

Final Examples

Announcements

Trees

Tree-Structured Data

```
def tree(label, branches=[]):
    return [label] + list(branches)

def label(t):
    return t[0]

def branches(t):
    return t[1:]

def is_leaf(t):
    return not branches(t)

class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
        self.branches = list(branches)

    def is_leaf(self):
        return not self.branches
```

A tree can contains other trees:

[5, [6, 7], 8, [[9], 10]]

(+ 5 (- 6 7) 8 (* (- 9) 10))

(S
 (NP (JJ Short) (NNS cuts))
 (VP (VBP make)
 (NP (JJ long) (NNS delays)))
 (. .))

```
<ul>
  <li>Midterm <b>1</b></li>
  <li>Midterm <b>2</b></li>
</ul>
```

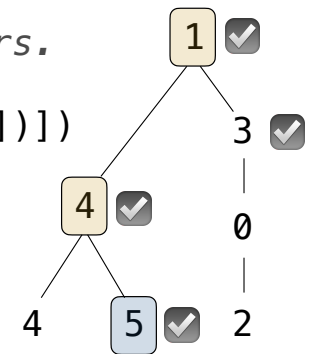
Tree processing often involves
recursive calls on subtrees

Tree Processing

Solving Tree Problems

Implement **big**s, which takes a Tree instance *t* containing integer labels. It returns the number of nodes in *t* whose labels are larger than all labels of their ancestor nodes.

```
def bigs(t):  
    """Return the number of nodes in t that are larger than all their ancestors.  
  
    >>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])  
    >>> bigs(a)  
    4
```



The root label is always larger than all of its ancestors

```
if t.is_leaf():  
    return ____  
else:  
    return ____([__ for b in t.branches])
```

Somehow increment
the total count

Somehow track a
list of ancestors

```
if node.label > max(ancestors):
```

Somehow track the
largest ancestor

```
if node.label > max_ancestors:
```

Solving Tree Problems

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>>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])
```

```
>>> bigs(a)
```

```
4
```

Somehow track the largest ancestor

```
def f(a, x):
```

```
    A node  $\swarrow$   $\nwarrow$  max_ancestor
```

```
    if a.label > x: node.label > max_ancestors
```

```
        return 1 + sum([f(b, a.label) for b in a.branches])
```

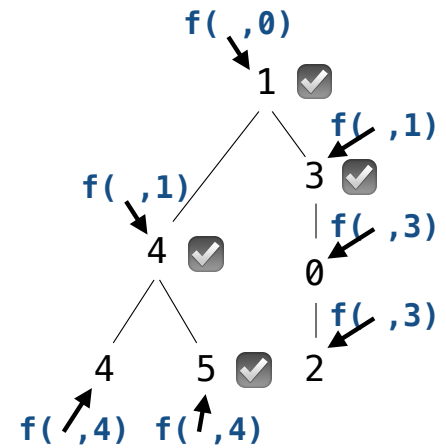
Somehow increment the total count

```
    else:
```

```
        return sum([f(b, x) for b in a.branches])
```

```
    return f(t, t.label - 1) Root label is always larger than its ancestors
```

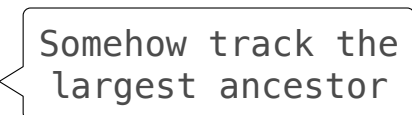
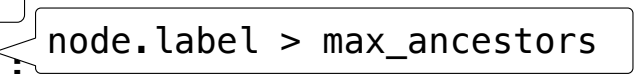
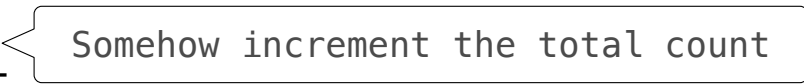

Some initial value for the largest ancestor so far...



Recursive Accumulation

Solving Tree Problems

Implement **big**s, which takes a Tree instance *t* containing integer labels. It returns the number of nodes in *t* whose labels are larger than any labels of their ancestor nodes.

```
def bigs(t):  
    """Return the number of nodes in t that are larger than all their ancestors."""  
    n = [0]  
  
    def f(a, x):  
         Somehow track the largest ancestor  
        if a.label > x:  node.label > max_ancestors  
             Somehow increment the total count  
            n[0] += 1  
        for b in a.branches:  
            f(b, max(a.label, x))  
    f(t, t.label - 1)  Root label is always larger than its ancestors  
  
    return n[0]
```

Designing Functions

How to Design Programs

From Problem Analysis to Data Definitions

Identify the information that must be represented and how it is represented in the chosen programming language. Formulate data definitions and **illustrate them with examples**.

Signature, Purpose Statement, Header

State what kind of data the desired function consumes and produces. Formulate a concise answer to the question *what* the function computes. Define a stub that lives up to the signature.

Functional Examples

Work through examples that illustrate the function's purpose.

Function Template

Translate the data definitions into an **outline** of the function.

Function Definition

Fill in the gaps in the function template. Exploit the purpose statement and the examples.

Testing

Articulate the examples as tests and ensure that the function passes all. Doing so discovers mistakes. Tests also supplement examples in that they help others read and understand the definition when the need arises—and it will arise for any serious program.

Applying the Design Process

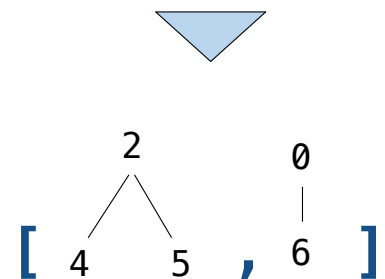
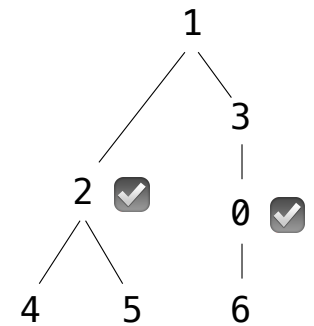
Designing a Function

Implement `smalls`, which takes a `Tree` instance `t` containing integer labels. It returns the non-leaf nodes in `t` whose labels are smaller than any labels of their descendant nodes.

```
def smalls(t): Signature: Tree -> List of Trees
    """Return the non-leaf nodes in t that are smaller than all their descendants.

    >>> a = Tree(1, [Tree(2, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(6)])])])
    >>> sorted([t.label for t in smalls(a)])
    [0, 2]

    """
    result = []
    Signature: Tree -> number
    def process(t): "Find smallest label in t & maybe add t to result"
        if t.is_leaf():
            return t.label
        else:
            return min(...)
    process(t)
    return result
```



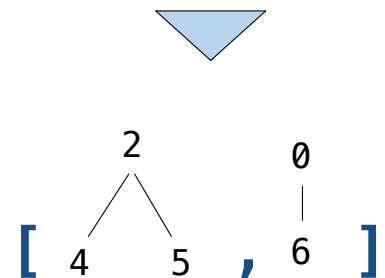
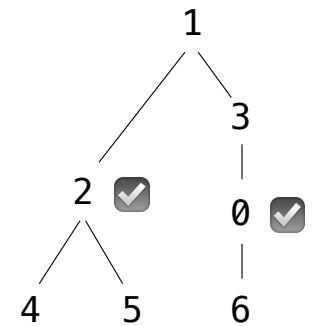
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    [0, 2]

    """
    result = []
    Signature: Tree -> number
    def process(t): "Find smallest label in t & maybe add t to result"
        if t.is_leaf():
            return t.label
        else:
            smallest = min([process(b) for b in t.branches])
            smallest label in a branch of t if t.label < smallest:
                result.append( t )
            return min(smallest, t.label)
    process(t)
    return result
```



Interpreters

Interpreter Analysis

What expressions are passed to `scheme_eval` when evaluating the following expressions?

(define x (+ 1 2))

(define (f y) (+ x y))

(f (if (> 3 2) 4 5))