Decimal Fractions

1. **Decimal Fractions:**

   Fractions in which denominators are powers of 10 are known as **decimal fractions.**

   Thus, $\frac{1}{10}$ = 1 tenth = .1; $\frac{1}{100}$ = 1 hundredth = .01;

   $\frac{99}{100}$ = 99 hundredths = .99; $\frac{7}{1000}$ = 7 thousandths = .007, etc.;

2. Conversion of a Decimal into Vulgar Fraction:

Put 1 in the denominator under the decimal point and annex with it as many zeros as is the number of digits after the decimal point. Now, remove the decimal point and reduce the fraction to its lowest terms.

Thus, $0.25 = \frac{25}{100} = \frac{1}{4}$; $2.008 = \frac{2008}{1000} = \frac{251}{125}$.

3. Annexing Zeros and Removing Decimal Signs:

Annexing zeros to the extreme right of a decimal fraction does not change its value. Thus, 0.8 = 0.80 = 0.800, etc.

If numerator and denominator of a fraction contain the same number of decimal places, then we remove the decimal sign.

Thus, $\frac{1.84}{2.99} = \frac{184}{299} = \frac{8}{13}$.

4. Operations on Decimal Fractions:

    1. **Addition and Subtraction of Decimal Fractions:** The given numbers are so placed under each other that the decimal points lie in one column. The numbers so arranged can now be added or subtracted in the usual way.

    2. **Multiplication of a Decimal Fraction By a Power of 10:** Shift the decimal point to the right by as many places as is the power of 10.

       Thus, $5.9632 \times 100 = 596.32$; $0.073 \times 10000 = 730$.

3. **Multiplication of Decimal Fractions:** Multiply the given numbers considering them without decimal point. Now, in the product, the decimal point is marked off to obtain as many places of decimal as is the sum of the number of decimal places in the given numbers.

   Suppose we have to find the product (.2 × 0.02 x .002).

   Now, 2 × 2 × 2 = 8. Sum of decimal places = (1 + 2 + 3) = 6.

   ∴ .2 x .02 x .002 = .000008

5. **Dividing a Decimal Fraction By a Decimal Fraction:** Multiply both the dividend and the divisor by a suitable power of 10 to make divisor a whole number.

   Now, proceed as above.

   Thus, $\frac{0.00066}{0.11} = \frac{0.00066 \times 100}{0.11 \times 100} = \frac{0.066}{11} = .006$

5. Comparison of Fractions:

Suppose some fractions are to be arranged in ascending or descending order of magnitude, then convert each one of the given fractions in the decimal form, and arrange them accordingly.

Let us to arrange the fractions $\frac{3}{5}$, $\frac{6}{7}$ and $\frac{7}{9}$ in descending order.

Now, $\frac{3}{5} = 0.6$, $\frac{6}{7} = 0.857$, $\frac{7}{9} = 0.777...$

Since, $0.857 > 0.777... > 0.6$. So, $\frac{6}{7} > \frac{7}{9} > \frac{3}{5}$.

6. **Recurring Decimal:**

If in a decimal fraction, a figure or a set of figures is repeated continuously, then such a number is called a **recurring decimal.**

n a recurring decimal, if a single figure is repeated, then it is expressed by putting a dot on it. If a set of figures is repeated, it is expressed by putting a bar on the set.

Thus, $\frac{1}{3} = 0.333... = 0.\dot{3}$; $\frac{22}{7} = 3.142857142857.... = 3.\overline{142857}$.

Pure Recurring Decimal: A decimal fraction, in which all the figures after the decimal point are repeated, is called a pure recurring decimal.

Converting a Pure Recurring Decimal into Vulgar Fraction: Write the repeated figures only once in the numerator and take as many nines in the denominator as is the number of repeating figures.

Thus, $0.\overline{5} = \frac{5}{9}$; $0.\overline{53} = \frac{53}{99}$; $0.\overline{067} = \frac{67}{999}$, etc.

Converting into Vulgar Fraction

$$0.\overline{39} = \frac{39}{99}$$

$$0.\overline{37} = \frac{37}{99}$$

$$0.\overline{053} = \frac{53}{999}$$

Mixed Recurring Decimal: A decimal fraction in which some figures do not repeat and some of them are repeated, is called a mixed recurring decimal.

Eg. $0.1733333.. = 0.17\overline{3}$.

Converting a Mixed Recurring Decimal Into Vulgar Fraction: In the numerator, take the difference between the number formed by all the digits after decimal point (taking repeated digits only once) and that formed by the digits which are not repeated. In the denominator, take the number formed by as many nines as there are repeating digits followed by as many zeros as is the number of non-repeating digits.

Thus, $0.1\overline{6} = \frac{16 - 1}{90} = \frac{15}{90} = \frac{1}{6}$; $0.22\overline{73} = \frac{2273 - 22}{9900} = \frac{2251}{9900}$.

Converting into Vulgar Fraction

$$0.2\dot{7}$$

$$= \frac{27-2}{90} = \frac{25}{90}$$

$$0.1\dot{7}$$

$$= \frac{17-1}{90} = \frac{16}{90}$$

$$0.12\overline{54}$$

$$= \frac{1254-12}{9900} = \frac{1242}{9900}$$

7. Some Basic Formulae:

1. $(a + b)(a - b) = (a^2 - b^2)$
2. $(a + b)^2 = (a^2 + b^2 + 2ab)$
3. $(a - b)^2 = (a^2 + b^2 - 2ab)$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
5. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
6. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
7. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
8. When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

$$\frac{0.0203 \times 2.92}{0.0073 \times 14.5 \times 0.7} = ?$$

$$\frac{0.0203 \times 2.92}{0.0073 \times 14.5 \times 0.7} = \frac{203 \times 292}{73 \times 145 \times 7} = \frac{4}{5} = 0.8$$

617 + 6.017 + 0.617 + 6.0017 = ?

A. 6.2963

B. 62.965

C. 629.6357

D. None of these

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  617.00
    6.017
    0.617
+   6.0017
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  629.6357
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If $\frac{144}{0.144} = \frac{14.4}{x}$, then the value of x is:

A. 0.0144

B. 1.44

C. 14.4

D. 144

$$\frac{144}{0.144} = \frac{14.4}{x}$$

$$\Rightarrow \frac{144 \times 1000}{144} = \frac{14.4}{x}$$

$$\Rightarrow x = \frac{14.4}{1000} = 0.0144$$

$$\frac{4.2 \times 4.2 - 1.9 \times 1.9}{2.3 \times 6.1}$$ is equal to:

A. 0.5

B. 1.0

C. 20

D. 22

$$\text{Given Expression} = \frac{(a^2 - b^2)}{(a + b)(a - b)} = \frac{(a^2 - b^2)}{(a^2 - b^2)} = 1.$$

Evaluate $0.\dot{6} + 0.\dot{7} + 0.\dot{8} + 0.\dot{3}$

$$= \frac{6}{9} + \frac{7}{9} + \frac{8}{9} + \frac{3}{9}$$

$$= 2.6666666 = 2.\dot{6}$$

Evaluate $3.\overline{87} - 2.\overline{59}$

$$\frac{387}{99} - \frac{259}{99} = \frac{128}{99} = 1.292929\ldots. = 1.\overline{29}$$

Simplify $\dfrac{(0.01)^2+(0.22)^2+(0.333)^2}{(0.001)^2+(0.022)^2+(0.0333)^2}$

$$\frac{(0.01)^2+(0.22)^2+(0.333)^2}{(0.001)^2+(0.022)^2+(0.0333)^2} = \frac{(0.01)^2+(0.22)^2+(0.333)^2}{(\frac{0.01}{10})^2+(\frac{0.22}{10})^2+(\frac{0.333}{10})^2} = 100$$

Find the value of $\dfrac{343 \times 343 \times 343 - 113 \times 113 \times 113}{343 \times 343 + 343 \times 113 + 113 \times 113}$

$(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$

$$\frac{a^3 - b^3}{a^2 + ab + b^2} = a - b = 343 - 113 = 230$$

If $\frac{b}{a} = 0.25$ then $\frac{2a - b}{2a + b} + \frac{2}{9} = ?$

b/a = 0.25 = b/a = 1/4

∴ By putting a = 4x and b = x in the given expression we get

⇒ (2 × 4x - x)/(2 × 4x + x) + 2/9

⇒ 7/9 + 2/9

⇒ 9/9 = 1

The product of two numbers is 0.008 and one is $\frac{1}{5}$ of the other, then the smaller number is

Let the smaller number is x

Then bigger number of is 5 times of smaller number because one number is $\frac{1}{5}$ another

Then product of number $= 5x \times x = 0.008$

$\Rightarrow x^2 = 0.0016$

$\Rightarrow x = 0.04$

Vishal donates blood thrice in 2 years-each time 350 ml. How many litres of blood will he donate in 6 years ?

Quantity of blood donated in 2 years

= (350 × 3) ml

= 1050 ml

= 1.05 litres

∴ Quantity of blood donated in 6 years

$$= \left(\frac{1.05}{2} \times 6\right)$$

= 3.15 litres

The numerator of a fraction is decreased by 25% and the denominator is increased by 250%. If the resultant fraction is 6/5 what is the original fraction ?

Let the original fraction = $\frac{x}{y}$

Numerator after 25% decrease = $x - \frac{25}{100}x = \frac{75x}{100}$

Denominator after 250% increase = $y + \frac{250}{100}y = \frac{350y}{100}$

Acc to ques,

=> $\frac{\frac{75x}{100}}{\frac{350y}{100}} = \frac{6}{5}$

=> $\frac{3x}{14y} = \frac{6}{5}$

=> $\frac{x}{y} = \frac{6}{5} \times \frac{14}{3}$

=> $\frac{x}{y} = \frac{28}{5}$

5/9 of a number is equal to twenty five percent of a second number. Second number is equal to 1/4 of a third number. The value of the third number is 2960. How much is the 30% of the first number?

Let's assume the first number is x and the second number is y:

$5/9\ x = 25/100\ y$

Multiplying both sides by 9/5, we get:

$x = 9/5 \times 25/100 \times y$

$\Rightarrow x = 0.45y$

Now we know that the second number is equal to 1/4 of a third number, and the value of the third number is 2960:

$y = 1/4 \times 2960$

$\Rightarrow y = 740$

Substituting y = 740 into the equation x = 0.45y, we get:

$x = 0.45 \times 740$

$\Rightarrow x = 333$

So, the first number is 333.

Now we need to find 30% of the first number:

30% of 333

$\Rightarrow 0.3 \times 333 = 99.9$

$\therefore$ 30% of the first number is 99.9.