Financial Mathematics Engine: NPV & Multiple IRR Solver

Anshuman Nema

January 4, 2026

1 Introduction

Capital Budgeting is the process by which businesses determine the viability of major projects. The two most critical metrics in this domain are **Net Present Value (NPV)** and the **Internal Rate of Return (IRR)**.

While standard financial calculators (like Excel's `XIRR`) function well for standard cash flows, they often fail when presented with **Non-Conventional Cash Flows** (projects with alternating periods of profit and loss). This project implements a robust Python-based engine capable of detecting **Multiple IRRs** using numerical analysis techniques, complete with a visualization dashboard to aid decision-making.

2 Mathematical Foundation

2.1 Net Present Value (NPV)

The NPV represents the sum of present values of incoming and outgoing cash flows over a period of time. It is calculated as:

$$NPV = \sum_{t=0}^{N} \frac{C_t}{(1+r)^t}$$

Where:

- C_t = Cash flow at time t
- r = Discount rate
- N = Total number of periods

2.2 The IRR Problem

The Internal Rate of Return is defined as the discount rate r^* at which $NPV = 0$. Mathematically, this is equivalent to finding the roots of a polynomial of degree N.

$$0 = C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_N}{(1+r)^N}$$

According to **Descartes' Rule of Signs**, the number of positive real roots (IRRs) is bounded by the number of sign changes in the cash flow stream. A project that invests, earns, and then pays cleanup costs (Signs: $-, +, -$) can theoretically have **two valid IRRs**. Standard iterative solvers (like Newton-Raphson) often converge to just one root and ignore the other, leading to dangerous financial blind spots.

3 Algorithmic Implementation

To solve the "Multiple IRR" problem, this system utilizes a hybrid **Incremental Search + Bisection Method** approach.

3.1 Phase 1: Incremental Search (Scanning)

The algorithm does not assume a single answer. Instead, it discretizes the search space (from -99% to 1000%) into small intervals (step size $\Delta = 0.01$).

$$\text{Check if } \text{NPV}(r_i) \times \text{NPV}(r_{i+1}) < 0$$

If the product of NPVs at the edges of an interval is negative, the Intermediate Value Theorem guarantees a root exists within that interval.

3.2 Phase 2: Bisection Method (Refinement)

Once a bracket $[a, b]$ containing a root is identified, the Bisection Method iteratively narrows the range:

$$c = \frac{a + b}{2}$$

The algorithm evaluates the midpoint c. If the sign changes between a and c, the new search range becomes $[a, c]$. This repeats until the error is within a tolerance of 1×10^{-6}.

4 Case Study Results

The engine was tested against three distinct financial scenarios to validate robustness.

4.1 Scenario 1: Standard Investment

Flows: $[-10,000, 3,000, 4,000, 5,000]$
Result: Single IRR at 8.90%.
Analysis: This represents a typical project with an initial outlay followed by steady returns. The single sign change leads to a unique solution.

4.2 Scenario 2: Financing (Loan)

Flows: $[10,000, -3,000, -3,000, -3,000, -3,000]$
Result: Single IRR at 7.71%.
Analysis: Here, cash flows enter first and leave later (a loan). The IRR correctly identifies the effective interest rate (cost of borrowing).

4.3 Scenario 3: The "Cleanup" Problem (Multiple IRRs)

Flows: $[-100, 230, -132]$
Result: Two IRRs found at 10.00% and 20.00%.
Analysis: This critical test case validates the engine's ability to detect multiple roots.

- At $r < 10\%$, the future liability (-132) weighs heavily, making NPV negative.
- Between $10\%-20\%$, the liability is discounted enough to make the project profitable.
- At $r > 20\%$, the heavy discounting erodes the value of the income ($+230$), turning NPV negative again.

Standard Excel functions would likely return an error or only the first value (10%) for this input.

5 Visualization Dashboard

To make these abstract numbers intuitive, the tool generates a dual-plot dashboard for every analysis:

1. **Cash Flow Diagram (Bar Chart):** Uses color-coding (Green for Inflow, Red for Outflow) to visualize the project structure immediately.
2. **NPV Profile (Curve):** Plots the NPV against a range of discount rates. The intersections with the X-axis (Zero line) are highlighted with red markers, providing visual confirmation of the calculated IRRs.

6 Conclusion

This project demonstrates that financial mathematics requires more than simple formula application. By implementing numerical methods from scratch, we created a tool that handles the "corner cases" of finance where traditional tools fail. The ability to identify multiple IRRs is crucial for risk management, ensuring that investors are aware of all boundary conditions that could turn a profitable project into a loss.