HW 7

Anshul Yadav

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Abstract

This report is written in response to HW 7. A thick walled pressure vessel is supported with an internal pressure. The vessel is simulated in ANSYS workbench. Only a quarter of the pressure vessel is modelled as the problem is axisymmetric. A preliminary theoretical analysis is performed using hand calculations and then the ansys results are summarised in this report.

Method

The Problem

<u>Your report for each problem should include:</u>

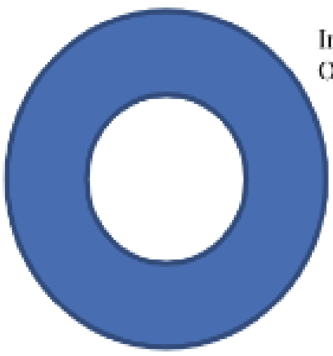
- Problem statement: your interpretation of the question with all inputs
- Mesh information: comment on element type, mesh error metric, picture of mesh
- Boundary Conditions: comment on symmetry, fixed displacements / rotations
- Loading: external forces, moments, pressures…. Include figure showing loading
- Solution: include requested stress information with figuring showing stress location
- Discussion: Comment on results. Do they make sense?
- Appendix: Hand calculations / Sanity checks

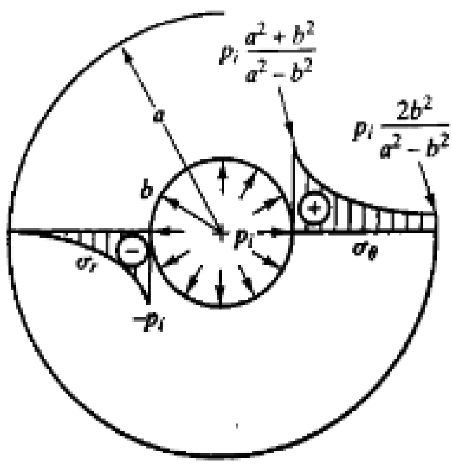
A thick walled pipe is subjected to an internal pressure such that yielding occurs throughout the entire cross sectional area. Assume that this cross section of the pipe is in plane stress, so that we can model it using 2D analysis.

The pipe material is assumed to behave as bilinear-kinematic hardening with a yield stress of 150MPa. The initial modulus is 200GPa and the tangent modulus after yielding is 2GPa. Assume a Poisson's ratio of 0.3. Include a figure of the ANSYS stress-strain curve in your report.

<u>Answer the following questions and provide stress figures</u>

- At what pressure will yielding first occur, and where does yielding first occur?
- What fraction of the cylinder area yields when the pressure increases to 1.2 times the yield pressure?
- At what pressure does the entire area yield?
- What happens if we remove the pressure after the entire area has just yielded? This should leave the pipe in a state of residual stress. Make sure to indicate compression and/or tension on your residual stress plots.





The figure on the right shows radial and hoop stresses for an internally pressurized thick walled pipe.

Theoretical Results:

Equations 1 and 2 summarise the hoop stress and radial stress due to internal pressure in a thick walled pressure vessel. Pi is the internal pressure, a is the inner radius, b is the outer radius, r is the radial distance from the centre,r is between a and. a = 0.2m, b = 0.3m. The equations were obtained from EMA 506.

$$\sigma_{rr} = \frac{P_i a^2}{b^2-a^2}(1 - b^2/r^2) = 0.8P_i - 0.072P_i/r^2$$

(eq1)

$$\sigma_{\theta\theta} = \frac{P_i a^2}{b^2-a^2}(1 + b^2/r^2) = 0.8P_i + 0.072P_i/r^2$$

(eq2)

Looking at eq1 and eq2, one may observe that the hoop stress is always going to be larger than the radial stress. Since the shear stress$\sigma_{r\theta}$is 0 throughout the body, the cylinder is already in principle orientation with the hoop stress being the first principle stress.

Part A:

Using the von mises criterion, yielding occurs when the equivalent stress exceeds yield stress. Since the cylinder in assumed to be in plane stress σ_{zz}(2nd or 3rd principal stress) is 0, $\sigma_{\theta\theta}$ is the first principle stress and $\sigma_{rr}$is the remaining principal stress.

$\sigma_{e,a} \geq \sigma_Y \Rightarrow (\sigma_{\theta\theta} - \sigma_{rr})^2 + \sigma_{\theta\theta}{}^2 + \sigma_{rr}{}^2 \geq 2\sigma_Y{}^2 \Rightarrow P_i = 46.6027MPa$ (eq3)

Thus yielding occurs at an internal pressure of 46.603 at the inner surface.

Part B:

If Pi = 1.2*46.603 MPa = 55.9232 MPa. Then the corresponding radial value is:

$\sigma_e = \sigma_Y \Rightarrow r = 0.2207m = 22.07cm$ (eq4)

Since the area internal to the radial value calculated by equation 4 has yielded, then fraction of the area yielded is

$\pi(r^2 - a^2)/\pi(b^2 - a^2) = (0.2207^2 - 0.2^2)/(0.3^2 - 0.2^2) = 0.17417 \approx 17.417\%$

(eq5)

This value may not entirely be correct as the part of the cylinder that has already yielded introduces non-linearities in the system.

Part C:

For the entire area to be yielded, the equivalent stress at the outer radius should be greater than or equal to the yield stress.

$$\sigma_{e,b} \geq \sigma_Y \Rightarrow 1.6P_i \geq 150MPa \Rightarrow P_i \geq 93.75MPa \qquad \text{(eq6)}$$

ANSYS Results:

A finite element model was made using cuboidal elements (solid 186). Only a quarter of the cylindrical surface was modelled due to symmetry. A height of 5 mm was assigned to the cylinder to simulate plane stresses and strains. The height does not affect the analysis, since a pressure is applied. Frictionless boundary conditions were applied on 2 surfaces to simulate symmetry. Displacement in the out of plane dimension(z) was constrained (Figure 2). The material applied is bilinear kinematic hardening steel.

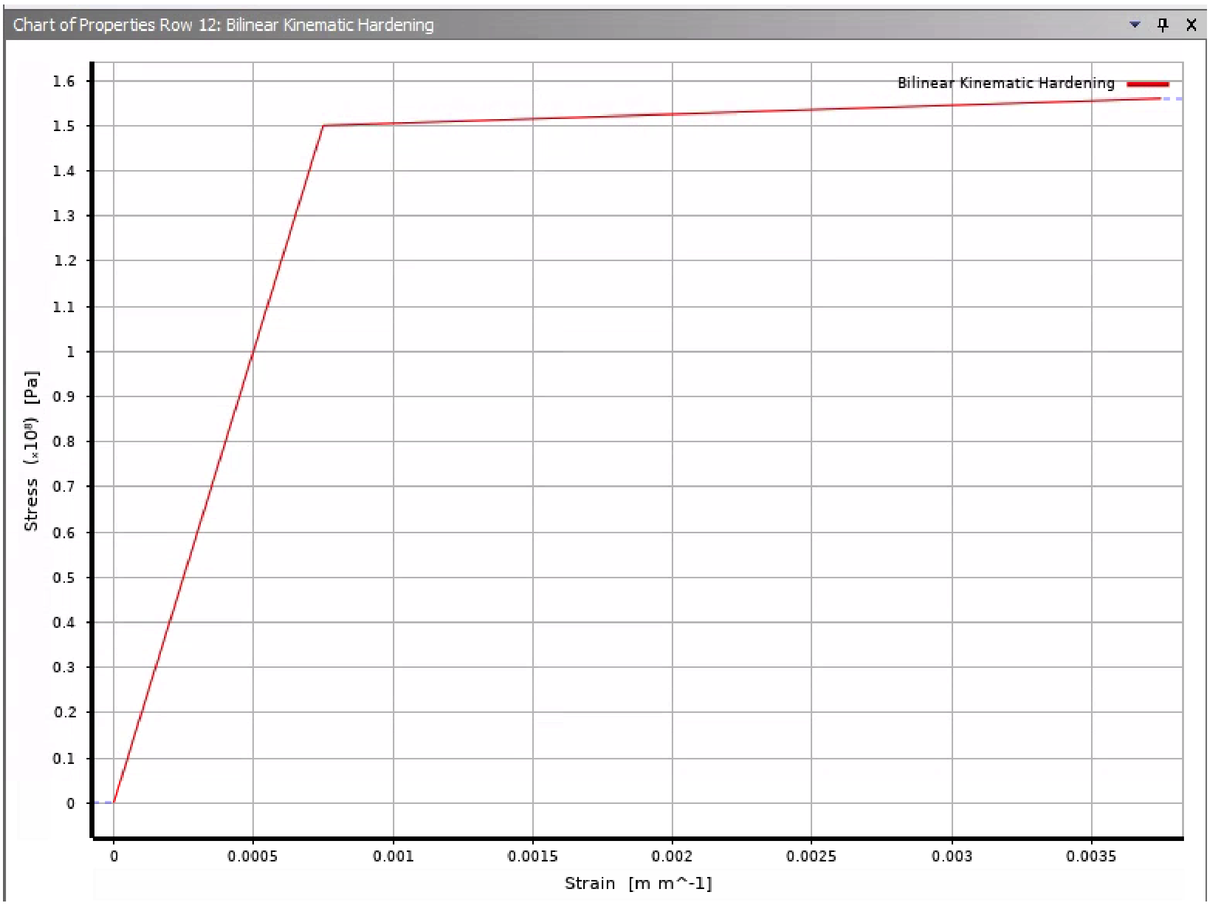


Figure 1: Stress-Strain plot for bilinear steel

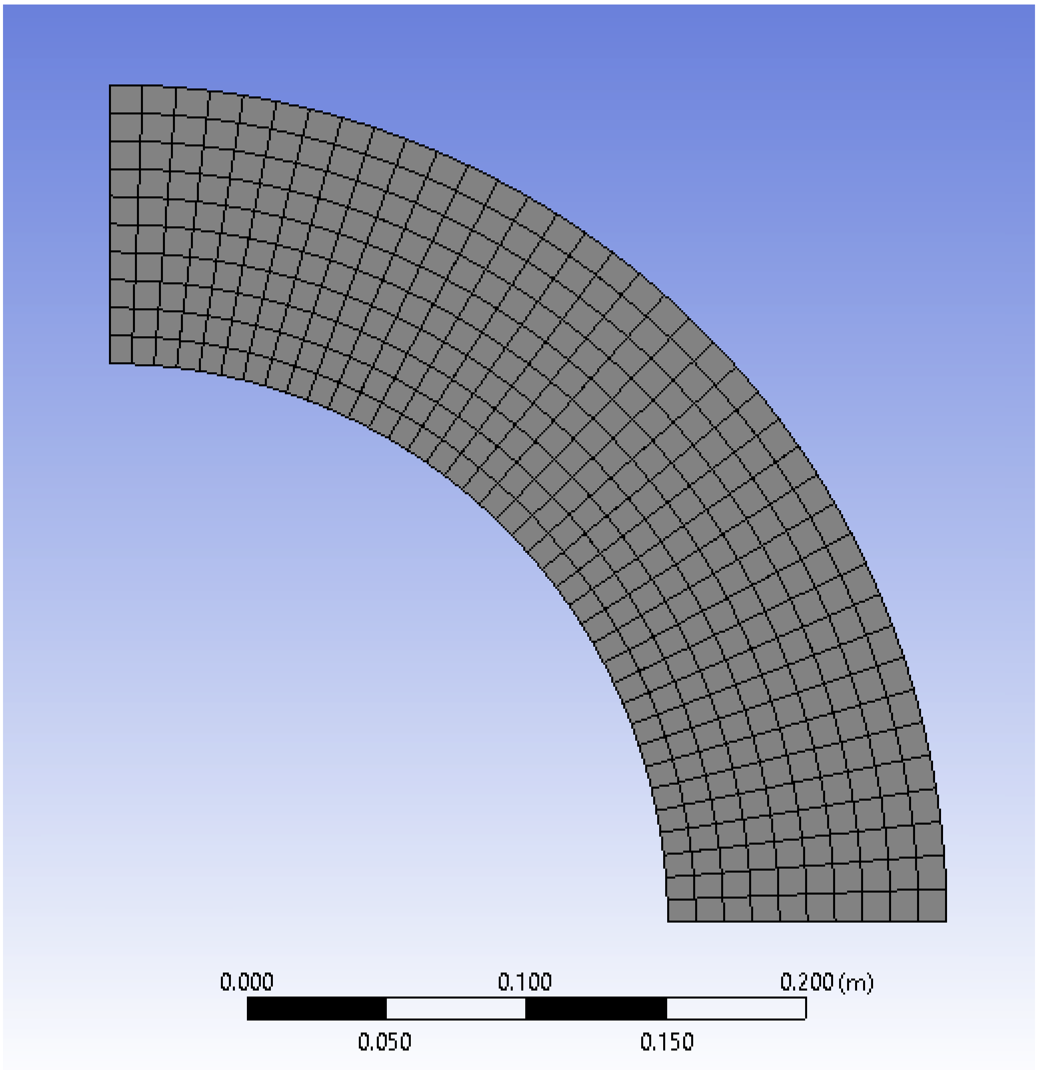


Figure 2: The Mesh

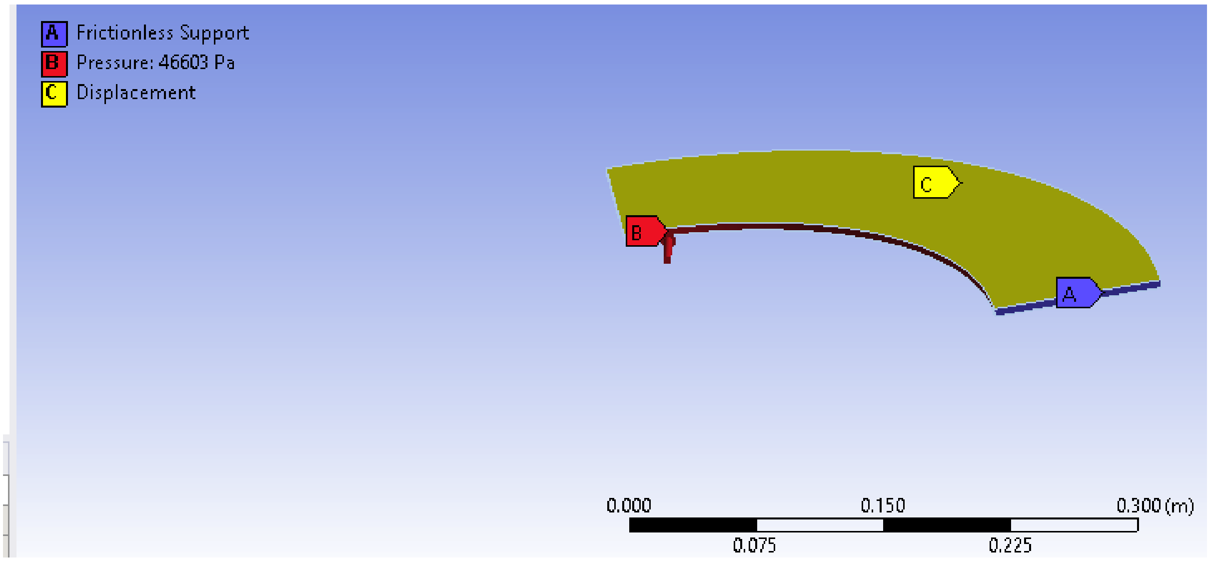


Figure 3: The modal.

Part A

A pressure of 46.6027 MPa is applied to vindicate theoretical results.

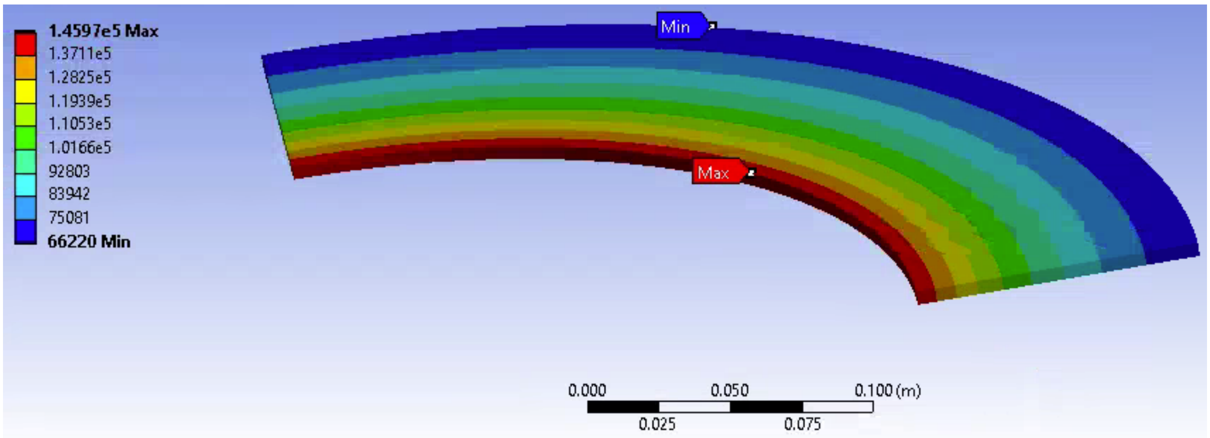


Figure 4: Part A

Figure 4 shows the variation of equivalent stress on the cylinder. As predicted the maximum occurs at the inner surface and minimum occurs at the outer surface. The value of the equivalent stress was observed to be 146 MPa which is very close to the theoretical prediction of 150 MPa.

Part B

A pressure of 55.9232 MPa was applied to find the area yielded.

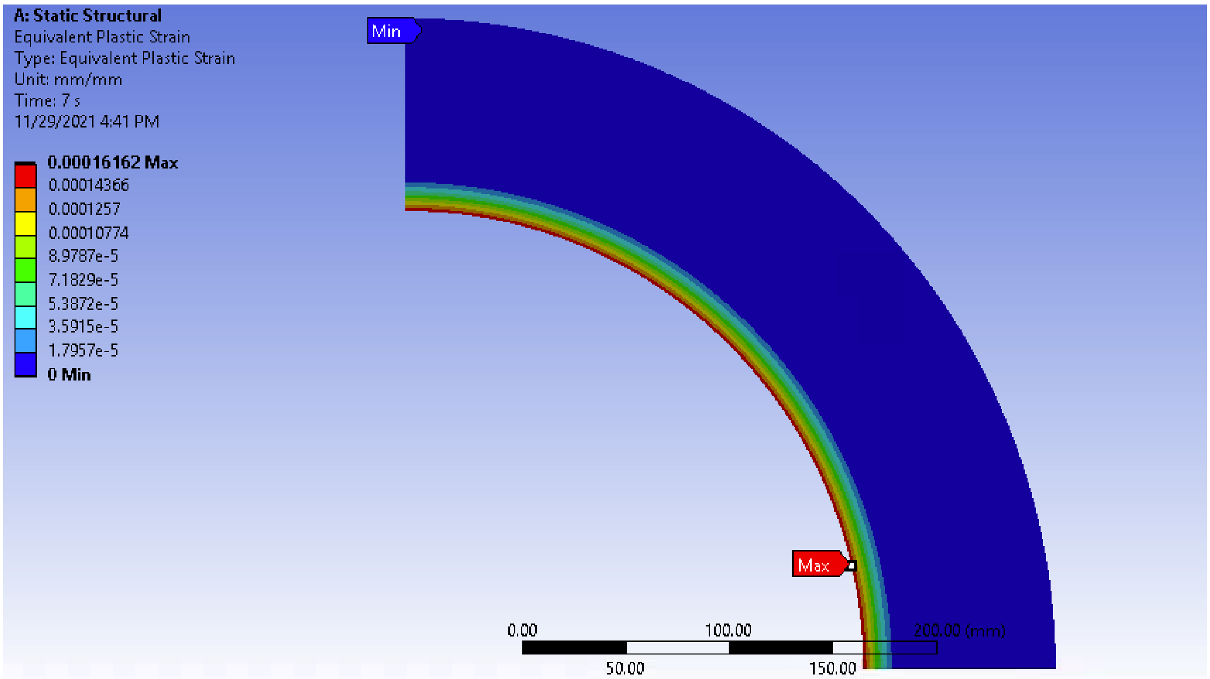


Figure 5: Part B

Figure 5 shows the variation of equivalent plastic strain along the cylinder. The deep blue region is where no plastic yield occurred. Since is no convenient way to find out the radius where the plastic deformation stops, a guess was performed using Figure 6.

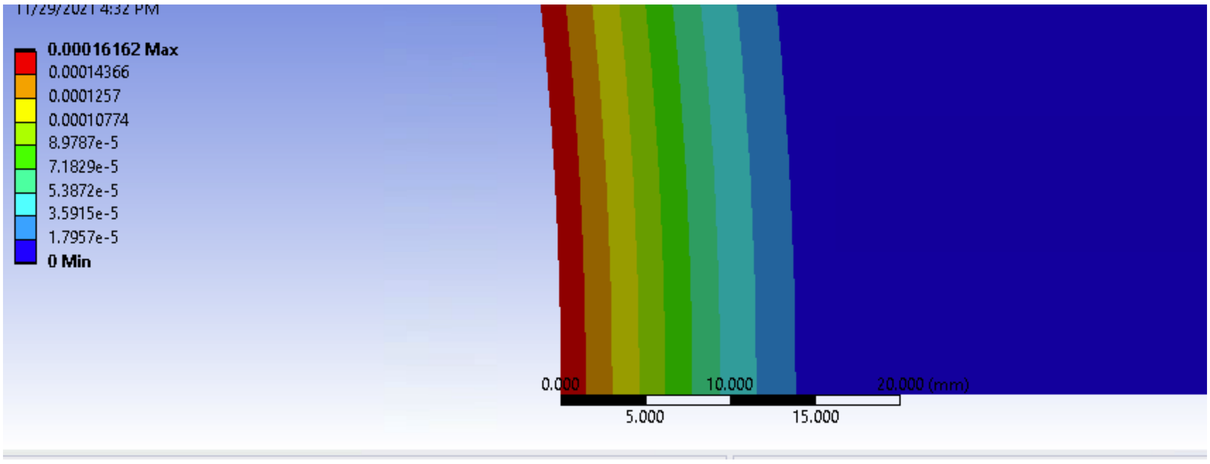


Figure 6: Estimate for part B

Using Figure 6 it seems like the plastic deformation stops about 14.2 mm inside the cylinder from the inner surface. Therefore, the radial value where plastic deformation stops is 20 + 1.42cm = 21.42 cm. The fraction of area yielded is:

$$A_Y = (0.2142^2 - 0.2^2)/(0.3^2 - 0.2^2) = 0.117633 = 11.76\% \quad \text{(eq7)}$$

This is about half of the theoretical prediction. Major reason for the difference is in mathematical calculation. Theoretical radial value of 22.07 cm is close to ansys value of 21.42cm but the way percent area yield is calculated makes it seem erroneous. The other reason could be that the theoretical result does not take plastic effects into account. Finally, a guess was performed to estimate the radius value which must introduce some errors.

Part C

A pressure of 93.75 MPa and the stress response is measured.

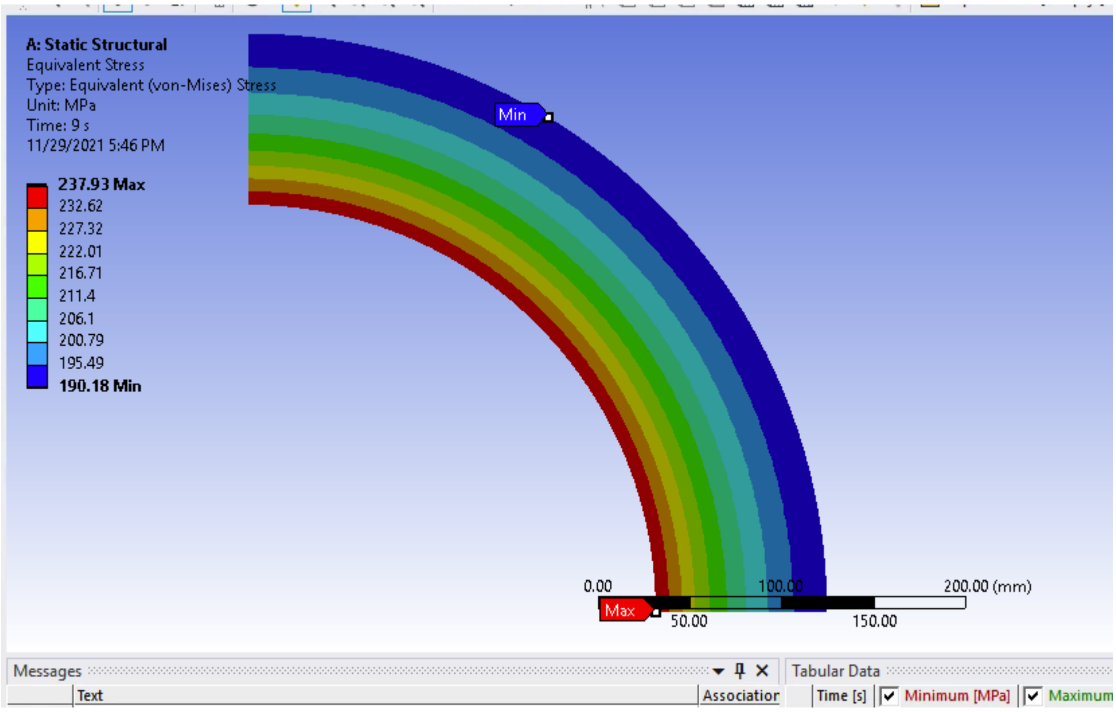


Figure 7: Part C

As expected the maximum occurs at the outer surface, but the value of the equivalent stress at the outer surface of 190.18 MPa is much higher than the theoretical value of 150 MPa. The main reason for this deviation is the non-linear behaviour of the material after yielding which is not taken into account by theory.

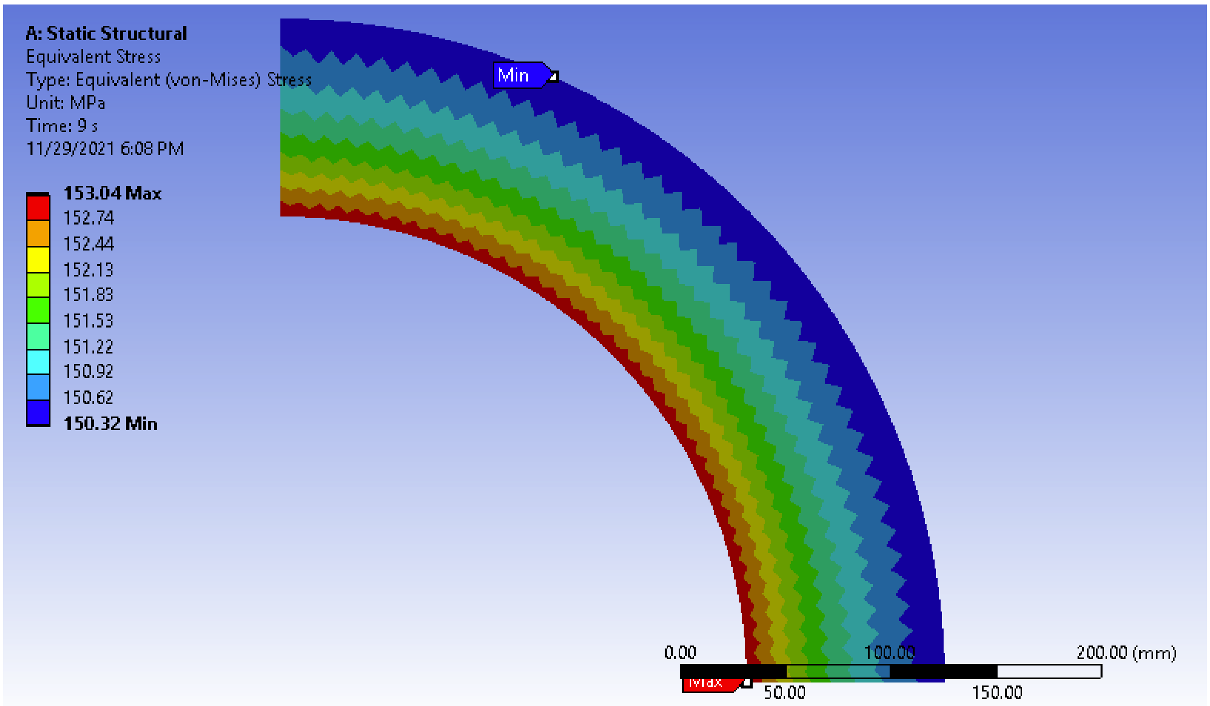


Figure 8: Variation of Stress when applied pressure is 70.5 MPa

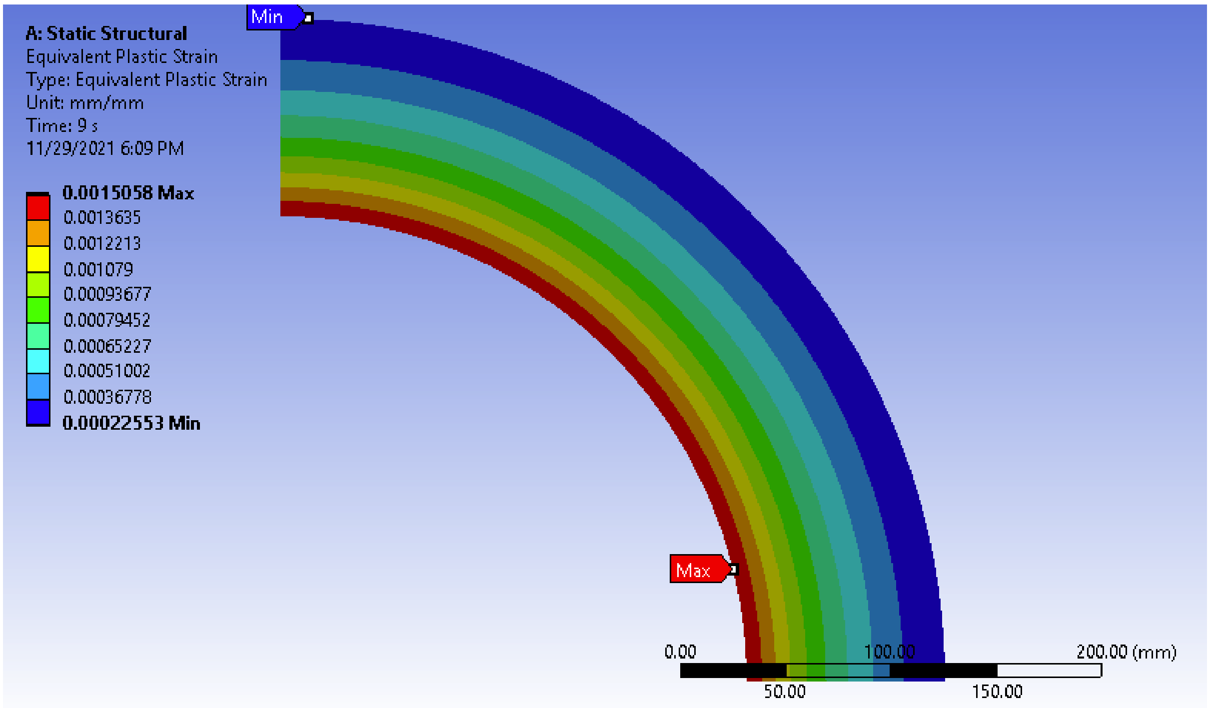


Figure 9: Variation of strain when applied pressure is 70.5 MPa

Through trial and error, I obtained the value of 70.5 MPa for applied pressure which just yielded the entire area. This can be seen in figure 8 when the von-mises stress at the outer surface is 150.32MPa and in figure 9 when the plastic yielding at the outer surface is just above 0 (0.000225 mm/mm). The difference between ansys values and theory could be explained as the theoretical results don't take plasticity into account.

Part D

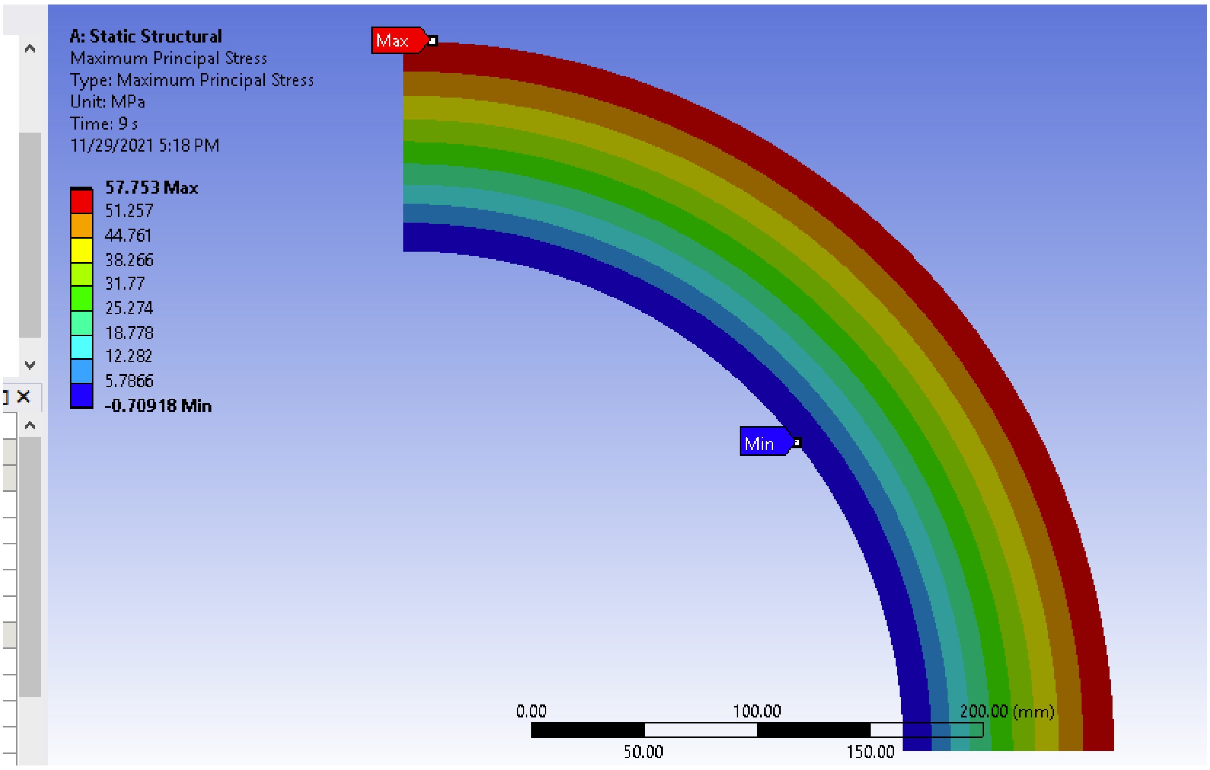


Figure 10: Variation of 1 principal stress along the pressure vessel

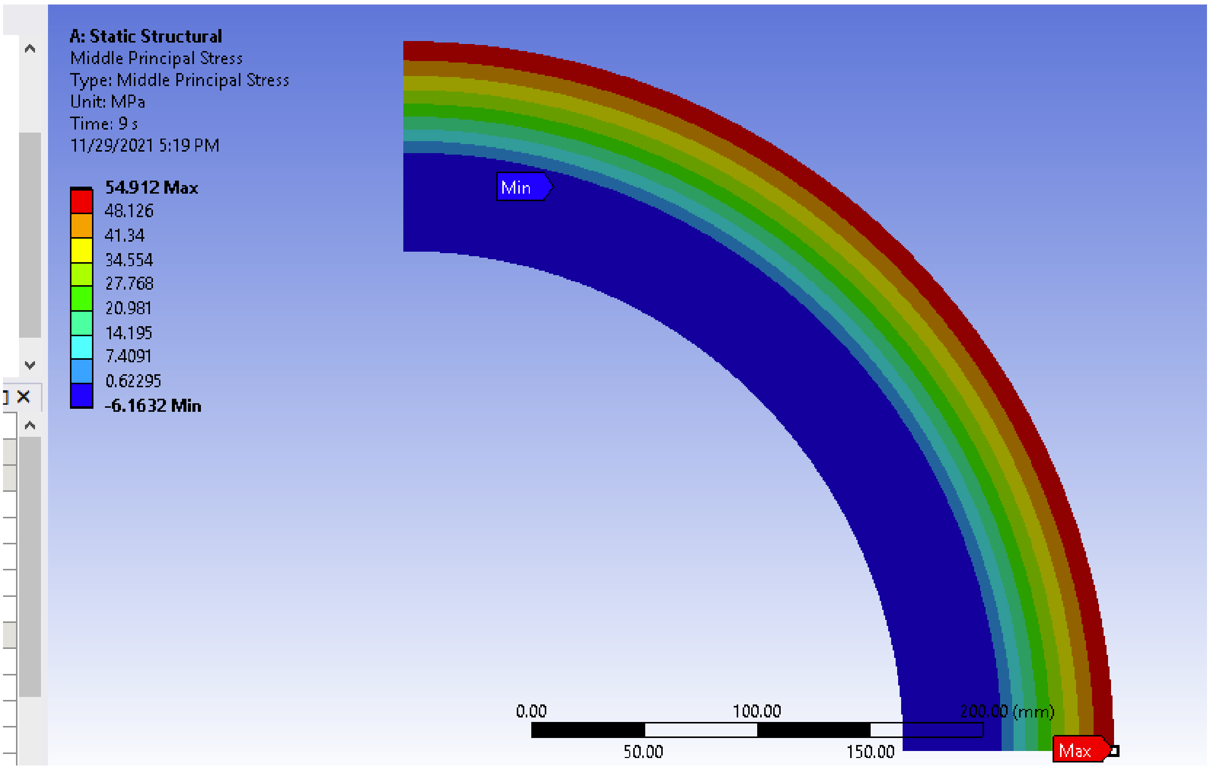


Figure 11: Variation of second principal stress along the pressure vessel

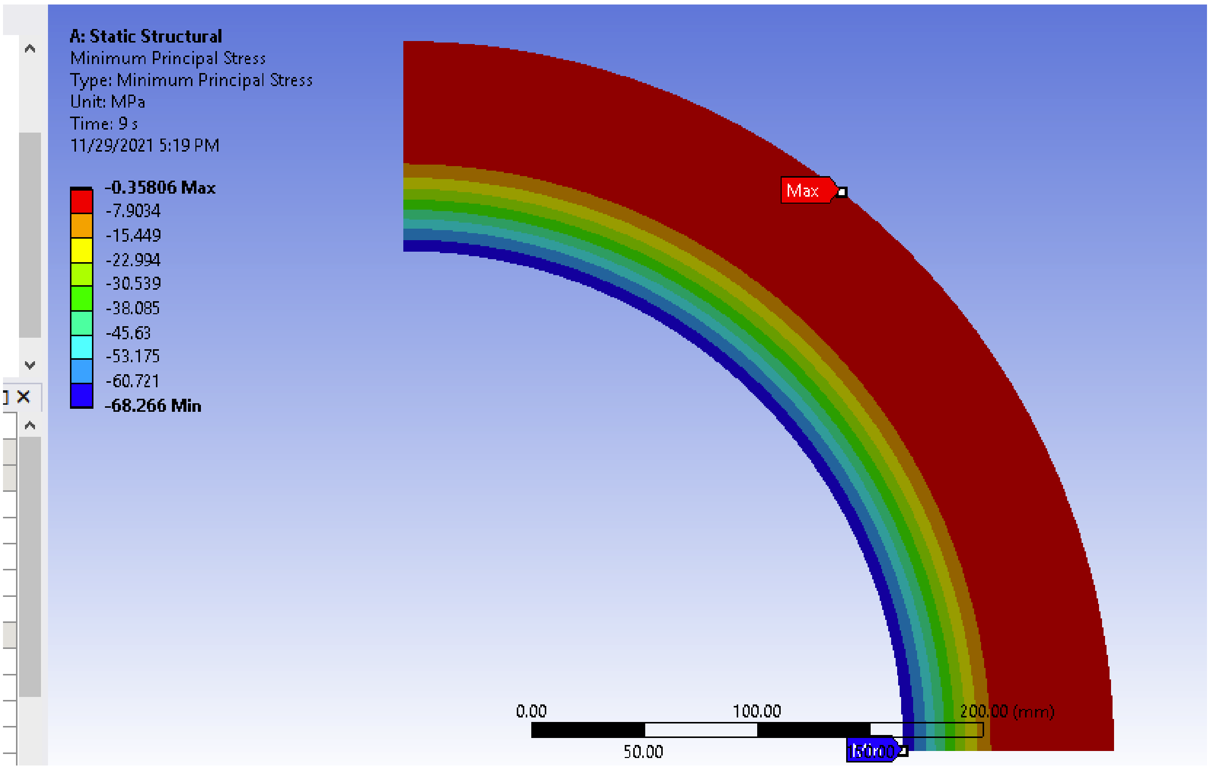


Figure 12: Variation of 3rd principal stress along the cylinder

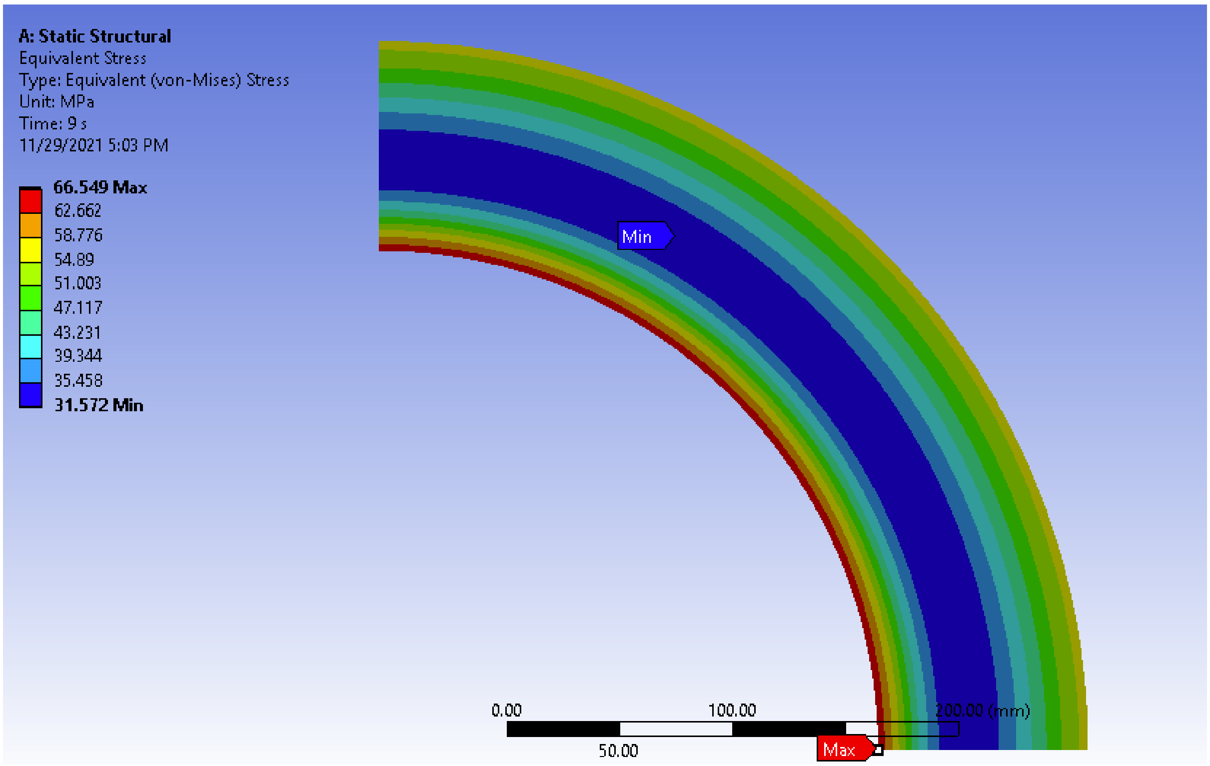


Figure 13: Von Mises Stress along the cylinder

Figures 10,11,12 show the variation of residual principal stresses along the cylinder. The max tensile residual stress (56.764 MPa) is observed on the outermost surface. This is in compliance with the theoretical understanding as the inner surfaces that are more plastically yielded as thus have a permanent offset apply an outward pressure on the outer surfaces which keeps the outer surfaces in tension. Thus, the maximum should occur at the outermost surface. The minimum occurs at the innermost surface since by Newton's second Law the outer surfaces apply a pressure on the inner surfaces. This is also shown by ansys whose max compressive stress of 66.549 MPa occurs at the innermost surface. Von Mises Stress plot (Figure 13) only shows the magnitude but does not decree tension/compression. Therefore, figures 10,11,12 were used to find out the above information.

Conclusion:

In conclusion theoretical results matched the ansys results closely for part a since no plastic deformation had taken place. As the plastic regime increases we observe more and more deviation in ansys from theory. This is mainly because theory precludes plastic effects and thus ansys results take precedence over theory.