Permutation and Combination

Permutations

Permutations are the different arrangements of a given number of things by taking some or all at a time.

Examples

All permutations (or arrangements) that can be formed with the letters a, b, c by taking three at a time are (abc, acb, bac, bca, cab, cba)

All permutations (or arrangements) that can be formed with the letters a, b, c by taking two at a time are (ab, ac, ba, bc, ca, cb)

Number of permutations of n things taking r at a time is given by

$$nP_r = n(n-1)(n-2)(n-3)......(n-r+1) = \frac{n!}{(n-r)!}$$

Ex.1

$$8P_2 = \qquad 8P_2 = \frac{8!}{(8-2)!} = 8\times 7 = 56$$

Ex.2

$$7P_3 = \qquad 7P_3 = \frac{7!}{(7-3)!} = 7\times 6\times 5 = 210$$

- Number of all permutations of n things, taking all at a time is *n!*
- If there are n objects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of r_{th} kind such that $(p_1 + p_2 + p_3 \ldots\ldots + p_r) = n,$

then number of permutations $= \dfrac{n!}{p_1! \cdot p_2! \cdots p_r!}$

Combinations

Each of the different groups or selections formed by taking some or all of a number of objects is called a combination.

Important note:

AB and BA are two different permutations

But they represent the same combination

Difference between Permutations and Combinations

If the order is important, problem will be related to permutations.

If the order is not important, problem will be related to combinations.

- Number of all combinations of n things, taken r at a time, is

$$nC_r = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)\text{....to r factors}}{\text{r!}}$$

$$nC_n = 1$$

$$nC_0 = 1$$

$$nC_r = nC_{(n-r)}$$

$$8C_3 = \frac{8\times7\times6}{3\times2\times1} = 56$$

$$16C_{13} = 16C_{(16-13)} = 16C_3 = \frac{16\times15\times14}{3\times2\times1} = 560$$

For permutations, the problems can be like "What is the number of permutations the can be made", "What is the number of arrangements that can be made", "What are the different number of ways in which something can be arranged", etc.

For combinations, the problems can be like "What is the number of combinations the can be made", "What is the number of selections the can be made", "What are the different number of ways in which something can be selected", etc.

Mostly problems related to word formation, number formation etc will be related to permutations.
Similarly most problems related to selection of persons, formation of geometrical figures, distribution of items (there are exceptions for this) etc will be related to combinations.

In how many different ways can the letters of the word “FIGHT” be arranged?

Total no.of word in fight = 5
No.of ways these can be arranged = 5!
$5\times4\times3\times2\times1 = 120$

How many ways can the letters in the word PENCIL be arranged?

Ans : 6!

How many ways can you order the letters in KEYBOARD if K and Y must always be kept together?

Ans :2! 7!

The number of permutations of the letters of the word "ENGINEERING" is

Given word ENGINEERING

no of times each letter of the given word is repeated

$E = 3$

$N = 3$

$G = 2$

$I = 2$

$R = 1$

So, the total no. of permutations

$$= \frac{11!}{3!3!2!2!1!}$$

$$= \frac{11!}{(3!2!)^2}$$

$$\therefore \text{the total no. of permutations} = \frac{11!}{(3!2!)^2}$$

Find the number of distinguishable permutations of the letters in
(a) OHIO and
(b) MISSISSIPPI.

a) 4 ! / 2 ! = 12.

b) 11 ! / 4! * 4! * 2! = 34,650.

In how many different ways can the letters of the word 'DESIGN' be arranged so that the vowels are at the two ends?

The word DESIGN consists of 6 distinct letters.
According to the question,
E I
I . . . . E
∴ Required number of arrangements
$=2!\times4!=2\times4\times3\times2=48$

In how many different ways can the letters of the word 'DETAIL' be arranged in such a way that the vowels occupy only the odd positions?

There are 6 letters in the given word, out of which there are 3 vowels and 3 consonants.
Let us mark these positions as under:

(1) (2) (3) (4) (5) (6)

Now, 3 vowels can be placed at any of the three places, marked 1, 3, 5.
Number of ways of arranging the vowels = ${}^3P_3 = 3! = 6$.
Also, the 3 consonants can be arranged at the remaining 3 positions.
Number of ways of these arrangements = ${}^3P_3 = 3! = 6$.
Total number of ways = $(6 \times 6) = 36$.

How many words can be formed from the letters of the word 'DAUGHTER' so that (i) the vowels always come together? (ii) the vowels never come together?

The letters of the word daughter are “d,a,u,g,h,t,e,r”.
So, the vowels are ‘a, u, e’ and the consonants are “d,g,h,t,r”.

(i)Now, all the vowels should come together, so consider the bundle of vowels as one letter, then total letters will be 6.

So, the number of words formed by these letters will be 6!

but, the vowels can be arranged differently in the bundle, resulting in different words, so we have to consider the arrangements of the 3 vowels.

So, the arrangements of vowels will be 3!

Thus, the total number of words formed will be equal to $(6! \times 3!) = 4320$

In how many different ways can the letters of the word “EXTRA ” be arranged so that the vowels are never together?

EXTRA → Total number of words = 5 and total number of vowels = 2

The word EXTRA can be arranged in 5! ways = 120 ways

The word EXTRA can be arranged in such a way that the vowels will be together = 4! × 2!

$\Rightarrow (4 \times 3 \times 2 \times 1) \times (2 \times 1)$

$\Rightarrow$ 48 ways

The letters of the words EXTRA be arranged so that the vowels are never together = (120 - 48)
= 72 ways.

∴ The letters of the words EXTRA be arranged so that the vowels are never together in 72 ways.

In how many ways can a cricket team of 11 players be chosen out of batch of 15 players?

Number of ways choosing 11 players from 15 is ${}^{15}C_{11} = {}^{15}C_4$

$$= \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}$$

$= 15 \times 7 \times 13$

$= 1365.$

In how many ways can a cricket eleven be chosen out of a batch of 15 players if a particular player is always chosen?

If a particular player is always chosen.

This means that 10 players are selected out of the remaining 14 players.

$\therefore$ Required number of way is $^{14}C_{10}$.................(In the form of $^{n}C_{r}$)

$= {}^{14}C_{14-10}$

$= {}^{14}C_{4}$

In how many ways a committee of six members be formed from 7 men and 5 women if the committee contains 4 men and 2 women?

Total men-7

Total women-5

Selecting 4 men from 7 men-$^7C_4$

Selecting 2 women from 5 women-$^5C_2$

Selecting 4 men and two women-$^7C_4 \times {}^5C_2$

A basketball team consists of two centers, five forwards, and four guards. In how many ways can the coach select a starting line up of one center, two forwards, and two guards?

Center:

$$ {}_2C_1 = \frac{2!}{1!1!} = 2 $$

Forwards:

$$ {}_5C_2 = \frac{5!}{2!3!} = \frac{5*4}{2*1} = 10 $$

Guards:

$$ {}_4C_2 = \frac{4!}{2!2!} = \frac{4*3}{2*1} = 6 $$

$$ {}_2C_1 * {}_5C_2 * {}_4C_2 $$

Thus, the number of ways to select the starting line up is 2*10*6 = 120.

4. In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

In a group of 6 boys and 4 girls, four children are to be selected such that at least one boy should be there. Hence we have 4 options as given below

We can select 4 boys ...(option 1)
Number of ways to this = $^6C_4$

We can select 3 boys and 1 girl ...(option 2)
Number of ways to this = $^6C_3 \times {}^4C_1$

We can select 2 boys and 2 girls ...(option 3)
Number of ways to this = $^6C_2 \times {}^4C_2$

We can select 1 boy and 3 girls ...(option 4)
Number of ways to this = $^6C_1 \times {}^4C_3$

Total number of ways
$= {}^6C_4 + {}^6C_3 \times {}^4C_1 + {}^6C_2 \times {}^4C_2 + {}^6C_1 \times {}^4C_3$
= 209

A box contains 4 red, 3 white and 2 blue balls. Three balls are drawn at random. Find out the number of ways of selecting the balls of different colours?

Ans

1 red ball can be selected in ${}^4C_1$ ways.

1 white ball can be selected in ${}^3C_1$ ways.

1 blue ball can be selected in ${}^2C_1$ ways.

Total number of ways

$$= {}^4C_1 \times {}^3C_1 \times {}^2C_1 = 4 \times 3 \times 2 = 24$$

A question paper has two parts P and Q, each containing 10 questions. If a student needs to choose 8 from part P and 4 from part Q, in how many ways an he do that?

Ans

Number of ways to choose 8 questions from part P = $^{10}C_8$
Number of ways to choose 4 questions from part Q = $^{10}C_4$

Total number of ways
= $^{10}C_8 \times {}^{10}C_4$
= 9450

In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together ?

The arrangement can be as follows:
__G__G__G__G__G__
5 girls can be seated in five places in 5 !=120 ways.
3 boys can be seated in six blank places in $6P3=6\times5\times4=120$ ways.
$\therefore$ Total number of ways $=120\times120=14400$

Circular Permutation

There are some arrangements which are circular in nature.

For example consider the roundtable conference, making of a necklace with different colored beads. These are like arranging the items in a closed loop. The number of ways of counting associated with the circular arrangement gives rise to a circular permutation.

Permutation in a circle is called circular permutation.

Total number of circular permutation of n elements taken all together = (n-1)!

Example,
if you consider 5 diamonds and you want to make a necklace.

In this case 5 diamonds can be arranged in a circle in $(5-1)! = 24$ ways.

How many ways are there to seat 4 people around a round table for lunch?
Ans: 6 ways

If 3 sisters and 8 other girls are together playing a game, In how many ways in which all the girls are seated around a circle such that the three sisters are not seated together.

The total number of ways that all girls seated around a circle is $10!$.

The number of ways the three sisters seated together along with the other girls is $8! \times 3!$.

So. required number of ways $= 10! - 8!3! = 3386880$ ways

The number of ways of arranging 8 men and 4 women around a circular table such that no two women can sit together, is
a)8! b)4! c)(8!)(4!) d)7!(8P4)

The number of ways of arranging 8 men = 7! The number of ways of arranging 4 women such that no two women can sit together = 8P4∴ Required number of ways =7!(8P4)