Sequence and Series

Sequence and Series Definition

A sequence is an arrangement of any objects or a set of numbers in a particular order followed by some rule. If a_1, a_2, a_3, a_4,..... etc. denote the terms of a sequence, then 1,2,3,4,.....denotes the position of the term.

A sequence can be defined based on the number of terms i.e. either finite sequence or infinite sequence.

If a_1, a_2, a_3, a_4, ... is a sequence, then the corresponding series is given by

$S_N = a_1+a_2+a_3 + .. + a_N$

Note: The series is finite or infinite depending if the sequence is finite or infinite.

	Arithmetic Progression	Geometric Progression
Sequence	$a, a+d, a+2d, \ldots\ldots, a+(n-1)d, \ldots$	$a, ar, ar^2, \ldots\ldots, ar^{(n-1)}, \ldots$
Common Difference or Ratio	Successive term – Preceding term Common difference = d = $a_2 - a_1$	Successive term/Preceding term Common ratio = r = $ar^{(n-1)}/ar^{(n-2)}$
General Term (nth Term)	$a_n = a + (n-1)d$	$a_n = ar^{(n-1)}$
nth term from the last term	$a_n = l - (n-1)d$	$a_n = l/r^{(n-1)}$
Sum of first n terms	$s_n = n/2(2a + (n-1)d)$ $S_n = \frac{n}{2}(a + l)$	$s_n = a(1 - r^n)/(1 - r)$ if $\lvert r \rvert < 1$ $s_n = a(r^n - 1)/(r - 1)$ if $\lvert r \rvert > 1$

*Here, a = first term, d = common difference, r = common ratio, n = position of term, l = last term

Sequences	Series
Set of elements that follow a pattern	Sum of elements of the sequence
Order of elements is important	Order of elements is not so important
Finite sequence: 1,2,3,4,5	Finite series: 1+2+3+4+5
Infinite sequence: 1,2,3,4,.....	Infinite Series: 1+2+3+4+.....

$$1+2+3+4.....\infty = \frac{n(n+1)}{2}$$

$$2+4+6+8.....\infty = n(n+1)$$

$$1+3+5+7.....\infty = (n)^2$$

$$(1)^2+(2)^2+(3)^2+(4)^2.....\infty = \frac{n(n+1)(2n+1)}{6}$$

$$(1)^3+(2)^3+(3)^3+(4)^3.....\infty = \left[\frac{n(n+1)}{2}\right]^2$$

How many natural numbers are there between 23 and 100 which are exactly divisible by 6?

Required numbers are $24, 30, 36, 40, \ldots . 96$

This is an A.P. in which $a = 24, d = 6, l = 96$

Let the number of terms in it be n.

Then $t_n = 96 \Rightarrow a + (n-1)d = 96$

$\Rightarrow 24 + (n-1) \times 6 = 96$

$\Rightarrow (n-1) = 12$

$\Rightarrow n = 13$

Required number of numbers = 13

How many natural numbers between 17 and 80 are divisible by 6

The sum of all even natural numbers between 1 and 31 is:

Required sum = $(2 + 4 + 6 + \ldots\ldots + 30)$

This is an A.P in which $a = 2, d = (4 - 2) = 2$ and $l = 30$.

Let the number of terms be n. Then,

$t_n = 30 \Rightarrow a + (n - 1)d = 30$

$\Rightarrow 2 + (n - 1) \times 2 = 30$

$\Rightarrow n - 1 = 14$

$\Rightarrow n = 15$

$\therefore S_n = \frac{n}{2}(a + l) = \frac{15}{2} \times (2 + 30) = 240$

Find the sum of all even natural numbers less than 75

Find the sum (6 + 15 + 24 +33 + ... +105)

The given series is:

$$6 + 15 + 24 + 33 + ... + 105$$

First term is $a = 6$

Common difference $d = 9$

It is an AP.

The last term is $l = 105$

Mathematically

$$l = a + (n - 1)d$$

Using values

$$105 = 6 + (n - 1)9$$

$$105 = 6 + 9n - 6 = 9n - 3$$

$$n = 108/9 = 12$$

Number of terms are 12

Sum of twelve terms are:

$$S = \frac{n}{2}(a + l)$$

$$S = \frac{12}{2}(6 + 105)$$

$$S = 111 * 6 = 666$$

How many terms are there in 3, 6, 12, 24, ... , 384 ?

Let n be the numbers in this G.P where a=3 & r=6/3 =2

Last term,tn = 384

⇒ a*r^(n-1) = 384

3*2^(n-1) = 384

2^(n-1) = 128 = 2^7

(n-1) = 7

Hence number of terms = 7+1=8

$$2^1 + 2^2 + 2^3 + 2^4 + \cdots \ldots . + 2^{10}$$

$$a = 2, r = 2, n = 10 \; S_n = \frac{a(r^n - 1)}{r - 1} = \frac{2(2^{10} - 1)}{2 - 1} = 2(1024 - 1)$$
$$= 1023 \times 2 = 2046$$

Find the wrong term(s) in the given series 9, 11, 15, 23, 39, 70, 135

The series was in the following form:

9+2=11

11+4=15

15+8=23

23+16=39

39+32=71

71+64=135

Hence 70 is wrong term and 71 should be in its place.

2, 6, 12, 20, 30, 42, 56,____

The sequence is $1*2, 2*3, 3*4, 4*5, 5*6, 6*7, 7*8$.

$\therefore$ Missing number $= 8*9 = 72$.

5, 10, 13, 26, 29, 58, 61, (….)

The logic here is to do the above operation on the alternate terms: Doubling the number and Add 3 to it.

Ans: 122

15, 31, 63, 127, 255, (....)

(1) Each number is double the preceding one plus 1 (255×2) + 1 = 511