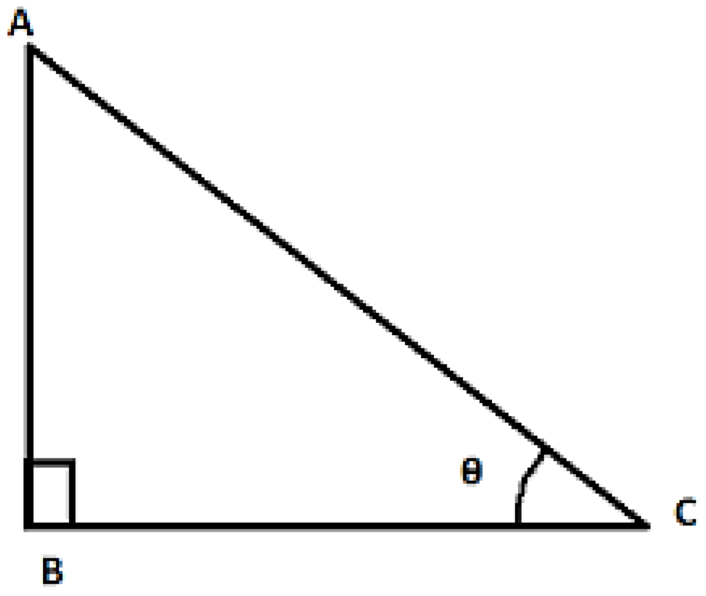
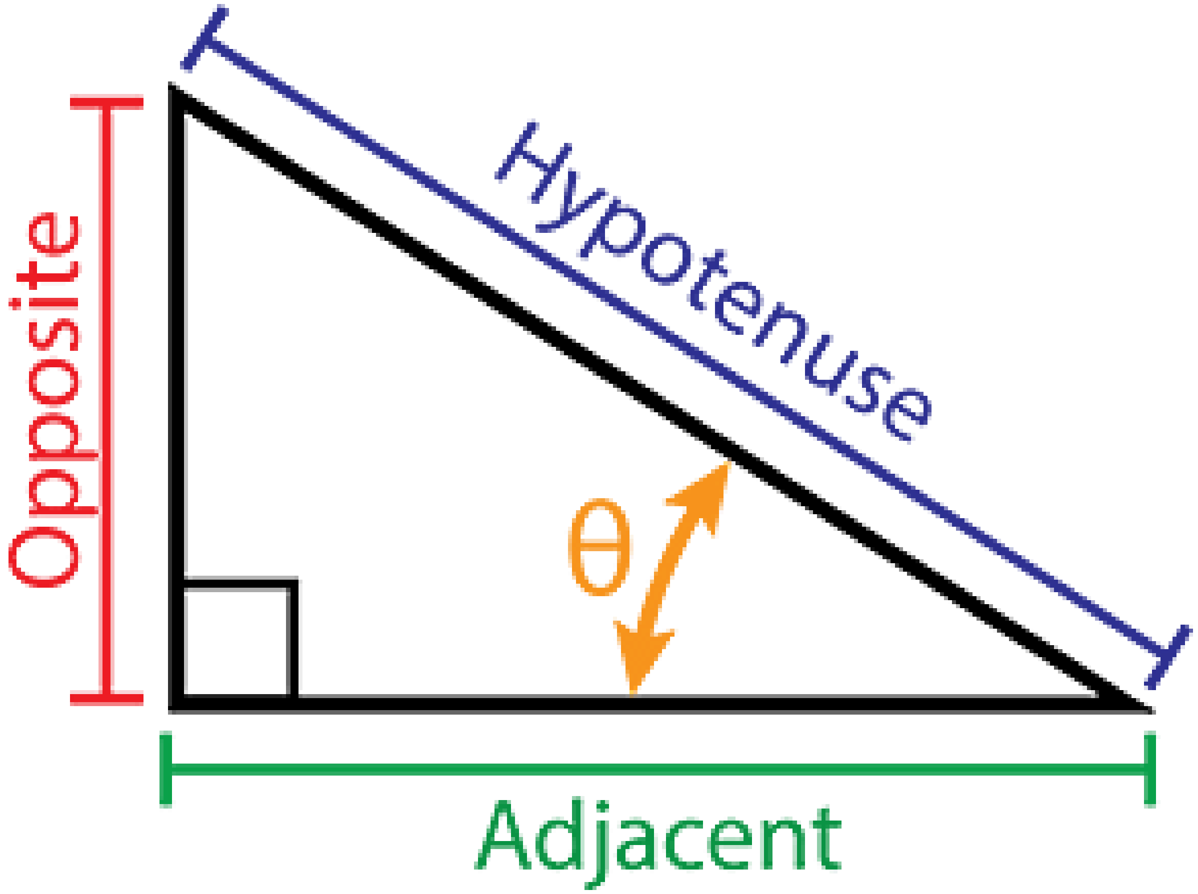
Trigonometry

What is Trigonometry

- ***Trigonometry is the branch of mathematics that deals with the study of relationships between sides and angles of a triangle. It is derived from the Greek word 'Trigonon' and 'metron' where , Tri meaning three and Gon means Angle and Metron means Measure.***





- AC = Hypotenuse of triangle
- AB = Side adjacent to angle A
- BC = Side opposite to angle A

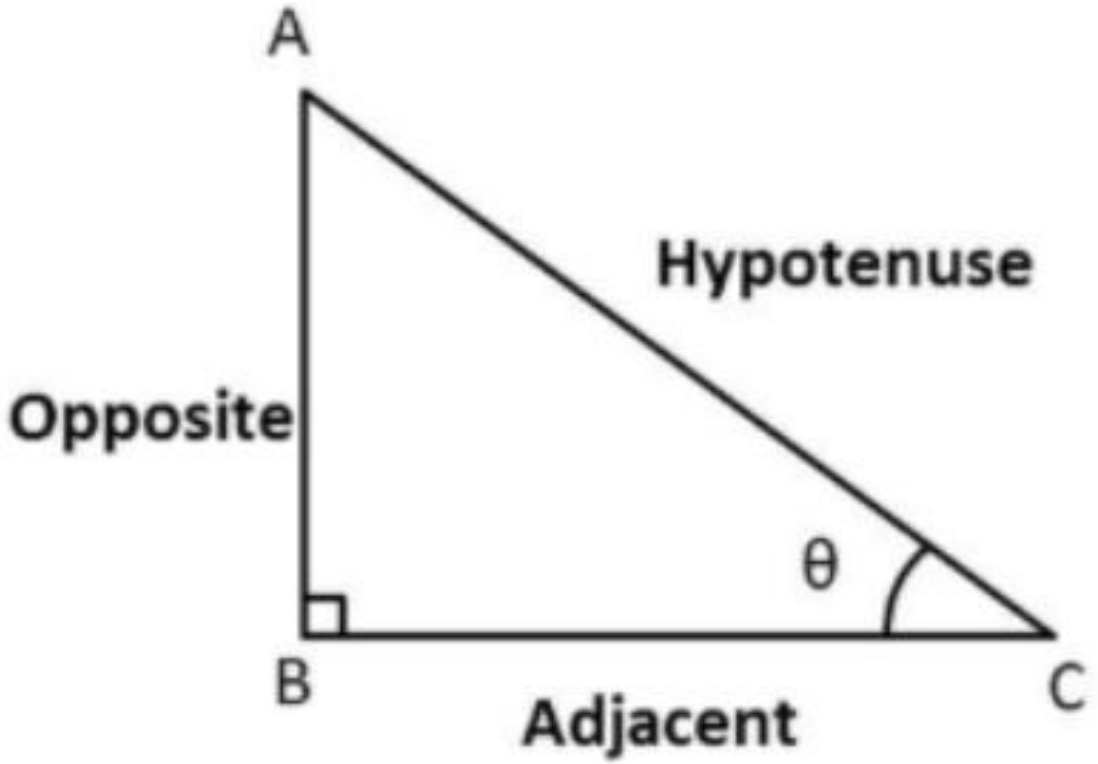
Sin, Cos, Tan – SOH CAH TOA

In right triangle ABC

$$\sin\theta = \frac{\textit{side opposite to angle } \theta}{\textit{Hypotenuse}}$$

$$\cos\theta = \frac{\textit{side adjacent to angle } \theta}{\textit{Hypotenuse}}$$

$$\tan\theta = \frac{\textit{side opposite to angle } \theta}{\textit{side adjacent to angle } \theta}$$



For cot, sec & cosec

$$cosec\,\theta = \frac{1}{\sin\theta}$$

$$sec\,\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Trigonometry Table

	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\operatorname{cosec}\theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Trigonometric Identities

(1) $\cos^2\theta + \sin^2\theta = 1$

(2) $1 + \tan^2\theta = \sec^2\theta$

(3) $1 + \cot^2\theta = \text{cosec}^2\theta$

	Quadrant I	Quadrant II	Quadrant III	Quadrant IV
sin	+	+	–	–
cos	+	–	–	–
tan	+	–	+	–

Sin (-x) = – Sin x

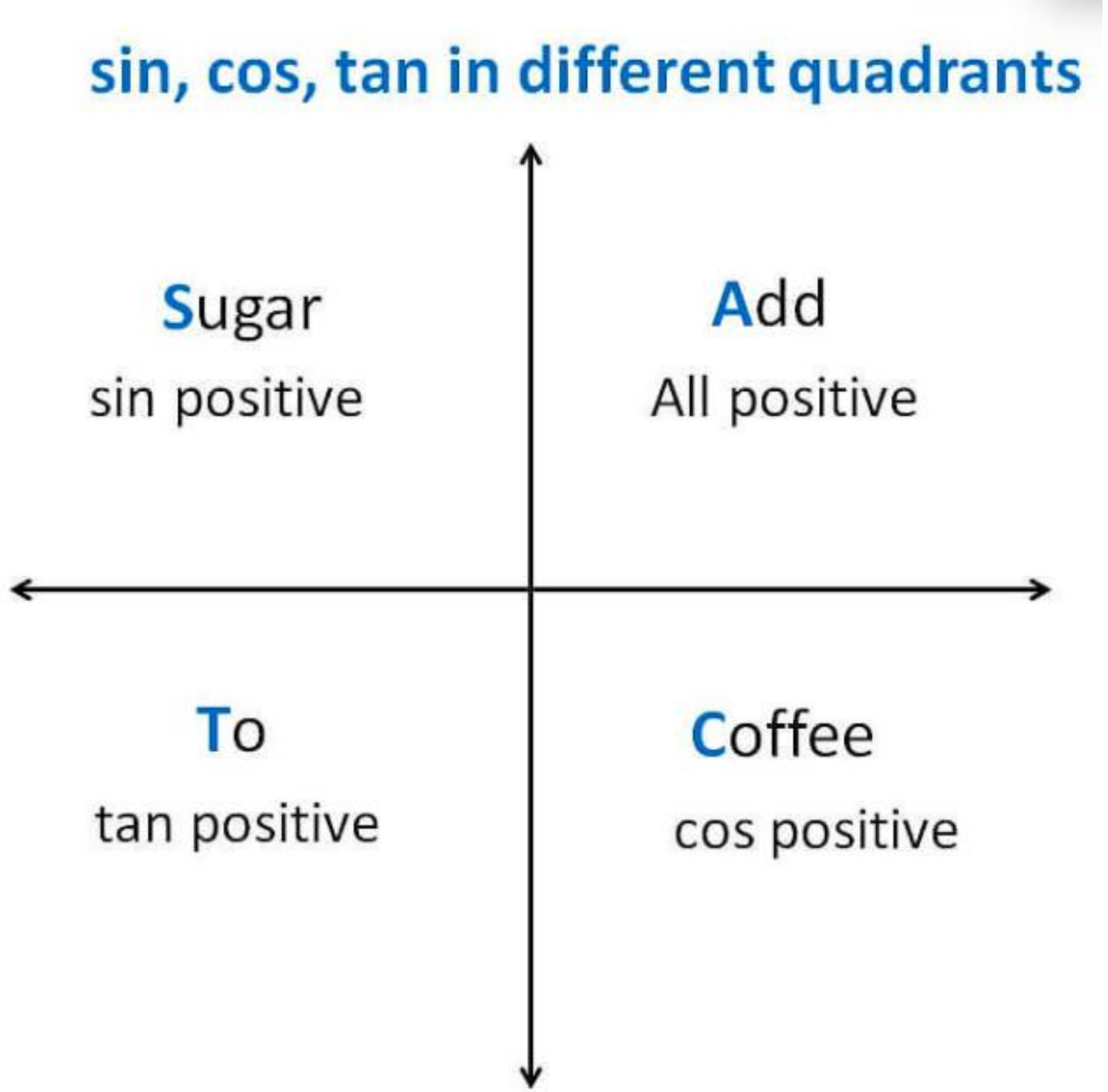
Cos (-x) = Cos x

Tan (-x) = – Tan x

Cot (-x) = – Cot x

Sec (-x) = Sec x

Cosec (-x) = – Cosec x



$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\sin 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1 = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\sin 3x = 3 \sin x - 4 \sin^3 x$$

$$\cos 3x = 4\cos^3 x - 3 \cos x$$

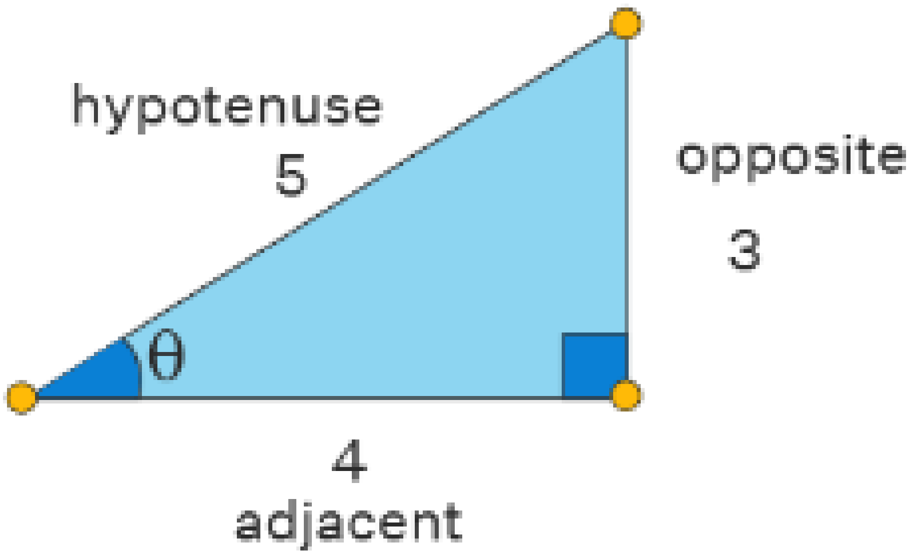
$$\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$



$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5} \qquad \csc(\theta) = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{5}{3}$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5} \qquad \sec(\theta) = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{5}{4}$$

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4} \qquad \cot(\theta) = \frac{\text{adjacent}}{\text{opposite}} = \frac{4}{3}$$

Find the value of (sin 30° + cos 30°) – (sin 60° + cos 60°).

(sin 30° + cos 30°) – (sin 60° + cos 60°)
= [(1/2) + (√3/2)] – [(√3/2) + (1/2)]
= (1/2) + (√3/2) – (√3/2) – (1/2)
= 0

If A = 30°, prove that tan 2A = 2 tan A/(1 – $\tan^2$A).

A = 30°

tan 2A = tan 2(30°) = tan 60°

As we know, tan 60° = √3.

So, tan 2A = √3

2 tan A/(1 – $\tan^2$A)

= [2 tan 30°/(1 – $\tan^2$(30°)]

= [2(1/√3)]/ [1 – $(1/\sqrt{3})^2$]

= (2/√3)/ [(3 – 1)/3]

= 3/√3

= √3

What is the value of $\tan(30°) / \cos(60°)$?

A) $\sqrt{3}/3$

B) $\sqrt{3}$

C) $1/\sqrt{3}$

D) $2/\sqrt{3}$

$\tan(30°) / \cos(60°) = (1/\sqrt{3}) / (1/2)$

$= (1/\sqrt{3}) \times (2/1)$

$= 2/\sqrt{3}$

$= (\sqrt{3} \times 2) / 3$

If (sin A – cos A) = 0 , then what is the value of cot A ?

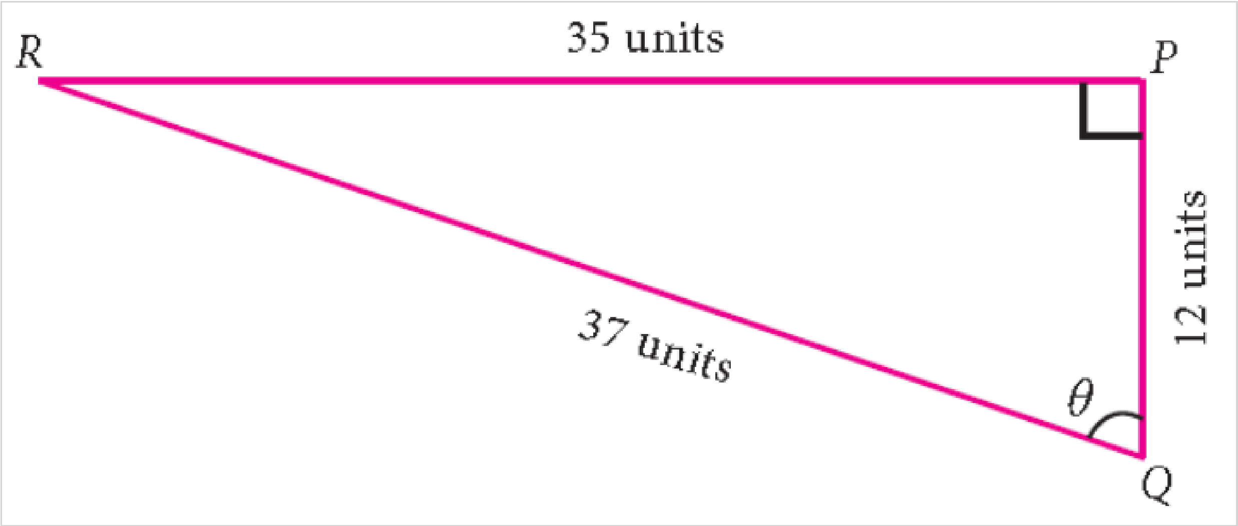
(sin A – cos A) = 0

sin A = cos A

$\frac{cosA}{sinA} = 1$

cot A = 1

For the measures in the figure shown below, compute sine, cosine and tangent ratios of the angle θ.

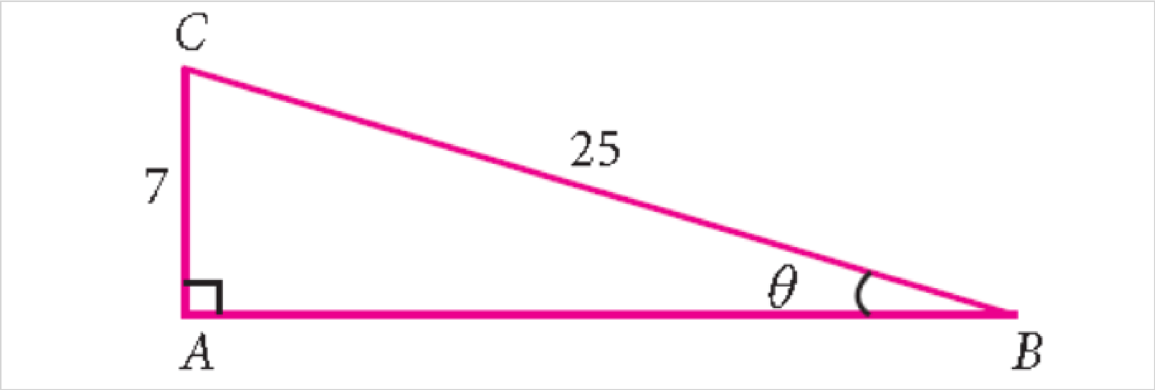


sinθ = opposite side/hypotenuse = PR/QR = 35/37

cosθ = adjacent side/hypotenuse = PQ/QR = 12/37

tanθ = opposite side / adjacent side = PR/PQ = 35/12

Find the six trigonometric ratios of the angle θ using the diagram shown below.



By Pythagorean Theorem,

$$BC^2 = AB^2 + AC^2$$

$$25^2 = AB^2 + 7^2$$

$$625 = AB^2 + 49$$

$\sin\theta$ = opposite side/hypotenuse = $AC/BC = 7/25$

$\cos\theta$ = adjacent side/hypotenuse = $AB/BC = 24/25$

$\tan\theta$ = opposite side/adjacent side = $AC/AB = 7/24$

$\csc\theta = 1/\sin\theta = 25/7$

$\sec\theta = 1/\cos\theta = 25/24$

$\cot\theta = 1/\tan\theta = 24/7$

If tanA = 2/3, then find all the other trigonometric ratios.

By Pythagorean Theorem,

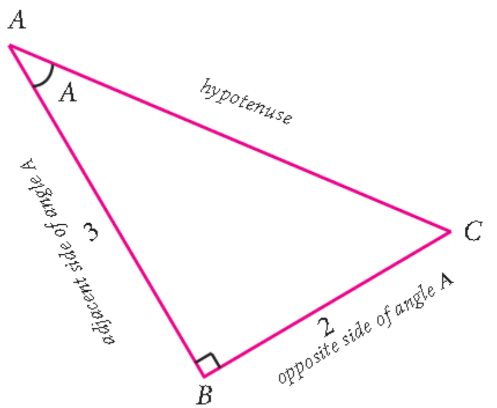
$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 3^2 + 2^2$$

$$AC^2 = 9 + 4$$

$$AC^2 = 13$$

$$AC = \sqrt{13}$$



sinA = opposite side/hypotenuse = BC/ AC = $2/\sqrt{13}$

cosA = adjacent side/hypotenuse = AB/AC = $3/\sqrt{13}$

cscA = 1/sinA = $\sqrt{13}/2$

secA = 1/cosA = $\sqrt{13}/3$

cotA = 1/tanA = 3/2

$$\sin 30° + \cos 30°$$

$$= \sin 30° + \cos 30°$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{1+\sqrt{3}}{2}$$

$$\frac{\tan 45^\circ}{\tan 30^\circ + \tan 60^\circ}$$

$$= \frac{1}{\frac{1}{\sqrt{3}} + \sqrt{3}}$$

$$= \frac{1}{\frac{1 + \left(\sqrt{3}\right)^2}{\sqrt{3}}}$$

$$= \frac{1}{\frac{4}{\sqrt{3}}}$$

$$= 1 \cdot \frac{\sqrt{3}}{4}$$

$$= \frac{\sqrt{3}}{4}$$

If $\sec\theta = \frac{13}{5}$, then what is the value of $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$?

(a) 1

(b) 3

(c) 2

(d) 4

Given, $\sec\theta = \frac{13}{5} \Rightarrow \sec^2\theta = \frac{169}{25}$

$\Rightarrow 1 + \tan^2\theta = \frac{169}{25}$ $\qquad (\because \sec^2\theta - \tan^2\theta = 1)$

$\Rightarrow \tan^2\theta = \frac{169}{25} - 1 = \frac{144}{25} \Rightarrow \tan\theta = \frac{12}{5}$

$$\therefore \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{\frac{2\sin\theta}{\cos\theta} - 3}{\frac{4\sin\theta}{\cos\theta} - 9}$$

(On dividing each term of numerator and denominator by $\cos\theta$)

$$= \frac{2\tan\theta - 3}{4\tan\theta - 9} = \frac{2\times\frac{12}{5} - 3}{4\times\frac{12}{5} - 9}$$

$$= \frac{24-15}{48-45} = \frac{9}{3} = 3.$$

Evaluate the following

$\tan^2 30° + \tan^2 60° + \tan^2 45°$

$\tan^2 30° + \tan^2 60° + \tan^2 45°$ ....(i)

By trigonometric ratios we have

$$\tan 30^\circ = \frac{1}{\sqrt{3}} \quad \tan 60^\circ = \sqrt{3} \quad \tan 45^\circ = 1$$

By substituting above values in (i), we get

$$\left[\frac{1}{\sqrt{3}}\right]^2 + \left[\sqrt{3}\right]^2 + [1]^2$$

$$\Rightarrow \frac{1}{3} + 3 + 1 \Rightarrow \frac{1}{3} + 4$$

$$\Rightarrow \frac{1+12}{3} = \frac{13}{3}$$

Evaluate the following

$$2\sin^2 30^2 - 3\cos^2 45^2 + \tan^2 60^\circ$$

$$2\sin^2 30^2 - 3\cos^2 45^2 + \tan^2 60^\circ \quad \text{....(i)}$$

By trigonometric ratios we have

$$\sin 30^\circ = \frac{1}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}} \quad \tan 60^\circ = \sqrt{3}$$

By substituting above values in (i), we get

$$2.\left[\frac{1}{2}\right]^2 - 3\left[\frac{1}{\sqrt{2}}\right]^2 + \left[\sqrt{3}\right]^2$$

$$2.\frac{1}{4} - 3.\frac{1}{2} + 3$$

$$\frac{1}{2} - \frac{3}{2} + 3 \Rightarrow \frac{3}{2} + 2 = 2$$

Evaluate the following

$$\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin^2 90^\circ - 2\cos^2 90^\circ + \frac{1}{24}\ \cos^2 0^\circ$$

$$\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin^2 90^\circ - 2\cos^2 90^\circ + \frac{1}{24}\ \cos^2 0^\circ \quad \text{....(i)}$$

By trigonometric ratios we have

$$\sin 30^\circ = \frac{1}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}} \quad \tan 30^2 = \frac{1}{\sqrt{3}} \quad \sin 90^\circ = 1 \quad \cos 90^\circ = 0 \quad \cos 0^\circ = 1$$

By substituting above values in (i), we get

$$\left[\frac{1}{2}\right]^2 \cdot \left[\frac{1}{\sqrt{2}}\right]^2 + 4\left[\frac{1}{\sqrt{3}}\right]^2 + \frac{1}{2}[1]^2 - 2[0]^2 + \frac{1}{24}[1]^2$$

$$\frac{1}{4.1}/2 + \frac{4}{1}/3 + \frac{1}{2} - 0 + \frac{1}{24}$$

$$\frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24} = \frac{48}{24} = 2$$

Evaluate the following

(cosec² 45° sec² 30°)(sin² 30° + 4 cot² 45° − sec² 60°)

By trigonometric ratios we have

$$\cos ec45^{\circ} = \sqrt{2} \quad \sec 30^{\circ} = \frac{2}{\sqrt{3}} \quad \sin 30^{\circ} = \frac{1}{2} \quad \cot 45^{\circ} = 1 \quad \sec 60^{\circ} = 2$$

By substituting above values in (i), we get

$$\left(\left[\sqrt{2}\right]^2 . \left[\frac{2}{\sqrt{3}}\right]^2\right)\left(\left[\frac{1}{2}\right]^2 + 4[1]^2 . [2]^2\right)$$

$$\Rightarrow \left[2.\frac{4}{3}\right]\left[\frac{1}{4} + 4 - 4\right] \Rightarrow 3.\frac{4}{3}.\frac{1}{4} = \frac{2}{3}$$

$\text{Sin}^4\theta - \cos^4\theta = 1 - 2\cos^2\theta$

$\text{Sin}^4\theta - \cos^4\theta = 1 - 2\cos^2\theta$

LHS = $\text{Sin}^4\theta - \cos^4\theta$

LHS = $(\text{Sin}^2\theta)^2 - (\cos^2\theta)^2$

LHS = $(\text{Sin}^2\theta + \cos^2\theta)(\text{Sin}^2\theta - \cos^2\theta)$ ...$[a^2 - b^2 = (a + b)(a - b)]$

LHS = $(\text{Sin}^2\theta - \cos^2\theta).(1)$ ...$(\text{Sin}^2\theta + \cos^2\theta = 1)$

LHS = $1 - \cos^2\theta - \cos^2\theta$ ...$(1 - \text{Sin}^2\theta = \cos^2\theta)$

LHS = $1 - 2\cos^2\theta$

RHS = $1 - 2\cos^2\theta$

LHS = RHS

Prove the following: $\frac{\sin\ \theta}{1+\cos\ \theta}+\frac{1+\cos\ \theta}{\sin\ \theta}=2\cos ec\ \theta$

$$\text{LHS}=\frac{\sin\ \theta}{1+\cos\ \theta}+\frac{1+\cos\ \theta}{\sin\ \theta}$$

$$=\frac{\sin^2\ \theta+(1+\cos\ \theta)^2}{\sin\ \theta\cdot(1+\cos\ \theta)}$$

$$=\frac{\sin^2\ \theta+\cos^2\ \theta+2\cos\ \theta+1}{\sin\ \theta\cdot(1+\cos\ \theta)} \qquad \left[\because\ (a+b)^2=a^2+2ab+b^2\right]$$

$$=\frac{1+2\cos\ \theta+1}{\sin\ \theta\cdot(1+\cos\ \theta)} \qquad \left[\because\ \sin^2\ \theta+\cos^2\ \theta=1\right]$$

$$=\frac{2(1+\cos\ \theta)}{\sin\ \theta\cdot(1+\cos\ \theta)}$$

$$=\frac{2}{\sin\ \theta}$$

$$=2\cos ec\ \theta \qquad \left[\because\ \frac{1}{\sin\ \theta}=\cos ec\ \theta\right]$$

$$=\text{RHS}$$

Prove that : √(1+sinθ/1-sinθ) + √(1-sinθ/1+sinθ) = 2sec θ

L.H.S = √(1+sinθ/1-sinθ) + √(1-sinθ/1+sinθ)

$$= \sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)} \times \frac{(1+\sin\theta)}{(1+\sin\theta)}} + \sqrt{\frac{(1-\sin\theta)}{(1+\sin\theta)} \times \frac{(1-\sin\theta)}{(1-\sin\theta)}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1+\sin\theta}{\cos\theta} + \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$$

$$= \frac{2}{\cos\theta}$$

$= 2\sec\theta =$ R.H.S.

Hence proved.

Prove the following trigonometric identities.

$$\frac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$$

We know that, $\sin^2\theta + \cos^2\theta = 1$

Multiplying both numerator and denominator by $(1-\sin\theta)$ we have

$$\frac{1-\sin\theta}{1+\sin\theta} = \frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$$= (\sec\theta - \tan\theta)^2$$

Evaluate

$\sin 27° \sin 63° - \cos 63° \cos 27°$

$\sin 27° \sin 63° - \cos 63° \cos 27°$

$= \sin(90° - 63°) \sin 63° - \cos 63° \cos(90° - 63°)$

$= \cos 63° \sin 63° - \cos 63° \sin 63°$

$= 0$