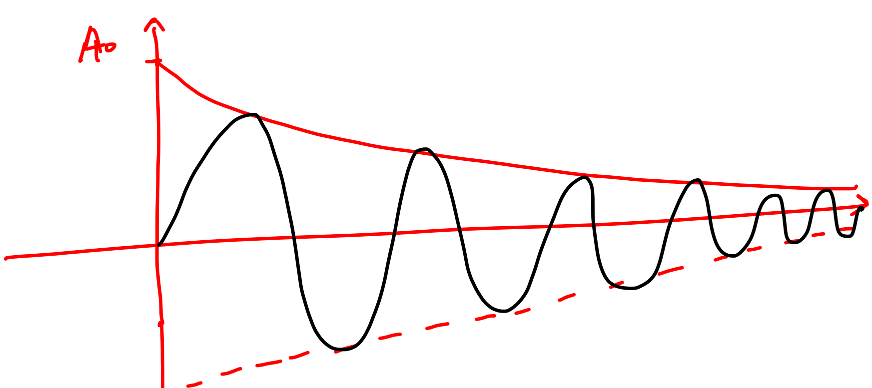
$$m\frac{d^2x}{dt^2} = -kx - \frac{b\,dx}{dt}$$

$$x(t) = A(t)\sin(\omega t + \phi_0)$$

$$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} \simeq \omega_0$$

ω_0^2 → k/m

$$\boxed{A(t) = A_0 e^{-bt/2m}}$$



Qu $\frac{A_0}{3}$ kab tak ?

$$\frac{A_0}{3} = A_0 e^{-bt/2m}$$

$$\frac{A_0}{3} = A_0 e^{-\lambda t} \quad \text{where} \quad \boxed{t = \frac{\ln 3}{\lambda}} \rightarrow b/2m$$

Total Energy $= \frac{1}{2} k A^2 + U_0$

$$= \frac{1}{2} k \left(A_0 e^{-bt/2m} \right)^2 + U_0$$

$A = \frac{1}{2} A_0$ $t = 4$ sec $\frac{A_0}{2} = A_0 e^{-\lambda t}$ ~ get λ

$A_0/3$ $t = ?$

$ma = -kx - \underbrace{6\pi\eta r}_{b} v$

$\boxed{\text{Simple Pendulum}}$

mass was given

$\frac{A_0}{2}$ in this much time

find coefficient of viscosity.

$$\frac{A_0}{2} = A_0 e^{-\frac{b}{2m} t}$$

$$\frac{A_0}{2} = A_0 e^{-\frac{6\pi\eta r}{2m} t}$$