

Ratio and Proportion

Important facts and Formulae

- ▶ **Ratio :** The ratio of two quantities a and b in the same units, is the fraction a/b and we write it as $a:b$.
- ▶ In the ratio $a : b$, we call 'a' as the first term or antecedent and 'b', the second term or consequent.
- ▶ **Ex.** The ratio $5 : 9$ represent $5/9$ with antecedent = 5, consequent = 9
- ▶ **Rule:** The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.
- ▶ **Ex.** $4:5 = 8 : 10 = 12: 15$ etc. Also, $4:6 = 2 : 3$.

- ▶ **Proportion** : The equality of two ratios is called proportion.
- ▶ If $a : b = c : d$, we write, $a : b :: c : d$ and we say that a, b, c, d are in proportion. Here a and d are called extremes, while b and c are called mean terms.
- ▶ Product of means = Product of extremes.
- ▶ Thus, $a : b :: c : d \leftrightarrow (b \times c) = (a \times d)$.
- ▶ **3. (i) Fourth Proportional** : If $a : b = c : d$, then d is called the fourth proportional to a, b, c .
- ▶ **(ii) Third Proportional** : If $a : b = b : c$, then c is called the third proportional to a and b .
- ▶ **(iii) Mean Proportional** : Mean Proportional between a and b is \sqrt{ab} .

If $a : b = 5:9$ and $b : c = 4 : 7$, find $a : b : c$.

► **Solution**

$$a : b = 5 : 9 \text{ and } b : c = 4 : 7$$

$$\frac{a}{b} = \frac{5}{9}, \quad a = \frac{5b}{9} \quad \frac{b}{c} = \frac{4}{7} \quad c = \frac{7b}{4}$$

$$a:b:c = \frac{5b}{9} : b : \frac{7b}{4} = \frac{5}{9} : 1 : \frac{7}{4} = 20:36:63$$

$$a:b:c = 20:36:63$$

a:	b:	c
5	9	
	4	7

a:	b:	c
5	9	9
× 4	4	7
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20	36	63

If $a : b = 3 : 4$ and $b : c = 8 : 9$, find $a : c$.

a:	b:	c
3	4	
	8	9

a:	b:	c
3	4	4
$\times 8$	8	9
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24	32	36
6	8	9

$$a:c = 6:9 = 2:3$$

If $a : b = 2 : 3$, $b : c = 4 : 5$ and $c : d = 6 : 7$ then
 $a : b : c : d$ is

a:	b:	c:	d
2	3		
	4	5	
		6	7

a:	b:	c :	d
2	3	3	3
$\times 4$	4	5	5
6	6	6	7
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48	72	90	105
16	24	30	35

Jo's collection contains US, Indian and British stamps. If the ratio of US to Indian stamps is 5 to 2 and the ratio of Indian to British stamps is 5 to 1, what is the ratio of US to British stamps?

US : INDAIN : BRITISH

5 : 2 : 2

5 : 5 : 1

25 : 10 : 2

US: British = 25:2

In an innings of a cricket match, three players A, B and C scored a total of 361 runs. If the ratio of the number of runs scored by A to that scored by B and also number of runs scored by B to that of C be 3 : 2, the number of runs scored by A was

Given total runs of A, B, C ($A + B + C = 361$)

$$\Rightarrow \begin{array}{l} A : B : C \\ 3 : 2 : 2 \\ 3 \quad 3 : 2 \\ \hline 9 : 6 : 4 \end{array}$$

$$9x + 6x + 4x = 361$$

$$19x = 361$$

$$x = 19$$

$$\therefore \text{Runs scored by A} = 9x = 9 \times 19 = 171$$

Find

- (i) The fourth proportional to 4, 9, 12.
- (ii) The third proportional to 16 and 36;
- (iii) The mean proportional between 0.08 and 0.18.

► (i) Let the fourth proportional to 4, 9, 12 be X.

$$a:b::c:d$$

Then, $4 : 9 :: 12 : X \leftrightarrow 4 \times X = 9 \times 12 \leftrightarrow x = (9 \times 12)/4 = 27$.

Forth proportional to 4, 9, 12, is 27.

► (ii) Let the third proportional to 16 and 36 be x.

$$a:b::b:c$$

Then, $16 : 36 :: 36 : X \leftrightarrow 16 \times X = 36 \times 36 \leftrightarrow X = (36 \times 36)/16 = 81$.

Third Proportional to 16 and 36 is 81.

(iii) Mean proportional between 0.08 and 0.18

$$= \sqrt{0.08 \times 0.18} = \sqrt{\frac{8}{100} \times \frac{18}{100}} = \sqrt{\frac{144}{100 \times 100}} = \frac{12}{100} = 0.12.$$

The ratio of three numbers is 3:4:7 and their product is 18144. The numbers are

Let the numbers are $3x$, $4x$ and $7x$ then

$$3x \times 4x \times 7x = 18144$$

$$84x^3 = 18144$$

$$x^3 = 216$$

$$x = 6$$

The numbers are 18, 24, 42

Divide Rs.672 in the ratio 5:3

Solution:

Sum of the ratio terms = $5+3=8$

First part = Rs. $672 \times \frac{5}{8} = Rs. 420$

Second part = Rs. $672 \times \frac{3}{8} = Rs. 252$

A bag contains 50p, 25p, and 10p coins in the ratio 5:9:4, amounting to Rs.206. Find the number of coins of each type.

- ▶ Let the number of coins be 50p , 25p and 10p coins be $5x$, $9x$ and $4x$

Rupees $206 = 20600p$

$$5x(50) + 9x(25) + 4x(10) = 20600$$

$$\text{or, } 250x + 225x + 40x = 20600$$

$$\text{or, } 515x = 20600$$

$$\text{or, } x = 20600/515$$

$$\text{or, } x = 40$$

- ▶ No. of 50p coins = $5x = 5 \times 40 = 200$ coins
No. of 25p coins = $9x = 9 \times 40 = 360$ coins
No. of 10p coins = $4x = 4 \times 40 = 160$ coins

In a bag the coins of 50p, 25p and 10p are in the ratio 2:3:5 and its total value is Rs.90. Find the number of coins of 25p in bag?

$$50p = x, 25p = y, 10p = z, 2:3:5 = a:b:c \quad \text{Rs } 90 = A$$

$$50p = \frac{a}{ax+by+cz} \times A$$

$$25p = \frac{b}{ax+by+cz} \times A$$

$$10p = \frac{c}{ax+by+cz} \times A$$

$$25p = \frac{3}{2(50)+3(25)+5(10)} \times 90 \times 100 = \frac{3}{25(4+3+2)} \times 9000$$

$$= \frac{3}{25(9)} \times 9000 = 120 \text{ coins.}$$

A mixture contains alcohol and water in the ratio 4:3. If 5 litres of water is added to the mixture, the ratio becomes 4:5. find the quantity of alcohol in the mixture.

Let the quantity of alcohol and water be $4x$ litres and $3x$ litres respectively.

$$\frac{4x}{3x + 5} = \frac{4}{5}$$

$$\Leftrightarrow 20x = 4(3x+5)$$

$$\Leftrightarrow 8x = 20$$

$$\Leftrightarrow x = 2.5$$

The quantity of alcohol = $4 \times 2.5 = 10$ litres

In a mixture 60 litres, the ratio of milk and water 2 : 1. If this ratio is to be 1 : 2, then the quantity of water to be further added is:

$$\text{Quantity of milk} = \left[60 \times \frac{2}{3} \right] \text{ litres} = 40 \text{ litres.}$$

$$\text{Quantity of water in it} = (60 - 40) \text{ litres} = 20 \text{ litres.}$$

$$\text{New ratio} = 1 : 2$$

Let quantity of water to be added further be x litres.

$$\text{Then, milk : water} = \left(\frac{40}{20 + x} \right).$$

$$\text{Now, } \left(\frac{40}{20 + x} \right) = \frac{1}{2}$$

$$\Rightarrow 20 + x = 80$$

$$\Rightarrow x = 60.$$

\therefore Quantity of water to be added = 60 litres.

The ages of A and B are in the ratio 3:1. Fifteen years hence, the ratio will be 2:1. Their present ages are

Let the ages of A and B be $3x$ years and x years respectively.

$$\text{Then, } \frac{3x+15}{x+15} = \frac{2}{1}$$

$$\Leftrightarrow 2x+30 = 3x+15$$

$$\Leftrightarrow x=15$$

So, A's age $3 \times 15 = 45$ years, B's age 15 years

Salaries of Ravi and Sumit are in the ratio 2:3. If the salary of each is increased by Rs. 4000, the new ratio becomes 40:57. What is Sumit's present salary ?

Let the original salaries of Ravi and Sumit be Rs. $2x$ and $3x$ respectively.

$$\text{Then, } \frac{2x+4000}{3x+4000} = \frac{40}{57}$$

$$\frac{2x + 4000}{3x + 4000} - 1 = \frac{40}{57} - 1$$

$$\frac{2x + 4000 - 3x - 4000}{3x + 4000} = \frac{40 - 57}{57}$$
$$\frac{-x}{3x + 4000} = \frac{-17}{57}$$

$$57x = 51x + 68000 = 6x + 68000, 3x = 34000$$

$$\text{Sumit's present salary} = 34000 + 4000 = 38000$$

The present ratio of students to teachers at a certain school is 30 to 1. If the student enrollment were to increase by 50 students and the number of teachers were to increase by 5, the ratio of the teachers would then be 25 to 1. What is the present number of teachers?

Assume the present students and teachers are $30x$, x

$$\text{Then } \frac{30x+50}{1x+5} = \frac{25}{1}$$

$$\Rightarrow 30x+50 / x+5 = 25 : 1 \Rightarrow 30x+50 = 25x+125$$

Solving we get $x = 15$

So present teachers are 15.

In a college, the ratio of the number of boys to girls is 8:5. if there are 160 girls, the total number of students in the college is

► Let the number of boys and girls be $8x$ and $5x$ respectively.

$$5x = 160, \quad x = 32$$

$$\text{Total number of students} = 13x = 13 \times 32 = 416.$$

The ratio of the number of boys and girls in a college is 7 : 8. If the percentage increase in the number of boys and girls be 20% and 10% respectively, what will be the new ratio?

Originally, let the number of boys and girls in the college be $7x$ and $8x$ respectively.

Their increased number is (120% of $7x$) and (110% of $8x$).

$$\Rightarrow \left(\frac{120}{100} \times 7x \right) \text{ and } \left(\frac{110}{100} \times 8x \right)$$

$$\Rightarrow \frac{42x}{5} \text{ and } \frac{44x}{5}$$

$$\therefore \text{ The required ratio } = \left(\frac{42x}{5} : \frac{44x}{5} \right) = 21 : 22.$$

Seats for Mathematics, Physics and Biology in a school are in the ratio 5 : 7 : 8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of increased seats?

Originally, let the number of seats for Mathematics, Physics and Biology be $5x$, $7x$ and $8x$ respectively.

Number of increased seats are (140% of $5x$), (150% of $7x$) and (175% of $8x$).

$$\Rightarrow \left(\frac{140}{100} \times 5x \right), \left(\frac{150}{100} \times 7x \right) \text{ and } \left(\frac{175}{100} \times 8x \right)$$

$$\Rightarrow 7x, \frac{21x}{2} \text{ and } 14x.$$

$$\therefore \text{ The required ratio} = 7x : \frac{21x}{2} : 14x$$

$$\Rightarrow 14x : 21x : 28x$$

$$\Rightarrow 2 : 3 : 4.$$

A dog takes 3 leaps for every 5 leaps of a hare. If one leap of the dog is equal to 3 leaps of the hare, the ratio of the speed of the dog to that of the hare is :

Dog: Hare = 3:5

One leap of the dog covers the same distance as 3 leaps of the hare. This means the dog covers more distance per leap

Dog : Hare = (3×3) leaps of hare : 5 leaps of hare = 9 : 5.

Sales manager of pepperfry.com was checking the inventory and concluded that the cost price of 2 tables and 3 chairs is Rs. 2200 and the cost price of 2 chairs and 4 tables is Rs. 2400. The question is what will be the ratio between the cost price of the chairs and the table ?

Let the cost price of one table = x Rs.,
the cost price of one chairs = y Rs.

According to the question,

$$2x + 3y = 2200 \dots 1$$

$$2y + 4x = 2400$$

$$2x + y = 1200 \dots 2$$

After solving these 2 equation,

$$x = 350 \text{ Rs.}, y = 500 \text{ Rs.}$$

$$\text{Ratio} = 500 : 350 = 10 : 7$$