Elementary Mensuration

1. CUBOID

   Let length = l , breadth = b and height = h units. Then

   1. **Volume** = (l x b x h) cubic units.

   2. **Surface area** = 2(lb + bh + lh) sq. units.

   3. **Diagonal** = $\sqrt{l^2 + b^2 + h^2}$ units.

2. CUBE

   Let each edge of a cube be of length a . Then,

   1. **Volume** = a^3 cubic units.

   2. **Surface area** = $6a^2$ sq. units.

   3. **Diagonal** = $\sqrt{3}a$ units.

3. CYLINDER

Let radius of base = r and Height (or length) = h . Then,

1. **Volume** = $(\pi r^2 h)$ cubic units.
2. **Curved surface area** = $(2\pi rh)$ sq. units.
3. **Total surface area** = $2\pi r\,(h + r)$ sq. units.

4. CONE

Let radius of base = r and Height = h . Then,

1. **Slant height,** $l = \sqrt{h^2 + r^2}$ units.
2. **Volume** = $\left(\frac{1}{3}\pi r^2 h\right)$ cubic units.
3. **Curved surface area** = (πrl) sq. units.
4. **Total surface area** = $(\pi rl + \pi r^2)$ sq. units.

5. SPHERE

Let the radius of the sphere be r . Then,

1. **Volume** = $\left(\frac{4}{3}\pi r^3\right)$ cubic units.

2. **Surface area** = $(4\pi r^2)$ sq. units.

6. HEMISPHERE

Let the radius of a hemisphere be r . Then,

1. **Volume** = $\left(\frac{2}{3}\pi r^3\right)$ cubic units.

2. **Curved surface area** = $(2\pi r^2)$ sq. units.

3. **Total surface area** = $(3\pi r^2)$ sq. units.

Note: 1 litre = 1000 cm^3.

Carpeting a floor:

$$\text{Length of the carpet required} \equiv \frac{\text{Length of room} \times \text{breadth of room}}{\text{width of carpet}}$$

$$\text{Amount required} = \text{Rate per meter} \times \frac{\text{Length of room} \times \text{breadth of room}}{\text{width of carpet}}$$

1. **How many meters of a carpet 75cm wide will be required to cover the floor of a room which is 20 meter long and 12 meters board?**

Solution:

Length required $= \frac{\text{Length of room} \times \text{breadth of room}}{\text{width of carpet}}$

$= \frac{20 \times 12}{0.75}$

$= 320$ meter

2. **Find the cost of carpeting a room 13m long and 9m board with carpet 75cm wide at the rate of Rs.12.40 per meter.**

Solution:

$$\text{Amount required} = \text{Rate per meter} \times \frac{\text{Length of room} \times \text{breadth of room}}{\text{width of carpet}}$$

$$= \text{Rs. } 12.40 \times \frac{13 \times 9}{0.75}$$

$$= \text{Rs.} 12.40 \times 156$$

$$= \text{Rs. } 1934.40$$

Paving the tiles:

Number of tiles required

$$= \frac{\text{Length} \times \text{ breadth of the courtyard}}{\text{Length} \times \text{breadth of each tile}}$$

Side of largest possible square tile = H.C.F of length and breadth of the room

3**. How many paving stones each measuring 2.5m×2m are required to pave a rectangular courtyard 30m long and 16.5m wide?**

Solution:

Number of tiles required

$$= \frac{\text{Length} \times \text{ breadth of the courtyard}}{\text{Length} \times \text{breadth of each tile}}$$

$$= \frac{30 \times 16.5}{2.5 \times 2} = 99$$

4.A rectangular courtyard, 3.78m long and 5.25m broad, is to be paved exactly with square tiles, all of the same size. Find the least number of square tiles covered.

Side of largest possible square tile = H.C.F of length and breadth of the room = 378 cm, 525cm

$$= 21 \text{ cm}$$

Number of tiles required

$$= \frac{\text{Length} \times \text{breadth of the courtyard}}{\text{Length} \times \text{breadth of each tile}}$$

$$= \frac{378 \times 525}{21 \times 21}$$

$$= 450$$

5. **The length of the rectangle is twice its breadth. If its length is decreased by 5cm and breadth is increased by 5 cm the area of the rectangle is increased by 75sq.cm. Find the length of the rectangle.**

Solution:

Let breadth = x then length = 2x.

Then, $(2x-5)(x+5) - 2x \times x = 75$

$5x-25 = 75$

$x = 20$

Length of the rectangle = 40cm

6. **Find the area of a square, one of whose diagonal is 3.8m long**

Solution:

Area of the square = $\frac{1}{2} \times \text{diagonal}^2$

$= \frac{1}{2} \times 3.8 \times 3.8 \ \text{m}^2$

$= 7.22 \ \text{m}^2$

7. **If each side of a square is increased by 25%, find the percentage change in its area.**

Solution:

Let each side of the square be a. Then area = a^2

New side = $\frac{125a}{100} = \frac{5a}{4}$

New area = $\left(\frac{5a}{4}\right)^2 = \frac{25a^2}{16}$

Increase in area = $\frac{25a^2}{16} - a^2 = \frac{9a^2}{16}$

Increase percentage = $\frac{9a^2}{16} \times \frac{1}{a^2} \times 100\% = 56.25\%$

8. A rectangular block 6 cm by 12 cm by 15 cm is cut up into an exact number of equal cubes. Find the least possible number of cubes.

Volume of the block = (6 x 12 x 15) cu.cm = 1080 cu.cm

Side of the largest cube = H.C.F. of 6 cm, 12 cm, 15 cm
= 3 cm.

Volume of this cube = (3 x 3 x 3) cu.cm = 27 cu.cm

Number of cubes =1080/27 = 40.

9. Three cubes of sides 8 cm, 6 cm and 1 cm are melted to form a new cube. The surface area of the cube so formed is _____.

- Volume of the new cube = Sum of volumes of the three cubes

  $\Rightarrow a^3 = (8\text{ cm})^3 + (6\text{ cm})^3 + (1\text{ cm})^3$ [Volume of cube = $(\text{Side})^3$]

  $\Rightarrow a^3 = 512 + 216 + 1 = 729\text{ cm}^3$

  $\Rightarrow a^3 = (9\text{ cm})^3$

  $\Rightarrow a = 9\text{ cm}$

  $\therefore$ Surface area of the new cube = $6a^2 = 6 \times (9\text{ cm})^2 = 6 \times 81\text{ cm}^2 = 486\text{ cm}^2$

  Thus, the surface area of the new cube is 486 cm^2.

10. Two cubes have their volumes in the ratio 1:27. Find the ratio of their surface areas.

Given, $V_1:V_2=1:27$

$\Rightarrow V_1:V_2=a^3_1:a^3_2$

$\Rightarrow a^3_1:a^3_2=1:27$

$\Rightarrow a_1:a_2=1:3$

$\Rightarrow a^2_1:a^2_2=1:3^2=1:9$

$Sa_1:Sa_2=6a^2_1:6a^2_2=a^2_1:a^2_2=1:9$

Therefore, the ratio of surface areas is 1:9.

11. The radii of the bases of a cylinder and cone are in the ratio 3:4 and their heights are in the ratio 2:3 then their volumes are in the ratio is

$$\frac{r_2}{r_1} = \frac{3}{4}, \frac{h_2}{h_1} = \frac{2}{3}$$

$$\text{ratio} = \frac{\text{volumeofcylinder}}{\text{volumeofcone}}$$

$$= \frac{\pi r_2^2 h_2}{\frac{1}{3}\pi r_1^2 h_1}$$

$$= 3\left(\frac{r_2}{r_1}\right)^2 \left(\frac{h_2}{h_1}\right)$$

$$= 3\left(\frac{3}{4}\right)^2 \left(\frac{2}{3}\right)$$

$$= 3 \times \frac{9}{16} \times \frac{2}{3}$$

$$= \frac{9}{8}$$

$\therefore$ Ratio is 9 : 8.

12. Two right circular cones having same base radius of 2.1 cm but heights 4.1 cm and 4.3 cm are melted and recast into a single sphere Find the radius of the sphere so formed

Volume of the sphere

$\frac{4}{3}\pi R^3$=volumes of the cones of hights h_1 and h_2 with same radius r

$= \frac{1}{3}\pi r^2 (h_1 + h_2)$

$= \frac{1}{3} \times \pi \times (2.1)^2 \times (8.4)$

$R^3 = \frac{1}{4} \times (2.1)^2 \times (8.4)$

$= (2.1)^3$

or R = 2.1cm

13. How many spherical bullets can be made out of a lead cylinder 28cm high and with base radius 6cm each bullet being 1.5 cm in diameter

Diameter of sphere = 1.5 cm

$\Rightarrow$ Radius of sphere = 1.5/2 cm

Let assume that n bullets can be made.

According to the question:

$\Rightarrow \pi r^2 h = n(\frac{4}{3}\pi r^3)$

$\Rightarrow \pi(6)^2(28) = n[\frac{4}{3}\pi(1.5/2)^3]$

$\Rightarrow 36 \times 28 = n \times \frac{4}{3} \times \frac{27}{64}$

$\Rightarrow n = 36 \times 28 \times \frac{16}{9}$

$\Rightarrow n = 1792$

$\therefore$ 1792 bullets can be made.

The correct option is 2 i.e. 1792

14. Rohan Bought a House, which had a garden outside it, with sides measuring 12 meters and 8 meters, Find the perimeter of the garden ?

The formula to find Perimeter is , P= 2(L+W)
= 2(12+8)
=2(20)
=40m Ans.

15. To protect his house from Robbers, Mohan wrapped a wire around his square plot. If the wire's length is 80 meters, what is the perimeter of the square?

Answer: Perimeter of Square Plot = 4a(Where a = length of side)
= 4 x 80
= 320m Ans.