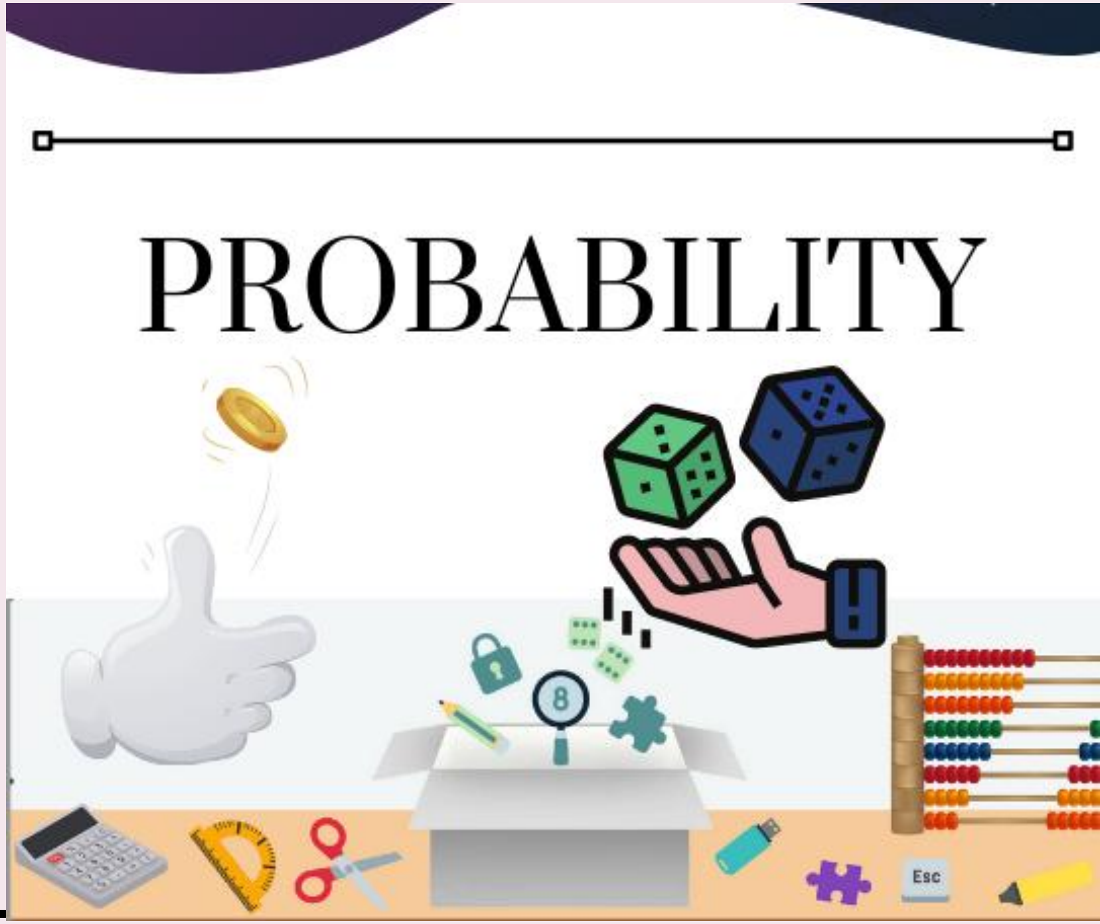
# MODULE 3
# PROBABILITY

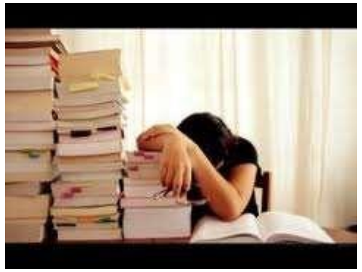


# What is probability?

Most people use terms like chance, likelihood, or probability to express the degree of uncertainty related to certain issues or events.
The following are some examples in which these terms are used:

- There is a 70% chance of rain tomorrow, according to forecasters, as you watch the news every day.

- When you decide to start a new business, an expert in the field tells you that the probability of making a profit in the first year is only 0.4, or there is a 40% chance that you will make a profit.



- As you take off a new course, you may be wondering whether you will pass or fail it.

# Experiment, outcome, event, and Sample Space

***Experiment:***

An experiment is a test, trial, or procedure for the purpose of discovering something unknown.

***An outcome:***

The result of a single trial of an experiment

***An event:***

A collection of one or more outcomes of an experiment

***Sample space:***

All possible outcomes taken together represent the ‘*sample space’* for the experiment.

**Example 1:** In rolling a six-sided die,

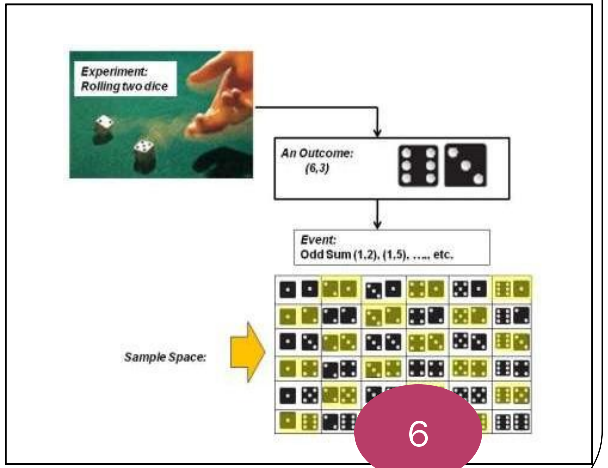
- What is an experiment?
- What is an outcome?
- What is an event?
- What is a sample space?

**Solution:**

Rolling a six-sided die is the experiment. A number such as 1, 2, 3, 4, 5, or 6 is the outcome. Specifying a certain number, such as odd or even number will be the event. The sample space for this experiment is $S = \{1, 2, 3, 4, 5, 6\}$.

**Example 2:** In the process of rolling a pair of fair dice,

- What is an experiment?
- What is an outcome?
- What is an event?
- What is a sample space?

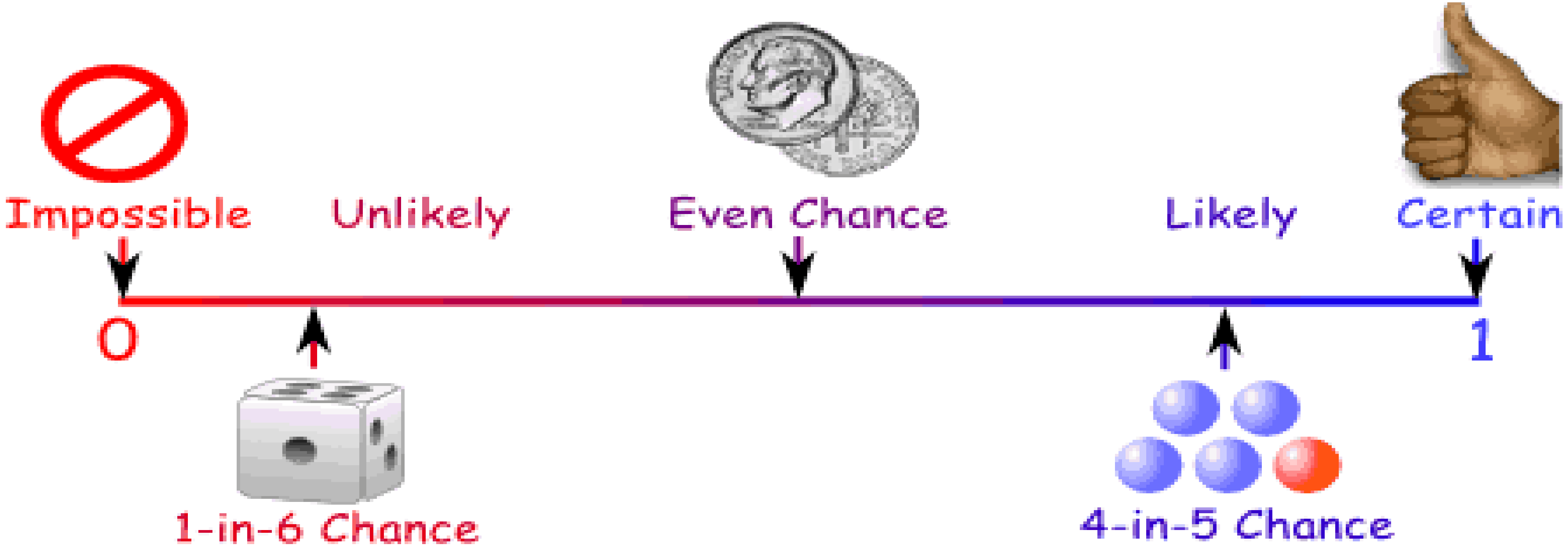


# Experiment, outcome, event, and Sample Space: More Examples

| Experiment | Outcomes | Event | Sample space |
|---|---|---|---|
| Rolling a die once | 1, 2, 3, 4, 5, or 6 | *Odd number* | $S$ ={1, 2, 3, 4, 5, 6} |
| Taking a test | *A, B, C, D, or F* | *Passing the test* | $S$ = {$A$, $B$, $C$, $D$, $F$} |
| Selecting an age | Old, middle, or young | *Getting older* | $S$ = {old, middle, young} |
| Birth | Male or female | *Gender choice* | $S$ = {male, female} |
| Tossing a coin twice | ($H$, $H$), ($H$, $T$), ($T$, $H$), or ($T$, $T$) | *Two heads* | $S$ = {($H$, $H$), ($H$, $T$), ($T$, $H$), ($T$, $T$)} |

# How do we describe probability?

A probability of an event can be represented on a number line.



# Mathematical Probability:

The classic probability of an event, $A$, is determined by the ratio between the number of favorable outcomes, $m$, and the total number of possible outcomes, $n$:

$$P(A) = \frac{number\ of\ favorable\ outcomes}{total\ number\ of\ outcomes} = \frac{m}{n}$$

The probability of non-occurrence of the event $A$ is called the probability of failure of occurrence and is denoted by:

$$P(A') = q(A) = P(not\ A) = \frac{(n-m)}{n} = 1 - p(A) = 1 - \frac{m}{n}$$

## Axioms

(I) For an event $A$, the probability will lie between 0 and 1, or

$$0 \leq P(A) \leq 1$$

(II) The sum of probabilities in a given experiment is always 1, $\sum P(A_i) = 1$

$\bar{A}$ This is read "*A* complement" and is the set of all elements in the sample space that are not in *A*

Remembering our second property of probability, "The sum of all the probabilities equals 1" we can determine that:

$$P(A) + P(\bar{A}) = 1$$

This is more often used in the form

$$P(A) = 1 - P(\bar{A})$$

If we know the probability of rain is 20% or 0.2 then the probability of the complement (no rain) is 1 - 0.2 = 0.8 or 80%

# Types of Events

1. **Mutually Exclusive Events** :- Events A and B are said to be mutually exclusive if and only if they cannot happen at the same time. ( Two events can't happen at the same time)

**Examples:** Turning left and right

Tossing a coin: Heads and Tails

Drawing a card: King and Queen

2. **Non-Mutually Exclusive Events** : if and only if they can happen at the same time.

**Examples:** Turning and scratching nose

Drawing a Card: King and Heart

3. **Independent Events :-** If event A cannot influence the outcome of event B

**Examples:** Flipping a coin 5 times

The chance that HEAD will occur does not affect the occurrence of the TAIL from one trial to another.

4. **Dependent Events :-** If event A can influence the occurrence of event B

**Examples:** Drawing 5 balls from the box

The chance of the next ball to be drawn is dependent on the outcome of the previous draw.

Let's roll a die once.

This is the sample space for all the possible outcomes

$$S = \{1, 2, 3, 4, 5, 6\}$$

probability an event will occur

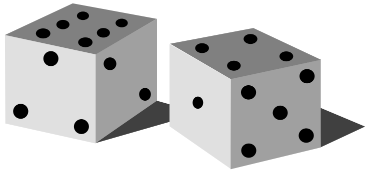
$$P(E) = \frac{\text{Number of ways that } E \text{ can occur}}{\text{Number of possibilities}}$$

What is the probability you will roll an even number?

There are 3 ways to get an even number, rolling a 2, 4 or 6

$$P(\text{Even number}) = \frac{3}{6} = \frac{1}{2}$$

There are 6 different numbers on the die.

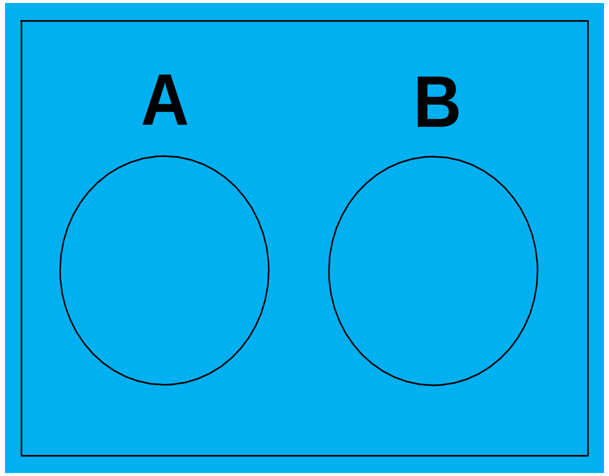


# Mutually Exclusive

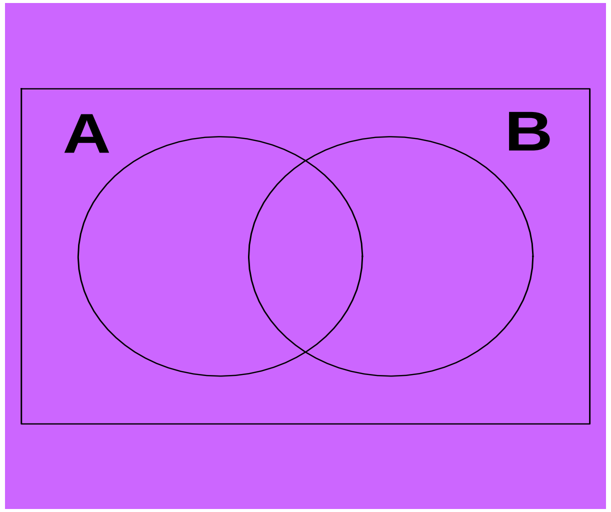
Two events A and B are said to be mutually exclusive if and only if A occur and B does not occur

$$P(A \cap B) = 0 \;\; and \;\; P(A \cup B) = P(A) + P(B)$$

In a Venn diagram this means that event A is disjoint from event B.




A and B are M.E.




A and B are not M.E.

# Addition Rule

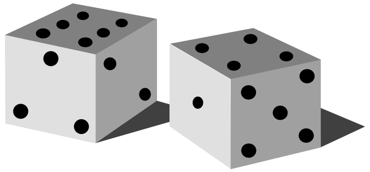
The probability that at least one of the events A or B will occur, P(A or B), is given by:

$$P(A\ or\ B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If events A and B are mutually exclusive, then the addition rule is simplified to:

$$P(A\ or\ B) = P(A \cup B) = P(A) + P(B)$$

This simplified rule can be extended to any number of mutually exclusive events.

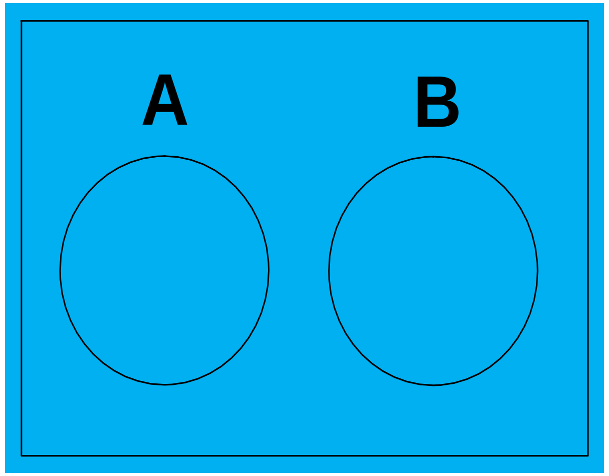


# Independent Events

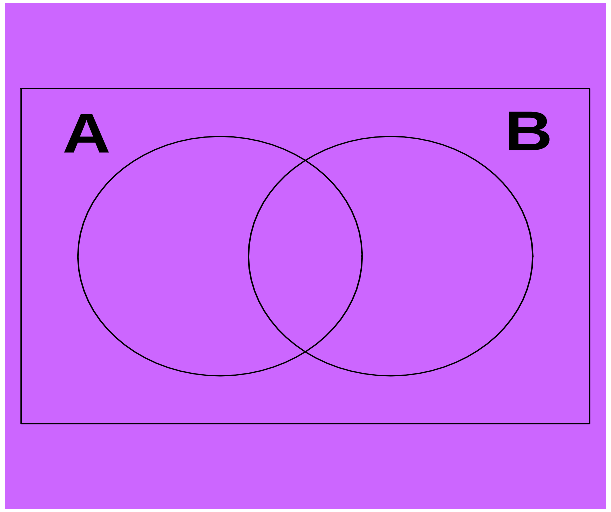
Two events A and B are said to be Independent events if and only if A and B are occurring in a simultaneously is

$$P(A \cap B) = P(A) * P(B) \text{ and } P(A \cup B) = 0$$

In a Venn diagram this means that event A is independent from event B




A and B are independent




A and B are not independent

# Multiplication Rule

If events A and B are independent, then the probability of two events, A and B occurring in a sequence is:

$$P(A\ and\ B) = P(A \cap B) = P(A) * P(B)$$

This rule can extend to any number of independent events.

Two events are independent if the occurrence of the first event does not affect the probability of the occurrence of the second event.

# Example

1. A coin is tossed at three times, what is the probability of getting (i) Exactly 2 heads (ii) At least 2 heads.

**Ans:** $S = \{HHH, HHT, HTT, HTH, TTT, TTH, THH, THT\}$

**P(Exactly 2 heads) = 3/8**

**P(At least 2 heads) = 4/8**

2. If A and B are mutually exclusive events,

P(A) = 0.26, P(B) = 0.45

Find (a) $P(\bar{A})$ (b) P (A ∪ B) (c) $P(\bar{A} \cap \bar{B})$

**Ans:**

$P(A) = 0.26\,; P(B) = 0.45$

**Find $P(\bar{A})$:**

$P(\bar{A}) = 1 - P(A) = 1 - 0.26 = 0.74$

**Find P (A ∪ B):**

P (A ∪ B) $= P(A) + P(B) = 0.26 + 0.45 = 0.71$ Where A and B are Mutually Exclusive

**Find $P(\bar{A} \cap \bar{B})$:**

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.71 = 0.29$

# Example

3. If P(A) = 0.29, P(B) = 0.43 then find $P(A\cap\bar{B})$, if A and B are mutually exclusive

**Ans: 0.29**

If A and B are Mutually Exclusive then $P(A \cap B) = 0$

$P(A\cap\bar{B}) = P(A) - P(A \cap B) = 0.29 - 0 = 0.29$

4. What is the probability of getting 53 Sundays in a leap year and non leap year?

**Ans :**

In a leap year (366) days, $\frac{364}{7} = 52$ are complete weeks, rst two days can be:-

Sun M, M T, T We, We Th, Th Fr, Fr Sa, Sa Su.

Out of which, 2 combinations would given 53 Sundays.

$\Rightarrow$ Probability = $\frac{2}{7}$.

For a non-leap year, the probability of getting 53 Sundays is **1/7**. This is because there are 52 complete weeks and one extra day in a non-leap year, and the extra day can be any of the seven days of the week. Therefore, the chance of the extra day being a Sunday is 1 out of 7.

5. If P(A) = 0.35 , P(B) =0.75 and P(AUB) =0.95 then find $P(\bar{A} \cup \bar{B})$

**Ans : 0.85**

$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - [P(A) + P(B) - P(A \cup B)]$

$P(\bar{A} \cup \bar{B}) = 1 - [0.35 + 0.75 - 0.95] = 1 - 0.15 = 0.85$

# Example

6. From the bag containing 3 red and 2 blacks balls, 2 balls are drawn at random. Find the probability that they are of the same colour.

**Ans:** Total no of balls = 5; 3 Red balls and 2 black balls. 2 balls are drawn at random. Therefore, the total number of possible ways, say n

$$n = 5C_2 = \frac{5*4}{1*2} = 10 \text{ Ways.}$$

P{ both of them same colour} = m $= 3C_2 + 2C_2 = 3 + 1 = 4$

Therefore, the required probability $P(A) = \frac{m}{n} = \frac{4}{10} = \frac{2}{5}$

7. When A and B are mutually exclusive events, such that P(A)=1/2 , P(B)=1/3 then find P(A ∪ B) and $P(A \cap B)$.

**Ans :** Given P (A) =1/2, P (B) =1/3.If A and B are mutually exclusive then $P(A \cap B) = 0$ and

P(A ∪ B) = P(A)+P(B) = 1/2+ 1/3 = 5/6

8. What is the probability of drawing an ace from a well shuffled deck of 52 playing cards?

**Ans :There are m= 4 ace among the n= 52 cards, so we get,**

**P(A) = m/n = 4/52 = 1/13**

# Example

9. If P(A)=0.65 , P(B)=0.4 and $P(A \cap B)$=0.24, Can A and B are an independent events?

**Ans:** If A and B are an independent events then $P(A \cap B) = P(A) * P(B)$, Therefore,

$0.24 = 0.65 * 0.4 = 0.26$; Here $0.24 \neq 0.26$ Hence A and B are not an independent events.

10. An urn contains 3 white balls, 4 red balls and 5 black balls. 2 balls are drawn at random. Find the probability that (i) both of them are of same colour(ii) they are different colour

**Ans:** Total no of balls = 12; 3 white balls, 4 red balls and 5 black balls. 2 balls are drawn at random.

Total no of possible ways = $n = 12C_2 = \frac{12*11}{1*2} = 66$ ways.

P{ Both of them are of same colour} = m = $3C_2 + 4C_2 + 5C_2 = 3 + 6 + 10 = 19$

Therefore, the required Probability is $P(A) = \frac{m}{n} = \frac{19}{66}$

P{They are different colour} = 1- P{ Both of them are of same colour} $= 1 - \frac{19}{66} = \frac{47}{66}$

# Example

11. A lot consists of 10 good articles, 4 with minor defects and 2 with major defects. Two articles are chosen at random. Find the probability that (i) both are good (ii) both have major defects (iii) atleast one is good (iv) atmost one is good (v) exactly one is good

## Answers

Total no of articles = 16; 10 good articles, 4 with minor defects and 2 with major defects.

2 articles are chosen at random. Therefore, total no of possible ways, n $=16C_2 = \frac{16*15}{1*2} = 120$ ways.

(i) $P\{\text{both are good}\} = m = \dfrac{\text{No of ways drawing 2 good articles}}{\text{Total no of ways of drawing 2 articles}} = \dfrac{10C_2}{120} = \dfrac{3}{8}$

(ii) $P\{\text{both have major defects}\} = m = \dfrac{\text{No of ways drawing 2 articles with major}}{\text{Total no of ways of drawing 2 articles}} = \dfrac{2C_2}{120} = \dfrac{1}{120}$

(iii) $P\{\text{atleast one is good}\} = P\{\text{Exactly 1 is good or both are good}\}\, m = \dfrac{10C_1 * 6C_1}{120} + \dfrac{10C_2}{120} = \dfrac{7}{8}$

(iv) $P\{\text{atmost one is good}\} = P\{\text{None is good or 1 is good}\} = m = \dfrac{10C_0 * 6C_2}{120} + \dfrac{10C_1 * 6C_1}{120} = \dfrac{5}{8}$

(v) $P\{\text{exactly one is good}\} = m = \dfrac{10C_1 * 6C_1}{120} = 0.5$

# Example

12. In a shooting test, the probability of hitting the target is 1/2 for A, 2/3 for B and 3/4 for C. If all of them fire at the target then find the probability that (a) None of them hits the target (b) Atleast one of them hits the target (c) Exactly two of them hits the target

**Ans:**

Given, P(A)=1/2 ; P(B) = 2/3 ; P(C)=3/4 ; $P(\bar{A}) = \frac{1}{2}; P(\bar{B}) = \frac{1}{3}$ ; $P(\bar{C}) = \frac{1}{4}$

(i) P{ None of them hits the target} $= P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A}) * P(\bar{B}) * P(\bar{C}) = \frac{1}{24}$

(ii) P{ Atleast one of them hits the target}=1- P{None of them hits the target}$= 1 - \frac{1}{24} = \frac{23}{24}$

(iii) P(Exactly two of them hits the target ) = $P(AB\bar{C}) + P(A\bar{B}C) + P(\bar{A}BC)$

$=P(A)\,P(B)\,P(\bar{C}) + P(A)\,P(\bar{B})\,P(C) + P(\bar{A}\,)\,P(B)\,P(C)$

=1/2 * 2/3 * 1/4 +1/2 * 1/3 * 3/4+1/2 * 2/3 * 3/ 4

= 11/24