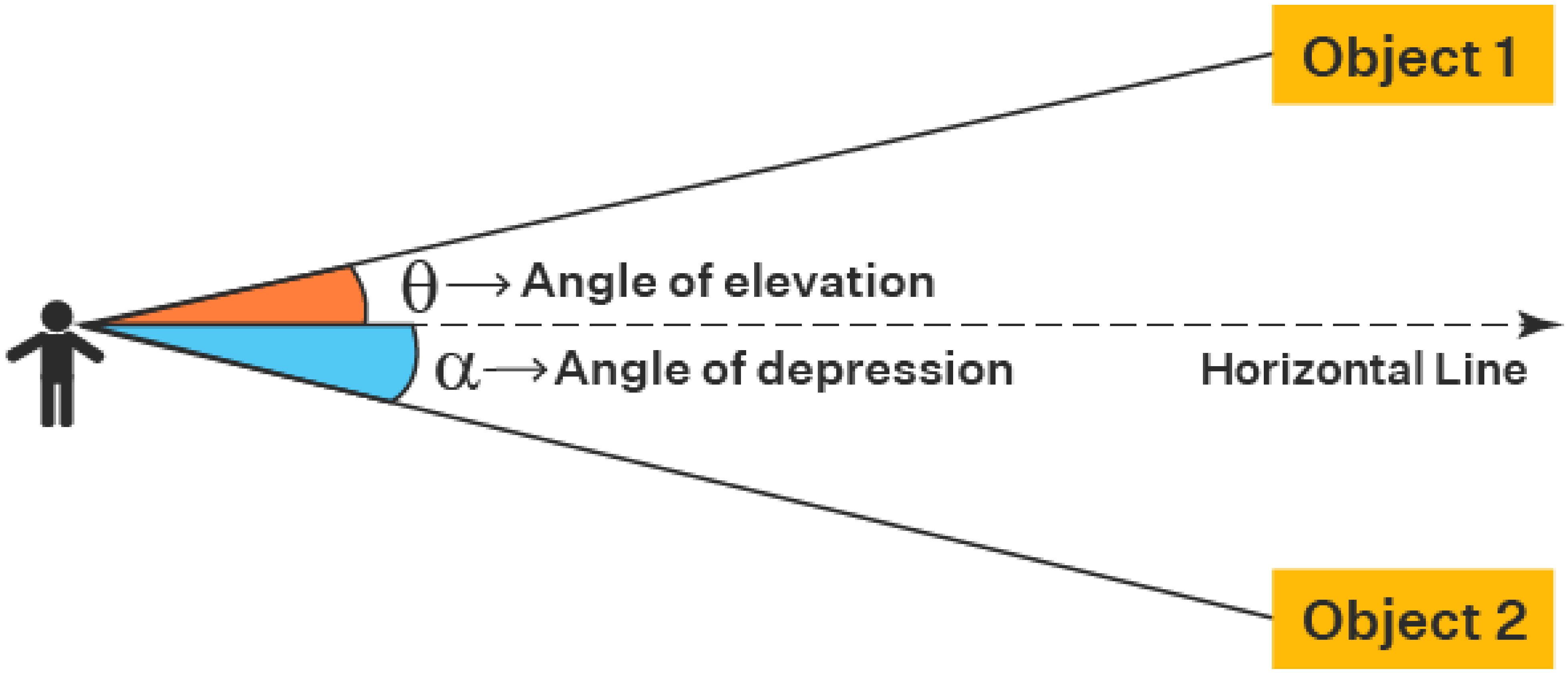
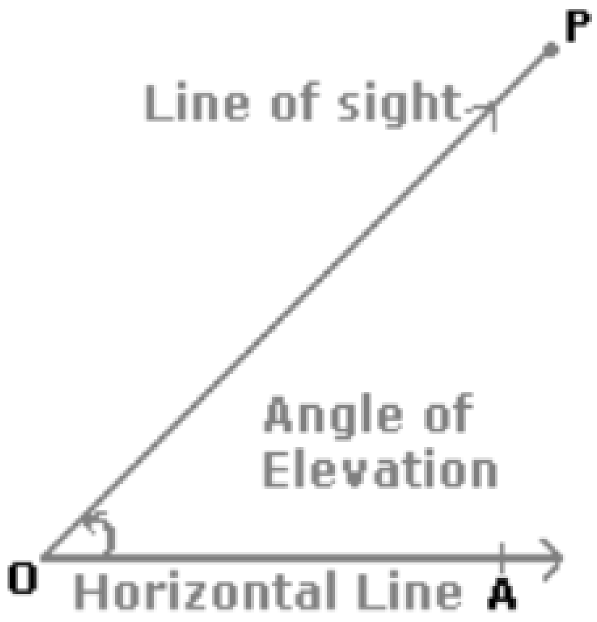
Height & Distances

Angle of Elevation and Depression



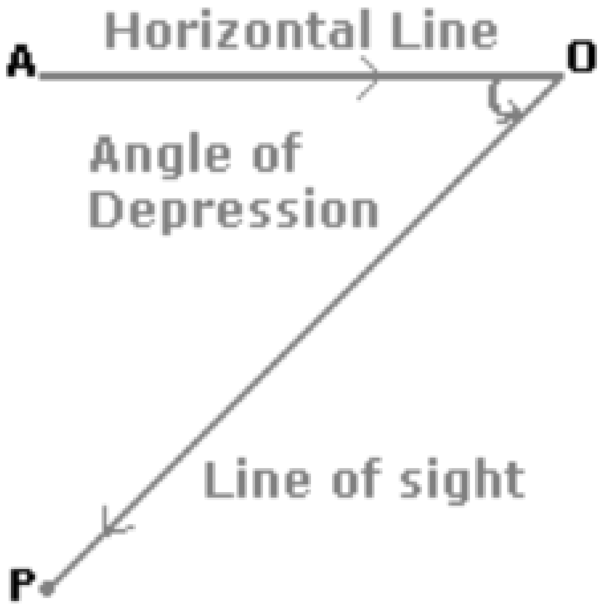
Angle of Elevation:



Suppose a man from a point O looks up at an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the **angle of elevation** of P as seen from O.

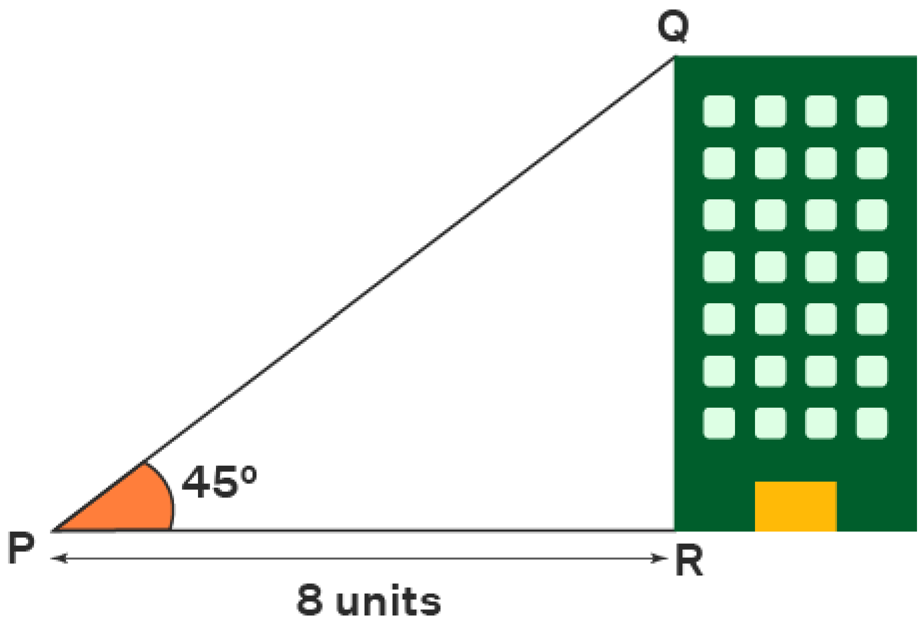
$\therefore$ Angle of elevation of P from O = $\angle AOP$.

Angle of Depression:



Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the **angle of depression** of P as seen from O.

If a girl is standing at point P, which is 8 units away from a building, making an angle of elevation of 45° with point Q, find the height of the building.



Solution: Given that PR=8 units, and ∠QPR=45°. To find the height of the building (QR), we can use the angle of elevation formula $\tan \theta = QR/PR$.

$\tan 45° = QR/8$

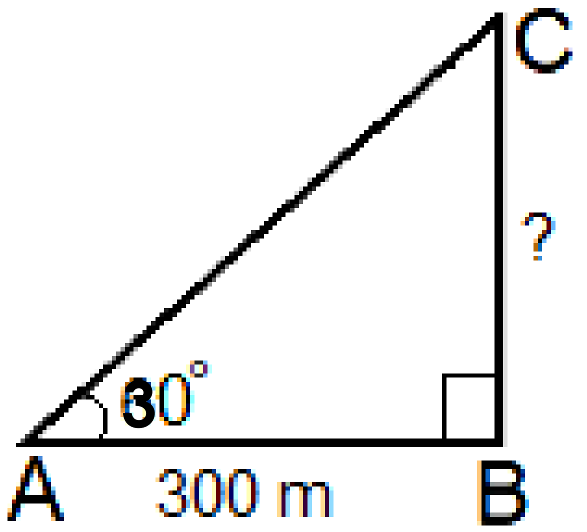
We know that $\tan 45°$ is 1, so,

$1 = QR/8$

QR=8 units

Answer: Therefore, the height of the building is 8 units.

300m from the foot of a cliff on level ground ,the angle of elevation of the top of a cliff is $30°$. Find the height of this cliff.



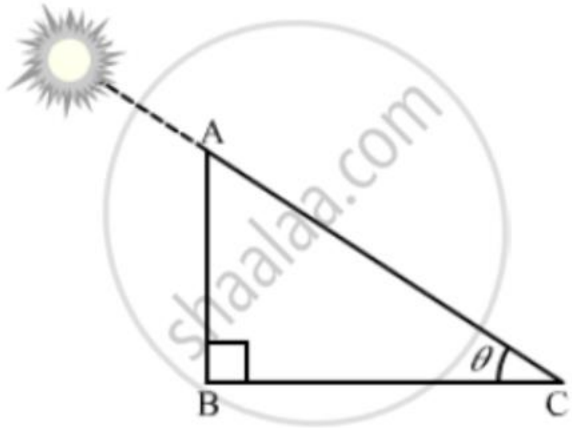
Let the foot of the cliff be B
Let the point of observer be A
Let the highest point of cliff be C

then ∠CBA=90°
∠CAB=30°

Applying tan to 30°,
we get-

BC/AB=tan 30°
BC/300=1/√3
BC=300/√3=100√3 m

The height of the vertical pole is $\sqrt{3}$ times the length of its shadow on the ground, then angle of elevation of the sun at that time is



Let the angle of elevation of the sun be θ.

Suppose AB is the height of the pole and BC is the length of its shadow.

It is given that, AB = $\sqrt{3}$BC

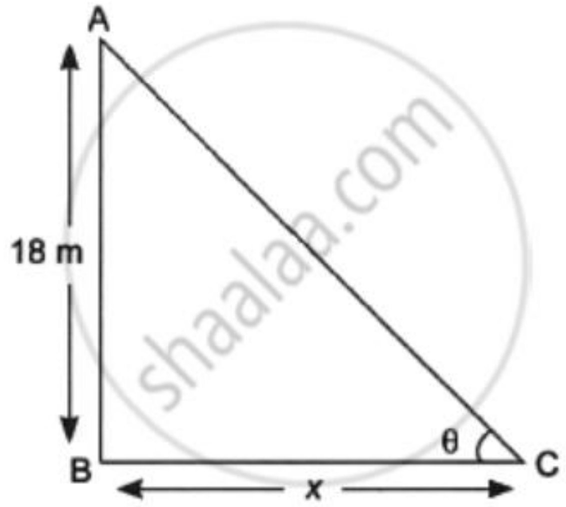
In right ΔABC,

$$\tan\theta = \frac{AB}{BC}$$
$$\Rightarrow \tan\theta = \frac{\sqrt{3}BC}{BC} = \sqrt{3}$$
$$\Rightarrow \tan\theta = \tan 60^\circ$$
$$\Rightarrow \theta = 60^\circ$$

Thus, the angle of elevation of the sun is 60º.

Find the length of the shadow on the ground of a pole of height 18m when angle of elevation θ of the sun is such that $\tan\theta = \frac{6}{7}$.

Let the length of the shadow be 'x' m



Given, $\tan\theta = \frac{6}{7}$

$= \frac{\text{Perpendicular}}{\text{Base}}$

$\Rightarrow \frac{AB}{BC} = \frac{6}{7}$

$$\Rightarrow \frac{AB}{BC} = \frac{6}{7}$$

$$\Rightarrow \frac{18}{x} = \frac{6}{7}$$

$$\Rightarrow x = \frac{7 \times 18}{6}$$

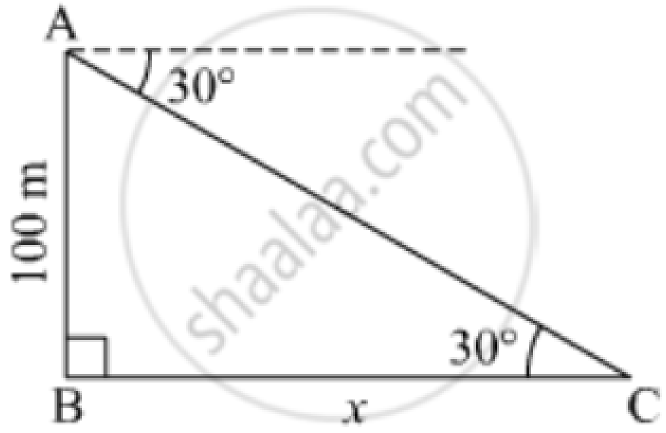
$= 7 \times 3$

$= 21$ m

From the top of the light house, an observer looks at a ship and finds the angle of depression to be 30°. If the height of the light-house is 100 meters, then find how far the ship is from the light-house.

Let AB be the lighthouse and C be the position of the ship from the lighthouse.

Suppose the distance of the ship from the lighthouse be *x* m.



Here, AB = 100 m and ∠ACB = 30°.

In right ΔABC,

$$\tan 30^\circ = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{x}$$

$$\Rightarrow x = 100\sqrt{3}m$$

Thus, the ship is $100\sqrt{3}$ m away from the lighthouse.

Two pillars of equal heights stand on either side of a roadway, which is 150 m wide. At a point in the roadway between the pillars the elevations of the tops of the pillars are 60∘ and 30∘, find the height of the pillars and the position of the point.

Let the height of the equal pillars be AB = CD = h

Given, width of the road is 150 m

Let BE = x, the DE = 150 - x

In right angle triangle ABE,

$\tan 60 = \frac{h}{x}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow h = \sqrt{3}x$ 1

In right angle triangle CDE,

$\tan 30 = \frac{h}{(150-x)}$

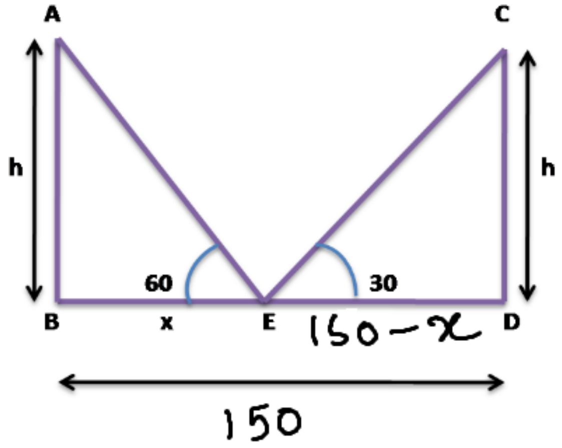
$\Rightarrow \sqrt{3}\,h = 150 - x$

$\Rightarrow \sqrt{3}\,h = 150 - \frac{h}{\sqrt{3}}$ {from equation 1}

$\Rightarrow h = \frac{150\sqrt{3}}{4}$

$\Rightarrow h = 37.5\sqrt{3}$m

So, the height of the equal pillars is $37.5\sqrt{3}$m



From a balloon vertically above a straight road, the angles of depression of two cars at an instant are found to be 45° and 60°. If the cars are 100 m apart, find the height of the balloon

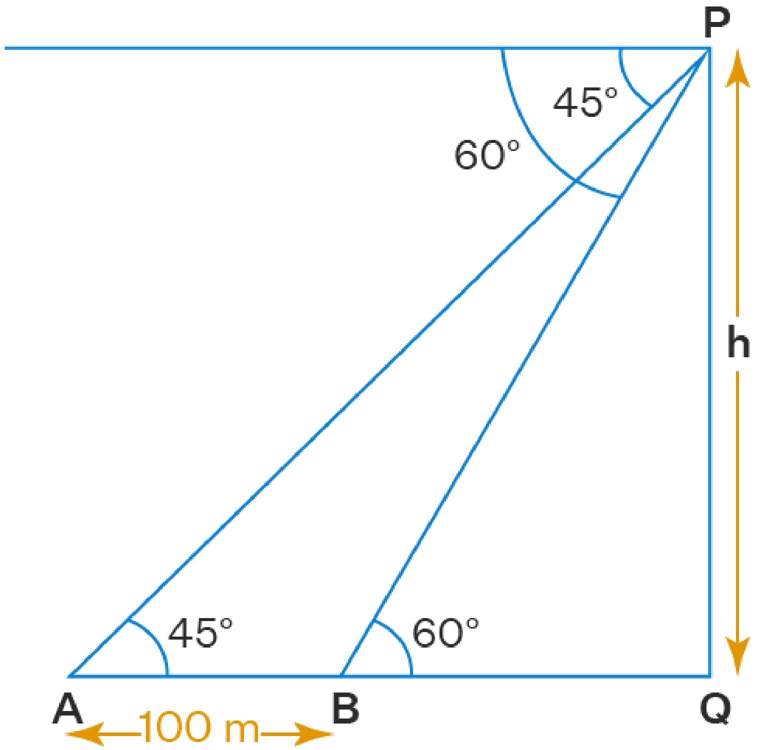
Consider the height of the balloon at P be h meters.

Consider A and B as the two cars.

So AB = 100 m.

In ΔPAQ,

AQ = PQ = h



In ΔPBQ,

$PQ/BQ = \tan 60° = \sqrt{3}$

From the figure

$h/ (h - 100) = \sqrt{3}$

By cross multiplication

$h = \sqrt{3}(h - 100)$

So we get

$h = 100\sqrt{3}/ (\sqrt{3} - 1)$

Let us multiply and divide by $(\sqrt{3} + 1)$

$= 100\sqrt{3}/ (\sqrt{3} - 1) \times (\sqrt{3} + 1)/ (\sqrt{3} + 1)$

By further calculation

$= (100 \times 3 + 100 \sqrt{3})/ (3 - 1)$

$= 100 (3 + \sqrt{3})/ 2$

So we get

$= 50(3 + \sqrt{3})$ m

Therefore, the height of the balloon is $50(3 + \sqrt{3})$ m.

Two villages are 2 km apart. If the angles of depression of these villages when observed from a plane are around to be 45∘ and 60∘ respectively , then height of the plane in km is:
(Plane is between two villages)

Let $PQ = h$ m be a plane

From right angled $\triangle PAQ$,

$$\tan 45^\circ = \frac{PQ}{AQ}$$

$$\Rightarrow 1 = \frac{h}{x}$$

$$\Rightarrow x = h \ldots (i)$$

Again from right angled $\triangle PBQ$

$$\Rightarrow \tan 60^\circ = \frac{PQ}{QB}$$

$$\Rightarrow \sqrt{3} = \frac{h}{2 - x}$$

$$\Rightarrow 2\sqrt{3} - \sqrt{3}x = h$$

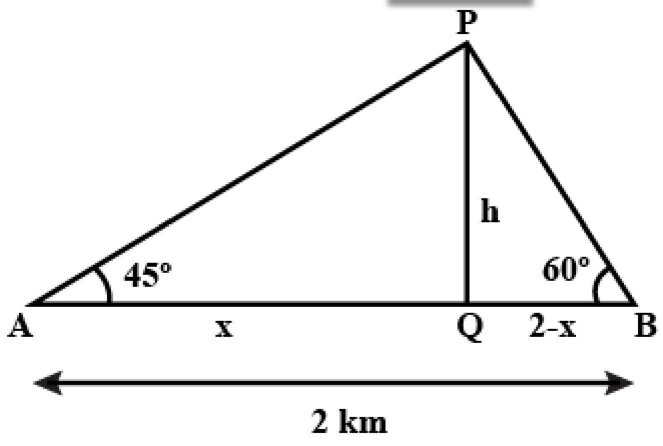
From equation (i)

$$\Rightarrow 2\sqrt{3} - \sqrt{3}h = h$$

$$\Rightarrow h + \sqrt{3}h = 2\sqrt{3}$$

$$\Rightarrow h(1 + \sqrt{3}) = 2\sqrt{3}$$

$$\Rightarrow h = \frac{2\sqrt{3}}{1 + \sqrt{3}}$$



$$\Rightarrow h = \frac{2\sqrt{3}}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$$

$$\Rightarrow h = \frac{2\sqrt{3} - 6}{1 - 3}$$

$$\Rightarrow h = \frac{-2(3 - \sqrt{3})}{-2}$$

$$\therefore \; h = (3 - \sqrt{3}) \text{ km}$$

The upper part of a tree is broken by the wind and makes an angle of 30° with the ground, The distance from the root of the tree to the point, where the top touches the ground is 5m. Find the height of the tree

Cos 30° = AB/BC

$$\frac{\sqrt{3}}{2} = \frac{5}{x}$$

$$x\sqrt{3} = 5 \times 2$$

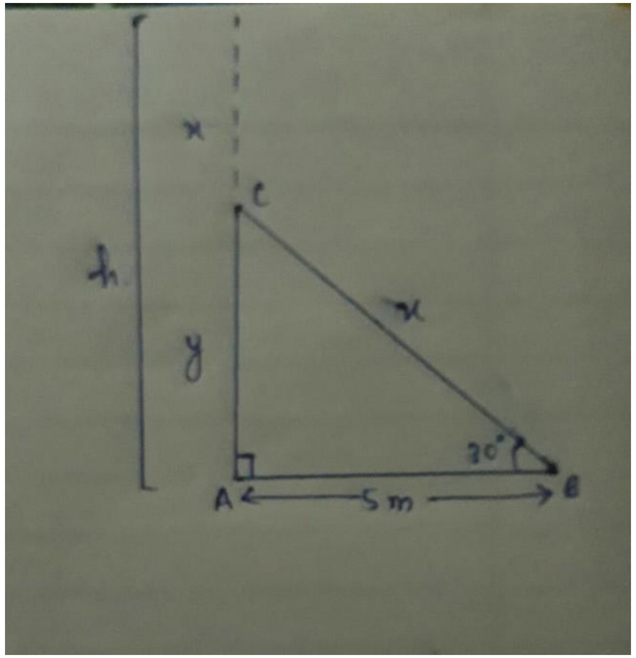
$$x = \frac{10}{\sqrt{3}}$$

Again , tan 30° = AC / AB

$$\frac{1}{\sqrt{3}} = \frac{y}{5}$$

$$y\sqrt{3} = 5$$

$$y = \frac{5}{\sqrt{3}}$$



The height h of the tree is given by :

h = x + y

$$h = \frac{10}{\sqrt{3}} + \frac{5}{\sqrt{3}}$$

$$h = \frac{15}{\sqrt{3}}$$

$$h = \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$h = \frac{15\sqrt{3}}{3}$$

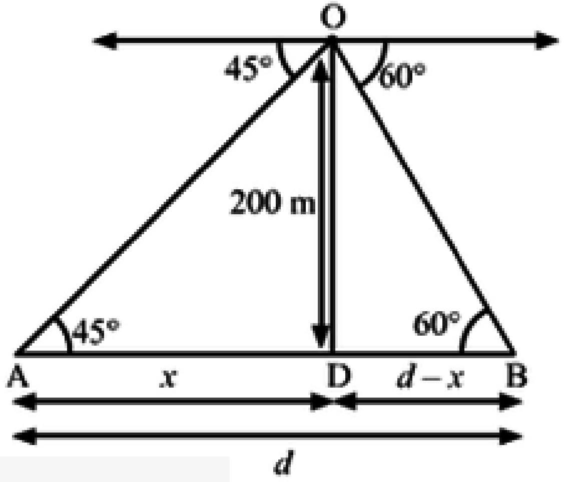
$$h = 5\sqrt{3}$$

$$h = 5 \times 1.73$$

$$\boxed{h = 8.65\,\mathrm{m}}$$

Two ships are there in the sea on either side of a light house in such a way that the ships and the light house are in the same straight line. The angles of depression of two ships as observed from the top of the light house are 60∘ and 45∘. If the height of the light house is 200 m, find the distance between the two ships. [Use √3=1.73]

Let d be the distance between the two ships. Suppose the distance of one of the ships from the light house is X meters, then the distance of the other ship from the light house is (d-x) meter.



In right-angled Δ ADO, we have.

$\tan 45^\circ = \frac{OD}{AD} = \frac{200}{X}$

$\Rightarrow 1 = \frac{200}{X}$

$\Rightarrow x = 200$ ....(1)

In right-angled ΔBDO, we have

$\tan 60^\circ = \frac{OD}{BD} = \frac{200}{d-x}$

$\Rightarrow \sqrt{3} = \frac{200}{d-x}$

$\Rightarrow d-x = \frac{200}{\sqrt{3}}$

Putting x=200. We have:

$d - 200 = \frac{200}{\sqrt{3}}$

$d = \frac{200}{\sqrt{3}} + 200$

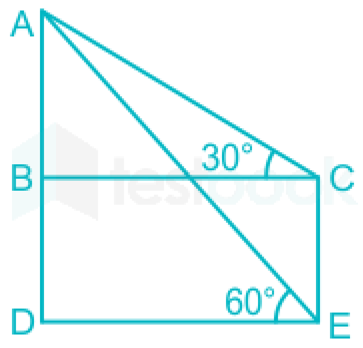
$\Rightarrow d = 200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$

$\Rightarrow d = 200 \times 1.58$

$\Rightarrow d = 316$ (approx.)

Thus, the distance between two ships is approximately 316 m.

From the top of a hill 240 m high, the angles of depression of the top and bottom of a pole are 30° and 60°, respectively. The difference (in m) between the height of the pole and its distance from the hill is:



As per the figure shown above, AD is the Hill of height 240 m.

CE is the height of the pole.

In Δ ADE $\tan 60° = AD/DE$

$\Rightarrow \sqrt{3} = 240/DE$

$\Rightarrow DE = 240/\sqrt{3}$

$\Rightarrow DE = 80\sqrt{3}$

In Δ ABC tan 30° = AB/BC = 1/√3

Where BC = DE = 80√3

$$\Rightarrow \frac{\sqrt{3} \times AB}{240} = \frac{1}{\sqrt{3}}$$

⇒ 3AB = 240

⇒ AB = 240/30

⇒ AB = 80.

The height of the hill is 240m and AB = 80

BD = 240 - 80

BD = 160. (The Height of the Pole)

The difference between the height of the pole and the distance between the pole and the hill

BD - DE

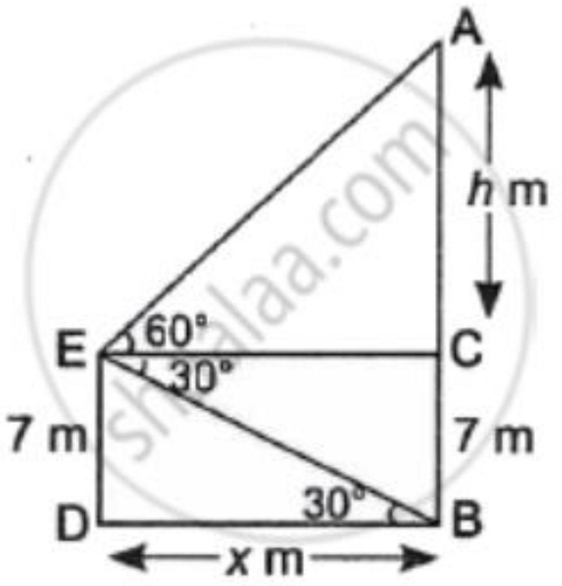
160 - 80√3

∴ **80(2 - √3)**

From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30°. Determine the height of the tower.

Given, height of building = 7 m

Let AC = h m and BD = x m



In ΔBDE,

$$\tan 30° = \frac{ED}{BD}$$

$$\Rightarrow = \frac{1}{\sqrt{3}} = \frac{7}{x}$$

$$\Rightarrow x = 7\sqrt{3} \text{ m}$$

In ΔACE,

tan 60° = $\frac{AC}{CE}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$ ...[∵ CE = BD]

$\Rightarrow$ h = $x\sqrt{3}$

= $7\sqrt{3} \times \sqrt{3}$

= 7 × 3

= 21 m

∴ Height of the tower = AB = AC + CB

= 21 + 7

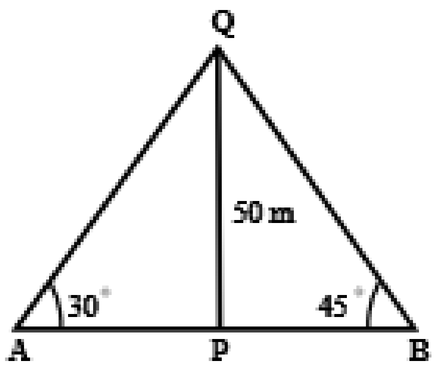
= 28 m.

Question : Two men are on opposite sides of a tower. They measure the angles of elevation of the top of the tower as 30° and 45°, respectively. If the height of the tower is 50 metres, the distance between the two men is: (take $\sqrt{3} = 1.732$)

Solution :

Let the positions of the two men be A and B.

Let PQ represent the tower.



∠QAP = 30° and ∠QBP = 45°

QP = 50 m

We get two right-angled triangles △QPB and △QPA

Now, From the △QPA:

$$\tan 30° = \frac{PQ}{AP}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50}{AP}$$

$\Rightarrow AP = 50\sqrt{3} = 50 \times 1.732 = 86.6$ metres

From the $\triangle QPB$:

$$\Rightarrow \tan 45° = \frac{PQ}{PB}$$

$$\Rightarrow 1 = \frac{50}{PB}$$

$\Rightarrow$ PB = 50 metres

Now, AB = AP + PB

= 86.6 + 50

= 136.6 metres

Hence, the correct answer is 136.6 metres.