Module 1
Elementary Algebra

H.C.F and L.C.M of Numbers

Least Common Multiple (L.C.M)

The least number which is exactly divisible by each one of the given numbers is called L.C.M

Example : Find out LCM of 8 and 14

Step 1 : Express each number as a product of prime factors.

$8 = 2^3$

$14 = 2 \times 7$

Step 2 : LCM = The product of highest powers of all prime factors.

Here the prime factors are 2 and 7

The highest power of 2 here = 2^3

The highest power of 7 here = 7

Hence LCM = $2^3 \times 7 = 56$

Example : Find out LCM of 18, 24, 9, 36 and 90

2	18,	24,	9,	36,	90
2	9,	12,	9,	18,	45
3	9,	6,	9,	9,	45
3	3,	2,	3,	3,	15
	1,	2,	1,	1,	5

Hence Least common multiple (L.C.M) of 18, 24, 9, 36 and 90

$= 2 \times 2 \times 3 \times 3 \times 2 \times 5$

$= 360$

Highest Common Factor(H.C.F)

Highest Common Factor (H.C.F) of two or more numbers is the greatest number which divides each of them exactly.
Greatest Common Measure(G.C.M) and Greatest Common Divisor(G.C.D) are the other terms used to refer HCF.

Example : Find out HCF of 60 and 75

Step 1 : Express each number as a product of prime factors.

$60 = 2^2 \times 3 \times 5$

$75 = 3 \times 5^2$

Step 2: HCF is the product of all common prime factors using the least power of each common prime factor.

Here, common prime factors are 3 and 5

The least power of 3 here = 3

The least power of 5 here = 5

Hence, HCF = $3 \times 5 = 15$

HCF using division method

To find out HCF of two given numbers using division method,

Step 1: Divide the larger number by the smaller number.

Step 2: Divisor of step 1 is divided by its remainder.

Step 3: Divisor of step 2 is divided by its remainder. Continue this process till we get zero as remainder.

Step 4: Divisor of the last step is the HCF.

To find out HCF of three given numbers using division method,

Step 1: Find out HCF of any two numbers.

Step 2: Find out the HCF of the third number and the HCF obtained in step 1.

Step 3: HCF obtained in step 2 will be the HCF of the three numbers.

In a similar way as explained for three numbers, we can find out HCF of more than three numbers also using division method.

Example : Find out HCF of 9, 27, and 48

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Take any two numbers and find out their HCF first. Say, let's find out HCF of 9 and 27 initially.

```
9) 27 (3
   27
   --
    0
```

```
9) 48 (5
   45
   --
    3) 9 (3
       9
       -
       0
```

Hence, HCF of 9 ,27, 48 = 3

Relation between HCF and LCM

- Product of two numbers = Product of their HCF and LCM.
- Least Common Multiple (LCM) of fractions

$$\text{LCM of fractions} = \frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

- Highest Common Multiple (HCF) of fractions

$$\text{HCF of fractions} = \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

- To calculate LCM and HCF of Decimals

Step 1: Make the same number of decimal places in all the given numbers by suffixing zero(s) in required numbers as needed.
Step 2: Now find the LCM/HCF of these numbers without decimal.
Step 3: Put the decimal point in the result obtained in step 2 leaving as many digits on its right as there are in each of the numbers.

Find the H.C.F and L.C.M of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$

1. **Find the H.C.F and L.C.M of** $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ and $\frac{10}{27}$

$$\text{H.C.F of given fraction} = \frac{\text{H.C.F. of } 2,8,16,10}{\text{L.C.M. of } 3,9,81,27} = \frac{2}{81}$$

$$\text{L.C.M of given fraction} = \frac{\text{L.C.M. of } 2,8,16,10}{\text{H.C.F. of } 3,9,81,27} = \frac{80}{3}$$

2. **Find the H.C.F and L.C.M of 0.63, 1.05 and 2.1**

The numbers can be written as 63, 105, 210

H.C.F of 63, 105, 210

H.C.F of 63, 105 is 21

63⟌105, R-42, 42⟌63, R-21, 21⟌42, R-0

H.C.F of 21, 210 is 21⟌210, R-0

H.C.F of 63, 105, 210 is 21

H.C.F of 0.63, 1.05 and 2.1 is 0.21

L.C.M of 63, 105, 210 is

3	63, 105, 210
7	21, 35, 70
5	3, 5, 10
	3, 1, 2

L.C.M = 3×7 ×5 ×3 ×2 = 630

L.C.M of 0.63, 1.05 and 2.1 is 6.30

The H.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77, then the other is:

A. 99

B. 98

C. 77

D. 78

Product of two numbers = Product of their HCF and LCM

Other number = $11 \times 693 / 77$

$= 99$

- In case of HCF, if some remainders are given, then firstly those remainders are subtracted from the numbers given and then their HCF is calculated.

Find the greatest number which on dividing 1657 and 2037 leaves remainder 6 and 5 respectively.

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Required number = H.C.F of (1657-6) and (2037-5)
= H.C.F of 1651 and 2032

2032 ÷ 1651,

R = 381

1651 ÷ 381

R = 127

381 ÷ 127

R = 0

Required number = 127

- Sometimes in case of HCF questions, when the **remainder is not given**, in those cases you will generally have three numbers given.
- For answering the question, you need to take the **difference of the three pairs of numbers**, now the HCF of these differences will become the answer

Find the greatest number which divides 62, 132 and 237 leaves the same remainder in each case.

Find the greatest number which divides 62, 132 and 237 leaves the same remainder in each case.

Required number = H.C.F of (132-62), (237-132) and (237-62)
= H.C.F of 70, 105, 175

H.C.F of 175,105

175 ÷ 105, R = 70

105 ÷ 70 R = 35

70 ÷ 35 R = 0

H.C.F of 175, 105 = 35

Now H.C.F of 70, 35 is

70 ÷ 35
R=0

Required number = 35

The greatest number which can divide 1356, 1868 and 2764 leaving the same remainder 12 in each case is

a) 64 b)124 c)156 d)260

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Required number = H.C.F of (1356-12),(1868-12),(2764-12)

= H.C.F of(1344, 1856, 2752)

H.C.F of 2752, 1856 is

64

H.C.F of 1344, 64 is 64

Ans a) 64

- In case of LCM, if a single remainder is given, then the LCM is calculated first and then that single reminder is added in that.

- In case of LCM, if for different numbers different remainders are given, then the difference between the number and its respective remainder will be equal. In that case, firstly the LCM is calculated, then that common difference between the number and its respective remainder is subtracted from that.

Example

Find the least number which when divided by 20,25, 35, and 40 leaves remainders 14,19,29 and 34 respectively.

Example

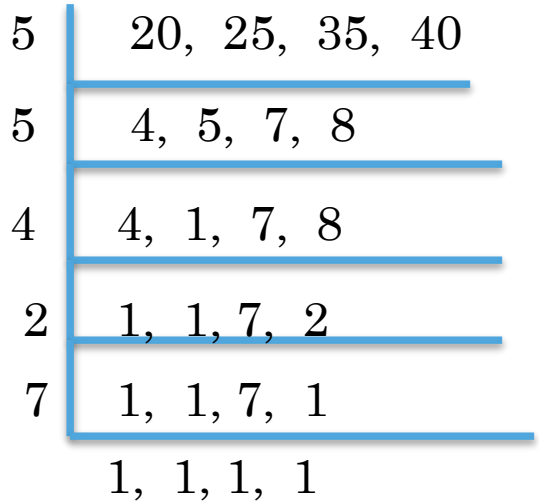
Find the least number which when divided by 20,25, 35, and 40 leaves remainders 14,19,29 and 34 respectively.

Solution

20-14 =6, 25-19 =6, 35-29 =6, 40-34 =6

L.C.M of 20,25,35 and 40 is

= $5 \times 5 \times 4 \times 2 \times 7 = 1400$



Required number =

L.C.M of 20,25,35 and 40 -6

= 1400 -6

= **1394**

- Whenever the question talks about the greatest or maximum, then in most of these cases it will be a question of HCF. Secondly, whenever the question is related to classification or distribution into groups, then in all the cases it is HCF only.

Example

The maximum number of students among them 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and the same number of pencils is

a) 91 b) 910 c) 1001 d) 1911

Required number of students = H.C.F of 1001, 910

= 91

- Whenever the question talks about the smallest or minimum, then in most of the cases it will be a question of LCM. Secondly, whenever the word ‘together’ or ‘simultaneous’ is used in the question, then in all the cases it is LCM.

- **The traffic lights at three different road crossings change after every 48 sec, 72 sec and 108 sec respectively. If they all change simultaneously at 8:20:00 hours, then at what time will they again change simultaneously?**

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Interval of change = L.C.M of (48, 72, 108) sec

= 432 sec

6	48, 72, 108
2	8, 12, 18
3	4, 6, 9
2	4, 2, 3
	2, 1, 3

So the lights will again change simultaneously
After every 432 sec, i.e 7 min 12 sec

Hence, next simultaneous change will take place at 8:27:12 hours.

Example

Four different electronic devices make a beep after 30 minutes, 1 hour, 1hour 30minutes, and 1 hour 45 minutes respectively. All the devices beeped together at 12 noon. They will again beep together at:

a) 12midnight b) 3 a.m. c) 6 a.m. d) 9 a.m.

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Solution

L.C.M of 30, 60, 90, 105 is 1260

i.e 1260 mins = 21 hours

So device will again beep together 21 hours after 12 noon i.e at 9 a.m.

Example

The greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm is:

a) 15 b)35 c)25 d)42

Example

The greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm is:

a) 15 b)35 c)25 d)42

Required length = H.C.F. of 700 cm, 385 cm and 1295 cm

= 35 cm.

Example

A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and c in 198 seconds, all starting at the same point. After what time will they meet again at the starting point ?

a) 26min18sec b)42min36sec c)45min d)46min12sec

Example

A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and c in 198 seconds, all starting at the same point. After what time will they meet again at the starting point ?

a) 26min18sec b)42min36sec c)45min d)46min12sec

L.C.M. of 252, 308 and 198 = 2772.

So, A, B and C will again meet at the starting point in 2772 sec. *i.e.*, 46 min. 12 sec.

Example

5 bells commence tolling together and toll at intervals 2, 4, 6, 8 and 10 seconds respectively. Find in 40 minutes, how many times do they toll together?

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Hence, find the L.C.M. of 2, 4, 6, 8 and 10 seconds.

L.C.M. of 2, 4, 6, 8 and 10

$= 2 \times 2 \times 2 \times 3 \times 5 \times 1 = 120$

2	2	4	6	8	10
2	1	2	3	4	5
2	1	1	3	2	5
3	1	1	3	1	5
5	1	1	1	1	5
	1	1	1	1	1

Hence, the bells toll together after 120 seconds = 2 min

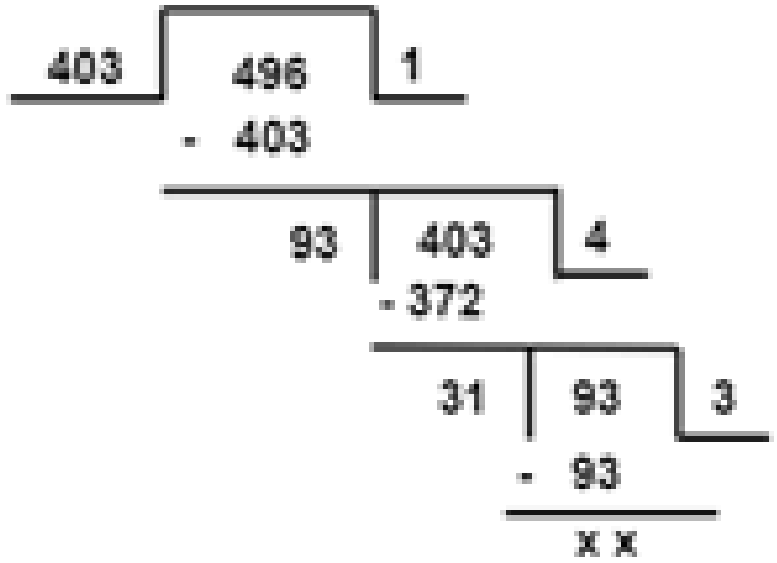
We are asked to find, how many times they will toll together in 40 min.

In 40 min, they toll together for $= 40/2 + 1 = 21$ times.

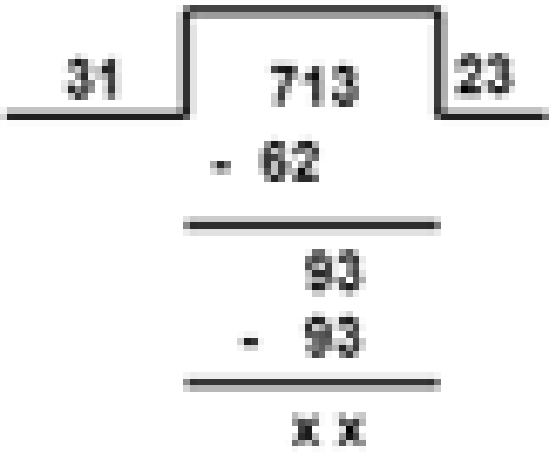
(i.e., they are tolling together at the 0th second)

- Three different containers contain 496 liters, 403 liters, and 713 liters of water, respectively. What is the capacity of the biggest container that can measure three different quantities evenly?
- 1 liter
- 7 liters
- 31 liters
- 41 liters

- For the capacity of the biggest container, we have to find the HCF.
  HCF: By Long Division method
  First find the HCF of two numbers, 496 and 403



The HCF of 496 and 403 = 31
Now find the HCF of 31 and 713



HCF of 713 and 31 is 31
So, the maximum capacity is 31 liters.

- Three bells toll at intervals of 36 sec, 40 sec, and 48 sec respectively. They start singing together at a particular time. When will they toll next together?

Firstly find the LCM of 36, 40, and 48

LCM = 2 x 2 x 2 x 2 x 3 x 3 x 5 = 720 seconds
Hence, after 720 seconds or 12 minutes three bells will toll together.

2	36 , 40 , 48
2	18 , 20 , 24
2	9 , 10 , 12
2	9 , 5 , 6
3	9 , 5 , 3
3	3 , 5 , 1
5	1 , 5 , 1
	1 , 1 , 1

Find the greatest number which divides 34, 90, and 104 and leaves the same remainder in each case.

1. 15
2. 17
3. 14
4. 18

- Difference between numbers = 90-34 = 56, and 104- 90=14
- HCF of 56 and 14 = 14
- Hence, 14 is the largest number which divides the given number and leaves the same remainder in each case.

Example

Find the greatest number that will divide 43,91 and 183 so as to leave the same remainder in each case.

Example

Find the greatest number that will divide 43,91 and 183 so as to leave the same remainder in each case.

a) 4 b) 7 c) 9 d) 13

Required number = H.C.F of (91-43, 183-91, 183-43)

= H.C.F of (48, 92, 140)

H.C.F =4

Required number =4