Square & Square root

Cube & Cube root

1. **Square Root:**

   If $x^2 = y$, we say that the square root of y is x and we write $\sqrt{y} = x$.

   Thus, $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{196} = 14$.

2. **Cube Root:**

   The cube root of a given number x is the number whose cube is x.

   We, denote the cube root of x by $\sqrt[3]{x}$.

   Thus, $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$, $\sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7$ etc.

   Note:

   1. $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$

   2. $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}} = \frac{\sqrt{x}}{\sqrt{y}} \times \frac{\sqrt{y}}{\sqrt{y}} = \frac{\sqrt{xy}}{y}$.

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

$9^2 = 81$

$10^2 = 100$

1. Find the square root of 6084 by prime factorization method.

The prime factors of 6084 are

$$6084 = 2 \times 2 \times 3 \times 3 \times 13 \times 13$$

Step 2: Find the square root

The square root of 6084 is

$$6084 = 2 \times 2 \times 3 \times 3 \times 13 \times 13$$

$$\Rightarrow \sqrt{6084} = \sqrt{2 \times 2 \times 3 \times 3 \times 13 \times 13}$$

$$\Rightarrow \sqrt{6084} = 2 \times 3 \times 13$$

$$\Rightarrow \sqrt{6084} = 78$$

Hence, the square root of 6084 is 78

2. Find the square root of 1471369 by long division method

	1213
1	$\overline{1}\ \overline{47}\ \overline{13}\ \overline{69}$
	1
22	0 47
	44
241	3 13
	2 41
2423	72 69
	72 69
	0

3. Find the square root of

i. 841
ii. 5625
iii. 1849
iv. 3481
v. 12996

Ans

i. 29
ii. 75
iii. 43
iv. 59
v. 114

4. Evaluate: √248+√52+√144.

We need to find value of $\sqrt{248 + \sqrt{52 + \sqrt{144}}}$

$= \sqrt{248 + \sqrt{52 + 12}}$

$= \sqrt{248 + \sqrt{64}}$

$= \sqrt{248 + 8}$

$= \sqrt{256}$

$= 16$

5. $\left(\frac{\sqrt{625}}{11} \times \frac{14}{\sqrt{25}} \times \frac{11}{\sqrt{196}}\right)$ is equal to:

Given Expression $= \frac{25}{11} \times \frac{14}{5} \times \frac{11}{14} = 5.$

6. The cube root of .000216 is:

$$(.000216)^{1/3} = \left(\frac{216}{10^6}\right)^{1/3}$$

$$= \left(\frac{6 \times 6 \times 6}{10^2 \text{ x } 10^2 \text{ x } 10^2}\right)^{1/3}$$

$$= \frac{6}{10^2}$$

$$= \frac{6}{100}$$

$$= 0.06$$

7.

What should come in place of both x in the equation $\frac{x}{\sqrt{128}} = \frac{\sqrt{162}}{x}$.

Let $\frac{x}{\sqrt{128}} = \frac{\sqrt{162}}{x}$

Then $x^2 = \sqrt{128 \times 162}$

$= \sqrt{64 \times 2 \times 18 \times 9}$

$= \sqrt{8^2 \times 6^2 \times 3^2}$

$= 8 \times 6 \times 3$

$= 144.$

$\therefore\ x = \sqrt{144} = 12.$

8. The least perfect square, which is divisible by each of 21, 36 and 66 is:

L.C.M. of 21, 36, 66 = 2772.

Now, $2772 = 2 \times 2 \times 3 \times 3 \times 7 \times 11$

To make it a perfect square, it must be multiplied by 7×11.

So, required number $= 2^2 \times 3^2 \times 7^2 \times 11^2 = 213444$

9. $\sqrt{1.5625} = ?$

Ⓐ 1.05

Ⓑ 1.25

Ⓒ 1.45

Ⓓ 1.55

$\sqrt{1.5625} = 1.25.$

10. If $x = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$, then the value of $(x^2 + y^2)$ is:

$$x = \frac{(\sqrt{3}+1)}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = \frac{(\sqrt{3}+1)^2}{(3-1)} = \frac{3+1+2\sqrt{3}}{2} = 2+\sqrt{3}.$$

$$y = \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} = \frac{(\sqrt{3}-1)^2}{(3-1)} = \frac{3+1-2\sqrt{3}}{2} = 2-\sqrt{3}.$$

$$\therefore\ x^2 + y^2 = (2+\sqrt{3})^2 + (2-\sqrt{3})^2$$

$$= 2(4+3)$$

$$= 14$$

11. 1250 oranges were distributed among a group of girls in a class. Each girl received twice as many oranges as the number of girls in that group. The number of girls in the group was _____.

Let the number of girls in the classroom be n.

Each girl gets = 2n oranges

Now, According to the question

$2n \times n = 1250$

$\Rightarrow 2n^2 = 1250$

$\Rightarrow n^2 = 625$

$\Rightarrow n = 25$

∴ The number of girls in the classroom is 25.

A gardener plants 17,956 trees such that the number of rows equals the number of trees in each row. Estimate the number of trees in a row.

Let the number of trees in each row be n.

The number of rows will also be equal to n.

Total number of trees planted = $n \times n = 17956$.

$\Rightarrow n^2 = 17956$

$\Rightarrow n = 134$.

$\therefore$ The number of trees in a row is 134.

A general wishes to arrange his 36,581 soldiers in the form of a solid square. After arranging them, he found that some soldiers were left over. Estimate the number of soldiers left.

	191
1	$3\,\overline{65}\,\overline{81}$
	1
29	265
	261
381	481
	381
	100

∴ Left solder are 100

12. Find the greatest four digit number which is a perfect square.

$$\begin{array}{r|l} & 9\,9 \\ \hline 9 & \overline{9\,9}\,\overline{9\,9} \\ & 8\,1 \\ \hline 189 & 1\,8\,9\,9 \\ & 1\,7\,0\,1 \\ \hline & \quad 1\,9\,8 \end{array}$$

Here,

Remainder = 198

Since *remainder* is ***not 0,***

So, 9999 is **not a perfect square**

Find the greatest number of 5 digits which is a perfect square.

Greatest number of 5-digits=99999

Finding square root,we see that 143 is left as remainder

	316
3	9 99 99
	9
61	99
	61
626	3899
	3756
	143

∴ Perfect square = 99999 – 143 = 99856

If we add 1 to 99999,it will because a number of 6 digits.

∴ Greatest square 5 – digits perfect square = 99856

$\sqrt{1.94 - 0.25} = ?$ $\sqrt{1.69} = 1.3$

$\sqrt{0.0009}$ $= \sqrt{\frac{0.0009 \times 10000}{10000}}$ $= \sqrt{\frac{9}{10000}} = \frac{3}{100} = 0.03$

$\sqrt{46.24}$ 6.8

$\sqrt{0.3969}$ 0.63

$$Find\ the\ sum:\quad 3+\frac{1}{\sqrt{3}}+\frac{1}{3+\sqrt{3}}-\frac{1}{3-\sqrt{3}}$$

Ans 3

Cube and Cube root

Find the cube root of 2744 by the prime factorization method.

The given number is 2744.

After resolving the prime factors, we get

$$2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$$

Upon grouping the factors we get

$$2744 = (2 \times 2 \times 2) \times (7 \times 7 \times 7)$$

Taking one factor from each group

$$\sqrt[3]{2744} = 2\ x\ 7$$

$$= 14$$

Hence, the required cube root is 14.

What is the Cube Root of 9.261?

$$\sqrt[3]{9.261} = \sqrt[3]{\frac{9261}{1000}}$$

$$= \sqrt[3]{\frac{3\times3\times3\times7\times7\times7}{10\times10\times10}}$$

$$= \frac{3\times7}{10}$$

$$= \frac{21}{10} = 2.1$$

By what number 4320 must be multiplied to obtain a number which is a perfect cube?

Prime factorising 4320, we get,

$4320 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$= 2^5 \times 3^3 \times 5^1$.

We know, a perfect cube has multiples of 3 as powers of prime factors.

Here, number of $2'$s is 5, number of $3'$s is 3 and number of $5'$s is 1.

So we need to multiply another 2, and 5^2 in the factorization to make 4320 a perfect cube.

Hence, the smallest number by which 4320 must be multiplied to obtain a perfect cube is $2 \times 5^2 = 50$.

By what least number 21600 must be multiplied to make it a perfect cube?

21600 can be factorized as $6 \times 6 \times 6 \times 10 \times 10$

To make it perfect cube, it must be multiplied by 10.

Evaluate $\sqrt{0.01} \times \sqrt[3]{0.008} - 0.02$

$=0.1 \times 0.2 - 0.02$

$=0.02 - 0.02$

$=0$

Find $\sqrt[3]{\sqrt{0.000064}}$

$= \sqrt[3]{0.008}$

$= 0.02$

$$Find\ x: 99 \times 21 - \sqrt[3]{x} = 1968$$

$$\sqrt[3]{x} = 99 \times 21 - 1968$$

=111

X = $(111)^3$

By what least number 675 be multiplied to obtain a number which is a perfect cube?

$675 = 3^3 \times 5^2$ So it has to be multiplied by 5.

By what smallest number should 3600 be multiplied to make it a perfect cube?

2	3600
2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

Prime factors of 3600 = $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

(2 marks)

Grouping the factors into triplets of equal factors, we get

$3600 = \underline{2 \times 2 \times 2} \times 2 \times 3 \times 3 \times 5 \times 5$

We know that, if a number is to be a perfect cube, then each of its prime factors must occur thrice.

We find that 2 occurs 4 times while 3 and 5 occurs twice only.

Hence, the smallest number, by which the given number must be multiplied in order that the product is a perfect cube = $2 \times 2 \times 3 \times 5 = 60$

The cube root of 2744 is

On prime factorization,

$14 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$

Hence, the cube root of 2744 is 14.

Cube root of 0.000216 is:

$$\sqrt[3]{0.000216} = \sqrt[3]{\frac{216}{1000000}} = \sqrt[3]{\frac{6^3}{100^3}} = \frac{6}{100} = 0.06.$$