# Finite Element Analysis: HomeWork 4

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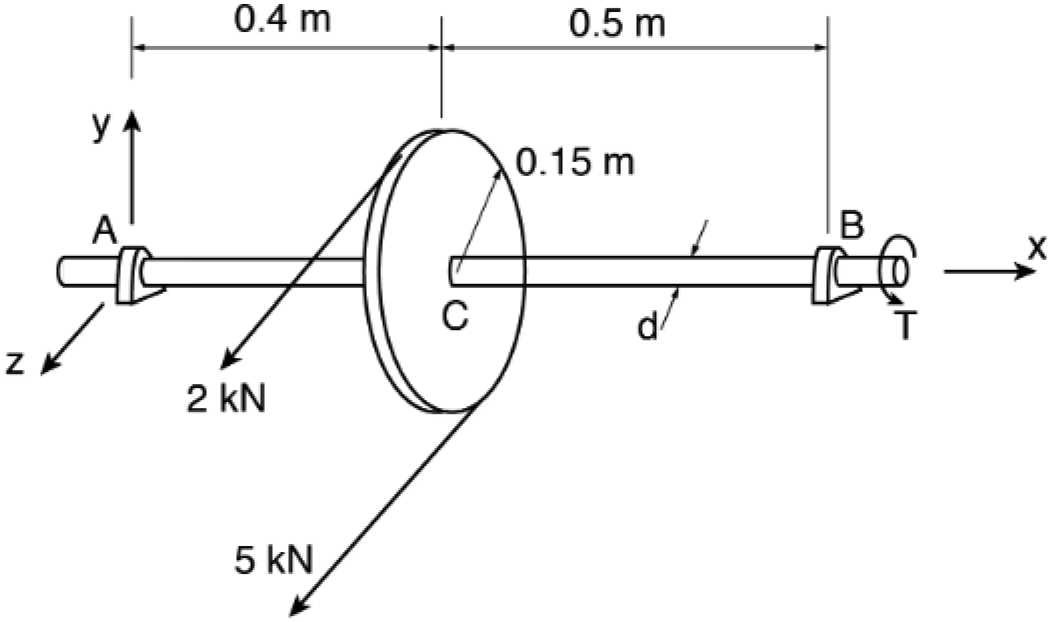
October 22 2021

# Abstract

The problem describes a pulley system with 2 horizontal forces being. The forces create a bending moment, a transverse shear force and a torsion on the shaft. The problem asks us to analyze these stresses to determine the size of the shaft that does fail. Max deflection of the shaft from the centre point is supposed to be less than 0.005 inches and the angle of twist is meant to be less than 0.26 degrees per meter. I analyse the problem theoretically using some simplifying assumptions and then use ANSYS to generate a finite element model to simulate the theoretical prediction.

Method:



# The Problem:

You are a newly hired structural engineer in BadgerDyn Corp. You were hired because you have taken EMA 405 at the University of Wisconsin-Madison and have demonstrated a great working knowledge in the finite element method and analysis during your interview. As your first task on the first day, you were asked to design the solid round shaft for the pulley-shaft system shown in the figure. The design must be submitted to the client within a week. The chief engineer has found the recommended angle of twist in driven shafts from *Machinery's Handbook, Oberg, E., et.al., 26th ed., Industrial Press, N.Y., 2000* and has asked you to design accordingly. Using APDL, determine the size of the shaft such that the maximum angle of twist between C and B is 0.26 degrees per meter of length and the deflection of the beam is less than 0.005 inches under the pulley C for the loads shown. You may assume simple supports at bearings A and B. The materials scientist has recommended that the most suitable material to make the shaft is cold-rolled AISI 1020 steel. Submit your recommendations in the form of a design report before the due date.

# The Exact Solution:

Forces/Moments on the shaft at point C:
A bending moment
A torsional torque
A transverse shear stress

The bending moment is responsible for deflection in the z direction and the torsional torque is responsible for creating the angle of twist. The transverse shear force contributes neither to the beam deflection nor to the angle of twist and thus is ignored from the discussion. Values for Young's Modulus and Poission's ratio used are: E= 186 GPa and ν=0.29 (ref1).

## Torsional Criterion:

The formula for angle of twist is presented in eq1 (ref2). T is the applied torque (450 Nm), L is the shaft length (0.5 m), J is the polar moment of inertia and G is the shear modulus (73.093 GPA).

$$\varphi = TL/JG \qquad \text{(eq1)}$$

$$G = E/2(1 + \nu) \qquad \text{(eq2)}$$

φis given to be 0.29 degrees per radian, which is converted to radians:

$$\varphi \le 0.29 * 0.4 * \pi/180 = 2.531 \times 10^{-3} rad \qquad \text{(eq3)}$$

Solving for J and consequently for radius r gives:

$$J = 0.5\pi r^4 \ge 450 * 0.5/(73.093 * 10^9 * 2.531 * 10^{-3}) = 4.9393 \times 10^{-7} m^4 \qquad \text{(eq5)}$$

$$r \geq 0.030012\, m \; = \; 30.012mm \qquad \text{(eq6)}$$

Thus the radius of the shaft must be at least 30.012mm

## Bending Criterion:

The formula for beam deflection δfor a doubly simply supported beam was obtained from roark's book (ref3). P is the applied point force (7 Kn), a is the location from the left end where the deflection is calculated (0.4 m), E is Young's Modulus (186 GPa), L is the beam length (0.9m) and I is the moment of inertia (m^4).

$$\delta = Pa((L^2 - a^2)/3)^{1.5}/(3EIL) \qquad \text{(eq7)}$$

For beam deflection less than 0.005 inches (1.27e-4 m), and solving eq7 for I and consequently for r, we get:

$$I \geq 7 * 10^3 * 0.4 * ((0.9^2 - 0.4^2)/3)^{1.5}/(3 * 180 * 19^9 * 0.9 * 1.27 * 10^{-4}) = 4.57516 * 10^{-6} m^4$$
(eq8)

$$I = \pi/4 * r^4 \Rightarrow r \geq 0.049128m \; = 49.128mm \qquad \text{(eq9)}$$

Thus, to satisfy the bending criterion radius has to be at least 49.128 mm.

Taking both criteria simultaneously the radius has to be at least 49.128mm.

# Finite Element Solution:

A finite element model was constructed with 9 beam 189 elements. A force of 7KN was applied at 0.4m from the left end in the -Z direction. A moment of -450N was applied about the x axis at the same point. The leftmost node was constrained in displacements but not in rotations. The rightmost node was constrained in displacements and x rotation to simulate a simply supported joint which is constrained to not rotate in x direction. The length of the shaft was 0.9m. The radius of the cross sectional area was 0.049 m.

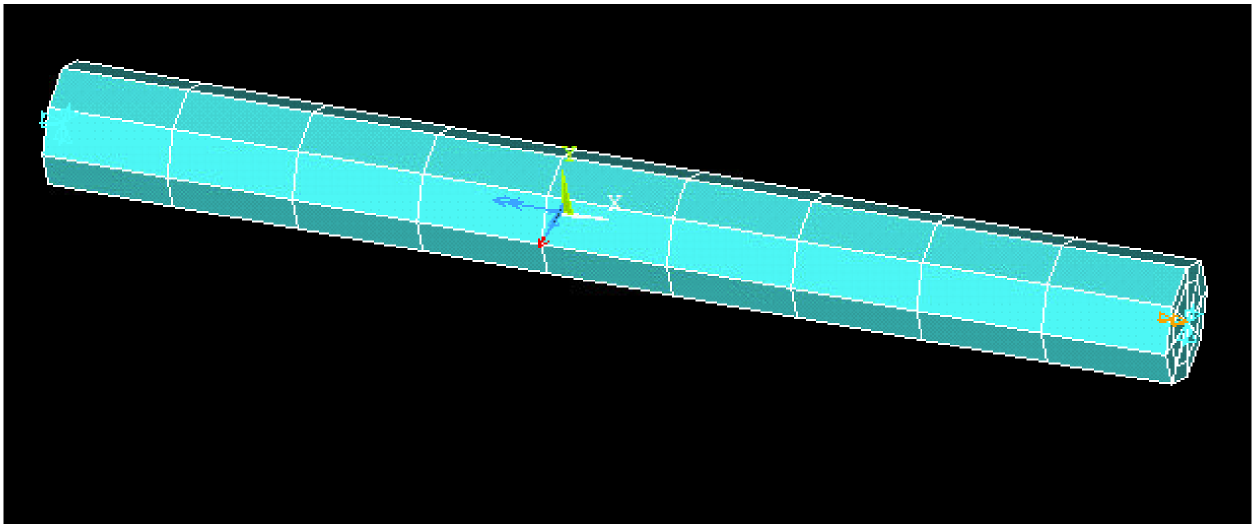



Figure 1: The FE model

## Discussion of Results:

Solving the model gives the following results:

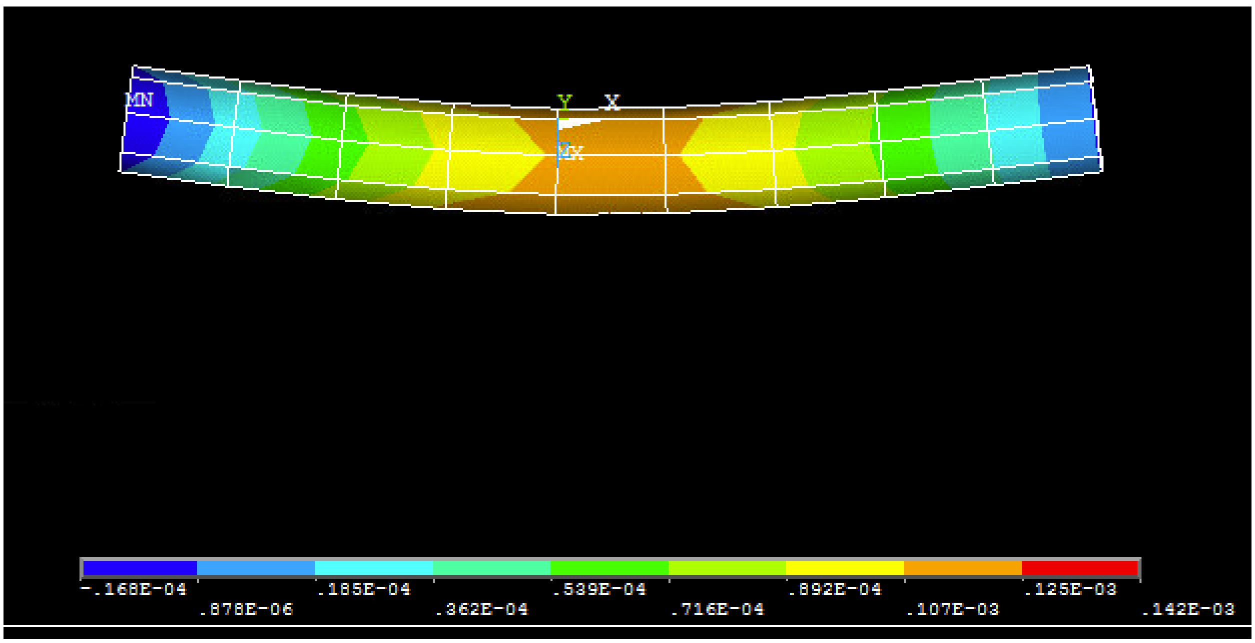



Figure 2: variation of displacement displacement

A max displacement of 1.259e-4 m was observed. Figure 2 shows the variation of displacement in the z direction from a top view. The value of max displacement in inches is 0.004957 which is slightly less than the expected value of 0.005 in but very close. Thus, a radial distance of 0.049mm satisfies the bending criterion, which was also predicted theoretically.

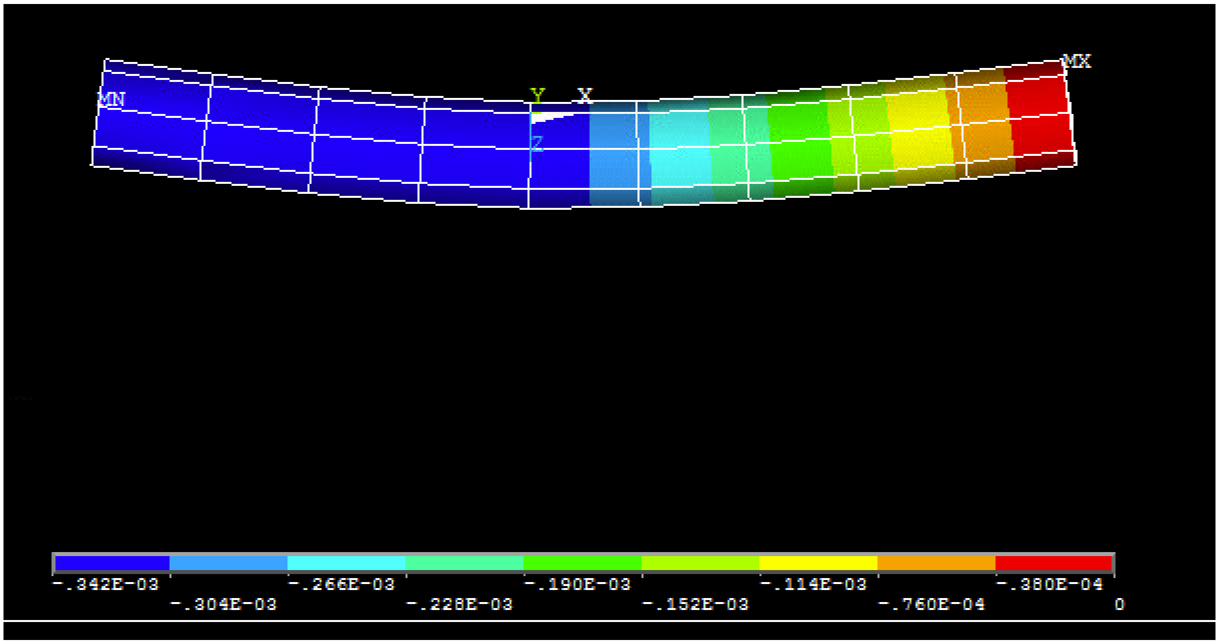


Figure 3: Variation of angle of twist.

Figure 3 shows the variation of angle of twist along the length of the shaft. As expected, the rotation at the right end is 0 rads. The absolute max value of angle of twist was observed to be 3.4178E-004 rads which is less than 2.531e-03 and thus within bounds for the angle of twist criterion. Thus, a radius of 0.049m satisfies the angle of twist criterion.

# Conclusion:

Therefore, as predicted theoretically, a radial value of 49.128mm satisfies both displacement and angle of twist criteria. One commonly available size of shaft available for is the 50mm circular shaft. As theoretically shown and experimentally borne any radial value greater than 49.128mm would work, but the 50mm radius shaft also minimizes the cost, since it consumes the least amount of material and is therefore the recommended shaft radius.

# References:

Ref1:
http://www.matweb.com/search/DataSheet.aspx?MatGUID=10b74ebc27344380ab16b1b69f1cffbb&ckck=1
Ref2: Roarks Formulas For Stress and Strain - Richard G Budynas (2020). Page 366, Table 10.1, Reference Number 1.
Ref3: Roarks Formulas For Stress and Strain - Richard G Budynas (2020). Page 204, Table 8.1, Reference Number 1e.