Elementary Algebra

An equation of the type $ax^2 + bx + c = 0$ is called the quadratic equation

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Solving Quadratic Equations:

1. By the factor method

2. By using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Roots of the quadratic equation:

The roots are denoted by α and β

1. Sum of the roots of a quadratic equation

$ax^2 + bx + c = 0$ is equal to $-\dfrac{b}{a}$. i.e $\alpha + \beta = -\dfrac{b}{a}$

2. Product of the roots is equal to $\dfrac{c}{a}$ i.e $\alpha\beta = \dfrac{c}{a}$

3. For the quadratic equation $ax^2 + bx + c = 0$

The roots will be equal if $b^2 = 4ac$

The roots will be unequal and real if $b^2 > 4ac$

The roots will be unequal and unreal if $b^2 < 4ac$

Given the roots of the quadratic equation,

The quadratic equation is:

$$x^2 - x(\text{sum of the roots}) + \text{Product of the roots} = 0$$

A quadratic equation $ax^2 + bx + c = 0$ has one root equal to zero, then c=0

A quadratic equation $ax^2 + bx + c = 0$ will have reciprocal roots, if a=c.

When the roots of the quadratic equation $ax^2 + bx + c = 0$
are negative reciprocals of each other, then c=-a

1. If one of the roots of the equation $x^2 - 19x + 88 = 0$ be 8 , find the other root.

Solution:

Product of two roots = $\dfrac{c}{a}$ = $\dfrac{88}{1}$ = 88

One root = 8,

Other root = $\dfrac{88}{8}$ = 11

Other root = 11.

2. Form a quadratic equation whose roots are (i) 3 and -5 (ii) 2 and 7

The quadratic equation is:

$$x^2 - x(\text{sum of the roots}) + \text{Product of the roots} = 0$$

$$x^2 + 2x - 15 = 0$$

3. Solve the equation $x^2 - 8x + 15 = 0$

$$15$$

-3 -5

-8

$$x^2 - 8x + 15 = 0$$

(x-3)(x-5)=0

x=3,5

a = 1, b=-8, c=15

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 1 \times 15}}{2 \times 1}$$

$$x = \frac{8 \pm \sqrt{64 - 60}}{2}$$

$$x = \frac{8 \pm 2}{2}$$

x=5,3

4. If $x + \dfrac{1}{x} = 2,$ find the value of $x^2 + \dfrac{1}{x^2}$

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 \times x \times \frac{1}{x} = 4 - 2 = 2$$

5. If $x^2 + \dfrac{1}{x^2} = 51$ then find $x - \dfrac{1}{x}$

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2 = 51 - 2 = 49 \qquad x - \frac{1}{x} = \pm 7$$

6.Evaluate $\sqrt{6+\sqrt{6+\sqrt{6+.....}}}$ to infinity

Solution: let $x = \sqrt{6+\sqrt{6+\sqrt{6+.....}}}$ to infinity

$$x = \sqrt{6+x}$$

Squaring both sides , we get

$$x^2 = 6 + x$$

$$x^2 - x - 6 = 0$$

x = 3 and -2

x = 3

7. A labourer was employed on the condition that he will get Rs. 1.50 for every day he works and will be fined 25p for every day he is absent. After 60 days he get Rs. 76 only. For how many days did he work?

Solution: Suppose he works for x days

He is absent for (60-x) days.

His salary = $x \times \dfrac{3}{2}$ = Rs. $\dfrac{3x}{2}$

and pays fine =RS. $(60 - x) \times \dfrac{1}{4}$

$$\frac{3x}{2} - \frac{60 - x}{4} = 76$$

$$6x - 60 + x = 76 \times 4$$

$$6x - 60 + x = 304$$

$$7x = 364$$

$$x = 52$$

He worked for 52 days

8. If a fraction has its numerator increased by 1, it becomes equal to 1/3, but if its denominator is increased by 1, it becomes equal to 1/4. Find the fraction.

Let the fraction is $\dfrac{x}{y}$

$$\dfrac{x+1}{y} = \dfrac{1}{3} \qquad\qquad \dfrac{x}{y+1} = \dfrac{1}{4}$$

$$3x - y = -3, \quad 4x - y = 1 \qquad\qquad x = 4, \; y = 15$$

The fraction is $\dfrac{4}{15}$

9. The sum of a positive number and its square root is 6 . The number is

a) 18   b) 9   c)4   d)None of these

Solution:

Let the number be x.

$$x + \sqrt{x} = 6$$

$$x + \sqrt{x} = 4 + 2$$

$$x + \sqrt{x} = 4 + \sqrt{4}$$

$$x = 4$$

10. Three consecutive numbers such that twice the first, 3 times the second and 4 times the third together make 182. The numbers are:

Let the first number be x.

∴ Second number = x+1

and, Third number = x+2

According to the question,

$2x+3(x+1)+4(x+2) = 182$

$\Rightarrow 2x+3x+3+4x+8 = 182$

$\Rightarrow 9x+11 = 182$

$\Rightarrow 9x = 182-11 = 171$

$\Rightarrow x = 171/9 = 19$

So, the first number = x = 19

Second number = x+1 = 19+1 = 20

Third number = x+2 = 19+2 = 21

So, the three numbers are 19, 20 and 21.

11. Mohan gets 3 marks for each correct answer and loses 2 marks for each wrong answer. He attempts 30 problems and obtains 40 marks. The number of problems solved correctly is _____.

Let the number of correct answers be x

$\Rightarrow$ Number of wrong answers = 30 – x

Marks obtained by Mohan = 40

$\Rightarrow$ 3x – 2(30 – x) = 5x – 60 = 40

$\therefore$ x = 20

Number of problems solved correctly = 20

12. 30 students can write a thesis in 18 days when they study for 7 hours. How many days will 21 students who study for 8 hours take to finish that thesis assignment?

Let the days be X
30*7*18 = 21*8*X
3780= 168X
Therefore,
X= 3780/168= 22.5 days

Question : Kobe Bryant wants to know that If $a \times b = 2a - 3b + ab$, then $3 \times 5 + 5 \times 3$ is equal to :

(1) 22

(2) 24

(3) 26

(4) 28

Solution : $a \times *b = 2a - 3b + ab$

$\Rightarrow 3 \times 5 = 2 \times 3 - 3 \times 5 + 3 \times 5 = 6$

$5 \times 3 = 2 \times 5 - 3 \times 3 + 3 \times 5$

$= 10 - 9 + 15 = 16$

Therefore, $3 \times 5 + 5 \times 3$

$= 6 + 16 = 22$

Question : Kareena Kapoor wants to know that if $p \times q = p + q + \frac{p}{q}$, then the value of 8×2 is :

Solution : $p \times q = p + q + \frac{p}{q}$

$\therefore \ 8 \times 2 = 8 + 2 + \frac{8}{2}$

=10 + 4 = 14

Question : Ranveer Singh wants to know that If a * b = 2 (a + b), then 5 * 2 is equal to :

(1) 3

(2) 10

(3) 14

(4) 20

Solution : a * b = 2 (a + b)

∴ 5 * 2 = 2 (5 + 2)

= 2 *7 = 14

Determine the value of 3(x)2 + 4(x)3 where x = 2

3×2 + 4×3
Substituting the value of x, we get
3(2)2 + 4(2)3
3 x 4 + 4 x 6
12 + 24
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