

**Table 1:** Key units in Physical Chemistry

|                        |                              |   |                           |                                       |
|------------------------|------------------------------|---|---------------------------|---------------------------------------|
| $N_{\text{Av}}$ :      | $6.02214 \times 10^{23}$     | $\text{mol}^{-1}$                             |                           |                                       |
| 1 amu:                 | $1.6605 \times 10^{-27}$     | kg  |                           |                                       |
| $k_{\text{B}}$ :       | $1.38065 \times 10^{-23}$    | $\text{J K}^{-1}$                             | $8.61734 \times 10^{-5}$  | $\text{eV K}^{-1}$                    |
| $R$ :                  | 8.314472                     | $\text{J K}^{-1} \text{mol}^{-1}$             | $8.2057 \times 10^{-2}$   | $\text{l atm mol}^{-1} \text{K}^{-1}$ |
| $\sigma_{\text{SB}}$ : | $5.6704 \times 10^{-8}$      | $\text{J s}^{-1} \text{m}^{-2} \text{K}^{-4}$ |                           |                                       |
| $c$ :                  | $2.99792458 \times 10^8$     | $\text{m s}^{-1}$                             |                           |                                       |
| $h$ :                  | $6.62607 \times 10^{-34}$    | $\text{J s}$                                  | $4.13566 \times 10^{-15}$ | $\text{eV s}$                         |
| $\hbar$ :              | $1.05457 \times 10^{-34}$    | $\text{J s}$                                  | $6.58212 \times 10^{-16}$ | $\text{eV s}$                         |
| $hc$ :                 | 1239.8                       | $\text{eV nm}$                                |                           |                                       |
| $e$ :                  | $1.60218 \times 10^{-19}$    | C   |                           |                                       |
| $m_e$ :                | $9.10938215 \times 10^{-31}$ | kg  | 1: 0.5109989              | $\text{MeV c}^{-2}$                   |
| $\epsilon_0$ :         | $8.85419 \times 10^{-12}$    | $\text{C}^2 \text{J}^{-1} \text{m}^{-1}$      | $5.52635 \times 10^{-3}$  | $e^2 \text{\AA}^{-1} \text{eV}^{-1}$  |
| $e^2/4\pi\epsilon_0$ : | $2.30708 \times 10^{-28}$    | $\text{J m}$                                  | 14.39964                  | $\text{eV \AA}$                       |
| $a_0$ :                | $0.529177 \times 10^{-10}$   | m   | 0.529177                  | $\text{\AA}$                          |
| $E_{\text{H}}$ :       | 1                            | Ha  | 27.212                    | eV                                    |

## 1 The Classical Foundations

### 1.1 Lecture 0: Introduction

1. Burning lighter
2. Foundations of Physical Chemistry
  - (a) Quantum mechanics
  - (b) Statistical mechanics
  - (c) Thermodynamics, kinetics, spectroscopy
  - (d) Physical and chemical properties of matter

### 1.2 Lecture 1: Basic statistics

1. Discrete probability distributions—Coin flip
  - (a) Example of Bernoulli trial,  $2^n$  possible outcomes from  $n$  flips
  - (b) Number of ways to get  $i$  heads in  $n$  flips,  ${}_nC_i = n!/i!(n-i)!$
  - (c) Probability of  $i$  heads  $P_i \propto {}_nC_i$
  - (d) Normalized probability,  $\tilde{P}_i = P_i / \sum_i P_i = {}_nC_i / 2^n$
  - (e) Expectation value  $\langle i \rangle = \sum_i i \tilde{P}_i$
2. Continuous distributions—temperature
  - (a) Probability density  $P(x)$  has units  $1/x$
  - (b) Normalized  $\tilde{P}(x) = P(x) / \int P(x) dx$
  - (c) (Unitless) probability  $a < x < b = \int_a^b \tilde{P}(x) dx$

(d) Expectation value  $\langle f(x) \rangle = \int f(x) \tilde{P}(x) dx$

(e) Mean =  $\langle x \rangle$

(f) Mean squared =  $\langle x^2 \rangle$

(g) Variance  $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$

(h) Standard deviation  $\Delta x = \sigma$

### 3. Boltzmann distribution

(a)  $P(E) \propto e^{-E/k_B T}$ , in some sense the definition of temperature

(b) Energy and its units

(c) Absolute temperature and its units

(d)  $k_B T$  as an energy scale, 0.026 eV at 298 K

(e) Gravity example

i.  $E(h) = mgh$ , linear, continuous energy spectrum

ii. molecule vs car in a gravitational field (Table 2)

iii. Barometric law for gases,  $P = P_0 e^{-mgh/k_B T}$

(f) Kinetic energy in 1-D example

i.  $KE = \frac{1}{2} m v_x^2$

ii.  $P_{1D}(v_x) = \left( \frac{m}{2\pi k_B T} \right)^{1/2} \exp \left( -\frac{m|v_x|^2}{2k_B T} \right)$

iii. Gaussian distribution, mean  $\mu$ , variance  $\sigma^2$

$$G(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left( -\frac{(x - \mu)^2}{2\sigma^2} \right)$$

iv. By inspection,  $\mu = \langle v_x \rangle = 0$ ,  $\sigma^2 = \langle v_x^2 \rangle = k_B T/m$

v. Molecule vs car again

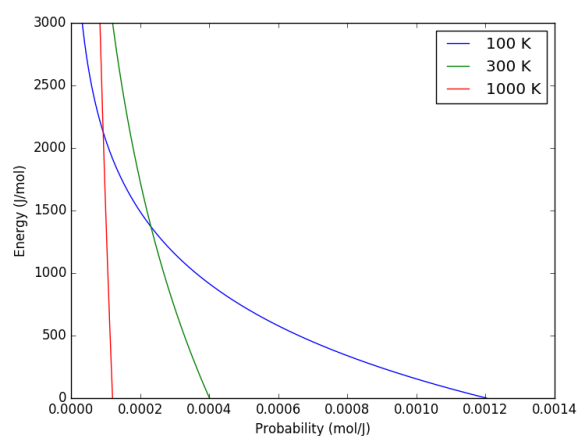
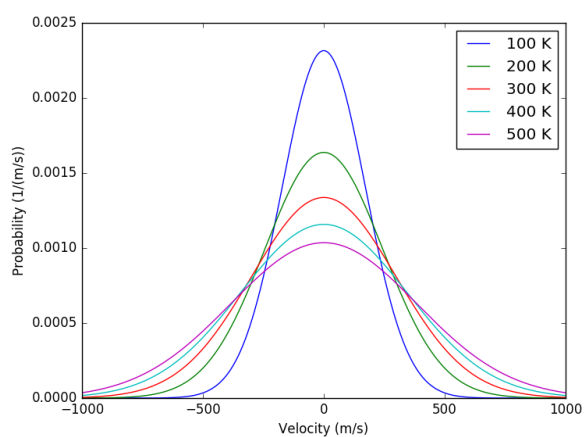
(g) Equipartition – energy freely exchanged between all degrees of freedom

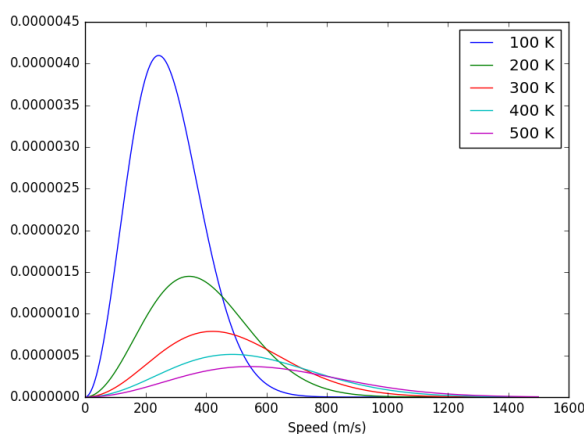
**Table 2:** Car vs gas molecule at the earth's surface

|                             | car                        | gas molecule            |
|-----------------------------|----------------------------|-------------------------|
| $m$                         | 1000 kg                    | $1 \times 10^{-26}$ kg  |
| $h$                         | 1 m                        | 1 m                     |
| $mgh$                       | 9800 J                     | $9.8 \times 10^{-26}$ J |
|                             | $6.1 \times 10^{22}$ eV    | $6.1 \times 10^{-7}$ eV |
| $T$                         | 298 K                      | 298 K                   |
| $k_B T$                     | 0.026 eV                   | 0.026 eV                |
| $mgh/k_B T$                 | $2.4 \times 10^{24}$       | $2.3 \times 10^{-5}$    |
| $P(1 \text{ m})/P(0)$       | $e^{-2.4 \times 10^{-24}}$ | 0.99998                 |
| $\langle h \rangle$         | 0 m                        | 42 km                   |
| $\langle v_x \rangle^{1/2}$ | $2 \times 10^{-12}$ m/s    | 640 m/s                 |

**Table 3:** Energy conversions and correspondences

|                          | J                        | eV                       | Hartree                  | $\text{kJ mol}^{-1}$    | $\text{cm}^{-1}$        |
|--------------------------|--------------------------|--------------------------|--------------------------|-------------------------|-------------------------|
| 1 J =                    | 1                        | $6.2415 \times 10^{18}$  | $2.2937 \times 10^{17}$  | $6.0221 \times 10^{20}$ | $5.0340 \times 10^{22}$ |
| 1 eV =                   | $1.6022 \times 10^{-19}$ | 1                        | 0.036748                 | 96.485                  | 8065.5                  |
| 1 Ha =                   | $4.3598 \times 10^{-18}$ | 27.212                   | 1                        | 2625.6                  | 219474.6                |
| 1 $\text{kJ mol}^{-1}$ = | $1.6605 \times 10^{-21}$ | 0.010364                 | $3.8087 \times 10^{-4}$  | 1                       | 83.5935                 |
| 1 $\text{cm}^{-1}$ =     | $1.986410^{-23}$         | $1.23984 \times 10^{-4}$ | $4.55623 \times 10^{-6}$ | 0.011963                | 1                       |

**Figure 1:** Boltzmann distribution at various temperatures**Figure 2:** One-dimensional (Gaussian) velocities of  $\text{N}_2$  gas



**Figure 3:** Maxwell-Boltzmann speed distribution of  $N_2$  gas

### 1.3 Lecture 2: Kinetic theory of gases

#### 1. Postulates

- (a) Gas is composed of molecules in constant random, thermal motion
- (b) Molecules only interact by perfectly elastic collisions
- (c) Volume of molecules is  $\ll$  total volume

#### 2. Maxwell-Boltzmann distribution of molecular speeds

- (a) Speed  $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$
- (b)  $P_{MB}(v)dv = P_{1D}(v_x)P_{1D}(v_y)P_{1D}(v_z) * \text{degeneracy}(v)dv$
- (c) mean speeds  $\propto \sqrt{T}$
- (d) mean energy  $U = \frac{3}{2}RT$  and heat capacity  $C_v = \frac{3}{2}R$

#### 3. Flux and pressure

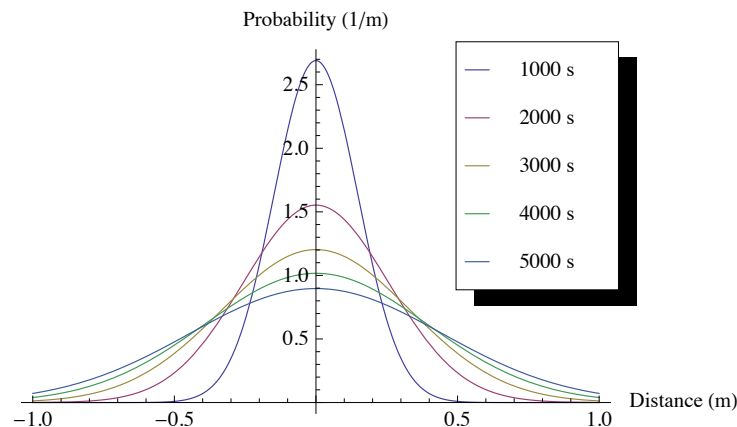
- (a) Velocity flux  $j(v_x)dv_x = v_x \frac{N}{V} P(v_x)dv_x$ , molecules /area /time  $/v_x$
- (b) Wall collisions,  $J_w$ , total collisions /area /time
- (c) Momentum exchange, pressure, ideal gas law

#### 4. Collisions and mean free path

- (a) Collision cross section  $\sigma = \pi d^2$ , size of molecule
- (b) Molecular collisions,  $z$  per molecule and  $z_{AA}$  per volume
- (c) Mean free path,  $\lambda$ , mean distance between collisions

**Table 4:** Kinetic theory of gases key equations

|  |   |
|--|---|
| Boltzmann distribution<br>( $g(E)$ : degeneracy of $E$ ) | $P(E) = g(E)e^{-E/k_B T}$   |
| Maxwell-Boltzmann distribution                           | $P_{\text{MB}}(v) = 4\pi v^2 \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_B T}\right)$                           |
| Mean and RMS speeds                                      | $\langle v \rangle = \left(\frac{8k_B T}{\pi m}\right)^{1/2} \quad \langle v^2 \rangle^{1/2} = \left(\frac{3k_B T}{m}\right)^{1/2}$   |
| Pressure   | $\langle P \rangle = \frac{\Delta p}{\Delta t} = m \frac{N}{V} \frac{1}{3} \langle v^2 \rangle = \frac{N k_B T}{V} = \frac{n R T}{V}$ |
| Wall collision frequency                                 | $J_W = \frac{1}{4} \frac{N}{V} \langle v \rangle = \frac{P}{(2\pi m k_B T)^{1/2}}$  |
| Molecular collision frequency                            | $z = \sqrt{2} \sigma \langle v \rangle \frac{N}{V} = \frac{4\sigma P}{(\pi m k_B T)^{1/2}}$   |
| Total collisions   | $z_{AA} = \frac{1}{2} \frac{N}{V} z$  |
| Mean free path   | $\lambda = \frac{\langle v \rangle}{z} = \frac{V}{\sqrt{2} \sigma N}$   |
| Graham's effusion law                                    | $\frac{dN}{dt} = \text{Area} \cdot J_w \propto 1/m^{1/2}$   |
| Effusion from a vessel                                   | $P = P_0 e^{-t/\tau}, \tau = \frac{V}{A} \left(\frac{2\pi m}{k_B T}\right)^{1/2}$   |
| Self-diffusion constant                                  | $D_{11} = \frac{1}{3} \langle v \rangle \lambda$  |
| Diffusion rate   | $\langle x^2 \rangle^{1/2} = \sqrt{2Dt} \quad \langle r^2 \rangle^{1/2} = \sqrt{6Dt}$   |
| Einstein-Smoluchowski equation                           | $D_{11} = \frac{\delta^2}{2\tau}$   |
| Stokes-Einstein equation for liquids                     | $D_{11} = \frac{k_B T}{4\pi\eta r}$ "Slip" boundary   |
|  | $D_{\text{Brownian}} = \frac{k_B T}{6\pi\eta r}$ "Stick" boundary   |



**Figure 4:** Diffusional spreading,  $\sqrt{\langle x^2 \rangle} = \sqrt{2Dt}$

### 1.4 Lecture 3: Transport

1. Effusion and Graham's law, effusion rate  $\propto MW^{-1/2}$
2. Fick's first law: net flux proportional to concentration gradient
  - (a)  $j_x = -D \frac{dc}{dx}$
  - (b) Self-diffusion constant,  $D = \frac{1}{3} \lambda \langle v \rangle$
3. Knudsen diffusion,  $D = \frac{1}{3} l \langle v \rangle$
4. Fick's second law: time evolution of concentration gradient
  - (a) Continuity with no advection:  $\frac{\partial c}{\partial t} = -\nabla \cdot \vec{j} + \text{gen}$
  - (b) One-dimension:  $\frac{dc}{dt} = D \frac{d^2 c}{dx^2}$
  - (c) Diffusion has Gaussian probability distribution:  $c(x, t)/c_0 = [2\sqrt{\pi Dt}]^{-1} \exp(-x^2/4Dt)$
5. Seeing is believing—Brownian motion
  - (a) Seemingly random motion of large particles (“dust”) due to “kicks” from invisible molecules
  - (b) Einstein receives Nobel Prize for showing:
    - i. Motion follows same Gaussian diffusion behavior
    - ii. From steady-state arguments in a field, diffusion constant is ratio of Boltzmann energy,  $k_B T$ , to mobility
    - iii. Mobility inversely related to viscosity
  - (c) Stokes-Einstein equation
  - (d) Allows measurement of Avogadro's number, final proof of kinetic theory
  - (e) Similar model for diffusion of liquid molecules, slip boundary
6. Random walk model of diffusion
  - (a) Binomial distribution
  - (b) Large  $N$  and Stirling approximation
  - (c) Einstein-Smoluchowski relation

**Table 5:** Classical waves

|                       |   |
|-----------------------|---|
| The wave equation     | $\frac{\partial^2 \Psi(x, t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \Psi(x, t)}{\partial t^2}$ |
| General solution      | $\Psi(x, t) = A \sin(kx - \omega t)$  |
| Wavelength (distance) | $\lambda = 2\pi/k$  |
| Frequency (/time)     | $\nu = \omega/2\pi$   |
| Speed                 | $v = \lambda\nu$  |
| Amplitude (distance)  | $A$   |
| Energy                | $E \propto A^2$   |
| Standing wave         | $\Psi(x, t) = A \sin(kx) \cos(\omega t), \quad k = n\pi/a$  |

## 2 Quantum Mechanics: Blurred Lines Between Particles and Waves

### 2.1 Lecture 4: Duality and demise of classical physics

#### 2.1.1 Properties of waves

1. Traveling waves, standing waves
2. interference, diffraction
3. Expected energy of a classical oscillator,  $\langle \epsilon \rangle_\nu = k_B T$  for all  $\nu$

#### 2.1.2 Blackbody radiation

1. Hohlraum spectrum (like the sun) empirically observed to obey:
  - (a) Stefan-Boltzmann law, total irradiance
  - (b) Wien's displacement law
2. Rayleigh-Jeans predicts spectrum using classical physics
  - (a) standing waves + classical oscillators  $\rightarrow$  ultraviolet catastrophe
3. Planck model
  - (a) Energy spectrum of oscillators are *quantized*,  $\epsilon_\nu = nh\nu$
  - (b) Expected energy of a quantized oscillator,  $\langle \epsilon \rangle_\nu = h\nu / (e^{h\nu/k_B T} - 1)$
  - (c) Correctly reproduces Stefan-Boltzmann and Wien Laws!

#### 2.1.3 Heat capacities of solids

1. Law of DuLong and Pettite,  $C_v = 3R$ , fails at low  $T$
2. Einstein model
  - (a) Atomic vibrations are *quantized*,  $\epsilon_n = nh\nu$
  - (b) Heat capacity goes to zero at low  $T$

**2.1.4 Photoelectric effect**

1. Stopping potential and work function,  $E_{\text{kinetic}} = h\nu - W$
2. Kinetic energy varies with light frequency, number of electrons varies with light intensity

**2.1.5 Compton effect**

1. light scattering of electrons changes  $\lambda$
2. Photon properties,  $\epsilon = h\nu, p = h/\lambda$

**2.1.6 Wave-particle duality****2.1.7 Rutherford, planetary model of atom**

1. Inconsistent with Maxwell's equations

**2.1.8 Bohr model of H atom**

1. Discrete H energy spectrum and Rydberg formula
2. Bohr model (the old quantum mechanics)
  - (a) Stable electron "orbits," quantized angular momentum
  - (b) Light emission corresponds to orbital jumps,  $\nu = \Delta E/h$
  - (c) Bohr equations
  - (d) Comparison with Rydberg formula
  - (e) Failure for larger atoms

**2.1.9 de Broglie relation**

1.  $\lambda = h/p$  *universally*
2. Relation to Bohr orbits
3. Davison and Germer experiment,  $e^-$  diffraction off Ni

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```

1  import numpy as np
2  import matplotlib.pyplot as plt
3
4  hc = 1239.8          eV nm
5  c = 2.9979e8 * 1.e9  nm/s
6  k = 8.61734e5       eV /K
7  hck = hc/k          nm K
8
9  def Irrad(wl,T):
10     return (8. * np.pi * hc * c * wl**5) / (np.exp(hck/(wl*T))1)
11  def PlanckEnergy(wl,T):
12     return (hc/wl) / (np.exp(hck/(wl*T))1)
13
14  plt.figure()
15  wl=np.linspace(100,5000,1000)
16  for T in [1000.,2000.,3000.,4000.,5000.]:
17     Intensity = Irrad(wl,T)
18     plt.plot(wl,Intensity,label= K.format(T))

```

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```

19
20 legend=plt.legend()
21 plt.xlabel(Wavelength (nm))
22 plt.ylabel(Irradiance (eV/nm3/s))
23 plt.title(Boltzmann distribution at various temperatures)
24 plt.savefig(./Images/BlackBody.png)
25
26 plt.figure()
27 color=[red,orange,green,blue,violet]
28 wl=np.linspace(100,20000,1000)
29 for T in [1000.,2000.,3000.,4000.,5000.]:
30     Energy = PlanckEnergy(wl,T)
31     plt.plot(wl,Energy,label= K.format(T),color=color[0])
32     kT = k*T
33     plt.plot([100,max(wl)], [kT,kT],ls=,color=color.pop(0))
34
35 legend=plt.legend()
36 plt.xlabel(Wavelength (nm))
37 plt.ylabel(Energy (eV))
38 plt.title(Boltzmann distribution at various temperatures)
39 plt.savefig(./Images/Planck.png)

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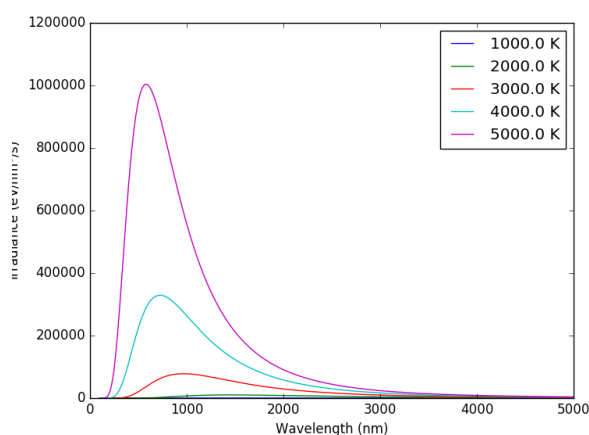


Figure 5: Blackbody irradiance

## 2.2 Lecture 5: Postulates of quantum mechanics

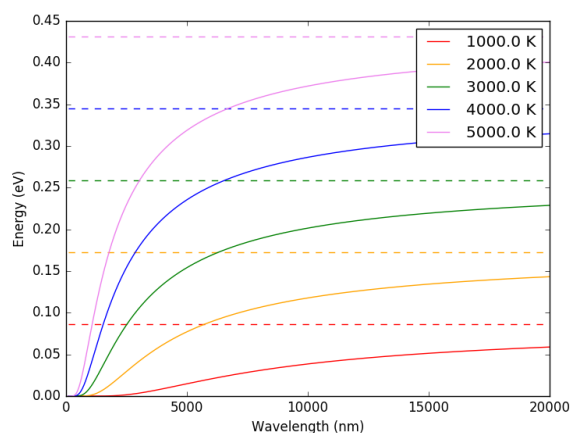
### 2.2.1 Schrödinger equation describes wave-like properties of matter

### 2.2.2 Born interpretation

1. wavefunction is a probability amplitude
2. wavefunction squared is probability density

### 2.2.3 Postulates

1. Wavefunction contains all information about a system
2. Operators used to extract that information
  - (a) QM operators are *Hermitian*



**Figure 6:** Average energy of a Planck quantized oscillator

**Table 6:** The new physics

|                      |  |
|----------------------|--|
| Stefan-Boltzmann Law | $\int I(\lambda, T) d\lambda = \sigma_{\text{SB}} T^4$                                       |
| Wien's Law           | $\lambda_{\text{max}} T = 2897768 \text{ nm K}$  |
| Rayleigh-Jeans eq    | $I(\lambda, T) = \frac{8\pi}{\lambda^4} k_B T c$   |
| Blackbody irradiance | $I(\lambda, T) = \frac{8\pi}{\lambda^5} \frac{hc^2}{e^{hc/\lambda k_B T} - 1}$               |
| Einstein crystal     | $C_v = 3R \left( \frac{h\nu}{k_B T} \right)^2 \frac{e^{h\nu/k_B T}}{(e^{h\nu/k_B T} - 1)^2}$ |
| Photon energy        | $\epsilon = h\nu = hc/\lambda$   |
| Rydberg equation     | $\nu = R_H c \left( 1/n^2 - 1/k^2 \right)$   |
| Bohr equations       | $l_n = n\hbar$   |
| $n = 1, 2, \dots$    | $r_n = n^2 \left( \frac{4\pi\epsilon_0 \hbar^2}{e^2 m_e} \right) = n^2 a_0$                  |
|                      | $E_n = -\frac{m_e e^4}{8\epsilon_0^2 h^2} \frac{1}{n^2} = -\frac{E_H}{2} \frac{1}{n^2}$      |
|                      | $p_n = \frac{e^2}{4\pi\epsilon_0} \frac{m_e}{\hbar} \frac{1}{n} = p_0 \frac{1}{n}$           |
| de Broglie equation  | $\lambda = \frac{h}{p}$  |

- (b) Have eigenvectors and real eigenvalues,  $\hat{O}\psi_i = o\psi_i$
  - (c) Are orthogonal,  $\langle\psi_i|\psi_j\rangle = \delta_{ij}$
  - (d) Always observe an eigenvalue when making an observation
3. Expectation values
  4. Energy-invariant wavefunctions given by Schrödinger equation
  5. Uncertainty principle

## 2.2.4 Particle in a box illustrations

## 2.3 Lecture 6: Particle in a box model

### 2.3.1 Particle between infinite walls, electron confined in a wire

### 2.3.2 Classical solution, either stationary or uniform bouncing back and forth

### 2.3.3 One-dimensional QM solutions

1. Schrödinger equation and boundary conditions
2. discrete, quantized solutions
3. standing waves,  $\lambda = 2L/n$ ,  $n - 1$  nodes, non-uniform probability
4. [Ho paper](#), STM of Pd wire
5. zero point energy and uncertainty
6. correspondence principle
7. superpositions

### 2.3.4 Finite walls and tunneling

1. Potential well of finite depth  $V_0$
2. Finite number of bound states
3. Classical region,  $\psi(x) e^{ikx} + e^{-ikx}$ ,  $k = \sqrt{2mE}/\hbar$
4. “Forbidden” region,  $\psi(x) e^{\kappa x} + e^{-\kappa x}$ ,  $\kappa = \sqrt{2m(V_0 - E)}/\hbar$
5. Non-zero probability to “tunnel” into forbidden region
6. Tunneling between two adjacent wells: chemical bonding, STM, nanoelectronics
7. H atom tunneling:  $\text{NH}_3$  inversion, H transfer, kinetic isotope effect

### 2.3.5 Multiple dimensions

1. separation of variables, one quantum number for each dimension

**Table 7:** Postulates of Non-relativistic Quantum Mechanics

**Postulate 1: The physical state of a system is completely described by its wavefunction  $\Psi$ .** In general,  $\Psi$  is a complex function of the spatial coordinates and time.  $\Psi$  is required to be:

- I. Single-valued
- II. continuous and twice differentiable
- III. square-integrable ( $\int \Psi^* \Psi d\tau$  is defined over all finite domains)
- IV. For bound systems,  $\Psi$  can always be normalized such that  $\int \Psi^* \Psi d\tau = 1$

**Postulate 2:** To every physical observable quantity  $M$  there corresponds a Hermitian operator  $\hat{M}$ . **The only observable values of  $M$  are the eigenvalues of  $\hat{M}$ .**

| Physical quantity             | Operator                    | Expression  |
|-------------------------------|-----------------------------|---|
| Position $x, y, z$            | $\hat{x}, \hat{y}, \hat{z}$ | $x, y, z$   |
| Linear momentum $p_x, \dots$  | $\hat{p}_x, \dots$          | $-i\hbar \frac{\partial}{\partial x}, \dots$  |
| Angular momentum $l_x, \dots$ | $\hat{p}_x, \dots$          | $-i\hbar \left( y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right), \dots$ |
| Kinetic energy $T$            | $\hat{T}$                   | $-\frac{\hbar^2}{2m} \nabla^2$  |
| Potential energy $V$          | $\hat{V}$                   | $V(\mathbf{r}, t)$  |
| Total energy $E$              | $\hat{H}$                   | $-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}, t)$   |

**Postulate 3:** If a particular observable  $M$  is measured many times on many identical systems in a state  $\Psi$ , the average results will be the expectation value of the operator  $\hat{M}$ :

$$\langle M \rangle = \int \Psi^* (\hat{M} \Psi) d\tau$$

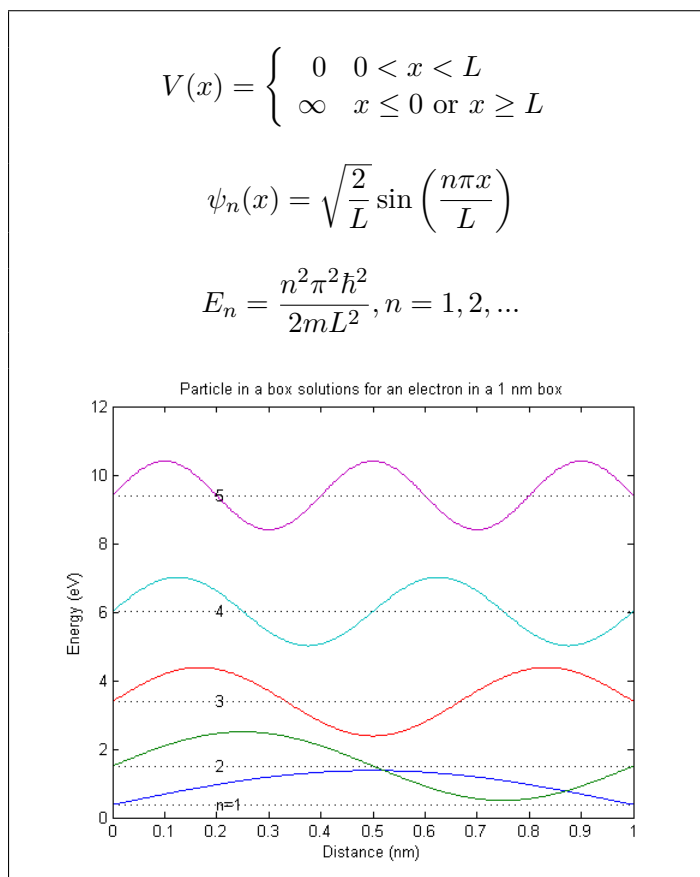
**Postulate 4:** The energy-invariant states of a system are solutions of the equation

$$\begin{aligned} \hat{H} \Psi(\mathbf{r}, t) &= i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) \\ \hat{H} &= \hat{T} + \hat{V} \end{aligned}$$

The time-independent, stationary states of the system are solutions to the equation

$$\hat{H} \Psi(\mathbf{r}) = E \Psi(\mathbf{r})$$

**Postulate 5:** (The **uncertainty principle**.) Operators that do not commute ( $\hat{A}(\hat{B}\Psi) \neq \hat{B}(\hat{A}\Psi)$ ) are called *conjugate*. Conjugate observables cannot be determined simultaneously to arbitrary accuracy. For example, the standard deviation in the measured positions and momenta of particles all described by the same  $\Psi$  must satisfy  $\Delta x \Delta p_x \geq \hbar/2$ .

**Table 8:** Particle-in-a-box model**2.3.6 Introduce Pauli principle for fermions?****2.4 Lecture 7: Harmonic oscillator****2.4.1 Classical harmonic oscillator**

1. Hooke's law,  $F = -k(x - x_0)$ ,  $k$  spring constant
2. Continuous sinusoidal motion
3.  $x(t) = A \sin(\frac{k}{\mu})^{1/2} t$ ,  $\nu = \frac{1}{2\pi}(\frac{k}{\mu})^{1/2}$ ,  $E = \frac{1}{2}kA^2$
4. Exchanging kinetic and potential energies

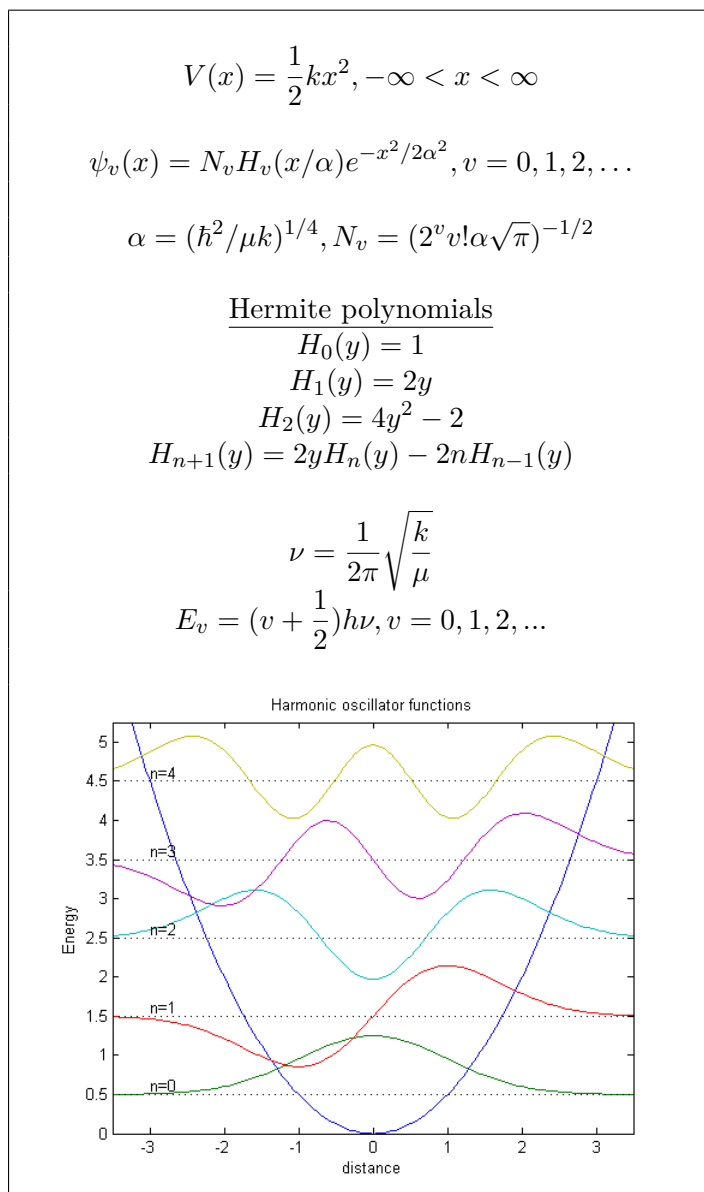
**2.4.2 Quantum harmonic oscillator**

1. Solutions like P-I-A-B, waves, nodes, even/odd symmetry
2. Zero-point energy
3. Expectation values:  $\langle x \rangle = 0$ ,  $\langle x^2 \rangle = \alpha^2(v + 1/2)$ ,  $\langle V(x) \rangle = \frac{1}{2}h\nu(v + \frac{1}{2})$
4. Classical turning point and tunneling

## 5. Classical limiting behavior

## 2.4.3 HCl example

1. Reduced mass,  $\frac{1}{\mu} = \frac{1}{m_A} + \frac{1}{m_B}$
2. ZPE, energy spacing in IR, Boltzmann probabilities

**Table 9:** Harmonic oscillator model

## 2.5 Lecture 8: Rigid Rotor

### 2.5.1 Classical rigid rotor

1. Compare rotation about an axis vs linear motion
2. Moment of inertia  $I = \mu r^2$
3. Angular momentum,  $\mathbf{l} = I\omega = \mathbf{r} \times \mathbf{p}$ ,  $T = l^2/2I$ 
  - (a) Angular momentum and energy continuous variables

### 2.5.2 Quantum rotor in a plane

1. Angular momentum and kinetic energy operators in polar coordinates,  $\hat{l}_z = -i\hbar \frac{d}{d\phi}$
2. Eigenfunctions degenerate, cw and ccw rotation
3. No zero point energy
4. Angular momentum eigenfunctions,  $l_z = m_l \hbar$
5. Energy superpositions and localization

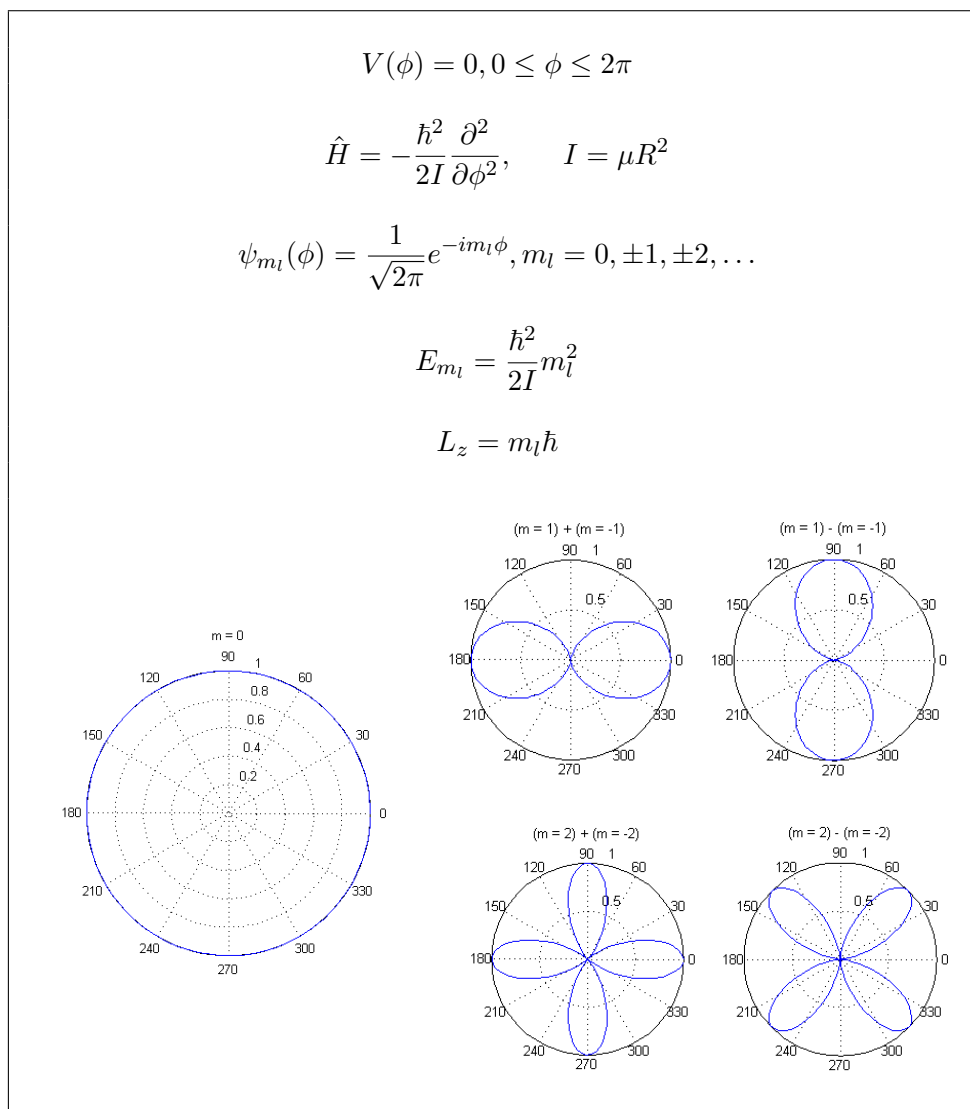
### 2.5.3 Quantum rotor in 3-D

1. Angular momentum and kinetic energy operators in spherical coordinates
2. Spherical harmonic solutions,  $Y_{lm_l}$
3. Azimuthal QN  $l = 0, 1, \dots$
4. Magnetic QN  $m_l = -l, -l+1, \dots, l$
5. Energy spectrum,  $2l+1$  degeneracy
6. Vector model - can only know total  $|L|$  and  $L_z$
7. Wavefunctions look like atomic orbitals,  $l$  nodes

---

```

1  import matplotlib.pyplot as plt
2  from matplotlib import cm, colors
3  from mpl_toolkits.mplot3d import Axes3D
4  import numpy as np
5  from scipy.special import sphharm
6
7  phi = np.linspace(0, np.pi, 100)
8  theta = np.linspace(0, 2*np.pi, 100)
9  phi, theta = np.meshgrid(phi, theta)
10
11  The Cartesian coordinates of the unit sphere
12  x = np.sin(phi) * np.cos(theta)
13  y = np.sin(phi) * np.sin(theta)
14  z = np.cos(phi)
15
16  m, l = 0, 0
17
```

**Table 10:** 2-D rigid rotor model

```

18  Calculate the spherical harmonic Y(l,m) and normalize to [0,1]
19  fcolors = sphharm(m, l, theta, phi).real
20  fmax, fmin = fcolors.max(), fcolors.min()
21  fcolors = (fcolors - fmin)/(fmax - fmin)
22
23  Set the aspect ratio to 1 so our sphere looks spherical
24  sfig = plt.figure(figsize=plt.figaspect(1.))
25  s = sfig.add_subplot(111, projection='3d')
26  s.plotsurface(x, y, z, rstride=1, cstride=1, facecolors=cm.seismic(fcolors))
27  Turn off the axis planes
28  s.setaxisoff()
29  plt.savefig('./Images/s.png')
30
31  m, l = 0, 1
32
33  Calculate the spherical harmonic Y(l,m) and normalize to [0,1]
34  fcolors = sphharm(m, l, theta, phi).real
35  fmax, fmin = fcolors.max(), fcolors.min()

```



**Table 11:** 3-D rigid rotor model

$$\begin{aligned}
 V(\theta, \phi) &= 0, 0 \leq \phi \leq 2\pi, 0 \leq \theta < \pi \\
 \hat{L}^2 &= -\hbar^2 \left[ \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) \right] \\
 \hat{H}_{\text{rot}} &= \frac{1}{2I} \hat{L}^2 \\
 Y_{lm_l}(\theta, \phi) &= N_l^{|m_l|} P_l^{|m_l|}(\cos(\theta)) e^{im_l \phi} \\
 l &= 0, 1, 2, \dots, \quad m_l = 0, \pm 1, \dots, \pm l \\
 E_l &= \frac{\hbar^2}{2I} l(l+1) \\
 |L| &= \hbar \sqrt{l(l+1)}, L_z = m_l \hbar
 \end{aligned}$$

```

36 fcolors = (fcolors - fmin)/(fmax - fmin)
37
38 Set the aspect ratio to 1 so our sphere looks spherical
39 fig = plt.figure(figsize=plt.figaspect(1.))
40 pfig = plt.figure(figsize=plt.figaspect(1.))
41 p = pfig.add_subplot(111, projection='3d')
42 p.plot_surface(x, y, z, rstride=1, cstride=1, facecolors=cm.seismic(fcolors))
43 Turn off the axis planes
44 p.setaxisoff()
45
46 plt.savefig('./Images/p.png')
47
48 m, l = 1, 2
49
50 Calculate the spherical harmonic Y(l,m) and normalize to [0,1]
51 fcolors = sphharm(m, l, theta, phi).real
52 fmax, fmin = fcolors.max(), fcolors.min()
53 fcolors = (fcolors - fmin)/(fmax - fmin)
54
55 Set the aspect ratio to 1 so our sphere looks spherical
56 fig = plt.figure(figsize=plt.figaspect(1.))
57 dfig = plt.figure(figsize=plt.figaspect(1.))
58 d = dfig.add_subplot(111, projection='3d')
59 d.plot_surface(x, y, z, rstride=1, cstride=1, facecolors=cm.seismic(fcolors))
60 Turn off the axis planes
61 d.setaxisoff()
62
63 plt.savefig('./Images/d.png')

```

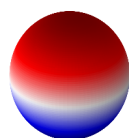
## 2.5.4 Particle angular momentum

### 1. Fermions, mass, half-integer spin

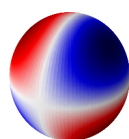
(a) Electron,  $s = 1/2, m_s = \pm 1/2$



**Figure 7:** Pythonic s spherical harmonic



**Figure 8:** Pythonic p spherical harmonic



**Figure 9:** Pythonic d spherical harmonic

2. Bosons, force-carrying, integer spin

## 2.6 Lecture 9: Spectroscopy

### 2.6.1 Spectroscopy is quantitative measurement of interaction of light with matter

1. Observed  $I(\nu)/I(\nu_0)$
2. Bohr condition,  $|E_f - E_i|/h = \nu = c\tilde{\nu} = c/\lambda$
3. Intensities determined by state populations and transition probabilities

### 2.6.2 Einstein coefficients

1. Stimulated absorption,  $dn_1/dt = -n_1 B \rho(\nu)$
2. Stimulated emission,  $dn_2/dt = -n_2 B \rho(\nu)$
3. Spontaneous emission,  $dn_2/dt = -n_2 A$ ,  $A = \left(\frac{8\pi h \nu^3}{c^3}\right) B$
4.  $1/A = \text{lifetime}$

### 2.6.3 Transition probability

1. Einstein coefficient  $B_{if} = \frac{|\mu_{if}|^2}{6\epsilon_0 h^2}$
2. Classical electric dipole,  $\vec{\mu} = q \cdot \vec{r}$ , quantum dipole operator  $\hat{\mu} = e \cdot \vec{r}$
3. Transition dipole moment,  $\mu_{if} = \left(\frac{d\mu}{dx}\right) \langle \psi_i | \hat{\mu} | \psi_f \rangle$
4. Selection rules—conditions that make  $\mu_{if}$  non-zero, “allowed” vs “forbidden” transitions

## 2.7 Lecture 10: Vibrational and rotational spectroscopy

### 2.7.1 Diatomic rotational spectroscopy

1. Rotational constant  $B = \hbar/4\pi I c \text{ cm}^{-1}$ ,  $I = \mu R^2$
2. Gross selection rule: dynamic dipole moment non-zero (heteronuclear, not homonuclear)
3. Specific selection rule:  $\Delta l = \pm 1$ ,  $\Delta m_l = 0, \pm 1$
4.  $\Delta \tilde{E}_l = 2B(l+1) \text{ cm}^{-1}$
5. Rotational state populations

### 2.7.2 Diatomic vibrational transitions

1. Gross selection rule: dynamic dipole  $d\mu/dx$  non-zero
2. Homo- vs. heteronuclear
3. Specific selection rule: dipole integral  $\langle \psi_v | \hat{\mu} | \psi_{v'} \rangle = 0$  unless  $\Delta v = \pm 1$
4. Allowed  $\Delta E = h\nu$
5. Boltzmann distribution implies  $v = 1$  states dominate at normal  $T$

### 2.7.3 Raman spectroscopy

1. Shine in light of arbitrary frequency  $\tilde{\nu}_0$ , mostly get out the same
2. Some light comes out at  $\tilde{\nu}_0 - \tilde{\nu}$  (Stoke's line)
3. Some light comes out at  $\tilde{\nu}_0 + \tilde{\nu}$  (anti-Stoke's line)
4. Gross selection rule: dynamic polarizability non-zero (homonuclear, not heteronuclear)

### 2.7.4 Anharmonicity, Morse potential

### 2.7.5 Vibration-rotation spectroscopy

1. Harmonic oscillator + rigid rotor
2. Selection rules:  $\Delta v = \pm 1, \Delta l = \pm 1$
3. *R* branch:  $\Delta \tilde{E} = \tilde{\nu} + 2B(l+1), \Delta l = 1$
4. *P* branch:  $\Delta \tilde{E} = \tilde{\nu} - 2B(l), \Delta l = -1$

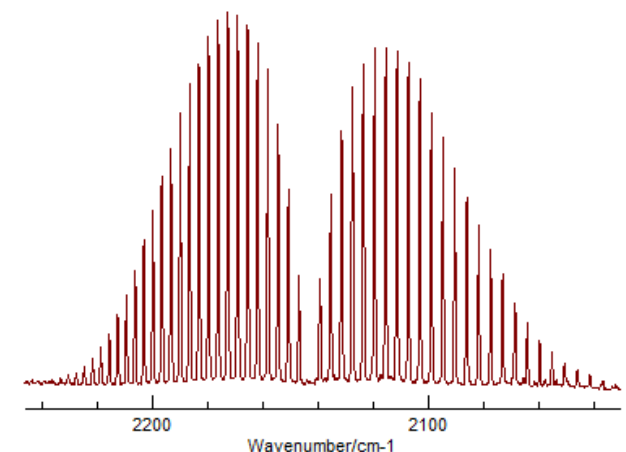


Figure 10: Rovibrational spectrum of carbon monoxide

### 2.7.6 Polyatomic vibrational spectroscopy

1. Polyatomics,  $3n - 6$  ( $3n - 5$  for linear polyatomic) vibrational modes
2. Selection rules and degeneracies affect number of observed features
3. CO<sub>2</sub> example

### 2.7.7 Polyatomic rotational spectroscopy

1. Three distinct moments of inertia ( $I_x, I_y, I_z$ )
2. Spectra more complex

## 2.8 Lecture 11: Hydrogen atom

### 2.8.1 Schrödinger equation

1. Spherical coordinates and separation of variables
2. Coulomb potential  $v_{\text{Coulomb}}(r) = -\frac{e^2}{4\pi\epsilon_0} \frac{1}{r}$
3. Centripetal potential  $v = \hbar^2 \frac{l(l+1)}{2\mu r^2}$

### 2.8.2 Solutions

1.  $\psi(r, \theta, \phi) = R_{nl}(r)Y_{lm}(\theta, \phi)$
2. Principle quantum number  $n = 1, 2, \dots$ 
  - (a)  $K, L, M, N, \dots$  shells
  - (b)  $n - 1$  radial nodes
3. Azimuthal quantum number  $l = 0, 1, \dots, n - 1$ 
  - (a)  $s, p, d, \dots$  orbital sub-shells
  - (b)  $l$  angular nodes
4. Magnetic quantum number  $m_l = -l, -l + 1, \dots, l$
5. Spin quantum number  $m_s = \pm 1/2$
6. Energy spectrum and populations
7. Electronic selection rules
  - (a)  $\Delta l = \pm 1 \quad \Delta m_s = 0 \quad \Delta m_l = 0, \pm 1$
8. Wavefunctions = “orbitals”
9. Radial probability function  $P_{nl}(r) = r^2 R_{nl}^2(r)$ 
  - (a)  $\langle r \rangle = \int r P_{nl}(r) dr = \left( \frac{3}{2} n^2 - l(l+1) \right) a_0$

### 2.8.3 Variational principle

1. Solutions of Schrödinger equation always form a complete set
2. True wavefunction energy is therefore lower bound on energy of any trial wavefunction

$$\langle \psi_{\text{trial}}^\lambda | \hat{H} | \psi_{\text{trial}}^\lambda \rangle = E_{\text{trial}}^\lambda \geq E_0$$

1. Optimize wavefunction with respect to variational parameter

$$\left( \frac{\partial \langle \psi_{\text{trial}}^\lambda | \hat{H} | \psi_{\text{trial}}^\lambda \rangle}{\partial \lambda} \right) = 0 \rightarrow \lambda_{\text{opt}}$$

**Table 12:** Hydrogen atom

$$\begin{aligned}
 V(r) &= -\frac{e^2}{4\pi\epsilon_0} \frac{1}{r}, 0 < r < \infty \\
 \hat{H} &= -\frac{\hbar^2}{2m_e} \frac{1}{r^2} \left[ \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \hat{L}^2 \right] + V(r) \\
 \psi(r, \theta, \phi) &= R(r) Y_{l, m_l}(\theta, \phi) \\
 \left\{ -\frac{\hbar^2}{2m_e} \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d}{dr} \right) + \frac{\hbar^2 l(l+1)}{2m_e r^2} - \frac{e^2}{4\pi\epsilon_0} \frac{1}{r} \right\} R(r) &= E R(r) \\
 R_{nl}(r) &= N_{nl} e^{-x/2} x^l L_{nl}(x), \quad x = \frac{2r}{na_0} \\
 P_{nl}(r) &= r^2 R_{nl}^2 \\
 n &= 1, 2, \dots, \quad l = 0, \dots, n-1 \quad m_l = 0, \pm 1, \dots, \pm l \\
 N_{nl} &= \sqrt{\left( \frac{2}{na_0} \right)^3 \frac{(n-l-1)!}{2n(n+l)!}} \\
 L_{10} = L_{21} = L_{32} = \dots &= 1 \quad L_{20} = 2 - x \quad L_{31} = 4 - x \\
 E_n &= -\frac{1}{2} \frac{\hbar^2}{m_e a_0^2} \frac{1}{n^2} = -\frac{E_H}{2} \frac{1}{n^2} \\
 |L| &= \hbar \sqrt{l(l+1)}, L_z = m_l \hbar
 \end{aligned}$$

## 2.9 Lecture 12: Many-electron atoms

### 2.9.1 Many-electron problem, Schrödinger equation not exactly solvable

1.  $e^- - e^-$  interaction terms prevent separation of variables
2. Independent electron model basis of all solutions, describes each electron by its own wavefunction, or “orbital”

### 2.9.2 Qualitative solutions

1.  $\psi_i$  look like H atom orbitals, labeled by same quantum numbers
2. *Aufbau principle*: “Build-up” electron configuration by adding electrons into H-atom-like orbitals, from bottom up

3. *Pauli exclusion principle*: Every electron in atom must have a unique set of quantum numbers, so only two per orbital (with opposite spin)
4. *Pauli exclusion principle (formally)*: The wavefunction of a multi-particle system must be anti-symmetric to coordinate exchange if the particles are fermions, and symmetric to coordinate exchange if the particles are bosons
5. *Hund's rule*: Electrons in degenerate orbitals prefer to be spin-aligned. Configuration with highest *spin multiplicity* is the most preferred

### 2.9.3 Structure of the periodic table

1. Electrons in different subshells experience different effective nuclear charge  $Z_{\text{eff}} = Z - \sigma_{nl}$
2. Inner (“core”) shells not shielded well at
3. Inner shell electrons “shield” outer electrons well
4. Within a shell, *s* shielded less than *p* less than *d* . . . , causes degeneracy to break down
5. Electrons in same subshell shield each other poorly, causing ionization energy to increase across the subshell

### 2.9.4 Quantitative solutions

1. Schrödinger equation

$$\hat{H}\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots) = E\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots)$$

$$\hat{H} = \sum_i \hat{h}_i + \frac{e^2}{4\pi\epsilon_0} \sum_i \sum_{j>i} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}$$

$$\hat{h}_i = -\frac{\hbar^2}{2m_e} \nabla_i^2 - \frac{Ze^2}{4\pi\epsilon_0} \frac{1}{|\mathbf{r}_i|}$$

2. Construct candidate many-electron wavefunction  $\Psi$  from one electron wavefunctions (mathematical details vary with exact approach)

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots) \approx \psi_1(\mathbf{r}_1)\psi_2(\mathbf{r}_2)\dots\psi_n(\mathbf{r}_n)$$

3. Calculate expectation value of  $E$  of approximate model and apply *variational principle* to find equations that describe “best” (lowest total energy) set of  $\psi_i$

$$\frac{\partial E}{\partial \psi_i} = 0 \quad \forall i$$

$$\hat{f}\psi = \left\{ \hat{h} + \hat{v}_{\text{Coul}}[\psi_i] + \hat{v}_{\text{ex}}[\psi_i] + \hat{v}_{\text{corr}}[\psi_i] \right\} \psi = \epsilon\psi$$

$$E = \sum_i \epsilon_i - \frac{1}{2} \langle \Psi | \hat{v}_{\text{Coul}}[\psi_i] + \hat{v}_{\text{ex}}[\psi_i] + \hat{v}_{\text{corr}}[\psi_i] | \Psi \rangle$$

4. Motivate as equation for an electron moving in a “field” of other electrons, adding an electron to a known set of  $\psi_i$

### 2.9.5 Electron-electron interactions

1. Coulomb ( $\hat{v}_{\text{Coul}}$ ): classical repulsion between distinguishable electron “clouds”
2. Exchange ( $\hat{v}_{\text{ex}}$ ): accounts for electron indistinguishability (Pauli principle for fermions). Decreases Coulomb repulsion because electrons of like spin intrinsically avoid one another
3. Correlation ( $\hat{v}_{\text{corr}}$ ): decrease in Coulomb repulsion due to dynamic ability of electrons to avoid one another; “fixes” orbital approximation
4. General form of exchange potential is expensive to calculate; general form of correlation potential is unknown

### 2.9.6 Popular models

1. *Hartree model*: Include only classical Coulomb repulsion  $\hat{v}_{\text{Coul}}$
2. *Hartree-Fock model*: Include Coulomb and exchange
3. *Density-functional theory* (DFT): Include Coulomb and approximate expressions for exchange and correlation
4. All the potential terms  $\hat{v}$  depend on the solutions, so equations must be solved *iteratively* to *self-consistency*

### 2.9.7 DFT calculations on atoms

1. See <http://www.chemsoft.ch/qc/fda.htm>

## 2.10 Lecture 13: Molecular orbital theory of molecules

### 2.10.1 Clamped nucleus (“Born-Oppenheimer”) approximation

1. Write one-electron equations parametrically in terms of positions of all atoms

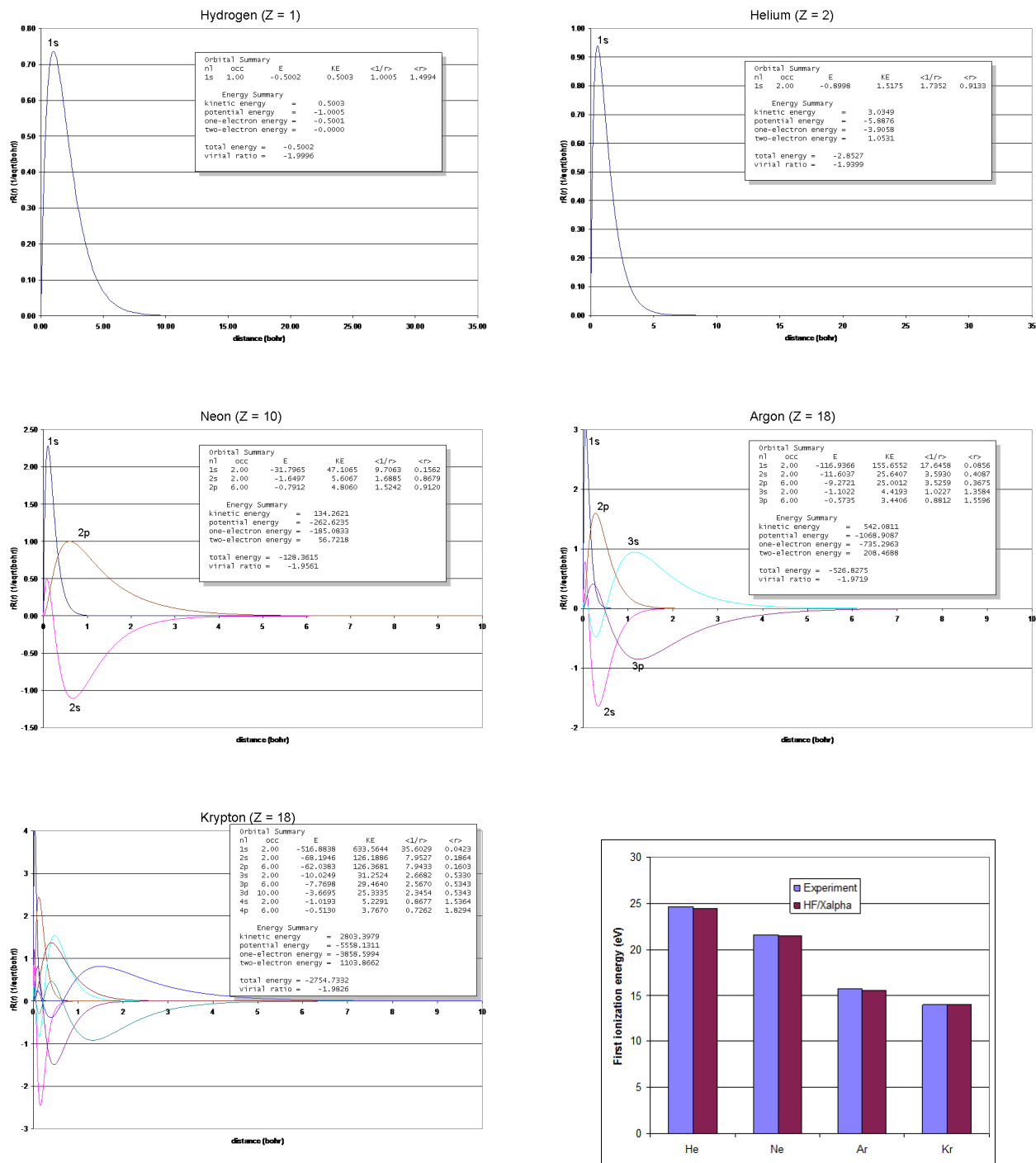
$$\hat{h} = -\frac{\hbar^2}{2m_e}\nabla^2 - \sum_{\alpha} \frac{Z_{\alpha}e^2}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{R}_{\alpha}|} \quad (1)$$

$$\hat{f}\psi = \left\{ \hat{h} + \hat{v}_{\text{Coul}}[\psi_i] + \hat{v}_{\text{ex}}[\psi_i] + \hat{v}_{\text{corr}}[\psi_i] \right\} \psi = \epsilon\psi \quad (2)$$

1. Solve as for atoms, using some model for electron-electron interactions
2. Potential energy surface (PES)

$$E(\mathbf{R}_{\alpha}, \mathbf{R}_{\beta}, \dots) = E_{\text{elec}} + \frac{e^2}{4\pi\epsilon_0} \sum_{\alpha} \sum_{\beta > \alpha} \frac{Z_{\alpha}Z_{\beta}}{|\mathbf{R}_{\alpha} - \mathbf{R}_{\beta}|}$$



**Table 13:** Numerical DFT Solutions for Atoms

**2.10.2 H<sub>2</sub> molecule as perturbation on two H atoms brought from infinite distance**

1. “Bonding” orbital,  $\sigma_g(\mathbf{r}) = 1s_A + 1s_B$
2. “Anti-bonding” orbital,  $\sigma_u(\mathbf{r}) = 1s_A - 1s_B$
3. Interaction scales with “overlap  $\langle 1s_A | 1s_B \rangle$ ”
4. Ground configuration =  $\sigma_g^2$
5. Bond order =  $\frac{1}{2}(n - n^*)$

**2.10.3 Secular equations**

1. Expand molecular orbitals in “basis” of atomic-like orbitals

$$\psi_{\text{MO}} = \sum_a c_a \phi_a(\mathbf{r}) \quad (3)$$

2. Problem reduces to finding set of  $c_a$  that give best molecular orbitals (MOs)
3. Substituting into Fock equation and integrating yields set of linear equations for the  $c_a$  for each MO

$$\begin{pmatrix} F_{11} - \epsilon S_{11} & F_{12} - \epsilon S_{12} & \dots \\ F_{21} - \epsilon S_{21} & F_{22} - \epsilon S_{22} & \dots \\ \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ \vdots \end{pmatrix} = 0$$

- (a)  $F_{ij} = F_{ji} = \langle \phi_i | \hat{f} | \phi_j \rangle$  are Fock “matrix elements”
  - (b)  $S_{ij} = S_{ji} = \langle \phi_i | \phi_j \rangle$  are overlaps
  - (c) Typically basis functions normalized such that  $S_{ii} = 1$
  - (d)  $\epsilon$  are molecular orbital energies (to be solved for, as many as there are equations)
4. From linear algebra, only possible solutions are those that make the determinant vanish

$$\begin{vmatrix} F_{11} - \epsilon S_{11} & F_{12} - \epsilon S_{12} & \dots \\ F_{21} - \epsilon S_{21} & F_{22} - \epsilon S_{22} & \dots \\ \vdots & \vdots & \ddots \end{vmatrix} = 0$$

5. Solve for  $\epsilon$ s and back-substitute to find correspond  $c_i$ s

**2.10.4 Qualitative solutions of secular equations**

1. Lot’s of insight into chemical bonding can be obtained from approximate solutions to secular equations, basis of “molecular orbital theory”
2. Two general assumptions
  - (a) Diagonal Fock elements are approximately equal to energies of corresponding atomic orbitals:  $F_{ii} \approx \epsilon_{i,\text{ao}}$
  - (b) Off-diagonal elements proportional to overlap and inversely proportional to energy difference:

$$F_{ij} \propto \frac{S_{ij}}{\epsilon_{i,\text{ao}} - \epsilon_{j,\text{ao}}}$$

- (c) (Often) set differential overlap  $S_{ij} = 0$

### 2.10.5 H<sub>2</sub> example, again

1. Assign one 1s atomic orbital (“basis function”) to each atom

$$F_{11} = F_{22} = \epsilon_{1s} = \alpha$$

$$F_{12} = F_{21} = \beta$$

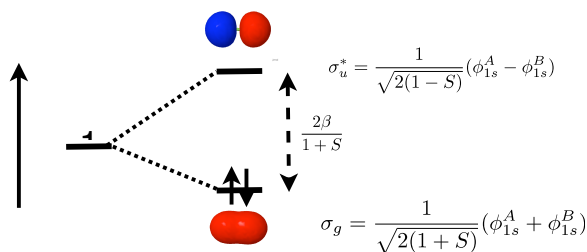
$$\alpha < \beta < 0 \text{ typically}$$

2. Set-up and solve secular matrix

$$\begin{vmatrix} \alpha - \epsilon & \beta - \epsilon S \\ \beta - \epsilon S & \alpha - \epsilon \end{vmatrix} = 0$$

$$\epsilon_+ = \frac{\alpha + \beta}{1 + S}, \quad c_1 = c_2 = \frac{1}{\sqrt{2(1 + S)}}$$

$$\epsilon_- = \frac{\alpha - \beta}{1 - S}, \quad c_1 = -c_2 = \frac{1}{\sqrt{2(1 - S)}}$$



3. From Taylor expansion get picture of atomic orbitals destabilized by electron repulsion  $\beta S$  and split by interaction  $\beta$

$$\epsilon_+ \approx \alpha - \beta S + \beta$$

$$\epsilon_- \approx \alpha - \beta S - \beta$$

4. Makes clear that bonding stabilization < anti-bonding destabilization

### 2.10.6 Heteronuclear diatomic: LiH, HF, BH example

1. Only AOs of appropriate symmetry, overlap, and energy match can combine to form MOs

$$\epsilon_+ \approx \alpha_1 - \beta S - \beta^2/|\alpha_1 - \alpha_2|$$

$$\epsilon_- \approx \alpha_2 - \beta S + \beta^2/|\alpha_1 - \alpha_2|$$

2. LiH: H 1s + Li 2s, bond polarized towards H
3. HF: H 1s + F 2p, bond polarized towards F, lots of non-bonding orbitals
4. BH: H 1s, B 2s and 2p<sub>z</sub> → bonding, non-bonding, anti-bonding orbitals

**2.10.7 Homonuclear diatomic: O<sub>2</sub>**

1. Assign aos, 1s, 2s, 2p for each atom (10 total)
2. In principle, solve  $10 \times 10$  secular matrix
3. In practice, matrix elements rules mean only a few off-diagonal elements survive
  - (a) 1s + 1s do nothing
  - (b) 2s + 2s form  $\sigma$  bond and anti-bond
  - (c)  $2p_z + 2p_z$  form second bond and anti-bond
  - (d)  $2p_{x,y} + 2p_{x,y}$  form degenerate  $\pi$  bonds and anti-bonds
  - (e) O<sub>2</sub> is a triplet, consistent with experiment!

**2.10.8 The Hückel/tight binding model: [Roberts, Notes on Molecular Orbital Theory](#)**

1.  $F_{ii} = \alpha, S_{ij} = \delta_{ij}, F_{ij} = \beta$  iff  $i$  adjacent to  $j$
2. Ethylene example
3. Butadiene example
4. Benzene example
5. Infinite chain example

---

```

1 from sympy import *
2 initprinting(useunicode=True)
3
4 alpha,beta = symbols(alpha beta)
5
6 M = Matrix([[alpha, beta, 0 , 0],[beta, alpha, beta, 0],[0,beta,alpha,beta],[0,0,beta,alpha]])
7
8 M = Matrix([[alpha,beta],[beta,alpha]])
9
10 eigs = M.eigenvects()
11 print(eigs)

```

---

$[(\alpha - \beta/2 + \sqrt{5}\beta/2, 1, [\text{Matrix}([ [4/((1 + \sqrt{5})*(-\sqrt{5}) + 1)]], [ -2/(1 + \sqrt{5})], [ 2/(1 + \sqrt{5})], [ 1]])], (\alpha + \beta/2 + \sqrt{5}\beta/2, 1, [\text{Matrix}([ [-4/((1 + \sqrt{5})*(-\sqrt{5}) + 1)]], [ -2/(-\sqrt{5}) + 1], [ -2/(-\sqrt{5}) + 1], [ 1]])], (\alpha - \sqrt{5}\beta/2 - \beta/2, 1, [\text{Matrix}([ [-4/((-1 + \sqrt{5})*(1 + \sqrt{5}))], [ 2/(-1 + \sqrt{5})], [ -2/(-1 + \sqrt{5})], [ 1]])], (\alpha - \sqrt{5}\beta/2 + \beta/2, 1, [\text{Matrix}([ [4/((-1 + \sqrt{5})*(1 + \sqrt{5}))], [ -2/(1 + \sqrt{5})], [ -2/(1 + \sqrt{5})], [ 1]])])]$

**2.10.9 Band structure of solids****2.11 Lecture 14: Computational chemistry****2.11.1 Numerical solvers of Schrödinger equation for molecules readily available to-day****2.11.2 Have to specify:**

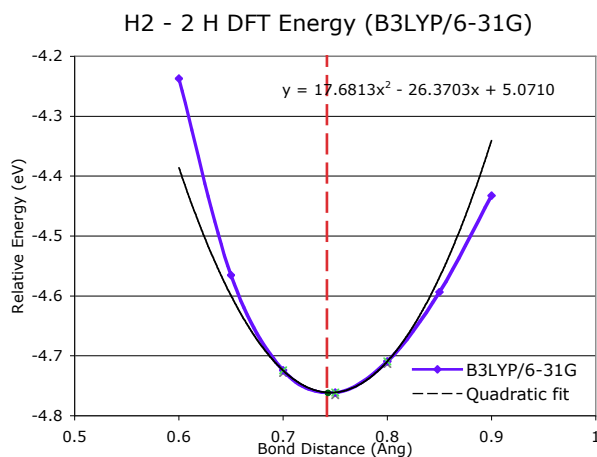
1. Identity of atoms

2. Positions of atoms (distances, angles, ...)
3. (spin multiplicity)
4. exact theoretical model (how are Coulomb, exchange, and correlation described?)
  - (a) Hartree, Hartree-Fock, DFT (various flavors), ...
5. basis set to express wavefunctions in terms of
6. initial guess of wavefunction coefficients (often guessed for you)

### 2.11.3 Secular equations solved iteratively until input coefficients = output coefficients

1. “self-consistent field”
2. Output
  - (a) energies of molecular orbitals
  - (b) occupancies of molecular orbitals
  - (c) coefficients describing molecular orbitals
  - (d) total electron wavefunction, total electron density, dipole moment, ...
  - (e) total molecular energy
  - (f) derivatives (“gradients”) of total energy w.r.t. atom positions
3. Plot total energy vs internal coordinates: potential energy surface (PES)
4. Search iteratively for minimum point on PES (by hand or using gradient-driven search): equilibrium geometry
5. Find second derivative of energy at minimum point on PES: harmonic vibrational frequency
6. Find energy at minimum relative to atoms (or other molecules): reaction energy

### 2.11.4 H<sub>2</sub> example



#### Bond length

|            |         |
|------------|---------|
| H-H exp't: | 0.742 Å |
| B3LYP opt: | 0.743   |

#### Bond energy

|  |            |
|--|------------|
| H-H exp't                                | = 4.478 eV |
| 2 H: 2(-0.4969) au = -27.041 eV          |            |
| H <sub>2</sub> : -1.1687 au = -31.803 eV |            |
| Dissociation E                           | = 4.762 eV |
| ZPE corrected                            | = 4.484 eV |

#### Vibrational frequencies

|                 |                       |
|-----------------|-----------------------|
| Experiment:     | 4401 cm <sup>-1</sup> |
| B3LYP harmonic: | 4487 cm <sup>-1</sup> |
| ZPE:            | 0.278 eV              |

### 2.11.5 Polyatomic molecules

1. Gradient-driven optimizations,  $3n - 6$  degrees of freedom
2. Hessian matrix for frequencies

## 2.12 Lecture 15: Electronic spectroscopy

## 2.13 Lecture 16: Electronic and magnetic properties

# 3 Statistical Mechanics: The Bridge from the Tiny to the Many

## 3.1 Lecture 17: Statistical mechanics

### 3.1.1 Need machinery to average QM information over macroscopic systems

### 3.1.2 Equal *a priori* probabilities

### 3.1.3 Two-state model

1. Box of particles, each of which can have energy 0 or  $\epsilon$
2. Thermodynamic state defined by number of elements  $N$ , and number of quanta  $q$ ,  $U = q\epsilon$
3. Degeneracy of given  $N$  and  $q$  given by binomial distribution:

$$\Omega = \frac{N!}{q!(N-q)!}$$

4. Allow energy to flow between two such systems
5. Energy of a closed system is conserved (first law!)
6. Degeneracy of total system is always  $\geq$  degeneracy of the starting parts!
7. Boltzmann's tombstone,  $S = k_B \ln \Omega$
8. Clausius: entropy of the universe seeks a maximum! Second Law...

### 3.1.4 Energy flow/thermal equilibrium between two large systems

1. Each subsystem has energy  $U_i$  and degeneracy  $\Omega_i(U_i)$
2. Bring in thermal contact,  $U = U_1 + U_2$ ,  $\Omega = \Omega_1(U_1)\Omega_2(U_2)$
3. If systems are very large, one combination of  $U_1$ ,  $U_2$  and  $\Omega$  will be much more probable than all others
4. What value of  $U_1$  and  $U_2 = U - U_1$  maximizes  $\Omega$ ?

$$\left(\frac{\partial \ln \Omega_1}{\partial U_1}\right)_N = \left(\frac{\partial \ln \Omega_2}{\partial U_2}\right)_N$$

$$\left(\frac{\partial S_1}{\partial U_1}\right)_N = \left(\frac{\partial S_2}{\partial U_2}\right)_N$$

5. Thermal equilibrium is determined by equal **temperature**!

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_N$$

6. When the temperatures of the two subsystems are equal, the entropy of the combined system is maximized!
7. (Same arguments lead to requirement that equal pressures ( $P_i$ ) and equal chemical potentials ( $\mu_i$ ) maximize entropy when volumes or particles are exchanged)

### 3.1.5 Two-state model in limit of large $N$

1. Large  $N$  and Stirling's approximation
2. Fundamental thermodynamic equation of two-state system:

$$S(U) = -k_B (x \ln x + (1-x) \ln(1-x)), \text{ where } x = q/N = U/N\epsilon$$

3. Temperature is derivative of entropy wrt energy yields

$$U(T) = \frac{N\epsilon}{1 + e^{\epsilon/k_B T}}$$

4.  $T \rightarrow 0, U \rightarrow 0, S \rightarrow 0$ , minimum disorder
5.  $T \rightarrow \infty, U \rightarrow N\epsilon/2, S \rightarrow k_B \ln 2$ , maximum disorder
6. Differentiate again to get heat capacity

### 3.1.6 Example of microcanonical (“NVE”) ensemble

1. Direct evaluation of  $S(U)$  is generally intractable, so seek simpler approach

## 3.2 Lecture 18: Canonical (NVT) ensemble

### 3.2.1 Partition function

1. Imagine a system brought into thermal equilibrium with a much larger “reservoir” of constant  $T$ , such that the aggregate has a total energy  $U$
2. Degeneracy of a given system microstate  $j$  with energy  $U_j$  is  $\Omega_{res}(U - U_j)$

$$T = \frac{dU_{res}}{k_B d \ln \Omega_{res}}$$

$$\Omega_{res}(U - U_j) \propto e^{-U_j/k_B T}$$

3. Probability for system to be in a microstate with energy  $U_j$  given by Boltzmann distribution!

$$P(U_j) \propto e^{-U_j/k_B T} = e^{-U_j \beta}$$

4. Partition function “normalizes” distribution,  $Q(T) = \sum_j e^{-U_j \beta}$
5. For system of identical (distinguishable) elements with energy states  $\epsilon_i$ , can factor probability to show

$$P(\epsilon_i) \propto e^{-\epsilon_i/k_B T} = e^{-\epsilon_i \beta}, \quad \beta = 1/k_B T$$

### 3.2.2 Energy factoring (sidebar)

1. If system is large, how to determine its energy states  $U_j$ ? There would be many, many of them!
2. One simplification is if we can write energy as sum of energies of individual elements (atoms, molecules) of system:

$$U_j = \epsilon_j(1) + \epsilon_j(2) + \dots + \epsilon_j(N) \quad (4)$$

$$Q(N, V, T) = \sum_j e^{-U_j \beta} \quad (5)$$

$$= \sum_j e^{-(\epsilon_j(1) + \epsilon_j(2) + \dots + \epsilon_j(N)) \beta} \quad (6)$$

3. If molecules/elements of system can be distinguished from each other (like atoms in a fixed lattice), expression can be factored:

$$Q(N, V, T) = \left( \sum_j e^{-\epsilon_j(1) \beta} \right) \dots \left( \sum_j e^{-\epsilon_j(N) \beta} \right) \quad (7)$$

$$= q(1) \dots q(N) \quad (8)$$

$$\text{Assuming all the elements are the same:} \quad (9)$$

$$= q^N \quad (10)$$

$$q = \sum_j e^{-\epsilon_j \beta} : \text{molecular partition function} \quad (11)$$



4. If *not* distinguishable (like molecules in a liquid or gas, or electrons in a solid), problem is difficult, because identical arrangements of energy amongst elements should only be counted once.
5. Approximate solution, good almost all the time:

$$Q(N, V, T) = q^N / N! \quad (12)$$

6. Sidebar: “Correct” factoring depends on whether individual elements are fermions or bosons, leads to funny things like superconductivity and superfluidity.

### 3.2.3 Distinguishable vs. indistinguishable particles

1. Distinguishable (e.g., in a lattice):  $Q(N, V, T) = q(V, T)^N$
2. Indistinguishable (e.g., a gas):  $Q(N, V, T) \approx q(V, T)^N / N!$

### 3.2.4 Two-state system again

1. Partition function,  $q(T) = 1 + e^{-\epsilon\beta}$
2. State probabilities
3. Internal energy  $U(T)$

$$U(T) = -N \left( \frac{\partial \ln(1 + e^{-\epsilon\beta})}{\partial \beta} \right) = \frac{N\epsilon e^{-\epsilon\beta}}{1 + e^{-\epsilon\beta}} \quad (13)$$

4. Heat capacity  $C_v$ 
  - (a) Minimum when change in states with  $T$  is small
  - (b) Maximize when change in states with  $T$  is large
5. Helmholtz energy,  $A = -\ln q / \beta$ , decreasing function of  $T$
6. Entropy

### 3.2.5 Thermodynamic functions in canonical ensemble

## 3.3 Lecture 19: Molecular Partition Functions

### 3.3.1 Ideal gas of molecules

$$Q_{ig}(N, V, T) = \frac{(q_{\text{trans}} q_{\text{rot}} q_{\text{vib}})^N}{N!}$$

**Table 14:** Equations of the Canonical ( $NVT$ ) Ensemble

| $\beta = 1/k_B T$                     | Full Ensemble  | Distinguishable particles<br>(e.g. atoms in a lattice)      | Indistinguishable particles<br>(e.g. molecules in a fluid)  |
|---------------------------------------|--|---|---|
| Single particle<br>partition function |  | $q(V, T) = \sum_i e^{-\epsilon_i \beta}$                    | $q(V, T) = \sum_i e^{-\epsilon_i \beta}$                    |
| Full partition<br>function            | $Q(N, V, T) = \sum_j e^{-U_j \beta}$                                     | $Q = q(V, T)^N$   | $Q = q(V, T)^N / N!$  |
| Log partition<br>function             | $\ln Q$  | $N \ln q$   | $N \ln q - \ln N!$<br>$\approx N(\ln Q - \ln N + 1)$        |
| Helmholtz energy<br>( $A = U - TS$ )  | $-\frac{\ln Q}{\beta}$   | $-\frac{N \ln q}{\beta}$                                    | $-\frac{N}{\beta} \left( \ln \frac{q}{N} + 1 \right)$       |
| Internal energy ( $U$ )               | $-\left( \frac{\partial \ln Q}{\partial \beta} \right)_{NV}$             | $-N \left( \frac{\partial \ln q}{\partial \beta} \right)_V$ | $-N \left( \frac{\partial \ln q}{\partial \beta} \right)_V$ |
| Pressure ( $P$ )                      | $-\left( \frac{\partial \ln Q}{\partial V} \right)_{N\beta}$             | $-N \left( \frac{\partial \ln q}{\partial V} \right)_\beta$ | $-N \left( \frac{\partial \ln q}{\partial V} \right)_\beta$ |
| Entropy ( $S/k_B$ )                   | $\beta U + \ln Q$  | $\beta U + N \ln q$   | $\beta U + N (\ln(q/N) + 1)$                                |
| Chemical potential ( $\mu$ )          | $-\frac{1}{\beta} \left( \frac{\partial \ln Q}{\partial N} \right)_{VT}$ | $-\frac{\ln q}{\beta}$                                      | $-\frac{\ln(q/N)}{\beta}$                                   |

**NOTE!** All energies are referenced to their values at 0 K. Enthalpy  $H = U + PV$ , Gibb's Energy  $G = A + PV$ .

**3.3.2 Particle-in-a-box (translational states of a gas)**

1. Energy states  $\epsilon_n = n^2\epsilon_0$ ,  $n = 1, 2, \dots$ ,  $\epsilon_0$  tiny for macroscopic  $V$
2.  $\Theta_{\text{trans}} = \epsilon_0/k_B$  translational temperature
3.  $\Theta_{\text{trans}} \ll T \rightarrow$  many states contribute to  $q_{\text{trans}} \rightarrow$  integral approximation

$$q_{\text{trans,1D}} = \int_0^\infty e^{-x^2\beta\epsilon_0} dx = L/\Lambda$$

$$\Lambda = \left( \frac{h^2\beta}{2\pi m} \right)^{1/2} \quad \text{thermal wavelength}$$

$$q_{\text{trans,3D}} = V/\Lambda^3$$

4. Internal energy
5. Heat capacity
6. Equation of state (!)
7. Entropy: Sackur-Tetrode equation

**3.3.3 Rigid rotor (rotational states of a gas)**

1. energy states and degeneracies
2.  $\Theta_{\text{rot}} = \hbar^2/2Ik_B$
3. “High” T  $q_{\text{rot}}(T) \approx \sigma\Theta_{\text{rot}}/T$

**3.3.4 Harmonic oscillator (vibrational states of a gas)**

1.  $\Theta_{\text{vib}} = h\nu/k_B$

**3.3.5 Electronic partition functions  $\rightarrow$  spin multiplicity****3.3.6 Non-ideality**

1. Real molecules interact through vdW interactions
2. Particle-in-a-box model breaks down, have to work harder but can still get at same ideas
3. See Hill, *J. Chem. Ed.* **1948**, *25*, p. 347 <http://dx.doi.org/10.1021/ed025p347>

**3.4 Lecture 20: Chemical reactions and equilibria****3.4.1 Standard states**

1. Translational partition function depends on concentration  $N/V$
2. “Standard state” corresponds to some standard choice for  $N/V$ ,  $c^\circ$
3. For ideal gas, related to pressure by  $P^\circ = c^\circ k_B T$

**Table 15:** Statistical Thermodynamics of an Ideal Gas**Translational DOFs** 3-D particle in a box model

$$\theta_{\text{trans}} = \frac{\pi^2 \hbar^2}{2mL^2 k_B}, \Lambda = h \left( \frac{\beta}{2\pi m} \right)^{1/2}$$

For  $T \gg \Theta_{\text{trans}}$ ,  $\Lambda \ll L$ ,  $q_{\text{trans}} = V/\Lambda^3$  (essentially always true)

$$U_{\text{trans}} = \frac{3}{2}RT \quad C_{v,\text{trans}} = \frac{3}{2}R \quad S_{\text{trans}}^\circ = R \ln \left( \frac{e^{5/2} V^\circ}{N^\circ \Lambda^3} \right) = R \ln \left( \frac{e^{5/2} k_B T}{P^\circ \Lambda^3} \right)$$

**Rotational DOFs** Rigid rotor model

**Linear molecule**  $\theta_{\text{rot}} = hcB/k_B$

$$q_{\text{rot}} = \frac{1}{\sigma} \sum_{l=0}^{\infty} (2l+1) e^{-l(l+1)\theta_{\text{rot}}/T} \approx \frac{1}{\sigma} \frac{T}{\theta_{\text{rot}}}, \quad T \gg \theta_{\text{rot}} \quad \sigma = \begin{cases} 1, & \text{unsymmetric} \\ 2, & \text{symmetric} \end{cases}$$

$$U_{\text{rot}} = RT \quad C_{v,\text{rot}} = R \quad S_{\text{rot}}^\circ = R(1 - \ln(\sigma \theta_{\text{rot}}/T))$$

**Non-linear molecule**  $\theta_{\text{rot},\alpha} = hcB_\alpha/k_B$

$$q_{\text{rot}} \approx \frac{1}{\sigma} \left( \frac{\pi T^3}{\theta_{\text{rot},\alpha} \theta_{\text{rot},\beta} \theta_{\text{rot},\gamma}} \right)^{1/2}, \quad T \gg \theta_{\text{rot},\alpha,\beta,\gamma} \quad \sigma = \text{rotational symmetry number}$$

$$U_{\text{rot}} = \frac{3}{2}RT \quad C_{v,\text{rot}} = \frac{3}{2}R \quad S_{\text{rot}}^\circ = \frac{R}{2} \left( 3 - \ln \frac{\sigma \theta_{\text{rot},\alpha} \theta_{\text{rot},\beta} \theta_{\text{rot},\gamma}}{\pi T^3} \right)$$

**Vibrational DOFs** Harmonic oscillator model

**Single harmonic mode**  $\theta_{\text{vib}} = h\nu/k_B$

$$q_{\text{vib}} = \frac{1}{1 - e^{-\theta_{\text{vib}}/T}} \approx \frac{T}{\theta_{\text{vib}}}, \quad T \gg \theta_{\text{vib}}$$

$$U_{\text{vib}} = \frac{R \theta_{\text{vib}}}{e^{\theta_{\text{vib}}/T} - 1} \quad C_{v,\text{vib}} = R \left( \frac{\theta_{\text{vib}}}{T} \frac{e^{\theta_{\text{vib}}/2T}}{e^{\theta_{\text{vib}}/T} - 1} \right)^2 \quad S_{\text{vib},i}^\circ = R \left( \frac{\theta_{\text{vib}}/T}{e^{\theta_{\text{vib}}/T} - 1} - \ln(1 - e^{-\theta_{\text{vib}}/T}) \right)$$

**Multiple harmonic modes**  $\theta_{\text{vib},i} = h\nu_i/k_B$

$$q_{\text{vib}} = \prod_i \frac{1}{1 - e^{-\theta_{\text{vib},i}/T}}$$

$$U_{\text{vib}} = R \sum_i \frac{\theta_{\text{vib},i}}{e^{\theta_{\text{vib},i}/T} - 1} \quad C_{v,\text{vib}} = R \sum_i \left( \frac{\theta_{\text{vib},i}}{T} \frac{e^{\theta_{\text{vib},i}/2T}}{e^{\theta_{\text{vib},i}/T} - 1} \right)^2 \quad S_{\text{vib},i}^\circ = R \left( \frac{\theta_{\text{vib},i}/T}{e^{\theta_{\text{vib},i}/T} - 1} - \ln(1 - e^{-\theta_{\text{vib},i}/T}) \right)$$

**Electronic DOFs**  $q_{\text{elec}}$  = spin multiplicity

**3.4.2 Chemical reaction  $A \rightarrow B$** 

1. Reaction entropy  $\Delta S^\circ(T) = S_B^\circ(T) - S_A^\circ(T)$
2. Reaction energy  $\Delta U^\circ(T) = U_B^\circ(T) - U_A^\circ(T) + \Delta E(0)$
3. Equilibrium condition—equate chemical potentials,  $\mu_A(N, V, T) = \mu_B(N, V, T)$
4. Equilibrium constant—evaluate from partition functions directly or indirectly from thermodynamic potentials

**3.4.3 Le’Chatelier’s principle**

1. Response to temperature: Boltzmann distribution favors higher energy things as  $T$  increases
2. Response to volume change: particle-in-a-box states increasingly favor side with more molecules as volume increases

**3.5 Lecture 21: Chemical kinetics****3.5.1 Kinetics and reaction rates**

1. Rate: number per unit time per unit something

**3.5.2 Empirical chemical kinetics**

1. Rate laws, rate orders, and rate constants
2. Arrhenius expression,  $k = Ae^{-E_a/k_B T}$

**3.5.3 Reaction mechanisms**

1. Elementary steps and molecularity
2. Collision theory—overpredicts rates

**3.5.4 Transition state theory (TST)**

1. Existence of reaction coordinate (PES)
2. Existence of dividing surface
3. Equilibrium between reactants and “transition state”
4. Harmonic approximation for transition state

**3.5.5 Locating transition states computationally****3.5.6 Thermodynamic connection****3.5.7 Diffusion-controlled reactions**

1. Intermediate complex
2. Steady-state approximation

**Table 16:** Equilibrium and Rate Constants**Equilibrium Constants**  $a A + b B \rightleftharpoons c C + d D$ 

$$K_{eq}(T) = e^{\Delta S^\circ(T,V)/k_B} e^{-\Delta H^\circ(T,V)/k_B T}$$

$$K_c(T) = \left(\frac{1}{c^\circ}\right)^{\nu_c + \nu_d - \nu_a - \nu_b} \frac{(q_c/V)^{\nu_c} (q_d/V)^{\nu_d}}{(q_a/V)^{\nu_a} (q_b/V)^{\nu_b}} e^{-\Delta E(0)\beta}$$

$$K_p(T) = \left(\frac{k_B T}{P^\circ}\right)^{\nu_c + \nu_d - \nu_a - \nu_b} \frac{(q_c/V)^{\nu_c} (q_d/V)^{\nu_d}}{(q_a/V)^{\nu_a} (q_b/V)^{\nu_b}} e^{-\Delta E(0)\beta}$$

**Unimolecular Reaction**  $[A] \rightleftharpoons [A]^\ddagger \rightarrow C$ 

$$k(T) = \nu^\ddagger \bar{K}^\ddagger = \frac{k_B T}{h} \frac{\bar{q}^\ddagger(T)/V}{q_A(T)/V} e^{-\Delta E^\ddagger(0)\beta}$$

$$E_a = \Delta H^{\circ\ddagger} + k_B T \quad A = e^1 \frac{k_B T}{h} e^{\Delta S^{\circ\ddagger}}$$

**Bimolecular Reaction**  $A + B \rightleftharpoons [AB]^\ddagger \rightarrow C$ 

$$k(T) = \nu^\ddagger \bar{K}^\ddagger = \frac{k_B T}{h} \frac{q^\ddagger(T)/V}{(q_A(T)/V)(q_B(T)/V)} \left(\frac{1}{c^\circ}\right)^{-1} e^{-\Delta E^\ddagger(0)\beta}$$

$$E_a = \Delta H^{\circ\ddagger} + 2k_B T \quad A = e^2 \frac{k_B T}{h} e^{\Delta S^{\circ\ddagger}}$$

3. Diffusion-controlled limit ( $k_D = 4\pi(r_A + r_B)D_{AB}$ )4. Reaction-controlled limit ( $k_{app} = (k_D/k_{-D})k_r$ )

### 3.6 Lecture 22: Conclusion

1. Do you think about the burning lighter any differently now?