CSCD18 A1 written part

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1 Assignment Part I Answer:

1. As the equation of the parabola is $y = x^2$, we can change it to implicit form:

$$f(x,y) \Leftrightarrow y - x^2 = 0.$$

Thus, in this case, we have:

$$\vec{n}(\overline{p}) = (\frac{\partial f(x,y)}{\partial x}, \frac{\partial f(x,y)}{\partial y}) \Rightarrow \vec{n} = (-2x, 1)$$

2. As $x = c$, we put c into the function $y = x^2$, we can get $y = c^2$, and therefore the intersection point will be

$$[c, c^2]$$

3. Given the intersection point is $[c, c^2]$ and the equation of the normal $\vec{n} = (-2x, 1)$, We can have

$$\vec{n_c} = (-2c, 1)$$

After normalizing it:

$$\vec{n'_c} = (\frac{-2c * \sqrt{4c^2+1}}{4c^2+1}, \frac{\sqrt{4c^2+1}}{4c^2+1})$$

4. Given the direction of the ray which is

$$\vec{d} = [0, -1]$$

the equation of the reflected ray is

$$(c, c^2) + \lambda(\frac{-4c}{4c^2+1}, \frac{2}{4c^2+1} - 1)$$

5. we can find lambda by setting x=0 at the intersection point between the light ray and y-axis so we have the equation

$$0 = c + \lambda \frac{-4c}{4c^2+1}$$

$$\lambda = \frac{4c^2+1}{4}$$

Applying this to calculate y:

$$y = c^2 + \frac{4c^2+1}{4} * \frac{2-(4c^2+1)}{4c^2+1}$$

$$\Rightarrow c^2 + \frac{2-(4c^2+1)}{4}$$

$$\Rightarrow c^2 + \frac{-4c^2+1}{4}$$

$$\Rightarrow c^2 + \frac{-4c^2}{4} + \frac{1}{4} = \frac{1}{4}$$

Therefore we have the focal point:

$$(0, \frac{1}{4})$$

2 Assignment Part II Answer:

1. As we have a

$$x(u,v) = Cos(u)$$
$$y(u,v) = Sin(u) + Cos(v)$$
$$z(u,v) = Sin(v)$$

Given an arbitrary point $\bar{s}(s,t) = \bar{p_0}$, we can have two tangent vector computed by:

$$\frac{\partial \bar{s}}{\partial u} \& \frac{\partial \bar{s}}{\partial v}$$

And we have $\frac{\partial \bar{s}}{\partial u}(1)$ as:

$$(\frac{\partial cos(u)}{\partial u}, \frac{\partial(sin(u) + cos(v))}{\partial u}, \frac{\partial sin(v)}{\partial u})$$
$$\Rightarrow (-sin(u), cos(u), 0) = \vec{\alpha}$$

where substitute u with s.
And $\frac{\partial \bar{s}}{\partial v}(2)$ as:

$$(\frac{\partial cos(u)}{\partial v}, \frac{\partial(sin(u) + cos(v)))}{\partial v}, \frac{\partial sin(v)}{\partial v})$$
$$\Rightarrow (0, -sin(v), cos(v)) = \vec{\beta}$$

where substitute v with t.
Therefore, we can get the parametric equation of the ***tangent plane***:

$$\bar{t}(\lambda_1, \lambda_2) = \bar{p_0} + \lambda_1 \vec{\alpha} + \lambda_2 \vec{\beta}$$

2. Normal to the plane:

$$\vec{n}(s,t) = (\frac{\partial \bar{s}}{\partial u}) \times (\frac{\partial \bar{s}}{\partial v})$$
$$\Rightarrow (cos(s)cos(t), -sin(s)cos(t), sin(s)sin(t))$$

where s and t are given.
So the implicit equation of the ***tangent plane***:

$$f(\bar{p}) = cos(s)cos(t) \times (p_x - cos(s)) + (-sin(s)cos(t))$$
$$\times (p_y - sin(s) - cos(t)) + sin(s)sin(t) \times (p_z - sin(t)) = 0$$

3. First, we will do a ***rotation*** transform about y-axis of angle ϕ which $\phi = arctan(\frac{\sqrt{(v_y{}^2+v_x^2)}}{v_z})$:

$$R_y(\phi) = \begin{bmatrix} cos(\phi) & 0 & sin(\phi) \\ 0 & 1 & 0 \\ -sin(\phi) & 0 & cos(\phi) \end{bmatrix}$$

Second, we will do another ***rotation*** transform about the z-axis of angle θ which $\theta = arctan(\frac{v_y}{v_x})$:

$$R_z(\theta) = \begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

After rotations, we make a ***translation*** transformation where the translate vector $\vec{t} = (p_x, p_y, p_z)^T$. So overall sequence would be:

$$\hat{M}\hat{p} = \hat{T}\hat{R}_z\hat{R}_y\hat{p}$$

where $\hat{p}$ are points in the original shape, $\hat{M}\hat{p}$ are points after transformed.