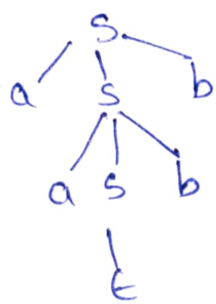
Theory of computation

1) Language L said to be unambigous if does not contain more than one parse tree, RMD, LMD for given input string.

$L = \{a^n b^n \mid n \geq 1\}$

R.E $S \rightarrow aSb \mid \epsilon$

aabb



we can generate only one parse tree for given string

L is unambigous

2) Given.

$S \rightarrow aA \mid a \mid Bb \mid cC \mid a$

$A \rightarrow aB$

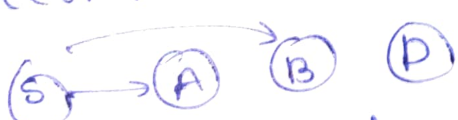
$B \rightarrow a \mid Aa$

$C \rightarrow cCD$

$D \rightarrow ddd$

Eliminating useless symbols

i) Here C is useless variable since we can't derive terminal.

2) 

we can eliminate D so grammer is.

$S \rightarrow aA \mid a \mid Bb$

$A \rightarrow aB$

$B \rightarrow a \mid Aa$

4)

a) Given, $L = \{ w, w^r \mid w \in (a+b)^* \}$

CFG $S \rightarrow aSa \mid bSb \mid \epsilon$

eliminating ϵ parameter

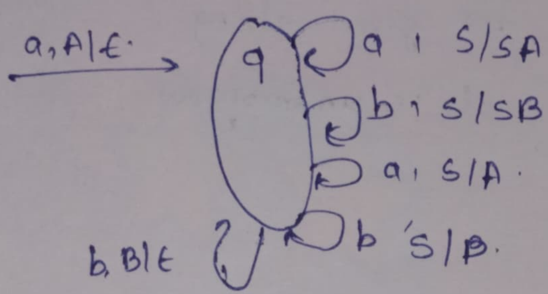
$S \rightarrow aSa \mid bSb \mid aa \mid bb$

CNF $S \rightarrow aSA \mid bSB \mid aA \mid bB$

$A \rightarrow a$

$B \rightarrow b$

PDA.



$m = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$

$Q = \{q_0\}$ $\Sigma = \{a, b\}$.

$\Gamma = \{S, A, B\}$ $z_0 = S$, $F = \phi$

b) given

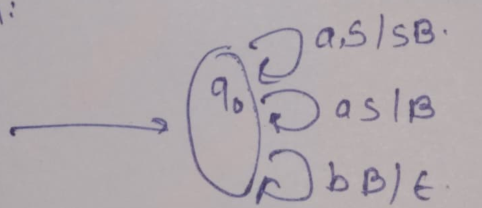
CFG: $S \rightarrow asb \mid ab$.

Converting it into CNF

$S \rightarrow aSB \mid aB$

$B \rightarrow b$

PDA:



$m = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$.

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5) Given.

CFG : $S \to aaaa\,S \mid aaa$

CNF : $S \to XS \mid AA$.

$X \to AA$.

$A \to BB$

$B \to a$

6) Given.

$S \to AA$.

$A \to B \mid BB$

$B \to abB \mid b \mid bb$.

Non unit productions.

$S \to AA$.

$A \to BB$

$B \to abB$

$B \to b$

$B \to bb$.

unit production

$A \to B$

$A \to B$ gives $A \to abB \mid b \mid bb$.

$\therefore$ grammer after removing unit productions.

$S \to AA$.

$A \to BB \mid abB \mid b \mid bb$

$B \to abB \mid b \mid bb$.

7) Given.

CFG. : $S \to ABA$.

$A \to aA \mid \epsilon$

$B \to bB \mid \epsilon$

i) removing ϵ-production.

prev state.	next state.	production.
ϕ	$\{A, B\}$	$A \to \epsilon, B \to \epsilon$.
$\{A, B\}$	$\{A, B, S\}$	$AS \to \epsilon$
$\{A, B, S\}$	—	—

given production

$S \to ABA$.

$A \to aA$.

$B \to bB$.

non ϵ production

$S \to ABA \mid AB \mid BA \mid A \mid B$

$A \to aA \mid a$

$B \to bB \mid b$.

ii) removing unit production.

non-unit production

S → ABA.

S → AB

S → BA.

A → aA.

A → a

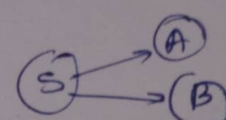
B → bB

B → b

unit productions

S → A.

S → B.



S → A gives S → aA | a

S → B gives S → bB | b.

minimized grammer is

S → ABA | AB | BA | aA | bB | b.

A → aA | a

B → bB | b

CNF

S → AB | BA | A'A | B'B | a | b | XA.

X → AB

A → A'A | a

B → BB | b

A → a

B' → a

8)

a) Given $L = \{a^n b^m a^n \mid n \geq 0, m \geq 1\}$

CNF : S → ABA.

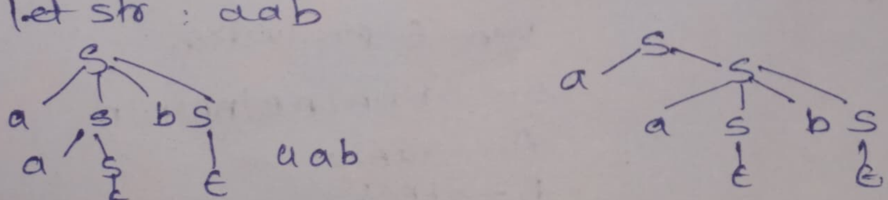
A → aA | ε

B → bB | b

b) given CNF:

q S → aS | aSbS | ε

let str : aab



we have more than 1 parse tree for given input.

∴ given grammer is ambigous.

9)

a) given.

$S \rightarrow AB$

$A \rightarrow BS$

$A \rightarrow b$

$B \rightarrow SA$

$B \rightarrow a$.

given grammer is Greekbank normal from.

PDA.

q_0

Converting into CNF.

$A_1 \rightarrow S$

$A_2 \rightarrow A$

$A_3 \rightarrow B$.

i) $A_1 \rightarrow A_2 A_3$

$A_2 \rightarrow A_3 A_1 \mid b$.

$A_3 \rightarrow A_1 A_2 / a$

ii) $A \rightarrow A_2 A_3$

$A_2 \rightarrow A_3\ a \mid A_2 A_3 A_2$

$A_3 \rightarrow a \mid A_3 A_1$

10)

a) given.

Ambigous grammer : A grammer said to be ambigous if we can generate more than one parse tree for the given input.

given.

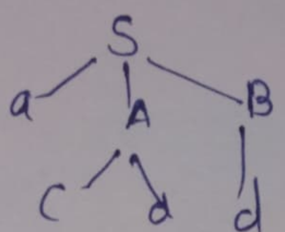
$S \longrightarrow aAB$

$A \longrightarrow bc \mid cd$

$C \longrightarrow cd$

$B \longrightarrow c \mid d$

let str : acdd



we can't generate more than two parser trees so given grammer is ambigous

11)

a) Closure properties of CFL:

i) union: If L_1 and L_2 are two Context free language their union $L_1 \cup L_2$ will also be CFL.

given grammer.

$S \rightarrow a$

$S \rightarrow Ab$

$S \rightarrow aBa$

$A \rightarrow b$

$B \rightarrow b$

$B \rightarrow A.$

Non E- productions

$S \rightarrow a$

$S \rightarrow Ab|b$

$S \rightarrow aBa|aa$

$A \rightarrow b$

$B \rightarrow b$

$B \rightarrow A.$

CFG without E productions is

$S \rightarrow a|Ab|b|aBa|aa$

$A \rightarrow b$

$B \rightarrow b|A.$

13) Given $E \rightarrow E+E|E-E|E*E|E/E|(E)|id.$

~~S → 0S1/A~~

a) ~~A → 1A0/S/ε~~

str - Id* Id + Id / Id

LMD.

$E \rightarrow E/E$

$\rightarrow E+E/E$

$\rightarrow E*E+E/E$

$\rightarrow id*E+E/E$

$\rightarrow id*id+E/E$

$\rightarrow id*id+id/E$

$\rightarrow id*id+id/id$

RMD.

$E \rightarrow E/E$

$\rightarrow E/id$

$\rightarrow E*E/id$

$\rightarrow E*E+E/id$

$\rightarrow E*E+id/id$

$\rightarrow E*id+id/id$

$\rightarrow id*id+id/id$

b) i. sentential forms: It is a string of symbols that is derivation of a grammer.

example $E \rightarrow E+E/E$

$E \rightarrow id*E+E/E$

ex: $L_1 = \{a^n b^n c^m \mid m, n \geq 0\}$ is CFL.

$L_2 = \{a^n b^m c^m \mid n, m \geq 0\}$ is CFL

$L_1 \cup L_2 = \{a^n b^n c^m \cup a^n b^m c^m \mid n, m \geq 0\}$ is also CFL.

2) Concatenation: If L_1 and L_2 are two CFL's then their Concatenation L_1 be will also context free language.

3) Kleen clousure: if L is context free the L^* is also Context free.

Ex: $L_1 = \{a^m b^n \mid n \geq 0\}$

$L_1 = \{a^n b^n \mid n \geq 0\}$ is also CFL.

4) Intersection: If L_1 and L_2 are two Context free language their intersection $L_1 \cap L_2$ not be Context free

$L_1 = \{a^n b^n c^m \mid n, m \geq 0\}$

$L_2 = \{a^m b^n c^n \mid n, m \geq 0\}$.

$L_1 \cap L_2 = \{a^n b^n c^n \mid n \geq 0\}$ need not be Context free

b)

Given

$S \rightarrow a \mid Ab \mid aBa$

$A \rightarrow b \mid \epsilon$

$B \rightarrow b \mid A$.

elimanating ϵ production

prev state	next state	production
$A \rightarrow \phi$	$\{A\}$	$A \rightarrow \epsilon$
$\{A\}$	$\{A, B\}$	$B \rightarrow A$.
$\{A, B\}$	$\{A, B\}$	

14) Deterministic push Down Automata.

A PDA is deterministic if there is never a choice for a next move in any instantaneous description it has only one move in each Condition,

ex:

$S \rightarrow aSA \,/\, bSB \,/\, C$

$A \rightarrow a$

$B \rightarrow b$

15) Given

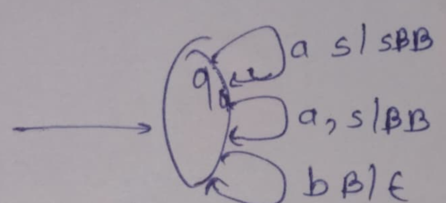
$L = \{ a^n b^{2n} \mid n \geq 0 \}$

CFG = $S \rightarrow aSbb \,/\, abb$

GNF : $S \rightarrow aSBB \,/\, aBB$

$B \rightarrow b$

PDA



str aaabbbbbb

$(q_0, aaabbbbbb, S)$

$(q_0, aabbbbbb, SBB)$ $(q_0, aabbbbbb, BB)$

$(q_0, abbbbbb, SBBBB)$ $(q_0, abbbbbb, BBBB)$ ✗

$(q_0, bbbbbb, SBBBBBB)$ ✗ $(q_0, bbbbbb, BBBBBB)$

$(q_0, bbbbb, BBBBB)$

(q_0, bbb, BBB)

(q_0, b, B)

$(q_0, \epsilon, \epsilon)$

"aaabbbbbb" is accepted by the PDA.

16)

a) Given

$S \rightarrow AaCb \mid ABa$

$A \rightarrow bAa \mid a$

$B \rightarrow BaB \mid b$

$C \rightarrow c$

i) given grammer doesn't Contain any ϵ production.

ii) ~~It doesn't~~ Contain any Contain production

iii) It doesn't Contain any useless symbols.

iv) CNF : $S \rightarrow xy \mid AB'$

$x \rightarrow AA'$

$y \rightarrow CE'$

$A' \rightarrow a'$

$C \rightarrow b$

$B' \rightarrow BA'$

$A \rightarrow a \mid c'z$

$z \rightarrow AA'$

$B \rightarrow B'B \mid b$

$C \rightarrow c$

b) Given

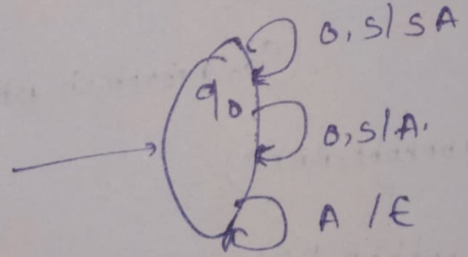
$L = \{0^n 1^n\}$

CFG : $S \rightarrow 0S1 \mid 01$

CNF : $S \rightarrow 0SA \mid 0A$

$A \rightarrow 1$

PDA



$M = (Q, \Sigma, \gamma, \delta, z_0, q_0, F)$

$Q = \{q_0\}$ $\Sigma = \{0, 1\}$

$\gamma = \{S, A\}$ $z_0 = S$ $q_0 = \{q_0\}$ $F = \phi$

17)

a) given

$L = \{0^{2n}1^n \mid n \geq 1\}$

CFG: $S \rightarrow 00S1 \mid \epsilon$

minimized CFG $S \rightarrow 00S1 \mid 001$

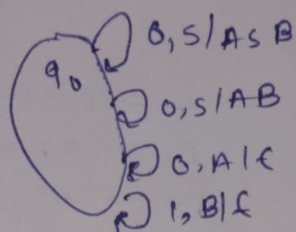
Converting into CNF

CNF $S \rightarrow 0ASB \mid 0AB$

$A \rightarrow 0$

$B \rightarrow 1$

PDF



$m = (Q, \Sigma, \gamma, \delta, q_0, z_0, F)$

$Q = \{q_0\}$ $\Sigma = \{0, 1\}$

$\gamma = \{S, A, B\}$ $z = S$, $F = \phi$

b) given

$S \rightarrow ABaC$

$A \rightarrow BC$

$B \rightarrow b \mid \epsilon$

$C \rightarrow D \mid \epsilon$

$D \rightarrow d$

Removing ϵ-production

prev state	next state	production
ϕ	$\{B, C\}$	$B \rightarrow \epsilon$, $C \rightarrow \epsilon$
$\{B, C\}$	$\{A, B, C\}$	$A \rightarrow BC$
$\{A, B, C\}$	$\{A, B, C\}$	–

given productions.

$S \rightarrow ABaC$

$A \rightarrow BC$

$B \rightarrow b$

$C \rightarrow D$

$D \rightarrow d$

non ϵ-productions.

$S \rightarrow ABaC \mid ABa \mid AaC \mid BaC \mid aC \mid Aa$
$\mid Ba \mid a$

$A \rightarrow BC \mid B \mid C$

$B \rightarrow b$

$C \rightarrow D$

$D \rightarrow d$.

∴ CFG without ε-productions:

$S \to ABaC \mid ABa \mid AaC \mid BaC \mid aC \mid Aa \mid Ba \mid a$

$A \to BC \mid B \mid C$

$B \to b$

$C \to D$

$D \to d$

18)

a) given.

$S \to ASB \mid \epsilon$

$A \to aAS \mid a$

$B \to SbS \mid A \mid bb.$

i) removing ε productions.

$S \to ASB$

$A \to aAS \mid a$

$B \to SbS \mid A \mid bb$

$S \to ASB \mid AB$

$A \to aAS \mid aA \mid a$

$B \to SbS \mid Sb \mid bS \mid b \mid A \mid bb$

ii) removing unit productions.

non unit productions

$S \to ASB \mid AB$

$A \to aAS \mid aA \mid a$

$B \to SbS \mid Sb \mid bS \mid b \mid bb$

unit production.

$B \to A$

$B \to A$ gives $B \to aAS \mid aA \mid a$

iii) minimal grammer is

$S \to ASB \mid AB$

$A \to aAS \mid aA \mid a$

$B \to aAS \mid aA \mid a \mid SbS \mid Sb \mid bS \mid b \mid a \mid bb$

iv) Converting to CNF

$S \to AB \mid XB$

$X \to AS$

$A \to AX \mid A' \mid Ab$

$A' \to a$

$B \to AX \mid A'A \mid a \mid YS \mid SB' \mid B'S \mid b \mid B'B'$

$Y \to SB'$

$B' \to b.$

12) given

$S \to 0S1 / A$.
$A \to 1A0 / S / \epsilon$.

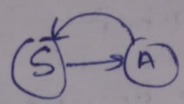
$S \to 0S1 / 01 / A$.
$A \to 1A0 / 10 / S$.

non-unit.

$S \to 0S1 / 01$
$A \to 1A0 / 10$

unit.

$S \to A$
$A \to S$



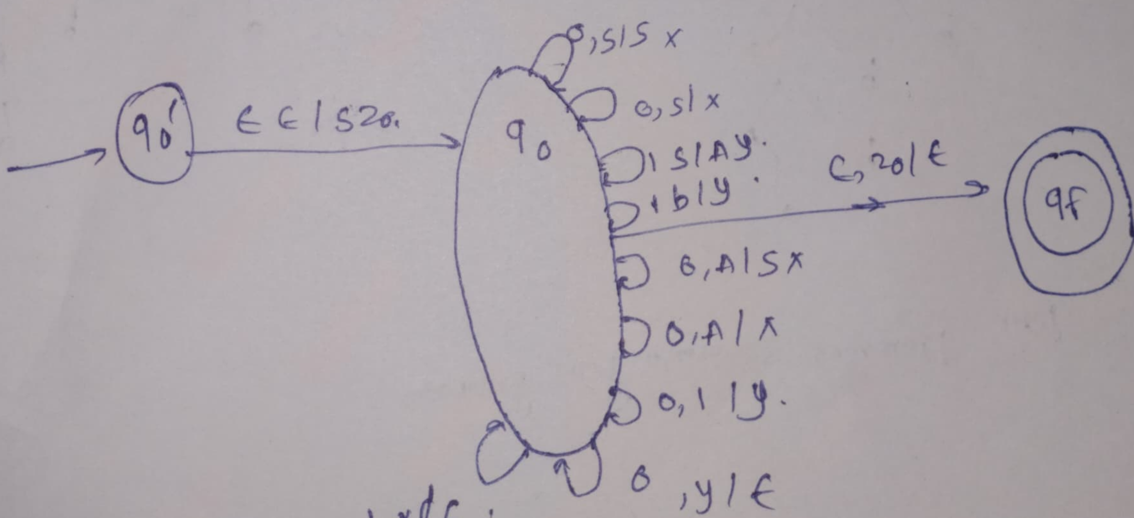
$S \to A$ give $S \to 1A0 / 10$
$A \to S$ gives $A \to 0S1 / 01$

$S \to 0S1 / 01 / 1A0 / 10$
$A \to 0S1 / 01 / 1A0 / 10$

CNF

$S \to 0SX / 0X / 1AY / 1Y$
$A \to 0SX / 0X / 1AY / 1Y$
$X = 1$
$Y \to 0$

PDA that accepts the language by empty stack is



b) Given

$S \rightarrow aB \mid bA$

$A \rightarrow a \mid aS \mid bAA$

$B \rightarrow b \mid bS \mid aBB$

String aabbaabbba

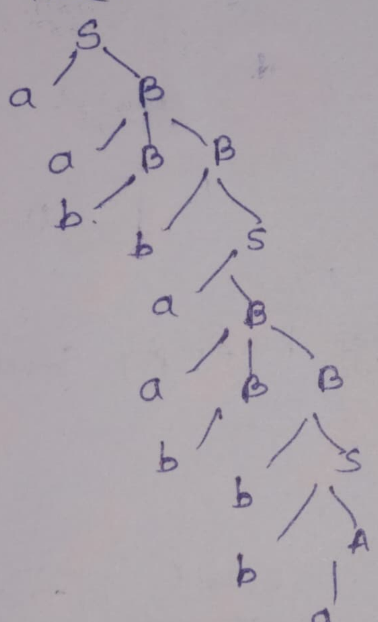
i) LMD

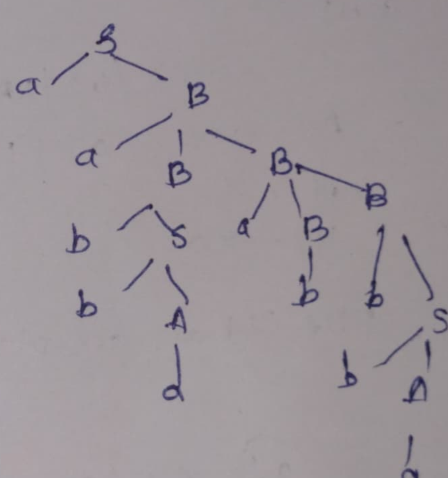
$S \rightarrow aB$
$\rightarrow aaBB$
$\rightarrow aabB$
$\rightarrow aabbS$
$\rightarrow aabbaB$
$\rightarrow aabbaaBB$
$\rightarrow aabbaabB$
$\rightarrow aabbaabbS$
$\rightarrow aabbaabbbA$
$\rightarrow aabbaabbba$

RMD

$S \rightarrow aB$
$\rightarrow aaBB$
$\rightarrow aaBbS$
$\rightarrow aaBbaB$
$\rightarrow aaBbaaBB$
$\rightarrow aaBbaaBbS$
$\rightarrow aaBbaaBbbA$
$\rightarrow aaBbaaBbba$
$\rightarrow aabbaabbba$

parse tree.





given grammar is ambiguous grammar.

14)

i) given $L = \{a^{m+n} b^m c^n \mid n, m > 0\}$

CFG $S \rightarrow A \mid B$

$A \rightarrow aA \mid abB \mid acC$

$B \rightarrow bB \mid abC$

$C \rightarrow cC \mid acB$

ii) $L = \{a^m b^n \mid m \neq \}$.

CFG:

$S \rightarrow aX \mid bX$ (or) $S \rightarrow asb \mid asa \mid bsa \mid bsb \mid a \mid b$

$X \rightarrow aS \mid bS \mid \epsilon$

iii) $L = \{0^n 1^n 2^n \quad n > 0\}$

CFG: $S \rightarrow A \mid 012$

$A \rightarrow 0AB2 \mid 012$.