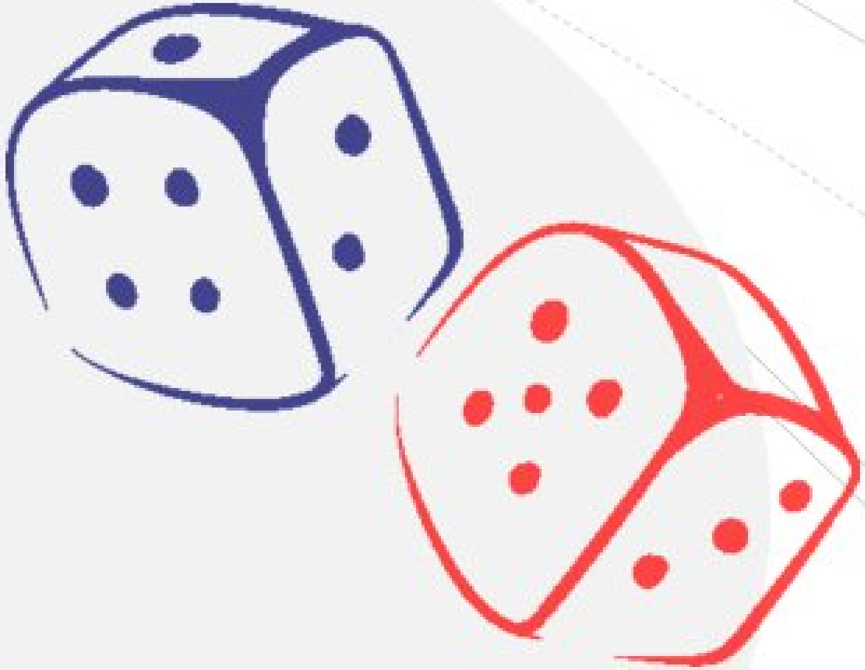
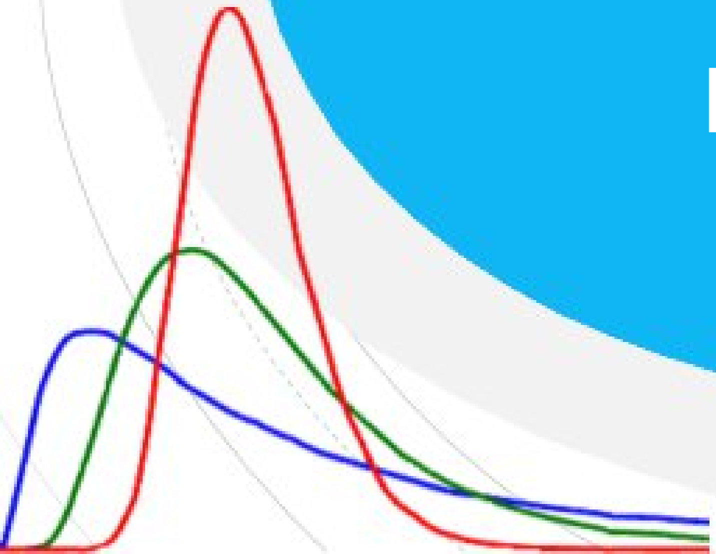


Probability

Week 09

Lecture 1&2



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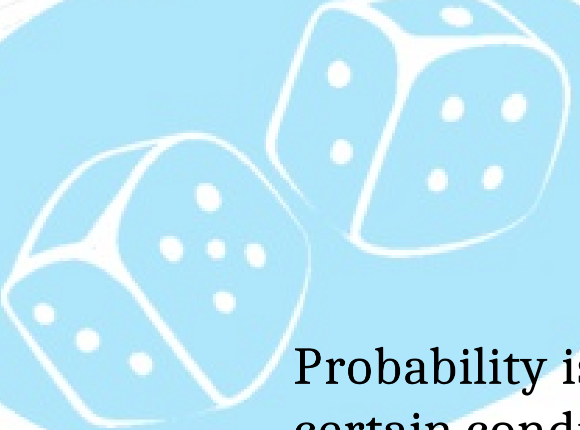
Probability



Probability is a numerical description of chance of occurrence of a given phenomena under certain condition.

Probability is a number that reflects a chance or likelihood that a particular event will occur

Probability always has a value from 0 and 1

0 indicates that there is no chance that event will occur

1 indicates that an event is certain to occur

$$P(A) = \frac{Number\ of\ Favourable\ Outcome}{Total\ Number\ of\ Favourable\ Outcomes}$$

Probability

Important Terms

- Experiment
- Random experiment
- Trial
- Sample space
- Event
- Equally likely events
- Exhaustive events
- Favorable events

Applying Set Theory to Probability

- Probability is a number that describes a set.
- The higher the number, the more probability there is. In this sense probability is like a quantity that measures a physical phenomenon; for example, a weight or a temperature.
- The basic model is a repeatable experiment. An experiment consists of a procedure and observations. There is uncertainty in what will be observed; otherwise, performing the experiment would be unnecessary.

Applying Set Theory to Probability

Experiment

It is a well defined operation or procedure that leads to an observable outcome

Outcome: The result of an experiment

Example : For a die rolled there are total of six outcomes

Applying Set Theory to Probability

Example 1.1 An experiment consists of the following procedure, observation, and model:

- Procedure: Flip a coin and let it land on a table.
- Observation: Observe which side (head or tail) faces you after the coin lands.
- Model: Heads and tails are equally likely. The result of each flip is unrelated to the results of previous flips.

Example 1.2 Flip a coin three times. Observe the sequence of heads and tails.
Example 1.3 Flip a coin three times. Observe the number of heads.

Outcome

- ♠ An **outcome** of an experiment is any possible observation of that experiment.
- ♠ An outcome is the notion that each outcome is distinguishable from every other outcome.
- ♠ As a result, we define the universal set of all possible outcomes.
- ♠ **In probability terms**, we call this universal set the sample space.

Sample Space

- The **sample space** of an experiment is the finest-grain, mutually exclusive, collectively exhaustive set of all possible outcomes.
- All possible distinguishable outcomes are identified separately.
- The requirement that outcomes be mutually exclusive says that if one outcome occurs, then no other outcome also occurs.
- For the set of outcomes to be collectively exhaustive, every outcome of the experiment must be in the sample space.

Sample Space

Example 1.4: The sample space in Example 1.1 is $S = \{h, t\}$ where h is the outcome "observe head," and t is the outcome "observe tail."

- ♠ The sample space in Example 1.2 is
- ♠ $S = \{hhh, hht, hth, htt, thh, tht, tth, ttt\}$
- ♠ The sample space in Example 1.3 is $S = \{0, 1, 2, 3\}$.

Example 1.5: *Develop a software and test it to determine whether it meets quality objectives. The possible outcomes are "accepted" (a) and "rejected" (r). The sample space is* $S = \{a, r\}$.

Event

- An **event** is a set of outcomes of an experiment.
- In common speech, an event is just something that occurs.
- In an experiment, we may say that an event occurs when a certain phenomenon is observed.
- To define an event mathematically, we must identify all outcomes for which the phenomenon is observed.
- That is, for each outcome, either the particular event occurs or it does not.
- In **probability terms**, we define an event in terms of the outcomes of the sample space.

Analogy B/W Sets & Probability

Set Algebra	Probability
Set	Event
Universal set	Sample space
Element	Outcome

Table 1.1 The terminology of set theory and probability.

Examples

Example 1.6 Suppose we roll a six-sided die and observe the number of dots on the side facing upwards. We can label these outcomes $i = 1, \ldots, 6$ where i denotes the outcome that i dots appear on the up face. The sample space is $S = \{1, 2, \ldots, 6\}$. Each subset of S is an event. Examples of events are

- ♠ The event $E1 = \{\text{Roll 4 or higher}\} = \{4, 5, 6\}$.
- ♠ The event $E2 = \{\text{The roll is even}\} = \{2, 4, 6\}$.
- ♠ $E3 = \{\text{The roll is the square of an integer}\} = \{1, 4\}$.

Example 1.7 Wait for someone to make a phone call and observe the duration of the call in minutes. An outcome x is a nonnegative real number. The sample space is $S = \{x|x \geq 0\}$. The event "the phone call lasts longer than five minutes" is $\{x|x > 5\}$.

Examples

Example 1.8 A software bug tester has a red light to indicate that there is a bug and a green light to indicate that there is no bug. Consider an experiment consisting of a sequence of three tests. In each test the observation is the color of the light that is on at the end of a test. An outcome of the experiment is a sequence of red *(r)* and green *(g)* lights. We can denote each outcome by a three-letter word such as *rgr*, the outcome that the first and third lights were red but the second light was green. We denote the event that light n was red or green by R_n or G_n. The event $R_2 = \{grg, grr, rrg, rrr\}$. We can also denote an outcome as an intersection of events R_i and G_j. For example, the event $R_1G_2R_3$ is the set containing the single outcome {rgr}.

Examples

- In Example 1.8, suppose we were interested only in the status of light 2.
- In this case, the set of events {G2, R2} describes the events of interest.
- Moreover, for each possible outcome of the three-light experiment, the second light was either red or green, so the set of events {G2, R2} is both mutually exclusive and collectively exhaustive.
- However, {G2, R2} is not a sample space for the experiment because the elements of the set do not completely describe the set of possible outcomes of the experiment.

Event Space

- An **event space** is a collectively exhaustive, mutually exclusive set of events.
- An **event space** and a **sample space** have a lot in common.
- The members of both are mutually exclusive and collectively exhaustive.
- They differ in the finest-grain property that applies to a sample space but not to an event space.
- Because it possesses the finest-grain property, a sample space contains all the details of an experiment.
- The members of a sample space are outcomes.
- By contrast, the members of an event space are events.
- The event space is a set of events (sets), while the sample space is a set of outcomes (elements).

Event Space

Example 1.9 Flip four coins, a penny, a nickel, a dime, and a quarter. Examine the coins in order (penny, then nickel, then dime, then quarter) and observe whether each coin shows a head (h) or a tail (t). What is the sample space? How many elements are in the sample space?

The sample space consists of 16 four-letter words, with each letter either h or t. For example, the outcome $tthh$ refers to the penny and the nickel showing tails and the dime and quarter showing heads. There are 16 members of the sample space.

Event Space

Example 1.10 Continuing Example 1.9, let Bi = {outcomes with i heads}. Each Bi is an event containing one or more outcomes. For example, *B1={ttth, ttht, thtt, httt}* contains four outcomes. The set *B={B0, B1, B2, B3, B4}* is an event space. Its members are mutually exclusive and collectively exhaustive. It is not a sample space because it lacks the finest-grain property. Learning that an experiment produces an event *B1* tells you that one coin came up heads, but it doesn't tell you which coin it was.

Event Space

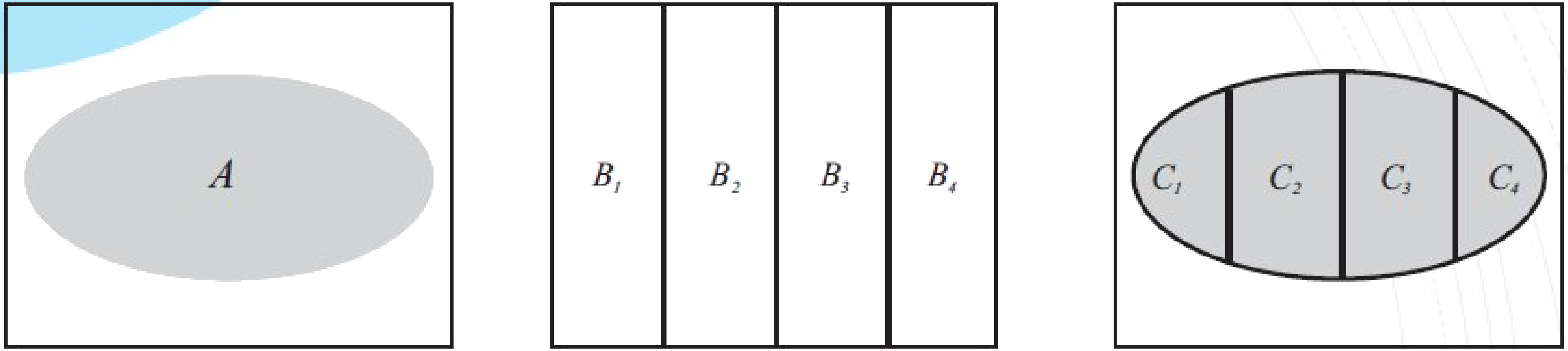


Figure 1.1 In this example of Theorem 1.2, the event space is B = {B1, B2, B3, B4} and $C_i = A \cap B_i$ for i = 1, . . . , 4. It should be apparent that $A = C_1 \cup C_2 \cup C_3 \cup C_4$.

The concept of an event space is useful because it allows us to express any event as a union of mutually exclusive events.

Theorem

Theorem 1.2 For an event space $B = \{B1, B2, \ldots\}$ and any event A in the sample space, let $Ci = A \cap Bi$. For $i \neq j$, the events Ci and Cj are mutually exclusive and
$A = C1 \cup C2 \cup \cdots$. Figure 1.1 is a picture of Theorem 1.2.
Example 1.11 In the coin-tossing experiment of Example 1.9, let A equal the set of outcomes with less than three heads:
A = {t t t t, ht t t, tht t, ttht, ttth, hhtt, htht, htth, tthh, thth, thht} . From Example 1.10, let Bi = {outcomes with i heads}.
Since $\{B0, \ldots, B4\}$ is an event space, Theorem 1.2 states that

$$A = (A \cap B0) \cup (A \cap B1) \cup (A \cap B2) \cup (A \cap B3) \cup (A \cap B4)$$

In this example, $Bi \subset A$, for i = 0, 1, 2. Therefore $A \cap Bi = Bi$ for i = 0, 1, 2. Also, for i = 3 and i = 4, $A \cap Bi = \varphi$ so that $A = B0 \cup B1 \cup B2$, a union of disjoint sets. In words, this example states that the event “less than three heads” is the union of events “zero heads,” “one head,” and “two heads.”

Problems

Monitor three consecutive phone calls going through a telephone switching office. Classify each one as a voice call *(v)* if someone is speaking, or a data call (*d*) if the call is carrying a modem or fax signal. Your observation is a sequence of three letters (each letter is either *v or d*). For example, two voice calls followed by one data call corresponds to *vvd*. Write the elements of the following sets:

A1 = {first call is a voice call}

A1 = {vvv, vvd, vdv, vdd}

B1 = {first call is a data call}

B1 = {dvv, dvd, ddv, ddd}

Problems

Monitor three consecutive phone calls going through a telephone switching office. Classify each one as a voice call *(v)* if someone is speaking, or a data call (*d*) if the call is carrying a modem or fax signal. Your observation is a sequence of three letters (each letter is either *v or d*). For example, two voice calls followed by one data call corresponds to *vvd*. Write the elements of the following sets:

A2 = {second call is a voice call}

A2 = {vvv, vvd, dvv, dvd}

B2 = {second call is a data call}

B2 = {vdv, vdd, ddv, ddd}

Problems

Monitor three consecutive phone calls going through a telephone switching office. Classify each one as a voice call *(v)* if someone is speaking, or a data call (*d)* if the call is carrying a modem or fax signal. Your observation is a sequence of three letters (each letter is either *v or d*). For example, two voice calls followed by one data call corresponds to *vvd*. Write the elements of the following sets:

A3 = {all calls are the same}

A3 = {vvv, ddd}

B3 = {voice and data alternate}

B3 = {vdv, dvd}

Problems

Monitor three consecutive phone calls going through a telephone switching office. Classify each one as a voice call *(v)* if someone is speaking, or a data call (*d)* if the call is carrying a modem or fax signal. Your observation is a sequence of three letters (each letter is either *v or d*). For example, two voice calls followed by one data call corresponds to *vvd*. Write the elements of the following sets:

A4 = {one or more voice calls}

A4 = {vvv, vvd, vdv, dvv, vdd, dvd, ddv}

B4 = {two or more data calls}

B4 = {ddd, ddv, dvd, vdd}


thank you