PIPES AND CISTERN

- Inlet:

A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.

- Outlet:

A pipe connected with a tank or cistern or reservoir, emptying it, is known as an outlet.

2. If a pipe can fill a tank in x hours, then:

   part filled in 1 hour = $\frac{1}{x}$.

3. If a pipe can empty a tank in y hours, then:

   part emptied in 1 hour = $\frac{1}{y}$.

4. If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $y > x$), then on opening both the pipes, then

the net part filled in 1 hour = $\left(\frac{1}{x} - \frac{1}{y}\right)$.

5. If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $x > y$), then on opening both the pipes, then

the net part emptied in 1 hour = $\left(\frac{1}{y} - \frac{1}{x}\right)$.

Pipe A can fill A tank in 30 hours and pipe B in 45 hour. If both the pipes are opened in an empty tank how much time will they take to fill it?

- Pipe A can fill the tank in 30 hours.
  - Therefore, the rate of Pipe A is 1/30 of the tank per hour.
- Pipe B can fill the tank in 45 hours.
  - Therefore, the rate of Pipe B is 1/45 of the tank per hour.
- $Part\ filled\ by\ A\ and\ B\ in\ 1\ hr = \frac{1}{30} + \frac{1}{45} = \frac{10}{180} = \frac{1}{18}$
- A and B take 18 hours to fill the tank.

A cistern can be filled by pipes A and B in 4 hours and 6 hours respectively. When full, the tank can be emptied by a pipe C in 8 hours. If all the taps be turned on at the same time , the cistern will be full in :

Pipe A can fill a tank in 4 hours
Pipe A in 1 hour fill 1/4 of the tank

Pipe B can fill a tank in 6 hours
Pipe B in 1 hour can fill 1/6 of the tank

Pipe C can empty a tank in 8 hours
Pipe C in 1 hour can empty 1/8 of the tank

All the three pipes are opened together.In 1 hour,
they fill: 1/4 + 1/6 - 1/8 = 6+4-3/24 = 7/24 of the tank [-1/8 coz it is emptying]
Hence tank will be full in 24/7 hours or 3 3/7 hours

A water tank is two-fifth full. Pipe A can fill a tank in 10 minutes and pipe B can empty it in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely?

Work done by pipe A in $1\ min = \frac{1}{10}$

Work done by pipe B in $1\ min = \frac{1}{6}$

Total work when A and B are opened $= \frac{1}{6} - \frac{1}{10}$

$= \frac{4}{60}$

$= \frac{1}{15}$

Part emptied in by A and B in $1\ min = \frac{1}{15}$

Time is taken to empty $\frac{2}{5}$ the tank $= \frac{\frac{2}{5}}{\frac{1}{15}}$

$= 6\ min$

Hence, the time taken to empty $\frac{2}{5}$ of the tank is $6\ min$.

A cistern has two taps which fill it in 12 minute and 15 minutes respectively. There is also a waste pipe in the cistern. When all the pipes are opened, the empty cistern is full 20 minutes. How long waste pipe take to empty a full cistern?

Work done by the waste pipe in 1 minutes

$$= \frac{1}{20} - \frac{1}{12} + \frac{1}{15}$$

$$= -\frac{1}{10} \qquad \text{[-ve sign means emptying]}$$

$\therefore$ Waste pipe will empty the full cistern in 10 minutes.

An electric pump can fill a tank in 3 hours. Because of a leak in the tank it took 31/2 hours to fill the tank. If the tank is full, how much time will the leak take to empty it?

Work done by the leak in 1 hour

$= \left(\frac{1}{3} - \frac{2}{7}\right) = \frac{1}{21}$

$\therefore$ Leak will empty the tank in 21 hours.

Two pipes can fill a cistern in 14 and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom, 32 minutes extra are taken for the cistern to be filled up. If the cistern is full, in what time would the leak empty it?

If both filling pipes are working together, portion of the cistern filled in an hour = 1/14 + 1/16 = (8+7)/112 = 15/112

Hence time taken to fill the cistern = 112/15 hours = 7 7/15 hours = 7 hours 28 minutes.

But due to leak it takes 32 min more.

Hence total time taken = 7 hours 28 minutes + 32 min = 8 hours.

Let the leak take n hours to empty the cistern.

So, 1 / 14 + 1 / 16 - 1 / n = 1 / 8

$\Rightarrow$ 15 / 112 - 1 / n=18

$\Rightarrow$ 1 / n = 15 / 112 - 1 / 8 = 1 / 112

Hence the cistern will be emptied in 112 hours.

If two pipes function simultaneously, the reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does it take the second pipe to fill the reservoir?

Let the slower pipe can fill the tank in x hours,

Then faster pipe can fill the tank in (x – 10) hours.

Tank filled by slower pipe in 1 hour = 1/x

Tank filled by faster pipe in 1 hour = 1/(x – 10)

Part filled by both the pipes in 1 hour = 1/12

According to the question

$1/x + 1/(x - 10) = 1/12$

$\Rightarrow [(x - 10) + x]/x(x - 10) = 1/12$

$\Rightarrow 12(x - 10) + 12x = x(x - 10)$

$\Rightarrow 12x - 120 + 12x = x^2 - 10x$

$\Rightarrow x^2 - 34x + 120 = 0$

$\Rightarrow x^2 - 30x - 4x + 120 = 0$

$\Rightarrow x(x - 30) - 4(x - 30) = 0$

$\Rightarrow (x - 30)(x - 4) = 0$

$\Rightarrow x = 30, 4$

But x = 4 is not possible since (x – 10) i.e. (4 – 10) will give a negative value

Hence slower pipe can fill the tank in 30 hours and

Faster pipe can fill the tank in = 30 – 10 hours = 20 hours

∴ The faster pipe alone take to fill the tank in 20 hours

Three pipes A, B and C can fill a cistern in 6 hours. After working together for 2 hours, C is closed and A and B fill the cistern in 8 hours. Then find the time in which the cistern can be filled by pipe C

$\Rightarrow$ A + B + C can fill in 1 hours = 1/6 of cistern

$\Rightarrow$ A + B + C can fill in 2 hours = 2/6 = 1/3 of cistern

$\Rightarrow$ Unfilled part = 1 - 1/3 = 2/3

$\Rightarrow$ 2/3 is filled by A + B in 8 hours

$\Rightarrow$ A + B can fill the cistern in 8 × 3/2 = 12 hours

$\Rightarrow$ we have A + B + C can fill the cistern in 6 hours

$\therefore$ C = (A + B + C) - (A + B) can fill the cistern = 1/6 - 1/12 = 12 hours

A, B and C are three pipes connected to a tank. A and B together fill the tank in 6 hours. B and C together fill the tank in 10 hours. A and C together fill the tank in 7 1/2 hours. In how much time will A, B and C fill the tank separately.

According to question,

$\Rightarrow a + b = 1/6$ ----(1)

$\Rightarrow b + c = 1/10$ ----(2)

$\Rightarrow a + c = 2/15$ ----(3)

Adding all the equation above we get,

$\Rightarrow 2(a + b + c) = (1/6) + (1/10) + (2/15)$

$\Rightarrow 2\{a + (1/10)\} = (5 + 3 + 4)/30$ [$\because$ equation (2)]

$\Rightarrow 2a + (1/5) = 2/5$

$\Rightarrow a = 1/10$

$\therefore$ A alone can fill the tank in 10 hours

B can fill in 15 hours
C can fill in 30 hours